# Quantum simulations with ultracold atoms: two examples

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# Outline

- Ultracold atoms as quantum simulators

- Non-abelian gauge potentials
  - Quantum Hall physics with ultracold atoms
  - Non-Abelian excitations at the degeneracy points of Landau levels

- Layered Fermi gases in optical lattices
  - Anisotropic Ginzburg-Landau & Lawrence-Doniach models
  - Effective DNLS dynamics close to $T_c$

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# Ultracold atoms as quantum simulators of:

- Strongly interacting lattice systems (e.g., Fermi and/or Bose Hubbard-like models )
- Quantum magnetism
- Disordered models
- Dirac and relativistic field theories
- Low-dimensional systems
- Quantum Hall physics
- (BCS) Superconductors with Cooper pairs
- ...

# Main available "ingredients"

- Bosons and/or fermions
- Geometry (1D / 2D)
- Long-range interactions
- Add disorder
- Time-dependence (and to a certain extent space-dependence) of the parameters of the Hamiltonian
- Explicit tuning of the interactions via Feshbach resonances
- Simulate a magnetic field through a rotation or with optical tools
- Optical lattices (i.e., periodic potentials and minima of the potential located on a lattice)
- ...

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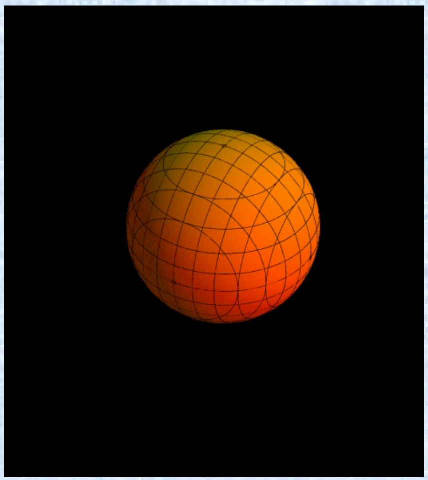
# What is needed to simulate quantum Hall physics with ultracold gases:

- 2D systems: obtained using the trapping potentials
- Strong - and possibly tunable - interactions between atoms: obtained using Feshbach resonances
- A (*fictitious*) magnetic field ...

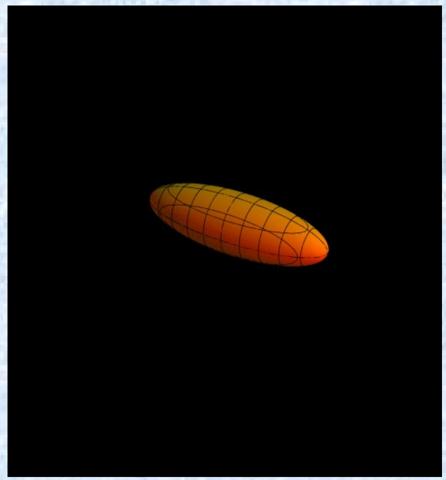
# Two-dimensional systems

Magnetic harmonic potential: $V(x,y,z)=\frac{1}{2}m\left(\omega_x^2\,x^2+\omega_y^2\,y^2+\omega_z^2\,z^2\right)$

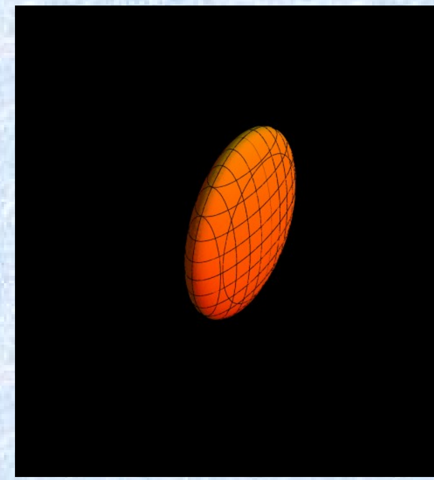
$\omega_x=\omega_y=\omega_z$



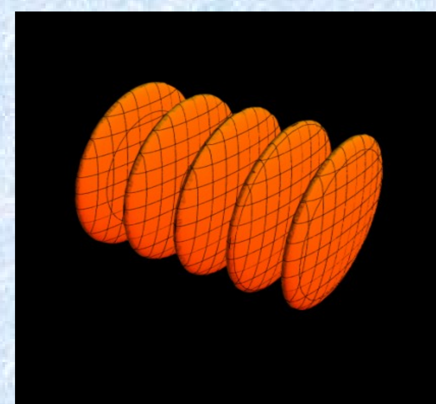
$\omega_x<<\omega_y=\omega_z$



$\omega_x>>\omega_y=\omega_z$



Using optical lattices:



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# What is needed to simulate quantum Hall physics with ultracold gases:

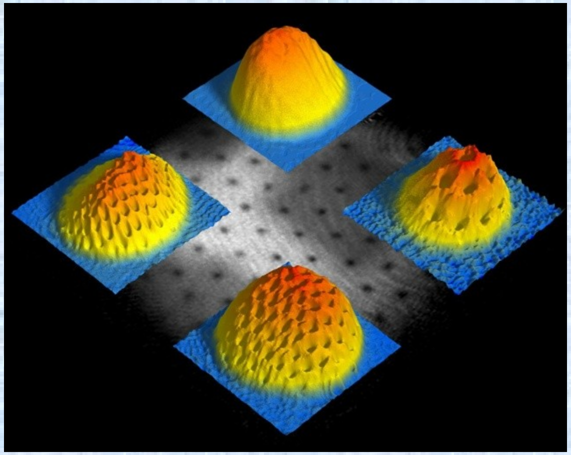
- 2D systems

- Strong - and possibly tunable - interactions between atoms

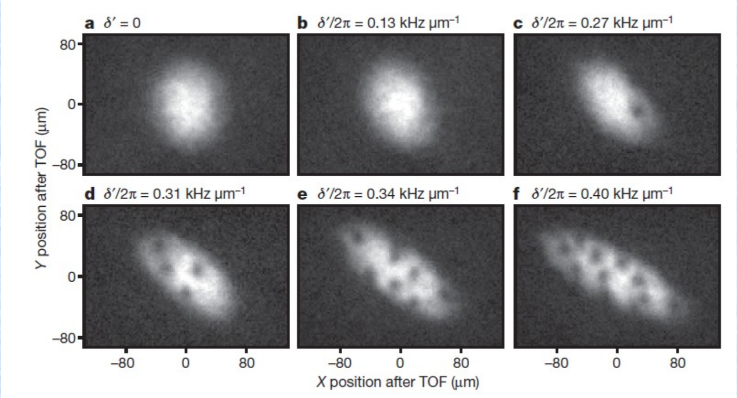
- A (*fictitious*) magnetic field ...how to do it?

# Simulating a magnetic field:

➢ Using rotating traps
[see the review Cooper, Adv. Phys. (2008)]



➢ With spatially dependent optical couplings between internal states of the atoms [Y.-J. Lin et al., Nature (2009)]



# Landau levels for 2D gases in rotation (I)

Single particle:

$$H = \frac{p^2}{2m} + V\left(R(t)\,\vec{r}\right); \qquad V\left(\vec{r}\right) = \frac{1}{2} m\omega^2\left(x^2 + y^2\right)$$

$$R(t) = \begin{pmatrix} \cos\Omega t & \sin\Omega t \\ -\sin\Omega t & \cos\Omega t \end{pmatrix}$$

$$\left|\tilde{\psi}\right\rangle = e^{i\Omega L_z t}\left|\psi\right\rangle \quad \Rightarrow \quad \tilde{H} = \frac{p^2}{2m} + V\left(\vec{r}\right) - \Omega L_z$$

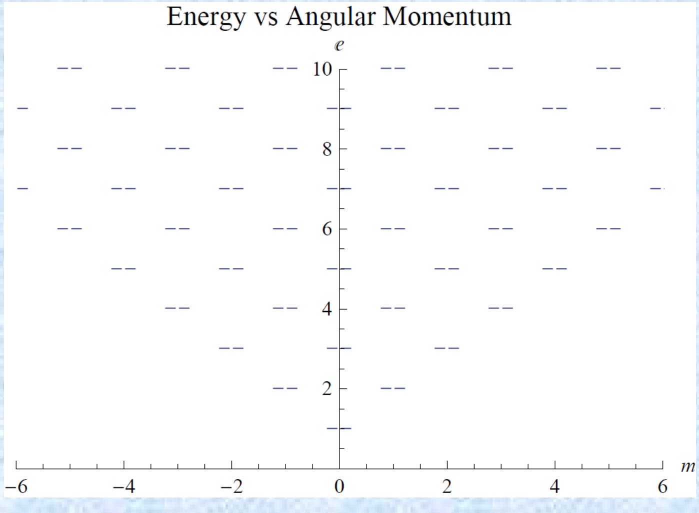
# Landau levels for 2D gases in rotation (II)

$$\tilde{H} = \frac{p^2}{2m} + V(\vec{r}) - \Omega L_z = H_0 - \Omega L_z$$

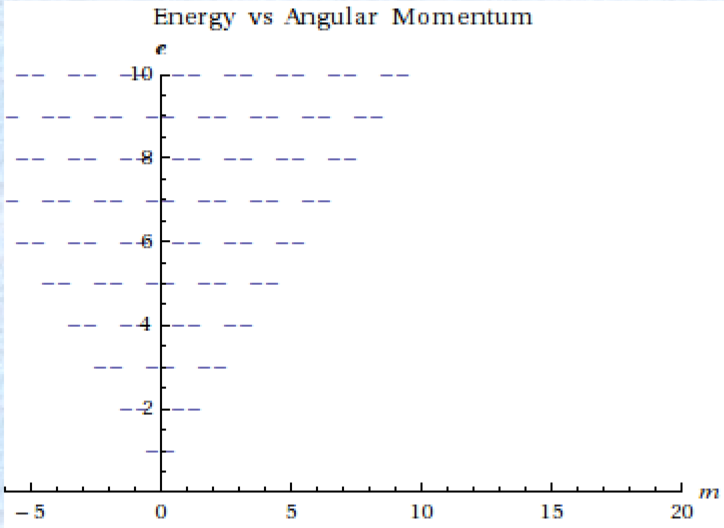
$$\begin{cases} H_0 |n,m\rangle = \hbar\omega(n+1)|n,m\rangle \\ L_z |n,m\rangle = m|n,m\rangle \end{cases}$$

$$(n \equiv n_x + n_y;\ m \equiv n_x - n_y)$$

$$\Omega = 0$$



# Landau levels for 2D gases in rotation (III)




For $\Omega$ close to $\omega$ (first Landau level):

$$|\psi\rangle = \sum_m c_m \, |m, m\rangle$$

$$\psi_{m,m}(x, y) = z^m \, e^{-|z|^2/4l^2} \qquad (l^2 \equiv \hbar / m\omega; \;\; z \equiv x + iy)$$

For many non-interacting particles:

$$\psi(z_1, \ldots, z_N) = \varphi(z_1, \ldots, z_N) \, e^{-\sum_i |z_i|^2/4l^2}$$

# Laughlin ground-states for 2D gases in rotation

Effect of a <u>strong</u> interaction:

$$H = \sum_i H_i + g_{2D}\,\hbar\omega \sum_{i<j} \delta\left(r_i - r_j\right)$$

For $g_{2D} \to \infty$

$$\varphi\left(z_1,\ldots,z_N\right) = \prod_{i<j}\left(z_i - z_j\right)\overline{\varphi}\left(z_1,\ldots,z_N\right) \;\Rightarrow\; \left(\omega - \Omega\right) L_z \overline{\varphi} = E\overline{\varphi}$$

$$\overline{\varphi} = \begin{vmatrix} 1 & z_1 & z_1^2 & \ldots & z_1^N \\ \ldots & & & & \\ 1 & z_N & z_N^2 & \ldots & z_N^N \end{vmatrix} = \prod_{i<j}\left(z_i - z_j\right)$$

$$\boxed{\psi(z_1,\ldots,z_N) = \prod_{i<j}\left(z_i - z_j\right)^2 e^{-\sum_i |z_i|^2/4l^2}}$$

Laughlin wavefunction

# Abelian excitations

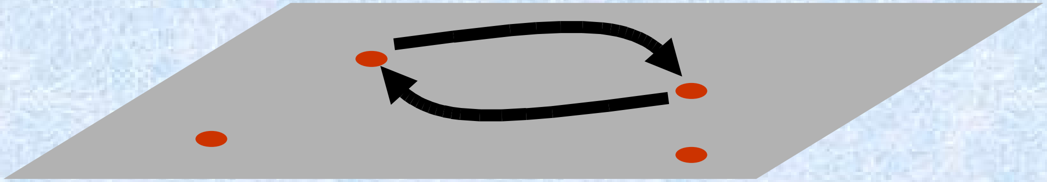
Excitations of the Laughlin ground-state:

$$\psi_{R_0}(z_1,\ldots,z_N) = \prod_{i<j}\left(z_i - R_0\right)\prod_{i<j}\left(z_i - z_j\right)^2 e^{-\sum_i |z_i|^2/4l^2}$$

For ultracold atomic systems, an <u>hole</u> in $R_0$ can be created with a laser centered in $R_0$; two excitations (say in $R_0$ and $R_1$) can be created

or

one excitation can be moved - i.e., $R_0=R_0(\mathit{time})$ – around another ➡ <u>a $\pi$ /2 phase is acquired</u>



# *Non* - Abelian excitations

[Moore and Read (1991

In a non-abelian quantum Hall state quasi-particles obey non-abelian statistics, meaning that (for example):

with 2N quasi-particles at fixed positions, the ground state is $2^N$-degenerate

the interchange of quasi-particles shifts between ground states (i.e., permutations between quasi-particles positions → unitary transformations in the ground state subspace)

An example: *Moore-Read states*

$$H = \sum_i \sum_{j \neq i} \sum_{k \neq i,j} \delta\left(z_i - z_j\right) \delta\left(z_i - z_k\right)$$

$$\Downarrow$$

$$\psi(z_1, \ldots, z_N) = \prod_{i<j} \left(z_i - z_j\right)^2 \prod_i e^{-\left|z_i\right|^2 / 4l^2} \cdot Pf\left(\frac{1}{z_i - z_j}\right)$$

# The quest for non-Abelian anyons with ultracold atoms

The Laughlin ground-state has abelian excitations: the Berry phase is a *number*.

How we can make it a *matrix*?

First (poor man) guess: add a degree of freedom,
i.e. consider a two-component Bose gas

Well, not so easy...

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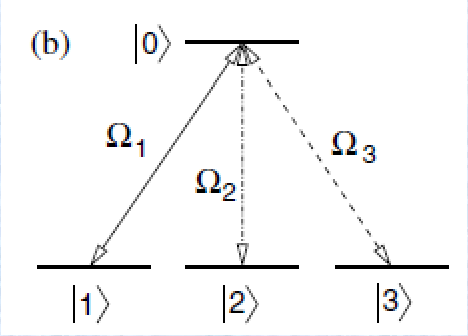
# Rotation & artificial magnetic fields (I)

| Rotation for neutral atoms | ~ | Magnetic field for charged particles |
|---|---|---|

One can rotate also systems with two components (possibly giving different rotations).

Another tool for simulate magnetic fields for a two-component Bose gas is purely optical: Rabi pulses couples two states with two dark states (tripod structure) [Ruseckas et al., PRL (2005)]




[see the review J. Dalibard, F. Gerbier, G. Juzelunas, and P. Ohberg, RMP (2011)]

# Rotation & artificial magnetic fields (II)

$$\begin{cases} \Omega_1 = \Omega \sin\theta \cos\phi \, e^{iS_1} \\ \Omega_2 = \Omega \sin\theta \sin\phi \, e^{iS_2} \\ \Omega_3 = \Omega \cos\theta \, e^{iS_3} \end{cases}$$

$$\boxed{i \,\blacksquare \frac{\partial \Psi}{\partial t} = \left[ \frac{1}{2m} \left( -i0\nabla - A \right)^2 + V \right] \Psi}$$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

A is a 2x2 matrix: $\begin{cases} A_{11} = \;\blacksquare\left( \cos^2\phi \cdot \nabla S_{23} + \sin^2\phi \cdot \nabla S_{13} \right) \\ A_{12} = \;\blacksquare\cos\theta \left( \frac{1}{2} \sin 2\phi \cdot \nabla S_{12} - i \nabla \phi \right) \\ A_{22} = H\cos^2\theta \left( \cos^2\phi \cdot \nabla S_{13} + \sin^2\phi \cdot \nabla S_{23} \right) \end{cases}$

# Rotation & artificial magnetic fields (III)

Other promising schemes:
-optical flux lattices [N. Cooper, PRL 2011]
-generating a geometric phase with specific arrangements on lattices using two stable states [K. Osterloh et al., PRL 2005; M. Aidelsburger et al., PRL 2011]

⬇

For this purpose Yb atoms can be extremely useful [F. Gerbier and J. Dalibard, New J. Phys. 2010]: two electronically-excited metastable states – 7 isotopic stable forms [5 bosons & 2 fermions] with different scattering properties – selective trapping of the two states

# Single-particle Hamiltonians

$$H = \left(p_x + A_x\right)^2 + \left(p_y + A_y\right)^2$$

For a single component: $A_x = 0;\;\; A_y = Bx \;\; (\mathit{Landau\ gauge})$

*or*

$$A_x = -\frac{B}{2}y;\;\; A_y = \frac{B}{2}x \;\; (\mathit{symmetric\ gauge})$$

ing two-component gases & rotations and/or tripod schem

s possible to have $A_x, A_y$ 2x2 matrices with

$$\left[A_x, A_y\right] \neq 0$$

Non-Abelian gauge potentials

# 2D atoms in a non-Abelian magnetic field: An example

$$H = \left(p_x + A_x\right)^2 + \left(p_y + A_y\right)^2$$

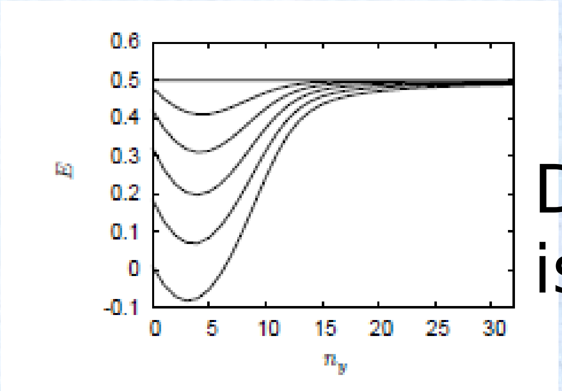
An example:

$$A_x = q\,M_x\,;\;\; A_y = B\,M_y\,x$$

2x2 matrices

with

$$M_y = \sigma_z;\;\; M_x = \sigma_y$$




Degeneracy of Landau levels is in general broken

[A. Jacob et al., New J. Phys. (2008)]

# 2D atoms in a symmetric non-Abelian magnetic field: single particle (I)

$$H = \left(p_x + A_x\right)^2 + \left(p_y + A_y\right)^2$$

## Our choice:

$$\boxed{A_x = q\sigma_x - \frac{B}{2}y;\;\; A_x = q\sigma_y + \frac{B}{2}x}$$

Reasons:

3)degeneracy of Landau levels not broken

2) analytical single particle energy levels

3) realistically implementable

[M. Burrello and A. Trombettoni, PRL (2010); PRA (2011)]

# 2D atoms in a symmetric non-Abelian magnetic field: single particle (II)

$$H = \left(p_x + A_x\right)^2 + \left(p_y + A_y\right)^2 \equiv H_{abel.} + H_{non-abel.}$$

$$A_x = q\sigma_x - \frac{B}{2}y; \;\; A_x = q\sigma_y + \frac{B}{2}x$$

One finds

$$H_{abel.} = 2q^2 + B + \frac{1}{4}d^+ d \qquad \left(d^+ = Bz^* - 4\partial/\partial z; \;\; z = x + iy\right)$$

$$H_{non-abel.} = q\begin{pmatrix} 0 & id \\ -id^+ & 0 \end{pmatrix}$$

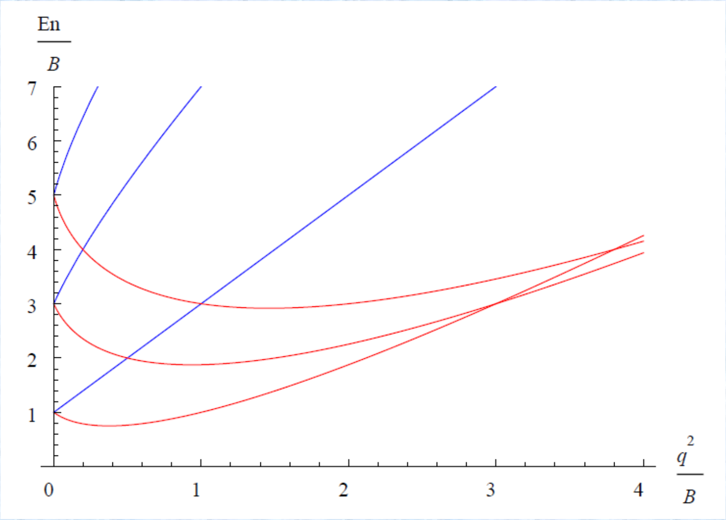
$$\boxed{\varepsilon_n^{\pm} = 2Bn + 2q^2 \pm \sqrt{B^2 + 8q^2 Bn}}$$

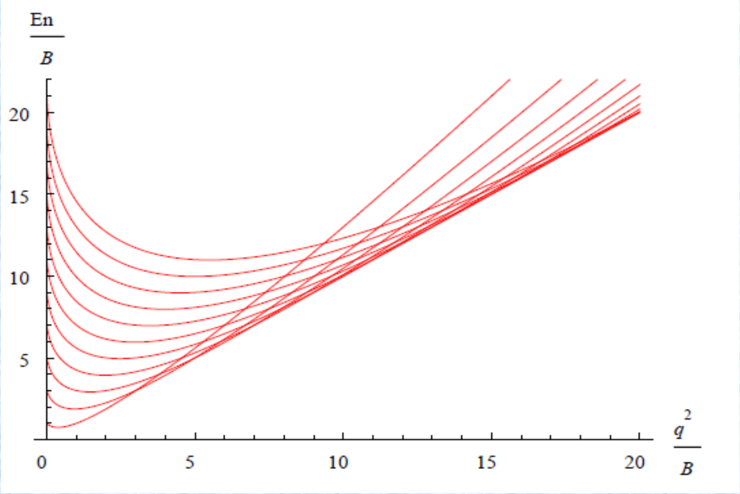
Degeneracy of Landau levels is preserved, although each level splits in two due to non-abelian term q

# 2D atoms in a symmetric non-Abelian magnetic field: single particle (III)

$$H = \left(p_x + A_x\right)^2 + \left(p_y + A_y\right)^2 = H_{abel.} + H_{non-abel.}$$

$$A_x = q\sigma_x - \frac{B}{2}y; \;\; A_y = q\sigma_y + \frac{B}{2}x$$




Landau levels split in two due to non-abelian term c

# 2D atoms in non-Abelian magnetic fields: Adding the interactions

q=0 → usual ("Abelian") case with lowest Landau levels; (strong) interaction gives the Laughlin state

q finite → *deformed* Laughlin state

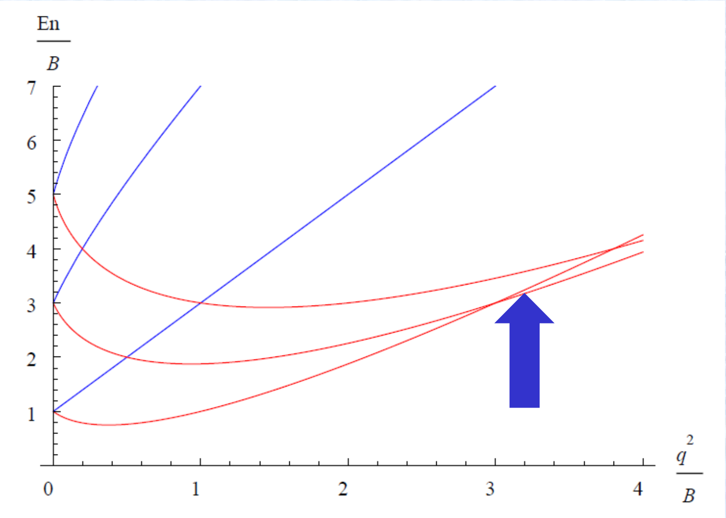
$$\Psi^m = \prod_j G_j \Psi_L^m; \qquad \Psi_L^m = \prod_{i<j} \left(z_i - z_j\right)^m e^{-\frac{B}{4}\sum_i |z_i|^2} \left|\downarrow\downarrow \ldots \downarrow\right\rangle$$

$$G \equiv c_\uparrow \sigma_x + c_\downarrow d^+; \quad d = Bz - 4\partial / \partial z^*$$

$$c_\uparrow = B + 2q\sqrt{2B} + \sqrt{B^2 + 8q^2 B}; \quad c_\downarrow = \frac{i}{2\sqrt{2B}}\left(B - 2q\sqrt{2B} - \sqrt{B^2 + 8q^2 B}\right)$$

...but excitations are Abelian...

# Non-Abelian excitations at the degeneracy points




At the degeneracy points, ground-states with non-Abelian excitations are found: a *deformed* Moore-Read

$$\Psi_{MR} = S\left(\prod_i G_{1,i}\prod_j G_{2,j}\right)Pf\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}\left(z_i - z_j\right)^2 e^{-\frac{B}{4}\sum_i |z_i|^2}\left|\downarrow\downarrow \ldots \downarrow\right\rangle$$

[M. Burrello and A. Trombettoni, PRL (2010)]

*N.B.* Requiring that for $H=\left(p_x+A_x\right)^2+\left(p_y+A_y\right)^2$

3)energy spectrum presents a Landau level structure (and the wavefunctions can be expressed as finite sum of terms)

2) each Landau level is degenerate with respect to the angular momentum

3) in the Abelian limit, the Landau levels become degenerate with respect of the spin degree of freedom

$\Downarrow$

only two classes of Hamiltonians are found

$$H=\left(E+h_z\sigma_z\right)d^+d+M_z\sigma_z-iq\sigma_+d+iq\sigma\_d^+$$

$$H=\left(E+h_z\sigma_z\right)d^+d+M_z\sigma_z-iq\sigma_+d^2+iq\sigma\_d^{+2}$$

[M. Burrello and A. Trombettoni, PRA (2011)]

# Part I: Conclusions & Perspectives

- Non-Abelian gauge potentials do not generally give non-Abelian excitations: rather *deformed* Laughlin states are found

- splitting of the Landau levels exactly determined in an experimentally realizable non-Abelian magnetic field

- however, at the degeneracy points of the Landau levels, ground-states with non-Abelian excitations can be found

- actual/future work:
- computation of pseudopotentials and numerical study of the (interacting) ground-states
- using more components, one finds lines of degeneracy
- study of the effect of an optical lattice
- equivalent study of non-Abelian anyons in other experimental proposals
- determine braiding and fusion rules (challenging...)

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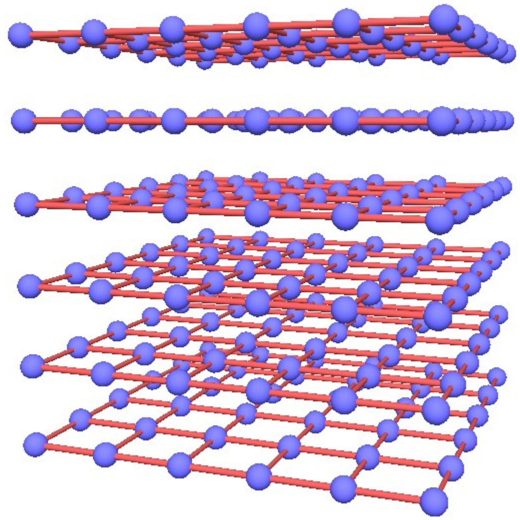
# Quantum simulation of layered superconductors

What is needed:
- interacting fermions
- a main optical lattice (say in the z direction)
- eventually two other lattices (in the x and y directions)

$t_{\perp} = 0$



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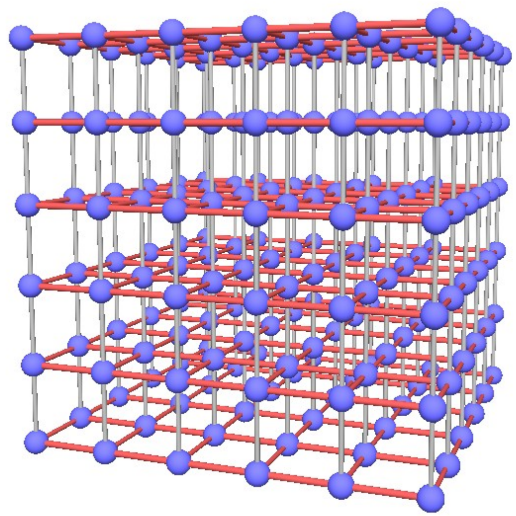


# Motivations

- Several superconducting systems have a layered structure:
  - transition-metal dichalocgenides ($TaS_2$) intercalated with organic molecules [F.R. Gamble et al., Science (1970)]
  - artificial layers of different metals [S.T. Ruggiero et al., PRL (1980)]
  - cuprates [J.G. Bednorz and K.A. Muller, Z. Phys. B (1986)]

- We therefore would like to have a system having tunable parameters and displaying the same phenomenology

- Ideal to study the crossover 2D/3D

- Realize Josephson physics (and Josephson arrays) with ultracold fermions [for ultracold bosons: F.S. Cataliotti et al., Science (2001)]

# What is “reasonably” known on cuprate superconductivity*

Superconductivity is due to the formation of Cooper pairs

Anisotropic Ginzburg-Landau well describes the electrodynamical
roperties

The locus of the superconductivity is the copper oxide planes

The order parameter is a spin singlet

0. The orbital symmetry of the order parameter is $d_{x^2-y^2}$

2. Formation of the Cooper pairs takes places indipendently within different
nultilayers

The electron-phonon interaction is not the principal mechanism of the
ormation of Coopr pairs

The size of the Cooper pairs is 10-30 Angstroms

see Nature Physics, vol. 2, March 2006 – also A.J. Leggett, Quantum Liquids, Chap. 7

# Phenomenological description of layered superconductors/superfluids

Anisotropic Ginzburg-Landau:

$$\delta F[\Psi]=\int d^3r\left\{\frac{\hbar^2}{2m_{\parallel}}\left(|\partial_x\Psi|^2+|\partial_y\Psi|^2\right)+\frac{\hbar^2}{2m_{\perp}}|\partial_z\Psi|^2+\alpha(T)|\Psi|^2+\beta|\Psi|^4\right\}$$

(anisotropy parameter: $\gamma=\sqrt{\frac{m_{\perp}}{m_{\parallel}}}$ )

Lawrence-Doniach model:

$$kinetic\ term=\sum_n\int d^2r\left\{\frac{0^2}{2m_{\parallel}}\left(|\partial_x\Psi_n|^2+|\partial_y\Psi_n|^2\right)+J_1|\Psi_n-\Psi_{n+1}|^2\right\}$$

*Both very successful for layered superconductors!*

$$\gamma\approx 7\ for\ YBCO\ \ and\ \ \gamma\approx 150\ for\ BSCCO$$

In the following we are going to obtain from the microscopics the (parameters of the) two effective models [M. Iazzi, S. Fantoni, and A. Trombettoni, arXiv:1112.6429]

# Effective models for ultracold bosons layered by an optical lattice at T=0

$$i\hbar\frac{\partial\Psi(r,t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{ext}(\vec{r})+g_0|\Psi|^2\right]\Psi$$

$$\psi(r,t)=\sum_j\psi_j(t)\,\phi_j(r)$$

$$\boxed{i\hbar\frac{\partial\psi_j}{\partial t}=-K\left(\psi_{j+1}+\psi_{j-1}\right)+U\,|\psi_j|^2\,\psi_j}$$

$$K=-\int d\vec{r}\left[\frac{\hbar^2}{2m}\vec{\nabla}\phi_j\cdot\vec{\nabla}\phi_{j\pm1}+\phi_jV_{ext}\phi_{j\pm1}\right]$$

$$U=\frac{4\pi\hbar^2a}{m}\int d\vec{r}\,\phi_j^4$$

Discrete non-linear Schroedinger equation (DNLS)

[A. Trombettoni and A. Smerzi, PRL (2001)]

# Hubbard model for layered fermions

$$H = -\sum_{<i,j>}\sum_{\sigma=\uparrow,\downarrow}\left(t_{ij}\,\phi_{i\sigma}^{+}\,\phi_{j\sigma} + h.c.\right) - U\sum_{i}\phi_{i\uparrow}^{+}\phi_{i\downarrow}^{+}\phi_{i\downarrow}\phi_{i\uparrow}$$

$t_{ij} = t_{\perp}, t_{\parallel}$

$n$ *filling*

($t_{\perp}, t_{\parallel}$ *and* $U$ *can be estimated from Wannier wavefunctions*)

# Saddle point + gaussian fluctuations: results (I)

$$\begin{cases} \dfrac{1}{U} = \displaystyle\sum_{\vec{k}\in BZ} \dfrac{\tanh\left(\beta E_{\vec{k}}/2\right)}{2E_{\vec{k}}} \\ n = -\dfrac{\partial F}{\partial \mu} \end{cases}$$

$$E_{\vec{k}} = \sqrt{\varepsilon_{\vec{k}}^2 + \Delta^2}$$

$$\varepsilon_{\vec{k}} = \varepsilon_{\vec{k}}^{(0)} - \mu$$

$$\varepsilon_{\vec{k}}^{(0)} = -2t_{\parallel}\left(\cos k_x + \cos k_y\right) - 2t_{\perp}\cos k_z$$

$$F = -\frac{1}{\beta_c}\sum_{\vec{k}\in BZ} \ln\left(1 + e^{-\beta_c \varepsilon_{\vec{k}}}\right) + \frac{1}{\beta_c}\sum_k \ln\left\{\frac{1}{U} + \chi(k)\right\}$$

$$\chi(k) = \frac{1}{\beta_c}\sum_k G_k^+ G_{k+q}^-$$

$$G_k^{\pm} = \frac{i\omega \pm \varepsilon_{\vec{k}}}{\omega^2 + \varepsilon_{\vec{k}}^2}$$

Two equations for the critical temperature and the chemical potential

# Saddle point + gaussian fluctuations: results (II)

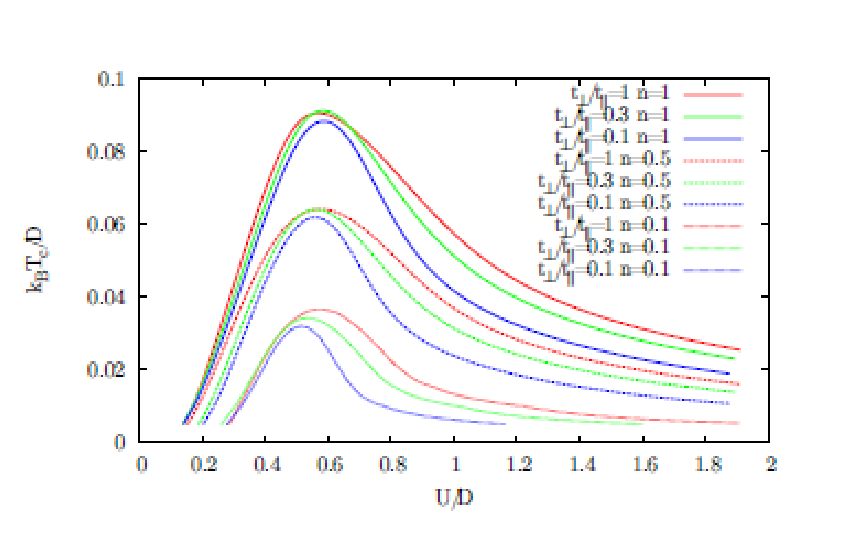
Once the critical temperature and the chemical potential have been determined we can compute the coefficients of the anisotropic Ginzburg-Landau:

$$\alpha = \frac{1}{U} - \sum_{\vec{k}} \frac{X}{2\varepsilon_{\vec{k}}} \qquad \beta = \sum_{\vec{k}} \left\{ \frac{X}{4\varepsilon_{\vec{k}}^{3}} - \frac{Y / k_B T_C}{8\varepsilon_{\vec{k}}^{2}} \right\}$$

$$\frac{\hbar^2}{2M_a} = \left[ \sum_{\vec{k}} \left\{ \frac{XY}{k_B T_C} + \frac{Y}{k_B T_C \varepsilon_{\vec{k}}} - \frac{2X}{\varepsilon_{\vec{k}}^{2}} \right\} \frac{\left( \partial_{q_a} \varepsilon_{\vec{k}+\vec{q}} \right)^2}{8\varepsilon_{\vec{k}}} + \sum_{\vec{k}} \left\{ \frac{X}{4\varepsilon_{\vec{k}}^{2}} - \frac{Y}{8 k_B T_C \varepsilon_{\vec{k}}} \right\} \partial_{q_a}^2 \varepsilon_{\vec{k}+\vec{q}} \right]_{\vec{q}=0}$$

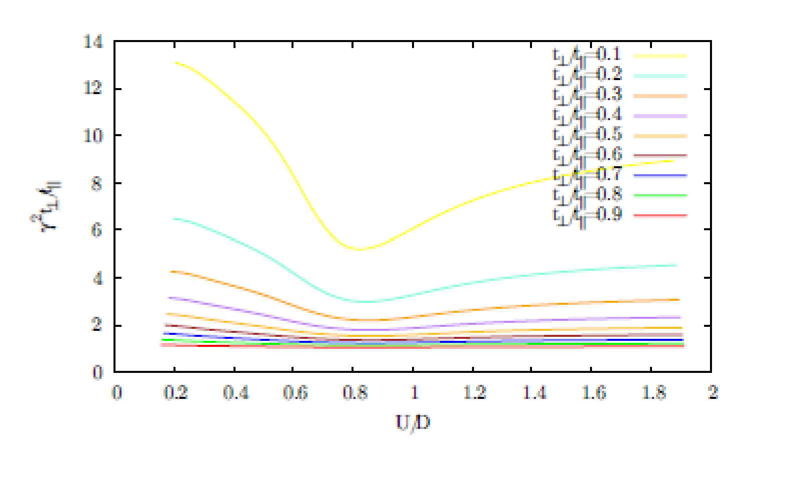
$$X = \tanh\left( \varepsilon_{\vec{k}} / 2k_B T_C \right); \quad Y = \frac{1}{\cosh^2\left( \varepsilon_{\vec{k}} / 2k_B T_C \right)}; \quad M_a = m_a U^2 \ \ (a = x, y, z)$$

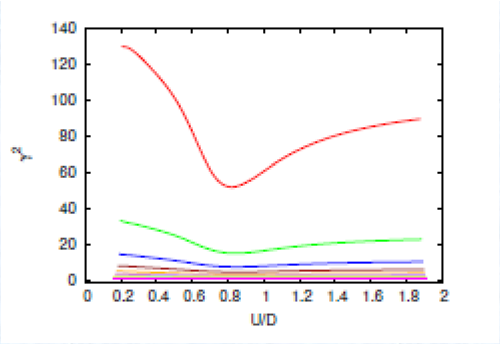
# Critical temperature




$$D = ۸ t_{\parallel} + ۴ t_{\perp} \quad total \quad bandwidth$$

# Anisotropy parameter




(n=0.5)

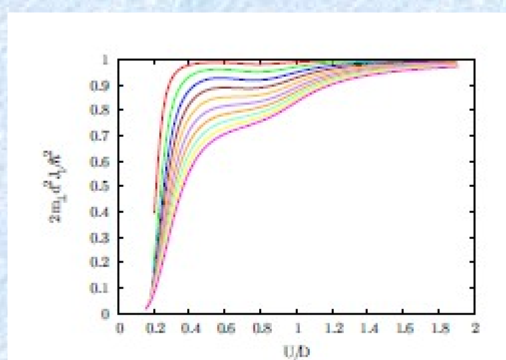
reduced at the unitary limit

# Josephson couplings in the Lawrence-Doniach model (I)

$$kinetic\ term\ along\ \ z = \sum_p \int d^2 r\, J_p \left|\Psi_n - \Psi_{n+p}\right|^2$$

$$J_p = -\frac{U^2 d}{2\pi} \int dq_z\, e^{-ipq_z d}\ \chi(i\omega = 0;0,0,q_z)$$

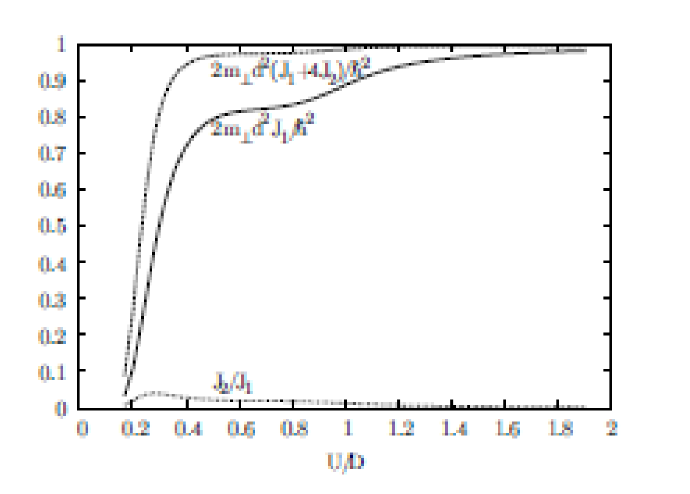
$$\boxed{\frac{{}^{2}}{2m_\perp d^2} = \sum_{p=1}^{\infty} p^2 J_p}$$




(n=0.5)

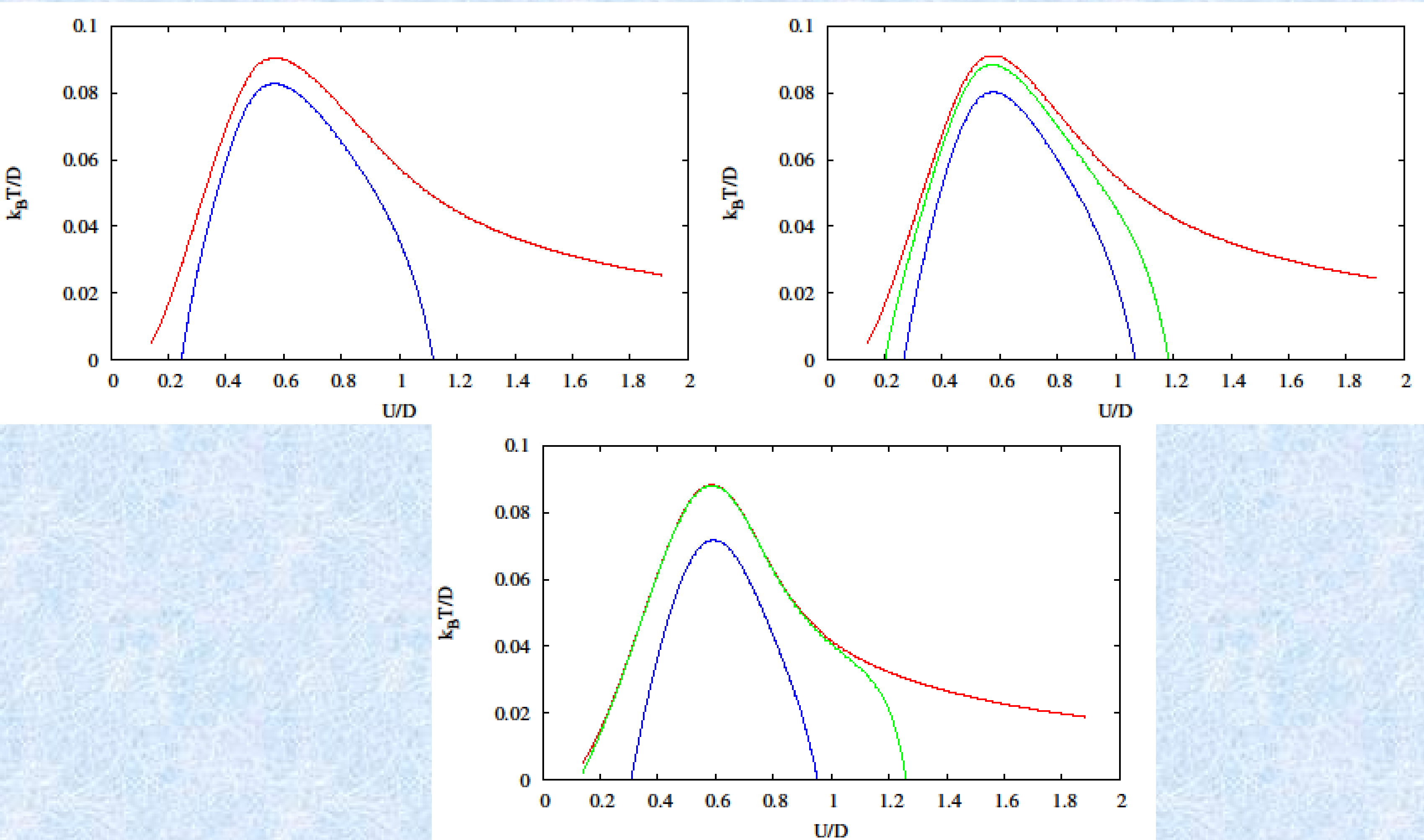
# Josephson couplings in the Lawrence-Doniach model (II)

At the unitary limit, $J_1$ and $J_2$ are enough: e.g.




$$n = 0.5; \quad t_\perp / t_\parallel = 0.6$$

# Crossover 3D/2D




Green (Blue): Temperature at which coeherence length becomes smaller than lattice spacing in the z (x,y) directions

# Experimental numbers...

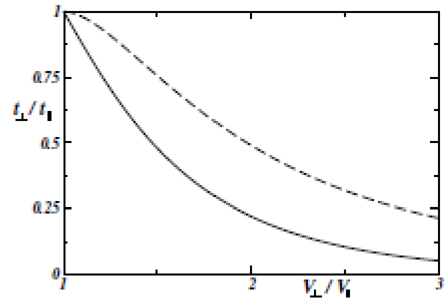



FIG. 1. Ratio of $t_\perp/t_\parallel$ vs. the ratio of the laser beam powers $V_\perp/V_\parallel$ for $V_\parallel = 5E_R$ (dashed line) and $V_\parallel = 7.5E_R$ (solid line).

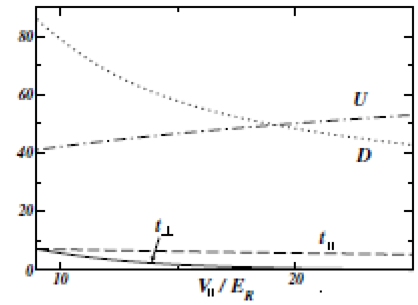



FIG. 2. Values in nanokelvins of the parameters $t_\perp$ (solid line), $t_\parallel$ (dashed) and $U$ (dot-dashed) as a function of $s_\perp = V_\perp/E_R$; the total bandwidth $D = 8t_\perp + 4t_\parallel$ (dotted) is reported as well. Estimates are for $^6$ Li atoms (the parameter $U$ is computed for $|a| = 300a_0$). The parameter $V_\parallel$ is fixed to be $V_\parallel = 9E_R$.

# Part II: Conclusions & Perspectives

- Anisotropic Ginzburg-Landau and Lawrence-Doniach models for layered ultracold fermions: parameters found from the microscopic Hamiltonian

- Response to (syntethic) magnetic field can be then studied as the electrodynamics of layered superconductors

- actual/future work:
  - time-dependent Ginzburg-Landau
  - study the crossover 2D/3D
  - study the Josephson at the unitary limit
  - extend the description at lower temperature (challenging)

Thank you!