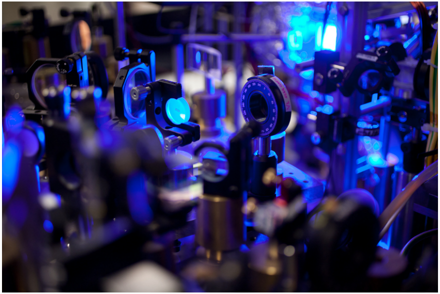
# Dynamics of impurities in a one-dimensional Bose gas



Francesco Minardi

Istituto Nazionale di Ottica-CNR
European Laboratory for Nonlinear Spectroscopy

**New quantum states of matter in and out of equilibrium**
Galileo Galilei Institute – May 14, 2012

# Acknowledgements

BEC$^3$ group at LENS, Firenze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University of Geneva

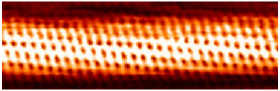
A. Kantian, T. Giamarchi

Scuola Normale Superiore, Pisa

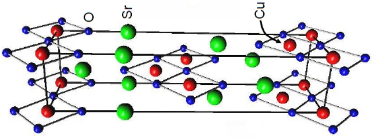
S. Peotta, D. Rossini, M. Polini
R. Fazio

# One-dimensional systems

- large quantum fluctuations + exactly solvable models (Lieb-Liniger, . . . ) + powerful numerics;
  time-dependent dynamics, out-of-equilibrium calculations
- real 1D systems do exist in our 3D world



carbon nanotubes

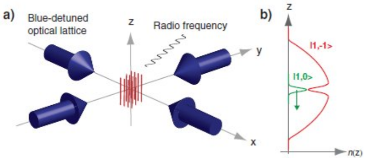



spin chains in cuprates

# One-dimensional systems

▷ Quantum gases

- experiments on (quasi)1D BEC: MIT, Hamburg, NIST, Orsay/Palaiseau, Amsterdam, ETHZ, Vienna ...
- strongly interacting (Tonks-Girardeau) regime: T. Kinoshita et al., Science 305, 1125 (2004); B. Parades *et al.*, Nature 429, 277 (2004); E. Haller, Science 325, 1124 (2009)
- relaxation dynamics: S. Trotzky *et al.*, Nature Physics (2012)



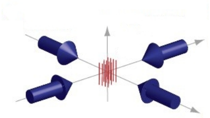

transport of spin impurities through a Tonks gas

impurity subject to constant force (gravity) + drag force due to host atoms

S. Palzer *et al.*, PRL 103, 150601 (2009)

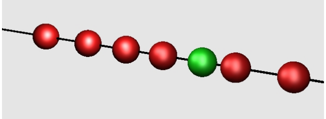
# Outline

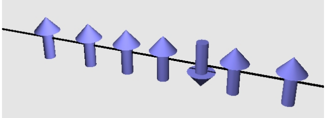
▷ diffusion and oscillations of an initially localized impurity (K atoms) in a harmonically trapped 1D Bose gas (Rb atoms),



▷ control of interaction of impurities (K) with host atoms (Rb), through Feshbach resonance

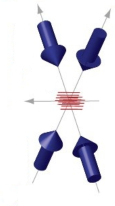
Analogous to spin excitation in a ferro-magnetic chain





# Outline

▷ diffusion and oscillations of an initially localized impurity (K atoms) in a harmonically trapped 1D Bose gas (Rb atoms), *horizontal*



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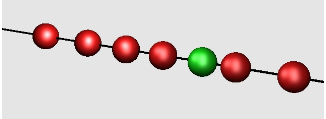
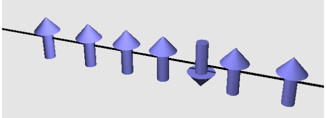




# Spin chain, Yang-Gaudin model

Lieb-Liniger model:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g\sum_{i<j}\delta(x_i - x_j), \quad \gamma = mg/(\hbar^2 n)$$

extended to (iso)spin $= 1/2 \rightarrow$ Yang-Gaudin model, SU(2) symmetric, only one coupling strength $g$

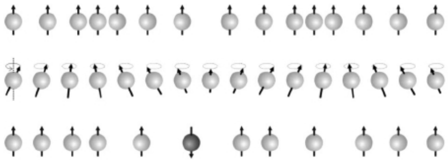
C. N. Yang, PRL 19, 1312 (1967); M. Gaudin, Phys. Lett. A 24, 55 (1967); J. N. Fuchs *et al.*, PRL 95, 150402 (2005)

Starting from ferromagnetic ground state:

– density excitations (phonons) $\epsilon_p = v_s p$

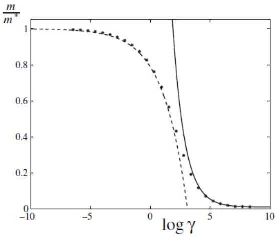
– spin excitations $\epsilon_p = p^2/(2m^*)$, $m/m^* = 1 - 2\sqrt{\gamma}/(3\pi)$ for weak coupling, $\gamma \ll 1$

$m/m^* = 1/N + 2\pi^2/(3\gamma)$ for strong coupling, $\gamma \gg 1$



# Effective mass, slow diffusion

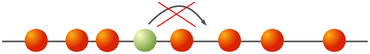
Effective mass for spin excitations




J. N. Fuchs *et al.*, PRL (2005)

For $\gamma \gg 1$ impurities move slowly, actually "subdiffuse" at short time, $x_{\rm rms} \sim \log(t)$

M. B. Zvonarev et al., PRL 99, 240404 (2009)



Beyond Luttinger-liquid description

About impurity motion in 1D also:

G. E. Astrakharchik *et al.*, PRA 70, 013608 (2004); M. D. Girardeau *et al.*, PRA 79, 033610 (2009); D. M. Gangardt *et al.*, PRL 102, 070402 (2009); A. Yu. Cherny *et al.*, PRA 80, 043604 (2009); T. H. Johnson *et al.*PRA 84, 023617 (2011)

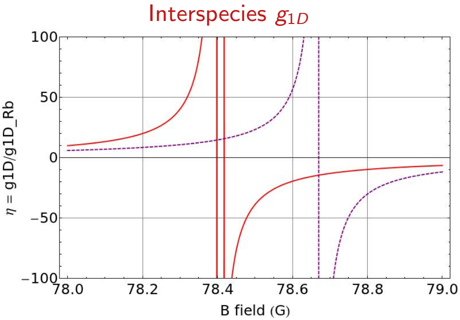
# Scattering of two unequal particles in 1D

Extension of Olshanii's analysis on Confinement Induced Resonances: V. Peano et al., NJP 7, 192 (2005)

No closed analytical expression for coupling strength of $\delta$-potential:

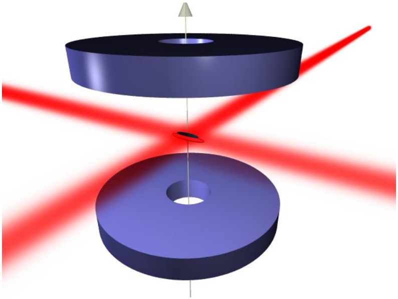
$$g_{1D} = \frac{1}{2\mu\pi a_\mu^2} \sum_n \frac{|\langle 0|e_n\rangle|^2}{\lambda_n + 1/(4\pi a)} \quad a_\mu = \sqrt{\frac{2\hbar}{\mu(\omega_1+\omega_2)}}$$

where $\lambda_n$, $|e_n\rangle$ eigenvalues/vectors of regular part of the Green's function



# Experiment

# Sample preparation, harmonic trap



Evaporation, both species in lowest hf state $|f = 1, m_f = 1\rangle$ featuring Feshbach resonances

B field controls of *interspecies* (K-Rb) interactions, while *intraspecies* (K-K, Rb-Rb) fixed

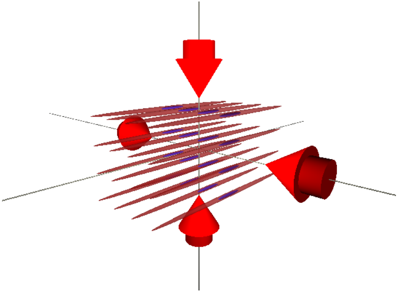
$\omega/2\pi = (39, 87, 81)$Hz for Rb ($\times 1.47$ for K)

At this point:

$T \simeq 140$nK

$N_{Rb} \simeq 1.5 \times 10^5$, $N_K \simeq 5 \times 10^3$

# Sample preparation, 2D lattice



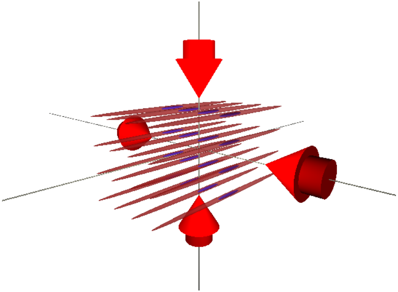
2D lattice $V$ =60(26) $E_r$ for Rb(K)

1st excited band gap = 29 kHz
i.e. 1.4 $\mu$K

tunneling time $\hbar/J$ =57(0.27)s

Non-homogenous 1D tubes, $\omega_x/2\pi$ =57(80)Hz

# Sample preparation, 2D lattice



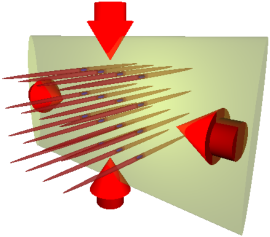
Max filling $=$ 180 (2) atoms/tube for Rb(K)

Rb $n_{1D} = 7$ atoms/$\mu$m
Lieb-Liniger parameter
$\gamma_{Rb} = g_{1D,Rb} m/(\hbar^2 n_{1D}) \simeq .5$

T$=(350 \pm 50)$ nK (from Rb time-of-flight images)

Rb degeneracy temperature
$T_d = \hbar\omega_x N = 520$nK $\rightarrow$ weakly interacting Rb condensates in central tubes

## Sample preparation, 2D lattice + "light-blade"



"Light-blade" $\lambda = 770$nm, elliptic $75 \times 15\mu$m

Species selective:
V $\simeq 0$ on Rb, $\simeq 6\mu$K on K
linear ramp in 50 ms

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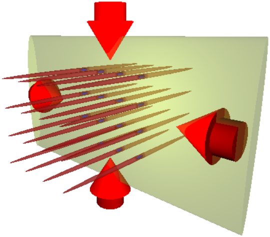

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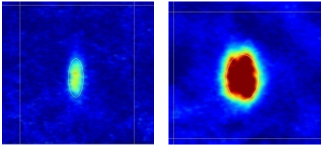
Initial configuration, $t = 0$ after light-blade off abruptly



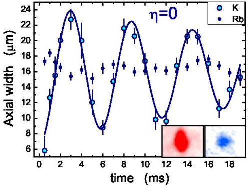
K    Rb

initial K size $<$ imaging resolution ($8\mu$m)

# Impurity oscillations

Longitudinal confinement along tubes $\rightarrow$ oscillations of K impurity rms size $\sigma(t)$

Interspecies interaction parameter: $\eta \equiv g_{1D}(KRb)/g_{1D}(Rb)$ $\qquad g_{1D}(Rb) = 2.36 \cdot 10^{-37}\,\mathrm{Jm}$




J. Catani et al., PRA 85, 023623 (2012)

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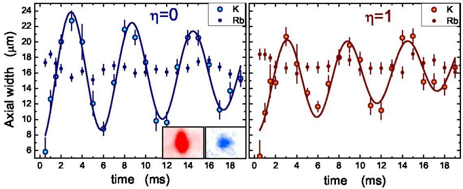


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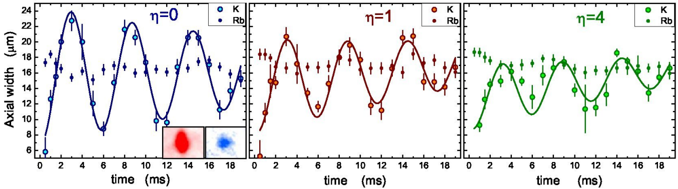


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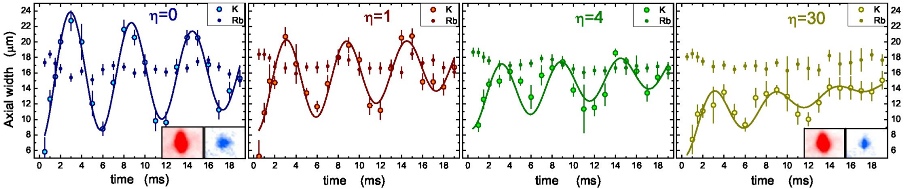


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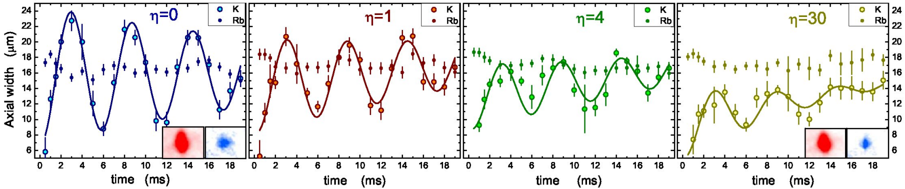


J. Catani et al., PRA 85, 023623 (2012)

▷ larger interactions $\rightarrow$ smaller oscillation amplitude of $\sigma(t)$

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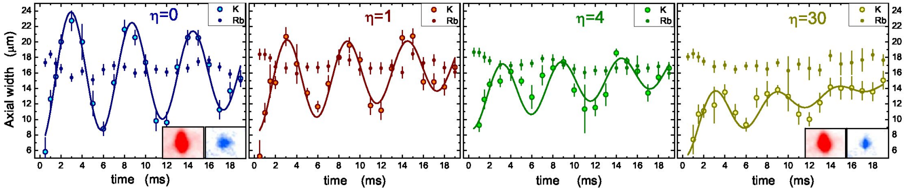



J. Catani et al., PRA 85, 023623 (2012)

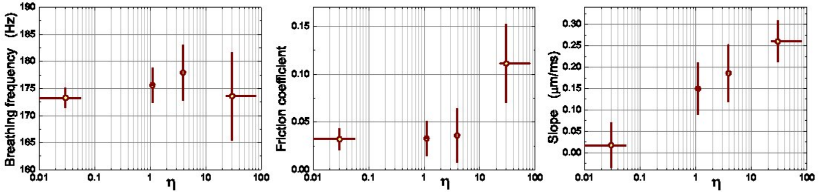
- larger interactions $\rightarrow$ smaller oscillation amplitude of $\sigma(t)$
- tilted oscillations

## Oscillation frequency, damping and slope

Fitting function:

$$\sigma(t) = \sigma_1 + \beta\, t - A\, e^{-\gamma\omega t} \cos(\sqrt{1-\gamma^2}\omega\, (t - t_0))$$

Fit results:




Oscillation frequency constant within errorbars

# Amplitude of first oscillation

Focus on the peak value of 1st oscillation: $\sigma_p \equiv \sigma(\text{t=3ms})$ vs $g_{1D}$ (exp. B field)

- $\sigma_p$ sensitive to coupling with Rb bath
- $\sigma_p$ least affected by Rb inhomogeneous density

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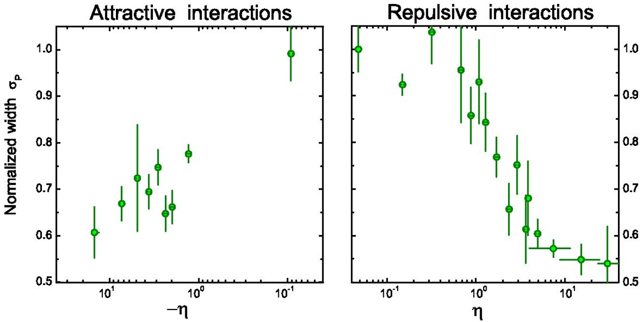



▷ NOT trivial mean-field pressure of bath

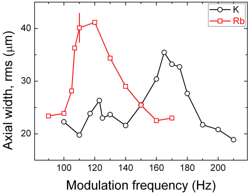
▷ saturation for $\eta > 5$

# Preparation of the sample, thermalization

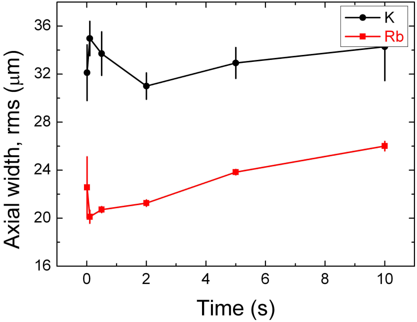
Compression of the "light-blade" expected to heat impurities

Does initial kinetic energy, thus $\sigma_p$, depend on $\eta$?
What is the time-scale for "thermalization"?

Selective heating of impurities in 1D by modulation of the axial confinement (parametric heating)




Vanishing interactions, $\eta = 0$



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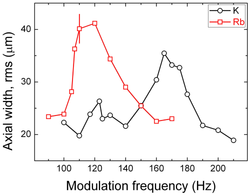
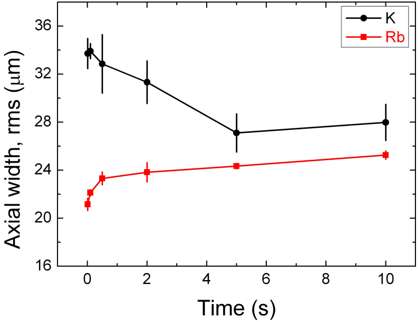



Resonant interactions, $|\eta|max$




Even at largest interaction strength, time scale for equilibration $>>$ preparation time ($\sim$ 50ms)

# Theoretical analysis (A. Kantian and T. Giamarchi, U. Geneve)

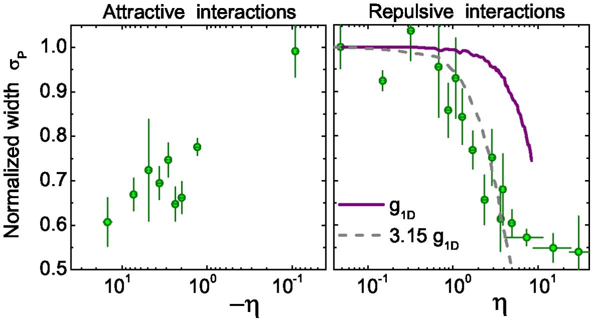
Semi-empirical model: quantum Langevin equation, damped harmonic oscillator in contact with a thermal bath

$$\begin{aligned}\dot{\hat{x}}(t) &= \hat{p}(t)/m_K^* \\ \dot{\hat{p}}(t) &= -m_K^*\omega^2\hat{x}(t) - \tilde{\gamma}\hat{p} + \hat{\xi}(t)\end{aligned}$$

- Rb density assumed to be uniform (weak dependence on exact value)
- mass is increased by polaronic coupling to the finite T bath, according to Feynmann
  R. P. Feyman, Phys. Rev. 97, 660 (1955)
- frequency is fixed, according to observation
  For 3D fermions, effective mass $m^*/m > 1$ measured from slowing frequency
  S. Nascimbene *et al.*, PRL 103, 170402 (2009)
- mass renormalization at fixed frequency → trapping potential renormalization (work in progress)

# Experiment/theory comparison

Good agreement, if interspecies $g_{1D}$ (i.e. $\eta$) increased by a factor $\sim 3$



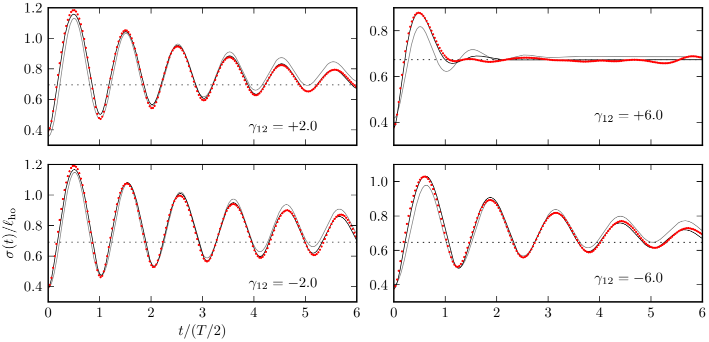

J. Catani et al., PRA 85, 023623 (2012)

# t-dependent DMRG calculations (preliminary)

Scuola Normale Superiore, Pisa: S. Peotta, D. Rossini, M. Polini and R. Fazio

Numerical simulations:

- ▷ homogeneous bath
- ▷ impurity and bath in gnd state before "quench" (T=0)
- ▷ quench $\omega/(2\pi) = 38 \to 12$ kHz (vs exp: $1 \to 0.08$ kHz)




$\gamma_{Rb} = 10$

$\eta = \gamma_{12}/\gamma_{Rb}$

$\ell_{\rm ho} = \sqrt{\hbar/m_K\omega_K}$

$T/2 = \pi/\omega_K$

# t-dependent DMRG calculations (preliminary)

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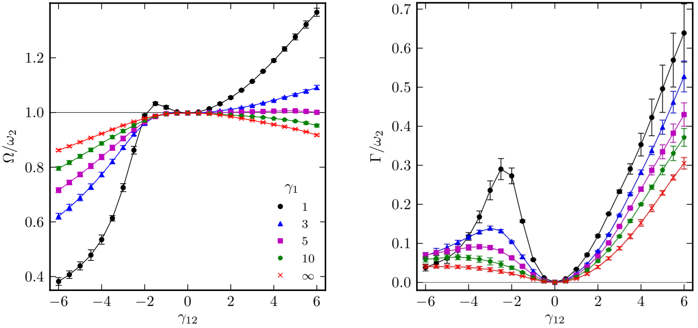
Notable results:

- asymmetric frequency shift
- strong and asymmetric damping

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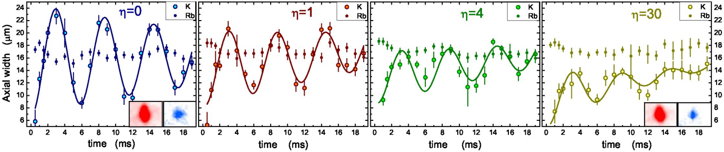
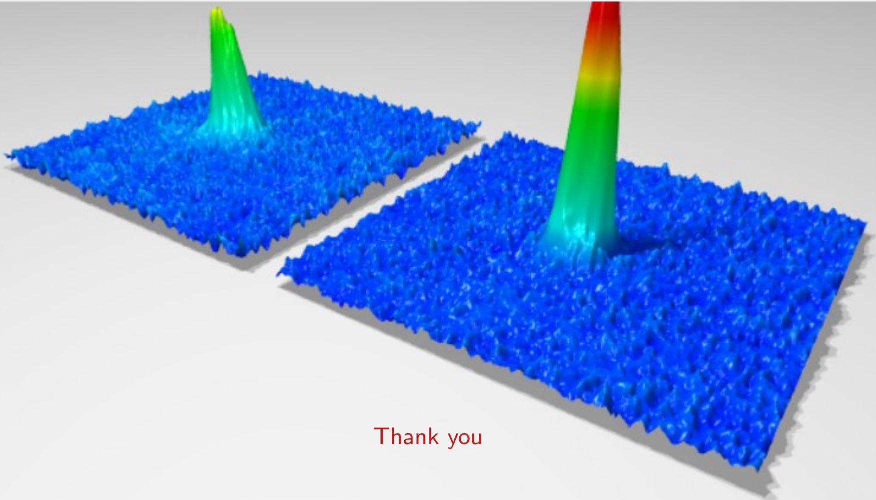



- ▷ diffusion/oscillations of impurities (rms size) in 1D Bose gas, as a function of their interaction with host medium
- ▷ frequency independent of interaction strength, amplitude decreases with interaction strength

- ▷ theoretical analysis in terms of Quantum Langevin eqn (A. Kantian, T. Giamarchi)
- ▷ polaronic mass shift calculated with Feynmann variational approach → amplitude reduction as observed in experiment
- ▷ tDMRG simulations (S. Peotta et al.): understanding in progress ...

## The end



Thank you

http://quantumgases.lens.unifi.it

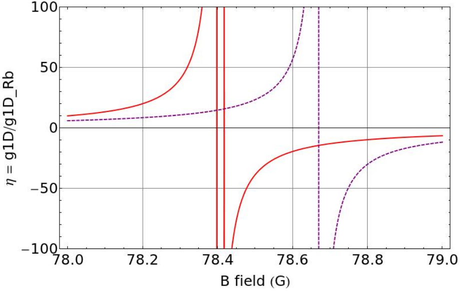
# Scattering of two unequal particles in 1D

Two-body scattering modified by confinement

Extension of Olshanii's CIR analysis: no analytic expression of the one-dimensional coupling strength $g1D$

V. Peano et al., NJP 7, 192 (2005)

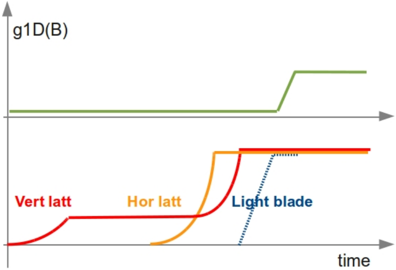
Interspecies coupling strength $\eta = g1D_{KRb}/g1D_{RbRb}$



# Time sequence


- ▷ $g_{1D} \simeq 0$, 2D lattice $s \to 60$
- ▷ light blade on slowly in 50ms, $g_{1D}$ to final value
- ▷ light blade off abruptly in 0.5ms, impurity expansion
  (then freeze+*in situ* imaging)

# Time sequence

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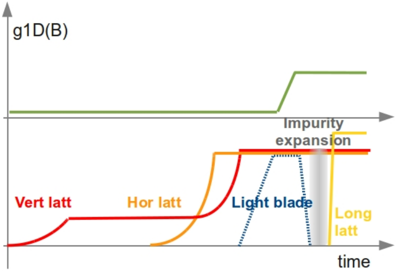


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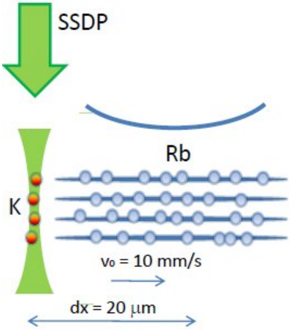
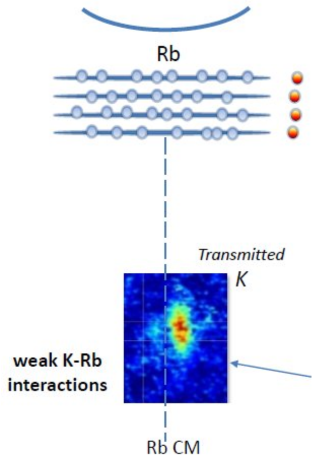
  (then freeze+*in situ* imaging)

# Impurity displaced




- Impurity displaced and released
- accelerated by harmonic potential

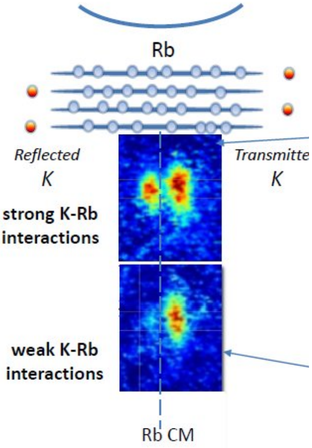
# Impurity displaced




- Impurity displaced and released
- accelerated by harmonic potential

◁ at small coupling strength, impurity transmitted

# Impurity displaced



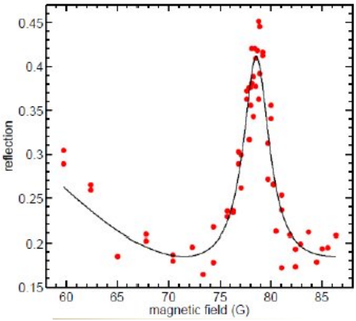

- Impurity displaced and released
- accelerated by harmonic potential

◁ at high coupling strength, partial reflection

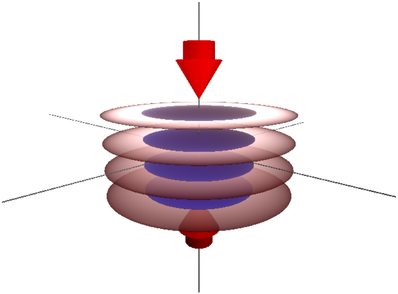
◁ at small coupling strength, impurity transmitted

# Impurity reflection

Quantum reflection, also at $g_{1D} < 0$



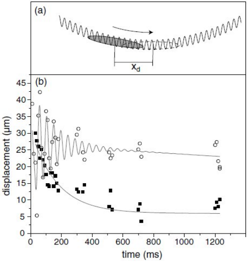
# Sample preparation, vertical lattice



Vertical lattice $V = 15(6.5)E_r$ [Rb(K)]

Tunneling time $\hbar/J = 80(4)$ms

Lighter K atoms fall under gravity, disrupted Bloch oscillations
similar to degenerate fermions colliding with bosons



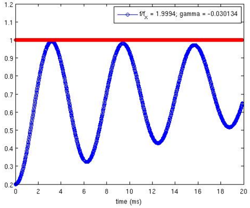

H. Ott et al., PRL 92, 160601 (2004)

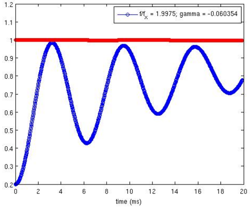
Collective oscillations for two colliding 1D normal, ideal gases

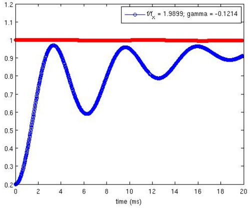
Transition from collisionless to hydrodynamic regime

D. Guery-Odelin et al., PRA 60, 4851 (1999); M. Anderlini et al., PRA 73, 032706 (2006)

Linear differential eqns for momenta of phase-space distribution: $\langle x_i^2\rangle, \langle x_i v_i\rangle, \langle v_i^2\rangle$



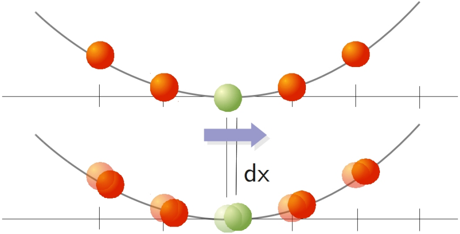



# Spring constant

Simple argument for impenetrable bosons ($\gamma \to \infty$)

Impurity moves by $dx$, hence forcing all particles to move




$$\begin{aligned}\Delta E &= \left(\int_{-L+dx}^{L+dx} - \int_{-L}^{L}\right)\frac{1}{2}kx'^2 n dx' \\ &= \frac{1}{2}k(dx)^2 N\end{aligned}$$

as if $k \to Nk$

At $\infty$ compressibility, rigid body $N$ particles all subject to same force $-kx$, total force $= -Nkx$