Simulations of Abelian lattice gauge theories with optical lattices

Luca Tagliacozzo

Alessio Celi
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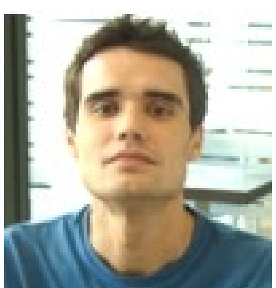
Maciej Lewenstein
ICFO



Alejandro Zamora
ICFO

Outlook

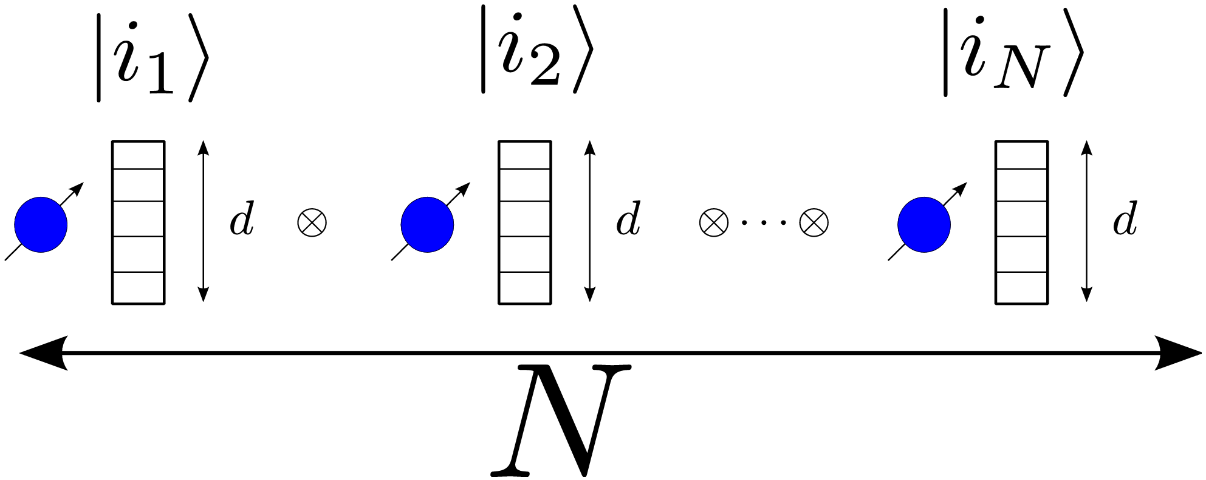
- Why simulations with optical lattices

- Why lattice gauge theories

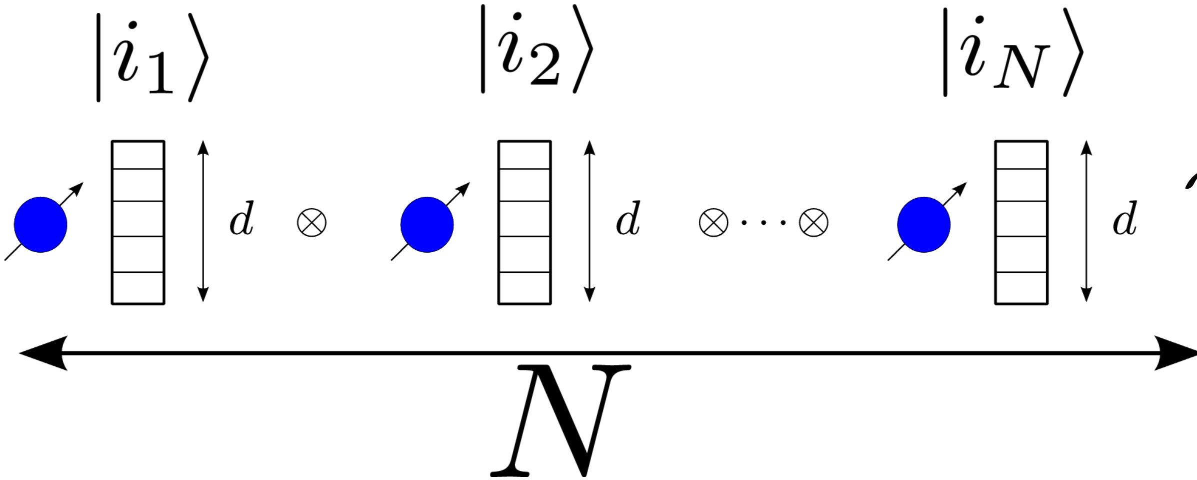
- Describe the specific work on Abelian LGT

Many body quantum systems

Many body quantum systems

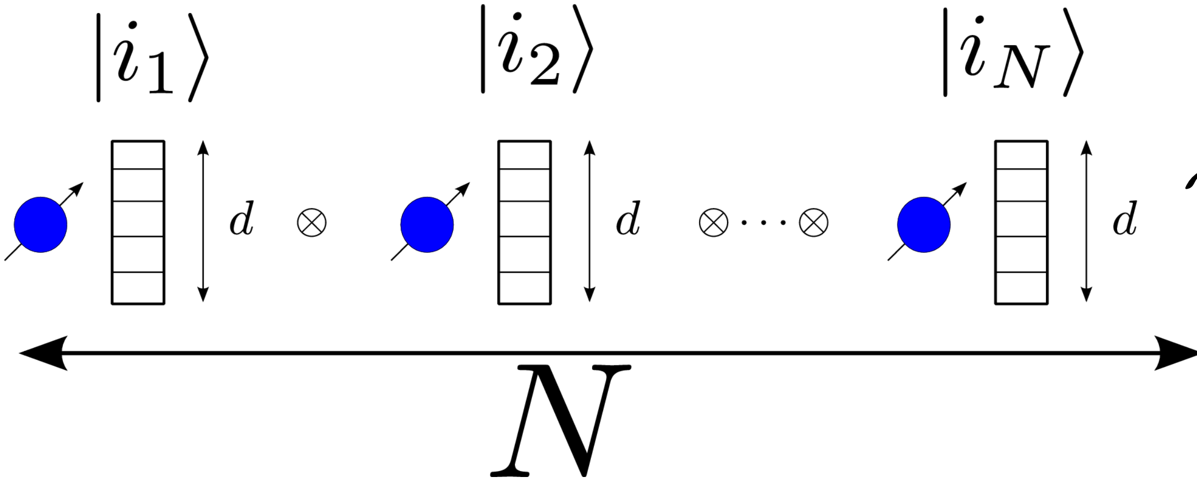


Many body quantum systems



$$\mathcal{H} = \mathcal{H}_1 \bigotimes \mathcal{H}_2 \cdots \mathcal{H}_N$$

Many body quantum systems



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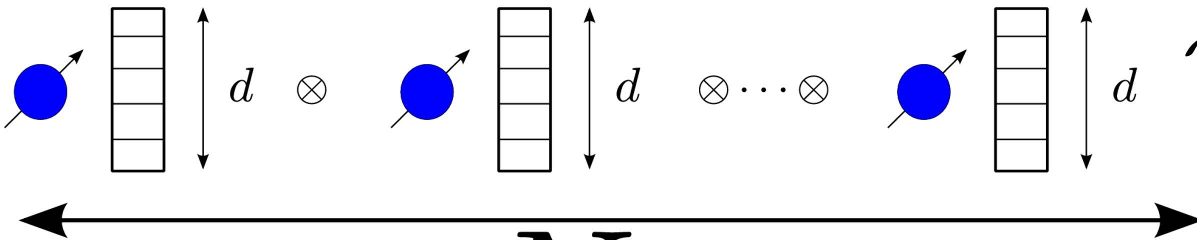
$$|\psi\rangle = c_{i_1 \ldots i_N} |i_1\rangle \bigotimes \cdots |i_N\rangle$$

Many body quantum systems

$|i_1\rangle$ $|i_2\rangle$ $|i_N\rangle$



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N

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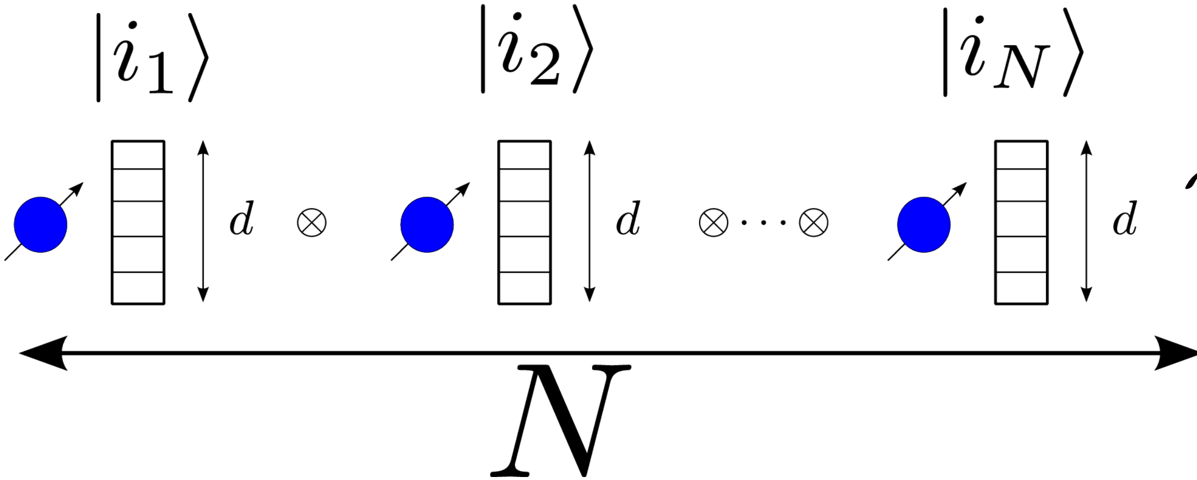
d^N

Many body quantum systems



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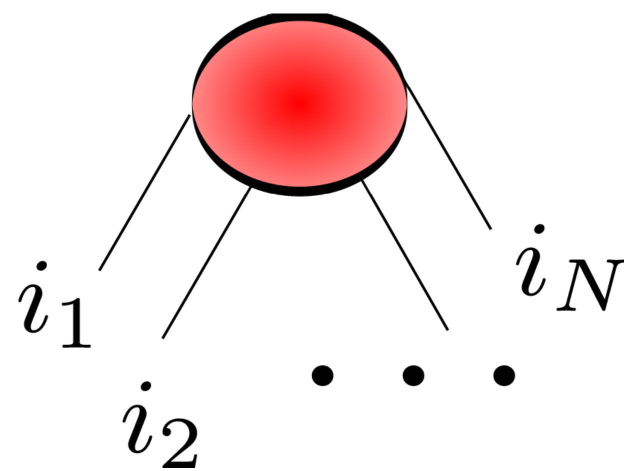


What would we like to know

- “More is different”
- Equilibrium, phases: strong interaction produces macroscopic phases very different from constituents (confinement, fractionalization, topological order) .
- Short time out of Equilibrium dynamic?
- Equilibration

Classical simulations

Classical simulations

- Density functional theory (small interaction)

- Monte Carlo (positive Hamiltonians)

- Tensor Networks (generic)

Controversial results

Stripes in the two-dimensional t-J model with infinite projected entangled-pair states

Philippe Corboz,[1,2] Steven R. White,[3] Guifré Vidal,[4] and Matthias Troyer[1]

[1] *Theoretische Physik, ETH Zurich, 8093 Zurich, Switzerland*
[2] *Institut de théorie des phénomènes physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland*
[3] *Department of Physics and Astronomy, University of California, Irvine, CA 92697-4575 USA*
[4] *School of Mathematics and Physics, The University of Queensland, QLD 4072, Australia*

(Dated: August 12, 2011)

Absence of static stripes in the two-dimensional $t-J$ model by an accurate and systematic quantum Monte Carlo approach

Wen-Jun Hu, Federico Becca, and Sandro Sorella

Democritos Simulation Center CNR-IOM Istituto Officina dei Materiali and International School for Advanced Studies (SISSA), Via Bonomea 265, 34136 Trieste, Italy

(Dated: November 23, 2011)

Frustrated antiferromagnets with entanglement renormalization: ground state of the spin-$\frac{1}{2}$ Heisenberg model on a kagome lattice

G. Evenbly[1] and G. Vidal[1]

[1] *School of Mathematics and Physics, the University of Queensland, Brisbane 4072, Australia*

(Dated: May 12, 2010)

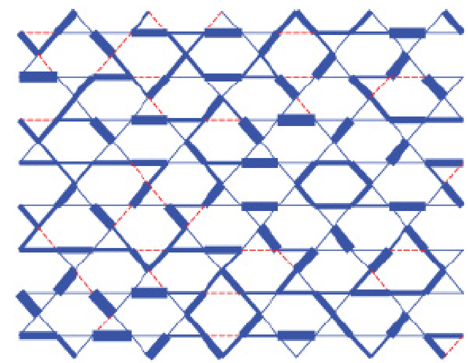
Spin Liquid Ground State of the $S = 1/2$ Kagome Heisenberg Model

Simeng Yan,[1] David A. Huse,[2,3] and Steven R. White[1]

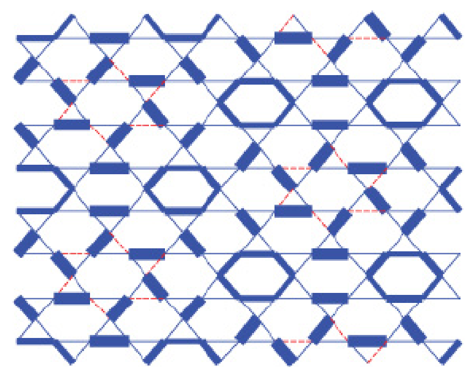
[1] *Department of Physics and Astronomy, University of California, Irvine, CA 92617*
[2] *Department of Physics, Princeton University, Princeton, NJ 08544*
[3] *Institute for Advanced Study, Princeton, NJ 08540*

(Dated: November 30, 2010)

N = 144, defective valence bond cystal



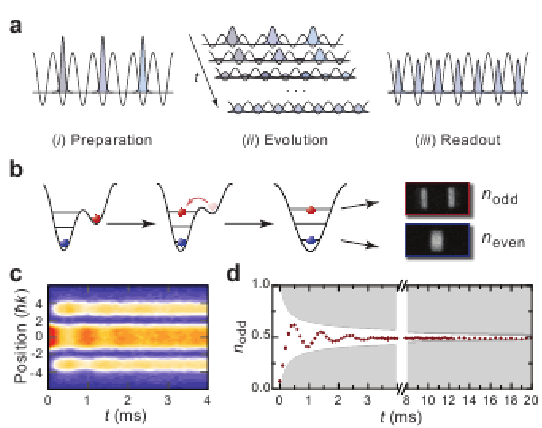
N = 144, honeycomb valence bond cystal

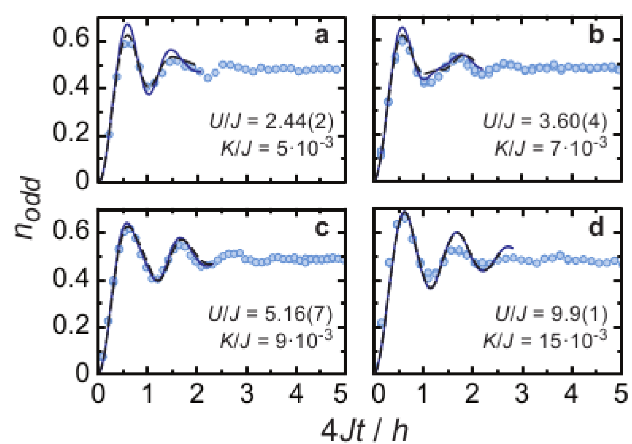


Time evolution

Probing the relaxation towards equilibrium in an isolated strongly correlated 1D Bose gas

S. Trotzky[1−3], Y.-A. Chen[1−3] , A. Flesch[4], I. P. McCulloch[5], U. Schollwöck[1,6], J. Eisert[6,7] and I. Bloch[1−3]
[1] *Fakultät für Physik, Ludwig-Maximilians-Universität, 80798 München, Germany*
[2] *Max-Planck Institut für Quantenoptik, 85748 Garching, Germany*
[3] *Institut für Physik, Johannes Gutenberg-Universität, 54099 Mainz, Germany*
[4] *Institute for Advanced Simulation, Forschungszentrum Jülich, 52425 Jülich, Germany*
[5] *School of Physical Sciences, The University of Queensland, Brisbane, QLD 4072, Australia*
[6] *Institute for Advanced Study Berlin, 14193 Berlin, Germany and*
[7] *Institute of Physics and Astronomy, University of Potsdam, 14476 Potsdam, Germany*





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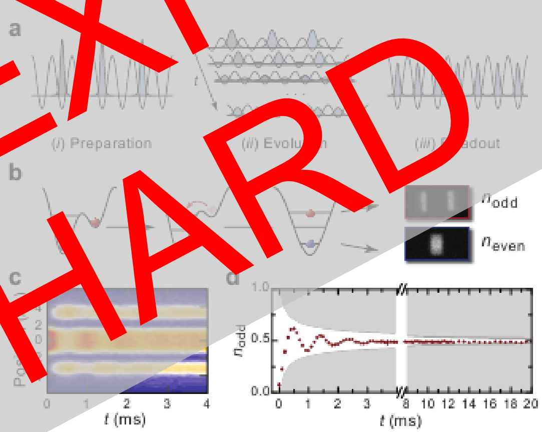
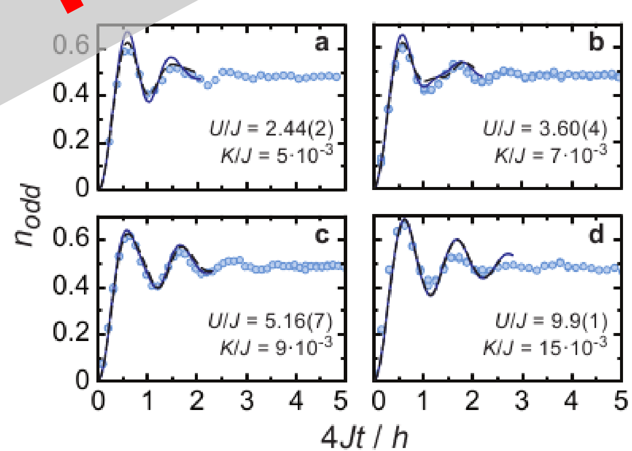
EXPONENTIALLY HARD PROBLEM

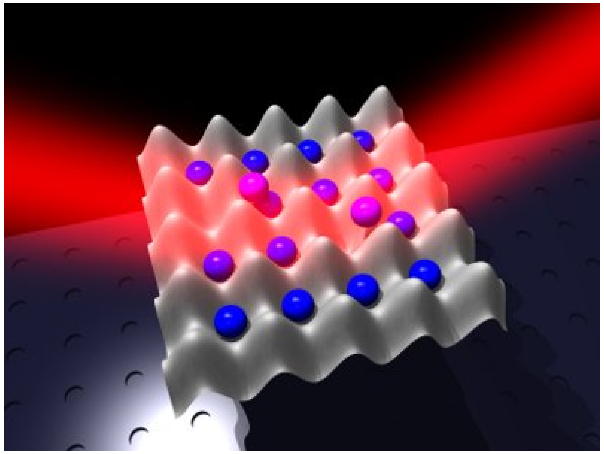
2





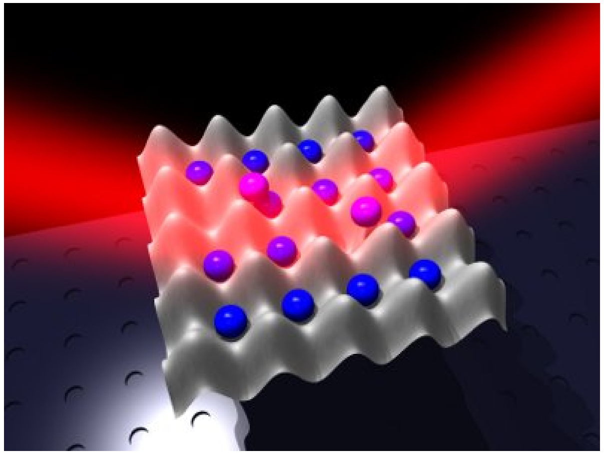
Quantum simulators

Quantum simulators



COLD ATOMS

Quantum simulators



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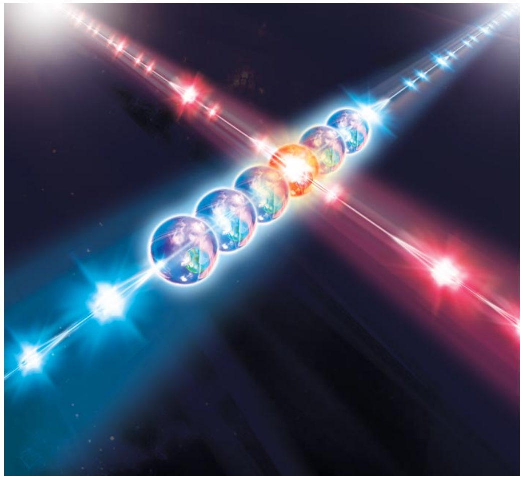
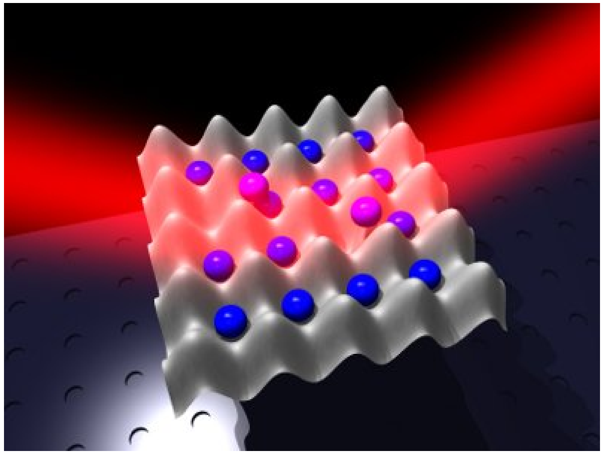


IONS TRAPS

Quantum simulators



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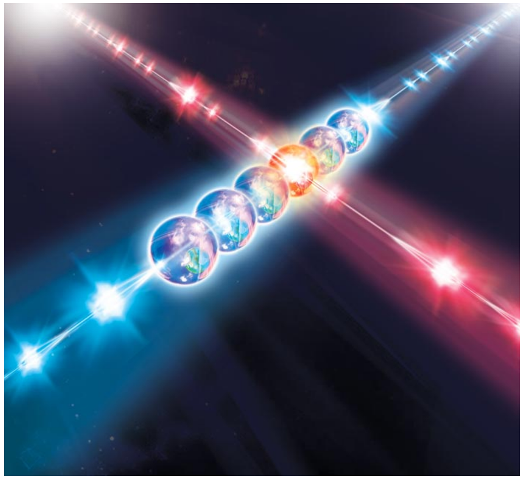

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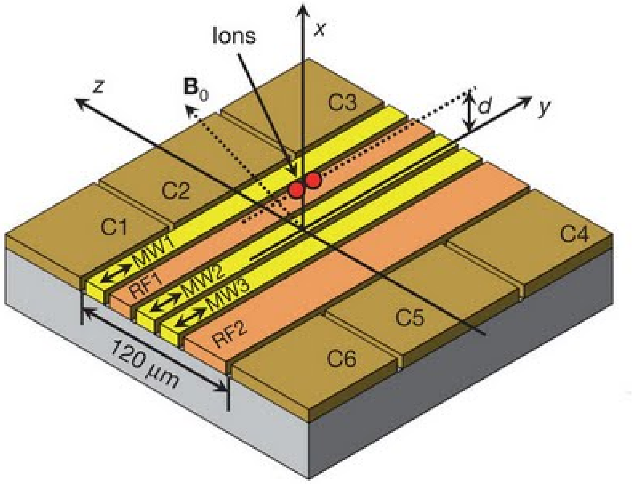
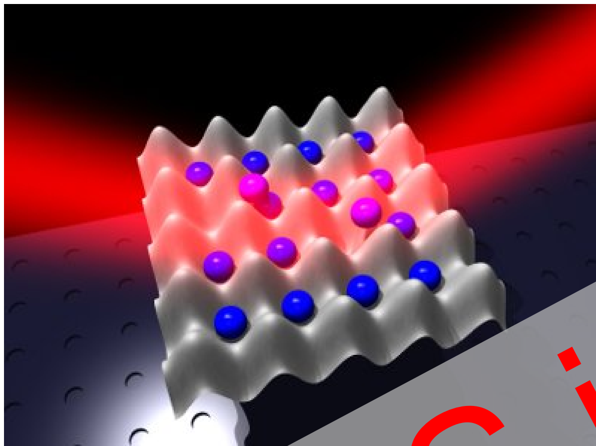


MICROWAVE ION CHIPS

Quantum simulators

IS it the way
To dynamic?



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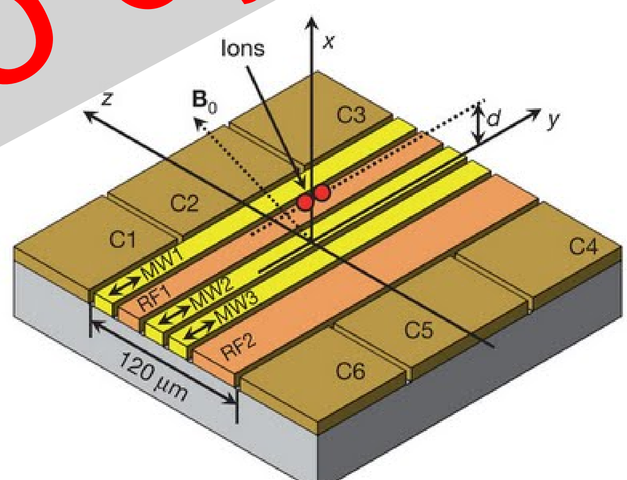


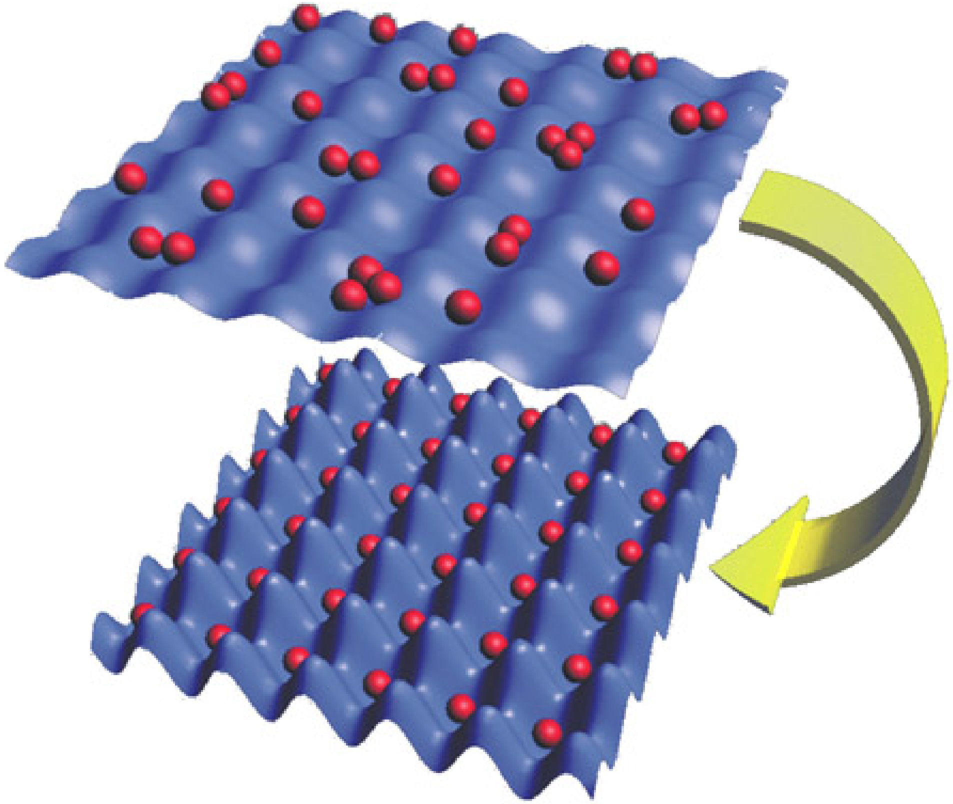
MICROWAVE ION CHIPS

Gauge theories

- Specific class of QMBS
- They are very important ingredients for the description of both high and low energies physics (QCD... antiferromagnets...)
- They typically involve more than 2 bodies interactions
- Their strongly coupled regime is still under debate

Natural interactions on optical lattices

Natural interactions on optical lattices



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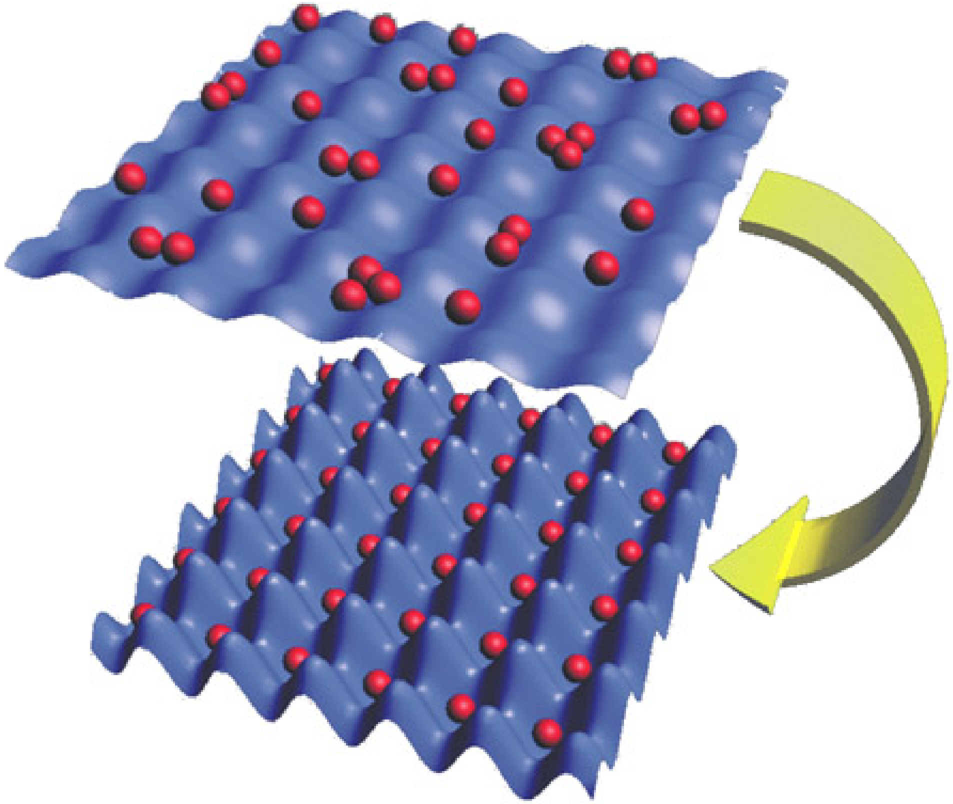
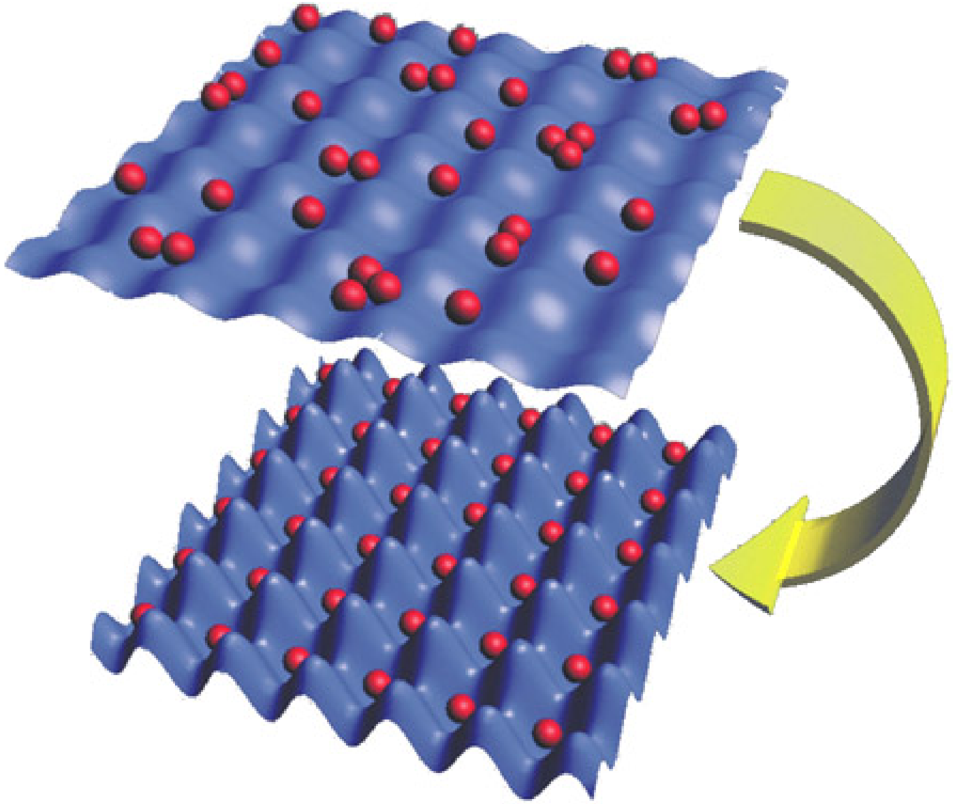


Difficult to engineer interactions different from

Natural interactions on optical lattices



$$c_i c_{i+1}^\dagger$$

Difficult to engineer interactions different from

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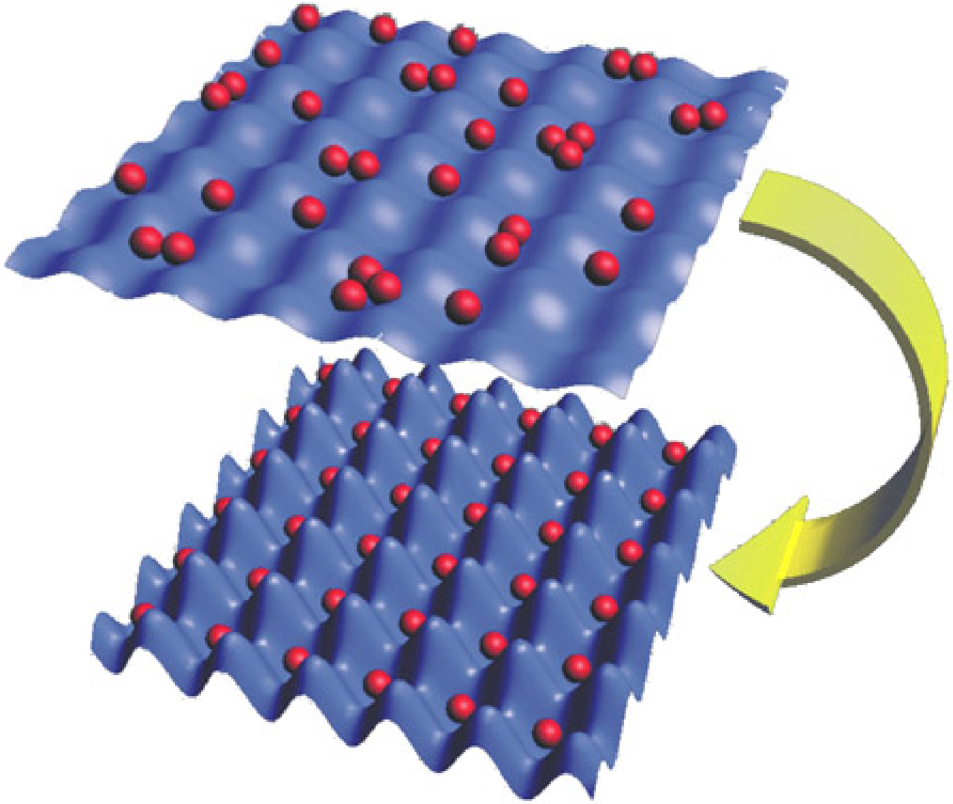

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$$n_i, n_i^2$$

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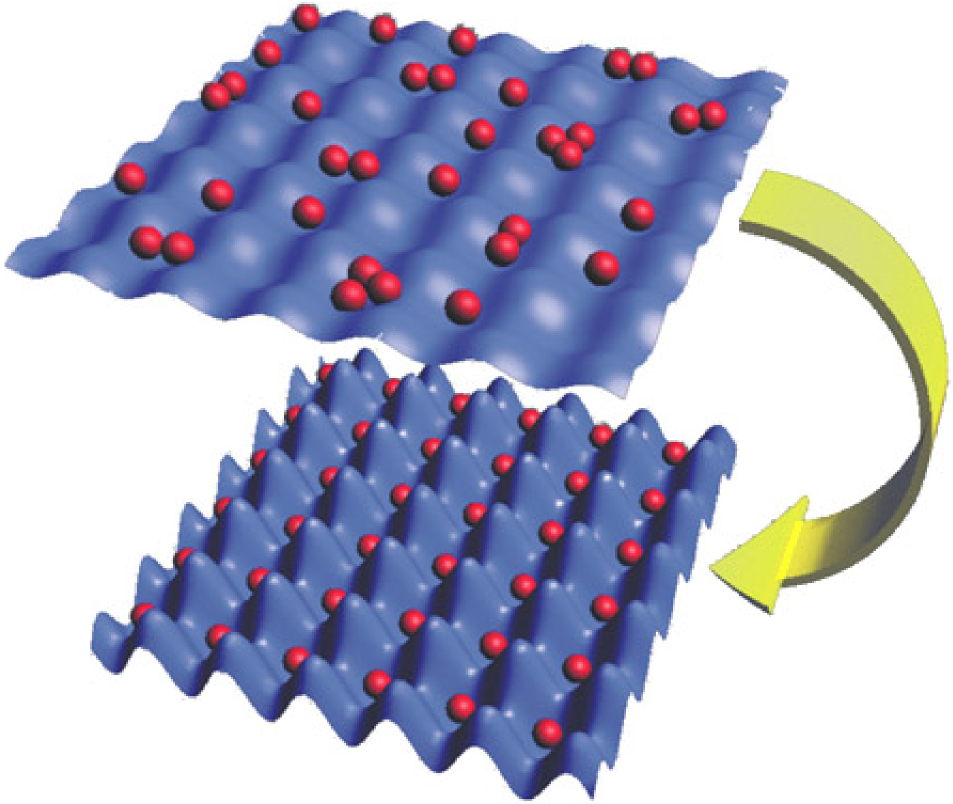

$$c_i c_{i+1}^{\dagger}$$

$$n_i, n_i^2$$

Difficult to engineer interactions different from

How do we get four body interactions ?

Four body interactions via the Rydberg gate

ARTICLES
PUBLISHED ONLINE: 14 MARCH 2010 | DOI: 10.1038/NPHYS1614
nature physics

A Rydberg quantum simulator

Hendrik Weimer[1]*, Markus Müller[2], Igor Lesanovsky[2,3], Peter Zoller[2] and Hans Peter Büchler[1]

A universal quantum simulator is a controlled quantum device that reproduces the dynamics of any other many-particle quantum system with short-range interactions. This dynamics can refer to both coherent Hamiltonian and dissipative open-system evolution. Here we propose that laser-excited Rydberg atoms in large-spacing optical or magnetic lattices provide an efficient implementation of a universal quantum simulator for spin models involving *n*-body interactions, including such of higher order. This would allow the simulation of Hamiltonians of exotic spin models involving *n*-particle constraints, such as the Kitaev toric code, colour code and lattice gauge theories with spin-liquid phases. In addition, our approach provides the ingredients for dissipative preparation of entangled states based on engineering *n*-particle reservoir couplings. The basic building blocks of our architecture are efficient and high-fidelity *n*-qubit entangling gates using auxiliary Rydberg atoms, including a possible dissipative time step through optical pumping. This enables mimicking the time evolution of the system by a sequence of fast, parallel and high-fidelity *n*-particle coherent and dissipative Rydberg gates.

Four body interactions via the Rydberg gate

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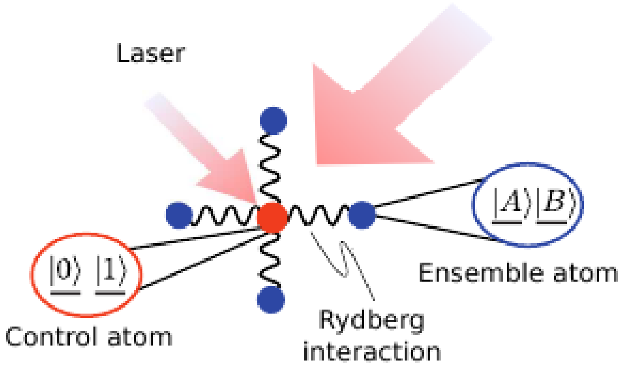

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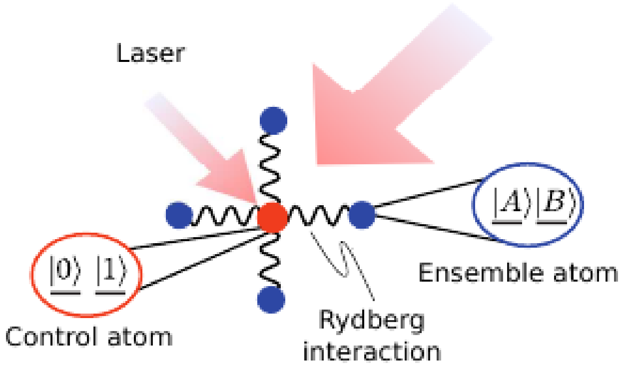

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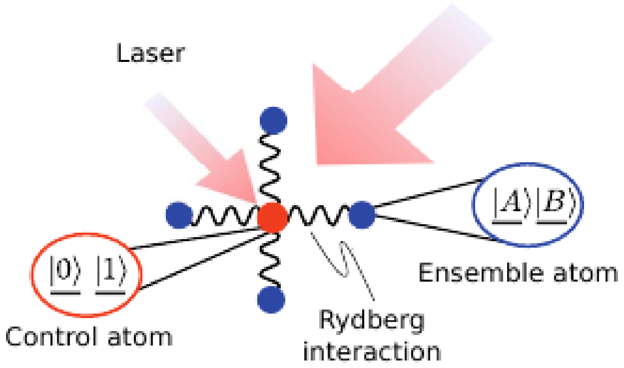

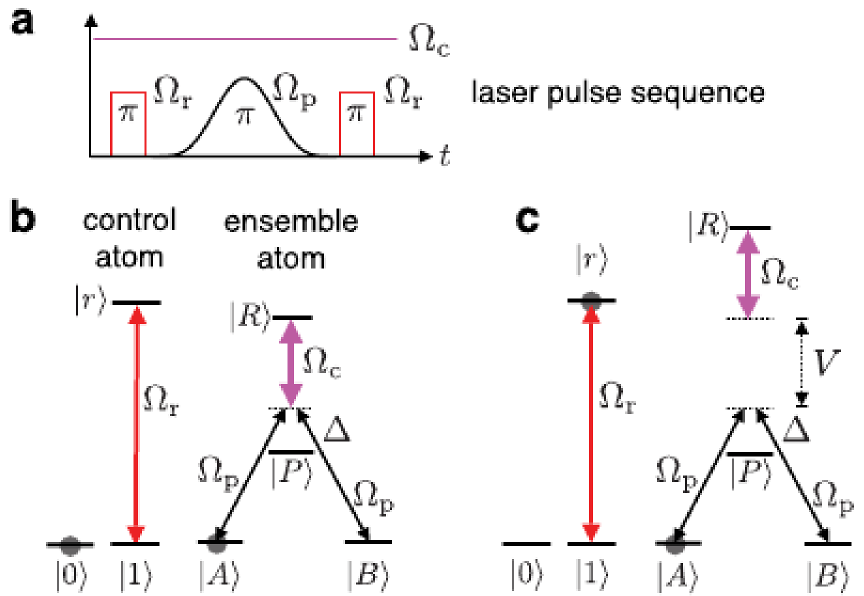

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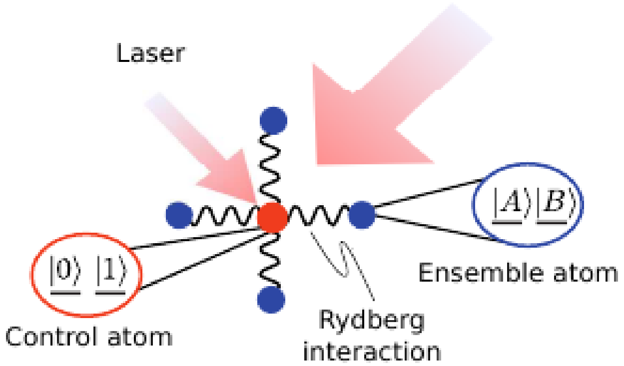

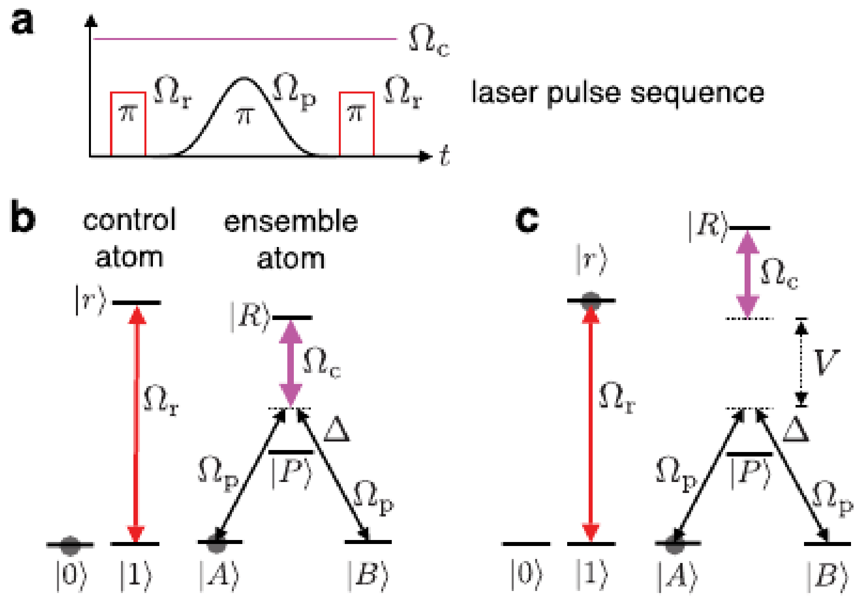

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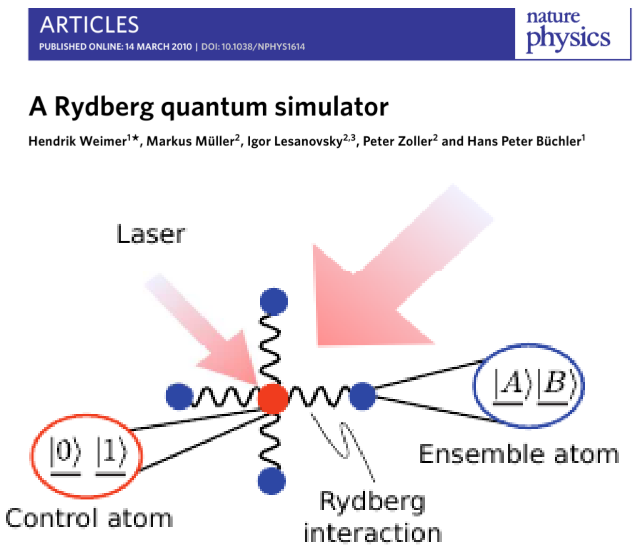
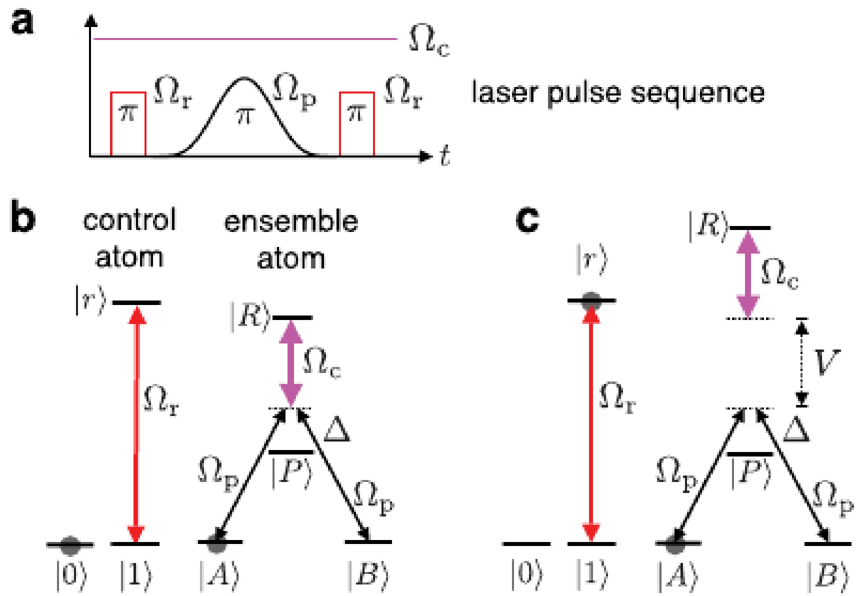
$$U_g = |0\rangle\langle 0|_c \otimes \mathbf{1} + |1\rangle\langle 1|_c \otimes \prod_{i\in p} \sigma_i^x$$

Four body interactions via the Rydberg gate





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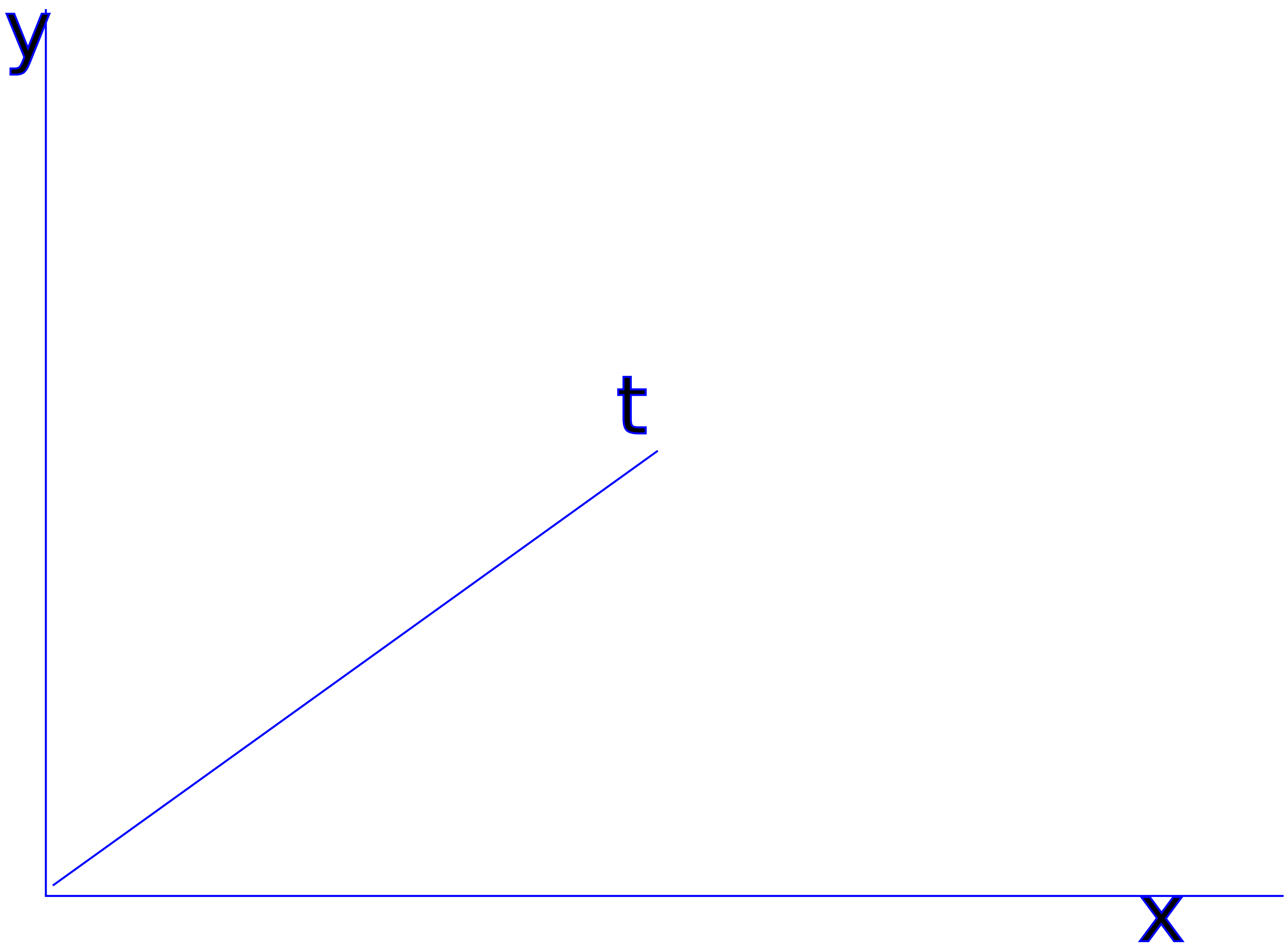
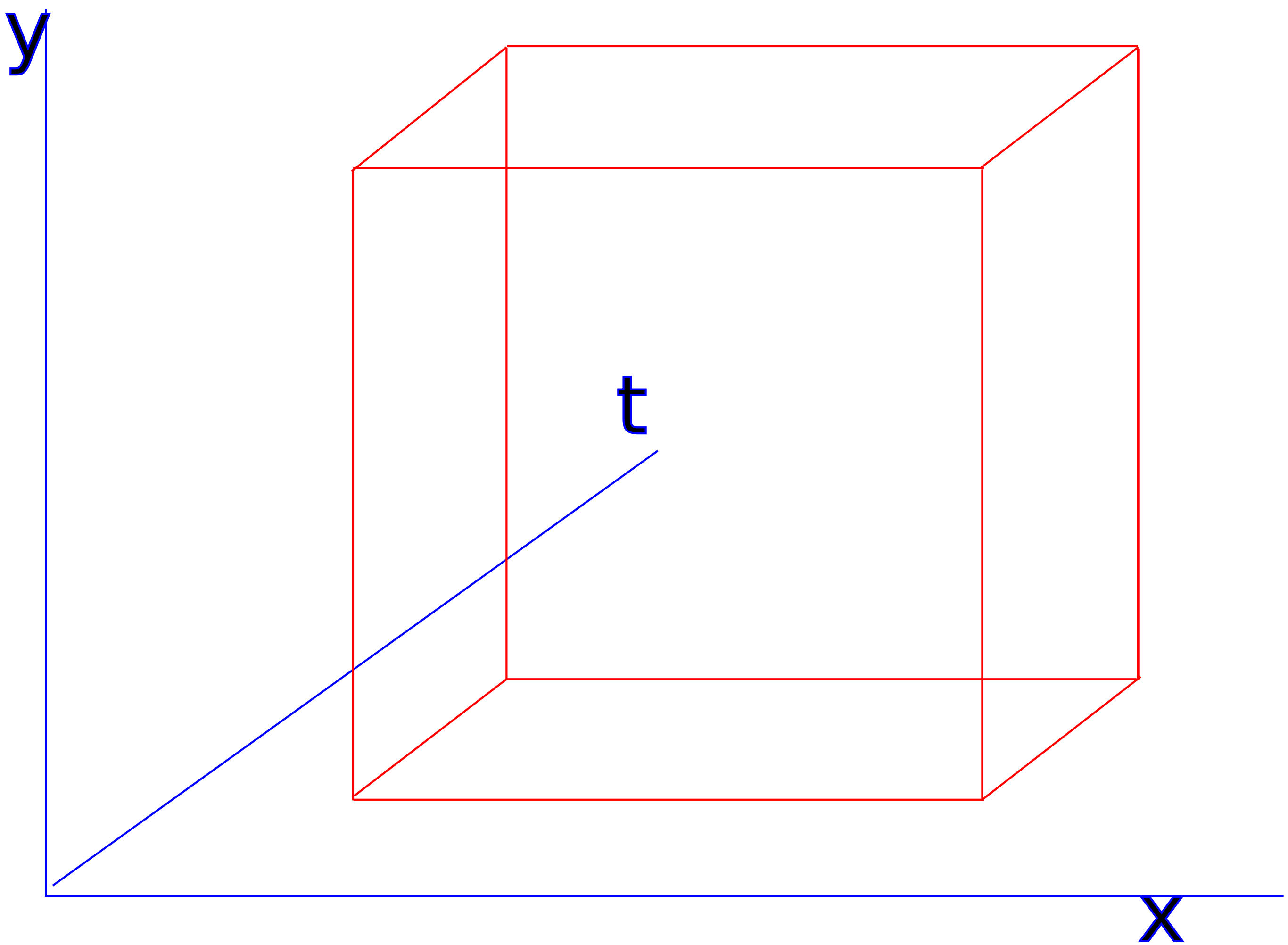
We need a two dimensional Hilbert space

Lattice gauge theories

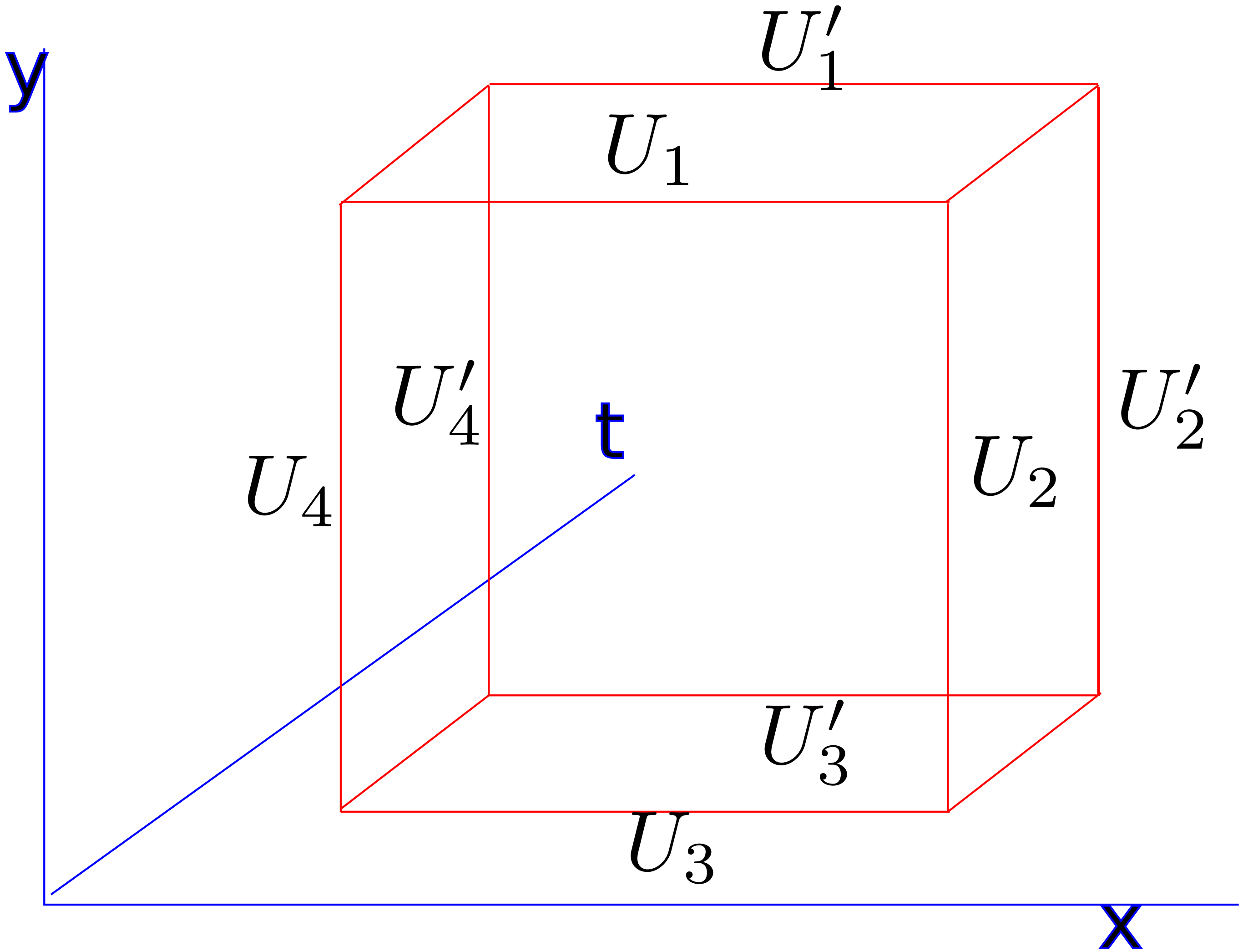


Lattice gauge theories



Lattice gauge theories

$$U_i \in G$$

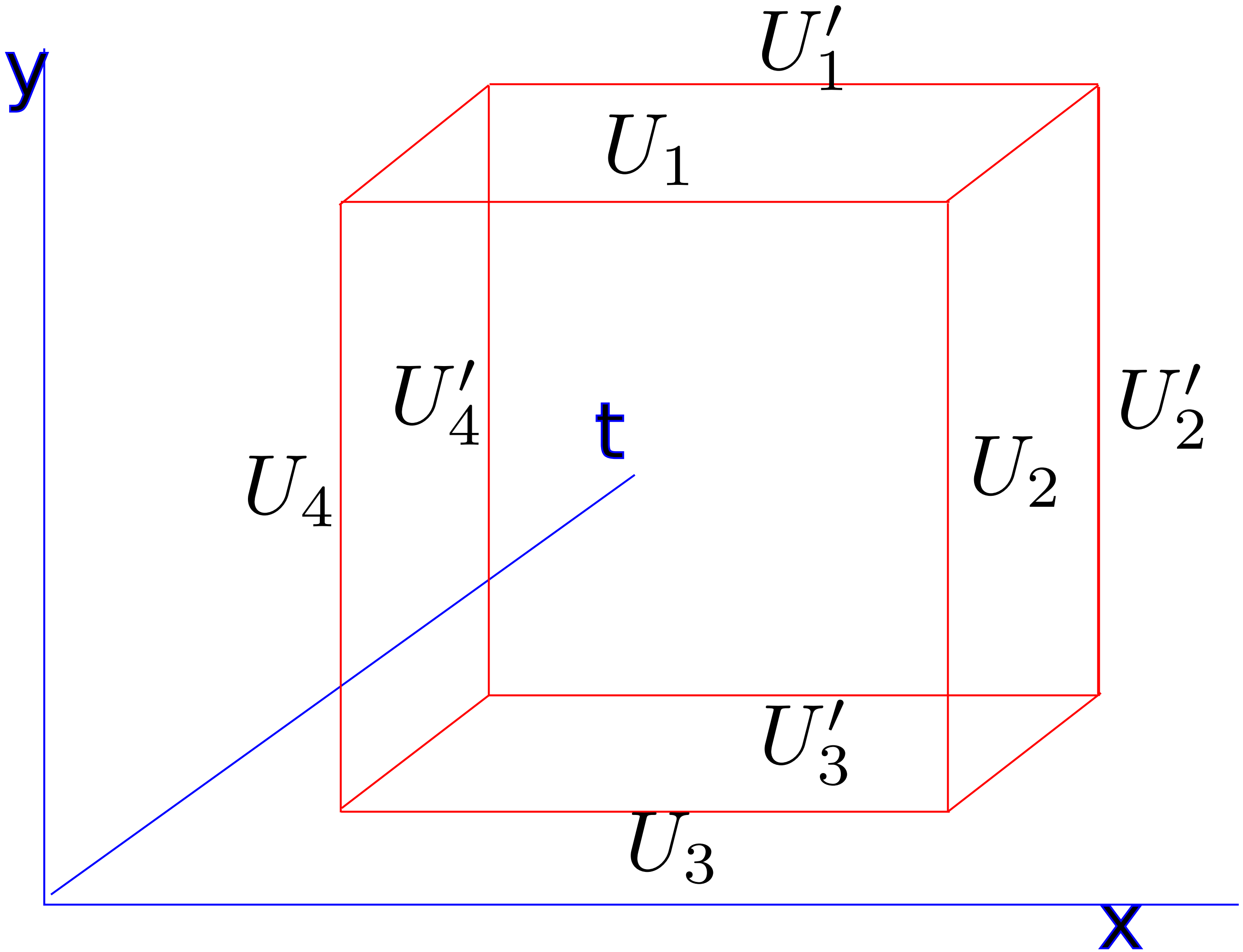


Lattice gauge theories

$$U_i \in G$$

$$U_T = tr(U_1 U_1') + tr(U_2 U_2') + \cdots$$

$$U_P = tr(U_1 U_2 U_3 U_4)$$



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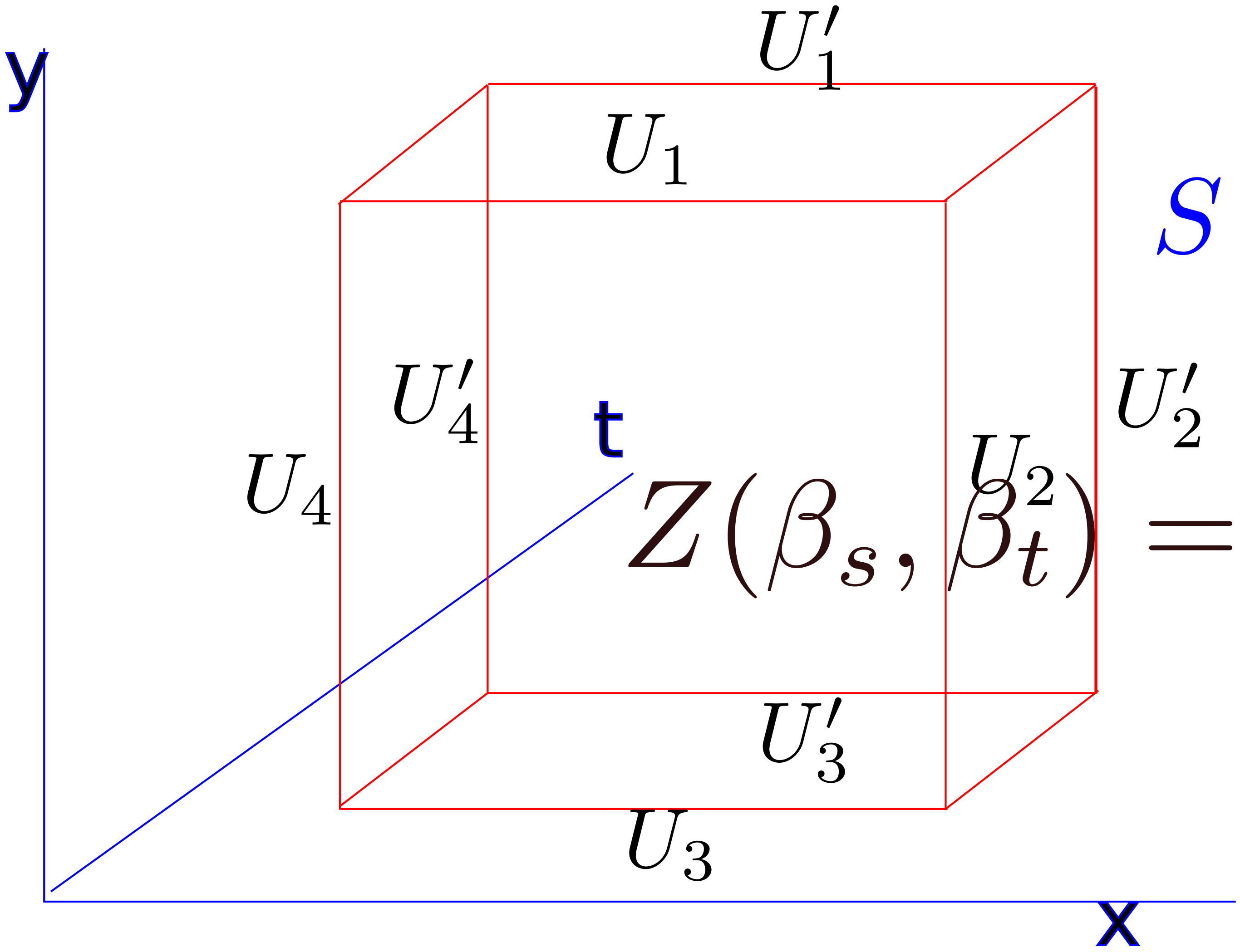
$$S = \beta_s U_P + \beta_t U_T$$



$$Z(\beta_s, \beta_t) = \sum_{U_i} expS$$

Lattice gauge theories

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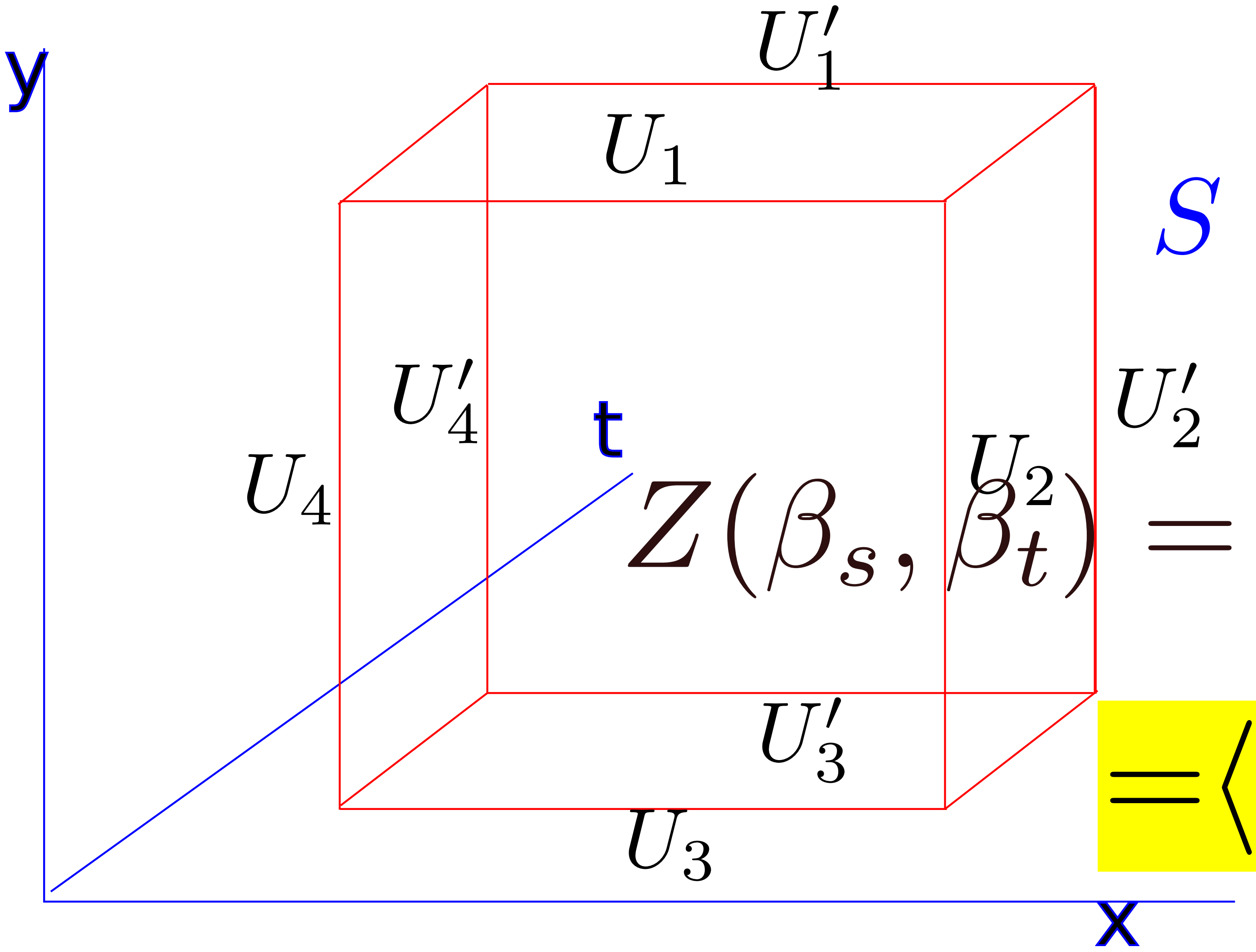

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$$= \langle I | T(U)^L | F \rangle$$

Hamiltonian formulation of LGT, Hilbert space

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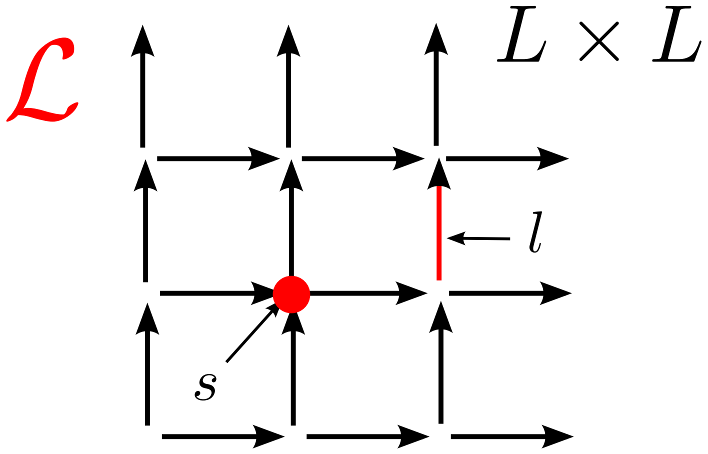
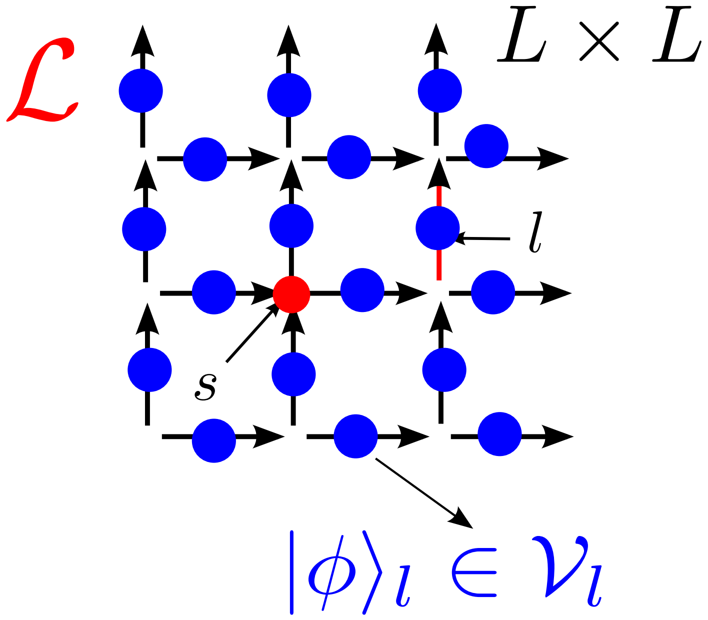


Hamiltonian formulation of LGT, Hilbert space



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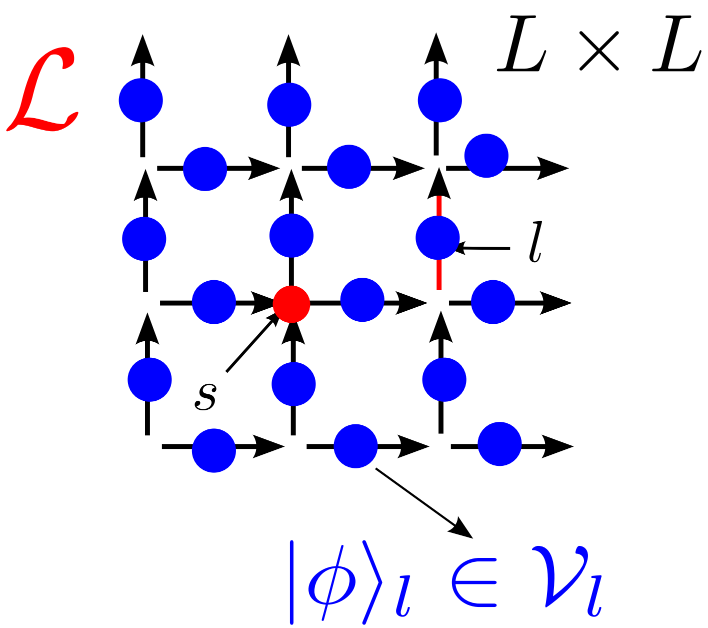
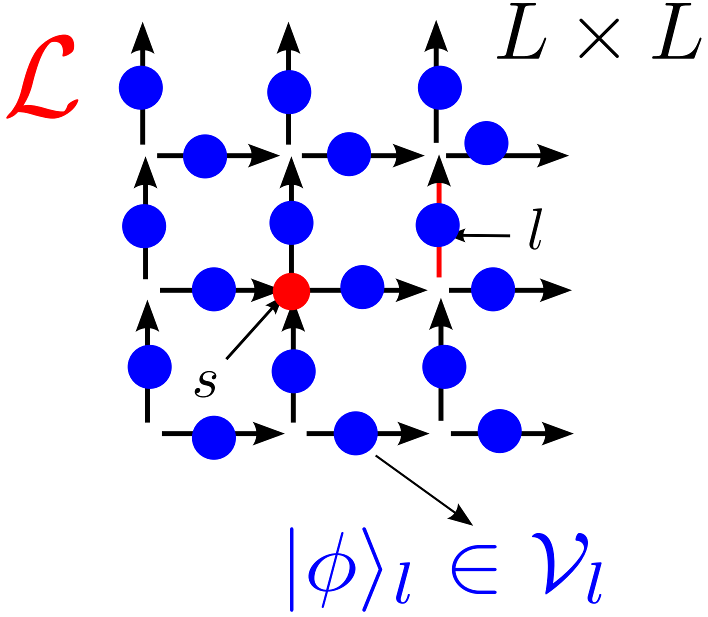


$$\mathcal{V}_l = C(G)$$

Hamiltonian formulation of LGT, Hilbert space



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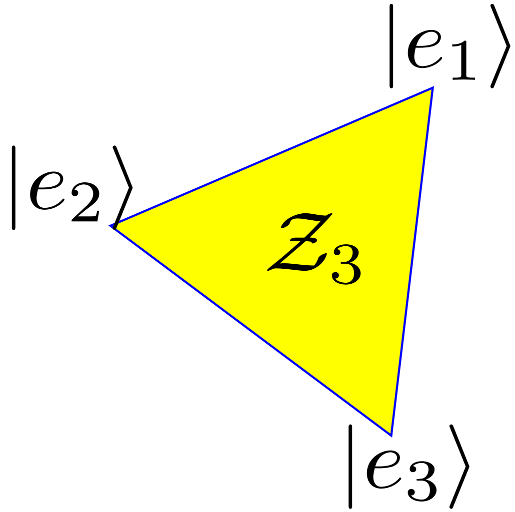
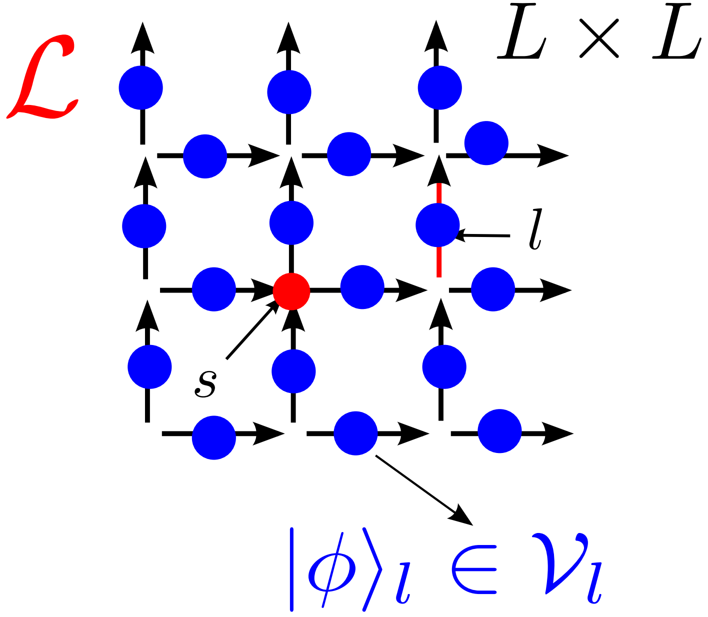


Hamiltonian formulation of LGT, Hilbert space



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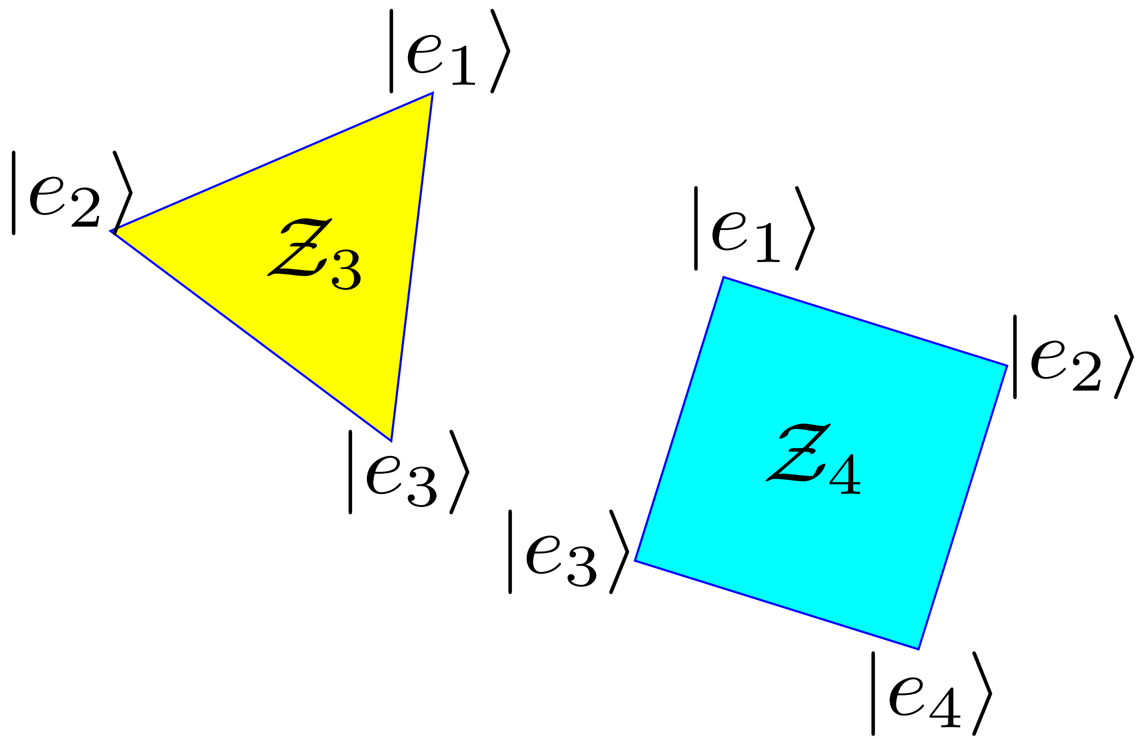

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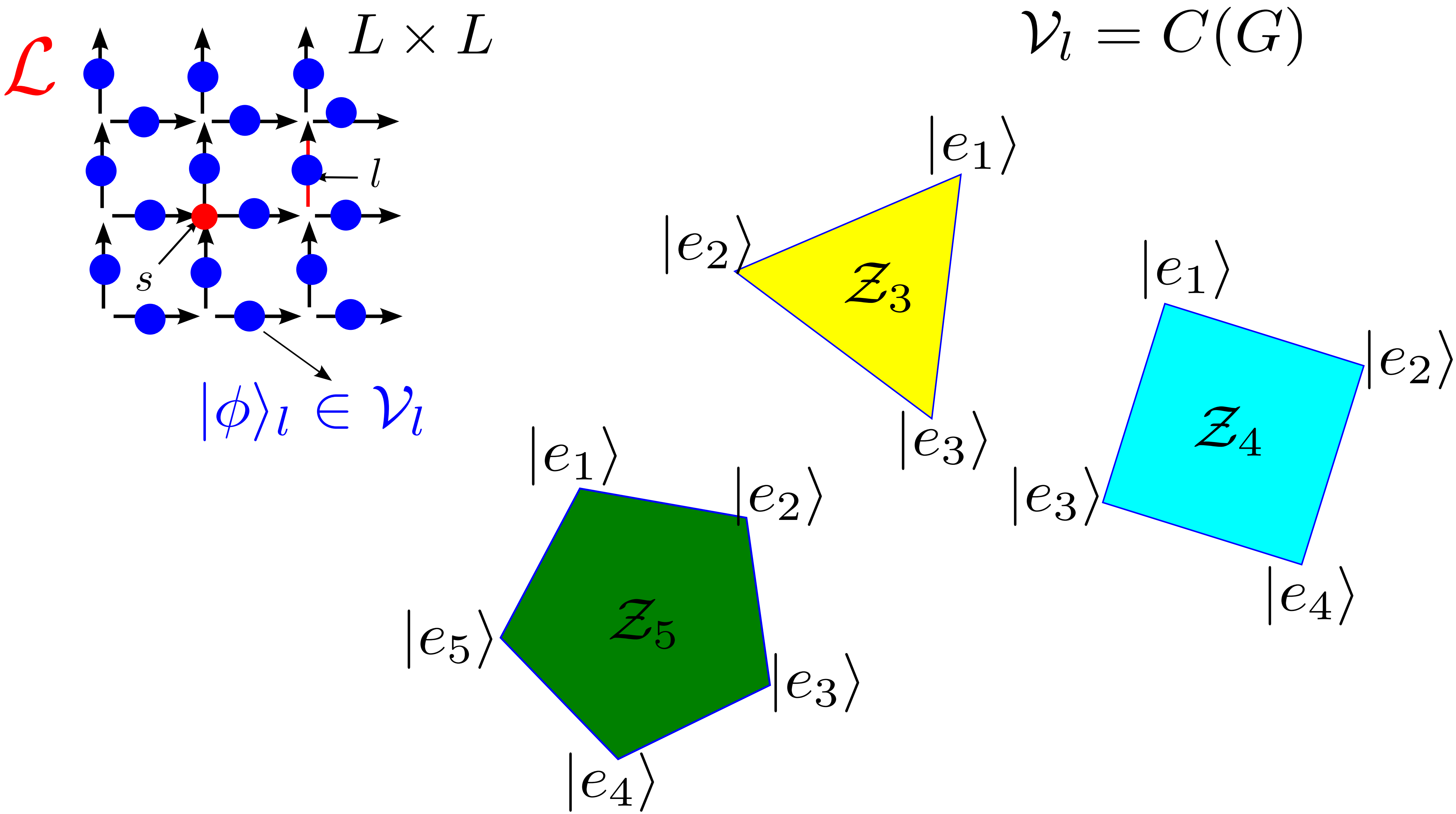

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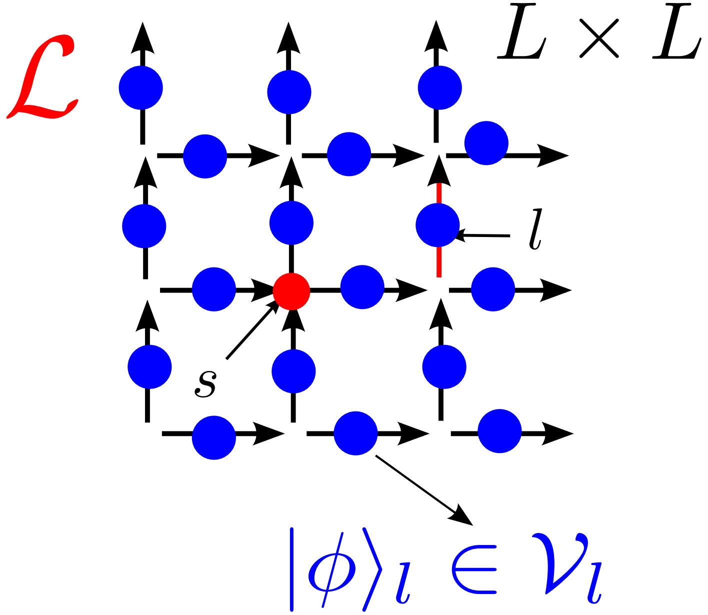

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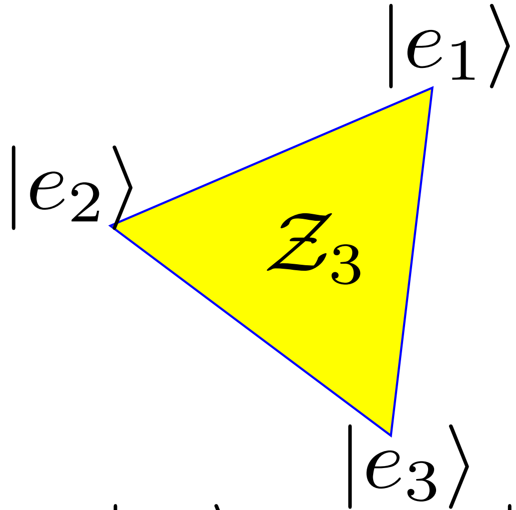

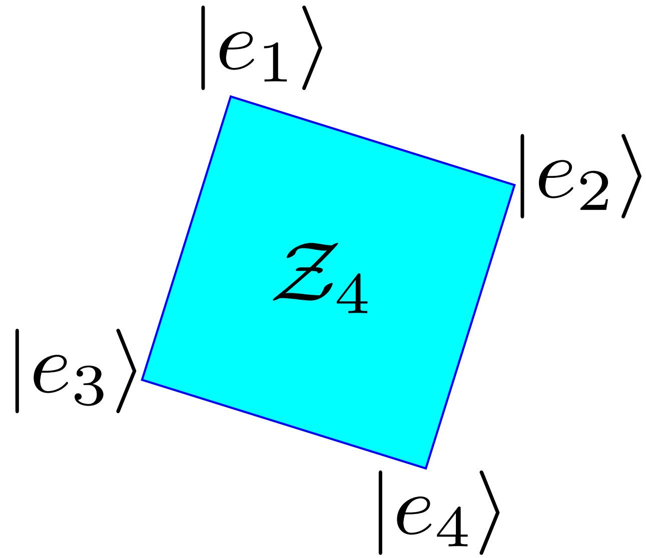

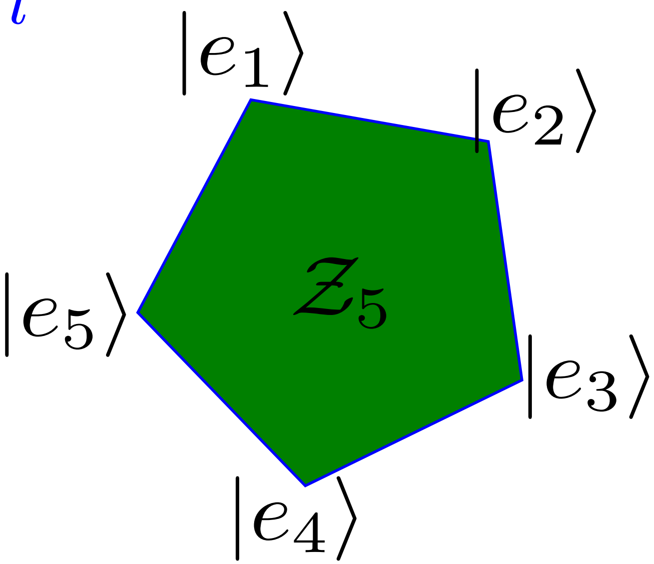

$$\langle e_i | e_j \rangle = \delta_{ij}$$

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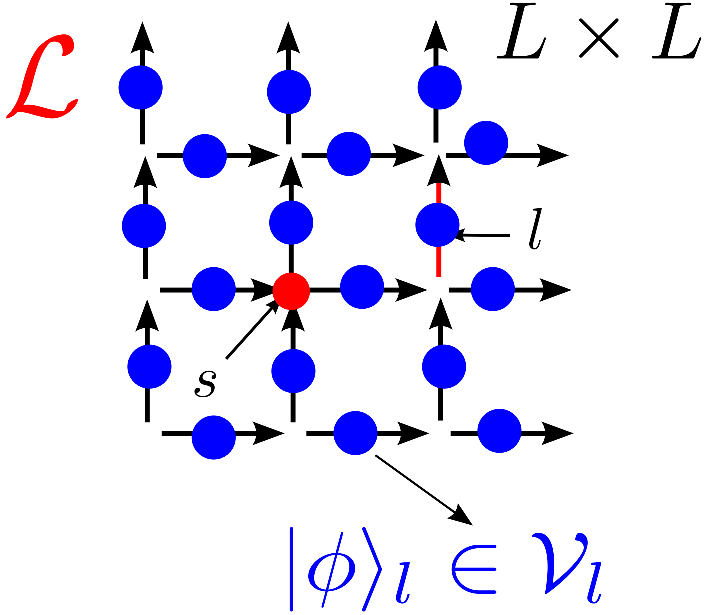

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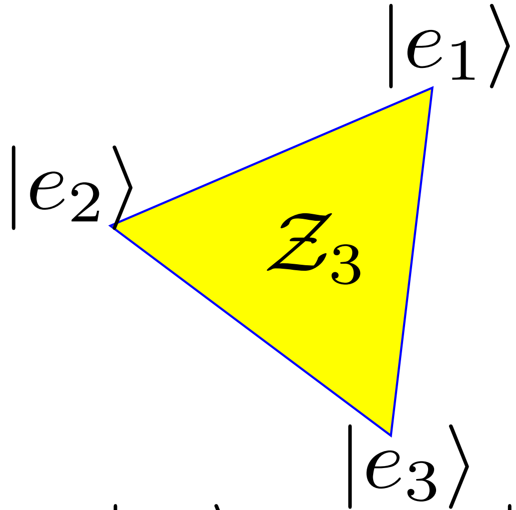
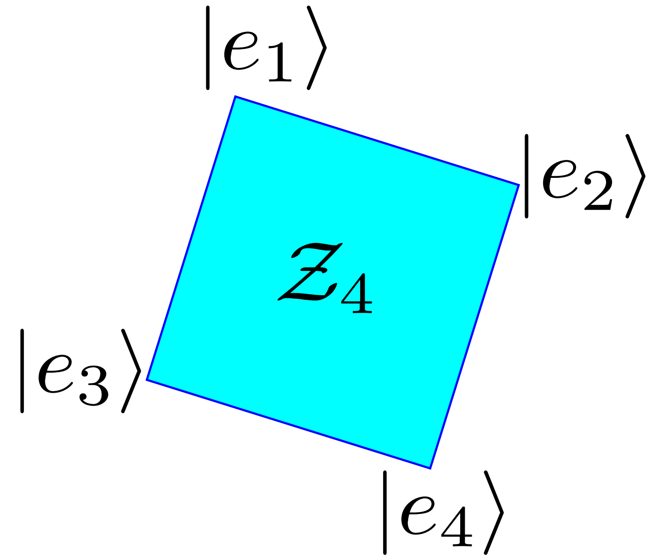
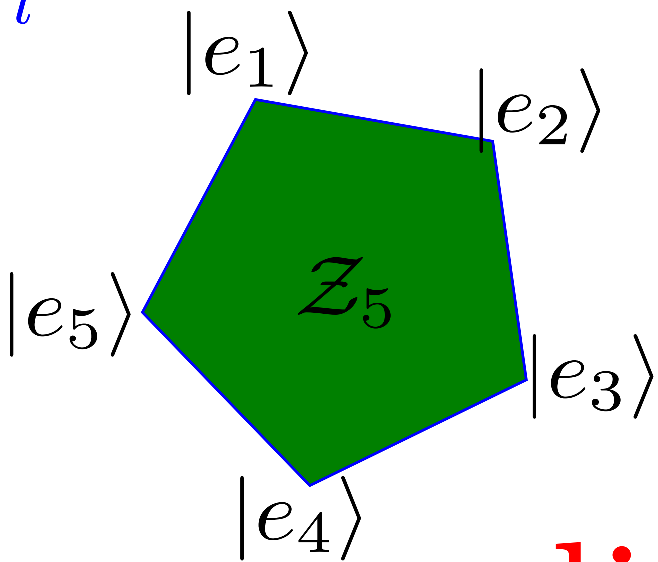




$U(1)$?



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$$\dim \mathcal{V}_l = |G|$$

Hamiltonian formulation of LGT, operators

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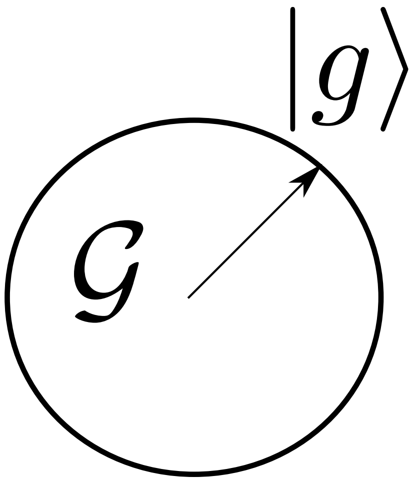

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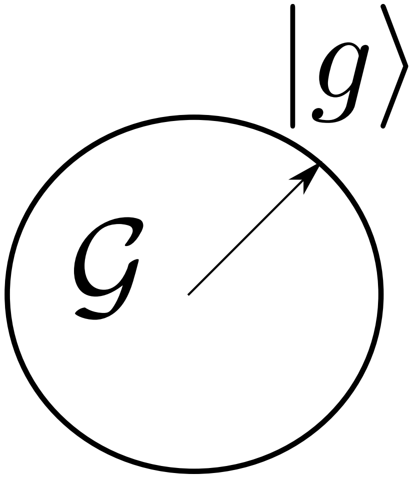
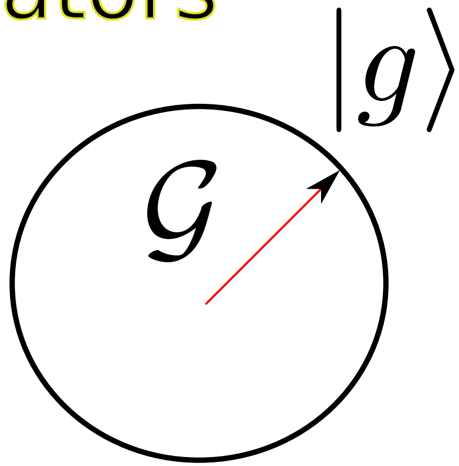


$$X(h)|g\rangle = |gh\rangle$$



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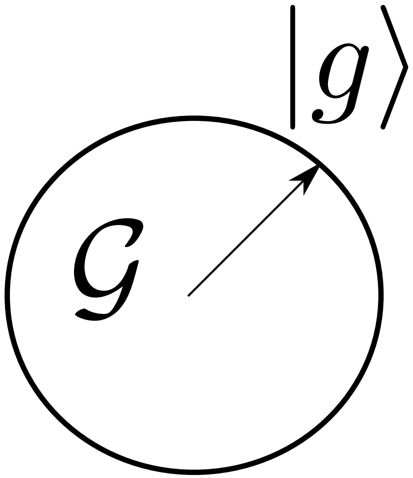

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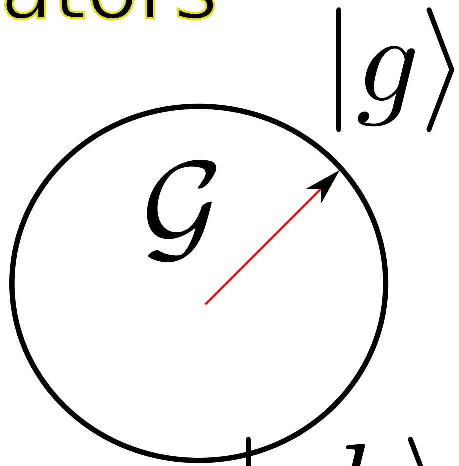
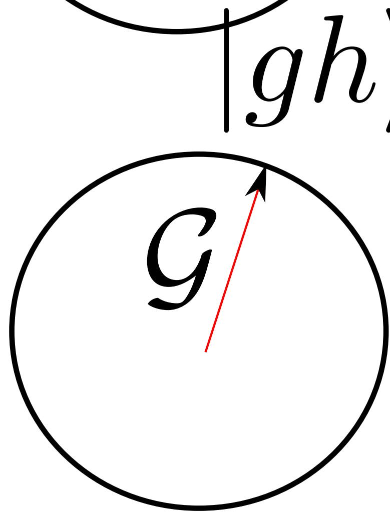


$$Z_r|g\rangle = R(g)|g\rangle$$



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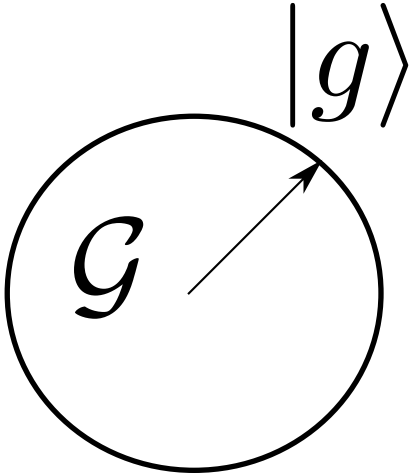

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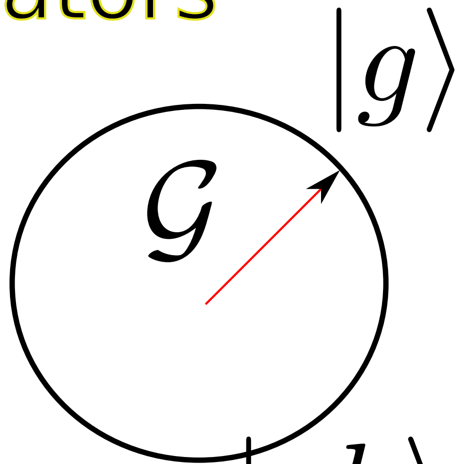
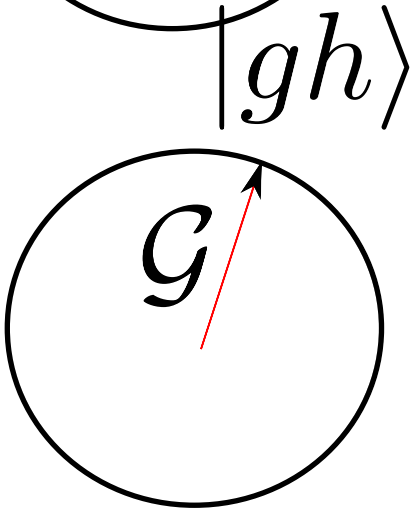




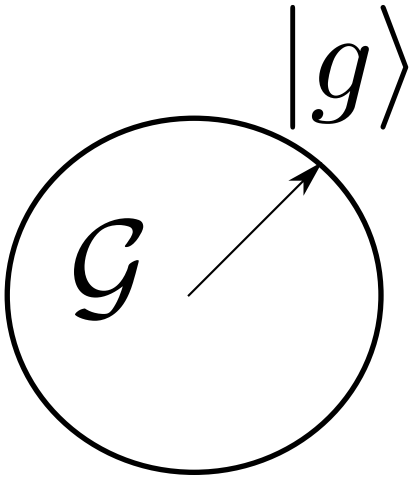
$$Z_r|g\rangle = R(g)|g\rangle$$

$$T(U) \simeq e^{-\delta t H}$$

$$H = cos(\theta)\sum_p tr(B_p + h.c.)$$

$$... + \sin(\theta)\sum_l X^l + h.c.$$

Hamiltonian formulation of LGT, operators



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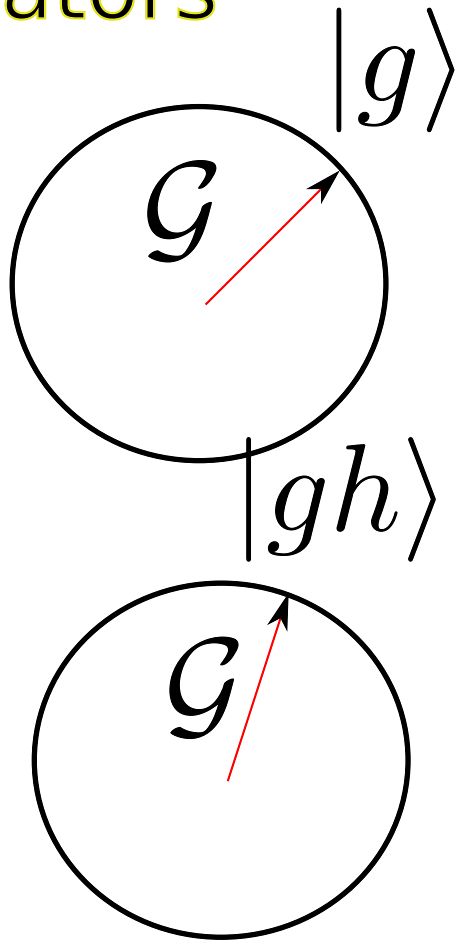

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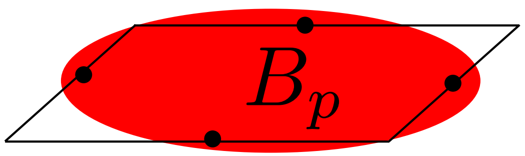

$$B_p = \otimes_{i\in\partial p} Z_r^i$$

The deformed toric code in parallel field

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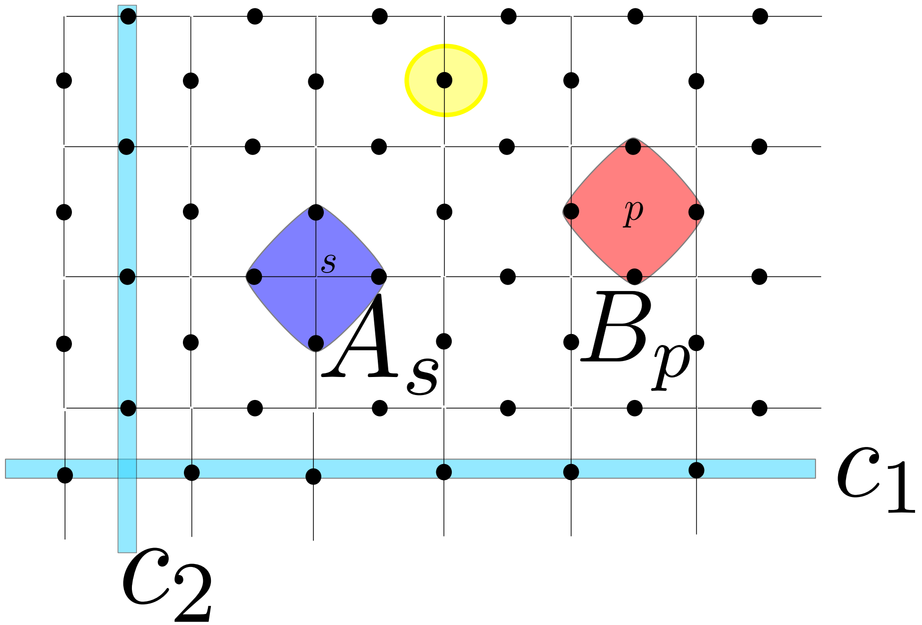

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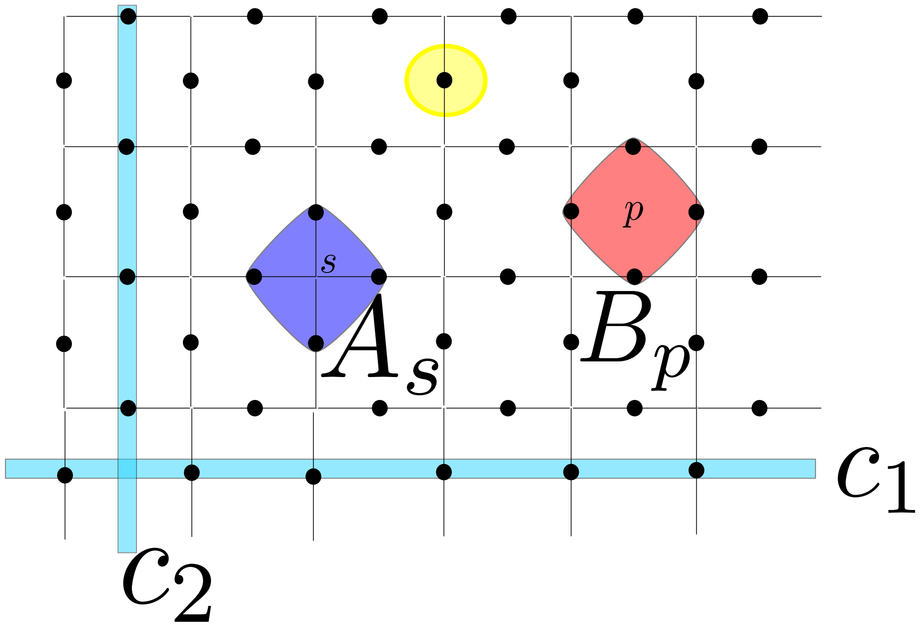

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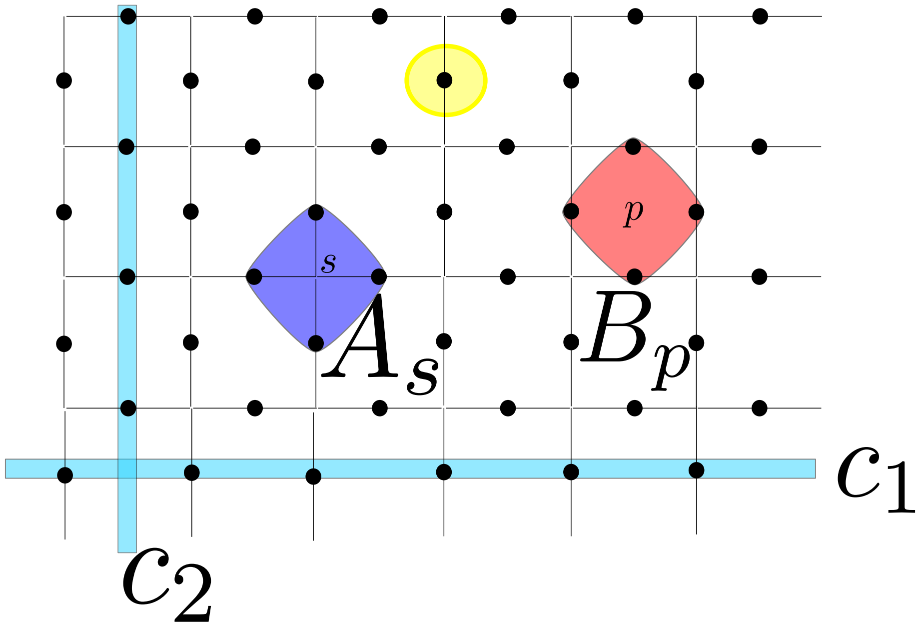

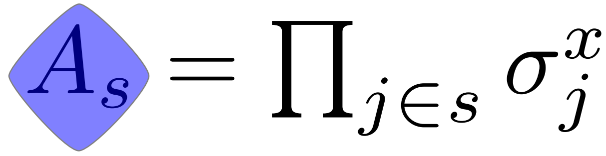

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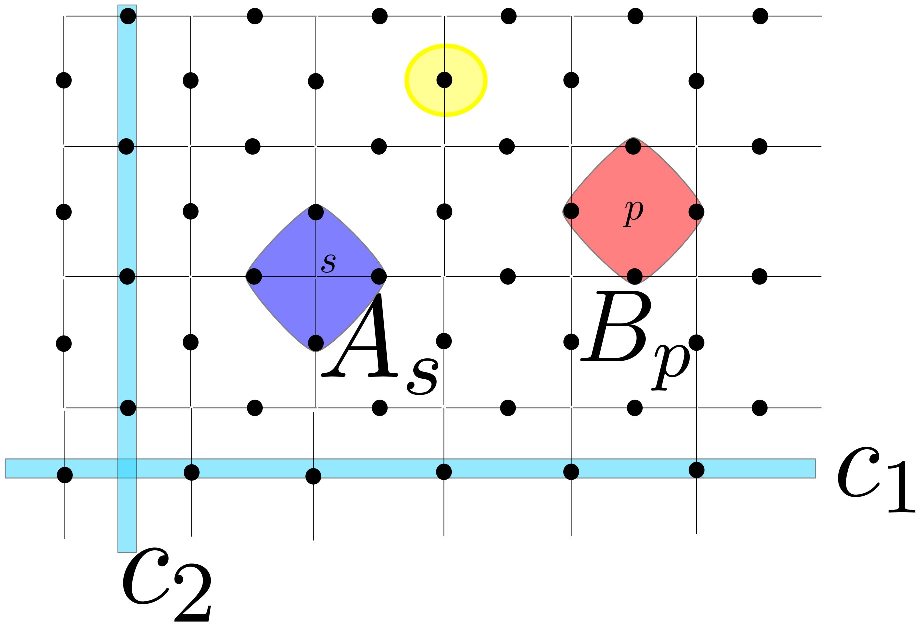

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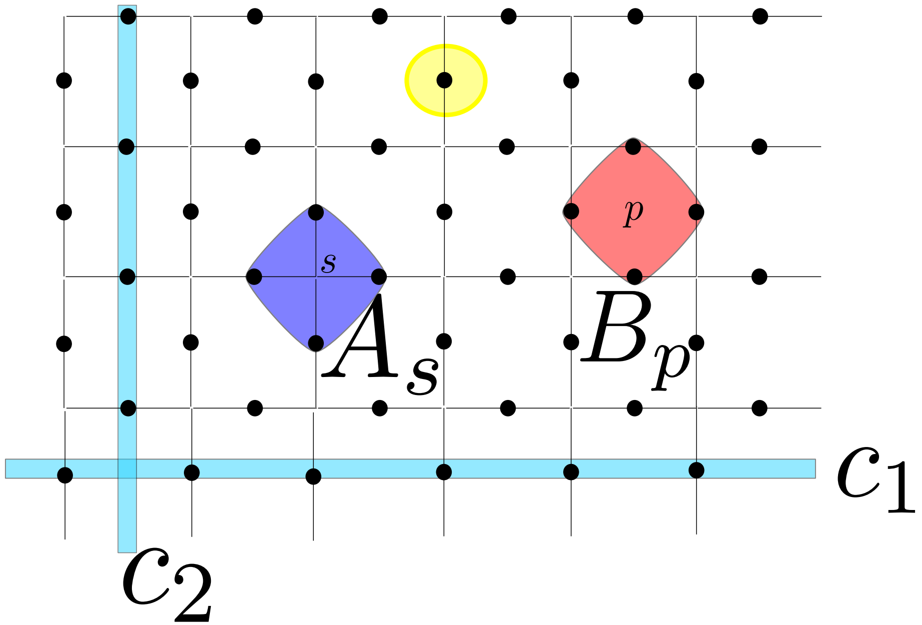

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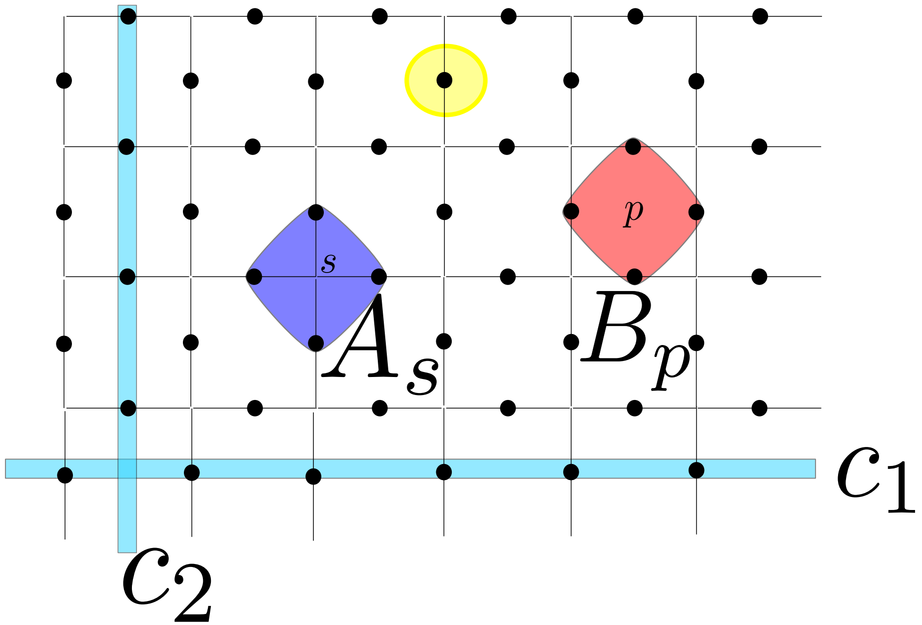

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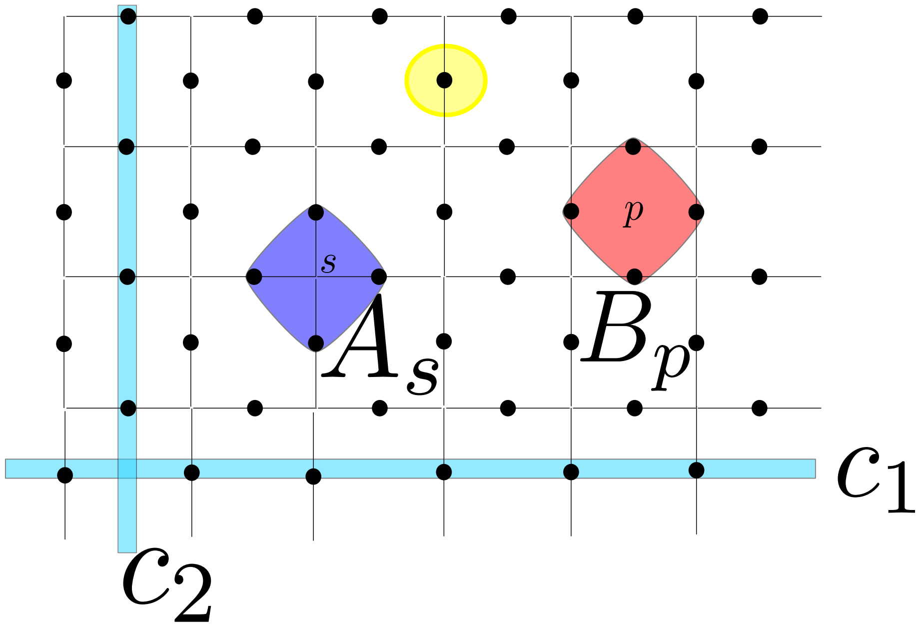

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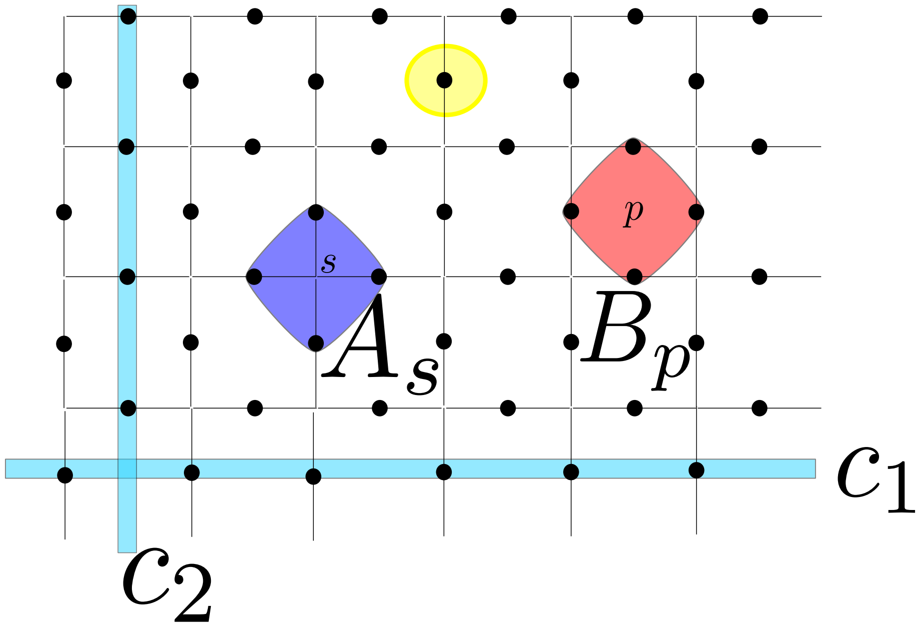

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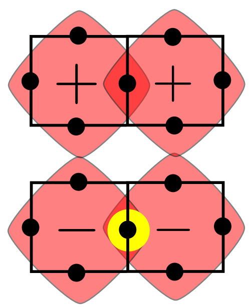
$$[B_p, \sigma_k^x] \neq 0$$



Pure Z2 lattice gauge theory as low energy of the deformed toric code

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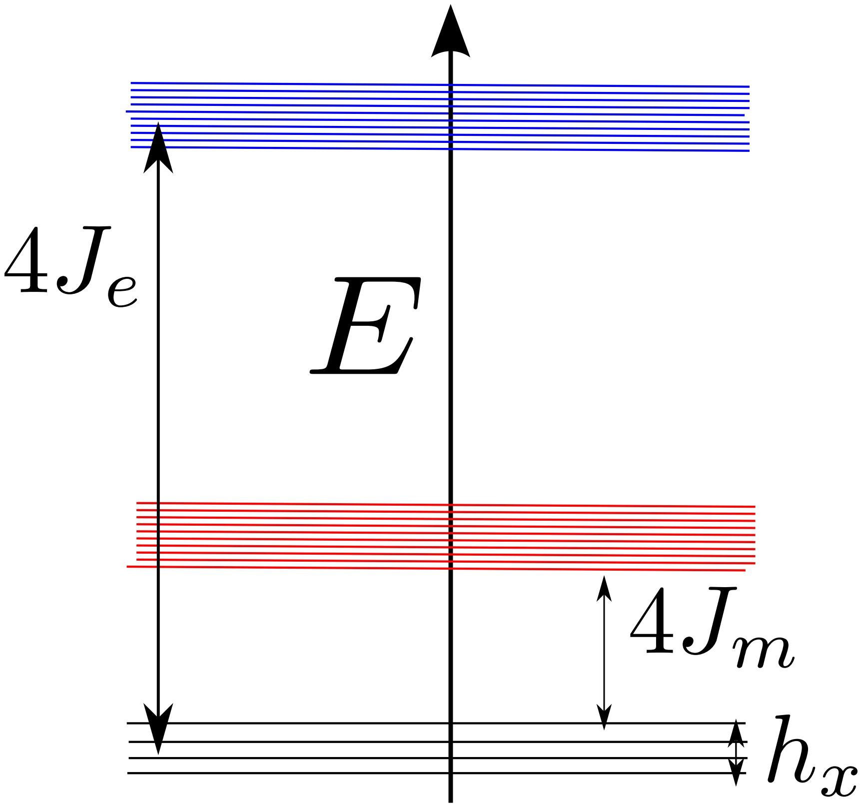

$$J_e \gg J_m \qquad J_m = 1$$

Pure Z2 lattice gauge theory as low energy of the deformed toric code

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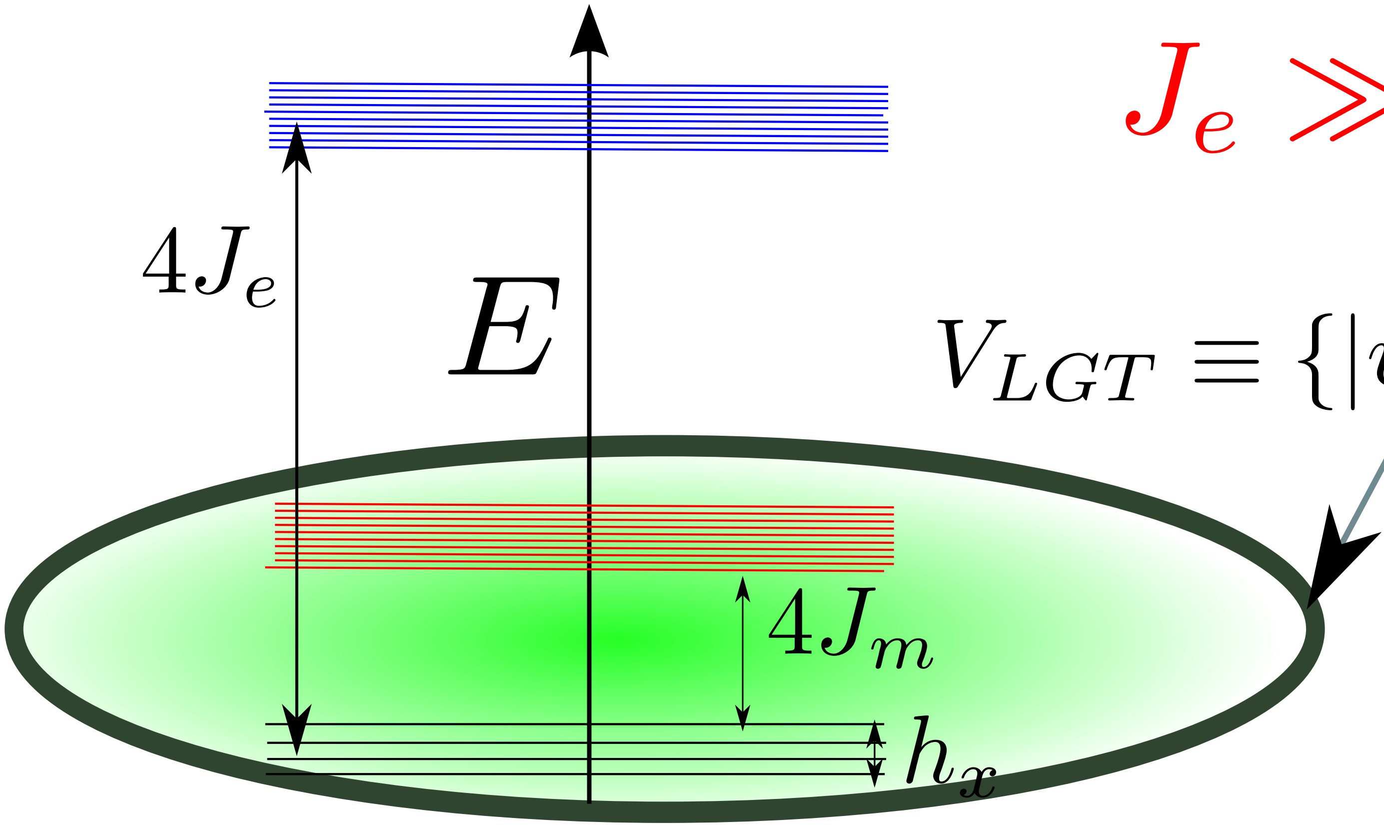
$$V_{LGT} \equiv \{|\psi\rangle : A_s|\psi\rangle = |\psi\rangle \forall s\}$$



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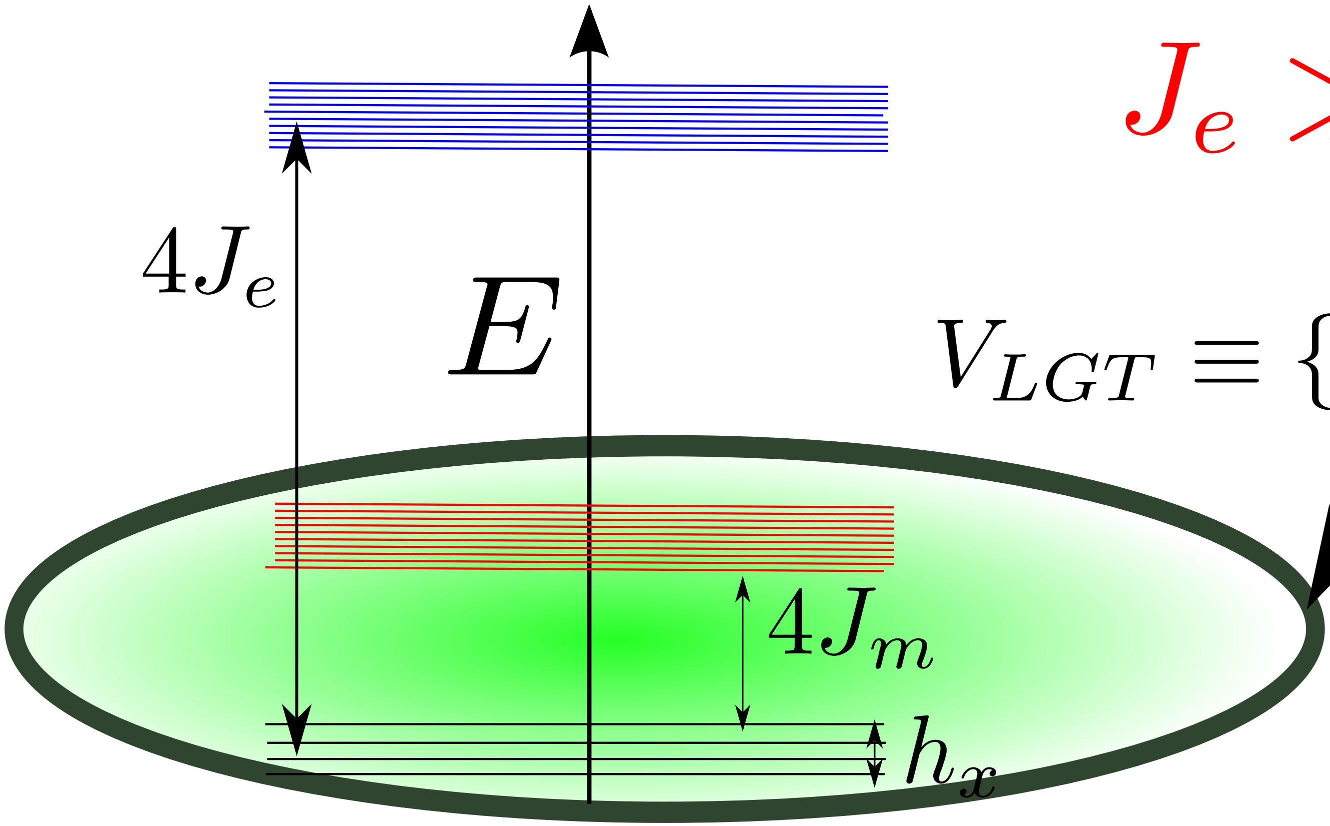

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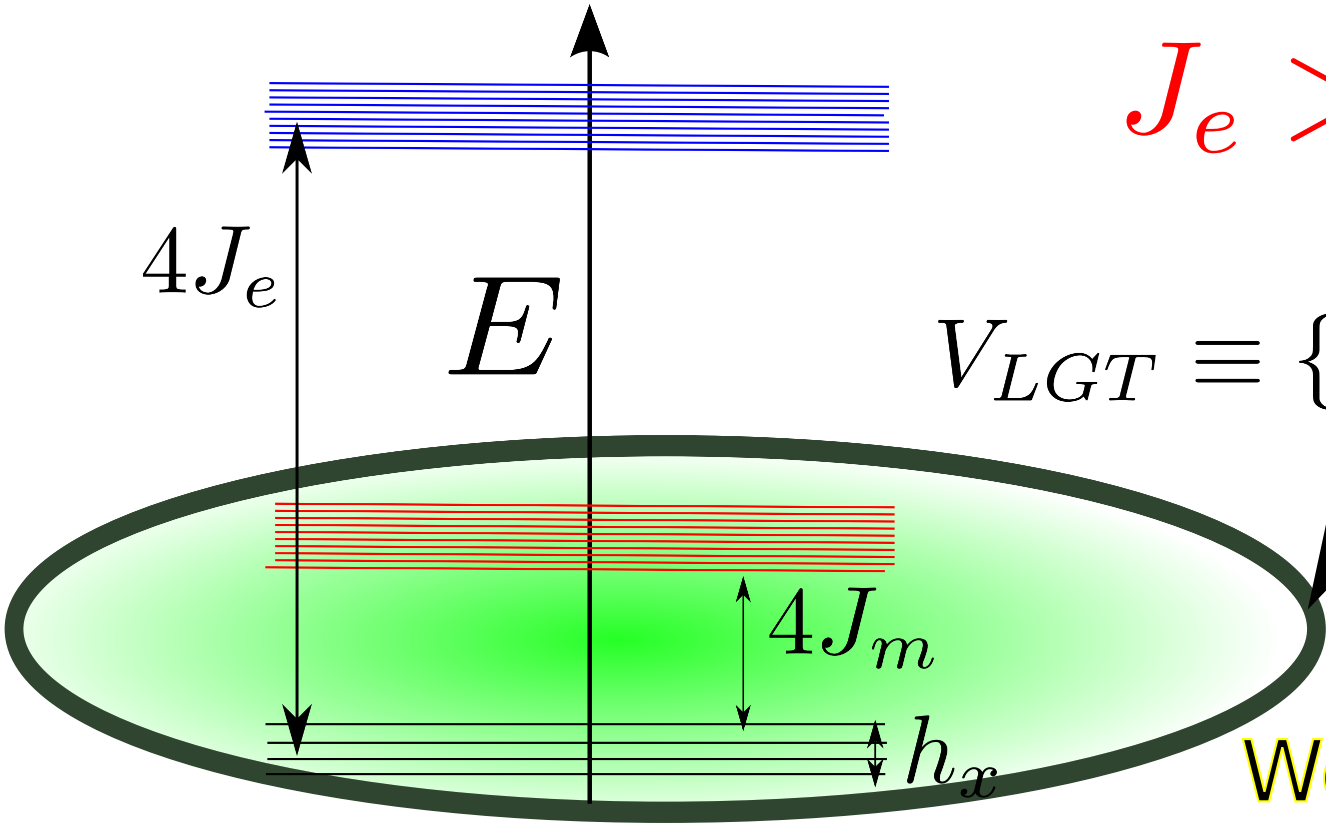

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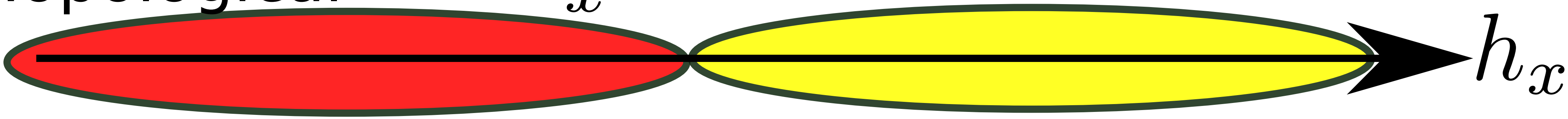
Wegner 71, Kogut 79

$$H \equiv -\sum_p B_p - h_x \sum_k \sigma_k^x$$

Deconfined/
Topological

$h_x^{crit} \simeq 0.33$

Confined



U(1) LGT with local Hilbert space of dim 2

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$$\mathcal{H}_G = \{|\psi\rangle\} \; s.t. \; \mathcal{T}(\{g_s\})|\psi\rangle = |\psi\rangle, \forall g_s \in G$$

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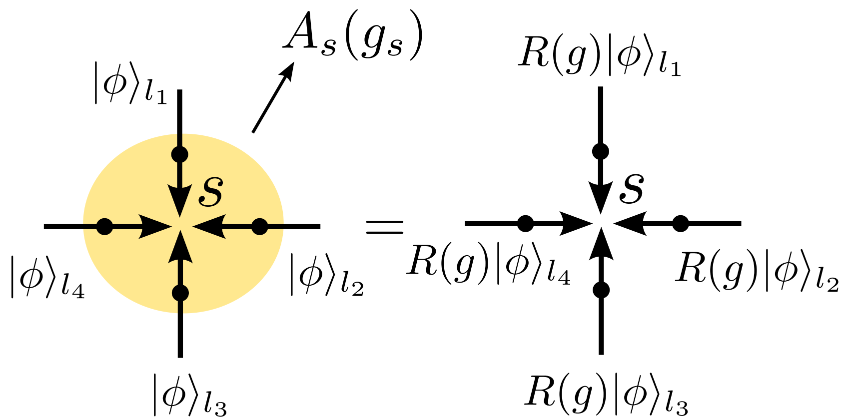

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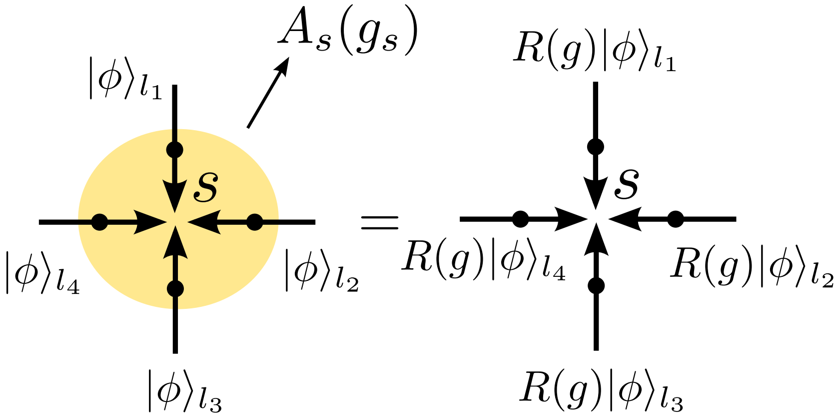

$|\phi\rangle_{l_i} \in C^2$

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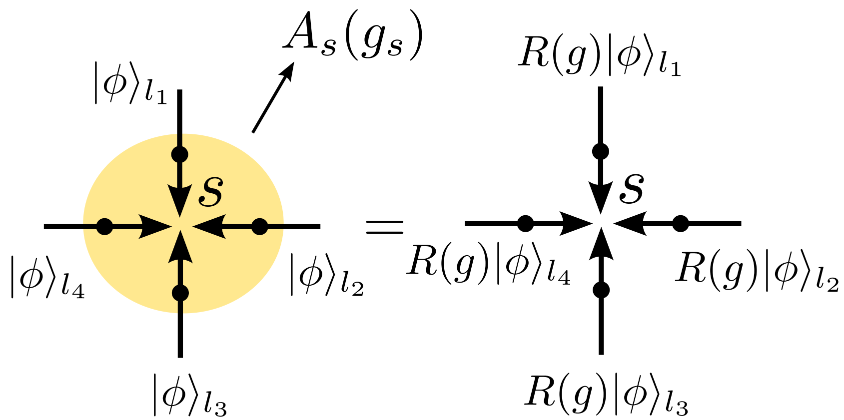

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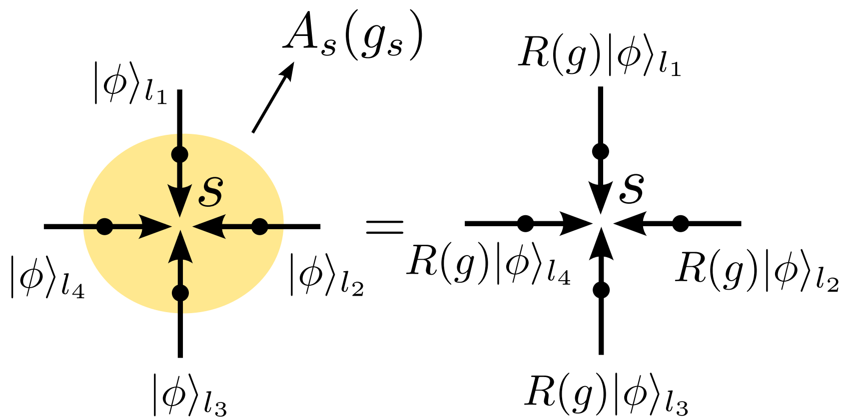

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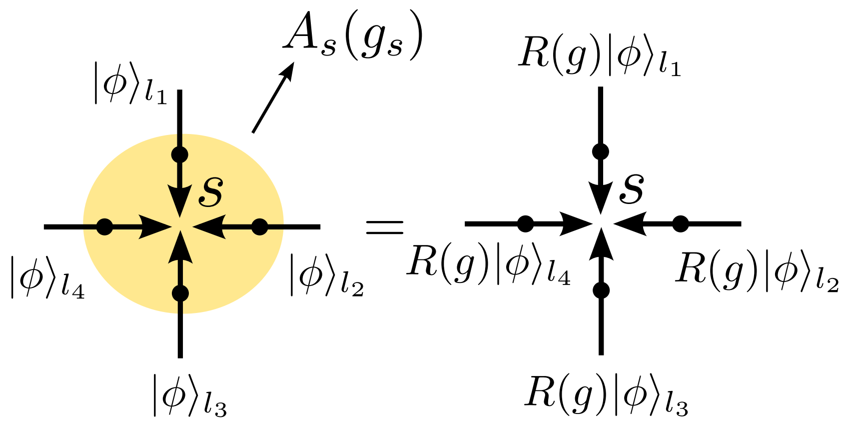

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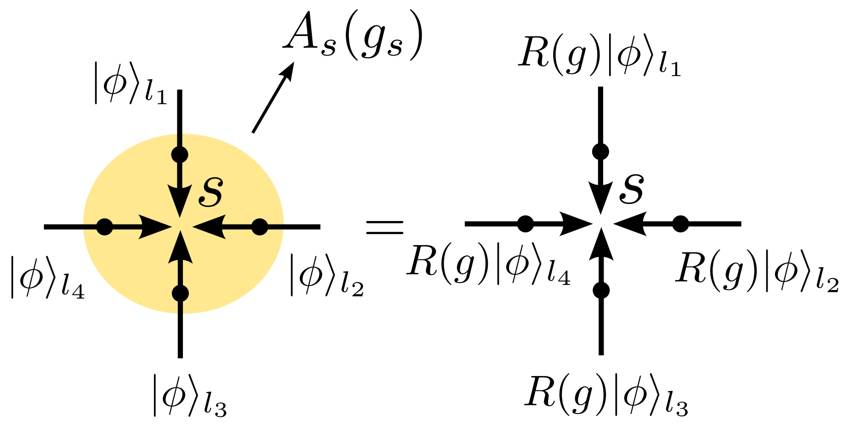

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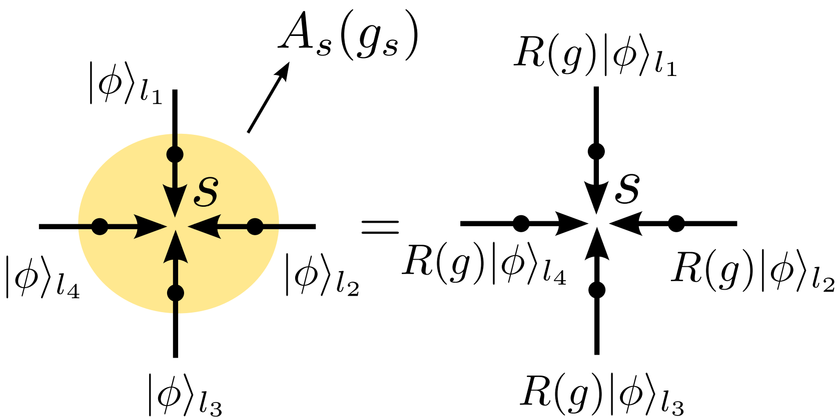

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Link models and gauge magnets

Link models and gauge magnets

Volume 100B, number 2 PHYSICS LETTERS 26 March 1981

FINITE MATRIX MODELS WITH CONTINUOUS LOCAL GAUGE INVARIANCE ☆

D. HORN
Department of Physics and Astronomy, Tel Aviv University, Ramat Aviv, Israel

Received 5 January 1981

We construct a hamiltonian lattice gauge theory which possesses local SU(2) gauge invariance and yet is defined on a Hilbert space of 5-dimensional real vectors for every link. This construction does not allow for generalization to arbitrary SU(N), but a small variation of it can be generalized to an SU(N) × U(1) local gauge invariant model. The latter is solvable in simple gauge sectors leading to trivial spectra. We display these by studying a U(1) local gauge invariant model with similar characteristics.

Link models and gauge magnets

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FINITE MATRIX MODELS WITH CONTINUOUS LOCAL GAUGE INVARIANCE ☆

D. HORN
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NUCLEAR PHYSICS B

Exact solution of a quantum gauge magnet in 2 + 1 dimensions

Peter Orland *
The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark
and
Physics Department, Baruch College, The City University of New York, 17 Lexington Avenue, New York, NY 10010, USA **

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Quantum Link Models:
A Discrete Approach to Gauge Theories *

S. Chandrasekharan and U.-J. Wiese

Center for Theoretical Physics,
Laboratory for Nuclear Science, and Department of Physics
Massachusetts Institute of Technology (MIT)
Cambridge, Massachusetts 02139, U.S.A.

MIT Preprint, CTP 2573

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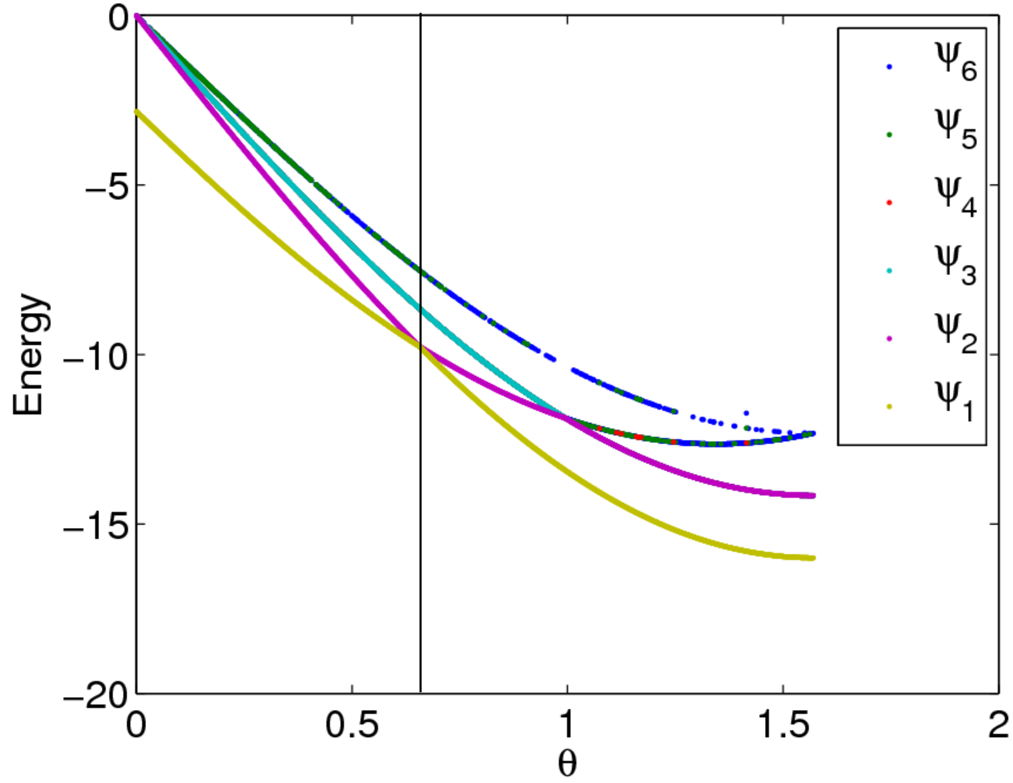

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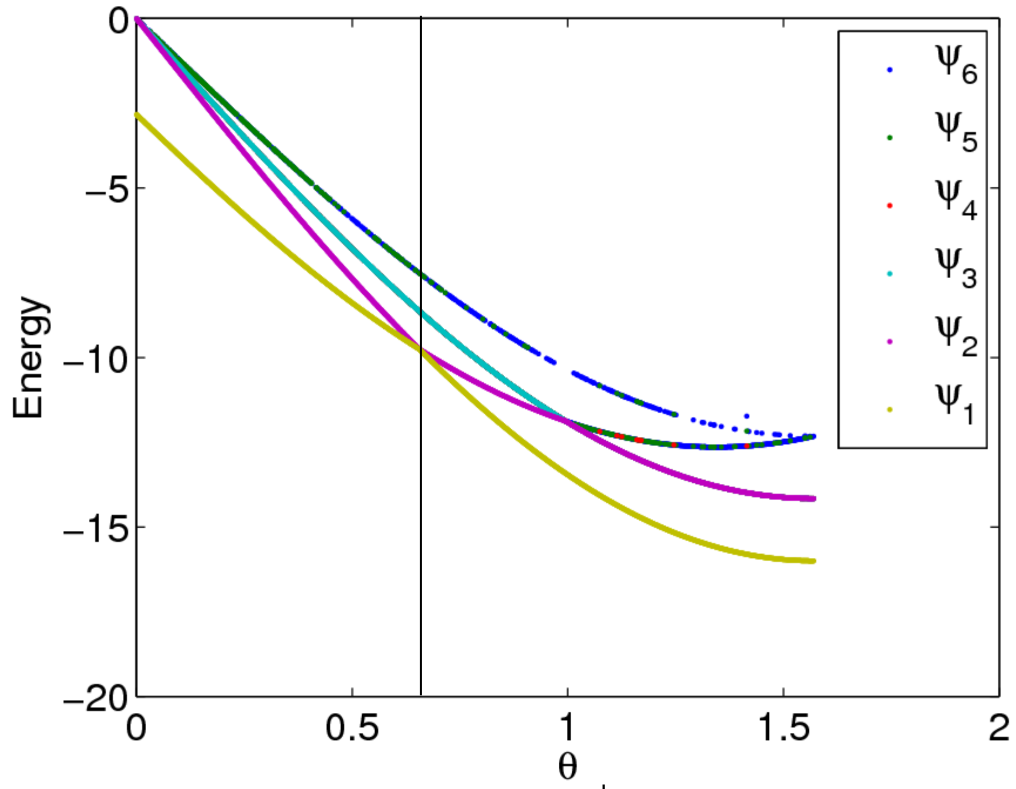

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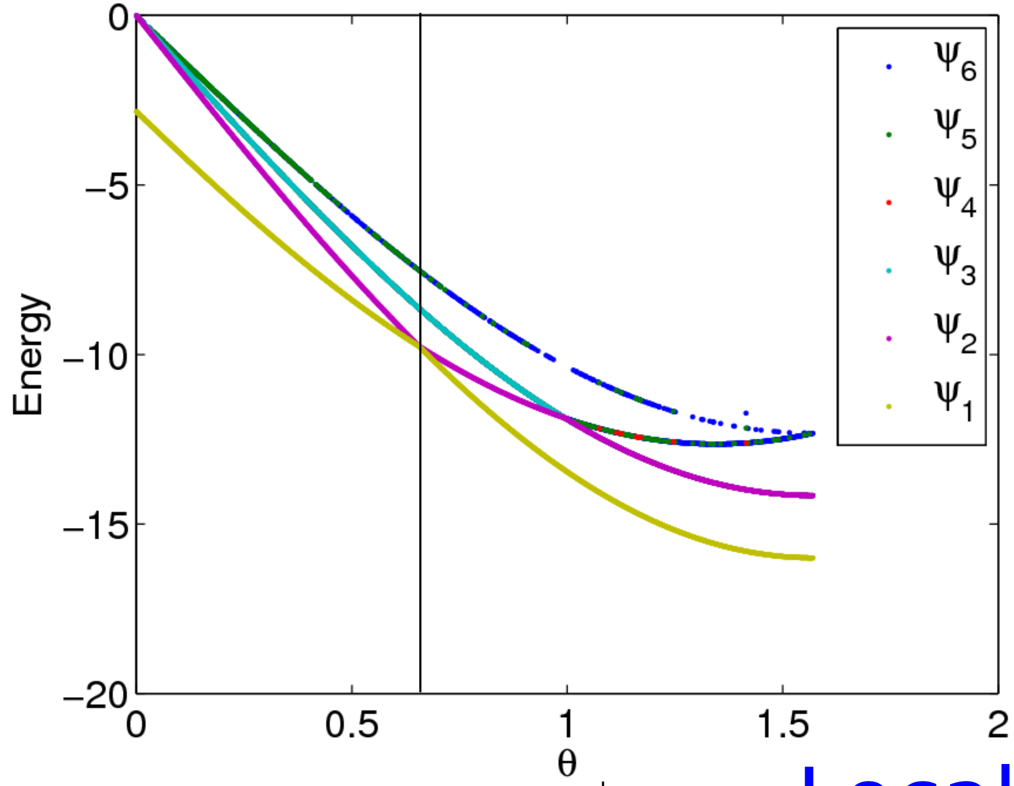

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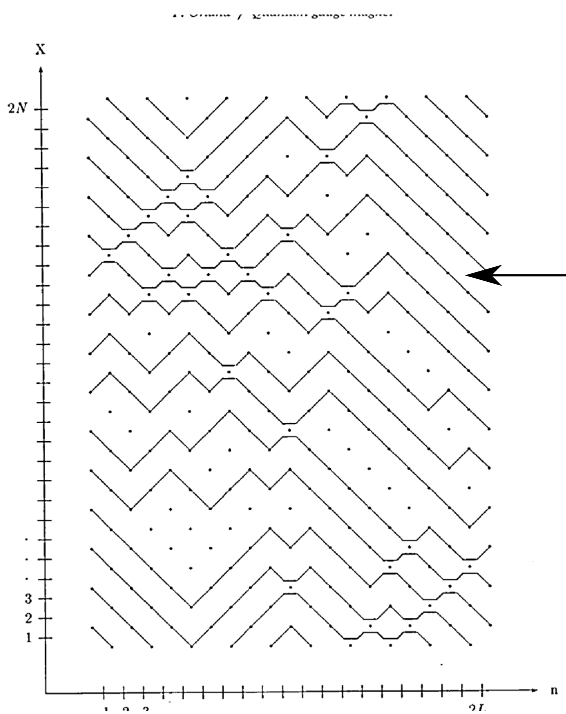

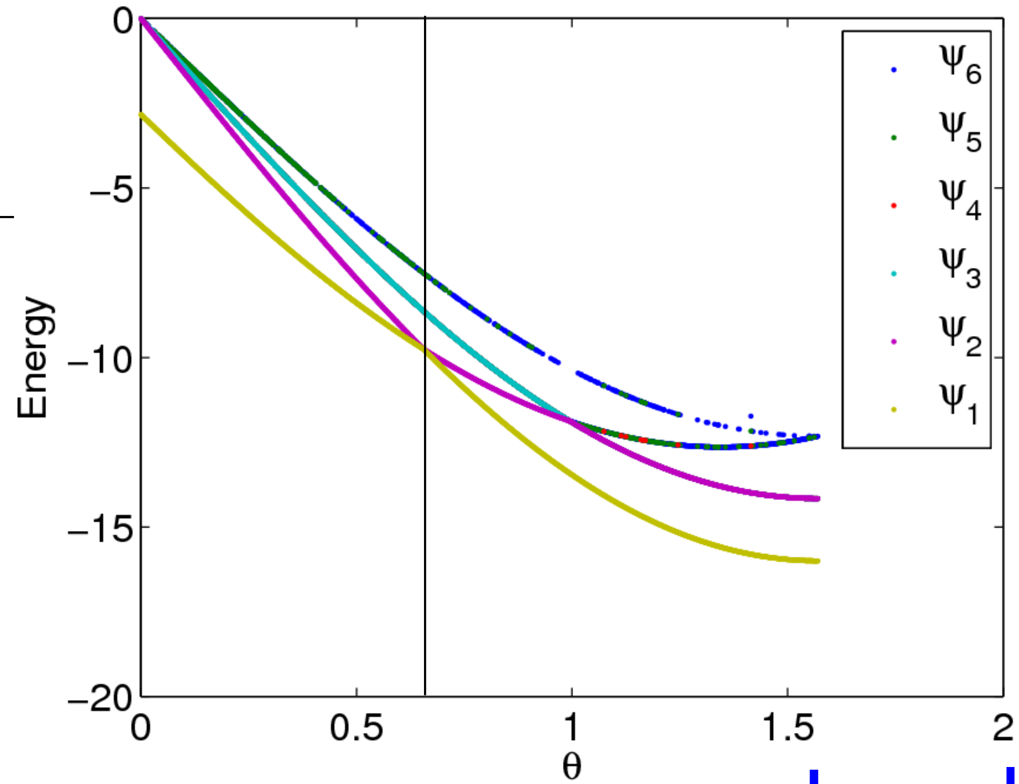

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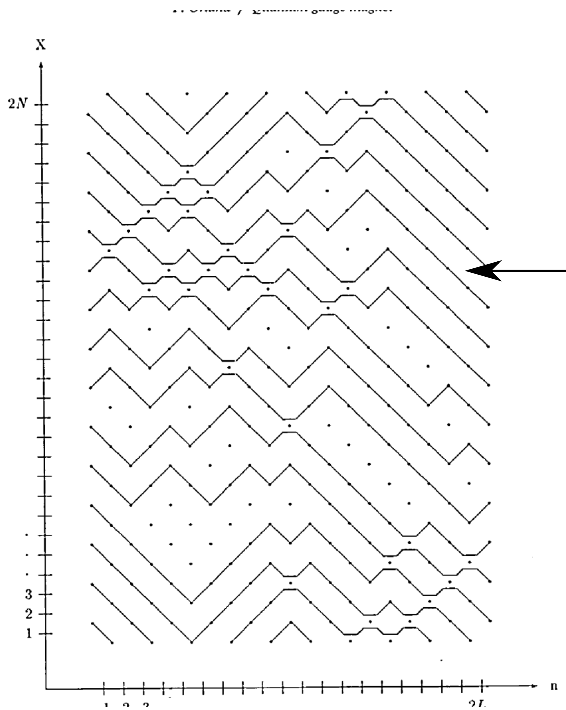

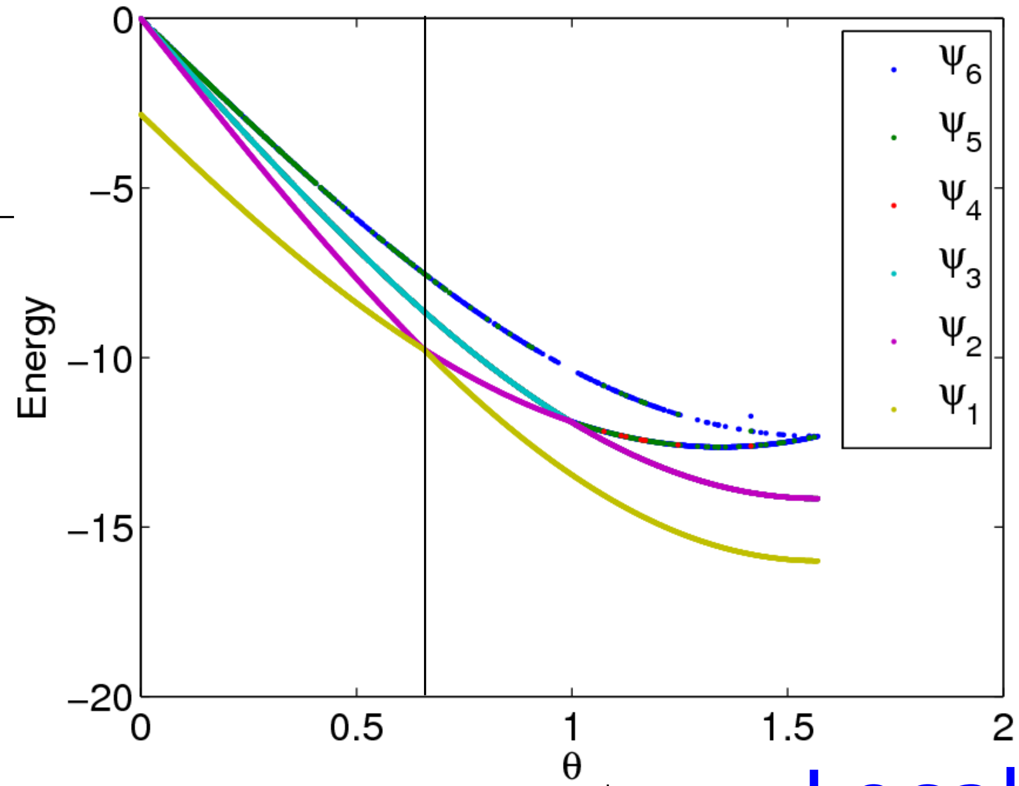

Topological
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Can we really simulate it with Rydberg?

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We can perform time evolution

two level system which dynamic can be implemented via Rydberg

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two level system which dynamic can be implemented via Rydberg

We cannot make dissipative state preparation

$$[H_P, H_{P'}] \neq 0, P \cap P' \neq 0$$

Hamiltonian is not frustration free
We do not know GS locally

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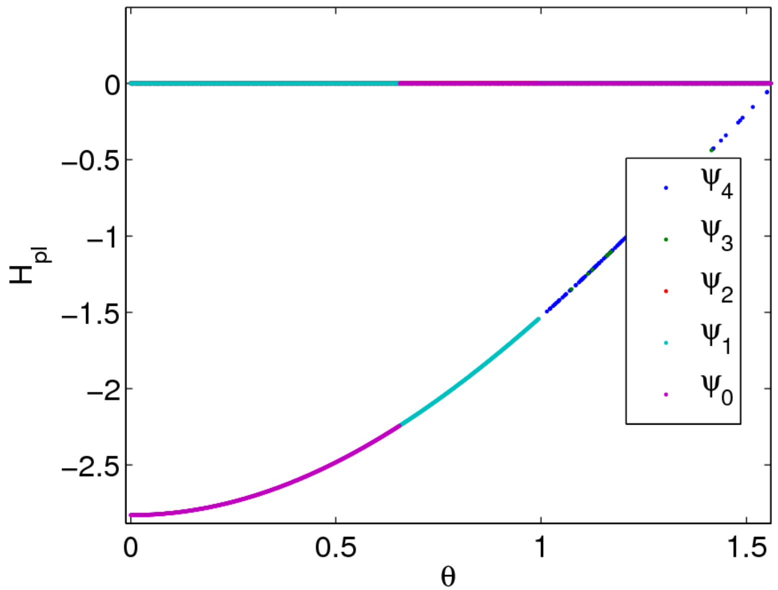
Gapless interesting phase no easy state

$$\Delta = 0$$



Level crossing with other phase where easy state

U(1) gauge magnet Rydberg state preparation

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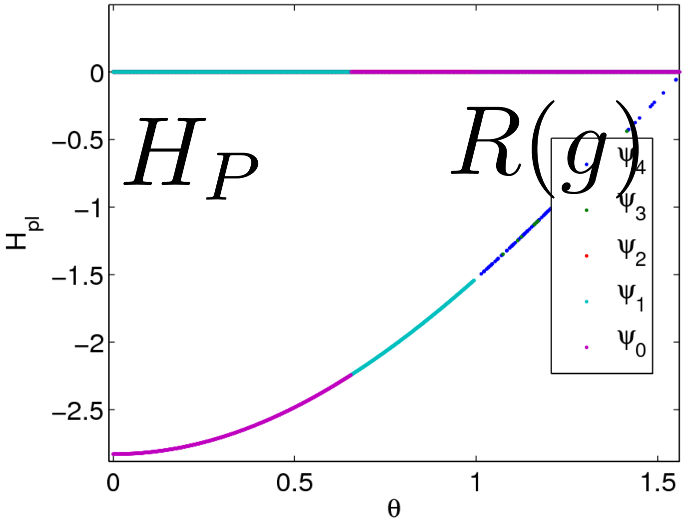

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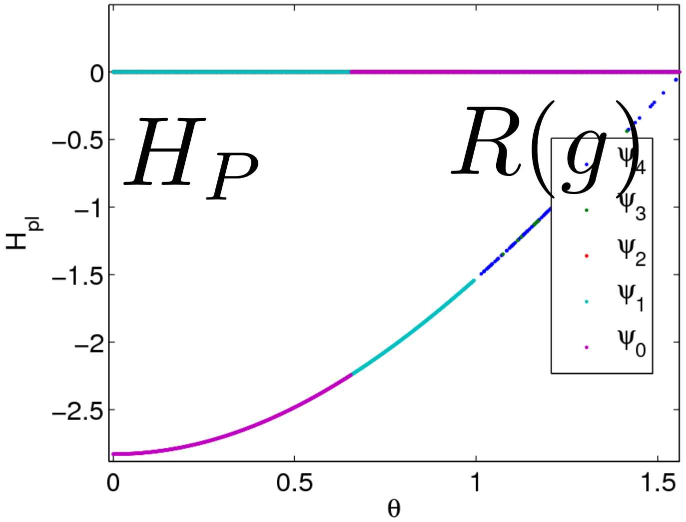

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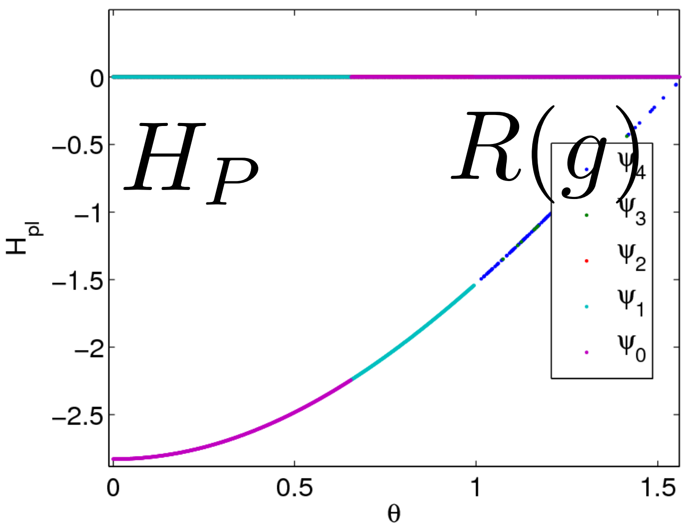

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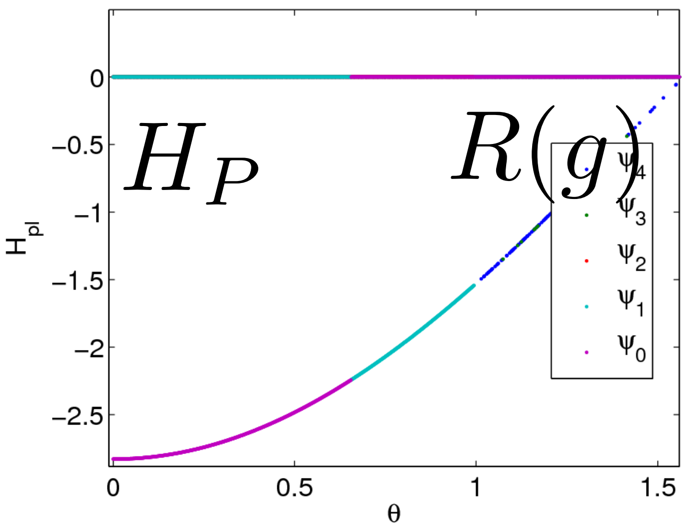

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$$H_P|GS\rangle \neq c|GS\rangle$$

State preparation, mixed strategy

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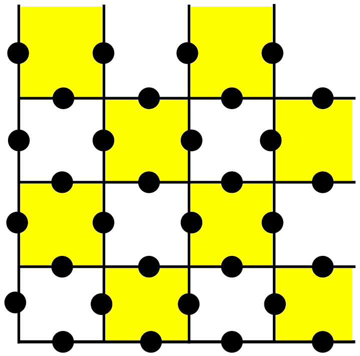
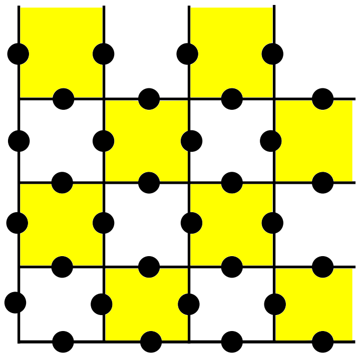


State preparation, mixed strategy

$$H = H_{P_w} + H_{P_y} + h.c.$$



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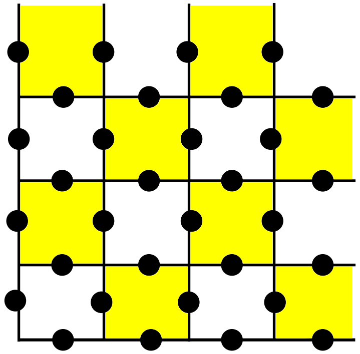

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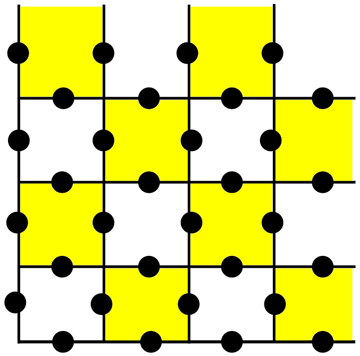

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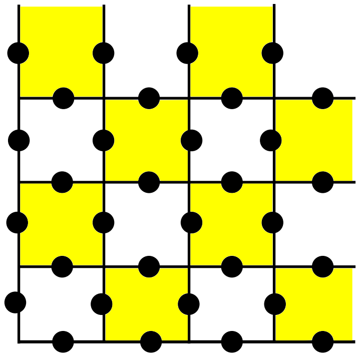

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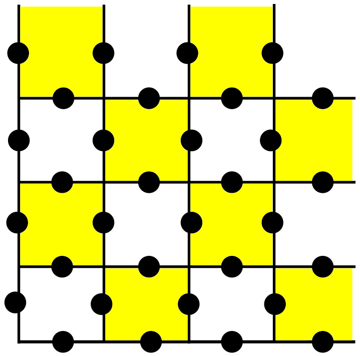

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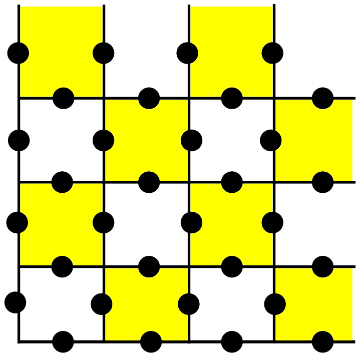

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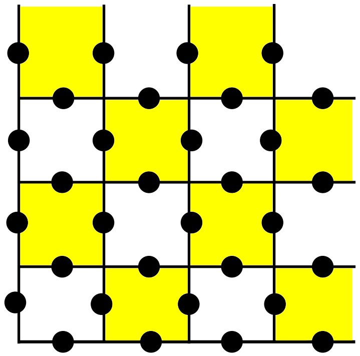

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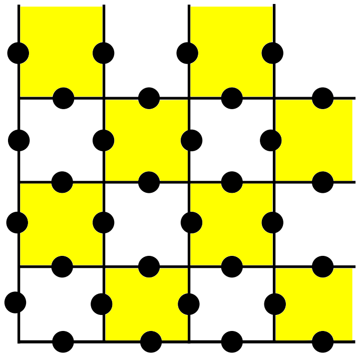

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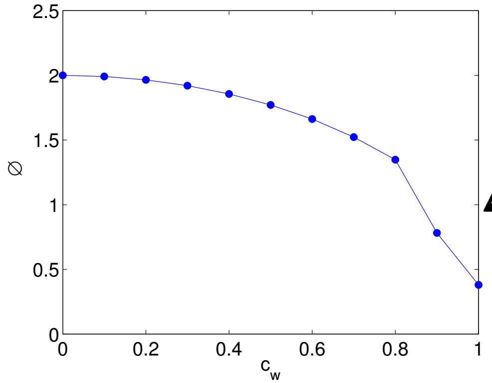

$$\Delta \propto (c_w - 1)^2$$

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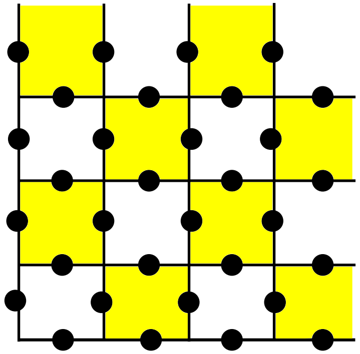

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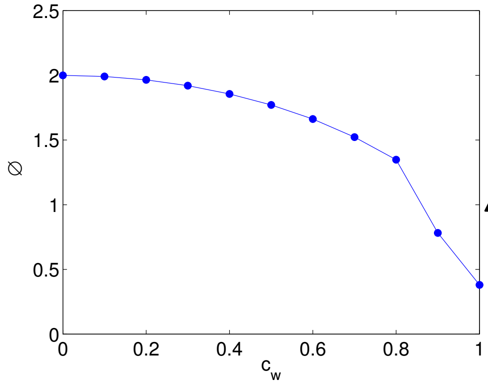

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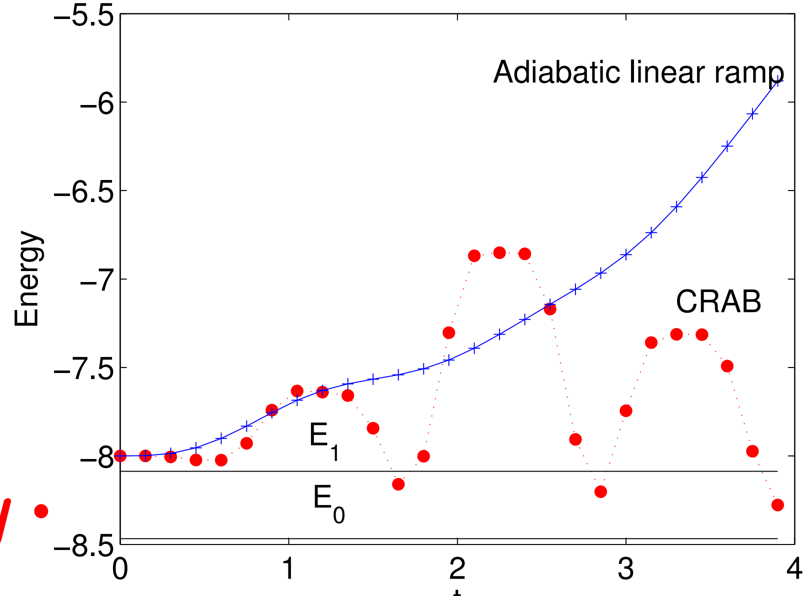

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U(1) gauge magnet Rydberg time evolution

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Conclusions

Intersting historical phase for MBQP

First quantum simulations (QS)

We propose a possible candidate for U(1) LGT QS

Now should be possible to perform
out of equilibrium time-evolution.

Further step towards complete QS LGT...
(need matter).