# Electronic transport in topological insulators

Reinhold Egger
Institut für Theoretische Physik, Düsseldorf
Alex Zazunov, Alfredo Levy Yeyati

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HEINRICH HEINE
UNIVERSITÄT DÜSSELDORF

# Overview

- Brief intro to Topological Insulators (TI)

  *M.Z. Hasan & C.L. Kane, Rev. Mod. Phys.* ***82****, 3045 (2010)*

- Helical Luttinger liquid in TI nanowires

  *RE, A. Zazunov & A. Levy Yeyati, PRL* ***105****, 136403 (2010)*

- Coulomb blockade of Majorana fermion induced transport

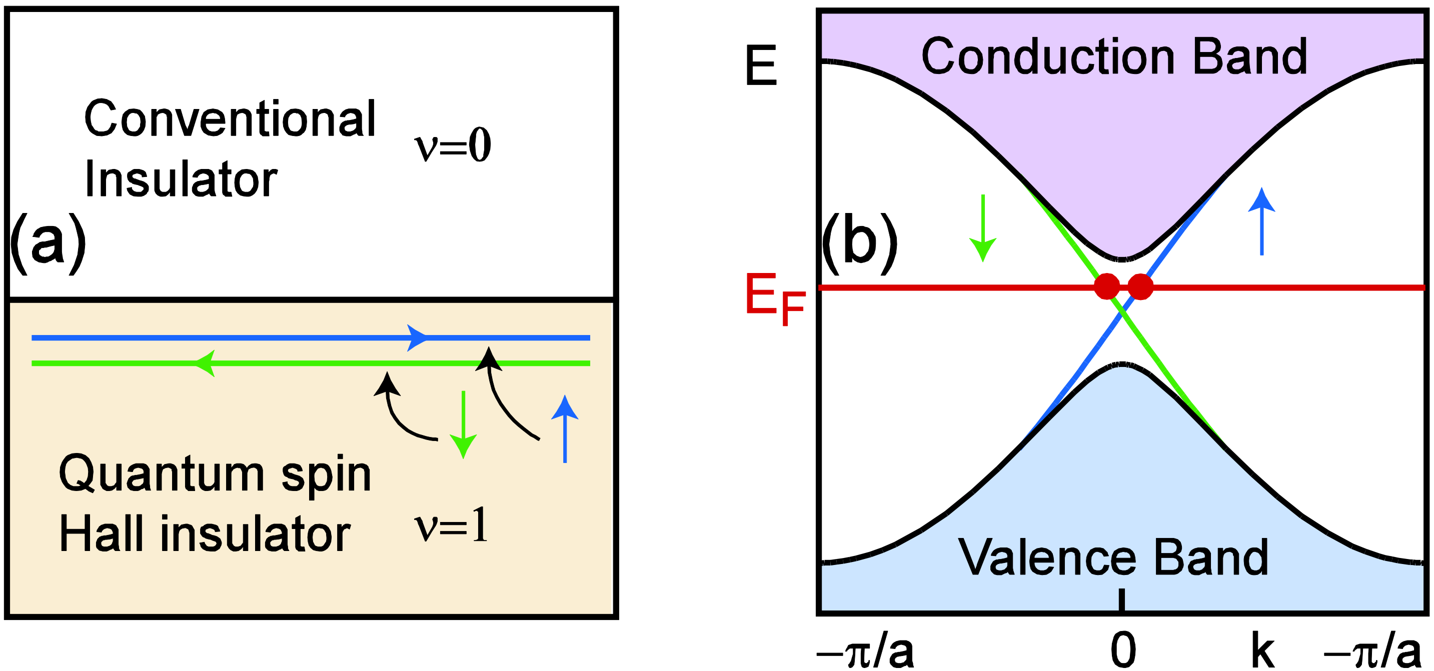
  *A. Zazunov, A. Levy Yeyati & RE, PRB* ***84****, 165440 (2011)*

# Topological Insulator

- Recently discovered new state of matter
  - Bulk band gap like ordinary insulator, but band inversion due to strong spin orbit coupling
  - Time reversal symmetry: topological protection of gapless modes at surface
- 2D „quantum spin Hall“ TI: helical edge states
  - Close cousin of integer quantum Hall state
  - Initially predicted for graphene *Kane & Mele, PRL 2005*
  - observed in HgTe quantum wells *König et al., Science 2007*
- 3D „strong“ topological insulator
  - Predicted independently by three groups *Moore & Balents, PRB 2007; Fu, Kane & Mele, PRL 2007; Roy, PRB 2009*
  - First observed via ARPES, present reference material is $Bi_2Se_3$ *Hasan & Kane, RMP 2010*

# Helical edge state of 2D TI

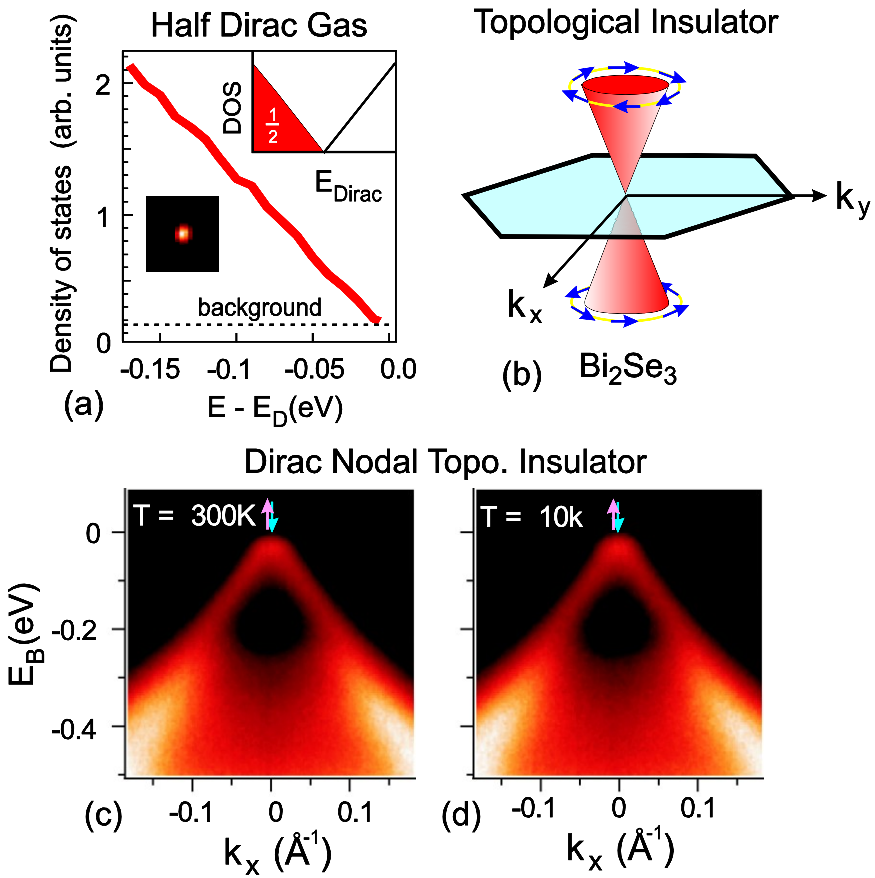
**… up and down spins propagate in different direction: a peculiar new spin-filtered 1D liquid**




© *Hasan & Kane, RMP 2010*

# 2D massless Dirac fermions as surface states of 3D topological insulators

- Map from Brillouin zone to Hilbert space has nontrivial topology
- Mathematically characterized by $Z_2$ invariant
- Avoid fermion doubling theorem
- Spin momentum locking
- Observed by ARPES in $Bi_2Se_3$

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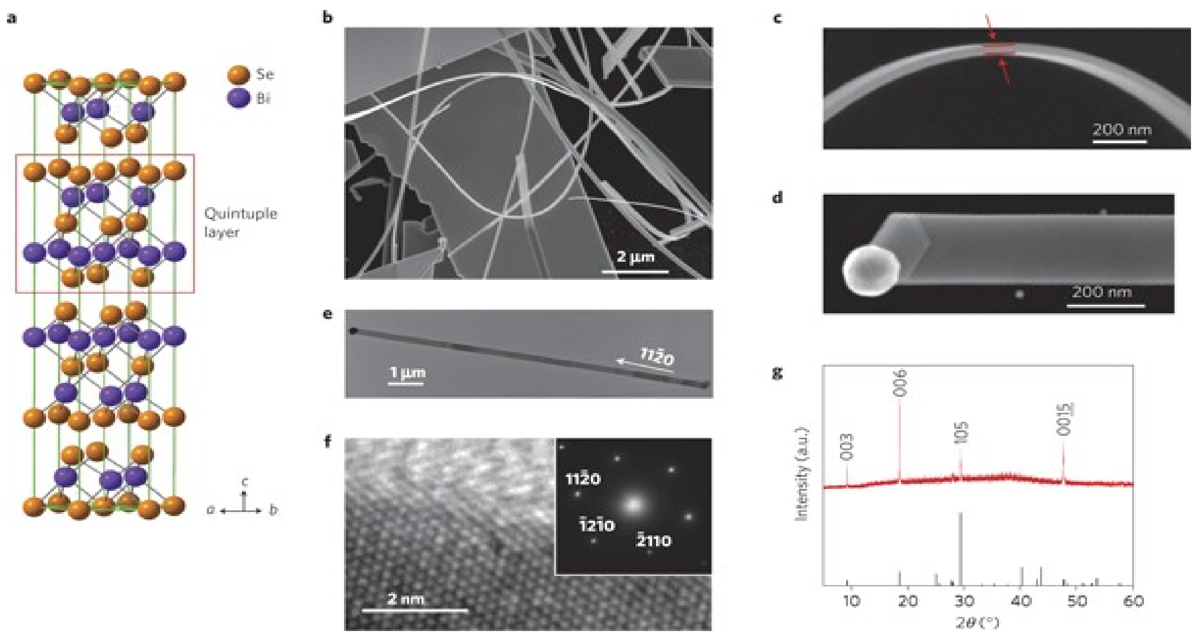
# $Bi_2Se_3$ nanowires: fabrication

Rhombohedral phase,
space group $D_{3d}^5$

Au catalysed VLS growth

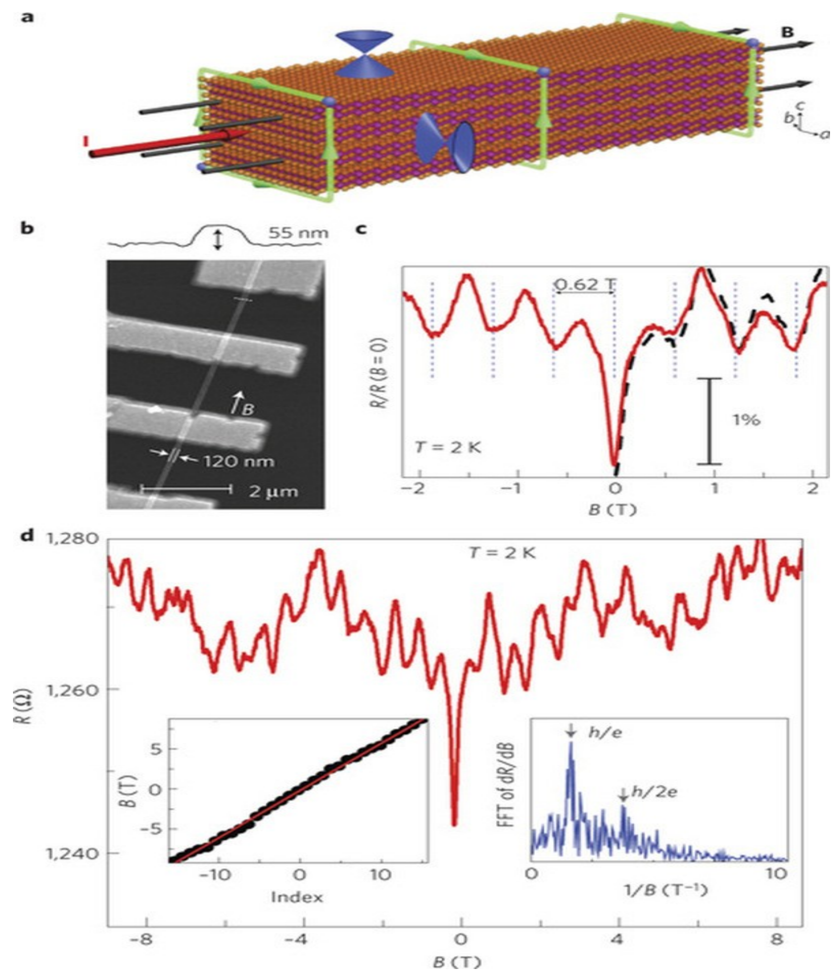



*Peng et al. (Stanford), Nature Materials 2010*

# Surface state in transport experiments

Clear evidence for the surface state: Aharonov-Bohm oscillations of conductance

- Upper bound for surface state width: 6 nm




*Peng et al., Nat. Mat. 2010*

# TI nanowires: theory

*Egger, Zazunov & Levy Yeyati, PRL 2010*

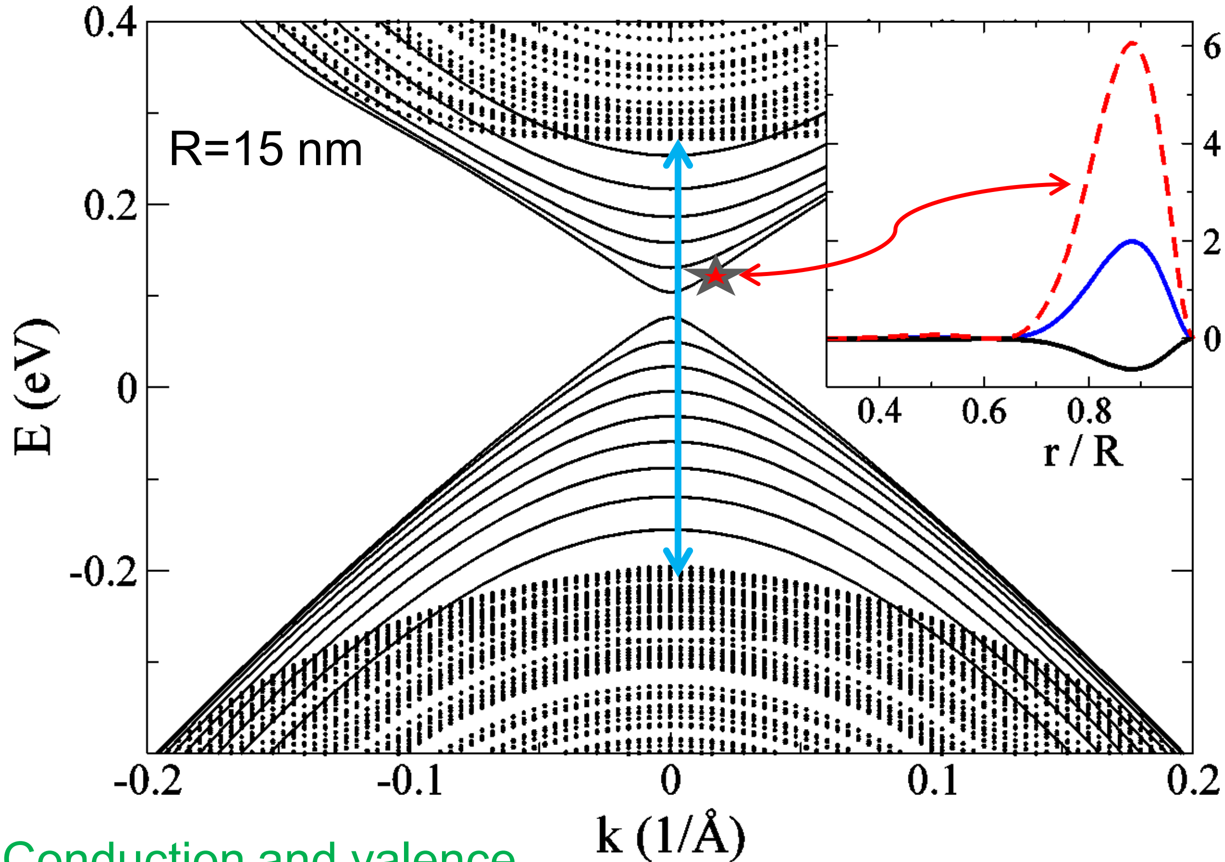
- Consider strong TI material (e.g. $Bi_2Se_3$): cylindrical nanowire of radius R
- Bandstructure from **kp** approach

*Zhang et al., Nature Phys. 2009*

  - anisotropy axis || z: conserved momentum k
  - Rotational xy symmetry: total angular momentum conservation, $J_z = -i\partial_\phi + \sigma_z/2 \;\; \rightarrow$ half-integer j
  - Wave function vanishes at boundary r=R: expand in orthonormal set of radial functions

$$u_{nj}(r) = \frac{\sqrt{2} J_{j-1/2}\left(\gamma_{n,j-1/2}\, r/R\right)}{R J_{j+1/2}\left(\gamma_{n,j-1/2}\right)}$$

# Bandstructure of TI nanowire




Density

Spin density $s_\phi$

Spin density $s_z$

Bulk gap ≈ 0.3 eV

Conduction and valence bands: indexed by j

surface states gapped…

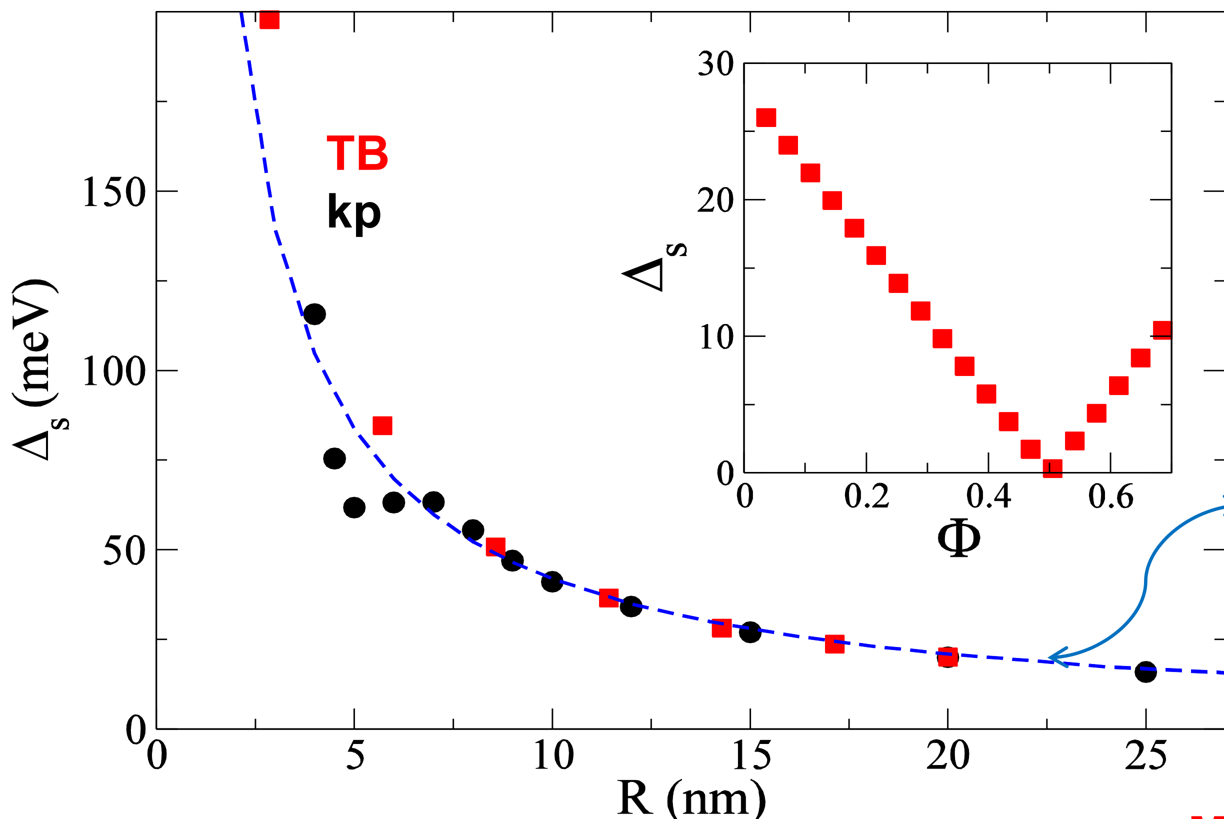
# Properties of surface states

- Kramers degeneracy: $E_{-j}(-k) = E_j(k)$
- Inversion symmetry: $E_{-j}(k) = E_j(k)$
- Qualitatively same results from tight-binding calculations
- Spin-momentum locking: Local spin is always oriented tangential to surface and perpendicular to momentum

$$\vec{s} \perp k\hat{e}_z + \frac{j}{R}\hat{e}_\phi \perp \hat{e}_r$$

  - Large |k|: R-mover counterclockwise spin-polarized
    L-mover clockwise spin-polarized

# Surface gap




**Dirac fermion description yields**

$$\Delta_s = \frac{\hbar v_\perp}{R}$$

**Magnetic flux can close the surface gap!**

# Analytical description

- Surface states well described by 2D massless Dirac fermions on cylinder surface

*Zhang & Vishwanath PRL 2010*

  - spin-momentum locking, spin direction tangential to surface
  - Dirac Hamiltonian in curved space:

$$H = -i\hbar v_F \left[\alpha^1 D_1 + \alpha^2 D_2\right] \qquad \alpha^{i=\phi,z} = \hat{e}_i \cdot \vec{\sigma}$$

  - Covariant derivative: $D_i = \partial_i + \Gamma_i$
  - Spin connection defined via relation involving Christoffel symbols $\left[\Gamma_i, \alpha^j\right] = \partial_i \alpha^j + \Gamma_{ik}^j \alpha^k$

# Surface Dirac fermion theory

- Take into account anisotropy & dim.less flux

  … yields surface Hamiltonian

$$H_{surf} = e^{-i\sigma_z\phi/2}\left[\hbar v_{\parallel} k \sigma_y - \frac{\hbar v_{\perp}}{R}\sigma_z\left(-i\partial_{\phi} + \Phi\right)\right]e^{i\sigma_z\phi/2}$$

  - Now perform unitary transformation $U = e^{-i\sigma_z\phi/2}$
  - Then: antiperiodic boundary conditions around circumference, i.e., half-integer j
  - Dispersion relation: waveguide modes

$$E_{\pm,j}(k) = \pm\sqrt{\left(\hbar v_{\parallel} k\right)^2 + \left(\frac{\hbar v_{\perp}(j+\Phi)}{R}\right)^2}$$

# Surface state properties

- Spin-momentum locking changes dispersion compared to nanotubes!
- Scattering of Kramers pair $[j+\Phi,k] \Leftrightarrow [-(j+\Phi),-k]$ forbidden (zero overlap of eigenstates!)
- Gap vanishes for half-integer flux and special band $j=-\Phi$
  - spin-conserving single-particle back-scattering forbidden, protected against weak disorder
  - chemical potential near zero: precisely one gapless helical mode, effectively time reversal invariant … from now on we focus on this case!

# Including interactions: Bosonization

- Chiral fermions with opposite spin

$$\Psi_{surf}(z,\phi)=\frac{1}{\sqrt{4\pi}}\left[e^{ik_F z}R(z)\begin{pmatrix}1\\ ie^{i\phi}\end{pmatrix}+e^{-ik_F z}L(z)\begin{pmatrix}-1\\ ie^{i\phi}\end{pmatrix}\right]$$

- Bosonize: $$(R/L)(z)\approx\frac{1}{\sqrt{2\pi\xi}}e^{i\sqrt{\pi}(\varphi\pm\theta)}$$

surface layer width

  - Spin momentum locking:

$$s_\phi=J=\frac{1}{\sqrt{\pi}}\partial_z\varphi(z)$$

dual boson fields

  - Particle density: $\rho=\Psi^{+}\Psi=\frac{1}{\sqrt{\pi}}\partial_z\theta$

# Interacting helical Luttinger liquid

- Include Coulomb interaction
  - Helical liquid protected against weak disorder
  - Long-ranged Coulomb tail dominates
  - Hard-core interaction subdominant (marginal)
- Single-mode helical Luttinger liquid

$$H = \frac{\hbar u}{2} \int dz \left[ K (\partial_z \varphi)^2 + K^{-1} (\partial_z \theta)^2 \right] \qquad u = {v_\parallel}/{K}$$

  - Interaction parameter K smaller than in HgTe:

$$K \approx \left( 1 + \frac{2e^2}{\pi \varepsilon v_\parallel} \left[ 0.51 + \ln \frac{L}{2\pi R} \right] \right)^{-1/2} \approx 0.4 \ldots 0.5$$

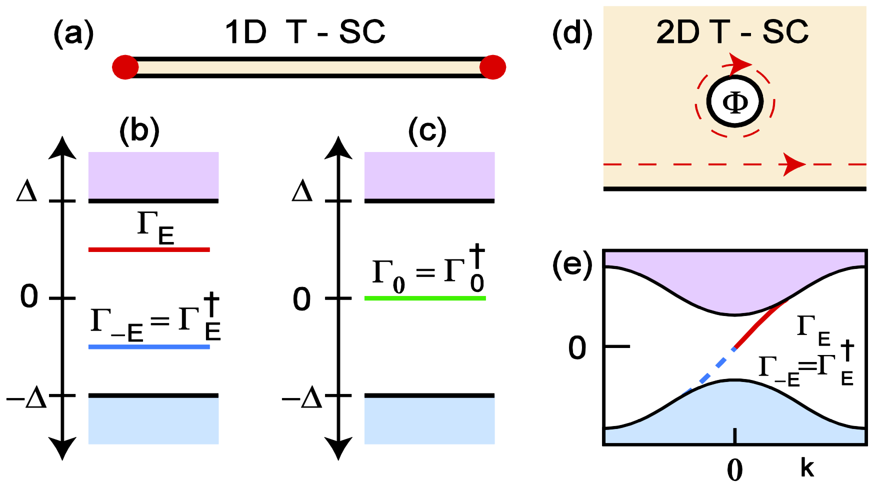
# Physical properties of helical LL

- Anisotropic spin correlations / RKKY interaction: Slow $z^{-2K}$ power law decay with $2k_F$ oscillations in z direction
  - Formally: anisotropic SDW phase
- Linear conductance G(T) very sensitive to presence of magnetic impurities

*Maciejko et al., PRL 2009, Tanaka et al., PRL 2011*

- Proximity induced superconductivity: Majorana fermion states are induced at ends of TI nanowire *Cook & Franz, PRB 2012*

# Majorana fermions: end states of 1D topological superconductors




*© Hasan & Kane, RMP 2010*

- BdG Hamiltonian has particle-hole symmetry
- Quasiparticle operator for zero mode (E=0) state bound to the end of the TSC must be Majorana fermion: Quasiparticle is its own antiparticle
- Recently observed for InSb nanowires

*Kouwenhoven group, Science Express 2012*

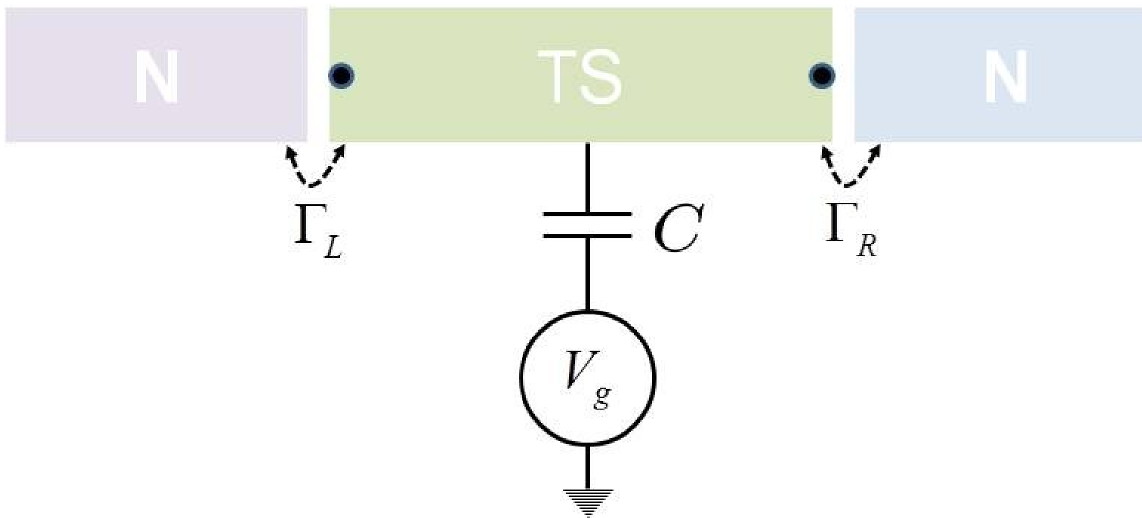
# Coulomb blockade of Majorana fermion induced transport

*Zazunov, Levy Yeyati & Egger, PRB **84**, 165440 (2011)*

Low-energy transport through topological superconductor wire with decoupled Majorana end states & source/drain metallic contacts

- Assume all energy scales small against proximity-induced gap, i.e., no quasiparticles on grain
- Zero mode Majorana fermions: $\gamma_{L/R} = \gamma_{L/R}^{+} \quad \{\gamma_i, \gamma_j\} = \delta_{ij}$
- Build auxiliary complex d fermion $d = \frac{1}{\sqrt{2}}(\gamma_L + i\gamma_R)$
- Assume „floating“ island (not grounded): current must be conserved

# Schematic setup




With Cooper pair number operator N & conjugate phase ϕ:

- instantaneous dot state can be described by (N,$n_d$) with auxiliary fermion occupation $n_d = d^+ d = 0{,}1$

# Model

- Wire has electrostatic charging energy

$$H_c = E_C\left[2N + n_d - n_{gate}\right]^2$$

  - Offset gate charge is continuous
- Full Hamiltonian: $H = H_c + H_t + H_{leads}$
  - Tunnel Hamiltonian: source contact couples to $\gamma_L$ & drain only to $\gamma_R$

$$H_t = \sum_k \left(\lambda_L c_{Lk}^{+}\left[d + e^{-i\phi} d^{+}\right] - i\lambda_R c_{Rk}^{+}\left[d - e^{-i\phi} d^{+}\right]\right) + h.c.$$

  - Fermi liquid leads: Wide band approximation & Fermi functions with chemical potential $\mu_{L/R}$
  - Hybridizations $\Gamma_{L/R} \sim \rho_0 \left|\lambda_{L/R}\right|^2$

# Noninteracting solution: resonant Andreev reflection

- Solution for $E_c$=0: $$I^{(0)}_{j=L/R} = \Gamma_j \int \frac{d\varepsilon}{2\pi} F\left(\varepsilon - \mu_j\right) A_j(\varepsilon)$$
  - Fermi functions: lead distribution $F(\varepsilon) = \tanh\left(\varepsilon/2T\right)$
  - Majorana spectral function $$A_j(\varepsilon) = -\mathrm{Im}\, G^{ret}_j(\varepsilon) = \frac{\Gamma_j}{\varepsilon^2 + \Gamma_j^2}$$
- Linear conductance @ T=0: $$G_{j=L/R} = \frac{e\partial I^{(0)}_j}{\partial \mu_j} = \frac{2e^2}{h}$$

  resonant Andreev reflection

*Bolech & Demler, PRL 2007*
*Law, Lee & Ng, PRL 2009*

# Strong Coulomb blockade: Electron teleportation

*Fu, PRL 2010*

- Resonant case: half-integer $n_{gate}$
- Charging energy allows only two configurations
  - For $n_{gate}$=1/2,5/2,…: fixed Cooper pair number N, states $n_d$=0,1 degenerate
  - For $n_{gate}$=3/2,7/2,…: (N-1,1) and (N,0) degenerate
- In both cases, model can be mapped to spinless resonant tunneling Hamiltonian
- Linear conductance (T=0): $G_{L/R} = e^2/h$
- Fu's interpretation: Electron teleportation

# Crossover from resonant Andreev reflection to electron teleportation

- Arbitrary charging energy: Keldysh approach with (Majorana-generalized) AES action
  - General expression: *Zazunov, Levy Yeyati & Egger, PRB 2011*
- Here study weak Coulomb blockade regime: interaction corrections to noninteracting result
- Full crossover from resonant Andreev reflection ($E_c$=0) to teleportation (large $E_c$) can be captured as well (work in progress)

# Weak Coulomb blockade regime

- Phase fluctuations are small & allow for semiclassical expansion
- Keldysh-AES functional is then equivalent to Langevin equation for „classical“ phase

$$\ddot{\phi}_c + \Omega \dot{\phi}_c = \xi(t)$$

  - Inverse RC time of effective circuit: $\Omega = \eta E_C$
  - Dimensionless damping strength

$$\eta = \frac{2}{\pi} \sum_j \frac{\Gamma_j^2}{\mu_j^2 + \Gamma_j^2}$$

    (higher energy scales: damping retardation!)
  - Gaussian random force $\langle \xi(t) \xi(t') \rangle = 4 E_C^2 K(t - t')$

# How to obtain the current…

K has lengthy expression…

- in equilibrium satisfies fluctuation dissipation theorem

$$K_{eq}(\omega) = \frac{\omega}{2}\coth\left(\frac{\omega}{2T}\right)\eta_{eq}(\omega)$$

- Current:

$$I_j = \Gamma_j \int d\tau \; G_j^{ret}(\tau)\sin(\mu_j \tau) F(\tau) e^{-J(\tau)}$$

$$J(t-t') = \frac{1}{2}\left\langle \left[\bar{\phi}_c(t) - \bar{\phi}_c(t')\right]^2 \right\rangle_{\xi} \geq 0$$

solution $\bar{\phi}_c(t)$ for given noise realization

- Some algebra:

$$J(\tau) = \frac{1}{\pi\eta^2}\int_0^{\infty} d\omega \; K(\omega)\frac{1-\cos\omega\tau}{\omega^2\left(1+\omega^2/\Omega^2\right)}$$

- Interactions always decrease current!

# Nonlinear conductance

- Symmetric system & T=0

$$\mu_L = -\mu_R = eV/2$$
$$\Gamma_L = \Gamma_R = \Gamma/2$$

- Observable:

$$g(V) = \frac{I(V)}{e^2 V/h}$$

  - Noninteracting case (resonant Andreev reflection):

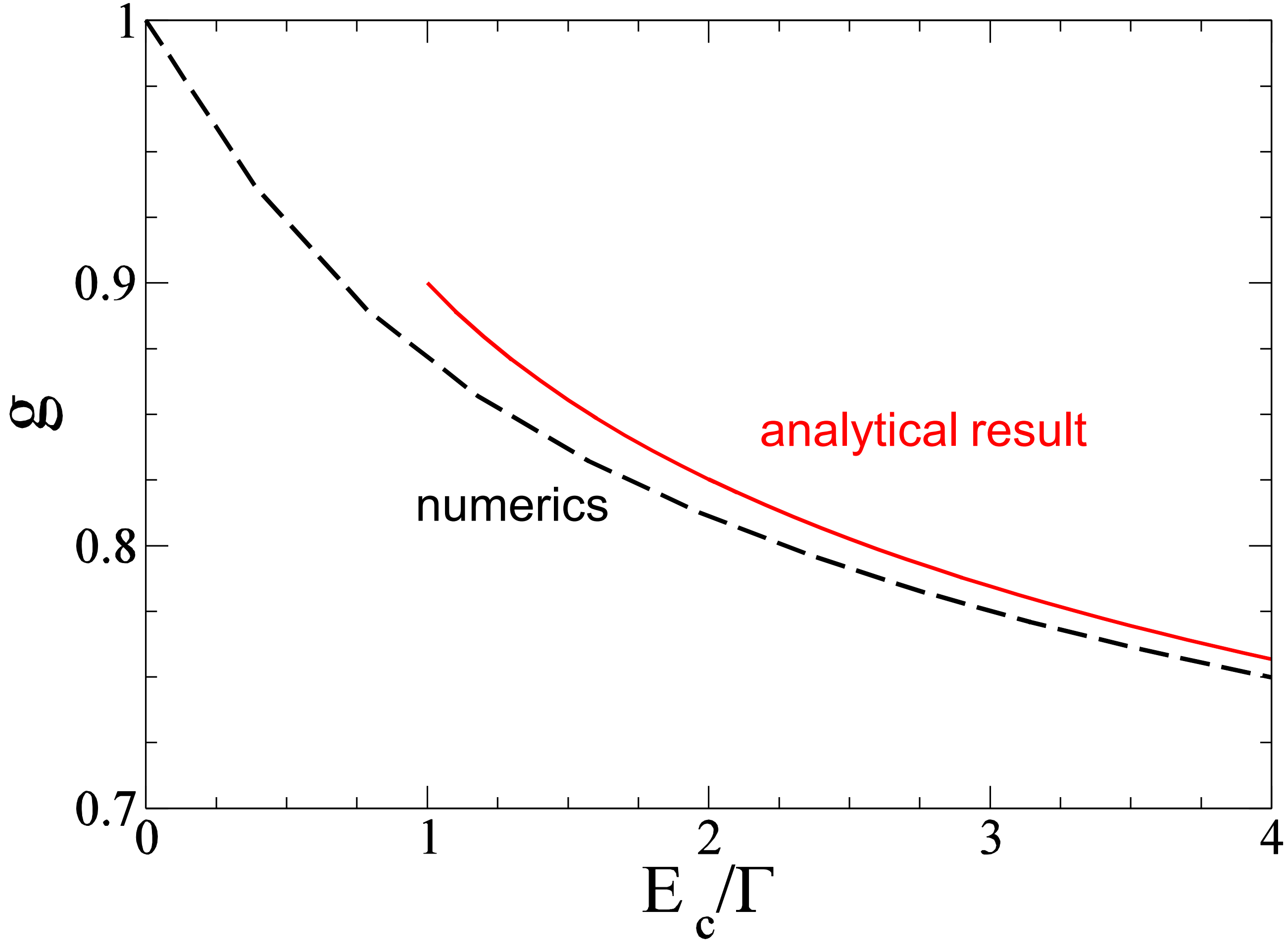
$$g^{(0)}(V) = \frac{\Gamma}{eV}\tan^{-1}\frac{eV}{\Gamma} \le 1$$

  - Analytical result for $\Gamma < E_C$ : universal power law suppression of linear conductance with increasing charging energy

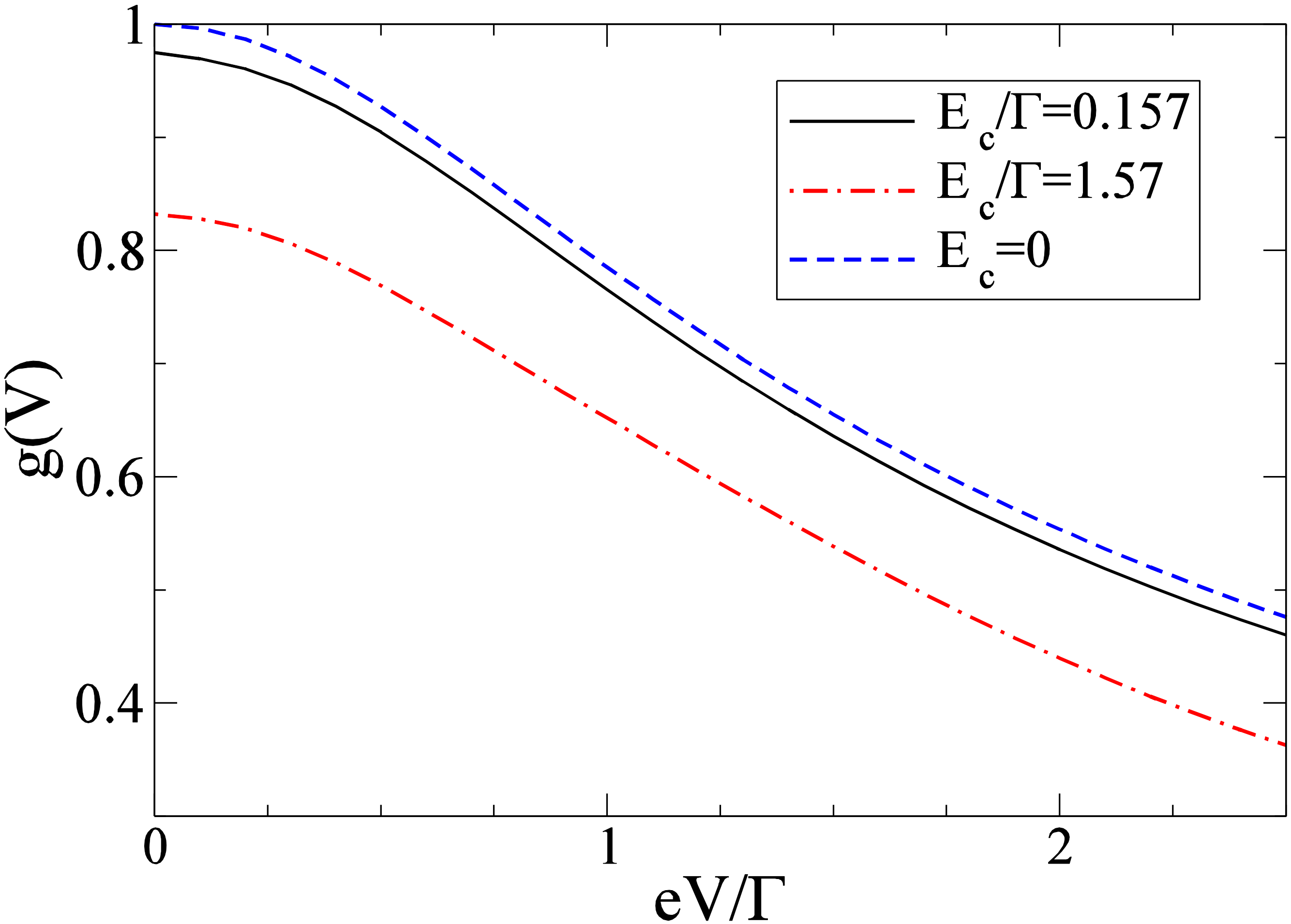
$$g(0) \approx 0.96\left( {E_C}/{\Gamma} \right)^{-1/8}$$

# Linear conductance: numerics

**interaction induced suppression**



# Nonlinear conductance



# Conclusions

- Topological insulators provide interesting playground for transport through interacting nanostructures
- TI nanowire as realization of single-mode helical Luttinger liquid with strong correlations

*Egger, Zazunov & Levy Yeyati, PRL **105**, 136403 (2010)*

- Coulomb blockade in Majorana fermion induced transport: from resonant Andreev reflection to electron teleportation

*Zazunov, Levy Yeyati & Egger, PRB **84**, 165440 (2011)*