Momentum-resolved spectroscopy of one-dimensional Bose gases

Nicole Fabbri

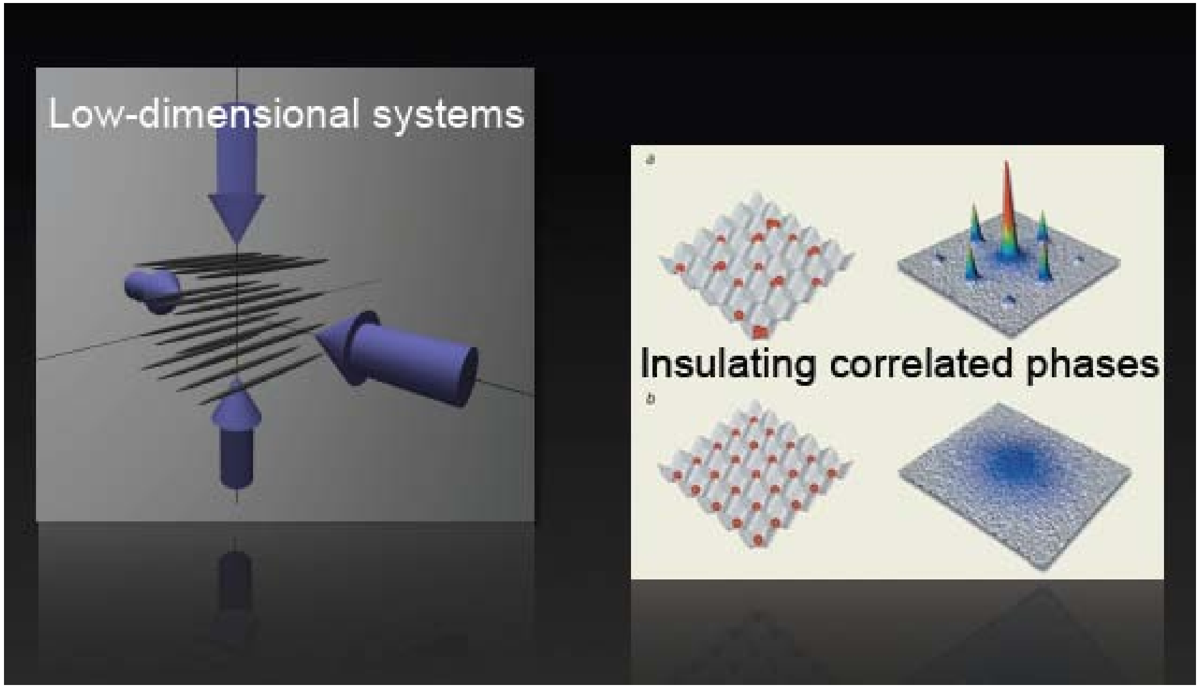
LENS - Università di Firenze and INO-CNR

GGI – Firenze, May 4 2012

Introduction

Atomic physics

Condensed matter



Outline

- Introduction:
  - some brief reminder of basic concepts about 1D physics
  - realizations of 1D systems in cold atom experiments

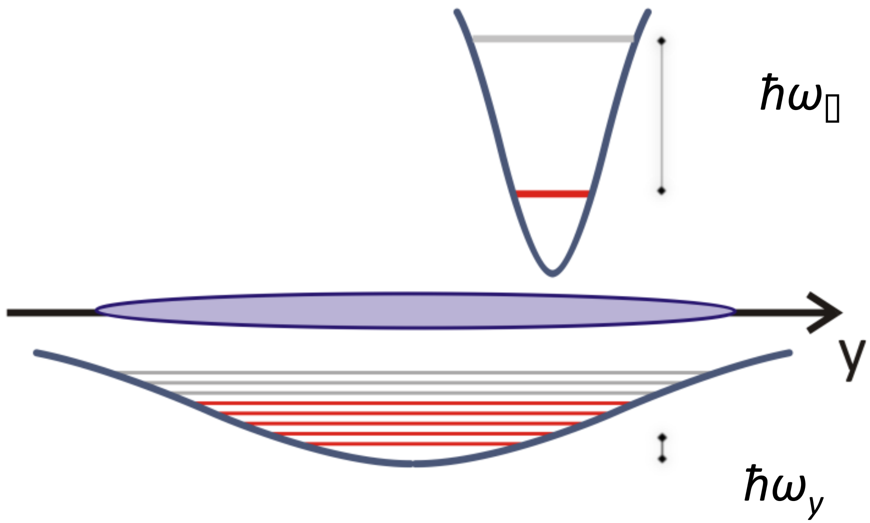
- Probing the coherence propeties of 1D atomic chains
  - inelastic light scattering (Bragg spectroscopy)
  - mapping momentum distribution via density distribution after free evolution

- Increasing interactions towards Tonks-Girardeau regime

One-dimensional systems

$$\hbar\omega_y \ll E_{int}, k_B T \ll \hbar\omega_\perp$$

The system occupies the transverse ground-state, which is degenerate since it includes several longitudinal modes.



Interacting particles in 1D: Tomonaga-Luttinger liquids

$$H = \frac{\hbar}{2\pi} \int dy \left(uK(\nabla\theta(y))^2 + \frac{u}{K}(\nabla\Phi(y))^2 \right)$$

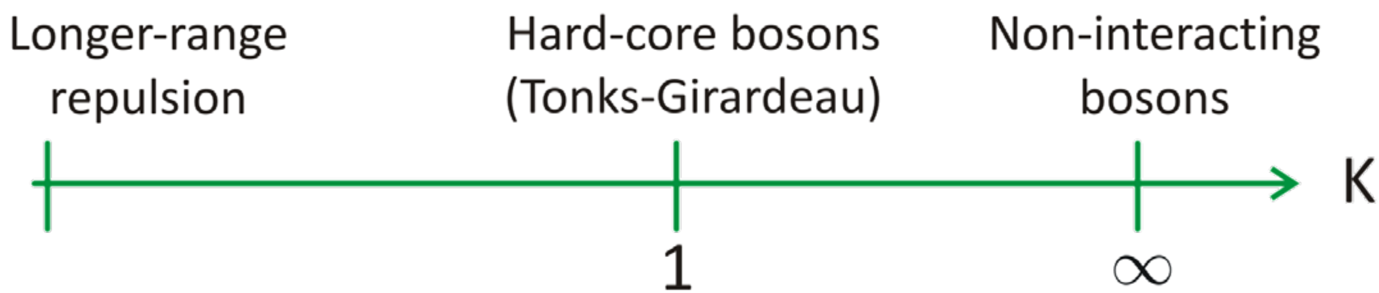
Single-particle creation operator $\psi^\dagger(y) \simeq \sqrt{\left(\rho - \frac{1}{\pi}\nabla\Phi(y)\right)} e^{i\theta(y)}$

- Low-energy effective description
- *u*, *K*: universal Luttinger parameters

S. Tomonaga, *Prog. Theor. Phys.* 5, 544 (1950)

Interacting particles in 1D: Tomonaga-Luttinger liquids

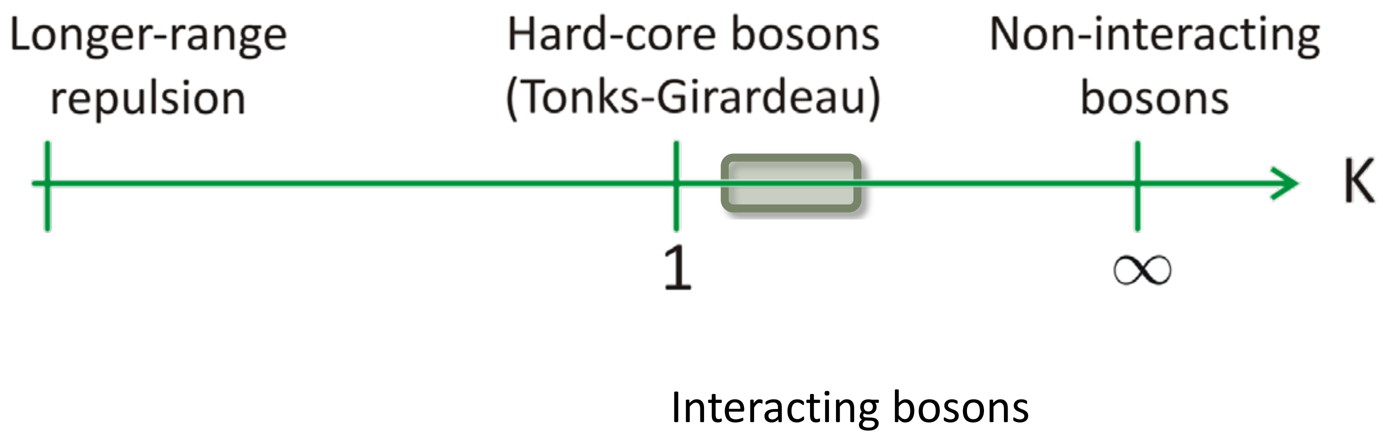
$$H = \frac{\hbar}{2\pi} \int dy \left(uK(\nabla\theta(y))^2 + \frac{u}{K}(\nabla\Phi(y))^2 \right)$$



gas of impenetrable bosons
|ψ| equivalent to a system of
spinless fermions
M. Girardeau, J. Math. Phys. (1960)

Interacting particles in 1D: Tomonaga-Luttinger liquids

$$H = \frac{\hbar}{2\pi} \int dy \left(uK(\nabla\theta(y))^2 + \frac{u}{K}(\nabla\Phi(y))^2 \right)$$



In our experiment: K ≈ 3-6

Advantage of ultra-cold atoms: Relate the Luttinger parameters with the microscopic properties

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial y_i^2} + g_{1D}\sum_{i<j=1}^{N}\delta(y_i - y_j)$$

E.H. Lieb e W. Liniger, PR 130 (4), 1963

many-body problem of bosons in 1D with pairwise interactions

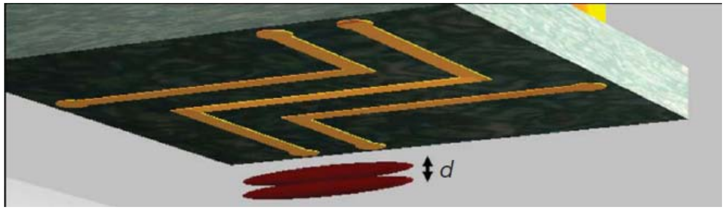
- T=0
- uniform
- contact interactions

$$\gamma = \frac{E_{int}}{E_{kin}} = \frac{m g_{1D}}{\hbar^2 \rho}$$

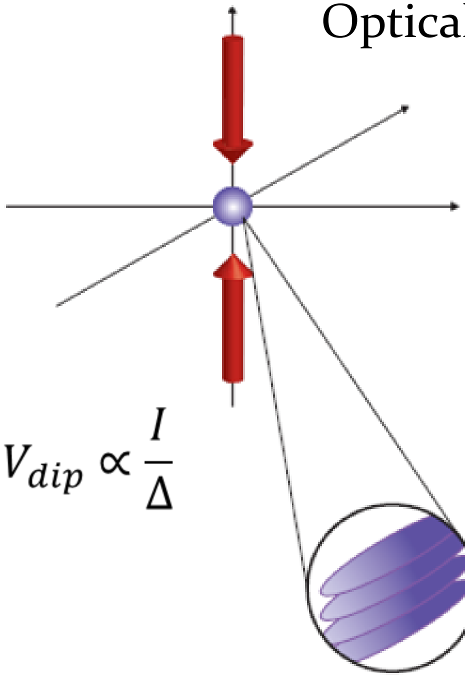
In 1D interactions dominate in the low density limit!

Realizations of 1D systems in cold atoms

Elongated magnetic traps



Optical lattices



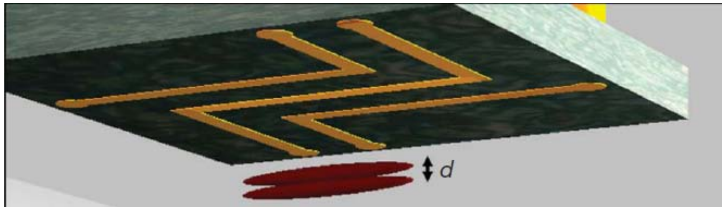
$$V_{dip} \propto \frac{I}{\Delta}$$

Modulation the laser intensity given by interference of a pair of counterpropagating off-resonant laser beams:

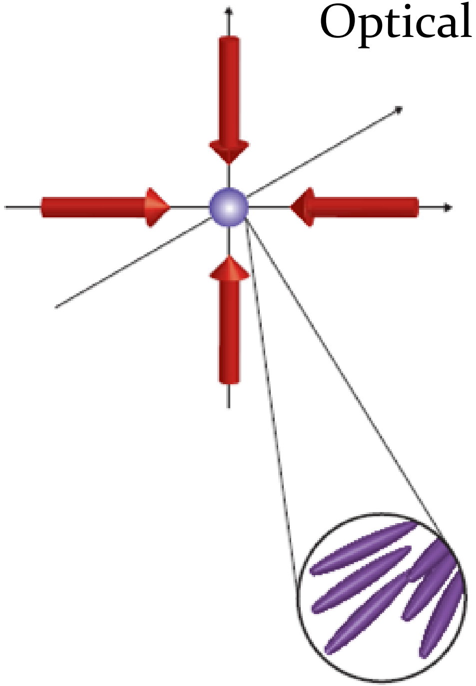
$$V_{dip}(\vec{r}) \propto s_{\perp} E_R \, sin^2(kx)$$

Realizations of 1D systems in cold atoms

Elongated magnetic traps



Optical lattices



2D lattice

$$V_{dip} \propto s_{\perp} E_R \left(sin^2(kx) + sin^2(kz)\right)$$

Our system

Bose-Einstein condensate:

^{87}Rb

$N_{BEC} \sim 10^5$

$T_C \cong 125\, nK$

$\mu/h \approx 600\, Hz$

$R_\perp \cong 4\, \mu m$

$R_y \cong 40\, \mu m$

$\omega_\perp / 2\pi \cong 90$ Hz

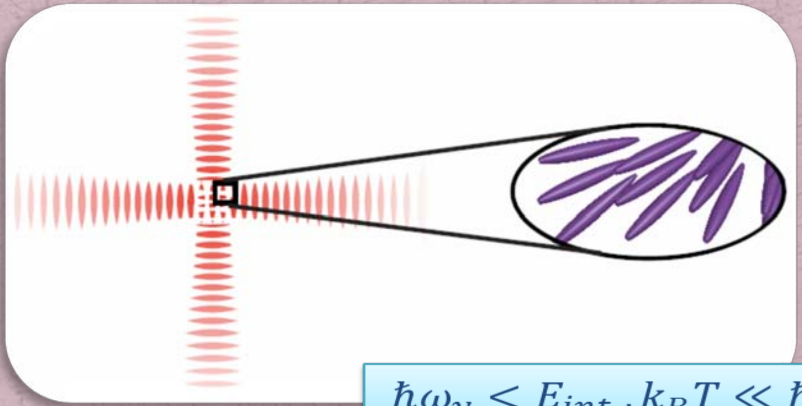
$\omega_y / 2\pi \cong 9$ Hz

Our system

Array of 1D gases:

$\lambda_L = 830.3\ nm$

Number of 1D gases $N \approx 1500$



$\hbar\omega_y < E_{int}, k_B T \ll \hbar\omega_\perp.$

$$V_{dip} \propto s_\perp E_R\,(sin^2(kx) + sin^2(kz))$$

$s_\perp = 40$:

$\omega_\perp/2\pi \approx 50\ kHz$

$\omega_y/2\pi \approx 50\ Hz$

$\mu_{1D}/h \approx 1\ kHz$

$k_B T/h \approx 2\ kHz$ $(T \sim 100\ nK)$

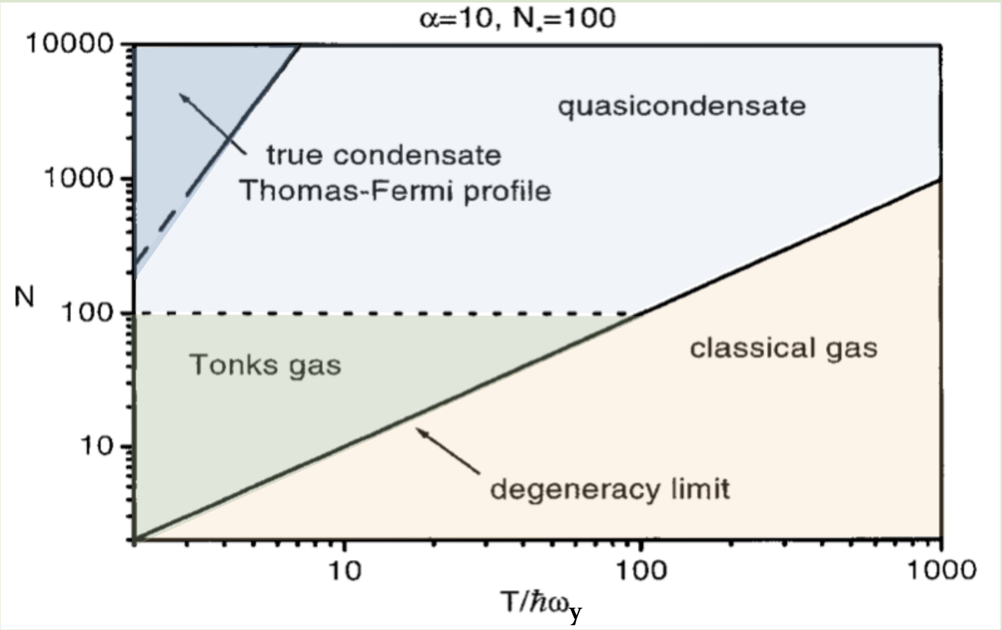
Typical size: $L_{//} \cong 30\ \mu m \times 50\ nm$

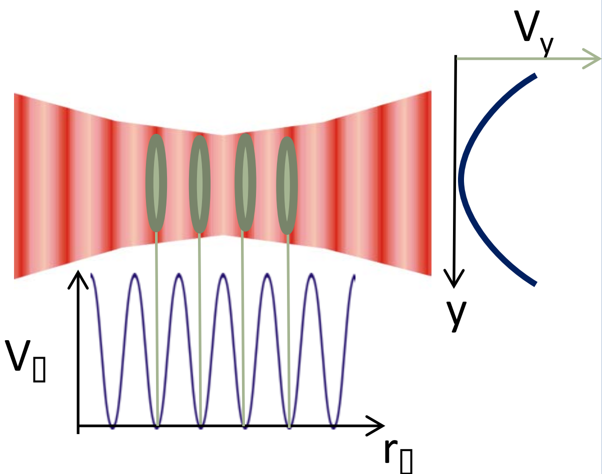
Phase diagram for trapped 1D interacting (repulsive) bosons

$$g_{1D} = 2\hbar\omega_\perp a$$

M. Olshanii PRL 81, 938 (1998)

$$\alpha = \frac{m g_{1D} a_y}{\hbar^2}$$





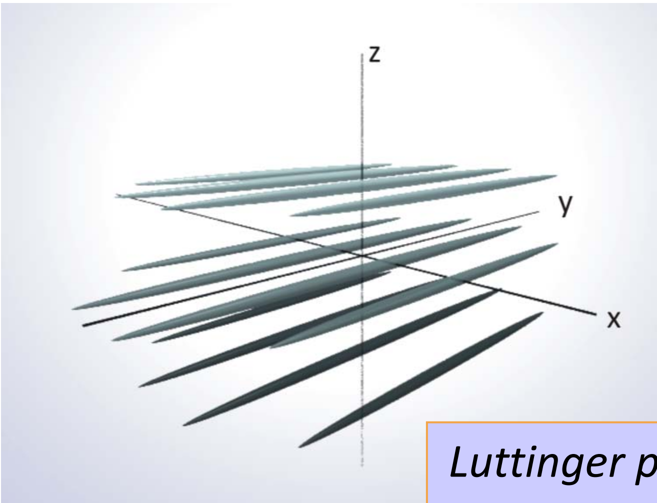
Quasicondensate: density fluctuations are suppressed but the phase still fluctuates

True condensate: phase fluctuations are suppressed due to the finite size of the system

Tonks-Girardeau gas: strongly interacting bosonic system map to non interacting fermionic system (interactions mimic Pauli exclusion principle)

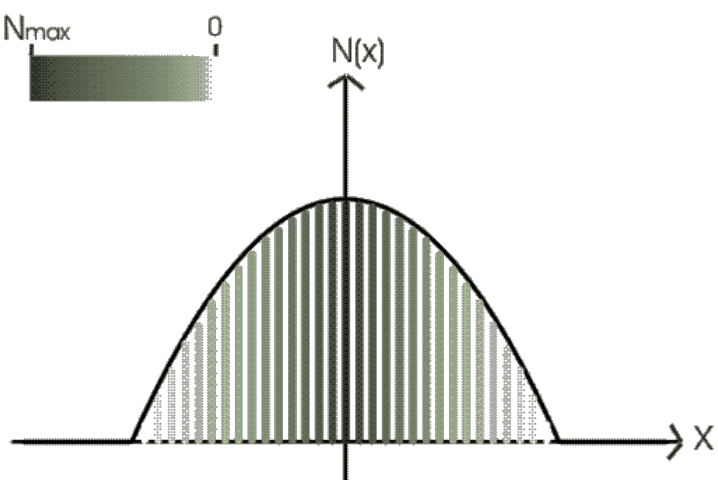
D. S. Petrov, G. V. Shlyapnikov and J. T. M. Walraven, Phys. Rev. Lett. 84, 3745 (2000).

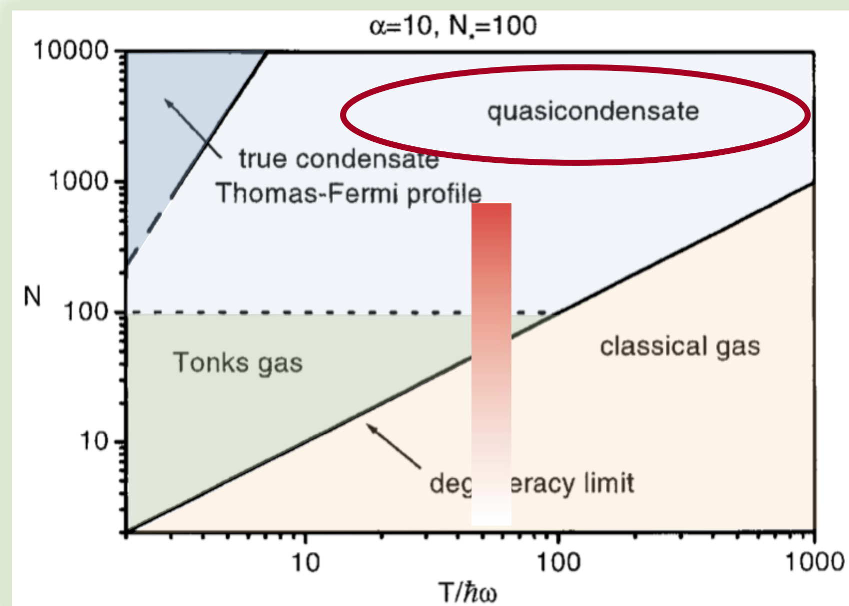
trapped 1D interacting (repulsive) bosons in a longitudinal harmonic trap



- interactions are beyond a mean-field picture but not too strong (Tonks *K*=1)
- finite temperature effects: activating 1D-excited modes

Luttinger parameter
K = 3 – 6
(γ = 0.2 - 0.8)





Investigating 1D systems in cold atoms

Some measurements realized in 1D gases:

- interference between two different 1D gases
Schmiedmayer (Nat. Phys. 2005)
- density modulations after ballistic expansion
Schmiedmayer (PRL 2001,PRA 2009)
- in situ measurement of density
van Druten (PRL 2008), Bouchoule (PRL 2006,2010,2011)
- three-Body recombination Porto, PRL 2004
- collective modes Esslinger, Phillips (Journal de Physique,2003) Naegerl (Science 2009)

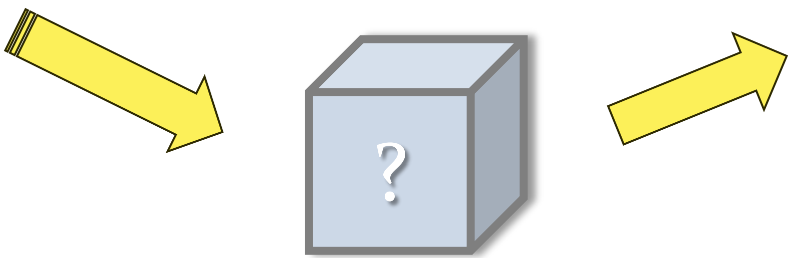
…

Inelastic light scattering

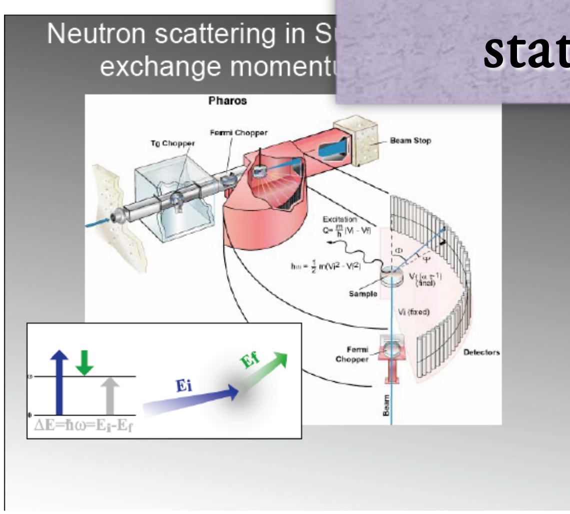
Inelastic scattering of waves or particles has been widely used to gain information on the structure of matter

ω, k

ω’, k’



Excitation δω, δk



Linear response of the many-body state to an external perturbation

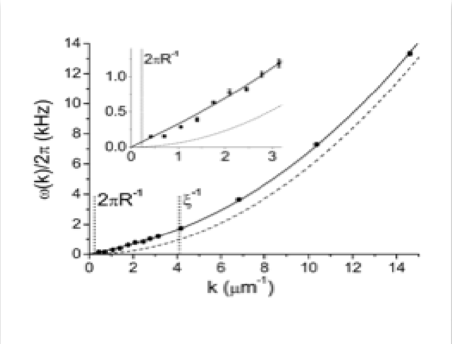
Create an excitation within the many-body ground state

Access the two-bodies correlation function

FT: Dynamical structure factor $\boldsymbol{S(q,\omega)}$

Bragg spectroscopy on quantum gases

First experiments:
M. Kozuma et al. PRL 82 871 (1999)
J. Stenger et al. PRL 82, 4569 (1999)

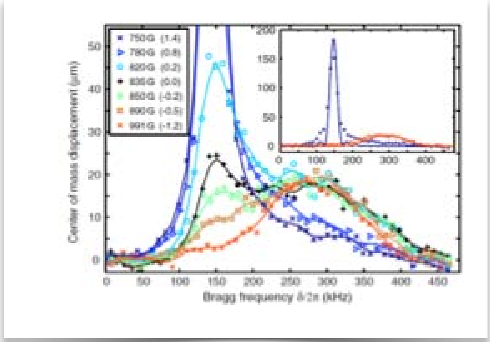


Weakly interacting 3D BEC

Excitation spectrum:
J. Stenger et al., PRL (1999)
J. Steinhauer et al., PRL (2002)
Phase fluctuations:
S. Richard et al., PRL (2003)
D. Helweg et al., PRL (2003)

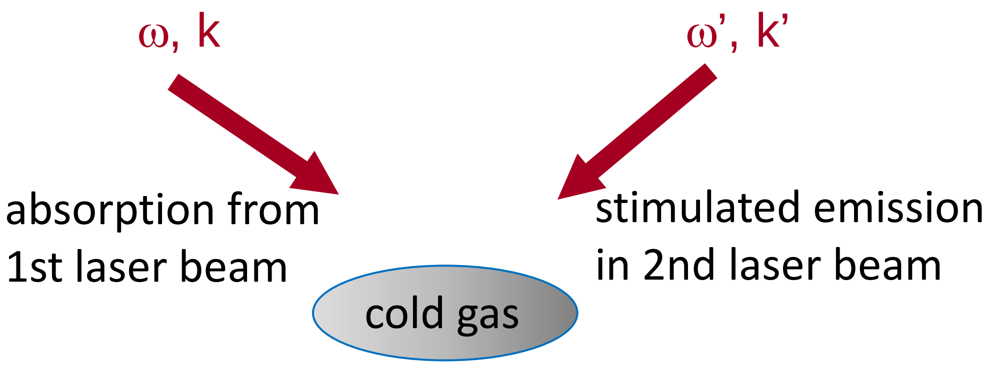
Strongly interacting gases close to Feshbach resonances

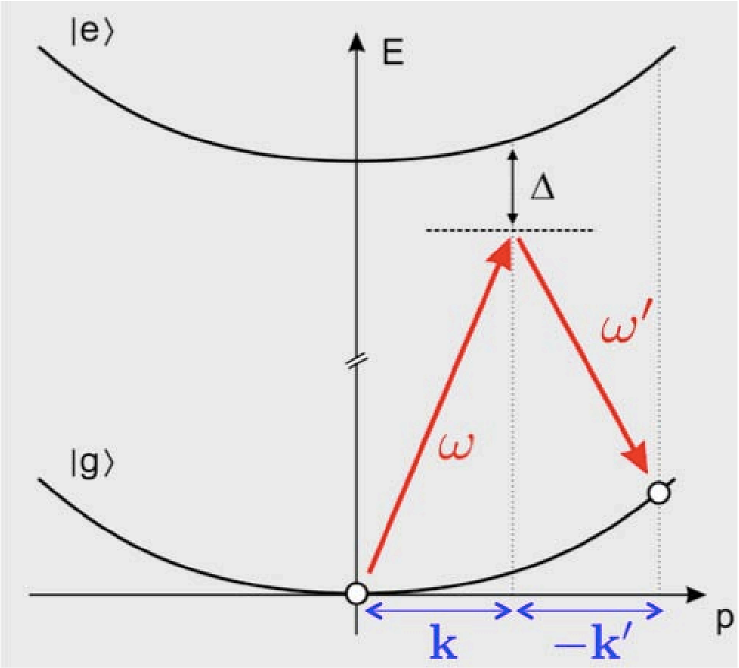
BEC of Rb-85
S.B. Papp et al., PRL (2008)
BCS-BEC crossover with Li-6:
G. Veeravalli et al. PRL (2008)



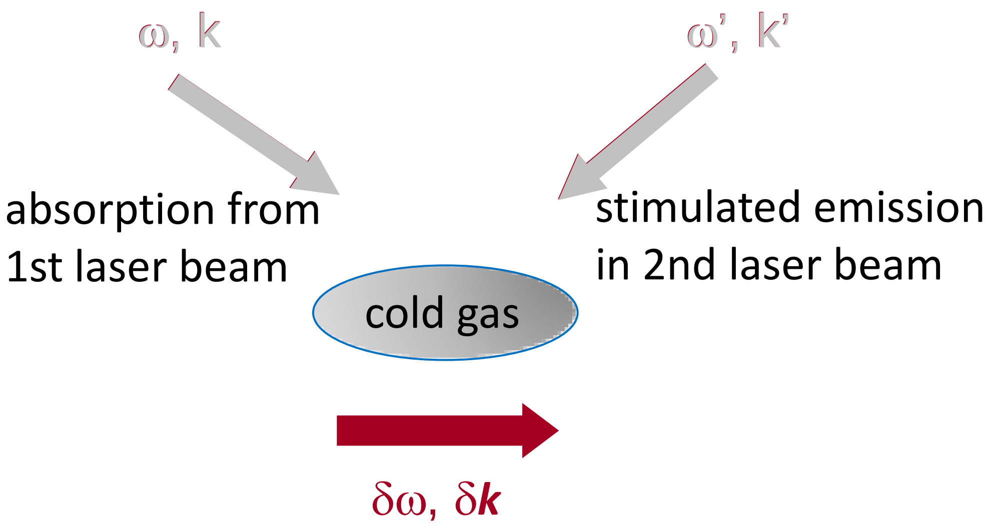
In periodic potentials:
Weakly interacting BECs in 3D OL:
Du et al., New J. Phys. 12, 083025 (2010)
Ernst et al. Nature Physics 5, 1 (2009)
Fabbri et al., PRA 79, 043623 (2009)
SF-MI Transition:
Clément et al., PRL (2009), New J. Phys (2009)

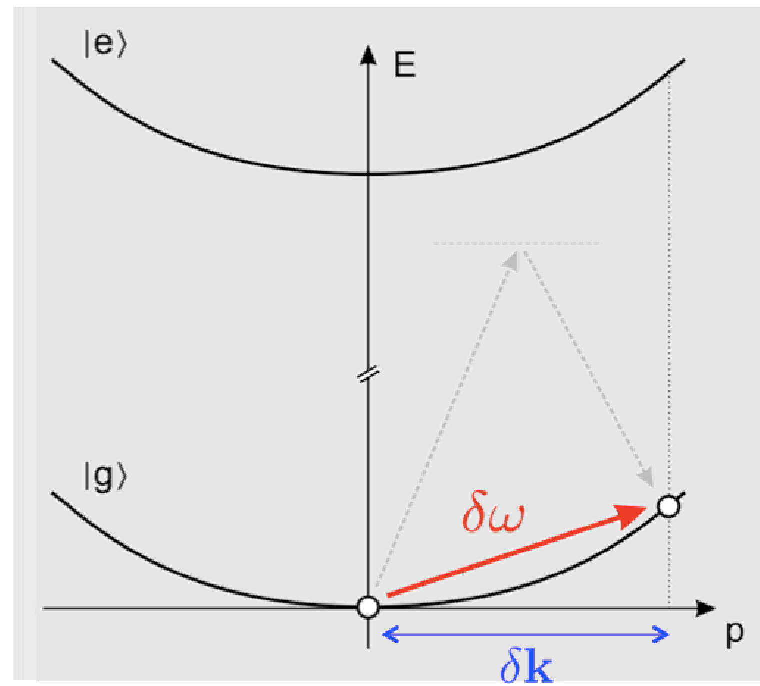
Scattering of light on ultracold atoms





Scattering of light on ultracold atoms



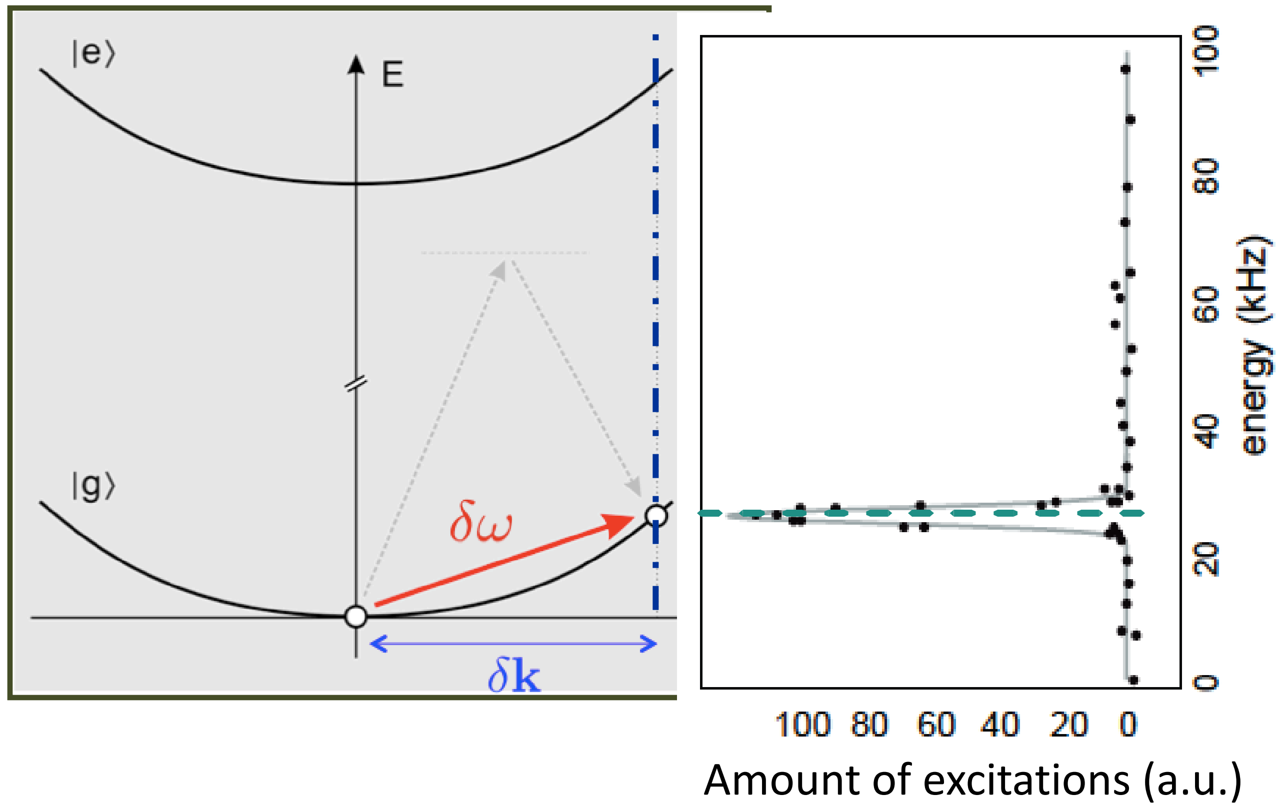


Energy of the excitation $\delta\omega = \omega - \omega'$

Momentum of the excitation $\delta \boldsymbol{k} = \boldsymbol{k} - \boldsymbol{k}'$

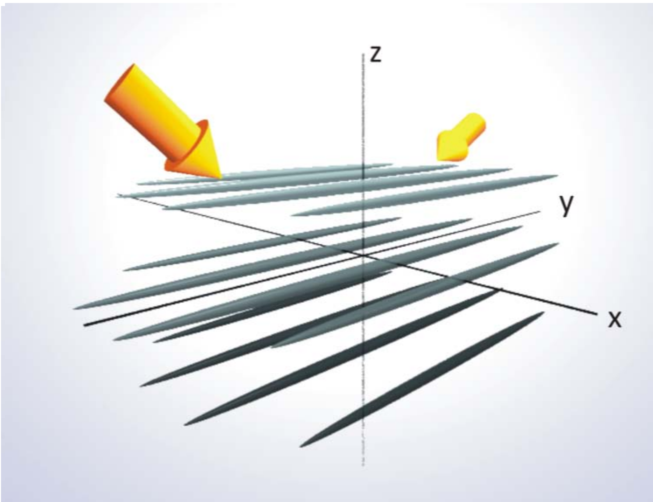
Scattering of light on ultracold atoms

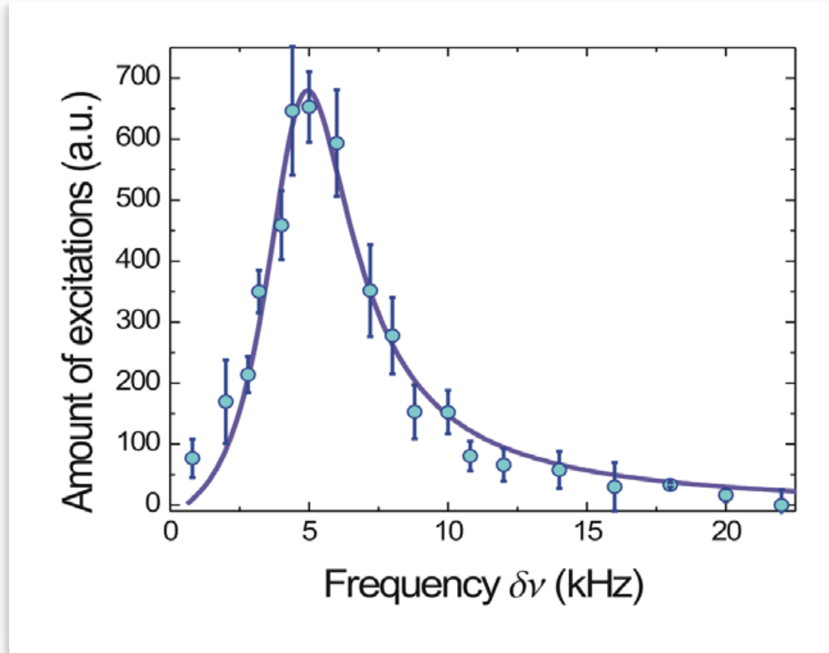
In practice, we choose δk and we scan $\delta\omega$

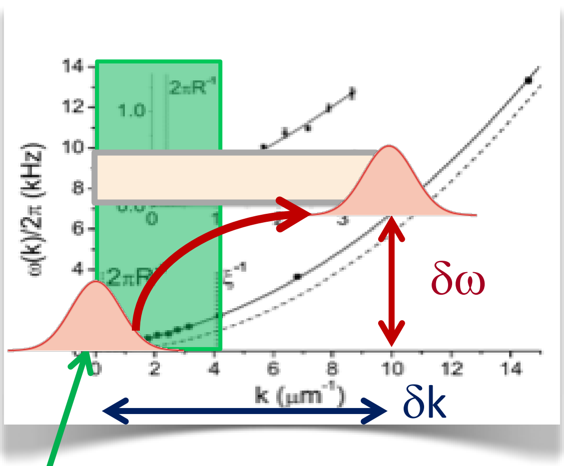


The energy transferred to the system is proportional to ω S(δk, $\delta\omega$)

Measuring momentum distribution via Bragg spectroscopy







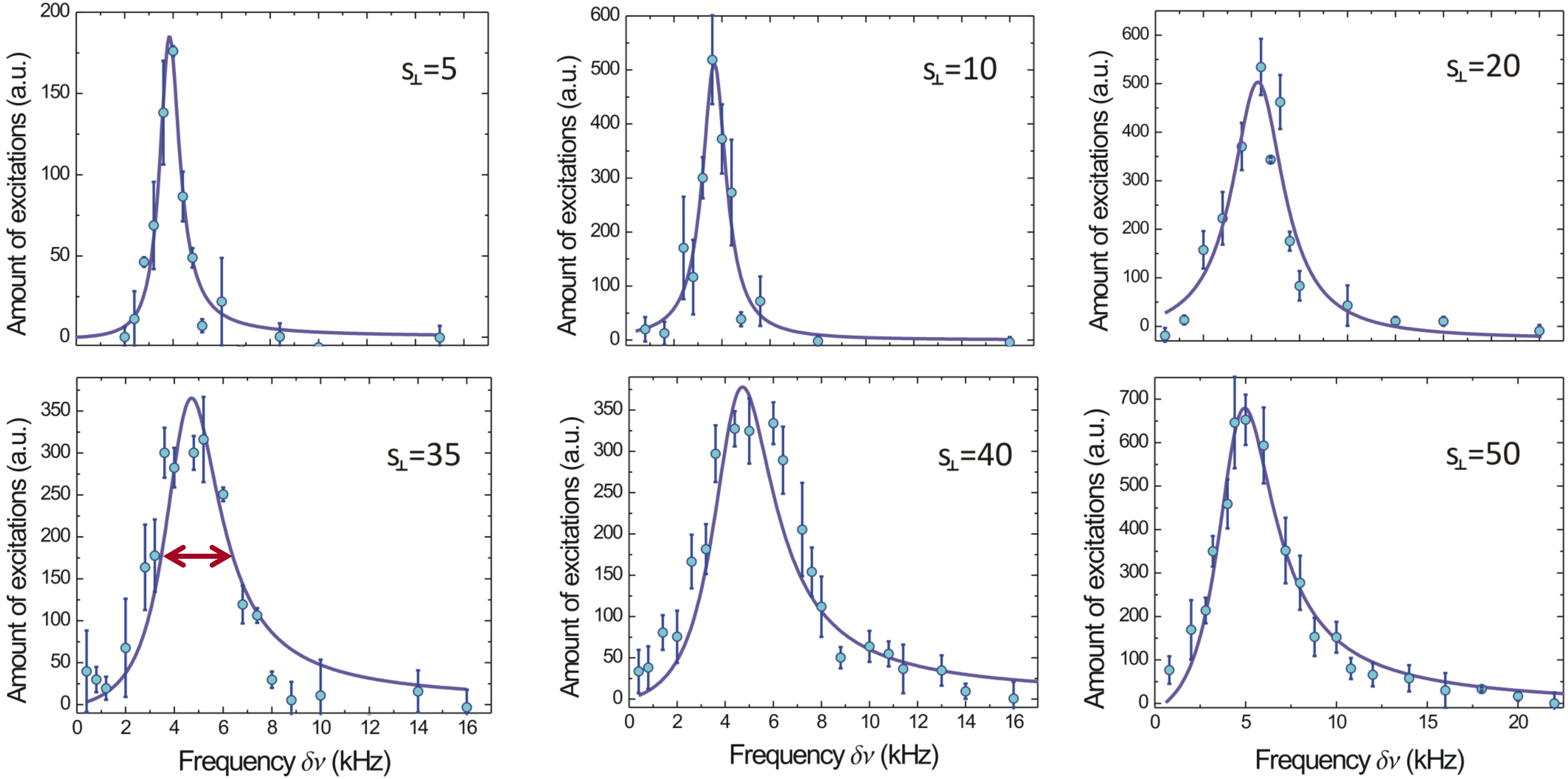
phononic collective regime

Bragg condition for energy and momentum conservation:

$$\hbar\delta\omega = 4E_{Rec}^{(B)} + \frac{\hbar^2 \delta k}{m} v_{at}$$

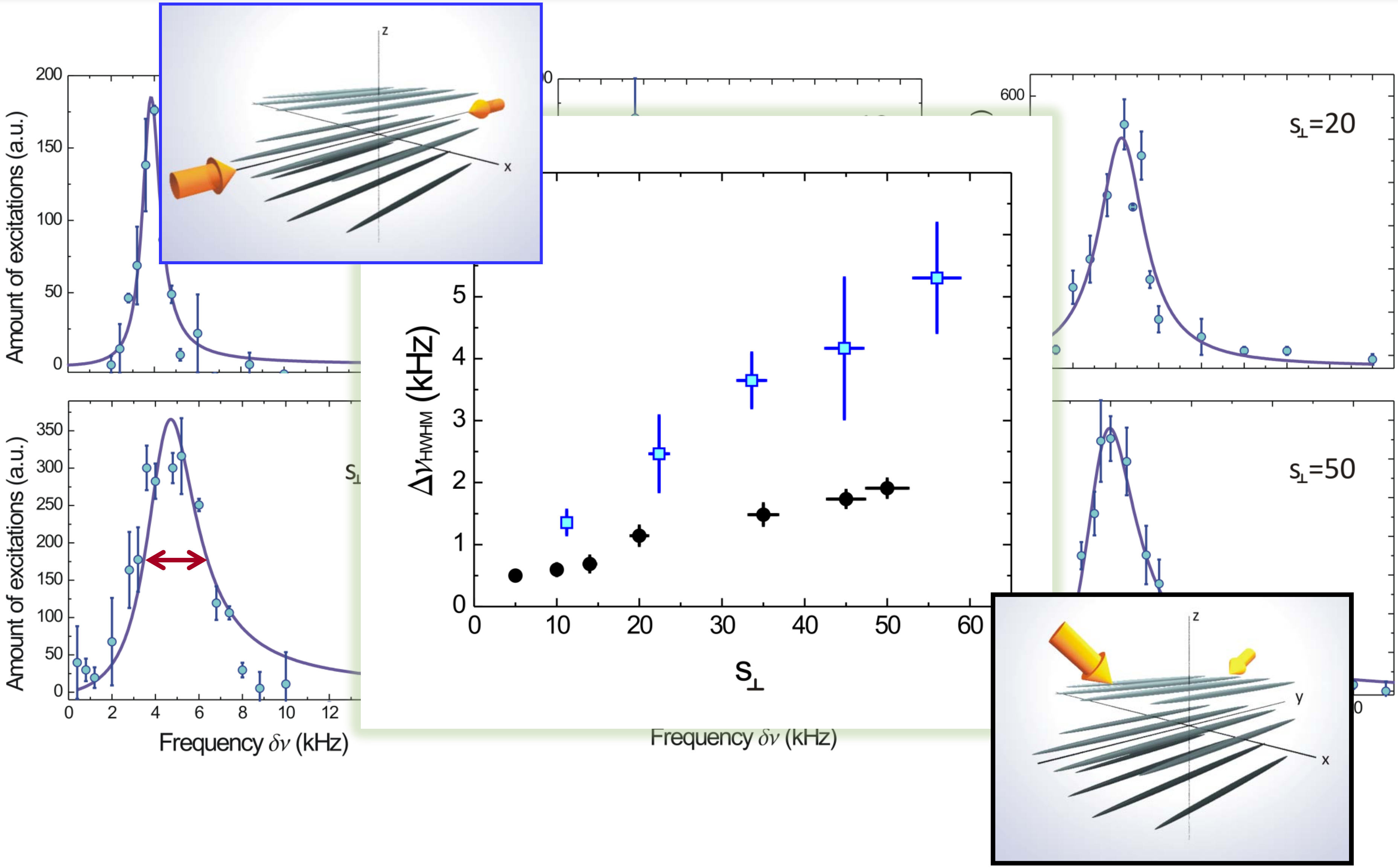
Doppler regime:
Bragg sensitive to the initial momentum distribution

Characterizing the array of 1D gases



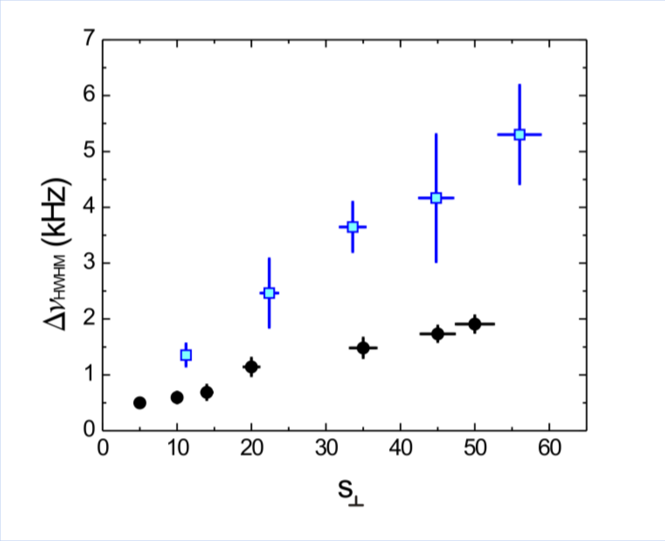
N. Fabbri, D. Clément, L. Fallani, C. Fort, M. Inguscio, Physical Review A 83, 031604(R) (2011)

Characterizing the array of 1D gases



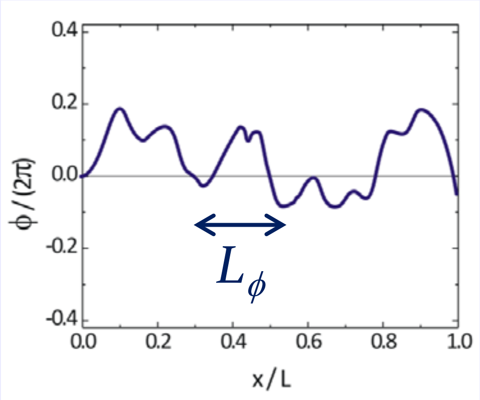
N. Fabbri, D. Clément, L. Fallani, C. Fort, M. Inguscio, Physical Review A 83, 031604(R) (2011)

Momentum distribution of 1D gases



Lorentzian shape:
width related to the inverse of the coherence length L_ϕ

$$L_\phi = \frac{\hbar^2 \rho}{m k_B T}$$



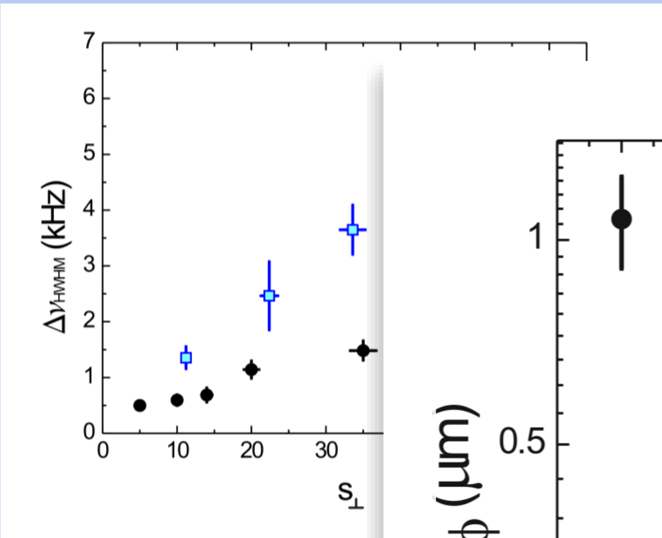
Momentum distribution of (not too strongly) interacting 1D gases at finite T:

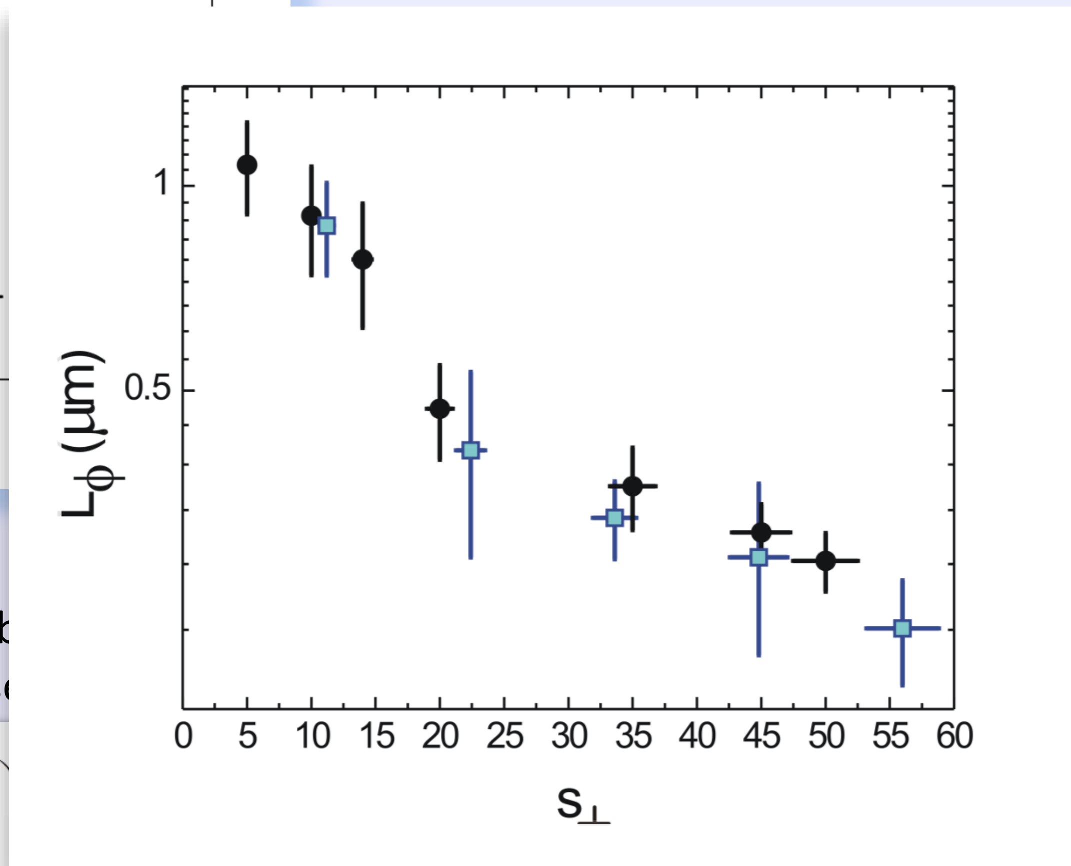
$$n(p,T) \approx 4\left(\frac{K^2}{\pi}\right)^{\frac{1}{2K}} \left(\rho L_\phi(T)\right)^{1-1/2K} \frac{1}{1+\left(2pL_\phi(T)\right)^2}$$

Cazalilla, New Journal of Physics (2006)

N. Fabbri, D. Clément, L. Fallani, C. Fort, M. Inguscio, Physical Review A 83, 031604(R) (2011)

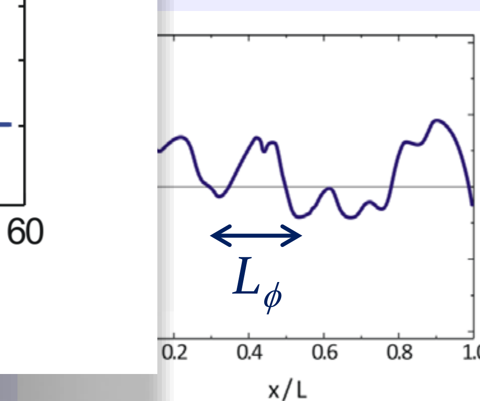
Coherence length of the 1D gases





ntzian shape:
ated to the inverse
herence length L_ϕ

$$= \frac{\hbar^2 \rho}{m k_B T}$$



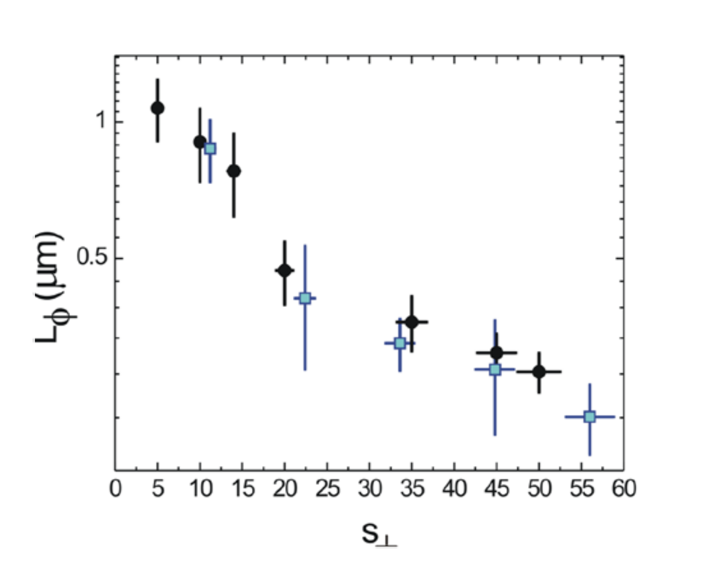
Momentum distrib
interacting 1D gas

$$n(p,T) \approx 4 \left(\frac{K^2}{\pi} \right.$$

$$1+(2pL_\phi(T))$$

N. Fabbri, D. Clément, L. Fallani, C. Fort, M. Inguscio, Physical Review A 83, 031604(R) (2011)

Coherence length of the 1D gases



$$L_\phi = \frac{\hbar^2 n}{m k_B T}$$

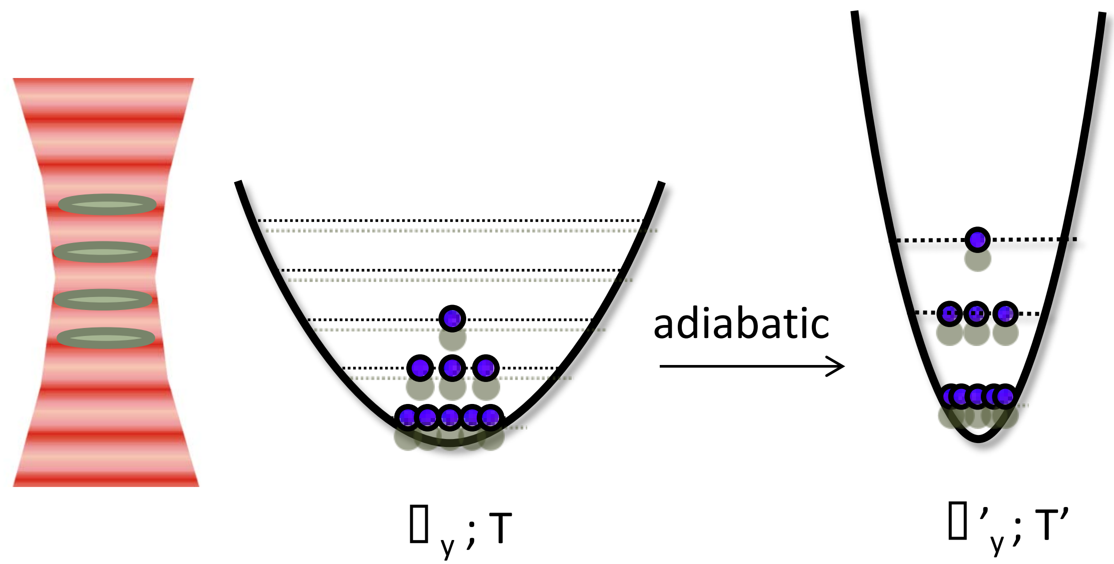
increasing the transverse confinement:

- anisotropy increases by ≈10%
- linear density decreases by ≈ 15%
- **T increases due to axial compression** (harmonic confinement along the tubes due to the radial effects of the optical lattice beams)

$$k_B T \gg \hbar\omega_y$$

$$T \sim \sum_j n_j \epsilon_j$$

$$\epsilon_j = \hbar\omega_y \sqrt{\frac{j(j+3)}{4}}$$



Measuring momentum distribution via TOF imaging

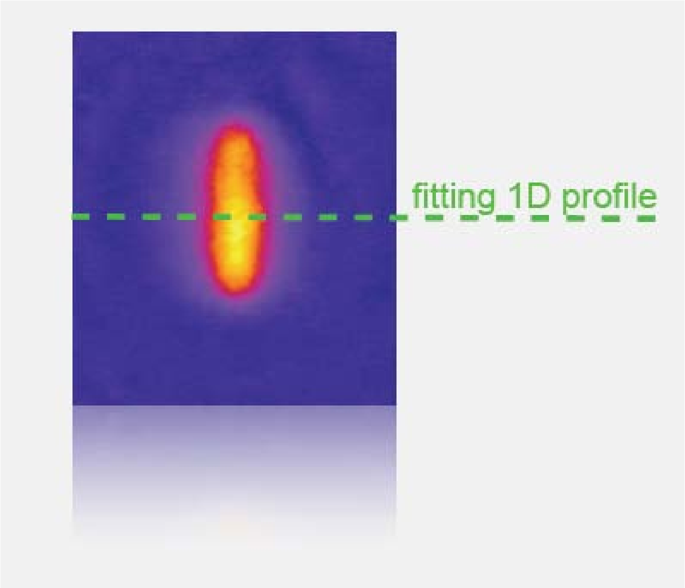
When the trap is switched off, the spatial density distribution of the atomic cloud after time-of-flight (TOF) reflects the *in-trap momentum distribution*

The expansion of the gas is governed by two kinds of kinetic energy:

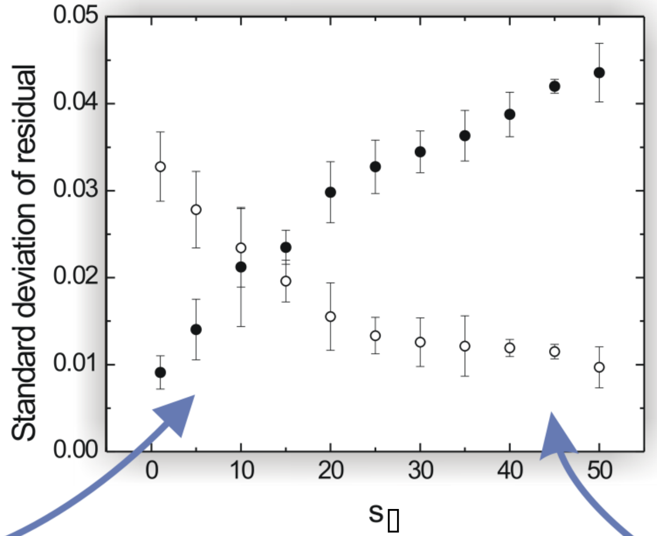
- interactions converts into kinetic energy
- local phase gradients produce a velocity field $\mathbf{v}_\phi = (\hbar/m)\nabla\phi$

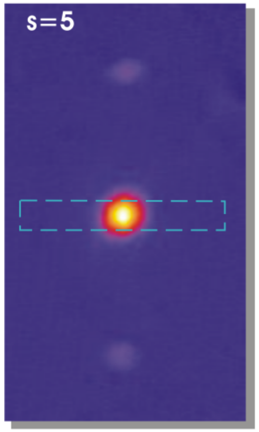
When strong enough, initial phase fluctuations dominate longitudinal dynamics during TOF

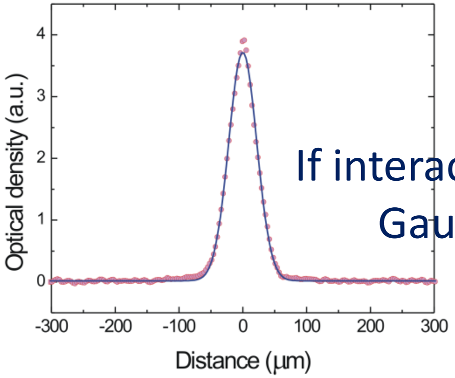
$$\frac{R^{\phi}_{\mathrm{TOF}}}{R^{\mathrm{int}}_{\mathrm{TOF}}} \sim \frac{\hbar t_{\mathrm{TOF}}}{m L_\phi R^{\mathrm{int}}_{\mathrm{TOF}}} > 1$$



Measuring momentum distribution via TOF imaging

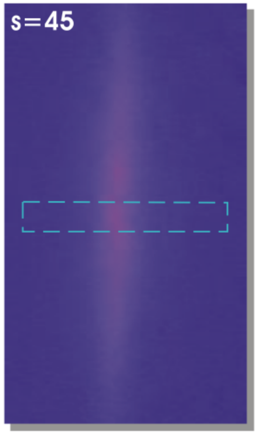


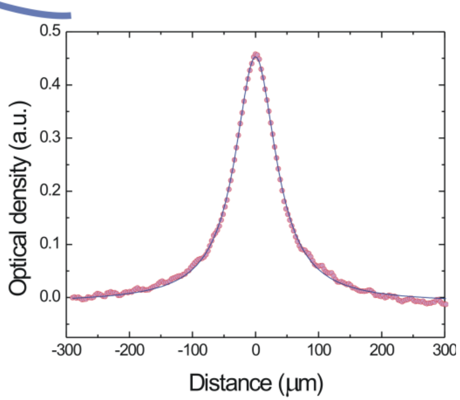




If interactions dominate:
Gaussian profile

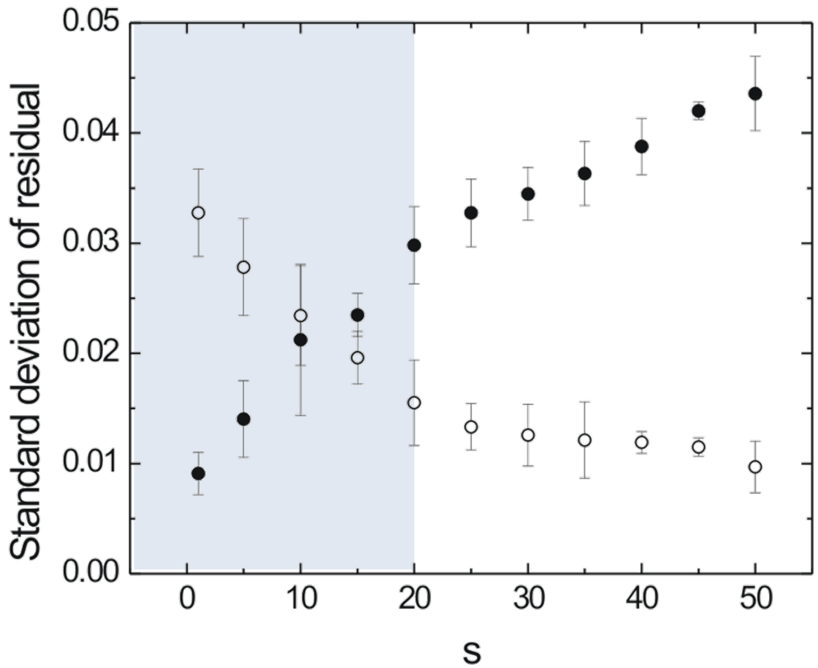
Phase fluctuations:
Lorentzian profile

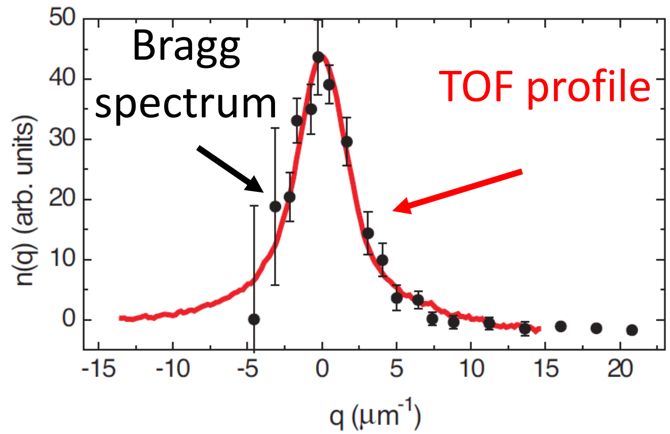


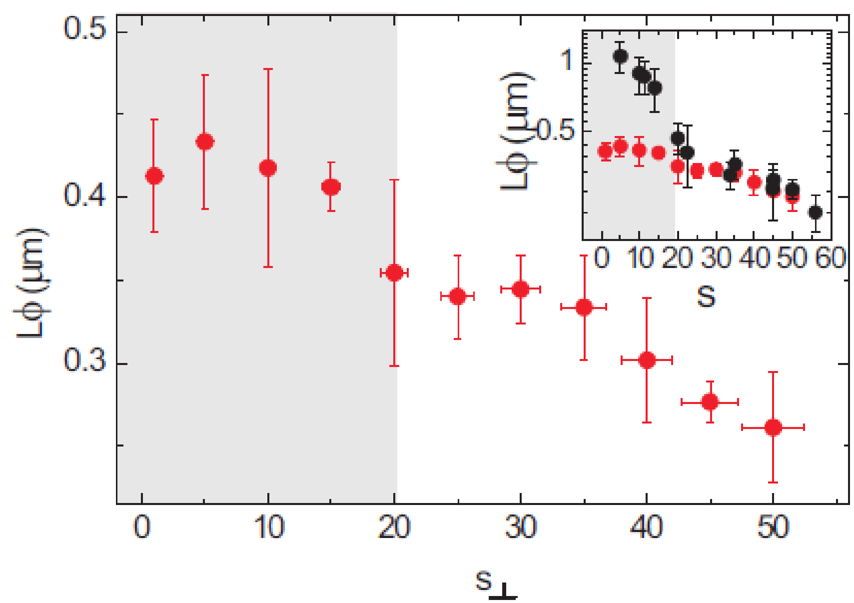


N. Fabbri, D. Clément, L. Fallani, C. Fort, M. Inguscio, Physical Review A 83, 031604(R) (2011)

Measuring momentum distribution via TOF imaging

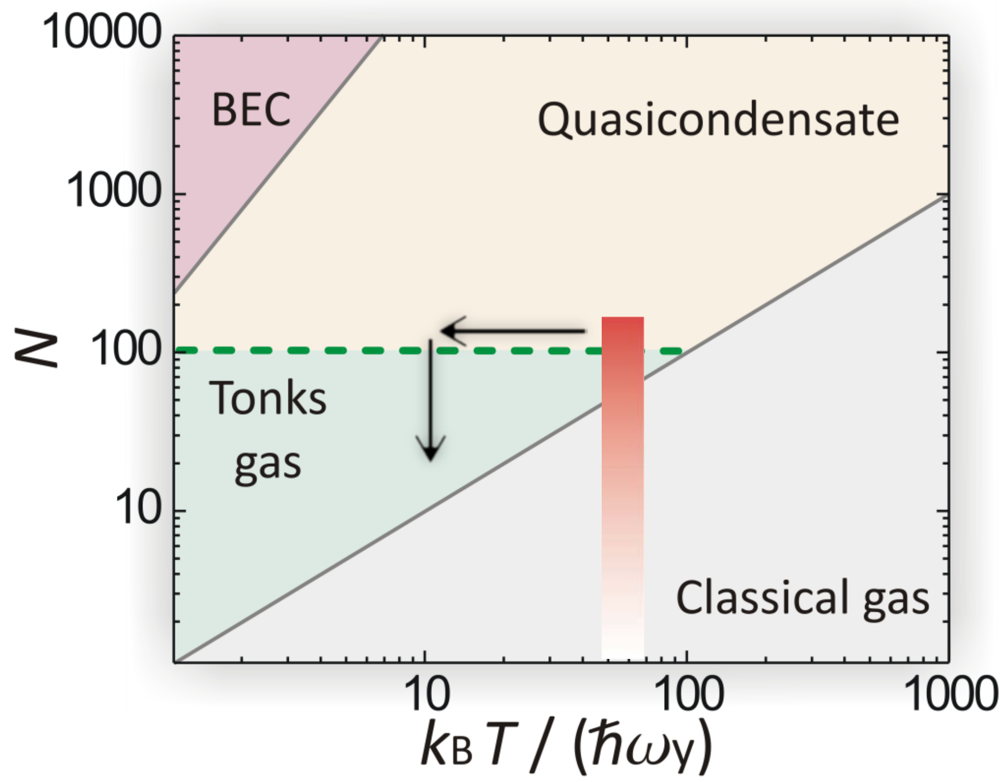






Direct imaging after TOF gives the temperature-dominated momentum distribution !

Towards strongly interacting regime

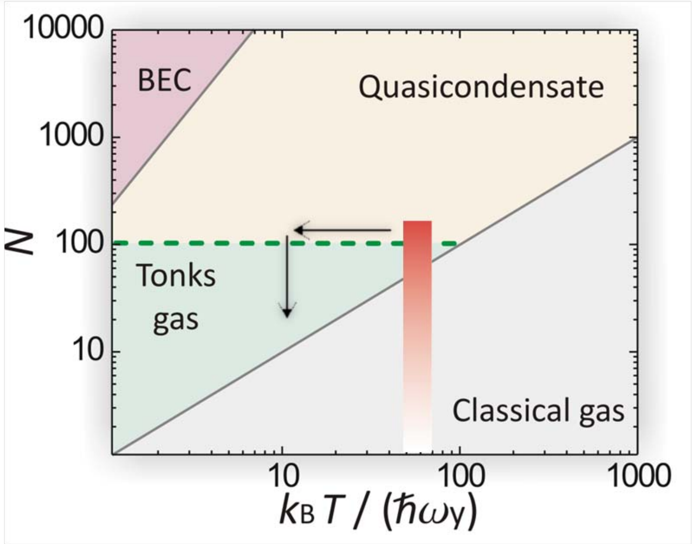


$$\gamma = \frac{m g_{1D}}{\hbar^2 n}$$

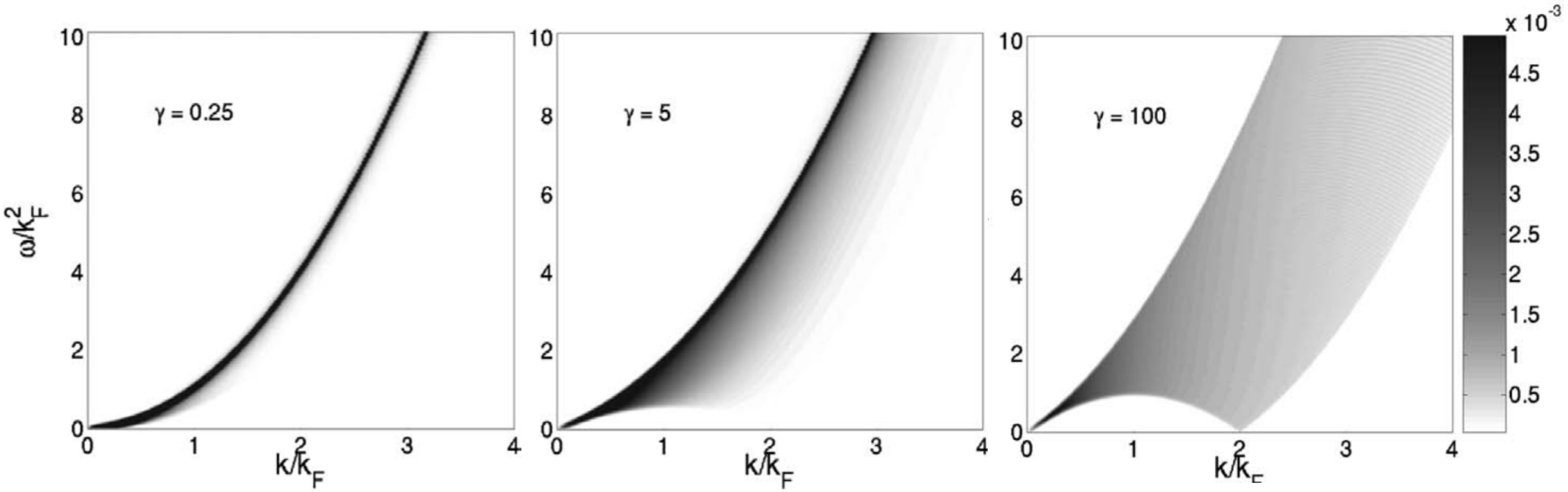
Strategy to approach the Tonks regime

- Increase γ by reducing density
- Reduce temperature

Dynamic structure factor of the one-dimensional Bose gas near the Tonks-Girardeau limit

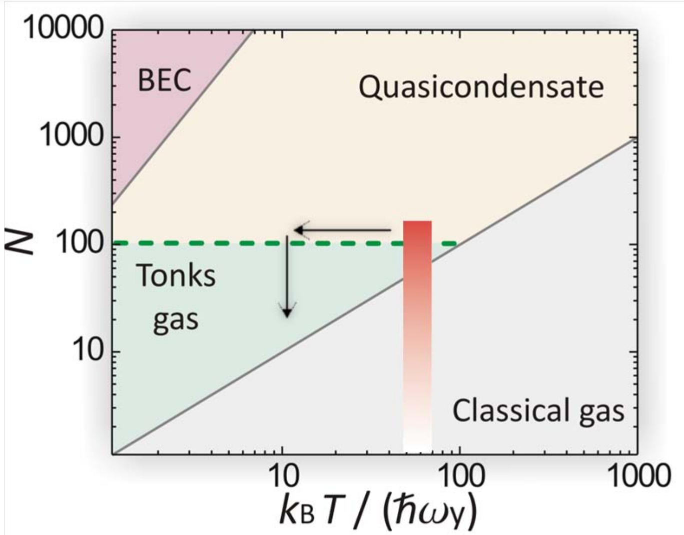


$$\gamma = \frac{m g_{1D}}{\hbar^2 n}$$

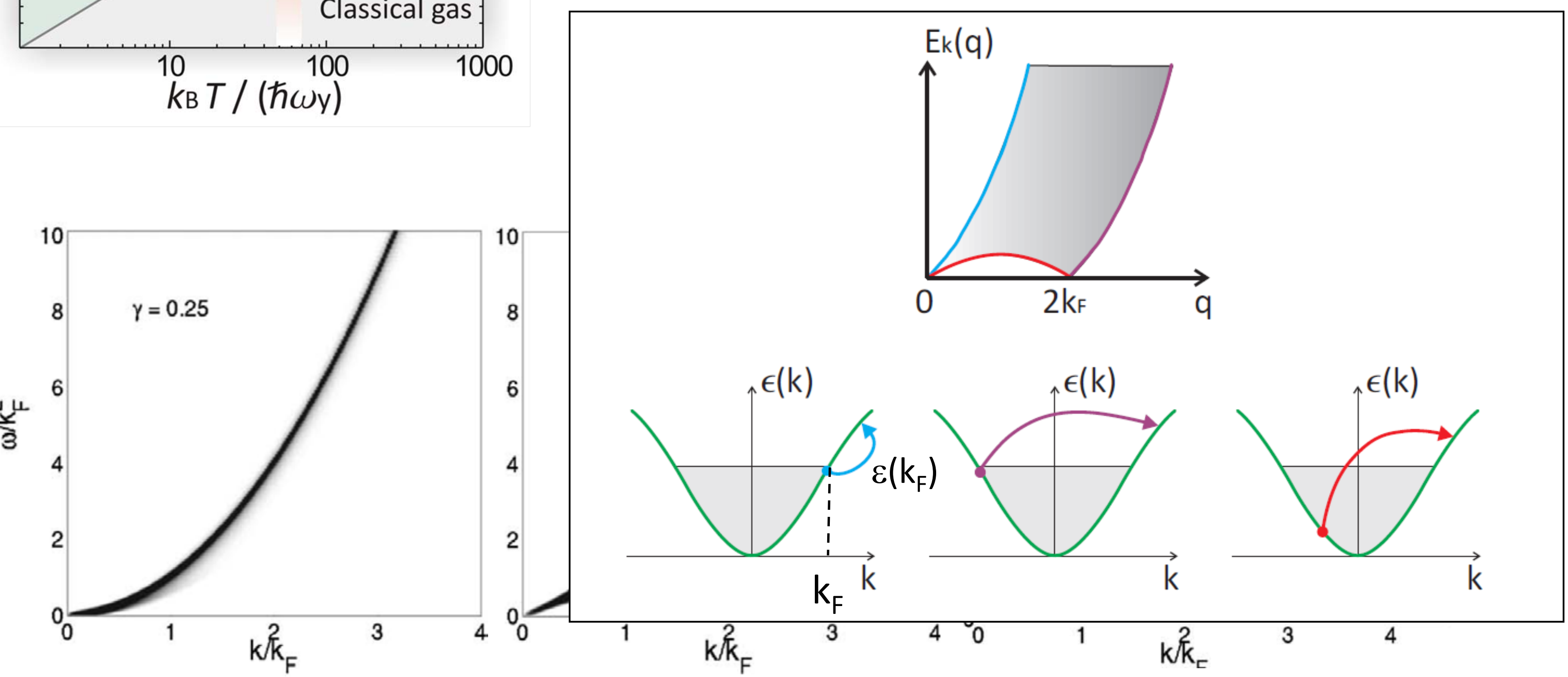


J.S. Caux and P. Calabrese PRA 74, 031605R (2006)

Dynamic structure factor of the one-dimensional Bose gas near the Tonks-Girardeau limit

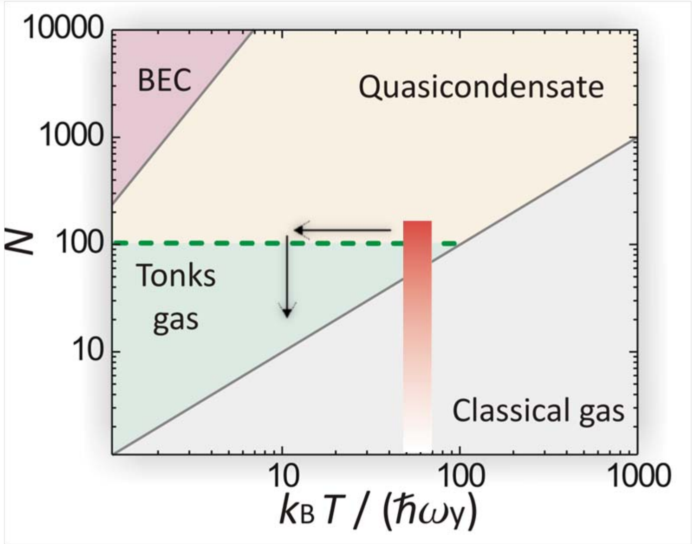


$$\gamma = \frac{m g_{1D}}{\hbar^2 n}$$

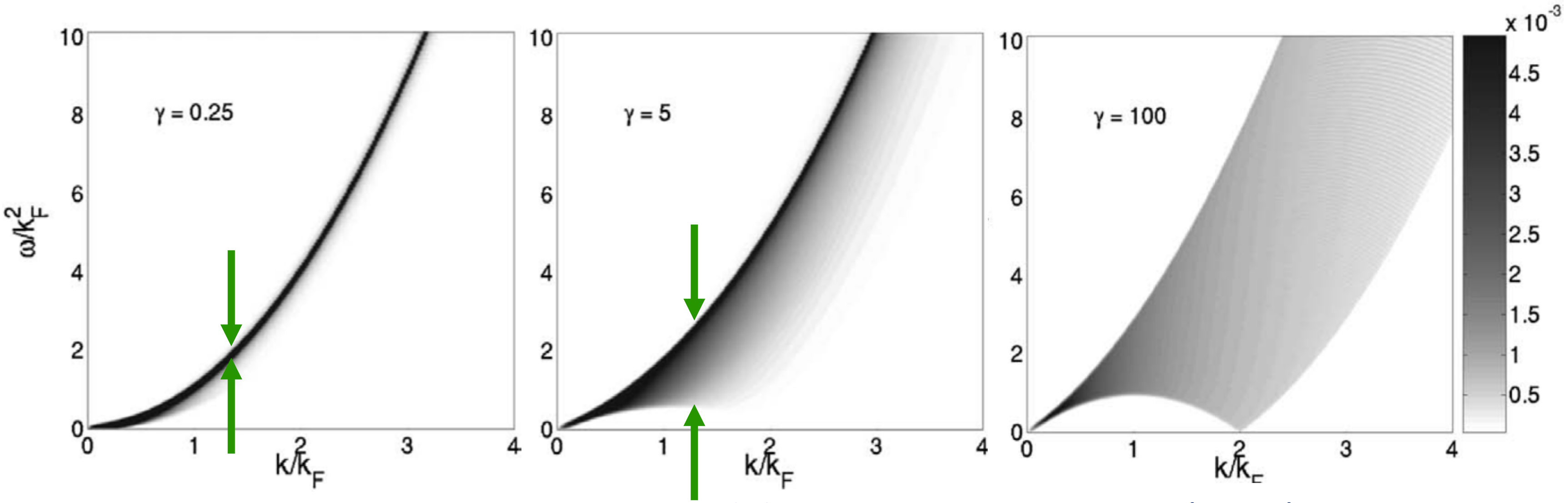


J.S. Caux and P. Calabrese PRA 74, 031605R (2006)

Dynamic structure factor of the one-dimensional Bose gas near the Tonks-Girardeau limit



$$\gamma = \frac{m g_{1D}}{\hbar^2 n}$$



J.S. Caux and P. Calabrese PRA 74, 031605R (2006)

To decrease density: decompressing the magnetic trap

compressed trap

$(\omega_x, \omega_y, \omega_z) = 2\pi\,(90, 8.7, 90)$ Hz

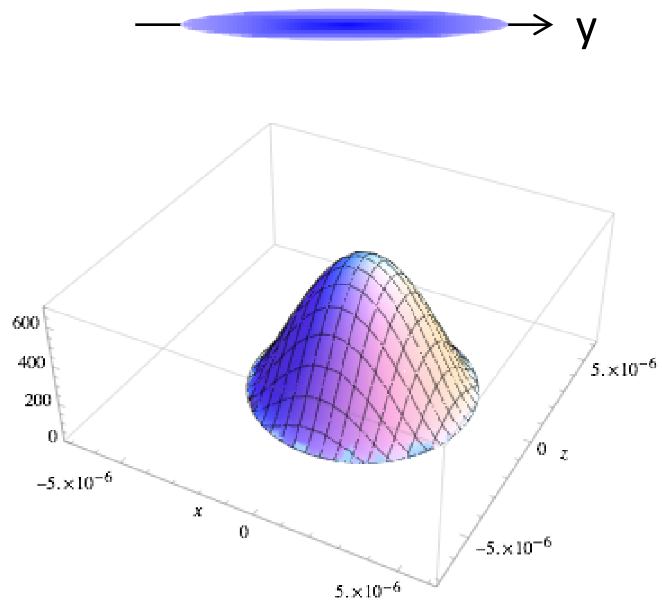
decompressed trap

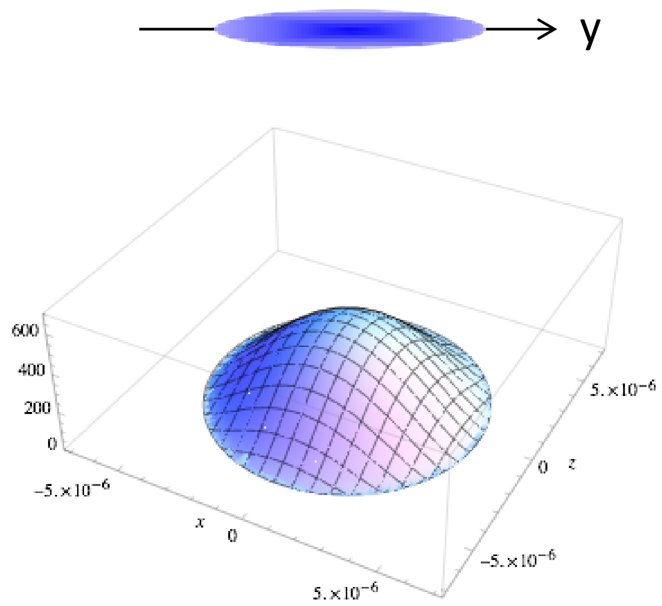
$(\omega_x, \omega_y, \omega_z) = 2\pi\,(57, 8.7, 57)$ Hz

BEC size (with N=80 000):

$(Rx,Ry,Rz) = (4, 41, 4)\ \mu m$

$(Rx,Ry,Rz) = (5, 34, 5)\ \mu m$



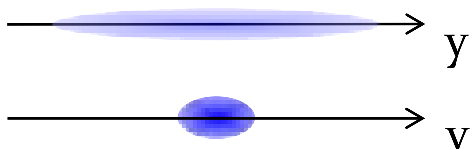


Decompressed magnetic trap + 2D lattice

compressed trap
+ 2D OL (s_x=s_z=40)

BEC size (with N=80 000):

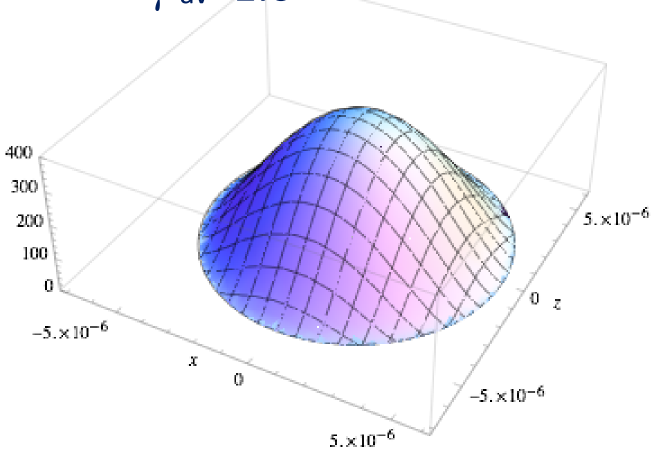
Rx,Ry,Rz = (8.7, 16, 8.7) μm



1400 occupied sites; N_{00}=144

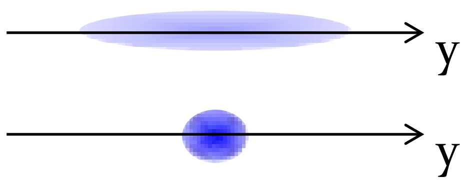
γ_{00}=0.6

γ_{av}=1.0



decompressed trap
+ 2D OL (s_x=s_z=40)

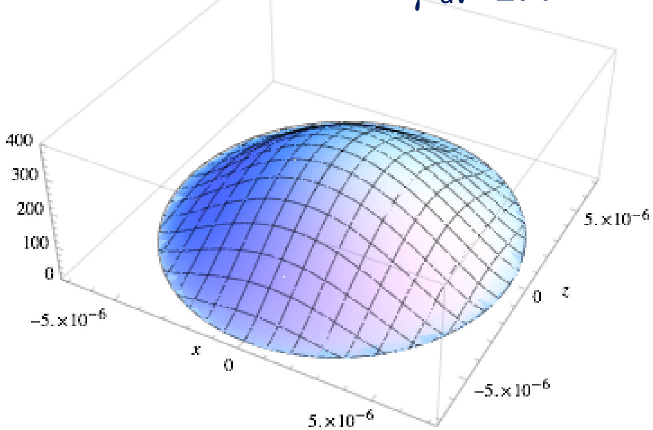
Rx,Ry,Rz = (11, 14, 11) μm



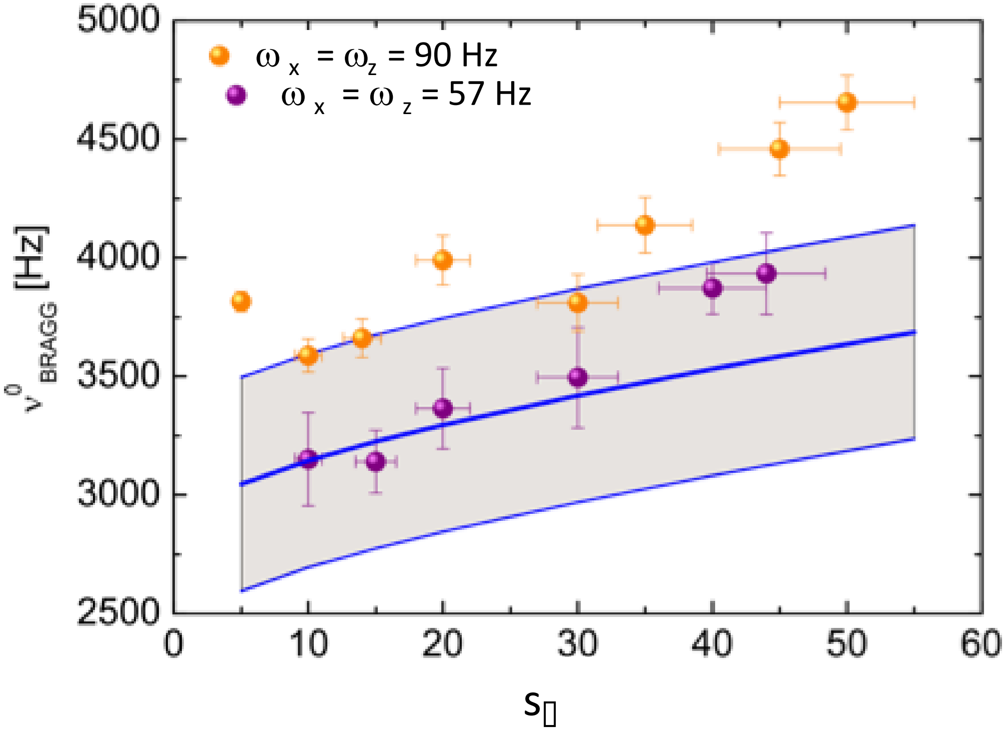
2000 occupied sites; N_{00}=95

γ_{00}=0.9

γ_{av}=1.4

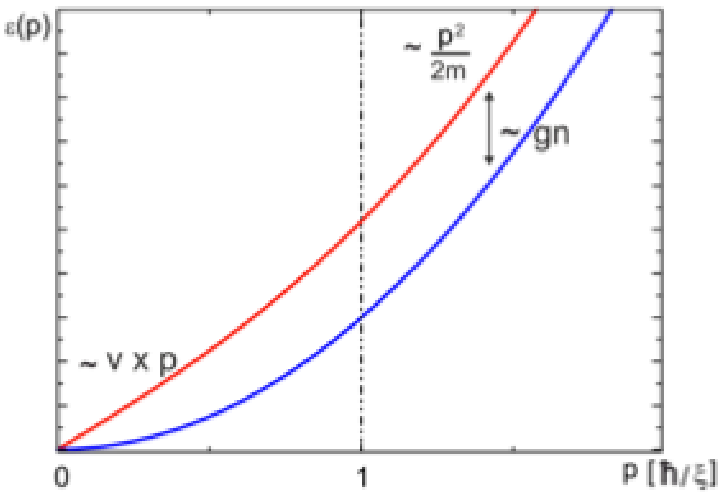


Decompressed magnetic trap + 2D lattice



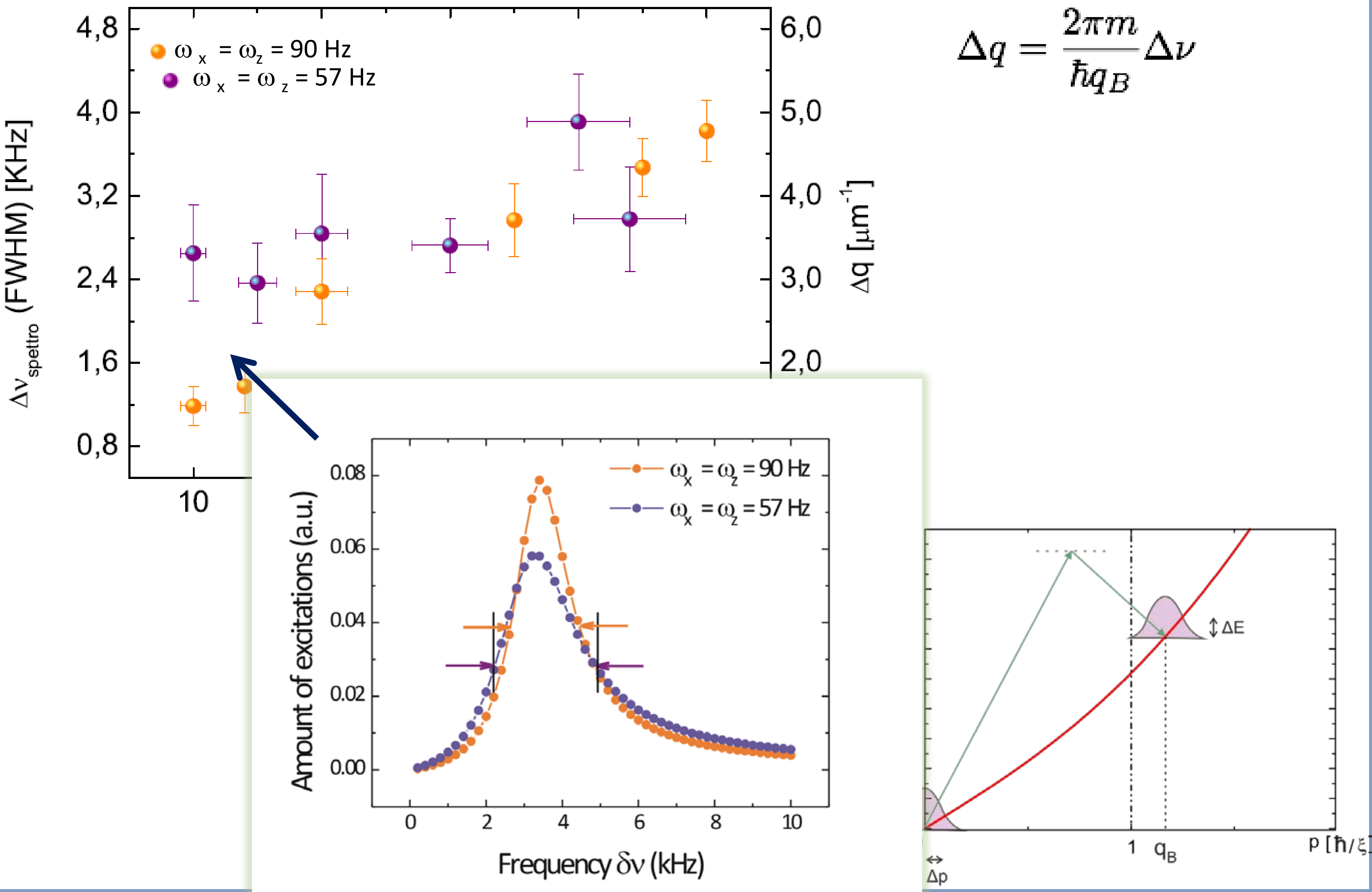
$$\nu_B^0 = \frac{hq_B^2}{4\pi m} + \frac{4\mu_{1D}}{7h}$$

($s_\perp$ = 45 : μ_{1D}/h = 2.7kHz → 2.0 kHz)



Decompressed magnetic trap + 2D lattice

$$\Delta q = \frac{2\pi m}{\hbar q_B}\Delta\nu$$



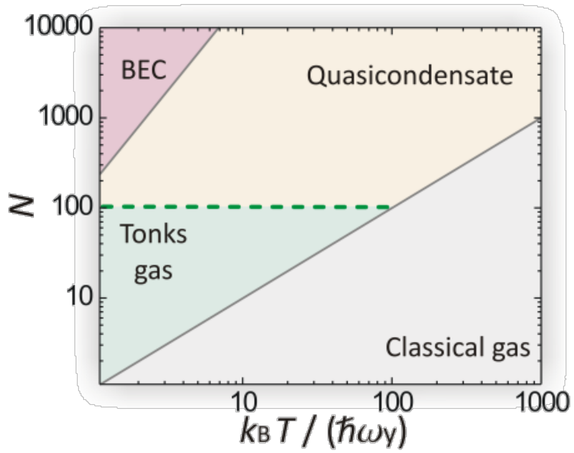
Outlook and Prospects

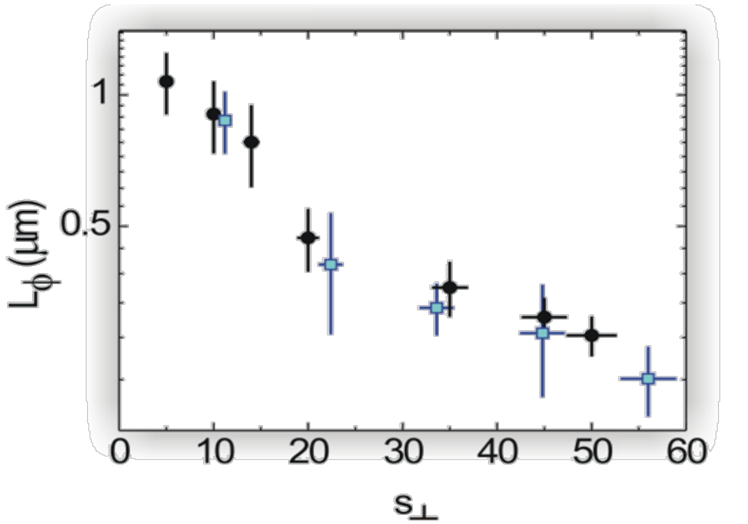
✓ Probe the coherence properties of 1D gases via Bragg spectroscopy (non-zero momentum + perturbative excitation)
✓Time-of-flight mapping of in-trap momentum distribution VS Bragg spectroscopy

→ Suitable for future momentum-resolved studies of the properties of 1D systems with short coherence length

➢ strongly correlated 1D bosons towards Tonks regime by descreasing density and temperature: produce the BEC in combined optical +magnetic trap (done); blue-detuned optical lattice (to do)

➢ disordered insulating phase





BEC 1 (Rb-87)

N. F.
Sara Rosi
Alain Bernard
Chiara Fort
Massimo Inguscio

David Clément
(now at Laboratoire Charles Fabry)
Leonardo Fallani
(now Yb lab)







http://quantumgases.lens.unifi.it