Dynamic correlations, fluctuation-dissipation relations and effective temperatures after a quantum quench (of the Ising chain)

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In collaboration with:

Laura Foini & Leticia F. Cugliandolo

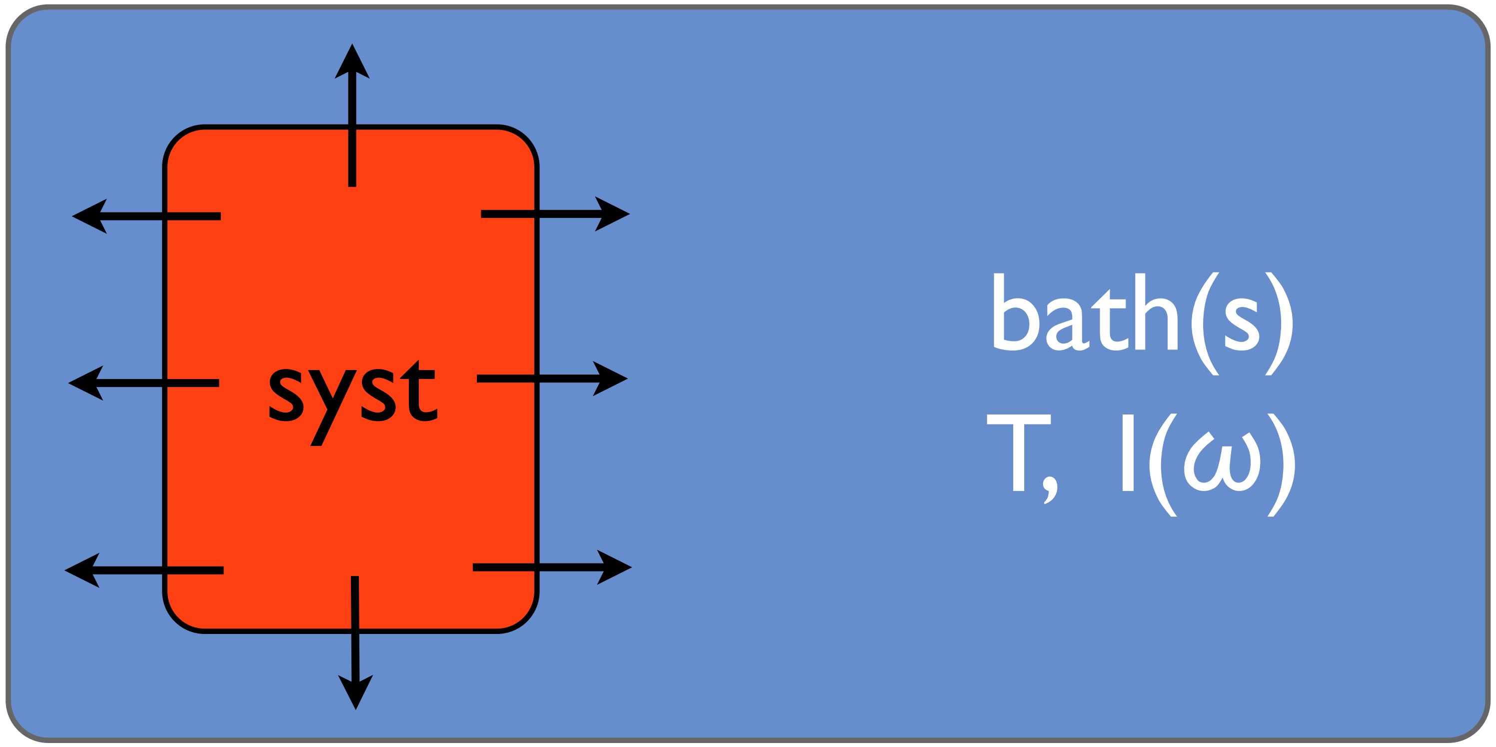
(LPTHE Paris)

GGI, Florence 18.05.12

Non-equilibrium dynamics

Classical/quantum open systems

$$H = H_{syst} + H_{bath} + H_{int}$$



- Transport
- Pumping
- Thermal quenches

  ➔ phase transitions, glasses

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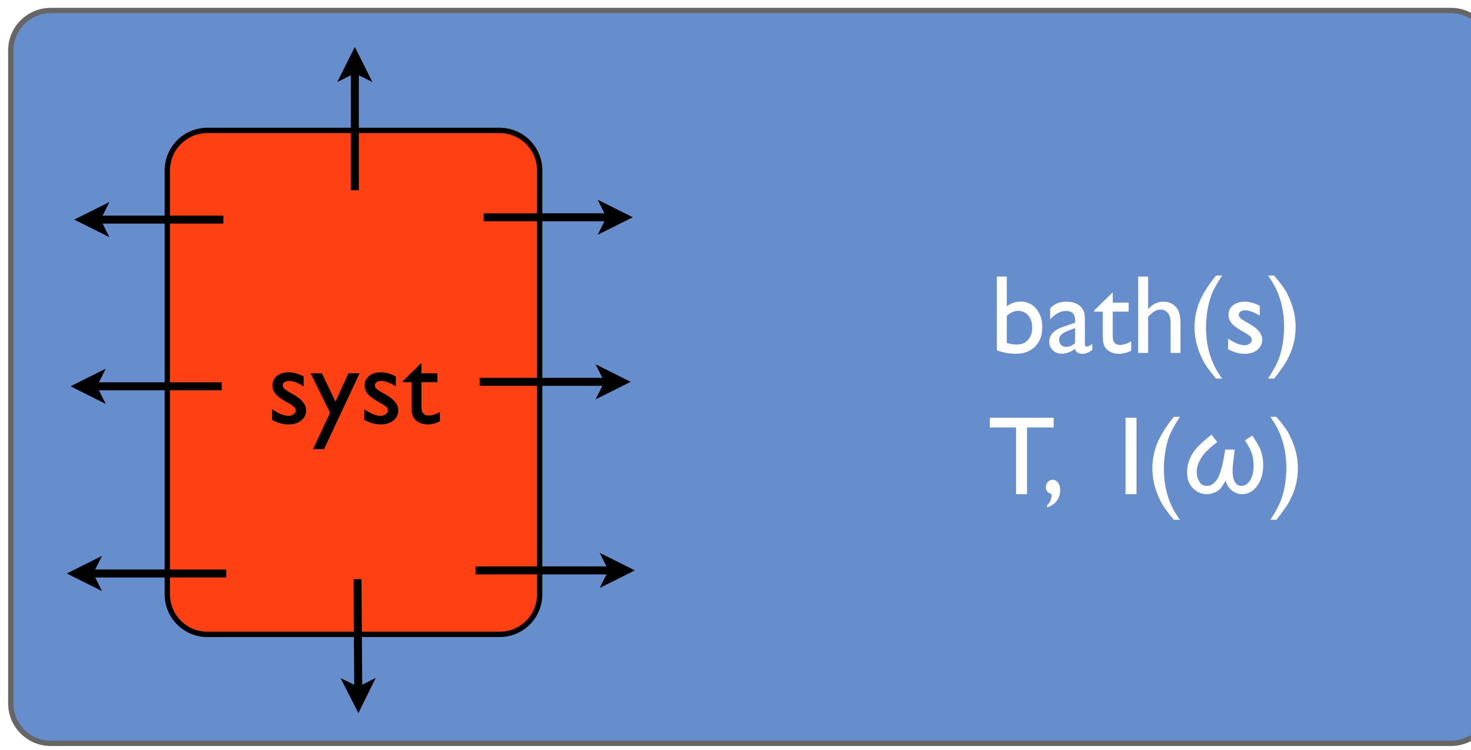

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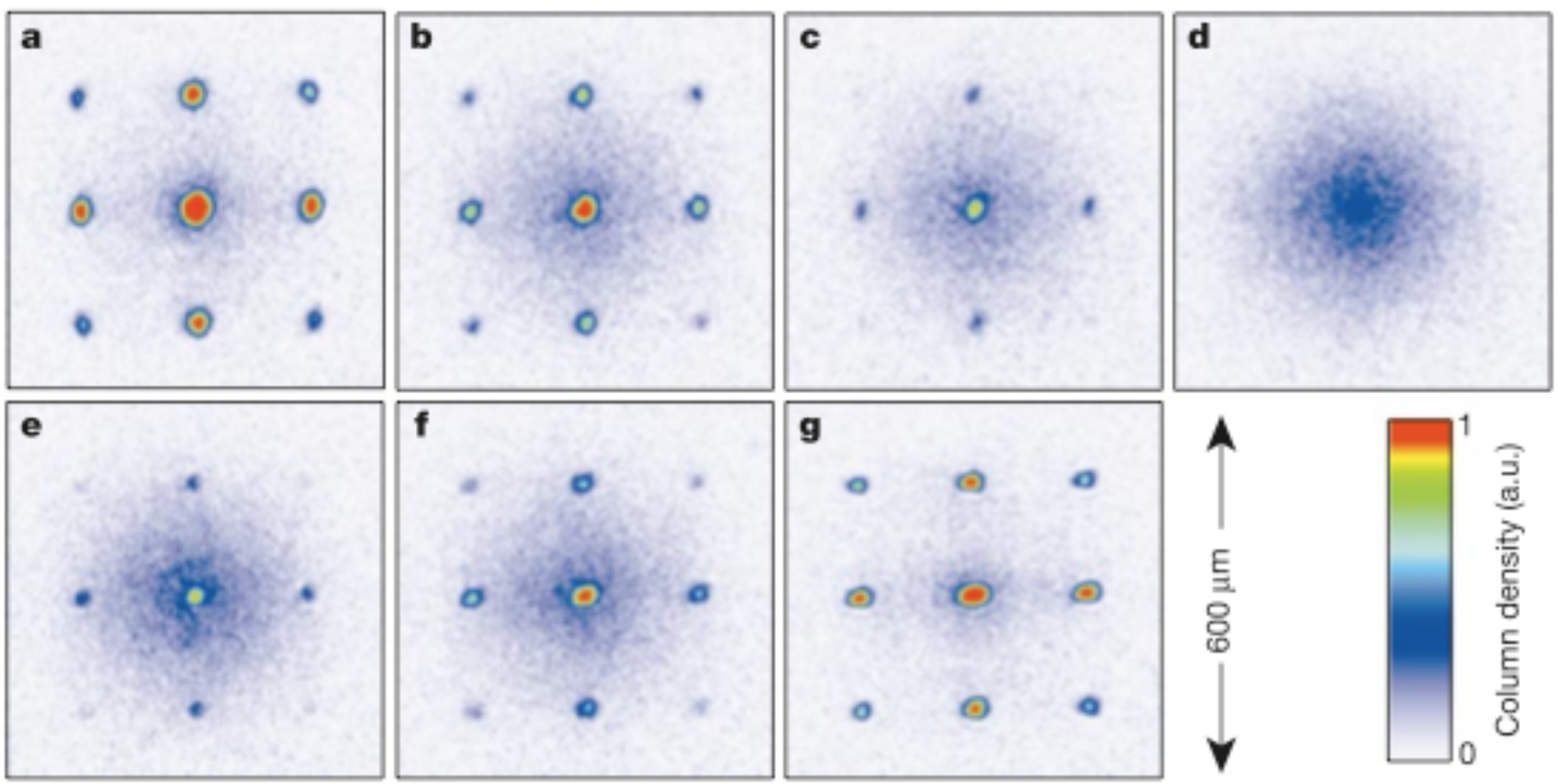
Non-equilibrium dynamics

Quantum isolated systems

Revived interest → ultra cold atoms

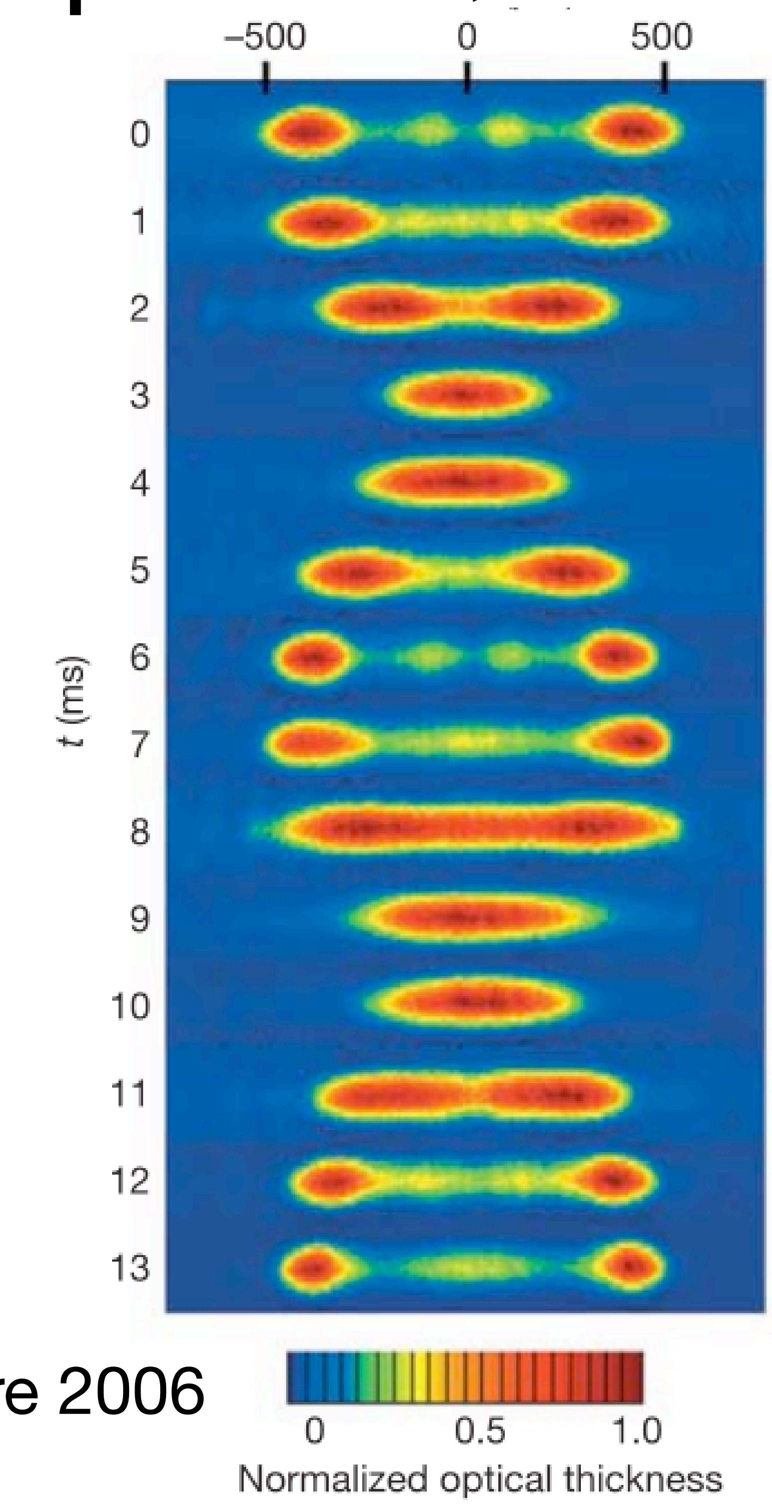
- Control of H
- Coherent dynamics



Greiner et al., Nature 2002

- Approach to equilibrium ➔ Foundation, cosmology
- Quantum annealing: quantum computation, Kibble-Zurek mechanism
- Lack of thermal behavior
  ➔ localization, integrability ...
- Exploration of
  highly excited states

[Polkovnikov,Sengupta,Silva,Vengalattore,RMP’11]



Kinoshita et al., Nature 2006

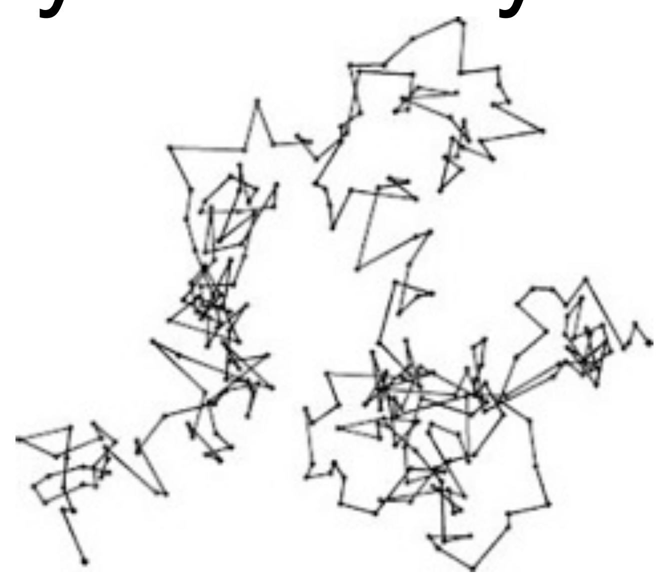
Equilibrium

Fluctuation-dissipation relations:

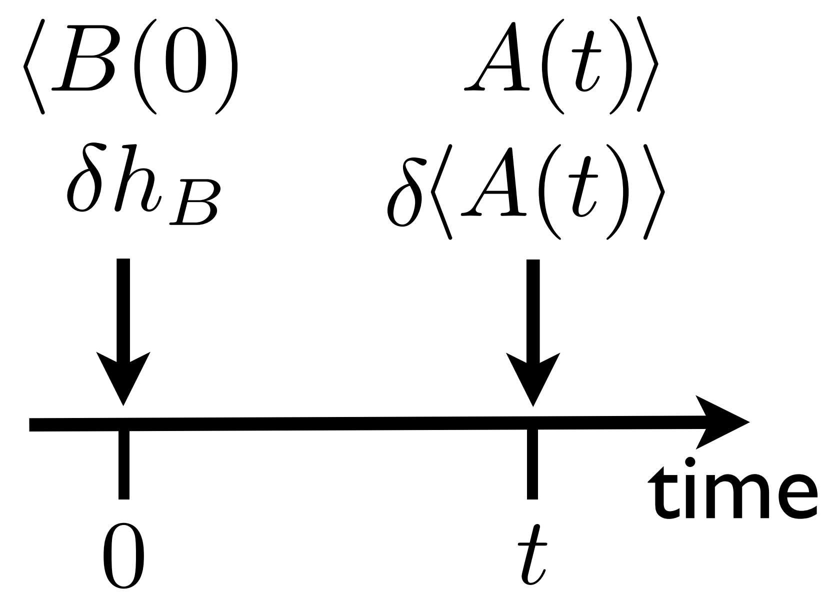
statistical averages ⇔ dynamical responses

(diffusivity/viscosity ...)

$$D \propto \frac{T}{\eta}$$



$$-\frac{\mathrm{d}}{\mathrm{d}t} C_{AB}(t) \underset{\forall A,B}{\propto} T R_{AB}(t)$$



Fluctuation-dissipation relations:

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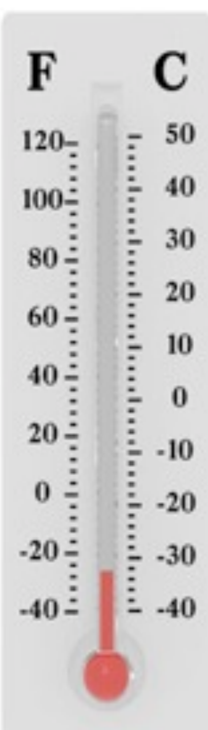

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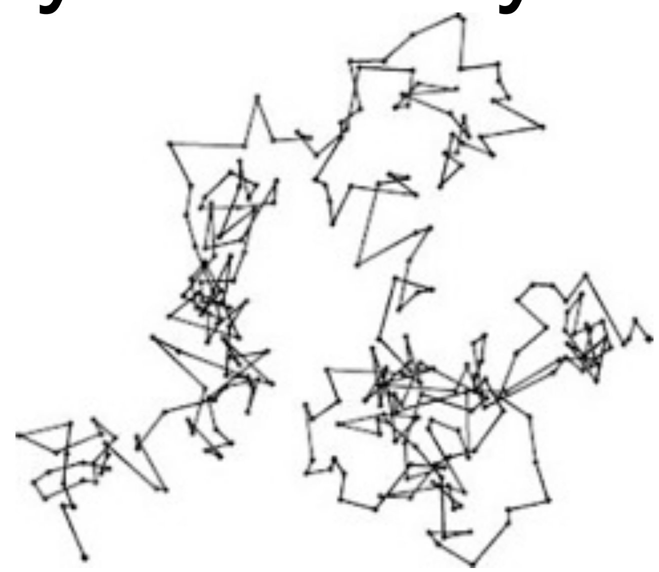

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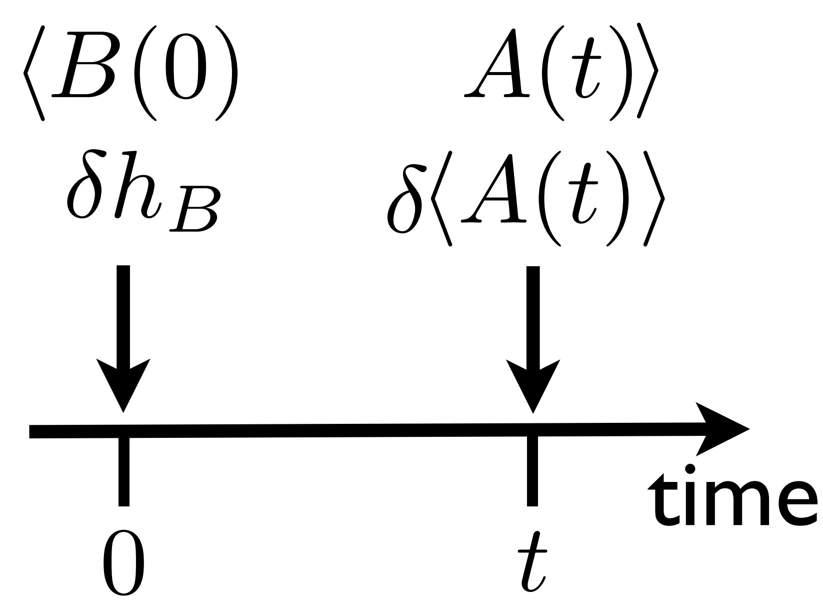

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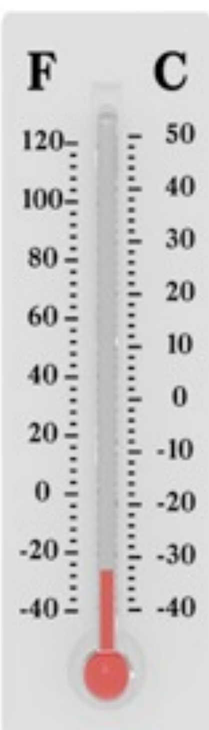

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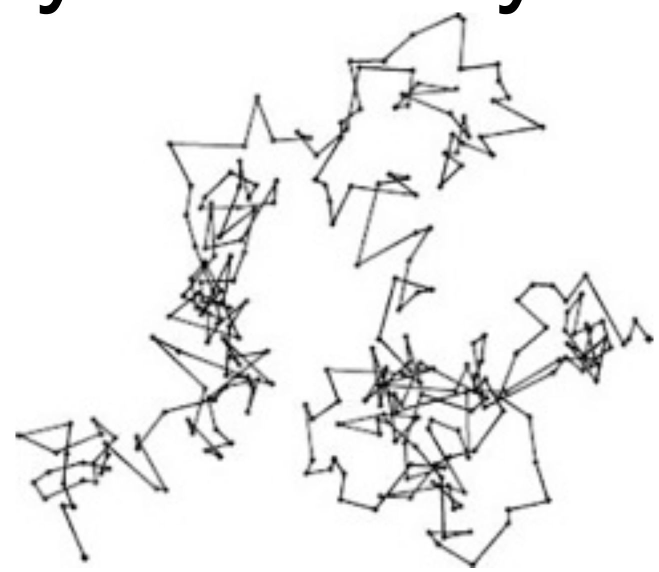

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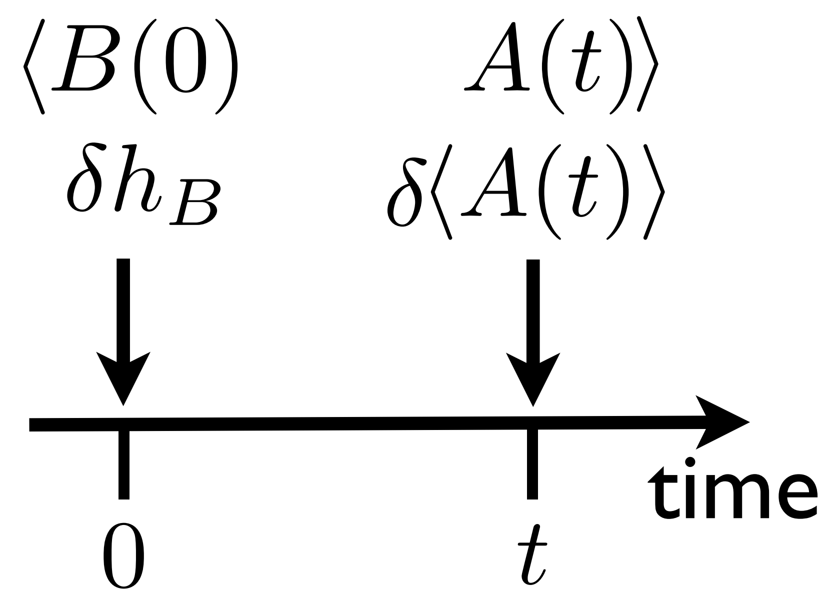


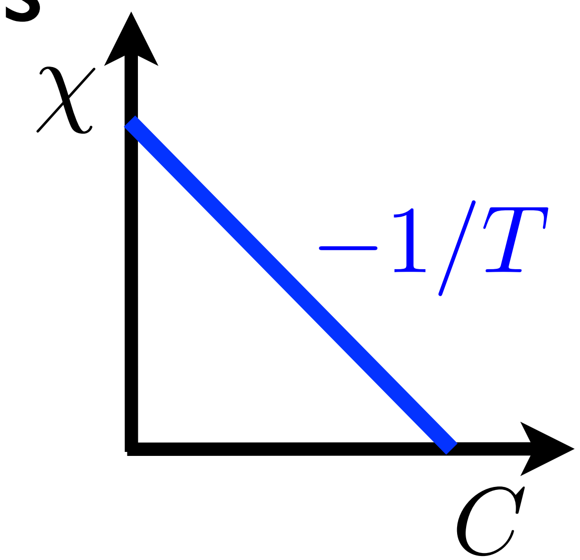
Non-equilibrium ⇒ ~~FDR~~

Temperature ?

Effective temperature

Dissipative (classical) dynamics

$$\chi(t-t_w) \equiv \int_{t_w}^{t} \mathrm{d}t' \, R(t,t') = \frac{1 - C(t-t_w)}{k_B T}$$



Effective temperature

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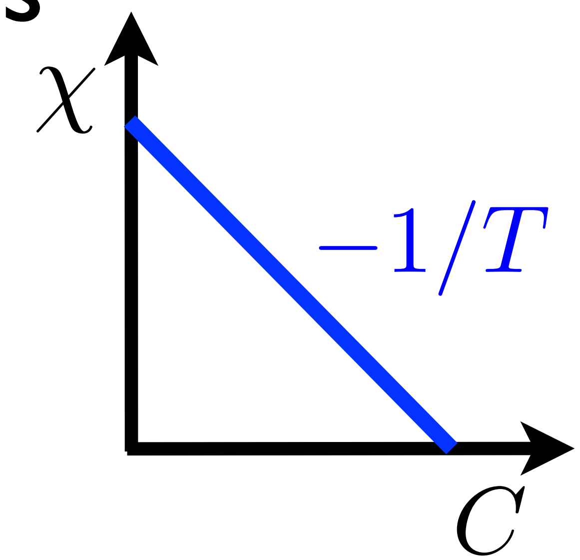

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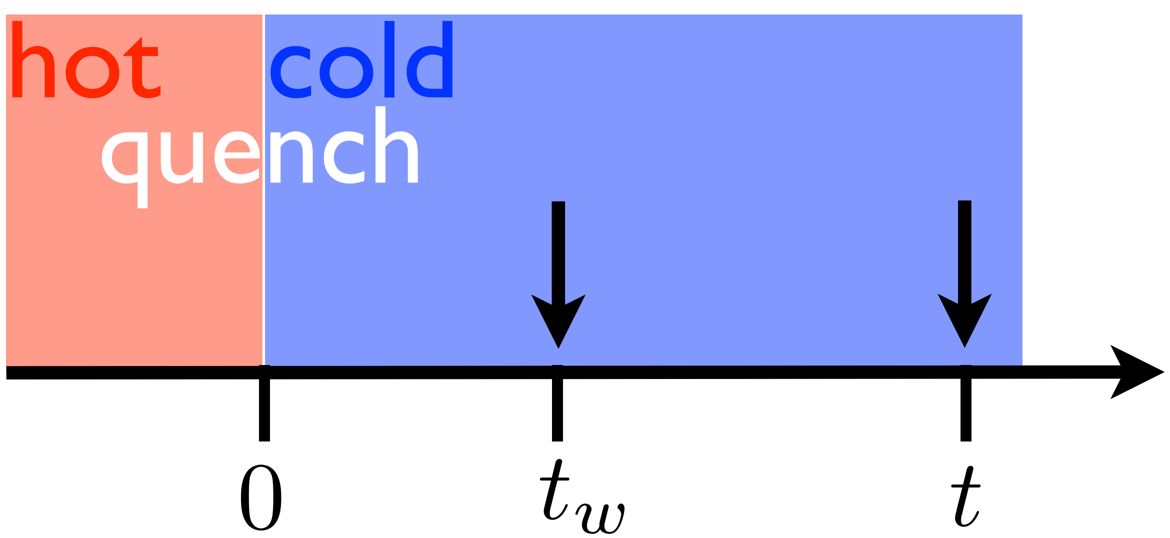
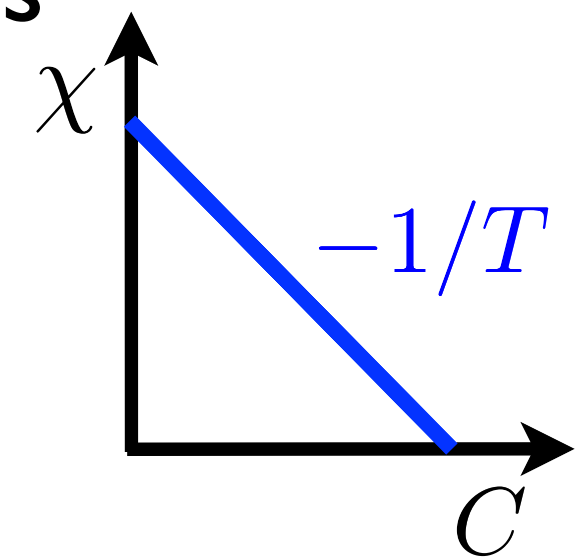
Glasses display slow dyn. & aging



$$\chi(t, t_w) \qquad C(t, t_w)$$

Effective temperature

Dissipative (classical) dynamics



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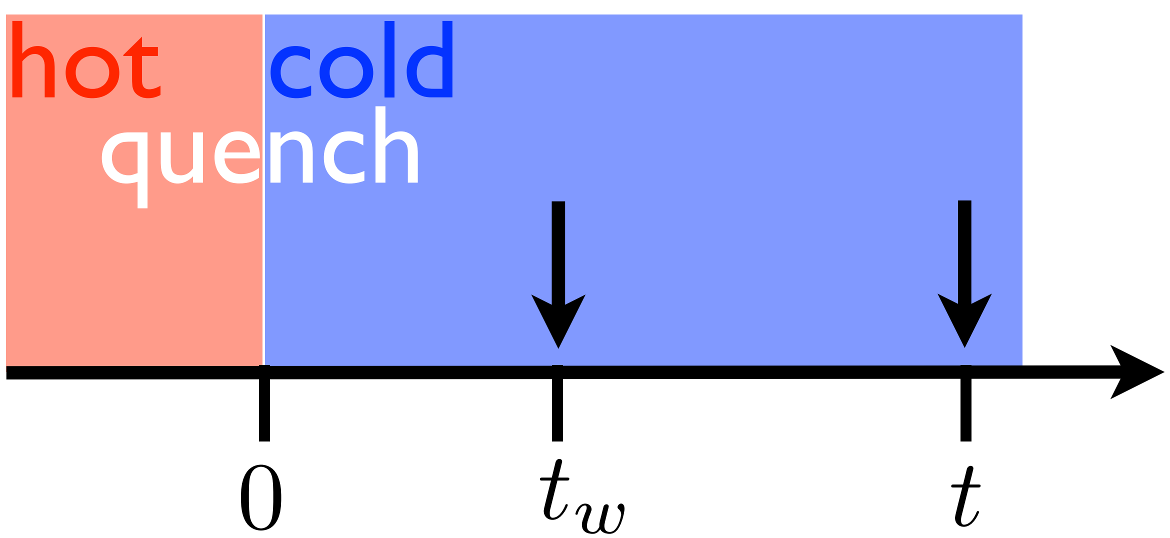

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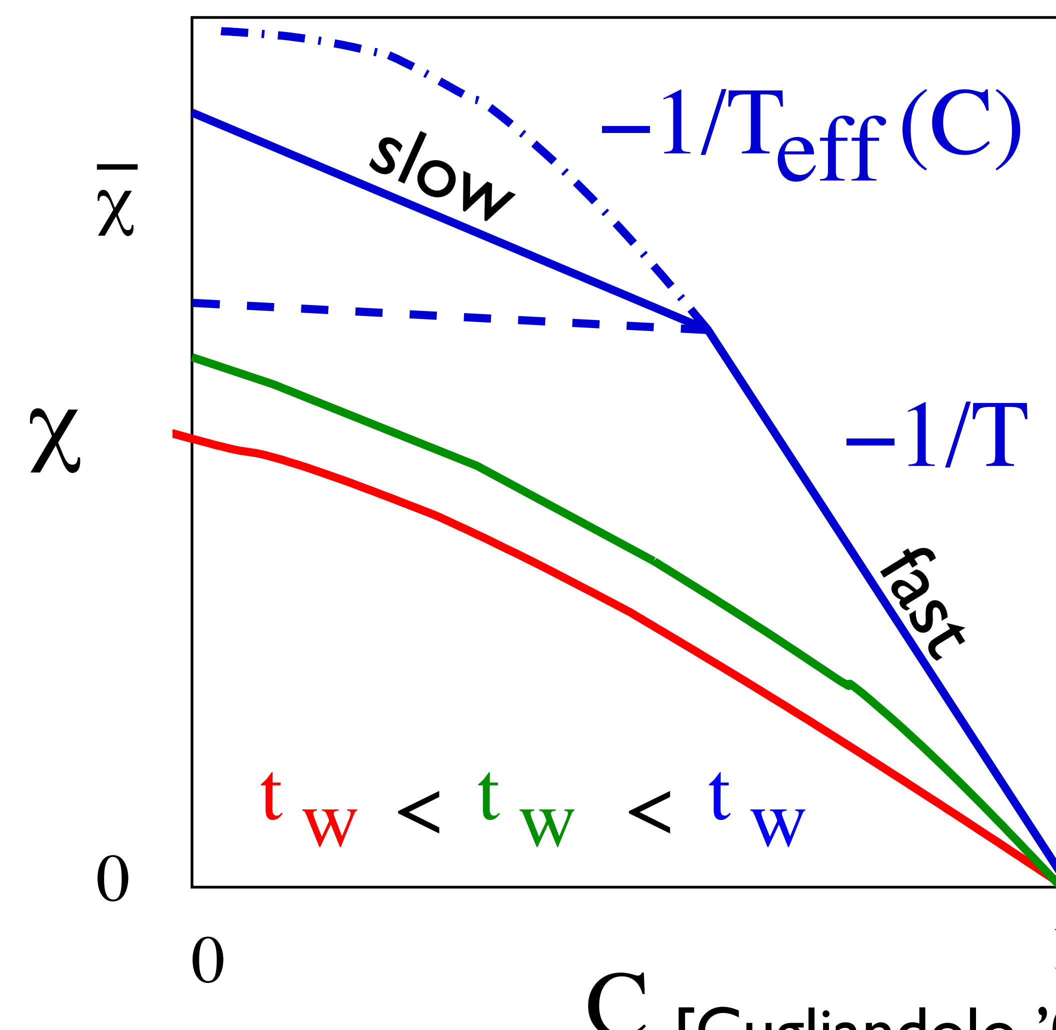


[Cugliandolo '02]

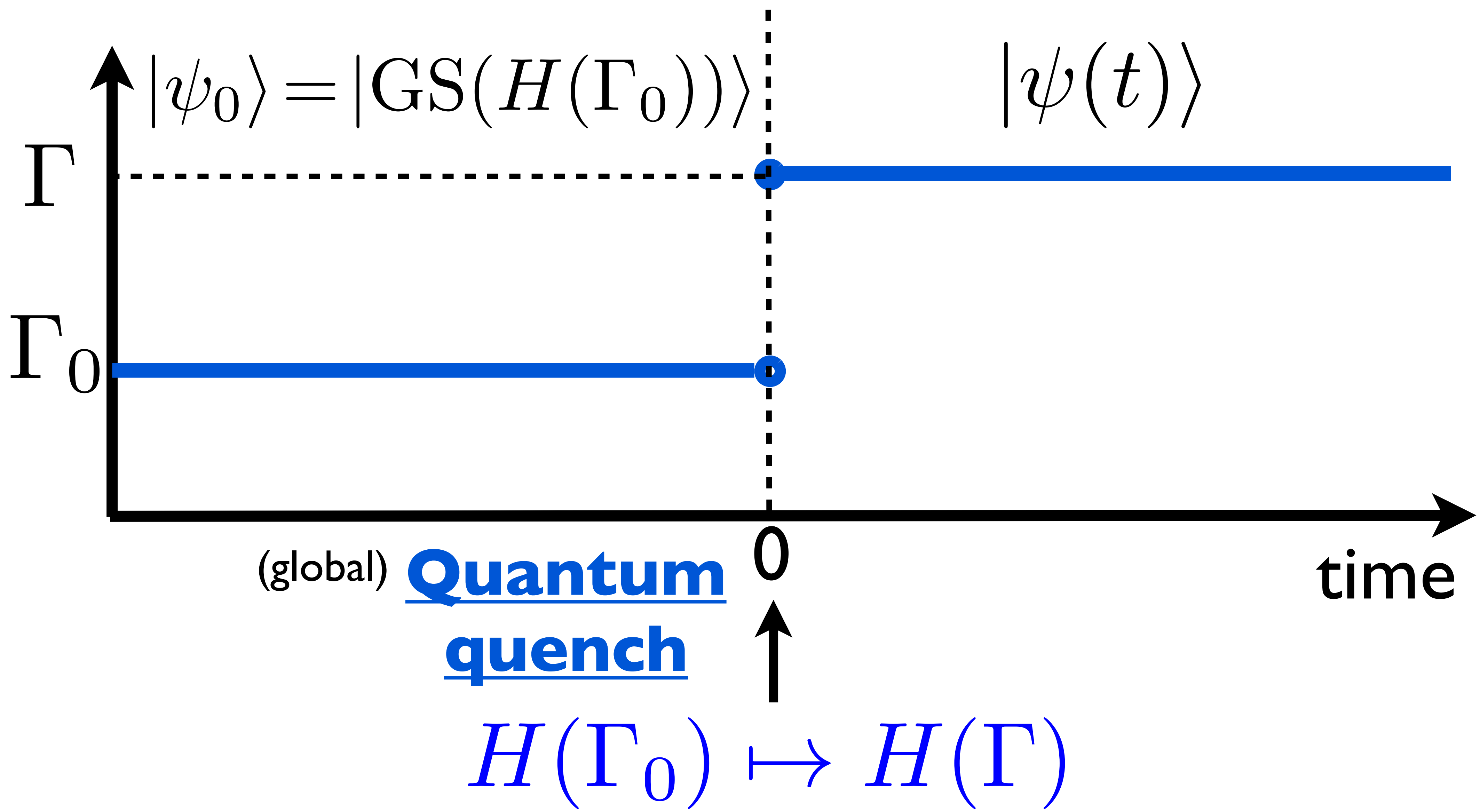
Partial equilibration ➔ T_{eff}

2 time scale regimes

Quantum dynamics of isolated systems

Unitary

$$|\psi(t)\rangle = \mathrm{e}^{-iH(\Gamma)t}|\psi_0\rangle$$



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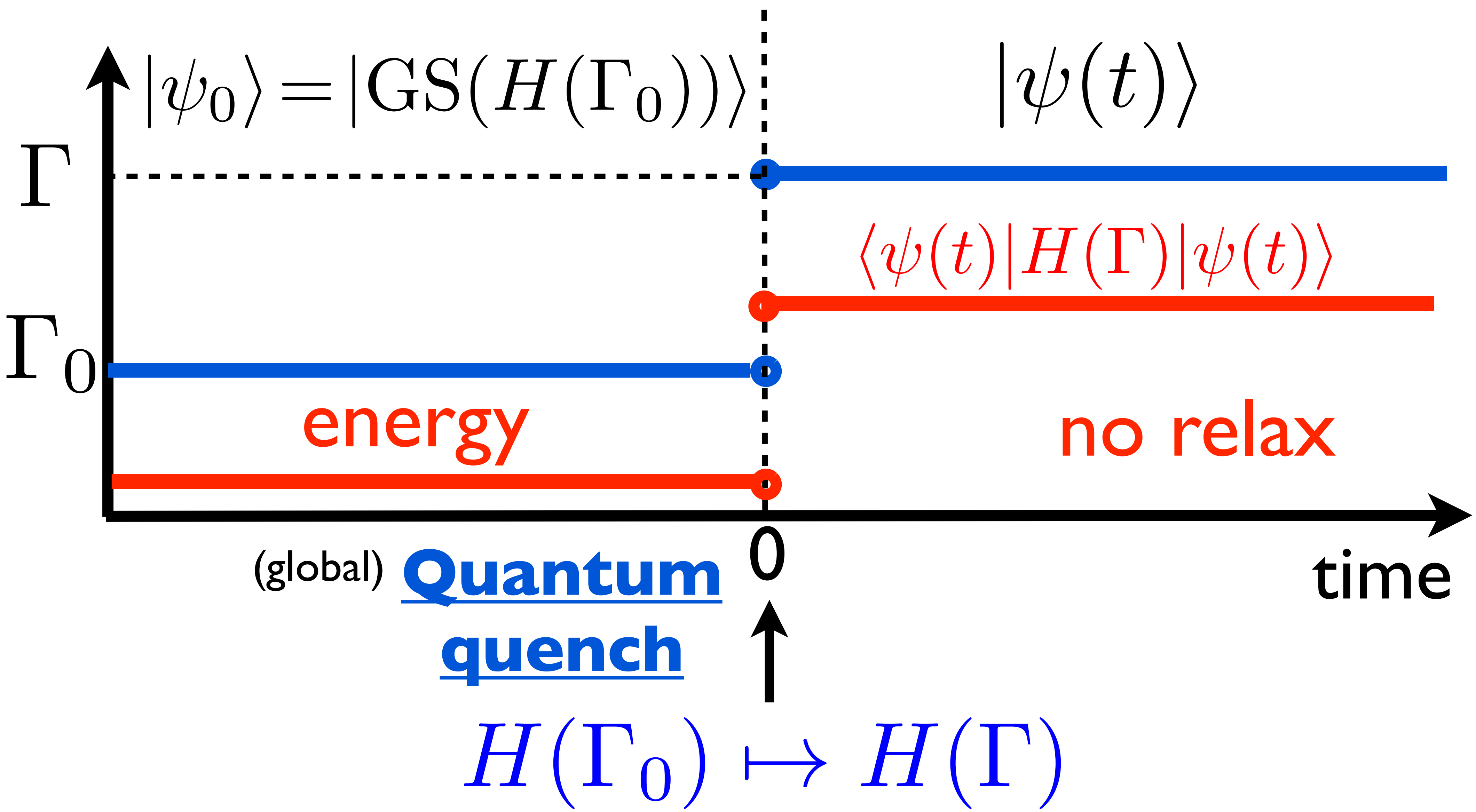

Issues...

- Dynamical transitions

Biroli, Sciolla - Schirò, Fabrizio - Gambassi, Calabrese -...

stationary state?

- Long-lived non-equilibrium states

Kinoshita, Wenger, Weiss - Kollath, Lauchli, Altman -...

dimensionality?

cons. laws?

- Relaxation of inhomogeneity, aging

Shutz, Trimper - Iglói, Rieger - Carleo, Becca, Schirò, Fabrizio - ...

integrability?

- Thermalization

Srednicki - Biroli,Kollath,Lauchli - Rigol,Dunjko,Olshanii - Berges - Calabrese,Cardy - Rossini,Silva,Mussardo,Santoro - Calabrese,Essler,Fagotti -...

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generic ⇒ Gibbs (self-bath)

T ?

effective temp.s ?

Effective temp.s

Quench

$L \to \infty$

$$\langle \psi(t) | \mathcal{O} | \psi(t) \rangle \xrightarrow{t \to \infty} \langle \mathcal{O} \rangle_{\mathcal{Q}}$$

$(?\exists)$

Gibbs

$$\rho \propto \mathrm{e}^{-H(\Gamma)/T}$$

$$\mathrm{Tr}[\rho \mathcal{O}]$$

Effective temp.s

Quench

Gibbs

$$\rho \propto \mathrm{e}^{-H(\Gamma)/T}$$

$$L \to \infty$$

$$\langle \psi(t)|\mathcal{O}|\psi(t)\rangle \xrightarrow{t\to\infty} \langle \mathcal{O} \rangle_{\mathcal{Q}} \quad = \quad \mathrm{Tr}[\rho\mathcal{O}] \quad \Rightarrow \quad T$$

$$(?\exists)$$

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$(?\exists)$

$H(\Gamma)$

=

Gibbs

$\rho \propto \mathrm{e}^{-H(\Gamma)/T}$

$\mathrm{Tr}[\rho\mathcal{O}] \Rightarrow T$

$\Rightarrow T^E$

(Γ, Γ_0)

Effective temp.s

Quench

$L \to \infty$

$$\langle \psi(t)|\mathcal{O}|\psi(t)\rangle \xrightarrow{t\to\infty} \langle \mathcal{O}\rangle_{\mathcal{Q}}$$

$(?\exists)$

$H(\Gamma)$

$\mathcal{O} \longrightarrow f_{\mathcal{Q}}(\Gamma, \Gamma_0)$

$=$

$\overset{?}{=}$

Gibbs

$\rho \propto \mathrm{e}^{-H(\Gamma)/T}$

$\mathrm{Tr}[\rho\mathcal{O}] \Rightarrow T$

$\Rightarrow T^E(\Gamma, \Gamma_0)$

$f_G(T^E(\Gamma, \Gamma_0))$

Effective temp.s

Quench | Gibbs

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$H(\Gamma)$

$$\Rightarrow \quad T^E(\Gamma, \Gamma_0)$$

$$\mathcal{O} \longrightarrow f_{\mathcal{Q}}(\Gamma, \Gamma_0) \quad \overset{?}{=} \quad f_G(T^E(\Gamma, \Gamma_0))$$

$$\Rightarrow \quad T^{\mathcal{O}}$$

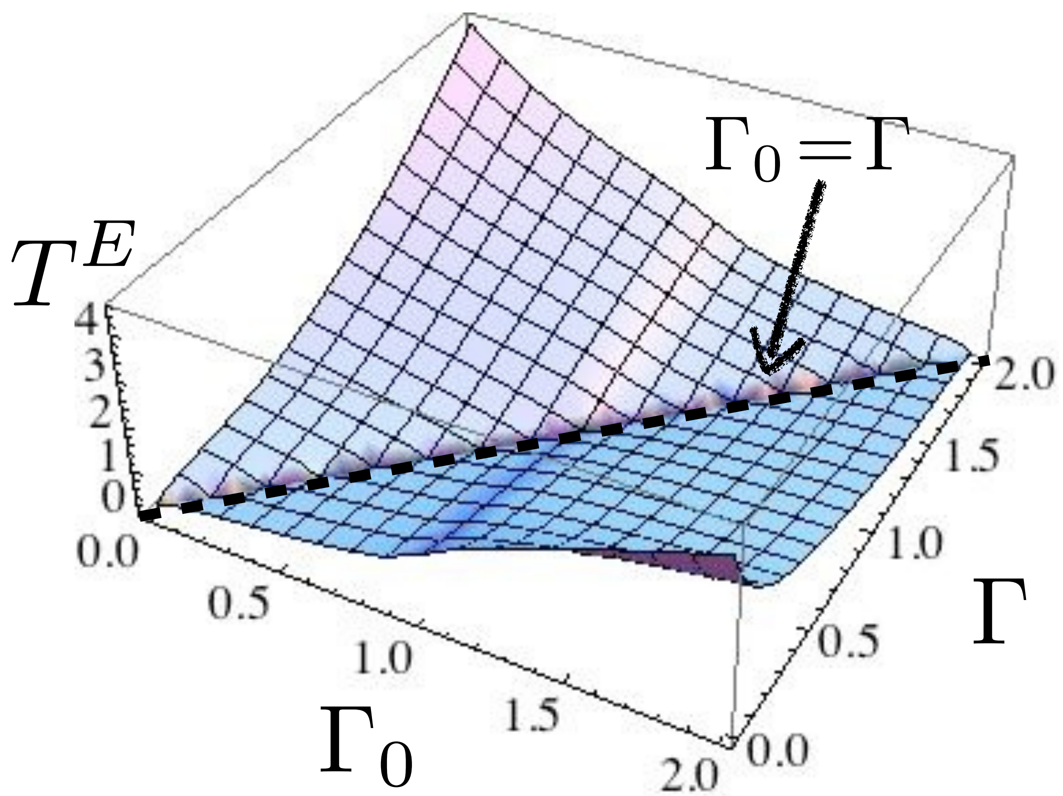
corr. length, coherence time

$\langle \sigma_i \sigma_j \rangle_{\mathcal{Q}} \quad \langle \psi_0 | \sigma_i(0) \sigma_i(t) | \psi_0 \rangle$

integrability - non-locality?

Rossini,Silva,Mussardo,Santoro

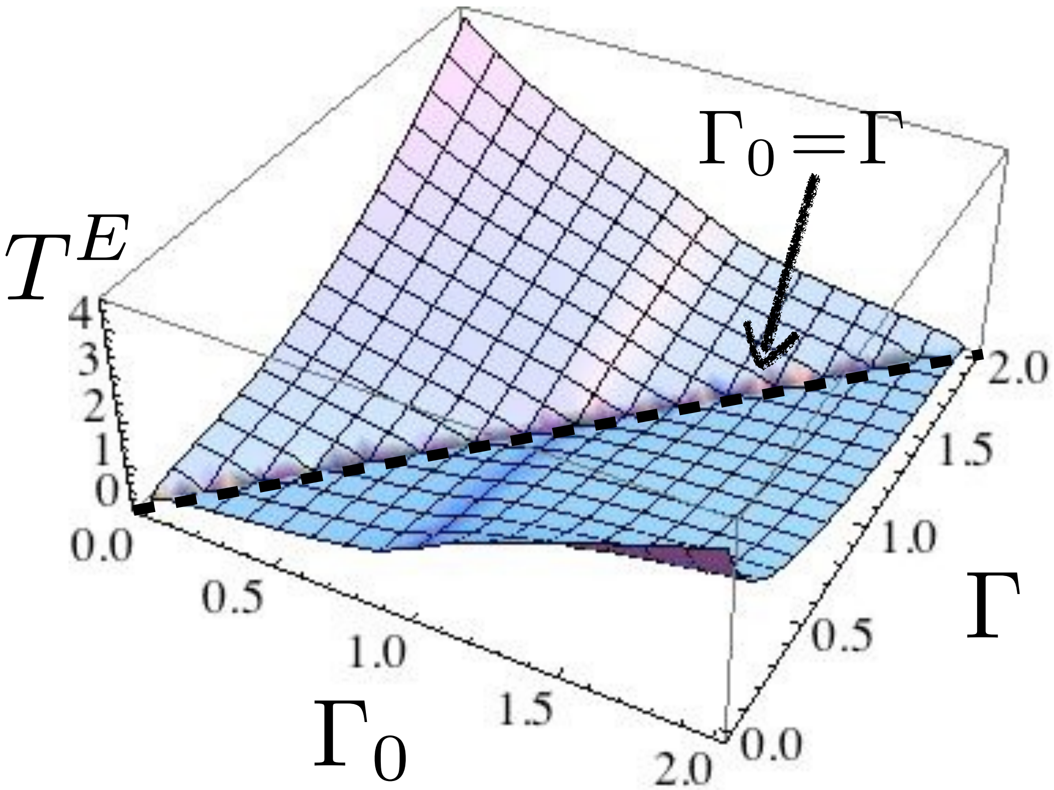
Effective temp.s



1d-lattice model

$\Gamma, \Gamma_0 > 0$

Effective temp.s



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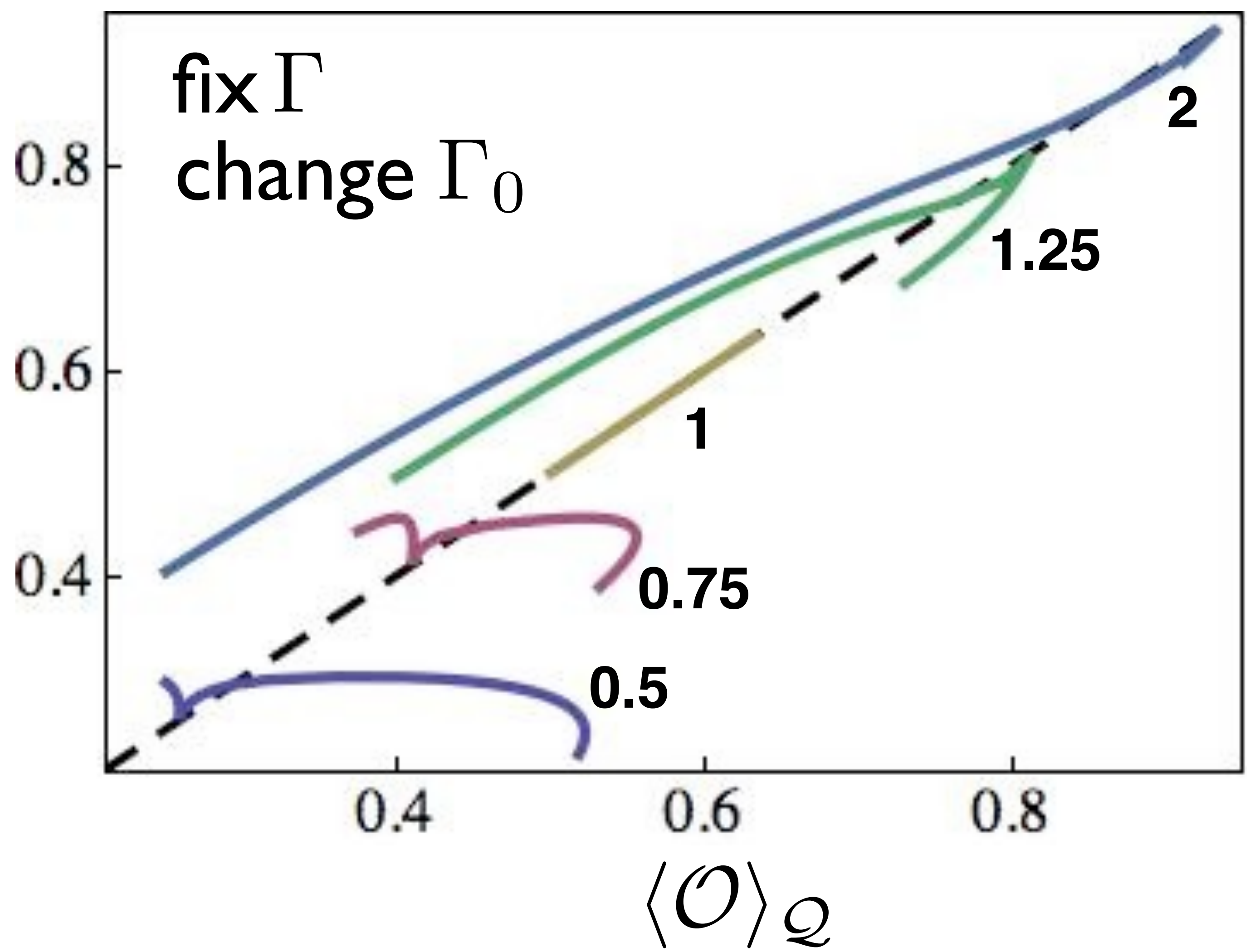
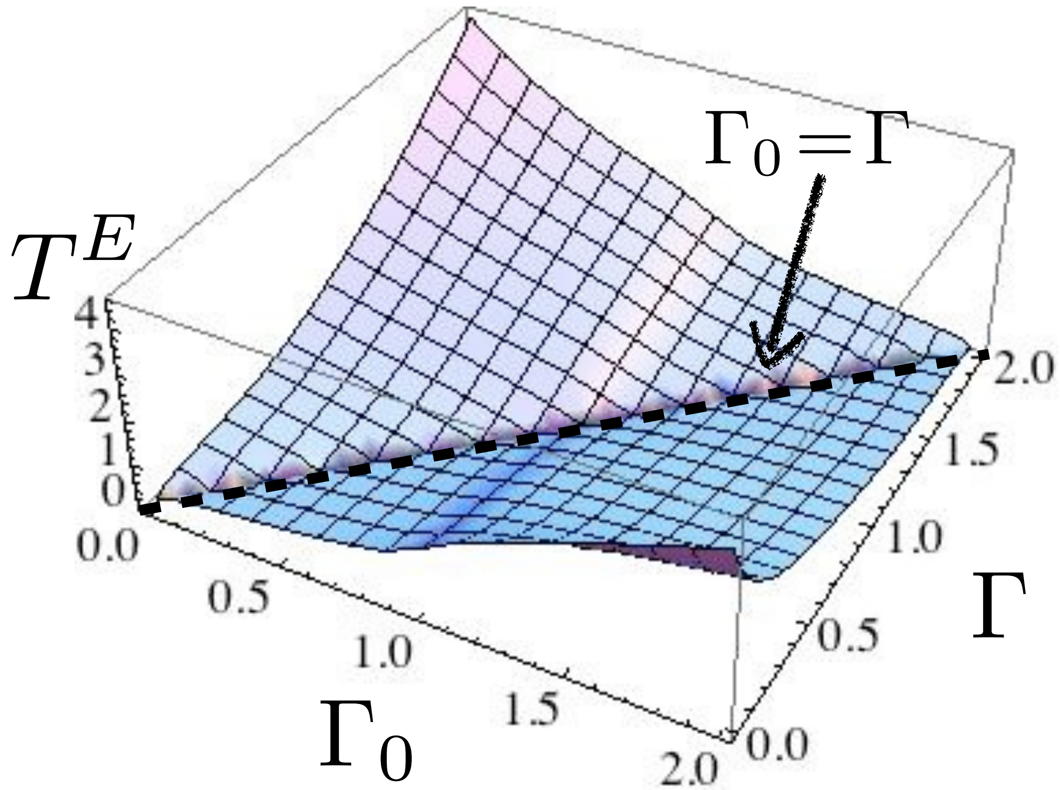
observable $\mathcal{O}$

$\langle \mathcal{O} \rangle_{\text{Gibbs}}$

@ $T = T^E(\Gamma_0, \Gamma)$



Effective temp.s



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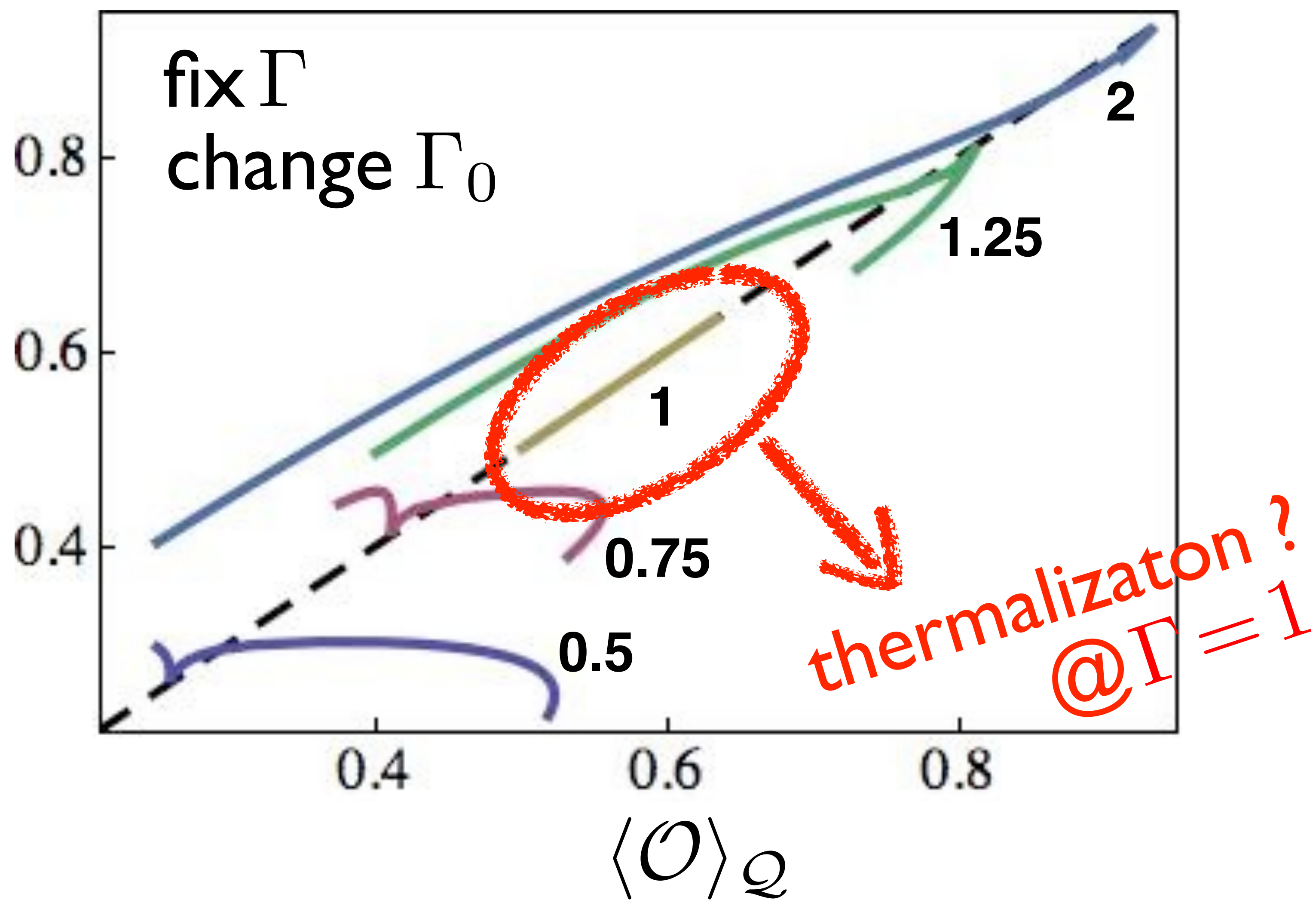

Effective temp.s

Quench

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$$U(t) = \mathrm{e}^{-iHt}$$

quantum FDT

$$\boxed{\frac{\hbar\,\mathrm{Im}\,R(\omega)}{\mathrm{Re}\,C(\omega)} = \mathrm{th}\left(\frac{\hbar\omega}{2T}\right)}$$

$+$

Foini,Gambassi,Cugliandolo PRB **84**, 212404 (2011) + coming soon

Effective temp.s

Quench

$$\langle \psi_0 | \mathcal{O}(t+t_0) \mathcal{O}(t_0) | \psi_0 \rangle \xrightarrow{t_0 \to \infty}$$

$$\theta(t)\,[\qquad , \qquad] \longrightarrow R(t)$$

$$\{\qquad , \qquad\} \longrightarrow C_+(t)$$

stationary regime

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stationary regime

$$\frac{\hbar \, \mathrm{Im}\, R(\omega)}{C_+(\omega)} = \mathrm{th}\left(\frac{\hbar\omega}{2T_{\rm eff}(\omega)} \right)$$

measure!

$\mathcal{O}$-dependent ?
dynamics !

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equilibrium

$\Downarrow$

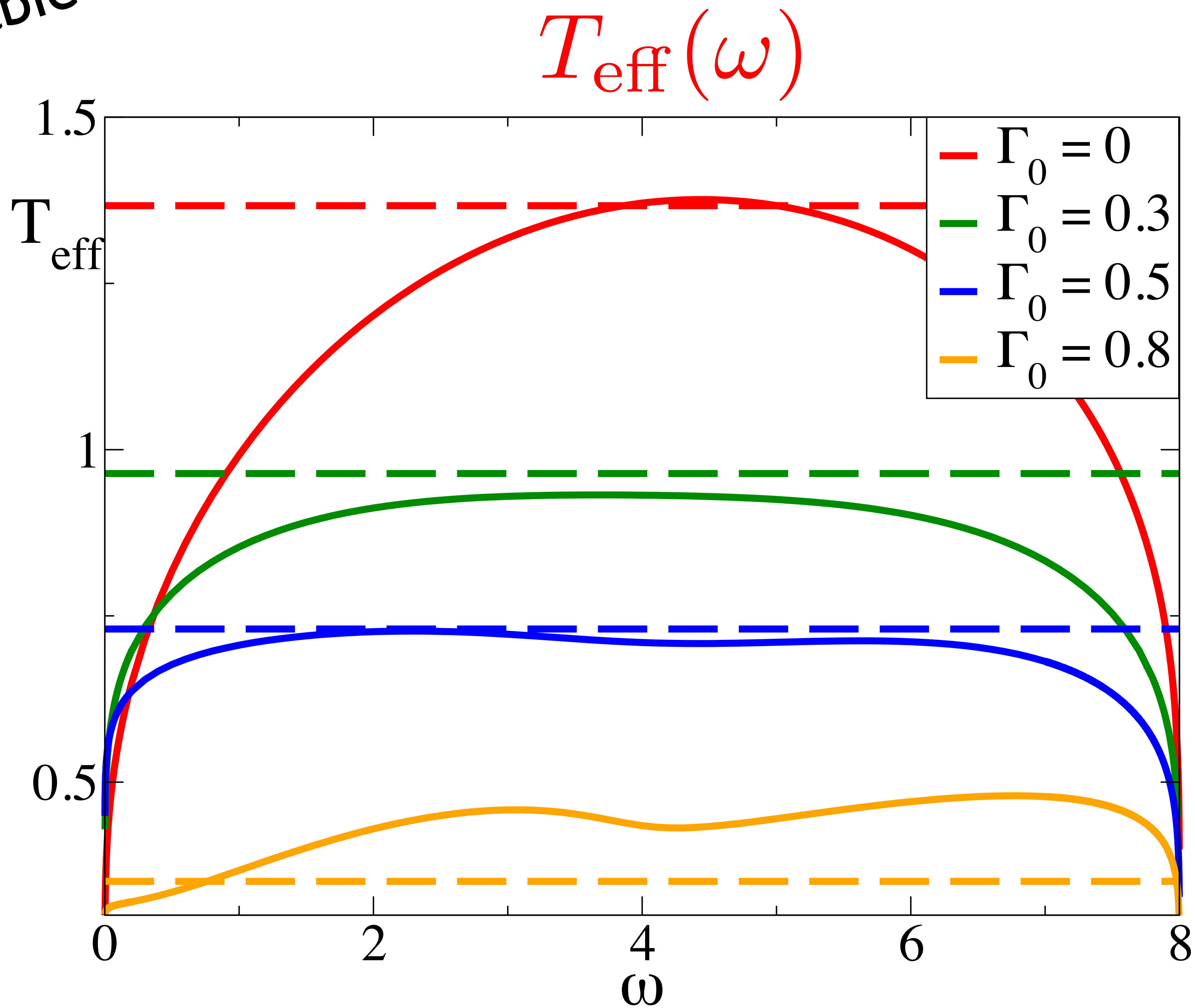
$$T_{\mathrm{eff}}\cancel{(\omega)} = T^E$$

$\mathcal{O}$-independent

Foini, Gambassi, Cugliandolo PRB **84**, 212404 (2011) + coming soon

Effective temp.s

focus on $\Gamma = 1$
observable $\mathcal{O}$



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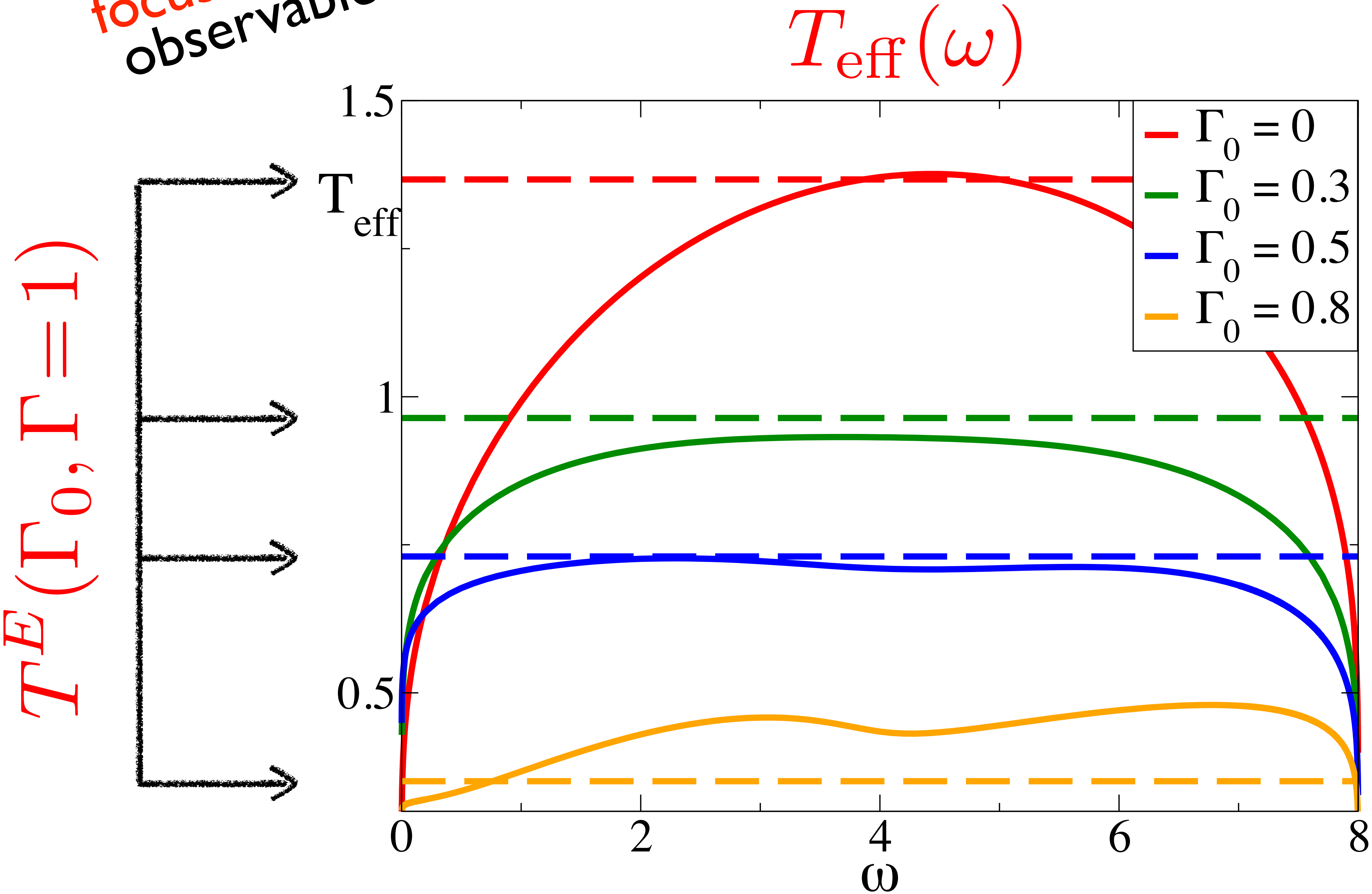
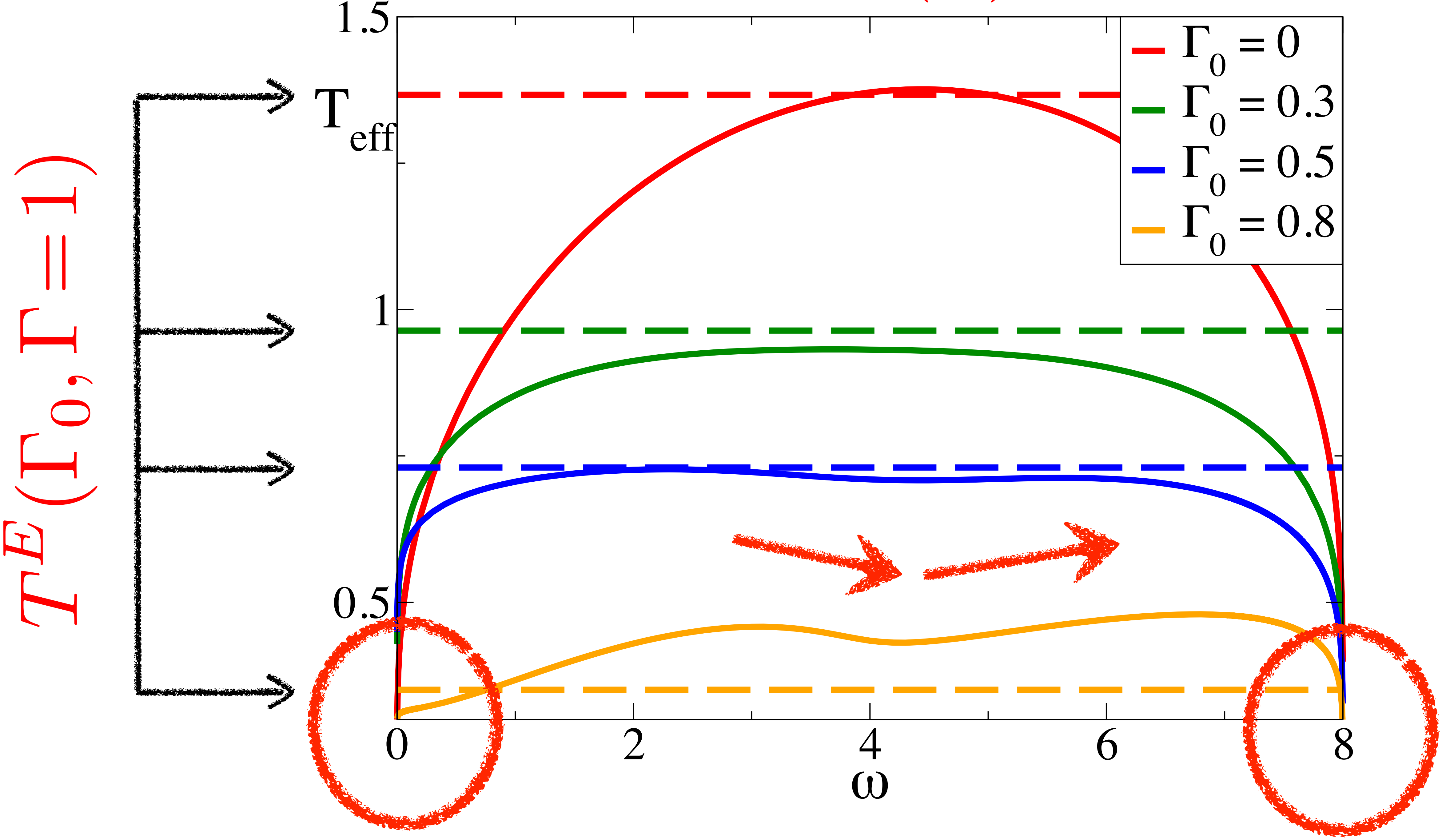


Effective temp.s

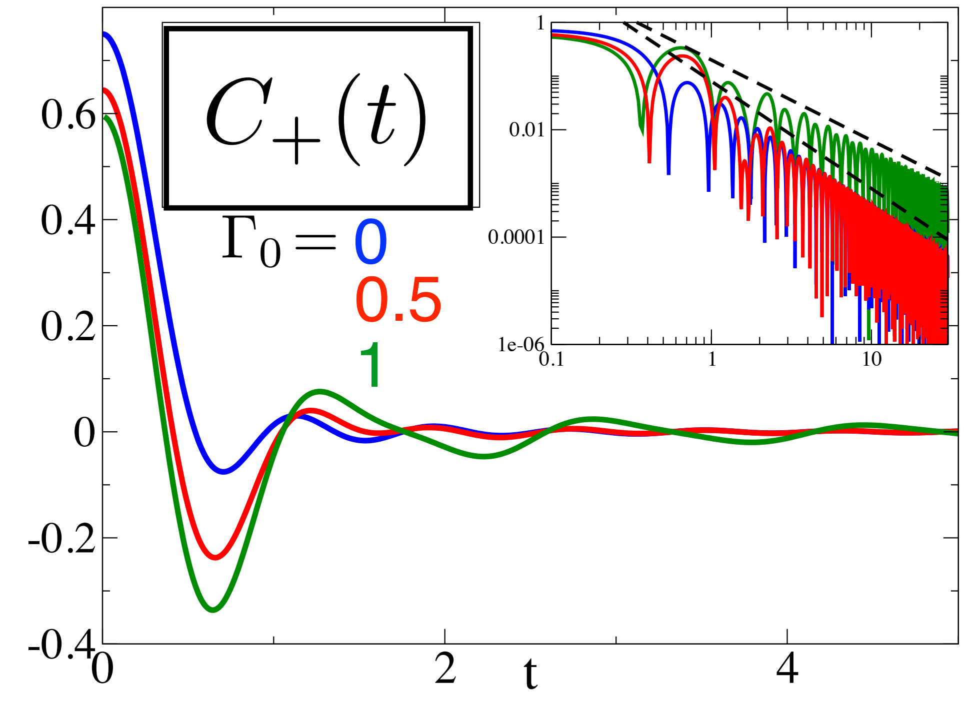
focus on $\Gamma = 1$
observable $\mathcal{O}$

$T_{\rm eff}(\omega)$



$R(t), C_+(t)$ *invar.* $\Gamma_0 \leftrightarrow 1/\Gamma_0$

focus on $\Gamma = 1$
observable $\mathcal{O}$



$$C_+(t) = -\frac{1}{4\pi t^2}\cos(8t) + O(t^{-3})$$

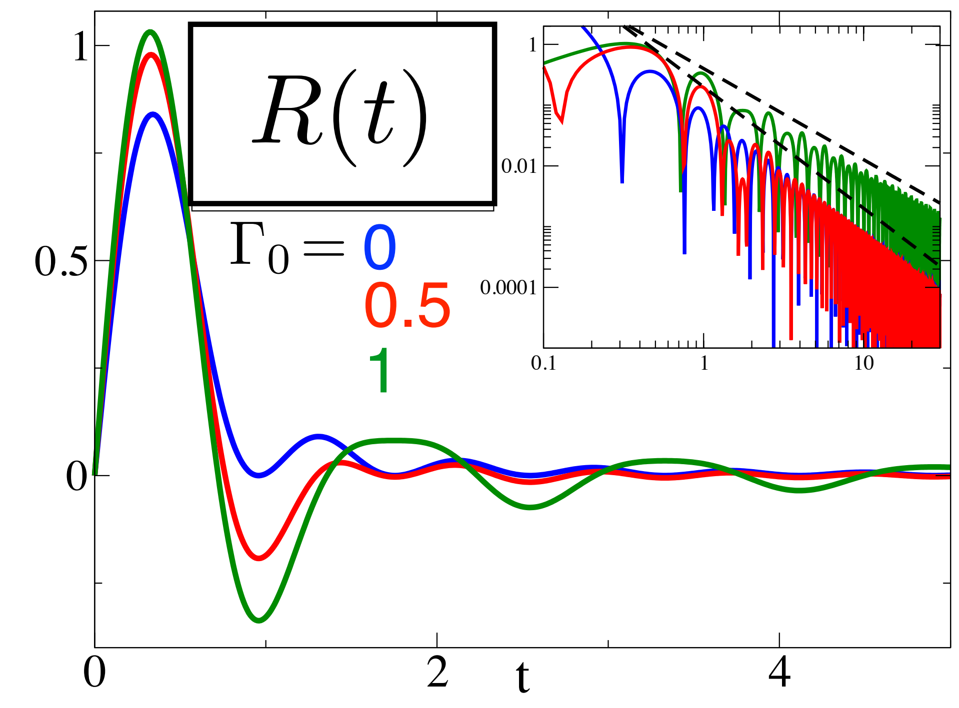
$\Gamma_0 = 1 : \sim t^{-3/2}$

$$R(t) = \frac{1}{4\pi t^2}[\Upsilon(\Gamma_0) - \sin(8t)] + O(t^{-3})$$

$$\Upsilon(\Gamma_0) = \left(\frac{1-\Gamma_0}{1+\Gamma_0}\right)^2$$

eq. @$T \neq 0$

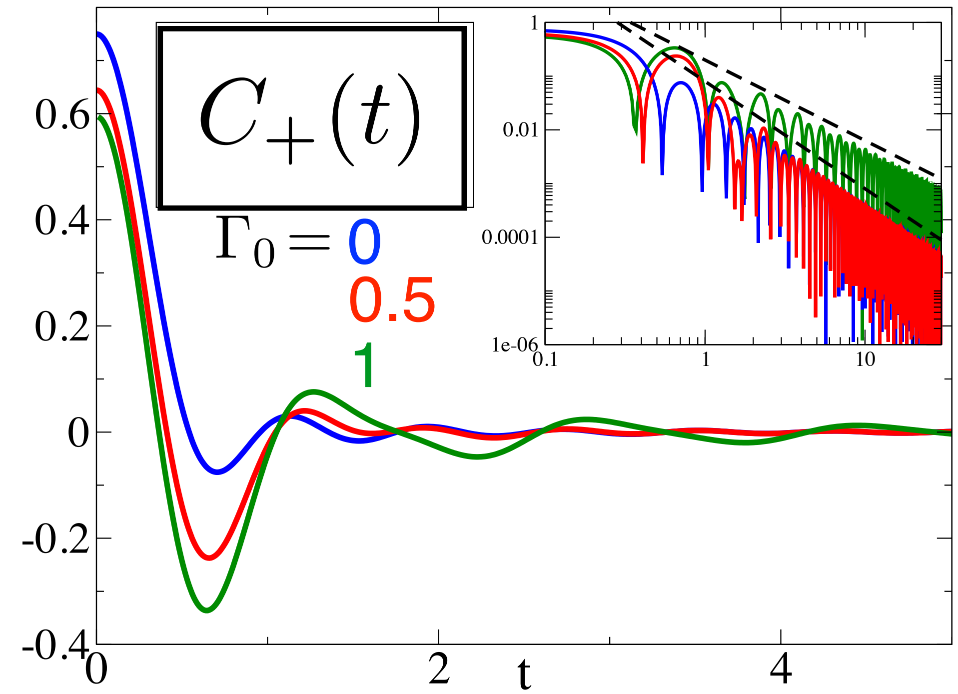
$$C_+(t), R(t) \propto \frac{1}{t}$$



[Niemeijer '67]

focus on $\Gamma = 1$
observable $\mathcal{O}$

slow relaxation



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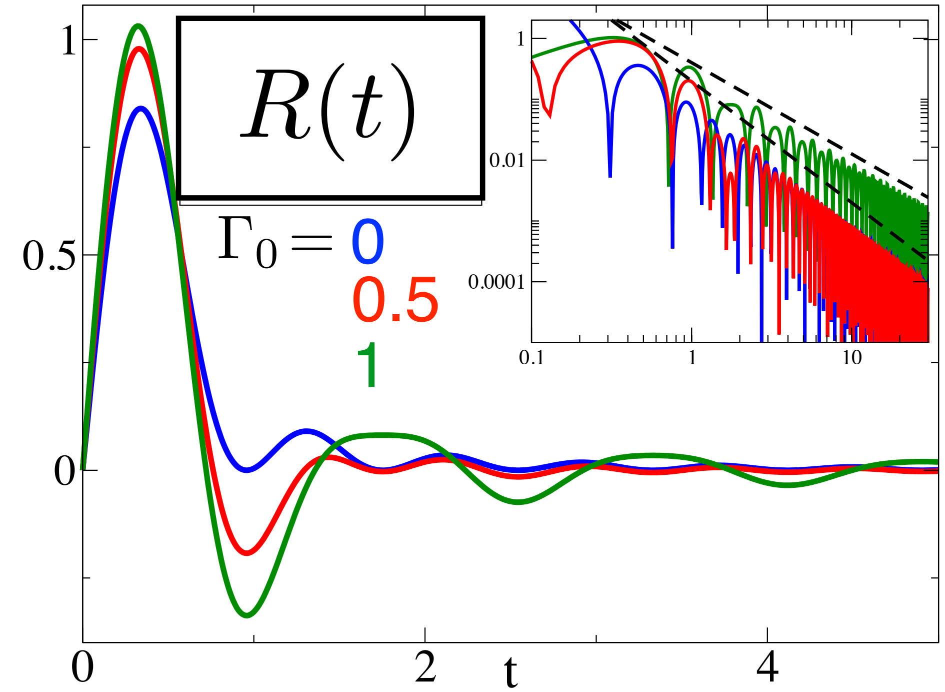

no coher. time
no match

[Niemeijer '67]

Effective temp.s

$$\langle H \rangle_{\mathcal{Q}} = \langle H \rangle_{\mathrm{Gibbs}} \quad \Longrightarrow T^E$$

static

observable $\mathcal{O}$

$$\langle \mathcal{O} \rangle_{\mathcal{Q}} = \langle \mathcal{O} \rangle_{\mathrm{Gibbs}@T=T^E} \quad \leadsto \text{thermalization ?}$$

Effective temp.s

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static

observable $\mathcal{O}$

$$\langle \mathcal{O} \rangle_{\mathcal{Q}} = \langle \mathcal{O} \rangle_{\text{Gibbs}@T=T^E} \qquad \rightsquigarrow$$ thermalization ?

dynamics

$R^{\mathcal{O}}(t), C^{\mathcal{O}}_{+}(t)$ Gibbs FDT $\Longrightarrow T^{\mathcal{O}}_{\text{eff}}(\omega)$

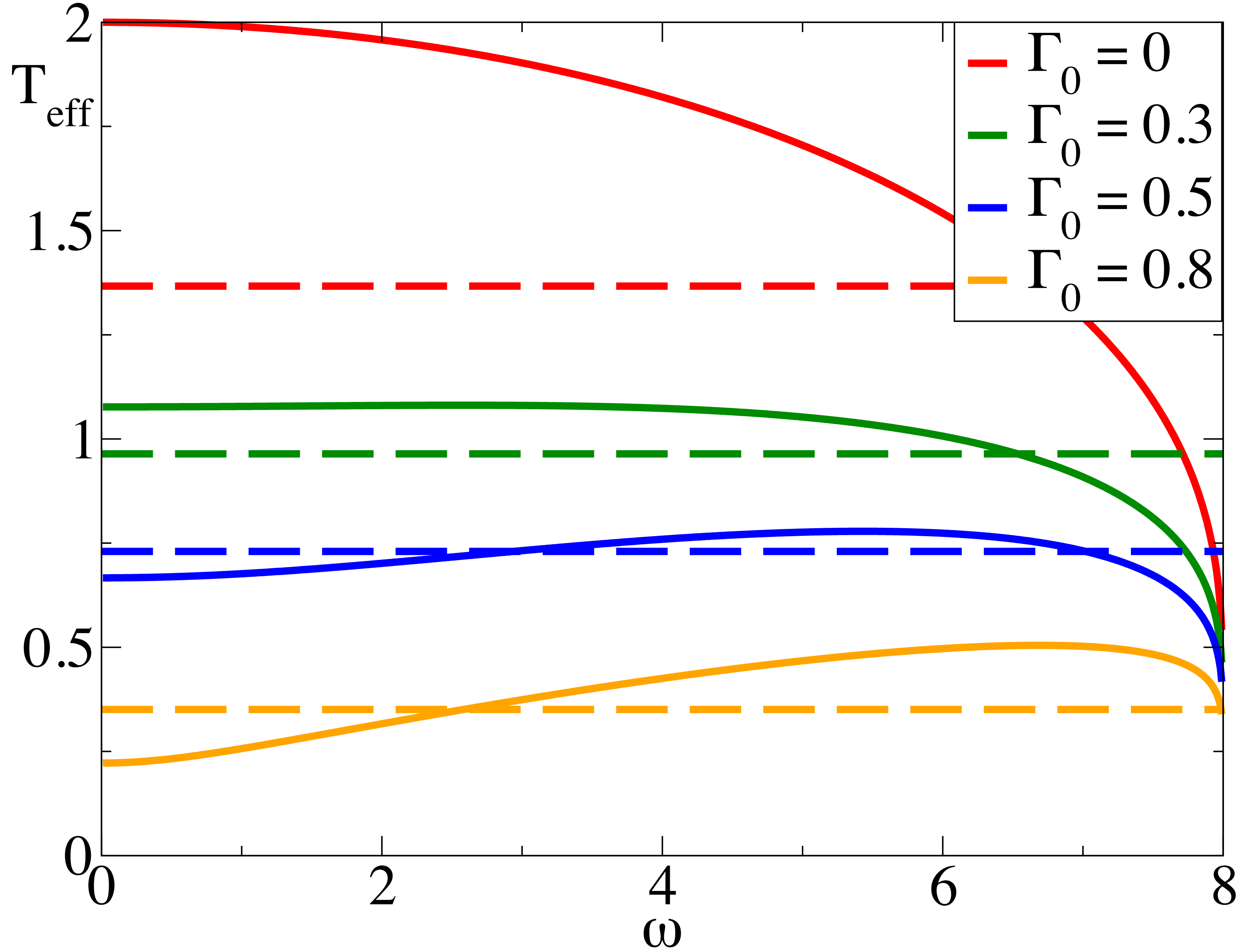
...not really....

... generic ?

Effective temp.s

focus on $\Gamma = 1$
observable $\mathcal{O}'$
$\mathcal{O}' \neq \mathcal{O}$

$T_{\rm eff}(\omega)$



Effective temp.s

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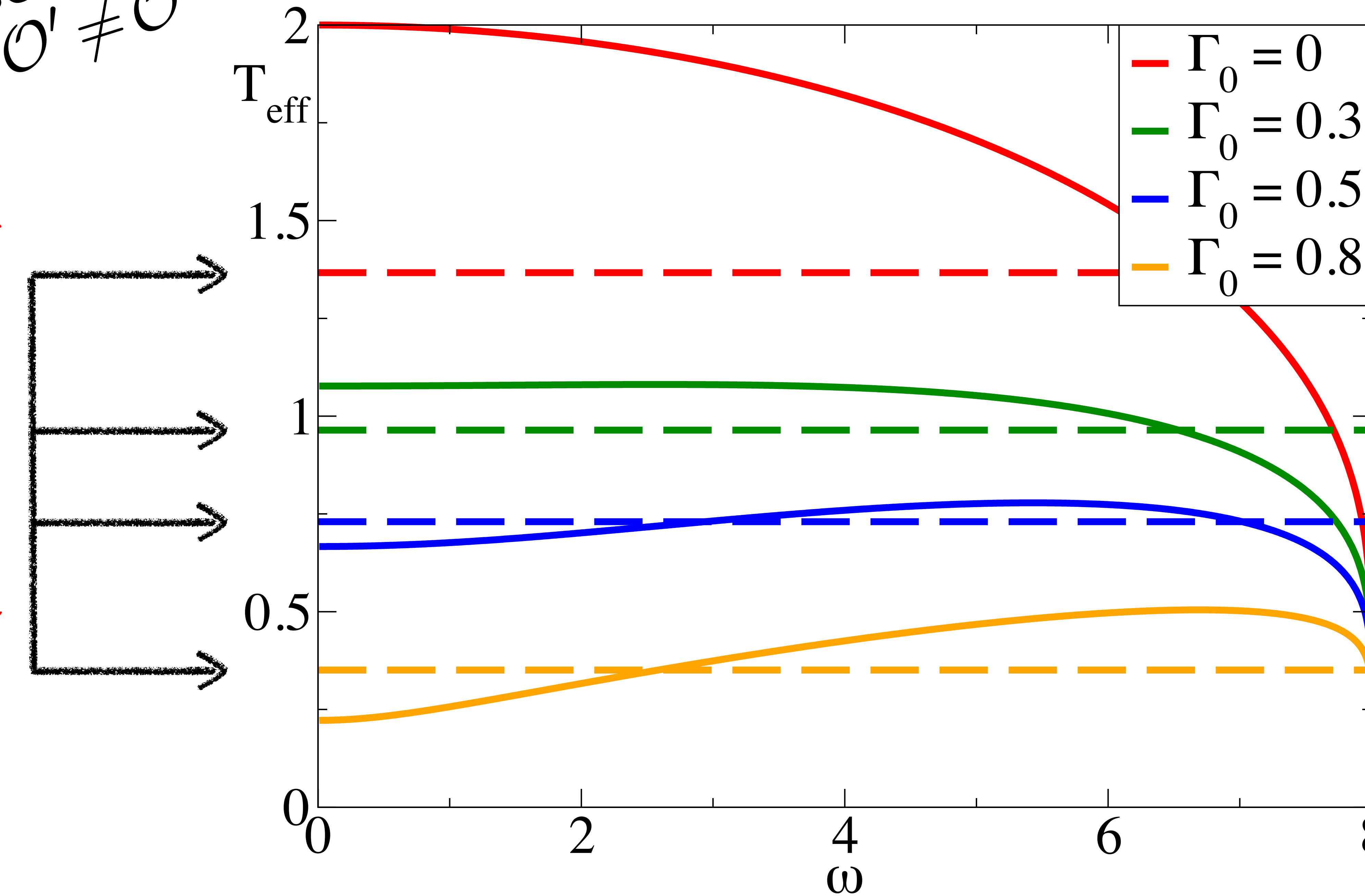
$T_{\rm eff}(\omega)$

$T^E(\Gamma_0, \Gamma = 1)$



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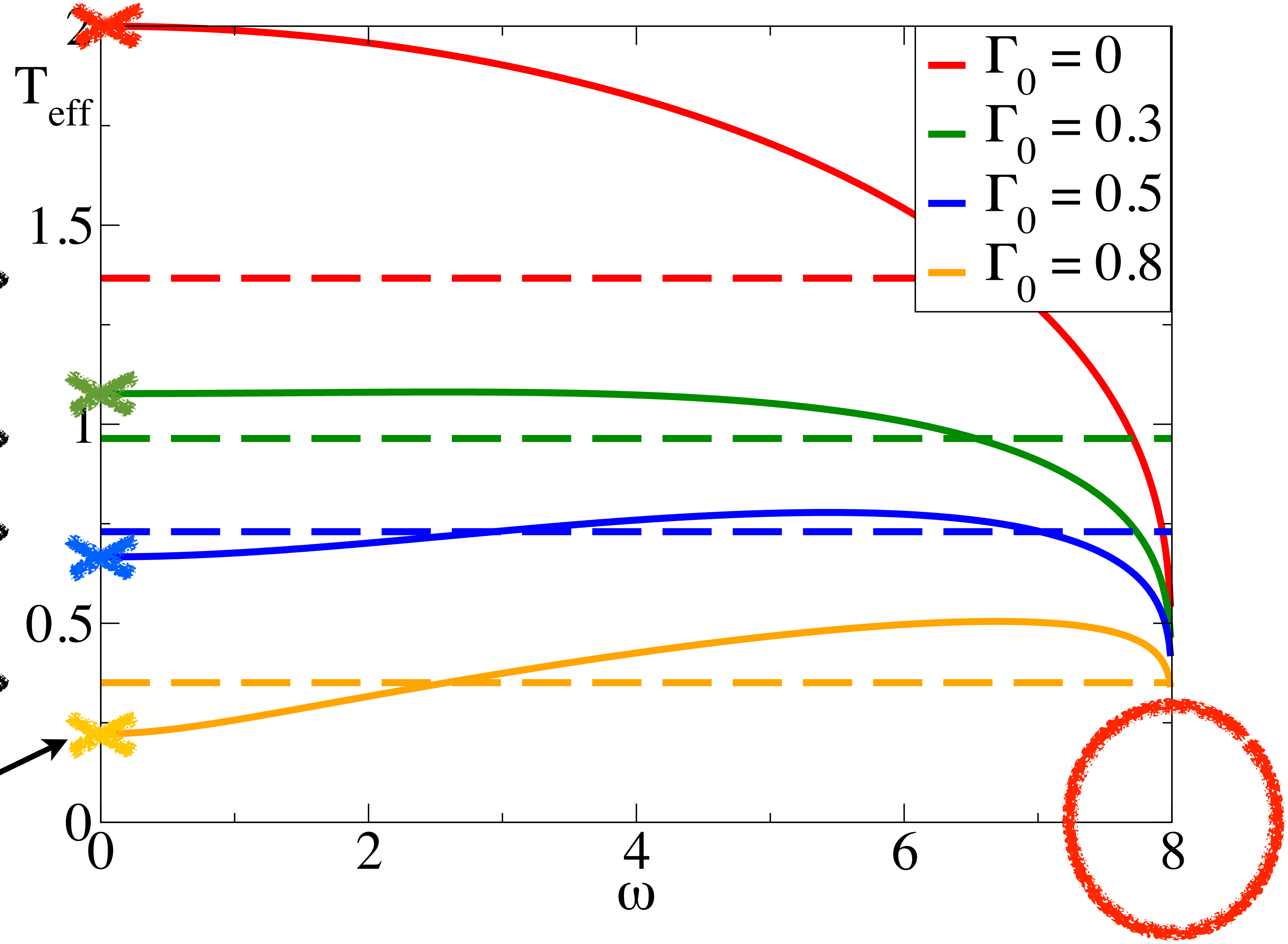
$T^E(\Gamma_0, \Gamma = 1)$



$2\frac{|1-\Gamma_0|}{1+\Gamma_0}$

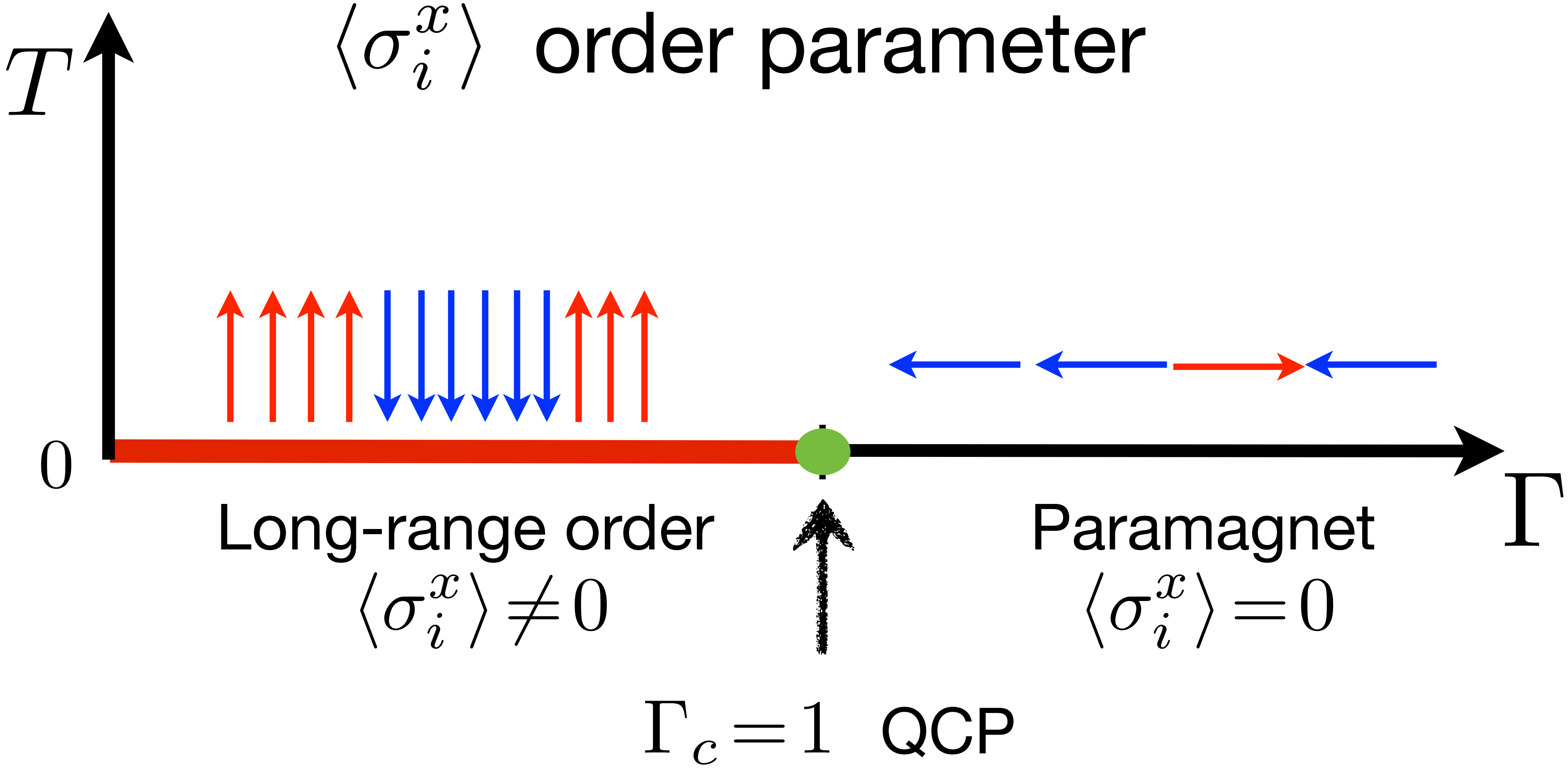
also for $\mathcal{O}' \neq \mathcal{O}$: $R(t), C_+(t)$ *invar.* $\Gamma_0 \leftrightarrow 1/\Gamma_0$

Transverse field Ising chain

$$H(\Gamma) = -J \sum_i \left[\, \sigma_i^x \sigma_{i+1}^x + \Gamma \sigma_i^z \,\right]$$

$T = 0$ QPT

$\langle \sigma_i^x \rangle$ order parameter



Transverse field Ising chain

$$H(\Gamma) = -J \sum_i \left[\sigma_i^x \sigma_{i+1}^x + \Gamma \sigma_i^z \right]$$

Diagonalization: fermions

1. Jordan-Wigner:

$$c_i = \left(\prod_{j=1}^{i-1} \sigma_j^z\right) \sigma_i^- \qquad \{c_i^\dagger, c_j\} = \delta_{i,j}$$

$$\{c_i, c_j\} = \{c_i^\dagger, c_j^\dagger\} = 0$$

2. Fourier transform (PBC):

$$c_k = \frac{1}{\sqrt{L}} \sum_j e^{-ikj} c_j$$

3. Bogoliubov rotation:

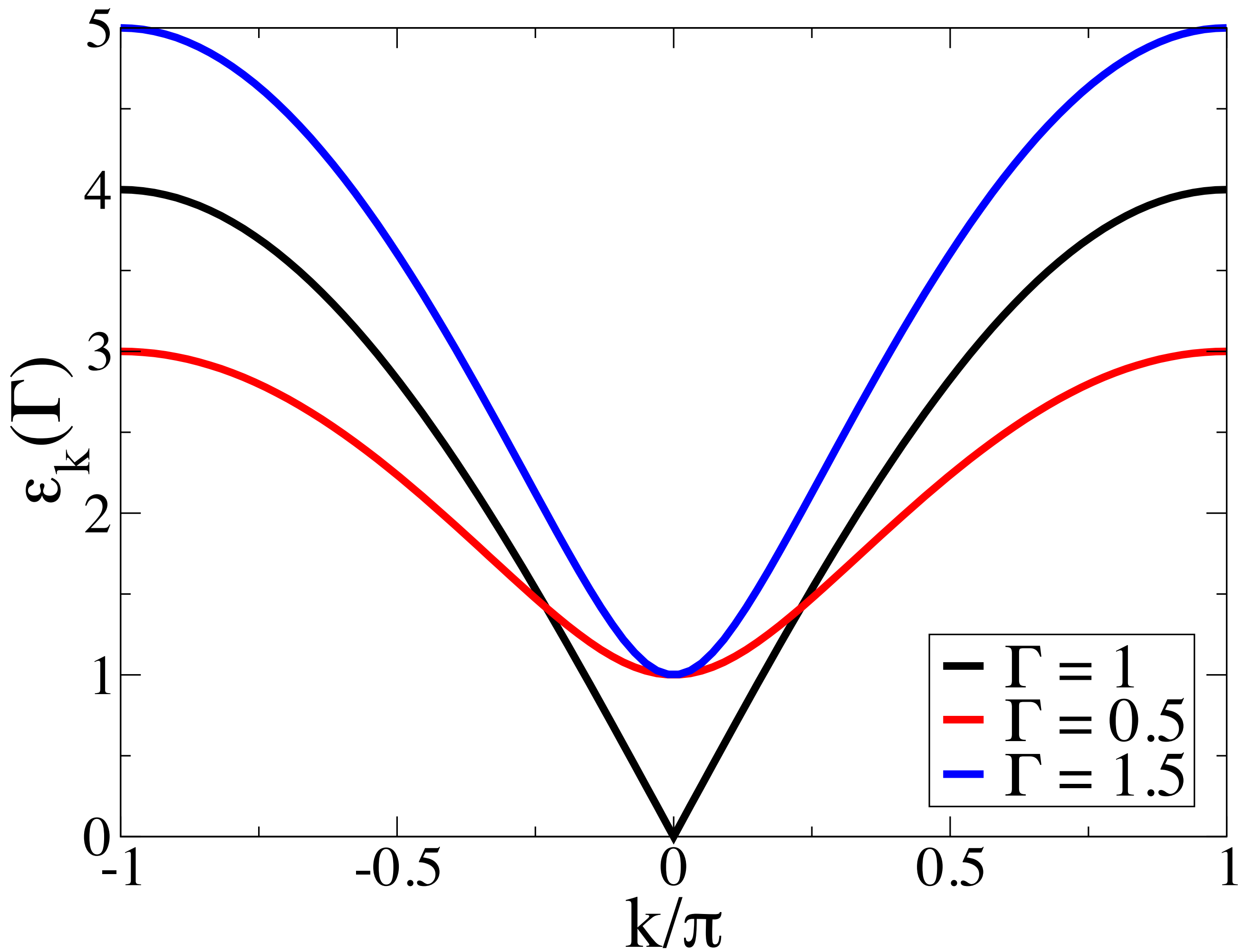
$$c_k = u_k \eta_k + v_k \eta_{-k}^\dagger$$

Spectrum

$$H(\Gamma) = \sum_k \epsilon_k(\Gamma)(\eta_k^\dagger \eta_k - 1/2)$$

$$\epsilon_k(\Gamma) = 2\sqrt{1+\Gamma^2 - 2\,\Gamma\cos k}$$

GS: $\eta_k|0\rangle = 0$



Dynamics

Heisenberg picture: $\mathcal{O}(t) = \mathrm{e}^{iHt}\mathcal{O}\mathrm{e}^{-iHt}$

Dyn. of excitations: $\eta_k(t) = \mathrm{e}^{-i\epsilon_k(\Gamma)t}\eta_k \implies n_k \equiv \eta_k^\dagger \eta_k$

conserved

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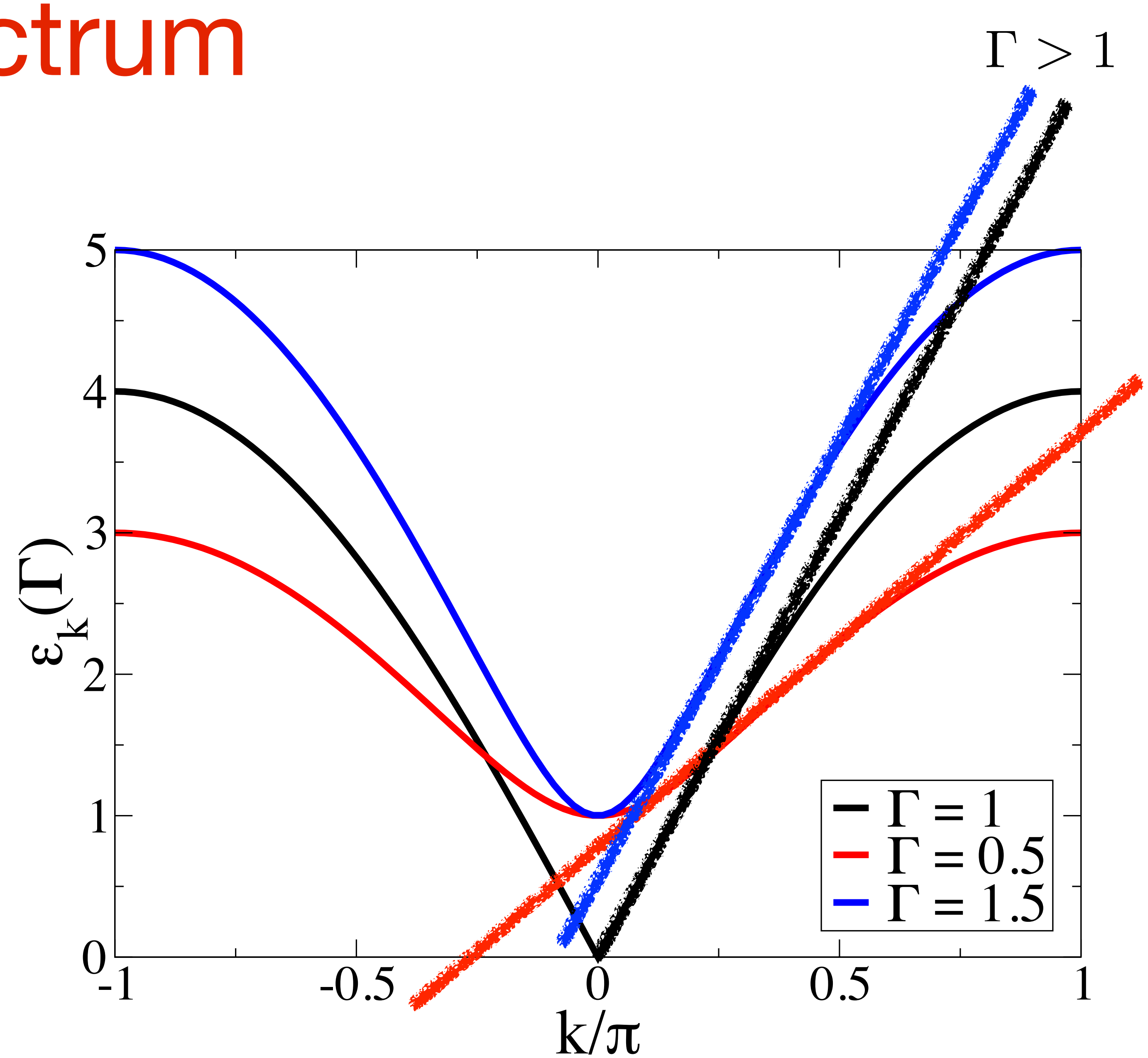

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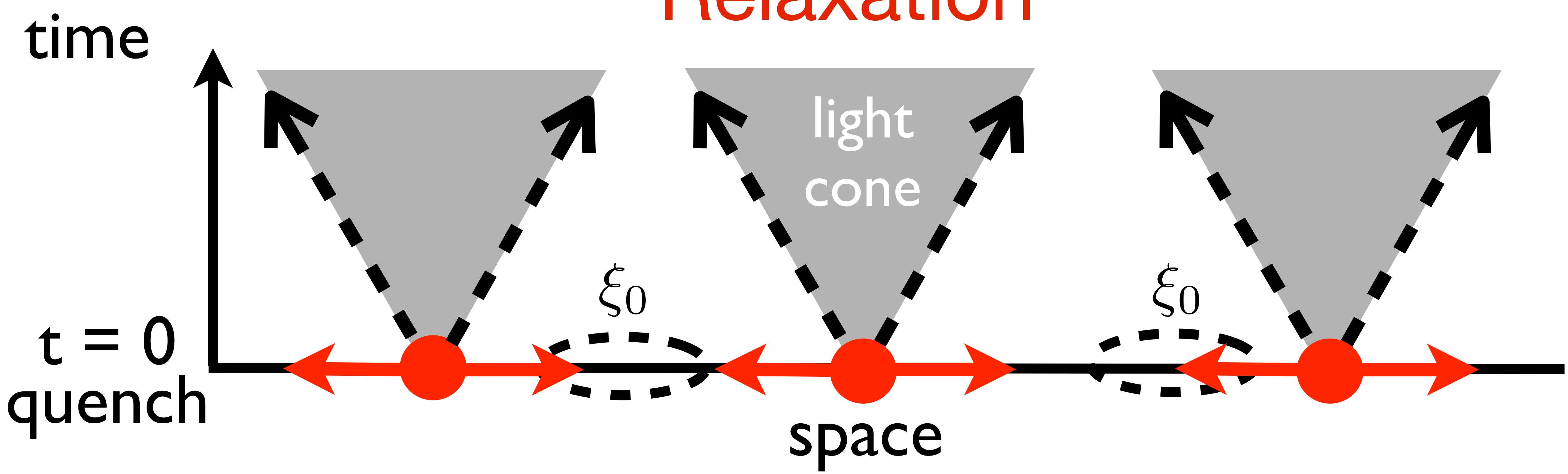
Relaxation



- $|\psi_0\rangle = |\mathrm{GS}(H(\Gamma_0))\rangle$ has extensive energy
- Ballistic excit.s, plane waves
- Finite systems ➔ recurrence

(eq.) Sachdev, Young '97
Igloi, Rieger, 2000,>2010
Calabrese, Cardy, >2006
Rossini, Mussardo, Santoro, Silva, Suzuki, >2009
Calabrese, Essler, Fagotti, >2011

observables: $\mathcal{O} \equiv \sigma_i^z$ ($\propto c_i^\dagger c_i$ density of fermions)

$$\mathcal{O}' \equiv M(t) = \frac{1}{L} \sum_i \sigma_i^z$$

$C, R \propto L \times \langle [M, M]_\pm \rangle$

Why critical quenches?

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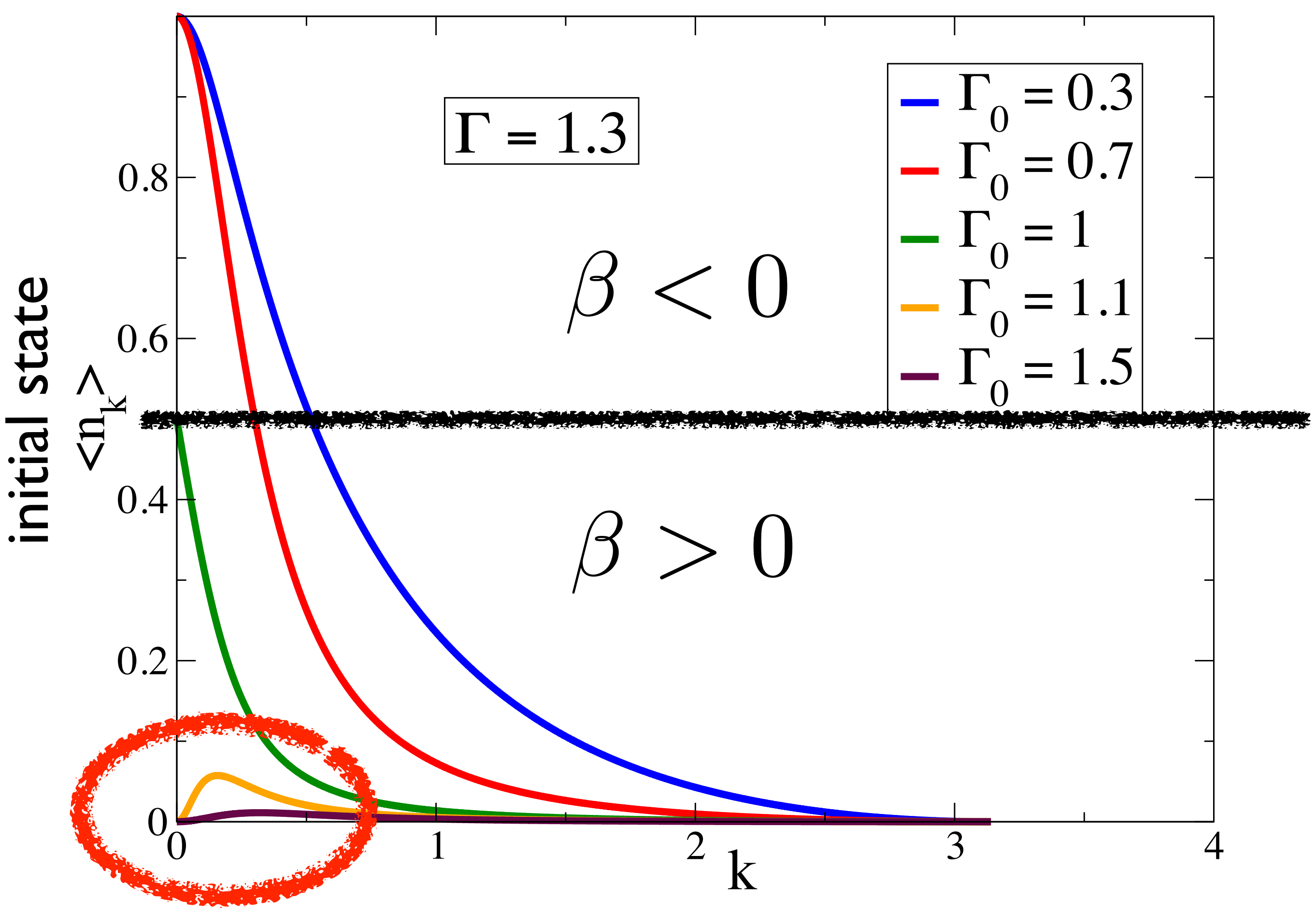

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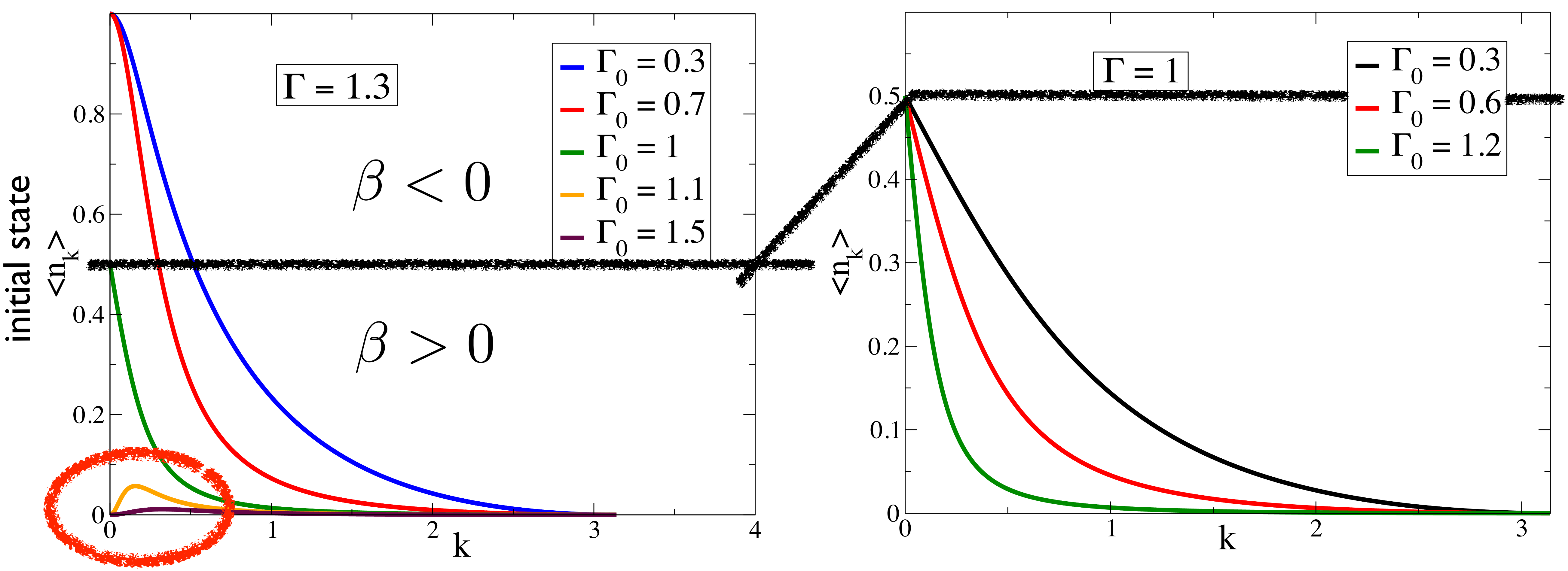

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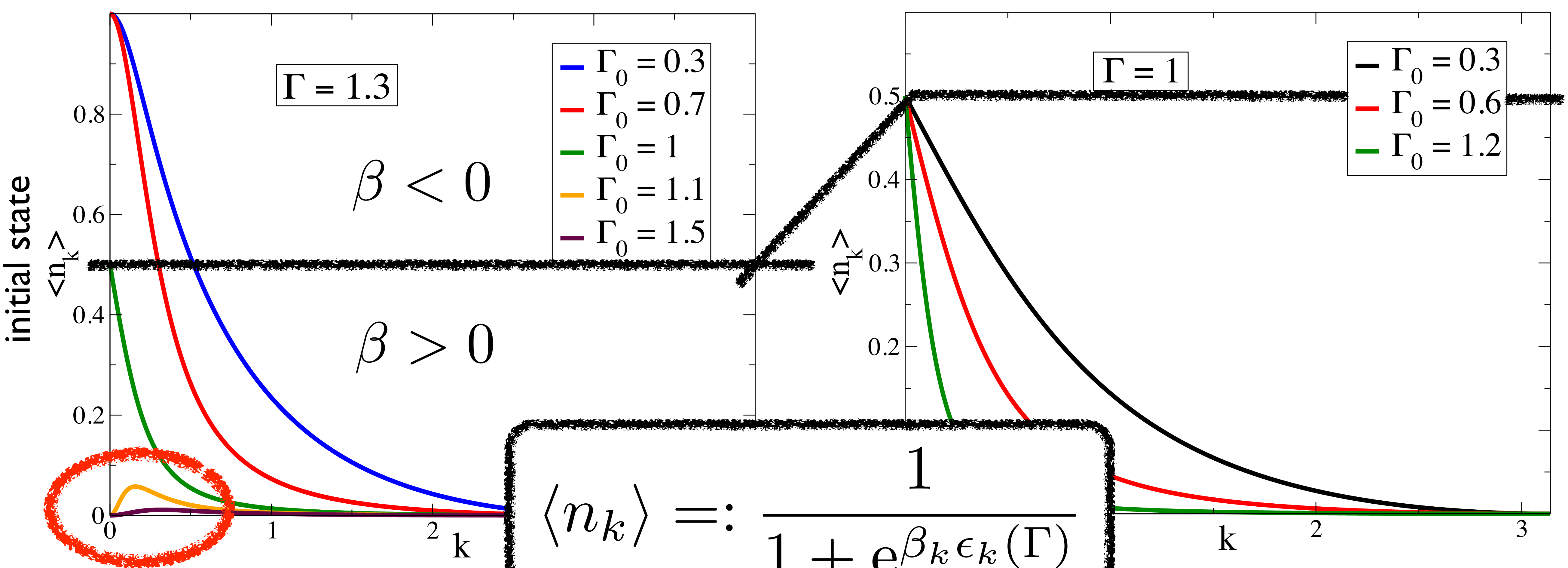

$$\langle n_k \rangle =: \frac{1}{1 + e^{\beta_k \epsilon_k(\Gamma)}}$$

GGE

because....

- $\{\langle\psi_0|n_k|\psi_0\rangle\}_k$ "more" thermal
- σ_i^z "special"
- no lattice $\Rightarrow$ $\epsilon_k \sim |k|$ CFT [Calabrese,Cardy '06]

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No Gibbs....

... GGE

[...-Calabrese,Essler,Fagotti-Blass,Rieger,Igloi]

what can we learn ?
effective "Gibbs"?

$$\beta_k^{-1} = T^M_{\text{eff}}(\omega = 2\epsilon_k(\Gamma))$$

"measure" temp.s of GGE

➤ local vs. non-local ? [Rossini et al. 2010]

σ_i^z, M σ_i^x

Role of "locality"

Transverse magnetization

$$\sigma_j^z = 2c_j^\dagger c_j - 1$$

Order parameter

$$\sigma_j^x = \exp\left[i\pi \sum_{l=1}^{j-1} c_l^\dagger c_l\right](c_j^\dagger + c_j)$$

[Rossini, Suzuki, Mussardo, Santoro, Silva '09]

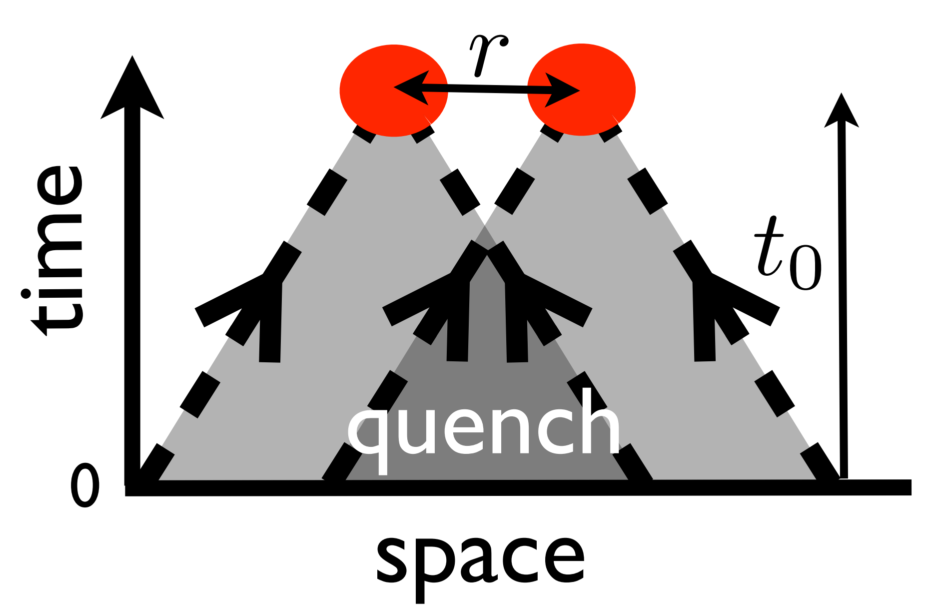
focus on $\Gamma = 1$

Order parameter

- $\langle \sigma^x(t) \rangle \xrightarrow[t \to \infty]{d > 1?} 0 \quad \forall\, \Gamma \neq \Gamma_0$

  $\sim \mathrm{e}^{-t/\tau}$

☑ **thermal behav.** @ $T \neq 0$



- $\langle \sigma_i^x(t_0) \sigma_{i+r}^x(t_0) \rangle \underset{(t_0 = \infty)}{\sim} \mathrm{e}^{-r/\xi}$

$$\tau^{-1} = -\int_0^\pi \frac{\mathrm{d}k}{\pi}\ \epsilon_k'(\Gamma)\ \ln|1 - 2\langle n_k \rangle|$$
$$\xi^{-1} = \ldots\ldots \quad 1 \quad \ldots\ldots\ldots\ldots\ldots$$

GGE

- $\langle \sigma_i^x(t+t_0) \sigma_i^x(t_0) \rangle \sim \mathrm{e}^{-t/\tau} \qquad \tau \simeq \tau_{\mathrm{eq}}(T = T^E)$

...we focus on the stationary regime

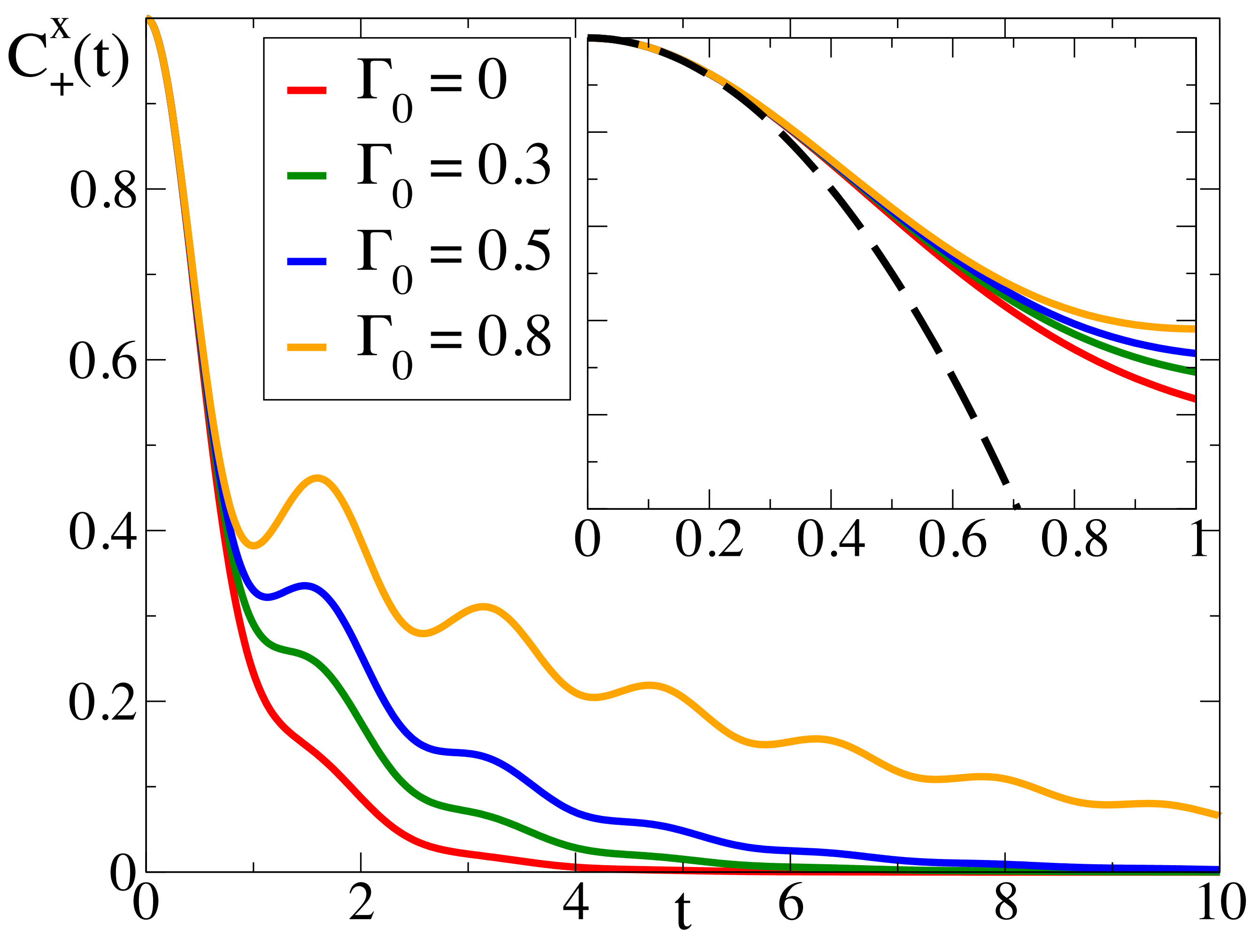
focus on $\Gamma = 1$

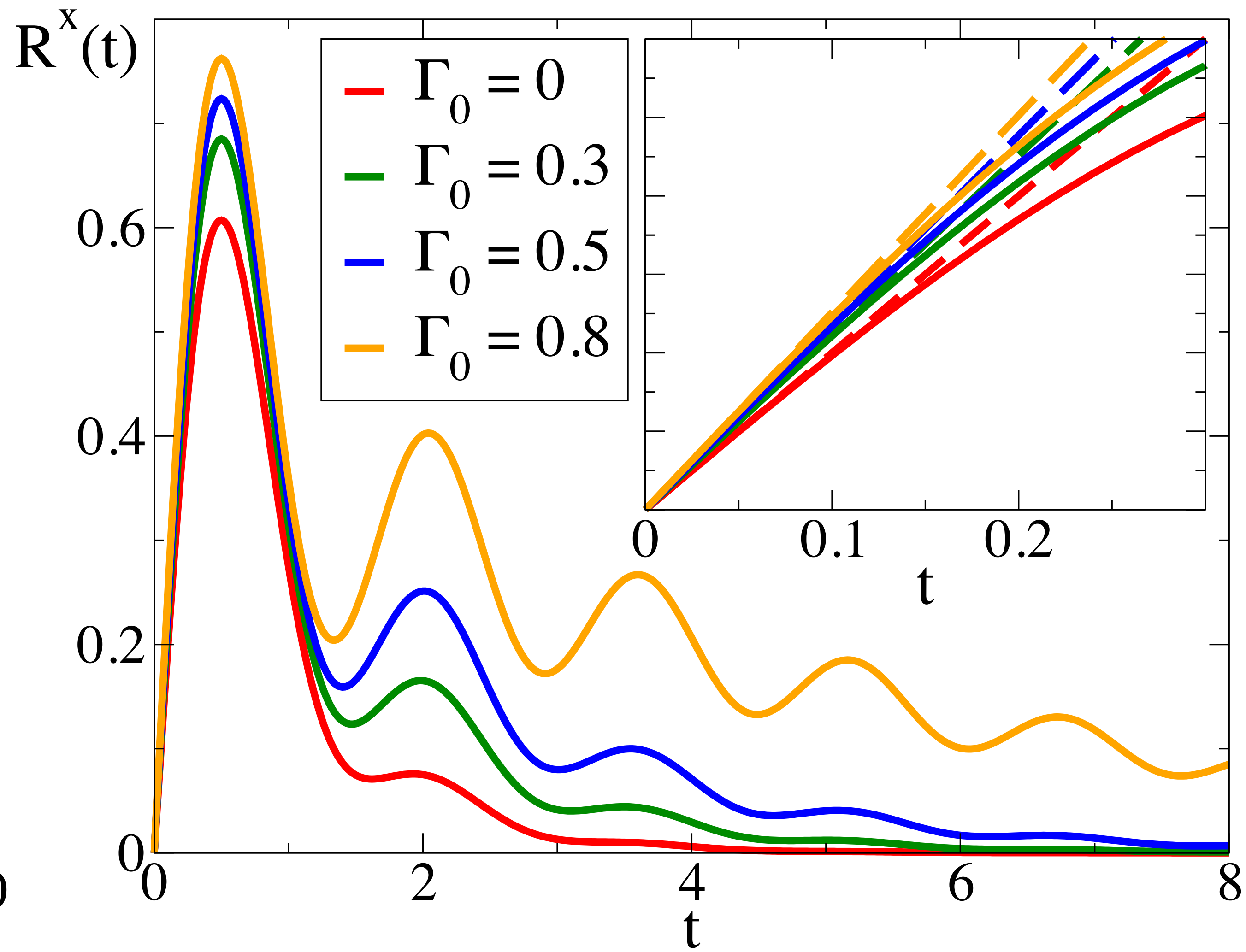
Order parameter $r = 0$

$$C^x(t \gtrsim 4) \simeq A_C(\Gamma_0)\, \mathrm{e}^{-t/\tau(\Gamma_0)}\, [1 + \frac{\overset{\simeq 0.21}{a_C}}{\sqrt{t}} \sin(4t + \phi)]$$

$A_C/A_R = 1.210(5)$

$$R^x(t \gtrsim 4) \simeq A_R(\Gamma_0)\, \mathrm{e}^{-t/\tau(\Gamma_0)}\, [1 - \frac{\overset{\simeq 0.52}{a_R}}{\sqrt{t}} \cos(4t + \phi)]$$





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fast relaxation

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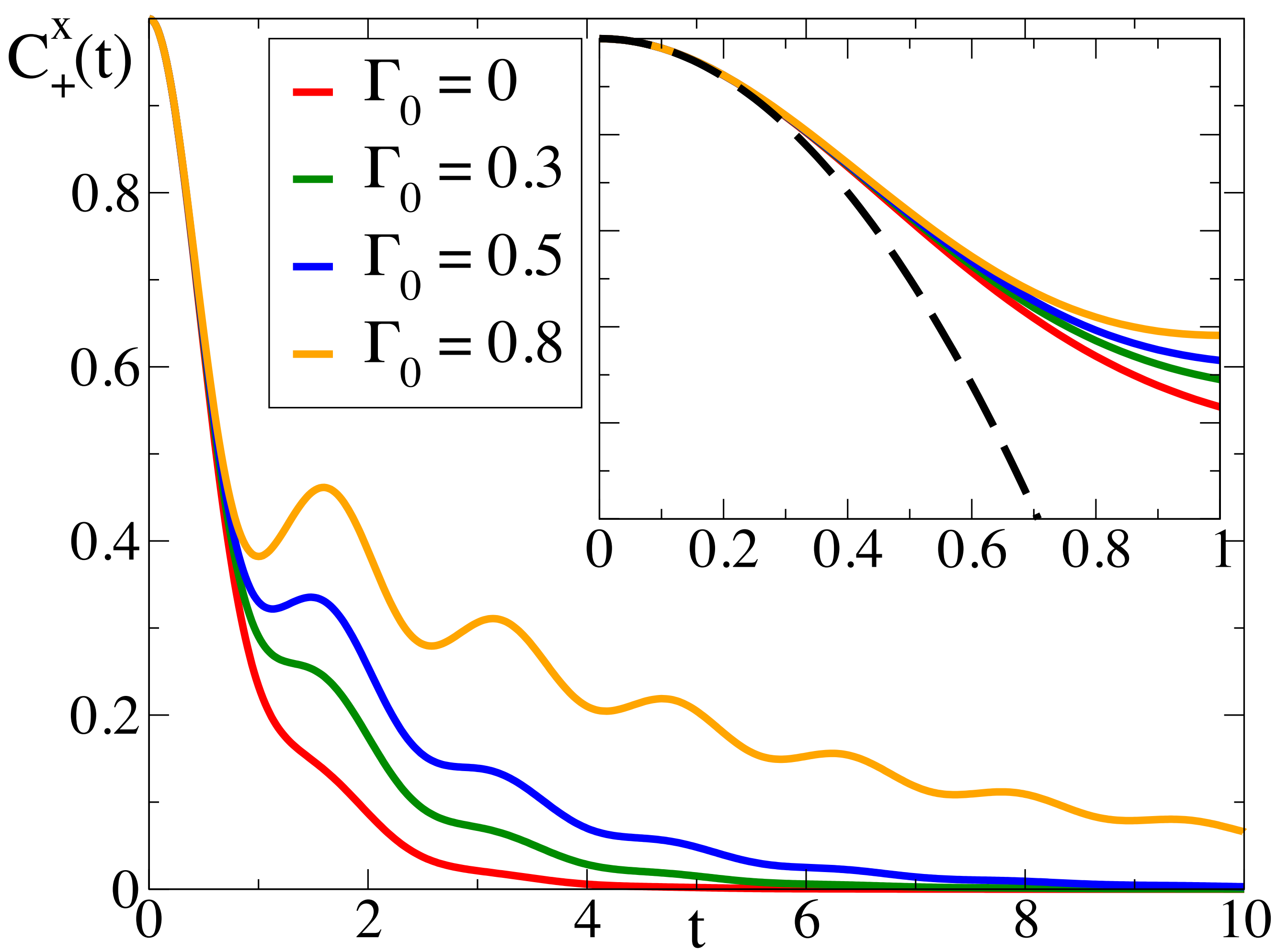

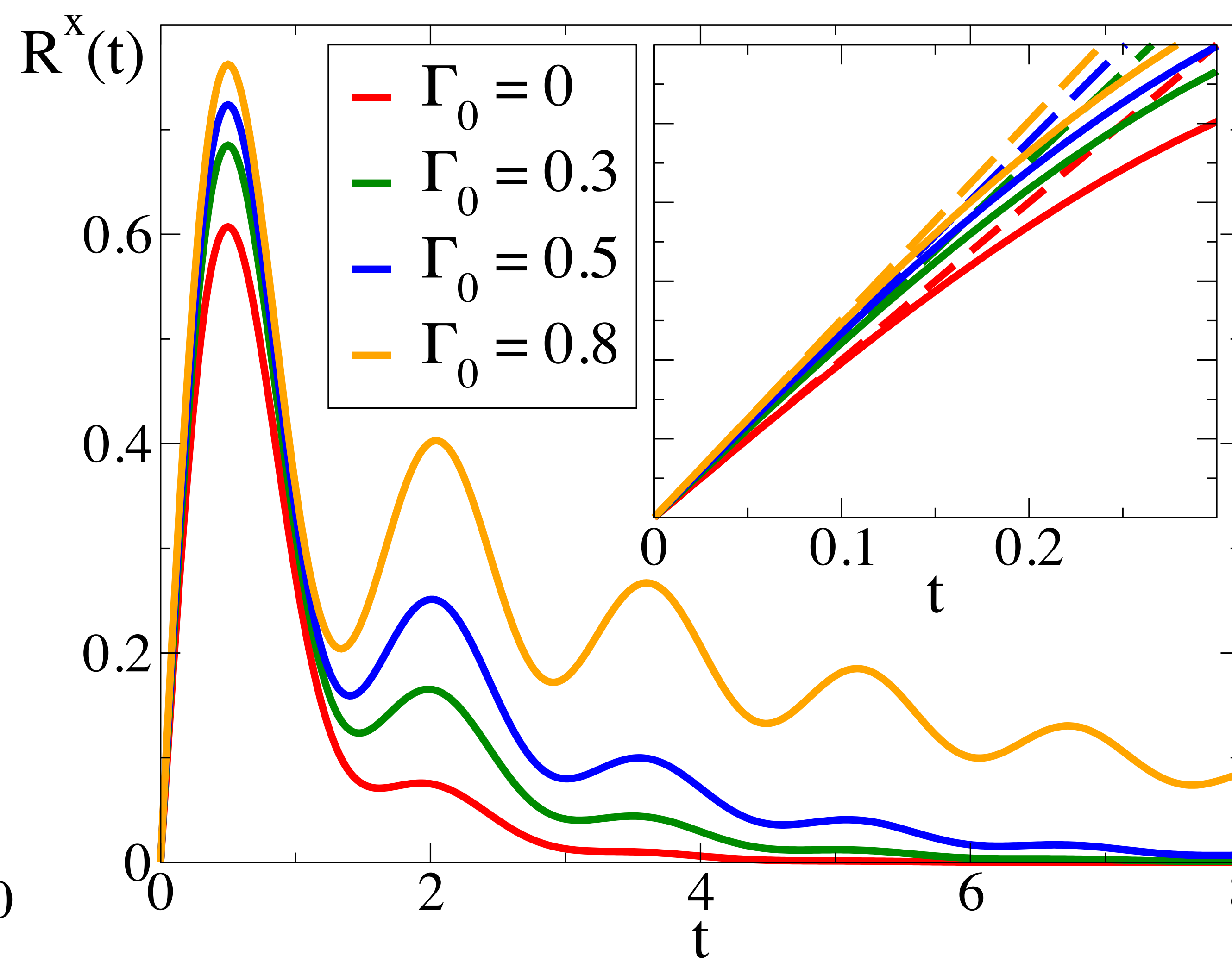

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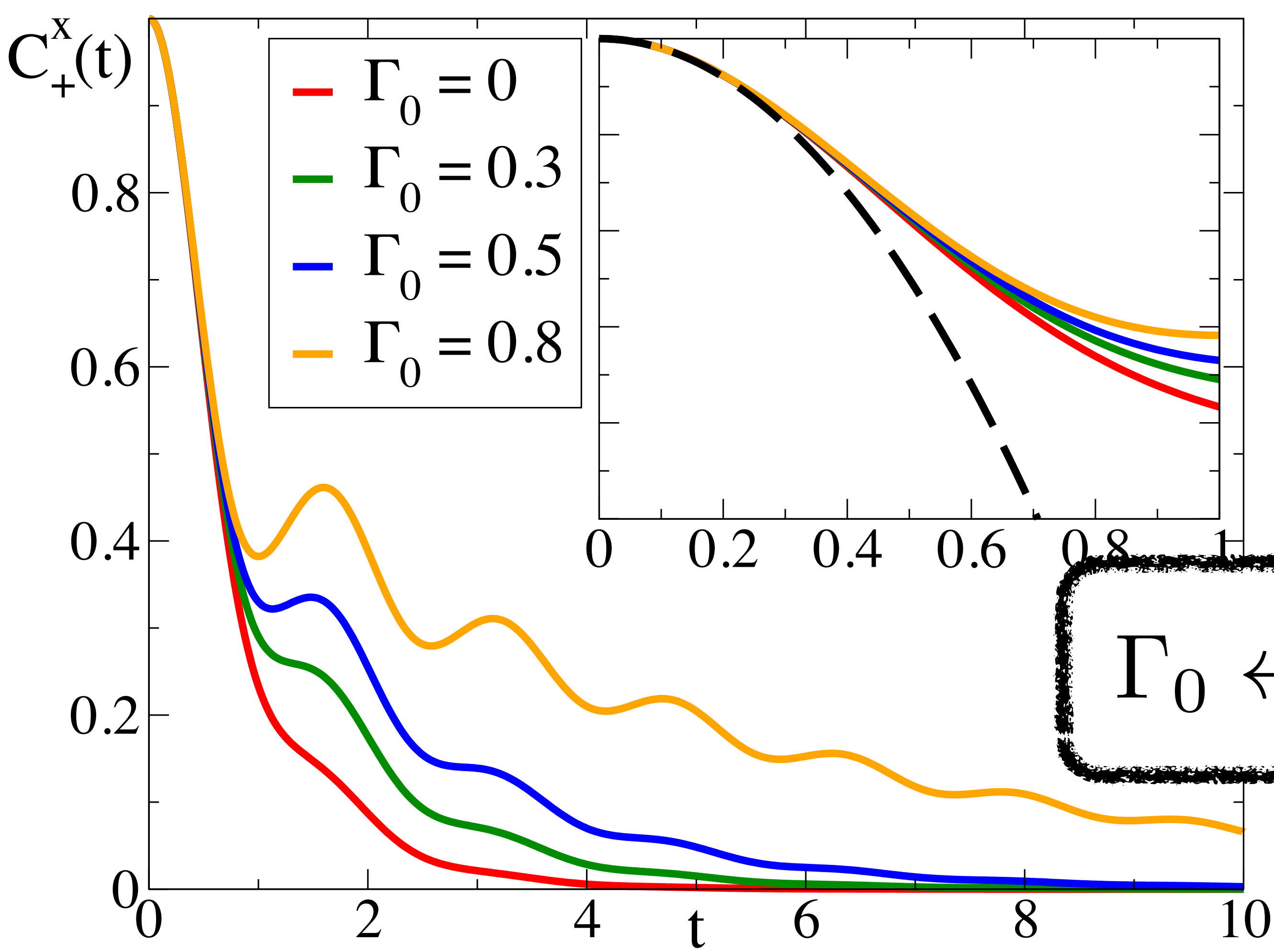
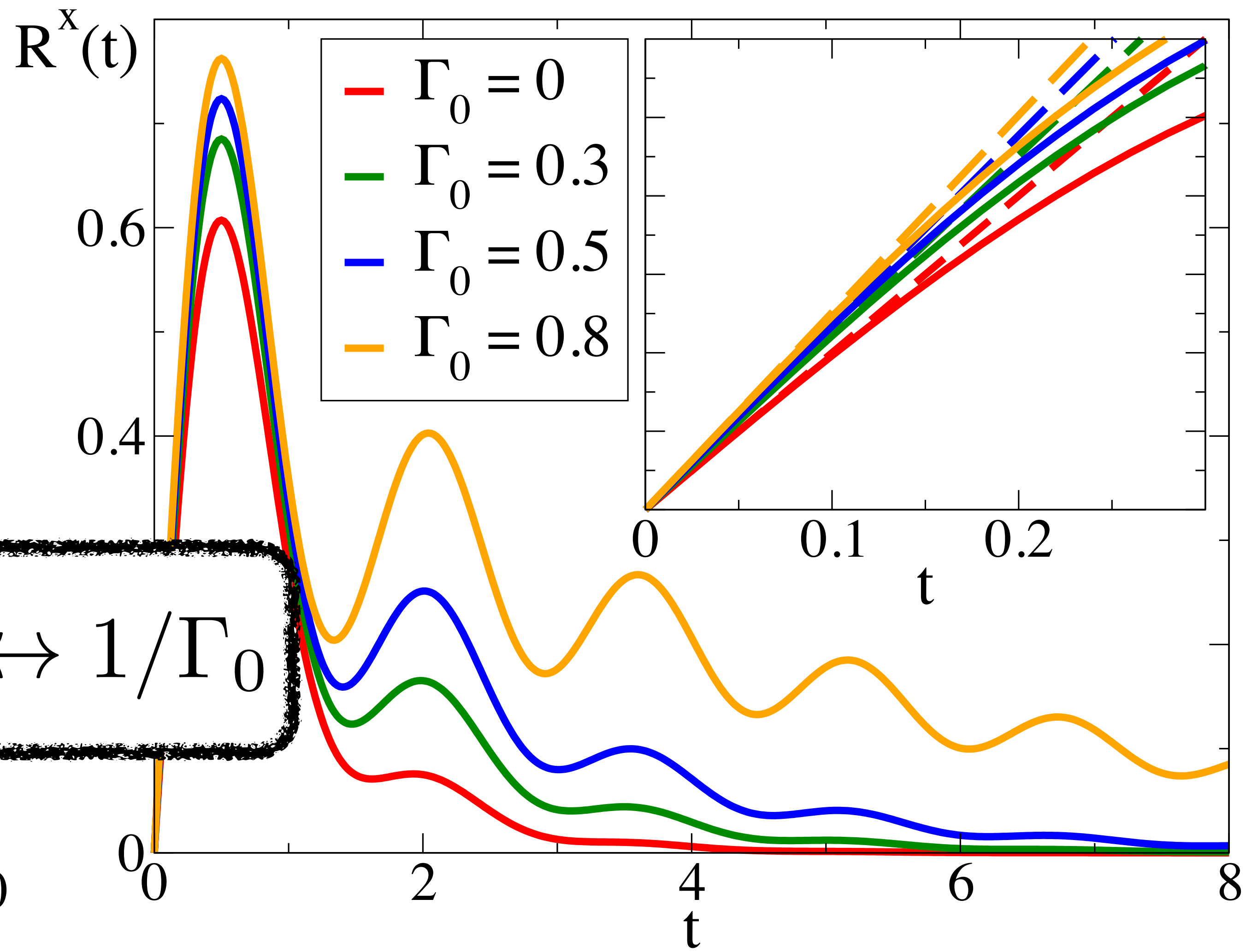


$\boxed{\Gamma_0 \leftrightarrow 1/\Gamma_0}$



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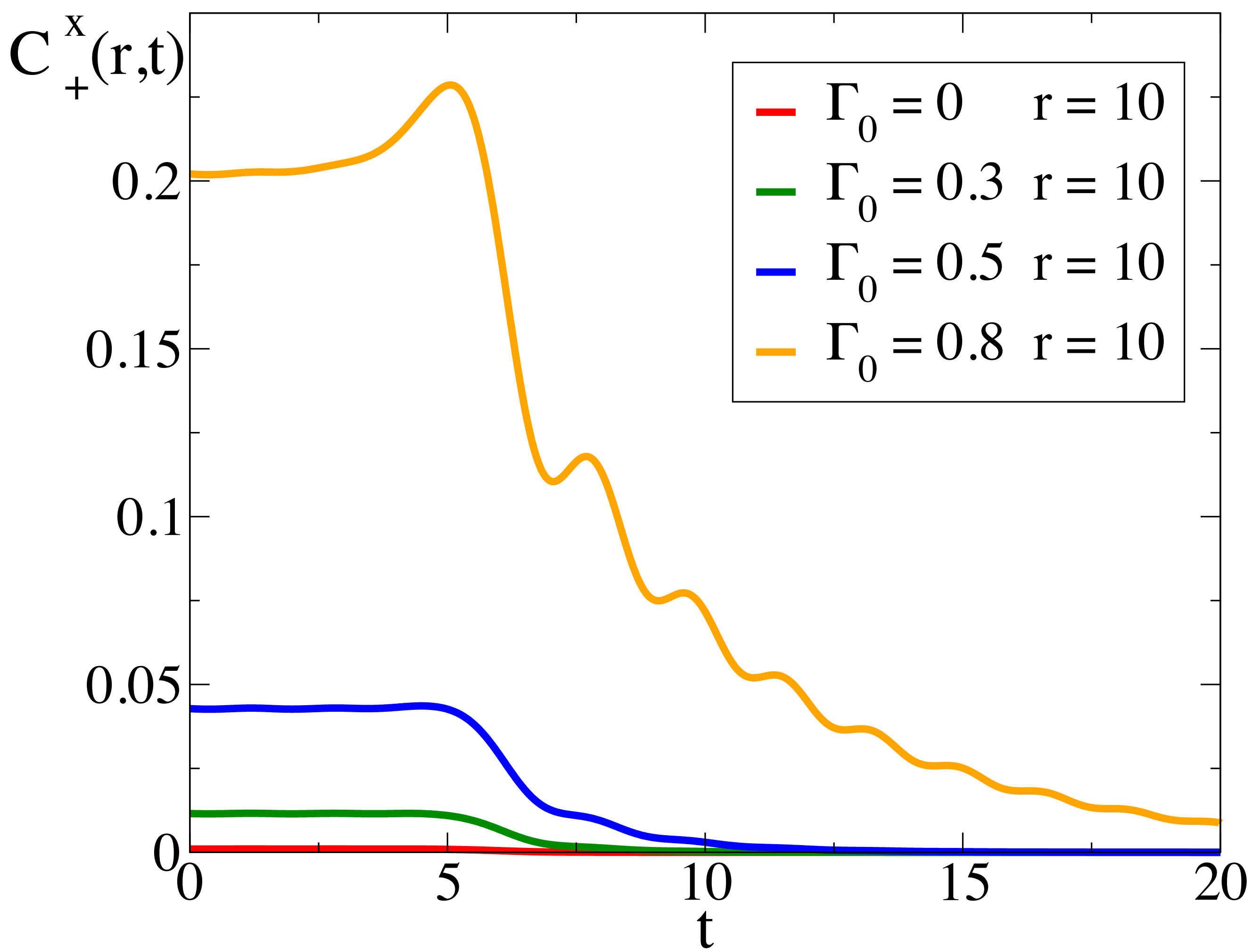

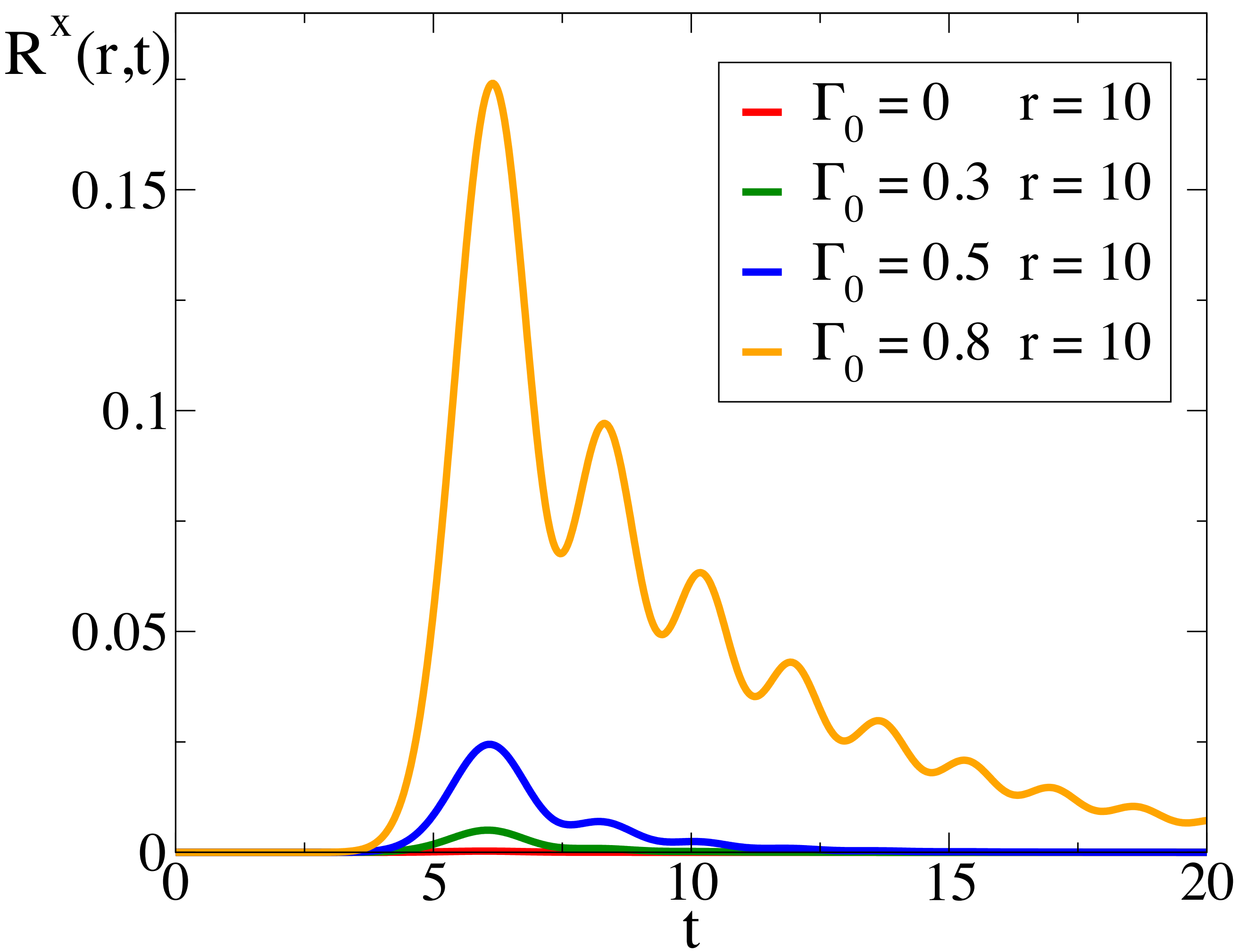

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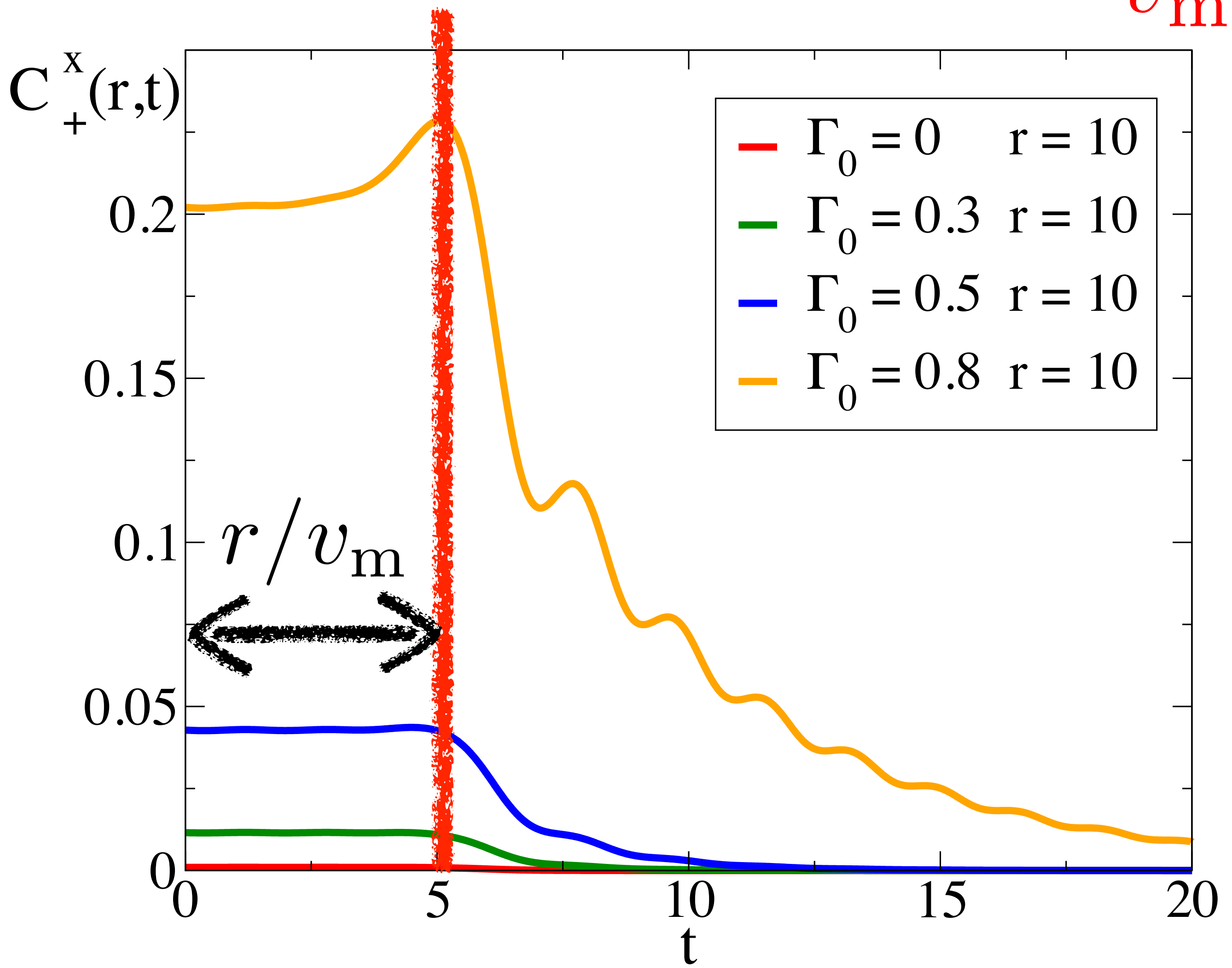
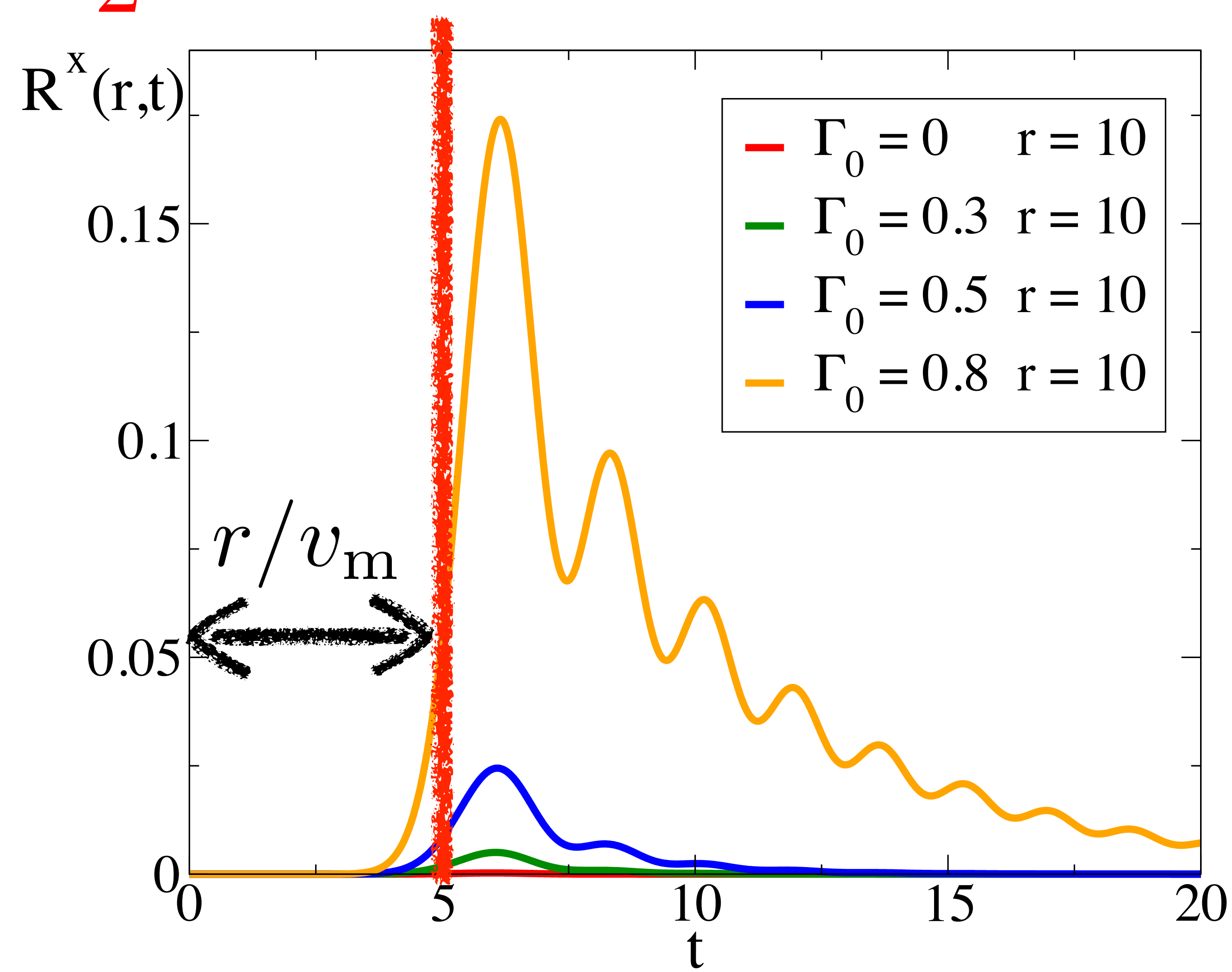
$v_{\rm m} = 2$





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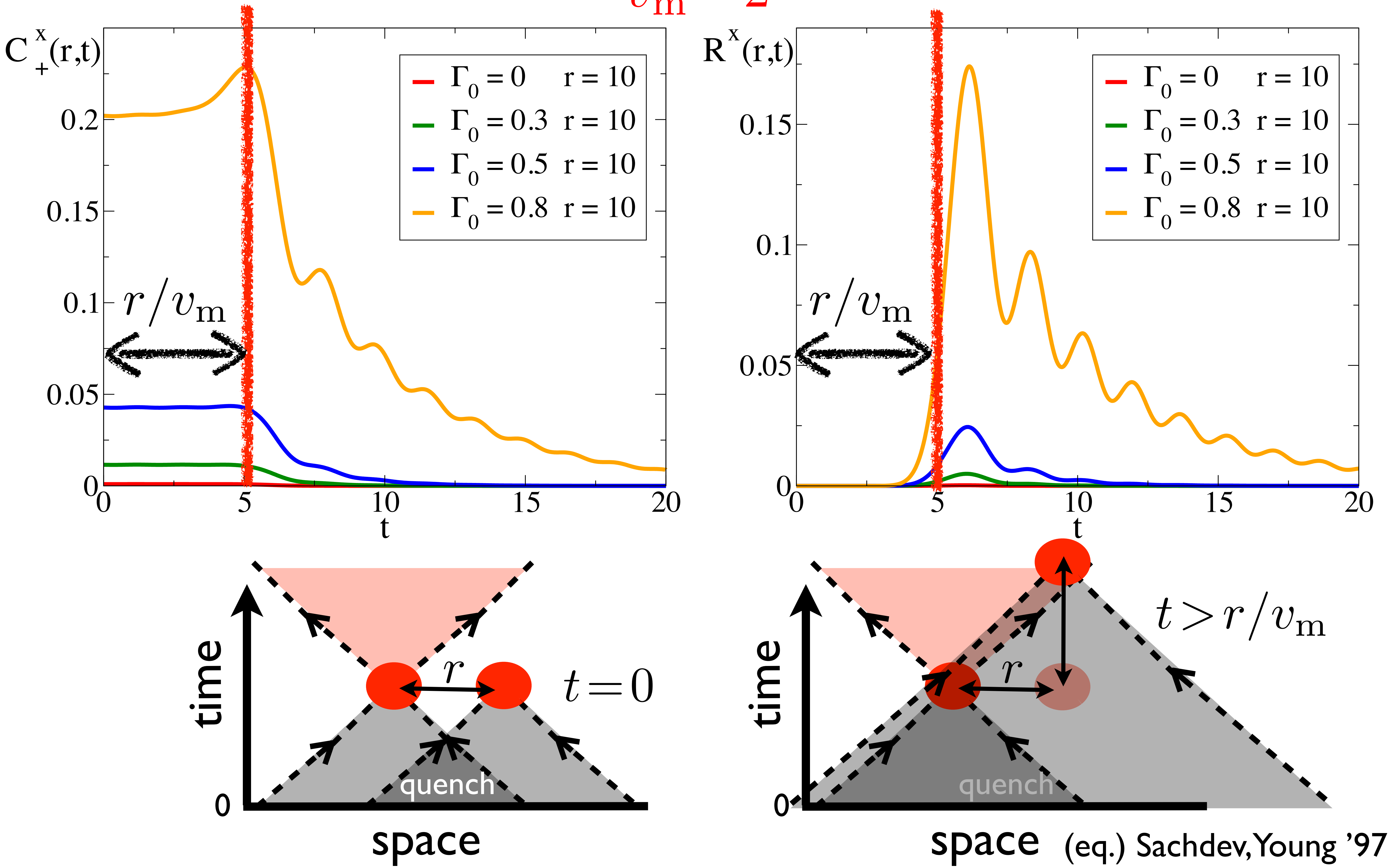

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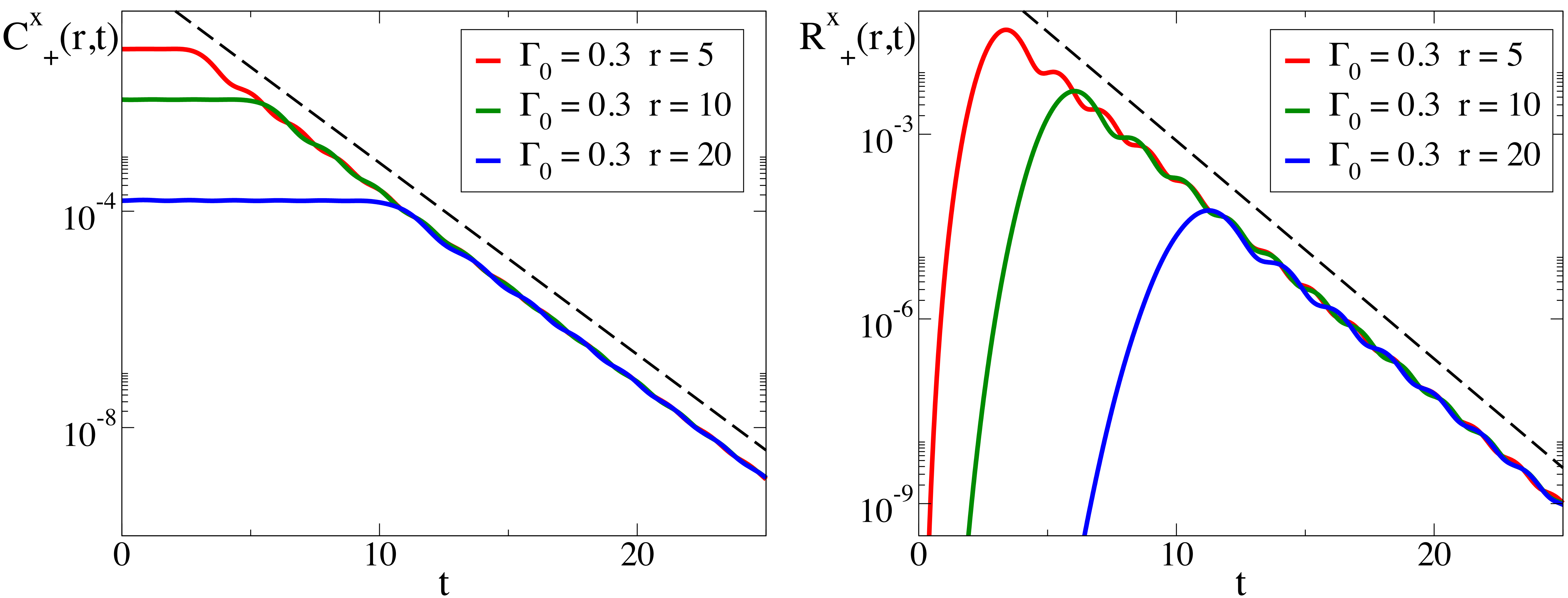

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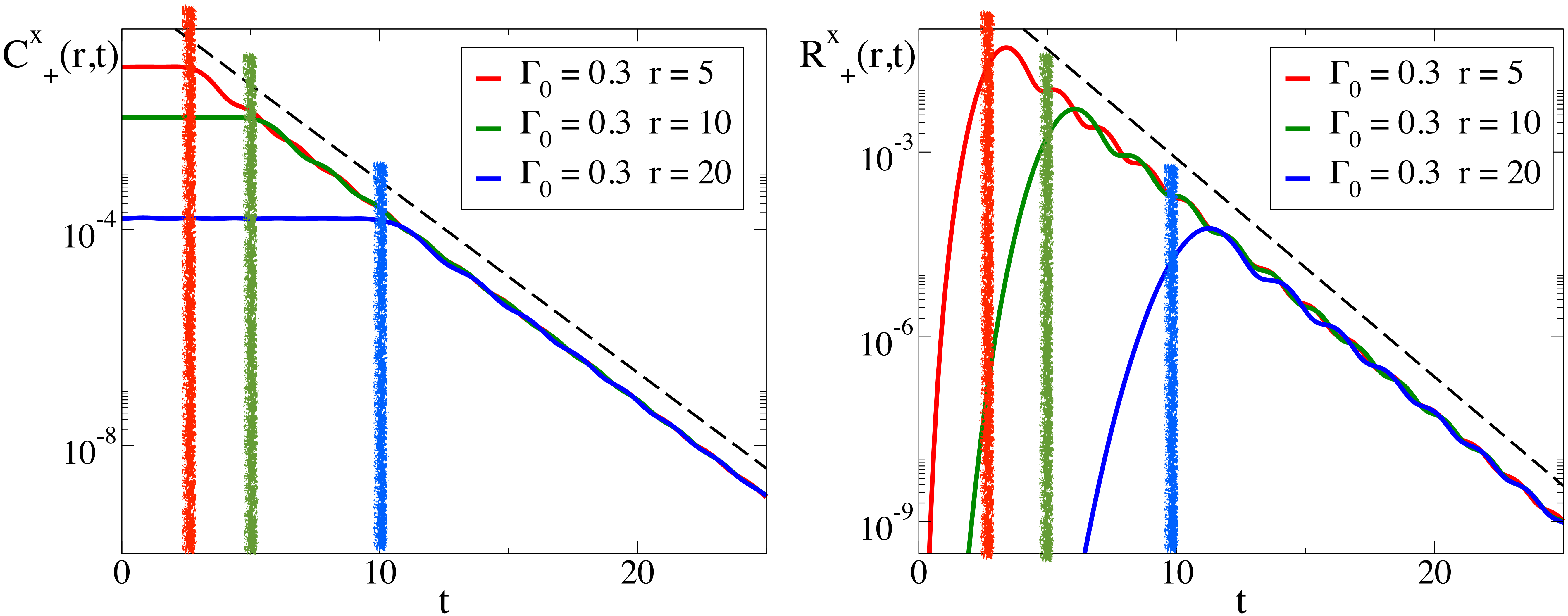

$t > r/2$ $\qquad$ $t > r/2$

Order parameter

FDT - effective temp.

$$\hbar\,\mathrm{Im}\,R(\omega) = \mathrm{th}\left(\frac{\beta\hbar\omega}{2}\right)\,\mathrm{Re}\,C(\omega)$$

$$R(t) = \frac{2i}{\hbar}\sum_{\mathrm{odd}\ n=1}^{\infty} c_n \left(\frac{i\beta\,\hbar}{2}\right)^n \frac{\mathrm{d}^n C(t)}{\mathrm{d}t^n}$$

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$$t \to \infty \quad \Longrightarrow \quad \frac{\hbar A_R}{2A_C} = \tan\left(\frac{\hbar}{2\tau T^x_{\text{eff}}}\right) \quad \Longrightarrow \quad T^x_{\text{eff}}$$

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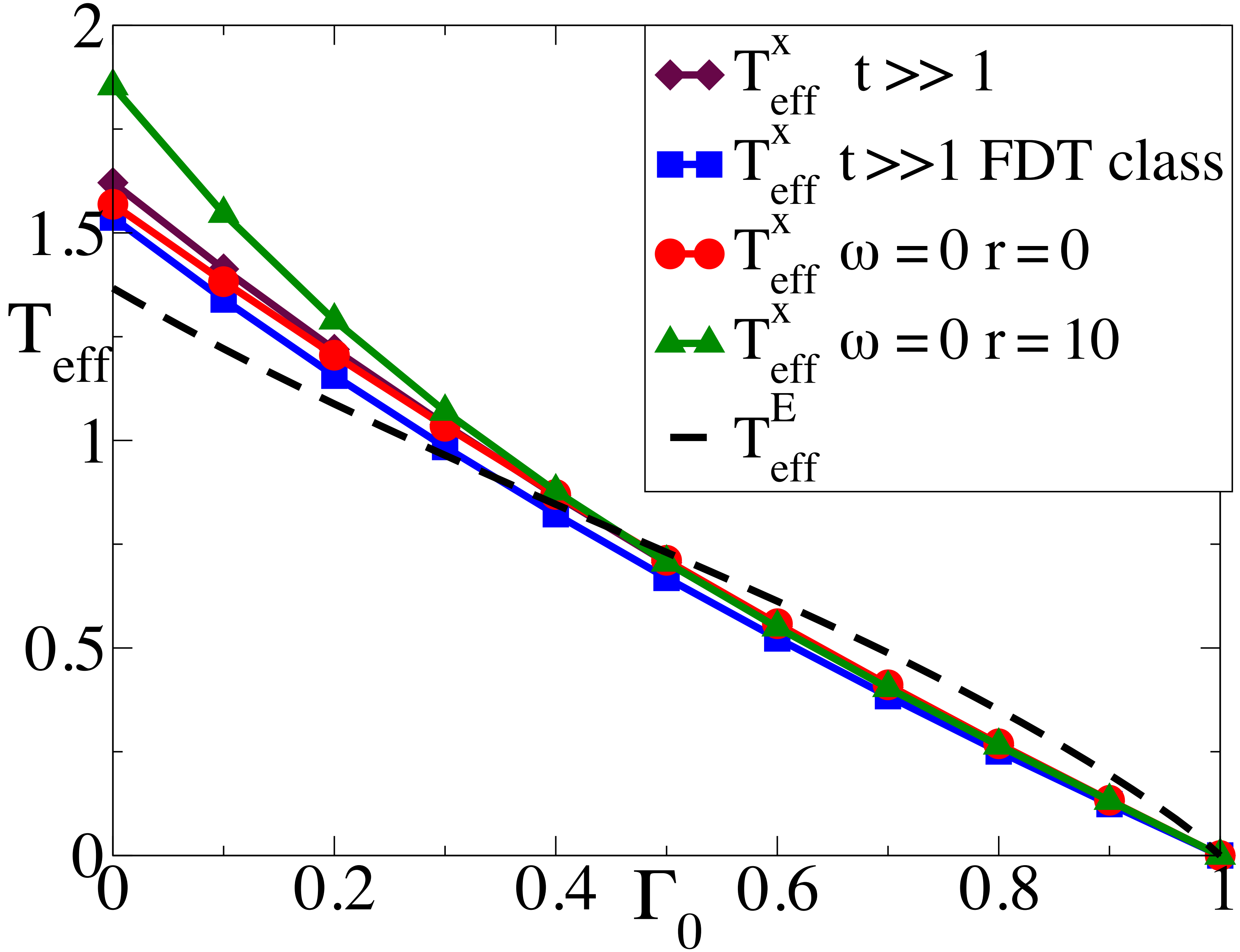

Take-home message:

Conclusions

- Effective temperatures via FDRs in *generic* **closed quantum** systems
- "Dynamic" probe of thermal behavior beyond expectation values or time/space length scales

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- Dynamics strongly depends on the observable
- Correlations spread ballistically - light cones
- Symmetries of stationary crit. dynamics
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Thank you!