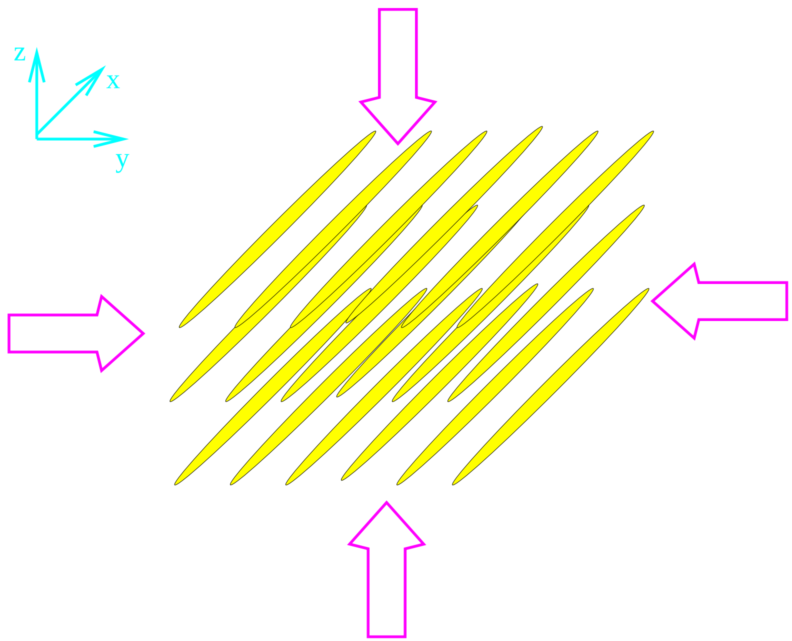
Exact solutions for inhomogeneous 1D quantum gases

Anna Minguzzi

Laboratoire de Physique et Modélisation des Milieux Condensés, Grenoble

1D quantum gases

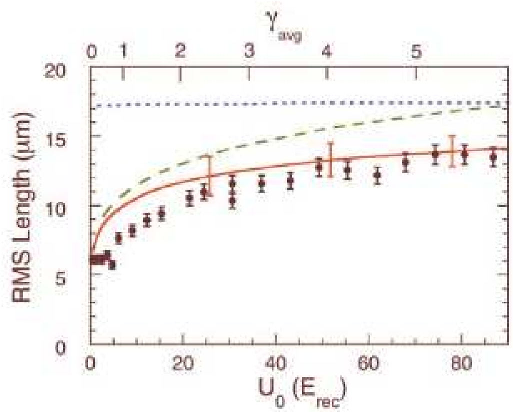
- Quasi-1D geometry:
  ultracold atoms in tight transverse confinement
  $$\mu, k_B T \ll \hbar\omega_\perp$$
  2D deep optical lattices, chip traps



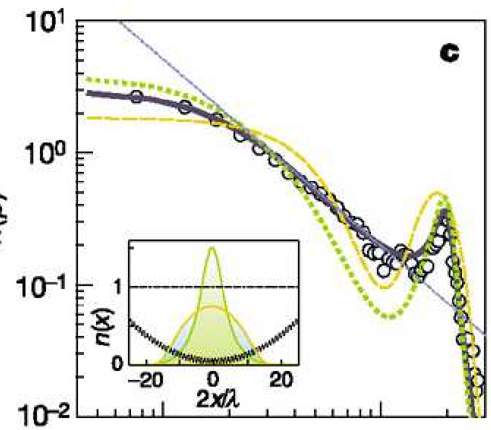
Experimental results

1D bosons in the strongly interacting regime

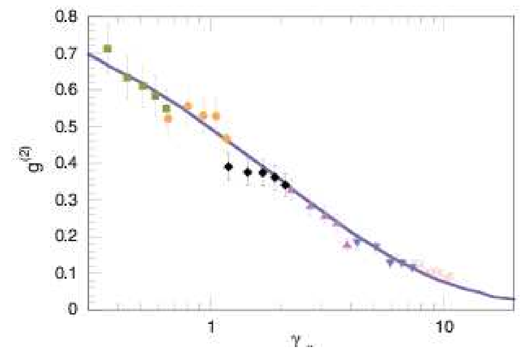
density profiles, momentum distribution, correlation functions, collective modes, transport...



[T Kinoshita et al (2004)]



[B. Paredes et al, 2004]



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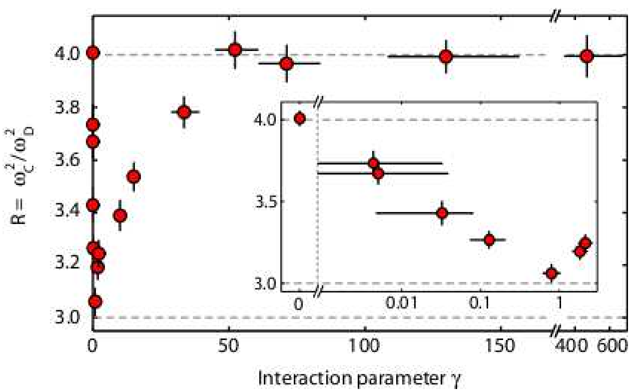
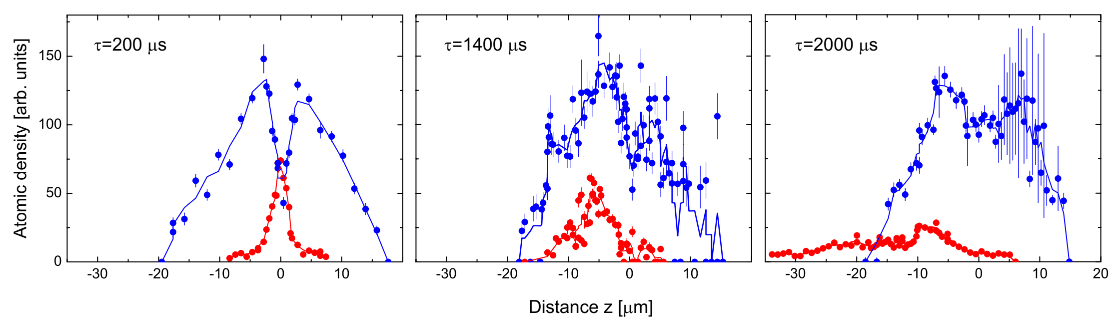


[E Haller et al, 2009]



[S. Palzer et al, 2009]

The model

- ultracold dilute bosonic gases:
  binary interactions through s-wave collisions
- for atoms in a tight waveguide *[Olshanii, 1998]*

$$v(x) = g\delta(x) \text{ with } g = 2a_s\hbar\omega_\perp(1 - 0.4602\, a_s/a_\perp)^{-1}$$

- model Hamiltonian *[Lieb and Liniger, 1963]*

$$\mathcal{H} = \sum_i -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V(x_i) + g\sum_{i<j}\delta(x_i - x_j)$$

Lieb-Liniger model with external potential
coupling strength:

$$\gamma = gn/(\hbar^2 n^2/m)$$

note: *strong* coupling at *weak* densities

From quasicondensate to TG

- Bose-Einstein condensation in 3D: off-diagonal long range order for $|\mathbf{x}-\mathbf{x}'| \to \infty$ *[Penrose and Onsager, 1965]*

$$\langle \Psi^\dagger(\mathbf{x})\Psi(\mathbf{x}')\rangle \to n_0$$

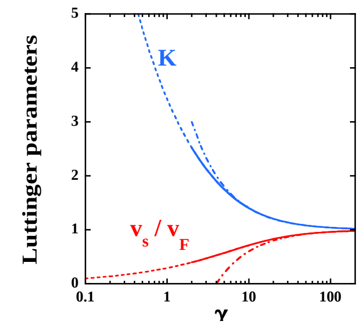
From quasicondensate to TG

quantum fluctuations: important in one-dimension

- in 1D *quasi*-long range order for $|x - x'| \to \infty$ *[Haldane, 1981]*

$$\langle \Psi^\dagger(x)\Psi(x') \rangle \to \frac{1}{|x - x'|^{1/2K}}$$

K: Luttinger parameter
depends on interactions



- Regimes of quantum degeneracy at $T = 0$:

  $\gamma \ll 1$ "quasicondensate"
  condensate with fluctuating phase, $K \gg 1$
  $\gamma \gg 1$ "Tonks-Girardeau" gas
  impenetrable-boson limit, $K = 1$

Impenetrable bosons: special features

- For $g \to \infty$ the many-body wavefunction vanishes at contact

$$\Psi(...x_j = x_\ell...) = 0$$

- Exact solution by mapping onto noninteracting fermions *[MD Girardeau, 1960]*

$$\Psi(x_1...x_N) = \Pi_{1\leq j<\ell\leq N}\mathrm{sign(x_j - x_\ell)}\frac{1}{\sqrt{N!}}\det(\psi_l(x_k))$$

with $\psi_l(x)$ single particle orbitals

for arbitrary external potential, also time dependent

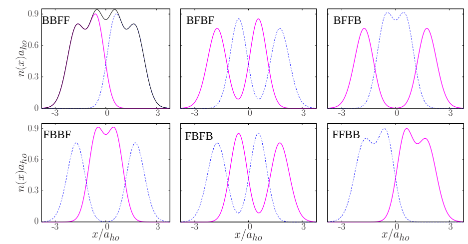
- *fermionization* $\Rightarrow$ impenetrable bosons are robust to two- and three-body particle losses

Plan

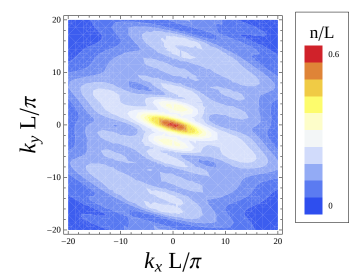
exact solutions for strongly interacting 1D gases:

external confinement and full quantum dynamics

- *TG gases in equilibrium:* extensions of the model, Bose-Fermi mixtures



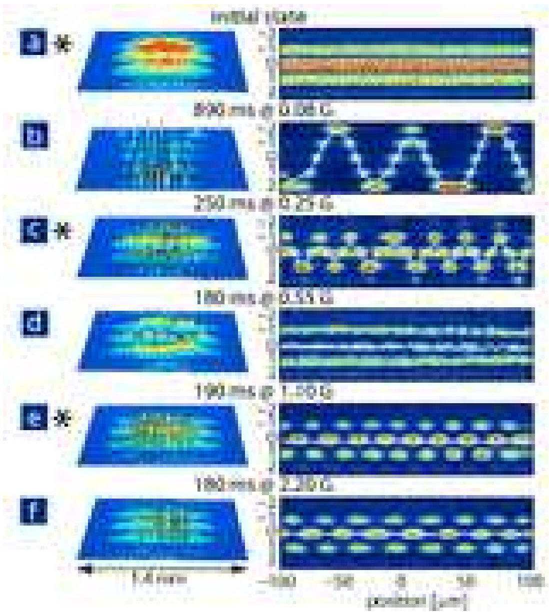
- *TG gases out-of-equilibrium:* sudden stirring of bosons on a ring



New solvable models : the Bose-Fermi mixture

1D spinors and mixtures

- Optical trapping allow for the study of multicomponent systems



spinor bosons [J. Kronjaeger et al PRL 105, 090402 (2010)]

- Extensions of the Girardeau solution for the strongly repulsive limit of Bose-Fermi mixtures *[M. Girardeau and A. Minguzzi PRL 99, 230402 (2007)]*, spin-1 bosons *[F. Deuretzbacher et al, PRL 100, 160405 (2008)]*, spin-1/2 fermions *[Liming Guan et al, PRL 102, 160402 (2009)]*

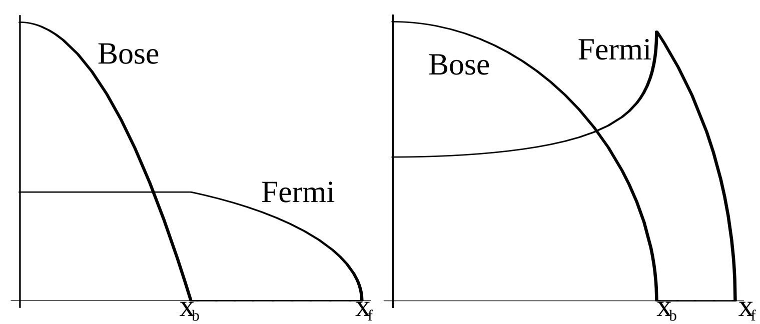
1D Bose-Fermi mixtures

with repulsive BB and BF interactions

- mean-field and Luttinger liquid analysis at weak coupling: *instability towards demixing*
- Homogeneous system with equal coupling constants and equal masses: *Bethe Ansatz solution – no demixing*

  [C.K. Lai and C.N. Yang, PRA 3, 393 (1971), A. Imambekov and E. Demler Ann. Phys. 321, 2390 (2006)]
- mixture in harmonic trap: partial demixing at intermediate interactions

  [A. Imambekov, E. Demler, ibid. (2006)]

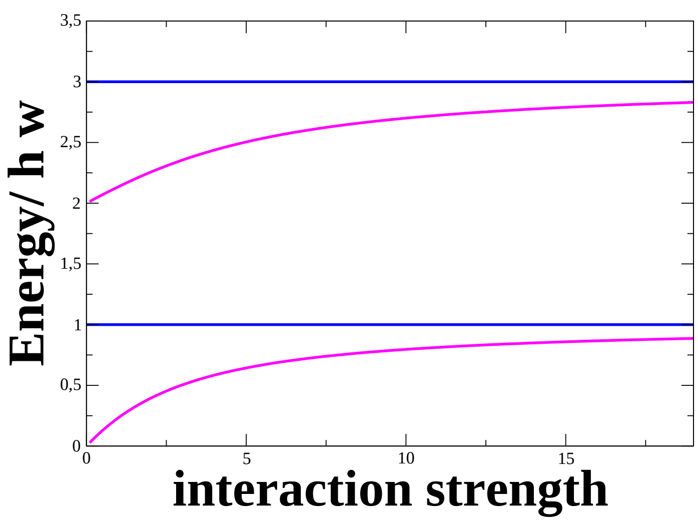


⇒ exact spatial structure in the trap at large interactions? ⇐

A symmetric model

with a large degeneracy

- Model: N_B bosons, N_F fermions with coupling constants $g_{BB} = g_{BF}$ and $m_B = m_F$, in harmonic trap
- BF mixture with small relative mass difference: ^{173}Yb-^{174}Yb
- In the TG limit $g_{BB}, g_{BF} \to \infty$: large degeneracy of the ground state $(N_B + N_F)!/N_B!/N_F!$



Energy levels for $N_B = 1$, $N_F = 1$: at increasing interactions, the even and odd levels approach

A basis set for the manifold

- We want to determine the wavefunction Ψ in each of the $N!$ coordinate sectors

$$x_{P(1)} < x_{P(2)} < \ldots < x_{P(N)}$$

with P a permutation, $P \in \mathcal{S}_N$

- TG limit: $\Psi = 0$ at each BB and BF contact $\Rightarrow$ in a given coordinate sector, $\Psi \propto \Psi_F$
- Constraint: satisfy bosonic and fermionic symmetry under particle exchange : $N_B!N_F!$ conditions
- note! degeneracy left: $N!/N_B!N_F!$ = ways you can order in a row N_B bosons and N_F fermions, eg

BBFF, BFBF, BFFB, FBBF, FBFB, FFBB

A basis set for the manifold

BBFF, BFBF, BFFB, FBBF, FBFB, FFBB

- Starting point: the snippet orthonormal basis

$$\langle x_1..x_N|P\rangle = \sqrt{N!}|\Psi_F(x_1..x_N)|$$

nonvanishing only in the coordinate sector P

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- idea! combine the snippets which correspond to the same BBFF sequence $\Rightarrow$ orthonormal basis

(since each snippet is used only once)

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- Example: x_1, x_2 bosons; x_3, x_4 fermions; coordinate sectors associated to $BBFF$:

$x_1 < x_2 < x_3 < x_4$

$x_2 < x_1 < x_3 < x_4$

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$$\Psi_{BBFF} = \langle x_1..x_N|(e + (12))(e - (34))\rangle$$

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$x_1 < x_4 < x_2 < x_3$

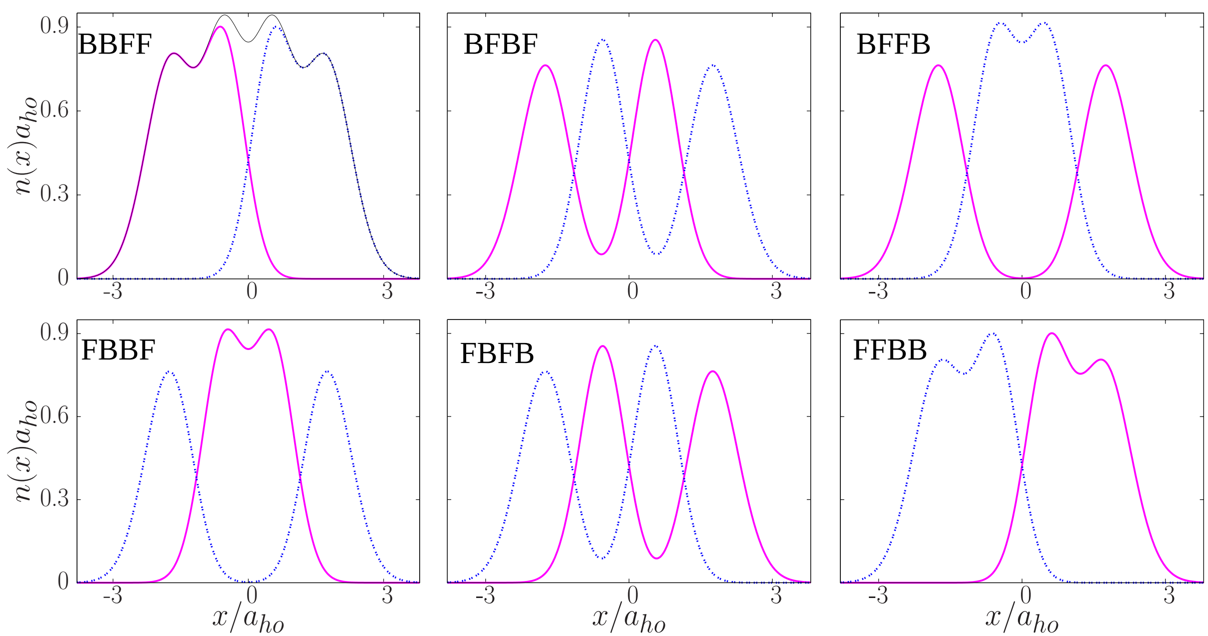
$x_2 < x_4 < x_1 < x_3$

$$\Psi_{BFBF} = \langle x_1..x_N|(23)(e + (12))(e - (34))\rangle$$

Density profiles for the BBFF basis

BBFF, BFBF, BFFB, FBBF, FBFB, FFBB

- Analogous to a system of distinguishable particles:



[B. Fang, P. Vignolo, M. Gattobigio, C. Miniatura, A. Minguzzi PRA 84, 023626 (2011)]

A special solution

- start from the Bethe Ansatz solution for the homogeneous system *[Lai and Yang (1971), Imambekov and Demler (2006)]*
- introduce $y_1, ...y_{N_B} = P^{-1}(1)..., P^{-1}(N_B)$ relative positions of the bosons in a sequence
- TG limit of the Bethe Ansatz solution: decoupling

$$\Psi_{BA} = \det[e^{i\frac{2\pi}{N}\kappa_i y_j}]\Psi_F(x_1, ...x_N)$$

where $\kappa = \{-(N_B-1)/2 + N/2, ..., N/2, ...(N_B-1)/2 + N/2\}$

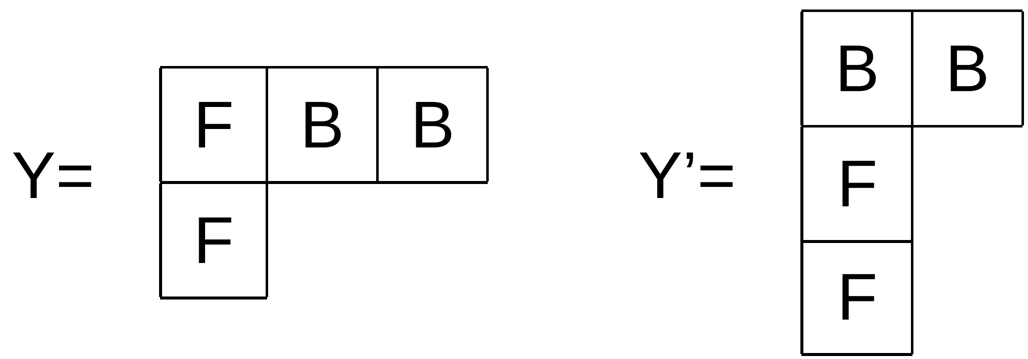
- *Generalize to the inhomogeneous case:*

use $\Psi_F(x_1, ...x_N)$ for harmonic trap

Conjecture: this solution is the one connected to the (nondegenerate) solution at finite interactions (with $g_{BB} = g_{BF}$)

Intermezzo: particle exchange symmetry

- Two possible Young tableaus



The ground state at finite interactions has the Y symmetry *[Lai, Yang (1971)]*

- to each tableau is associated a value of the Casimir invariant: $\hat{C} = \sum_{i<j} (i,j)$ with (i,j) particle permutation

$$c_Y = (N_B(N_B+1) - N_F(N_F-1))/2$$
$$c_{Y'} = (N_B(N_B-1) - N_F(N_F+1))/2$$

Casimir operator

- Representation of the Casimir operator on the BBFF basis for $N_B = 2$, $N_F = 2$:

$$\begin{pmatrix} 0 & 1 & -1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & -1 \\ -1 & 1 & 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ -1 & 0 & 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 1 & 1 & 0 \end{pmatrix}$$

- similar structure for $N_B = 3$ $N_F = 3$

Symmetry check

- Use the Casimir to "test" the symmetry of a wavefunction $\frac{\langle\Psi|\hat{C}|\Psi\rangle}{\langle\Psi|\Psi\rangle}$
- Check for $N_B = 3$ $N_F = 3$: the "BA" solution has the Y symmetry

$$\frac{\langle\Psi_{BA}|\hat{C}|\Psi_{BA}\rangle}{\langle\Psi_{BA}|\Psi_{BA}\rangle} = 3$$

F	B	B	B
F			
F			

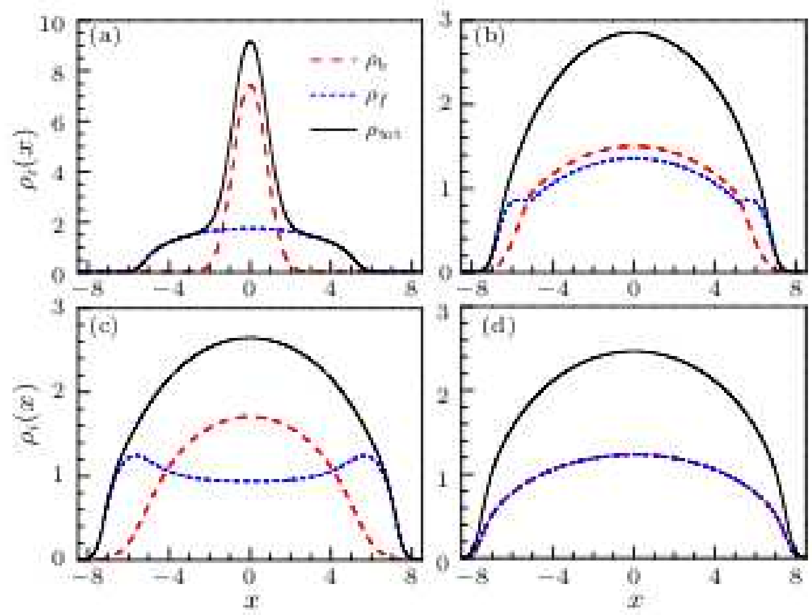
Ψ_{BA} has the symmetry of the ground state

Spatial structure of the BF mixture

- The BA solution yields a non-demixed density profile: connection with partial demixing at intermediate interactions?

Spatial structure of the BF mixture

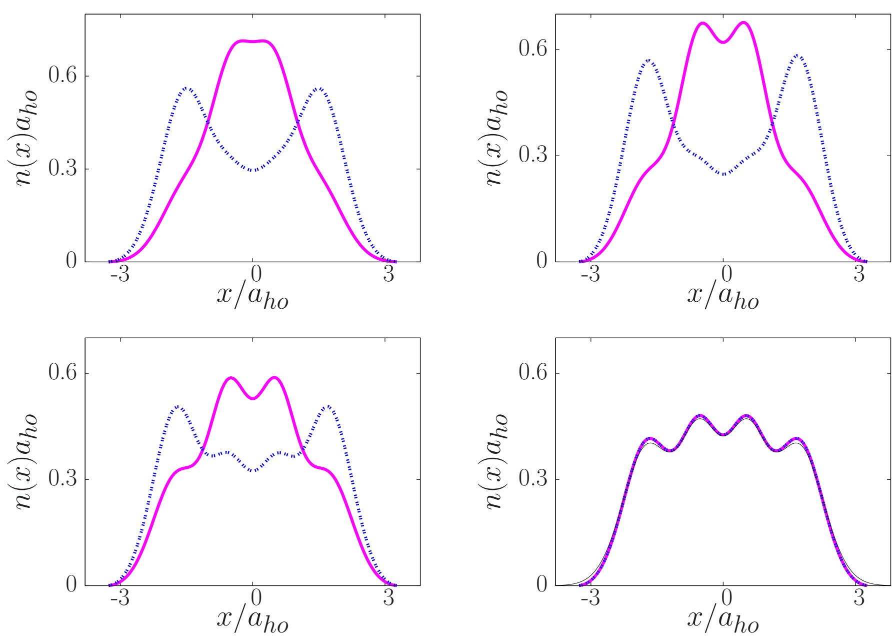
- The BA solution yields a non-demixed density profile: connection with partial demixing at intermediate interactions?
- A density functional study:



[Ya-Jiang Hao, Chin. Phys. Lett. 28 010302 (2011)]

Spatial structure of the BF mixture

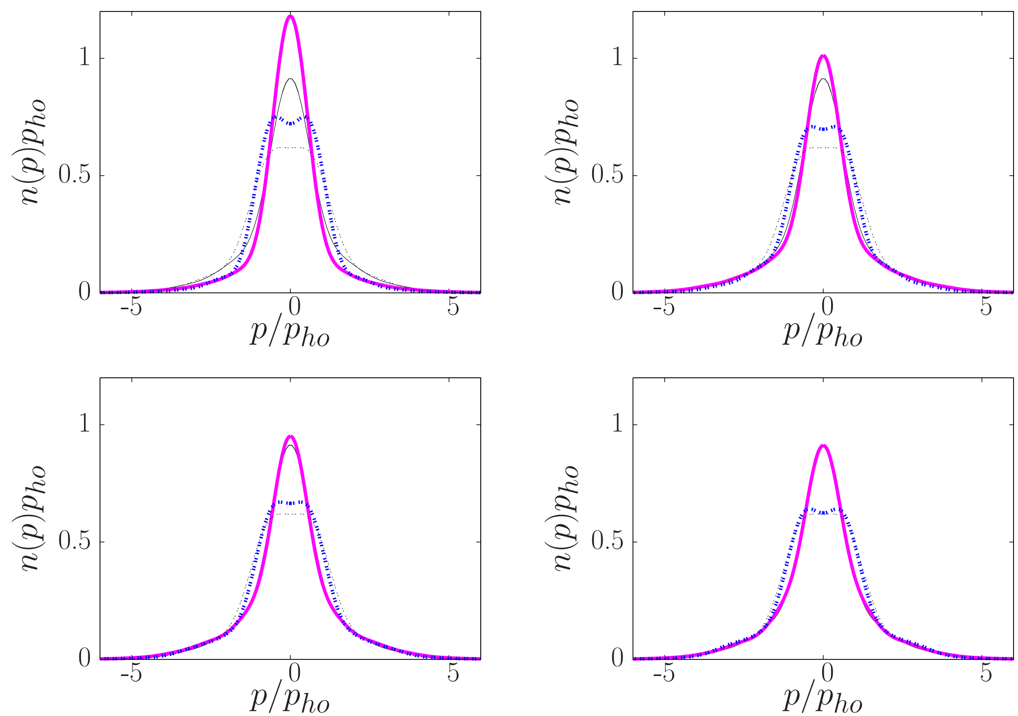
- The BA solution yields a non-demixed density profile: connection with partial demixing at intermediate interactions?
- our DMRG results *[B. Fang,P. Vignolo, M. Gattobigio, C. Miniatura, A. Minguzzi, PRA 84, 023626 (2011)]*



No demixing at very large interactions

Further comparisons

- our DMRG results for the momentum distribution

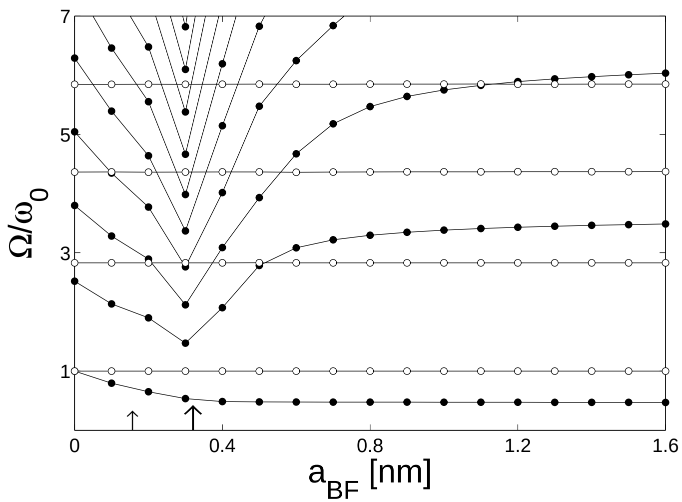


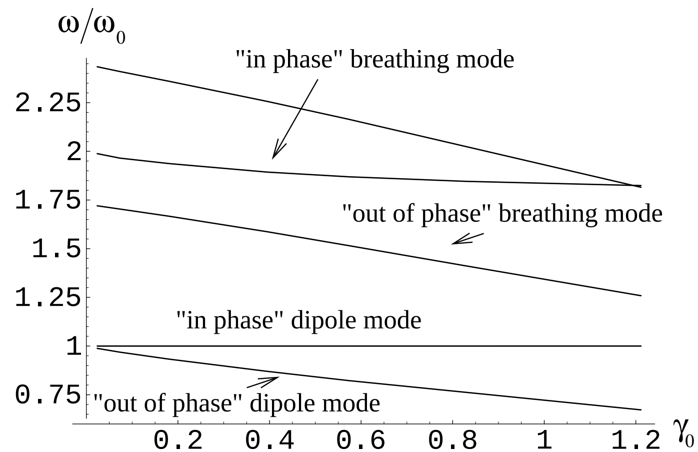
[B. Fang et al PRA 84, 023626 (2011)]

Ψ_{BA} *well describes the DMRG data at large interactions*

Dynamical aspects

- The spatial structure influences the collective mode spectrum: demixing $\Rightarrow$ frequency softening of out-of-phase modes





[P. Capuzzi, A. Minguzzi, M.P. Tosi PRA 67, 053605 (2003)]

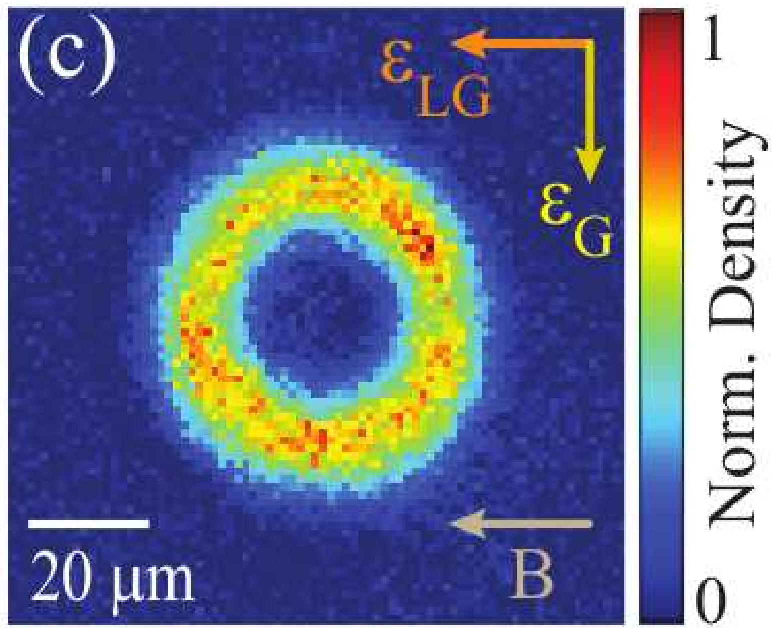
[A. Imambekov, E. Demler, Ann. Phys. 321, 2390 (2006)]

the crossover partial demixing - no demixing should also be observable on the frequencies of collective modes

1D bosons on a ring trap

Bosons on a ring trap

- New topology realized in experiments *(NIST, Oxford, Cambridge, Villetanneuse...)*
- Possibility to set into *rotation* a barrier potential

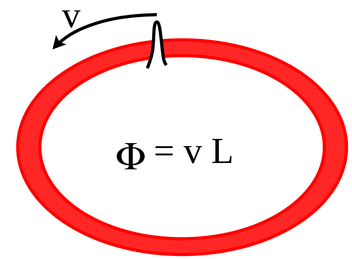


Ramanathan et al (2011)

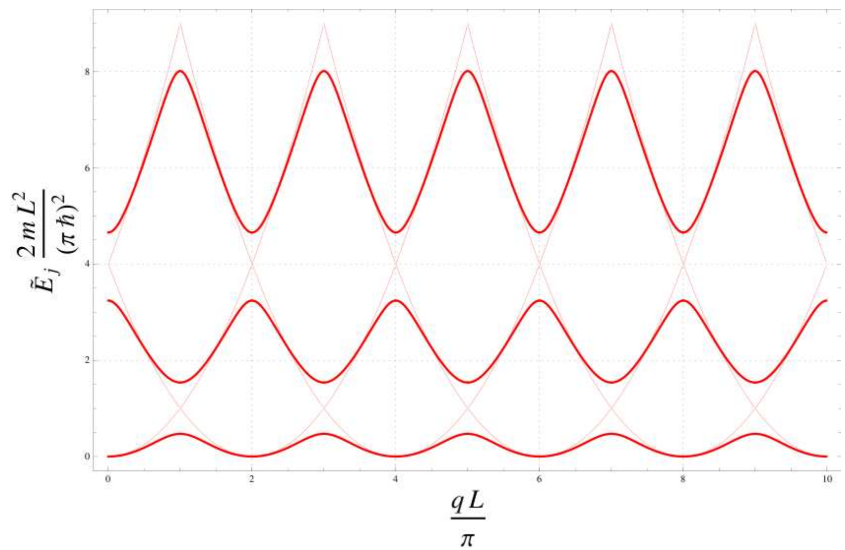
- small, tight rings under construction

Bosons on a 1D ring

stirred by a rotating localized barrier

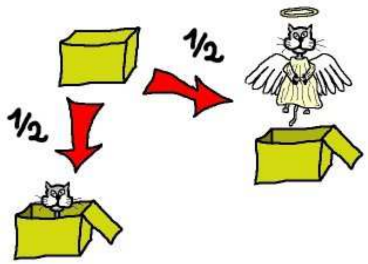


- artificial gauge fields – *rotation* ⇔ *magnetic field*
  $\mathcal{H} = \frac{1}{2m}(i\hbar\nabla - mv)^2 + V_{ext}$

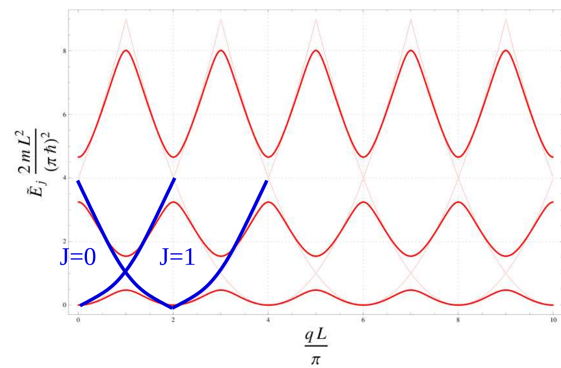


- Mesoscopic effects: energy levels depend on Coriolis flux $\Phi = Lv$, periodic in flux quantum $\Phi_0 = 2\pi\hbar/m$

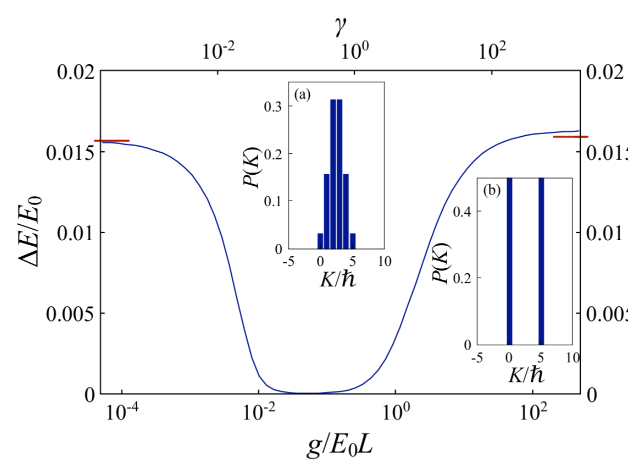
Macroscopic superposition states



- *the "Schroedinger cat"* : a quest with ultracold atoms; decoherence due to particle losses and magnetic fluctuations



- on a ring: superpositions of current states



- weak interactions are harmful; robust superpositions at strong interactions *[DW Hallwood et al (2010)]*

also: fermionization prevents two- and three-body losses

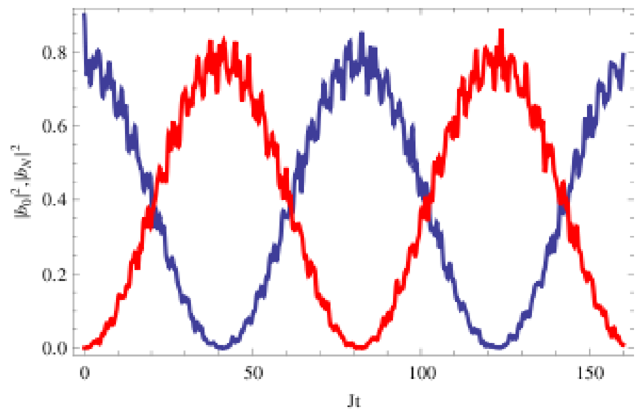
A close look to the superpositions

Rabi-like oscillations between current states induced by a velocity quench

- at zero (or weak) interactions: ”NOON” state, superposition of $q = 0$ and $q = q_0$

$$|NOON\rangle \propto [(b_0^\dagger)^N + (b_{q_0}^\dagger)^N |vac\rangle$$

[A Nunnenkamp et al (2008)]



- strong interactions prevent from multiple occupation of single particle state – *not a simple NOON*: nature of the superposition?

Superpositions with a TG gas

- width of the TG gas in momentum space v_F
  typical velocity at half Coriolis flux $v = \pi\hbar/mL$
  if $v \ll v_F$ difficult to resolve this superposition

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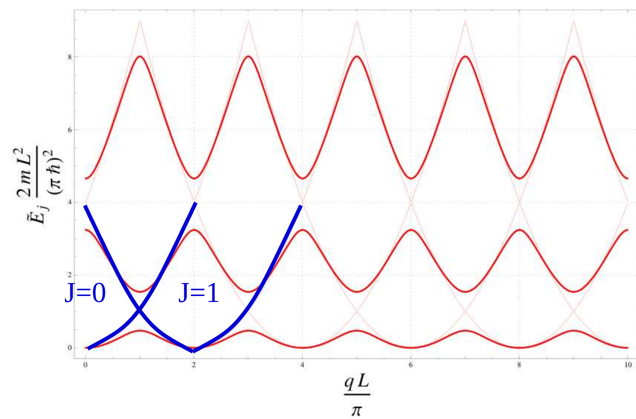

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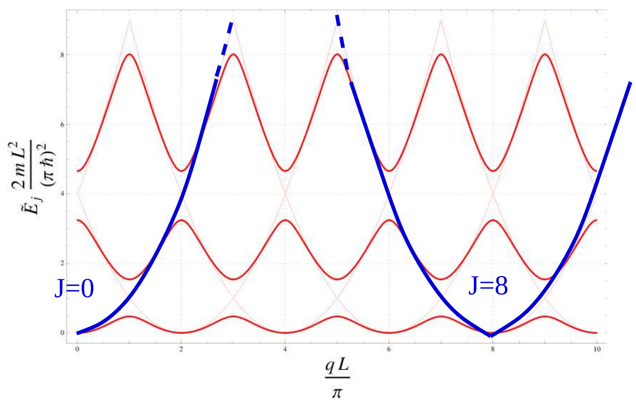
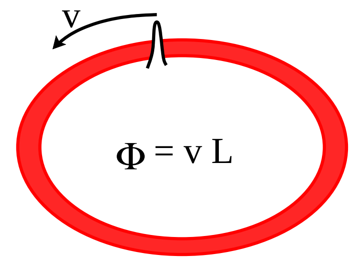


occupation of highly excited states: through a velocity quench!

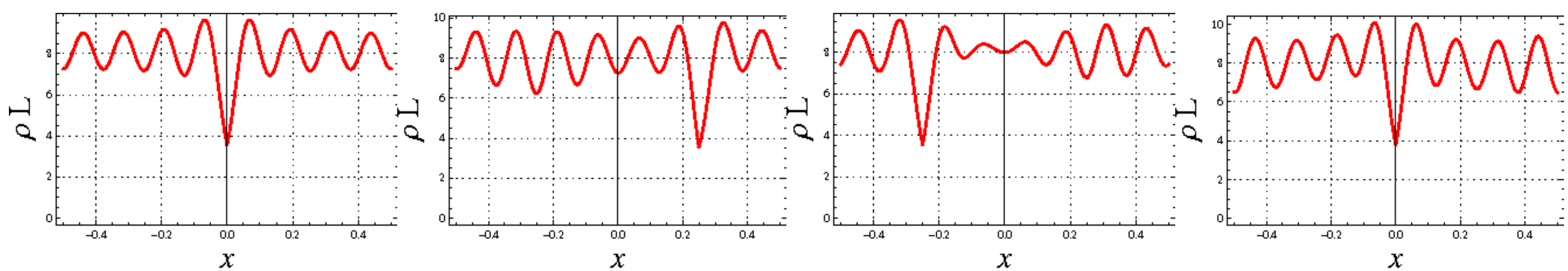
Stirring impenetrable bosons

TG bosons on a ring, with moving barrier

$U(x,t) = U_0 \delta(x - vt)$



- initial state: ground state of the static barrier problem
- *sudden quench* of the barrier velocity to its final value v

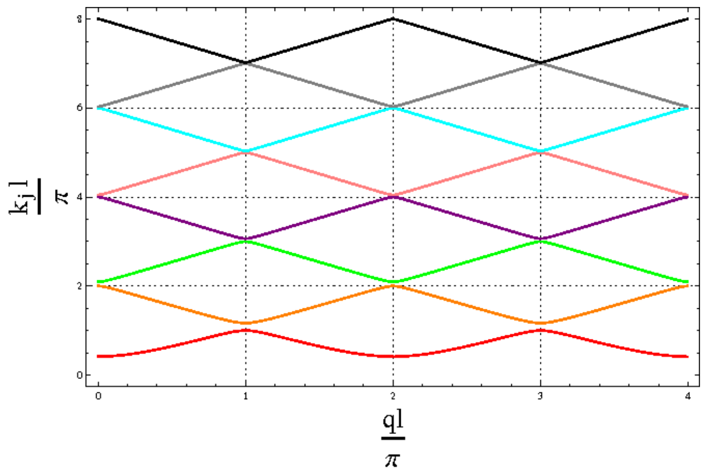


- *exact solution of the quantum non-equilibrium problem* by the time-dependent Bose-Fermi mapping

A novel superposition

Sudden quench to $v \geq v_F$: occupied states

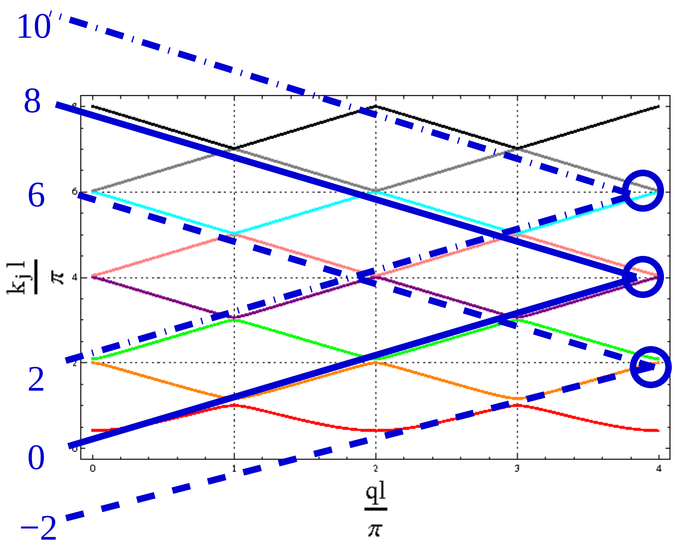
mapped Fermi problem at avoided level crossings



wavevector dispersion of the single particle problem

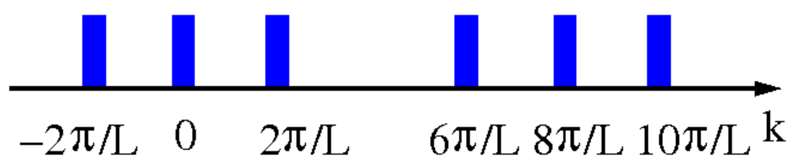
A novel superposition

Sudden quench to $v \geq v_F$: occupied states mapped Fermi problem at avoided level crossings



occupied states for N=3 TG bosons at $v = 4\pi\hbar/mL$

- occupation number distribution:

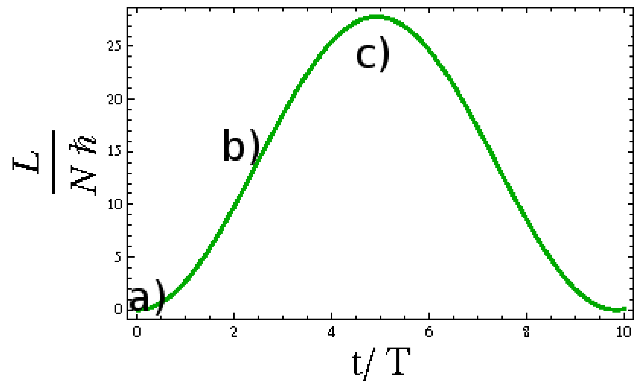


a superposition of two Fermi spheres

Exact quantum dynamics

following a sudden quench of barrier velocity

- spatially integrated particle current *vs* time



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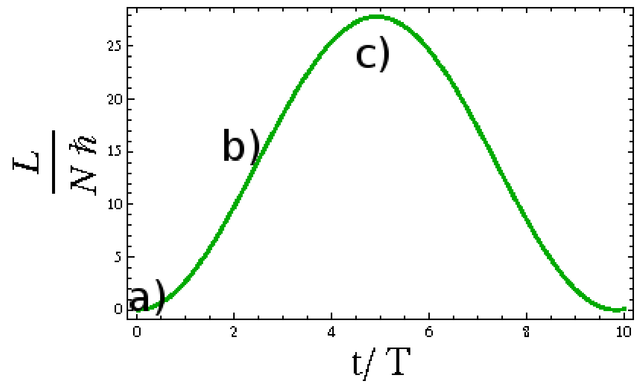
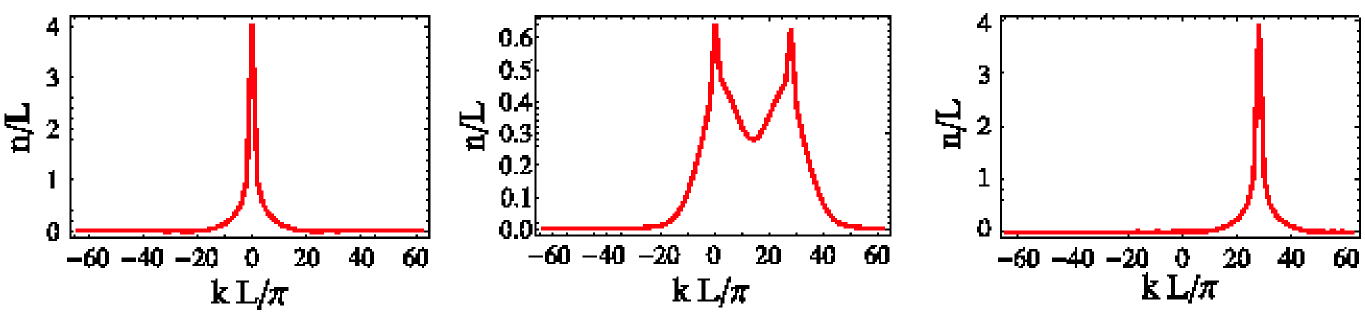


- momentum distribution: tomography of Rabi-like oscillations



superposition of current states with velocity 0 and $2v$

Quantum state engineering

- New! *exact many-body wavefunction for the superposition state of correlated bosons*

$$\Psi_B = \Pi_{1\leq j<\ell\leq N}\mathrm{sign}(x_j - x_\ell)\,\det[\alpha_i\phi_i(x_k) + \beta_i\phi_{i+2n}(x_k)]$$

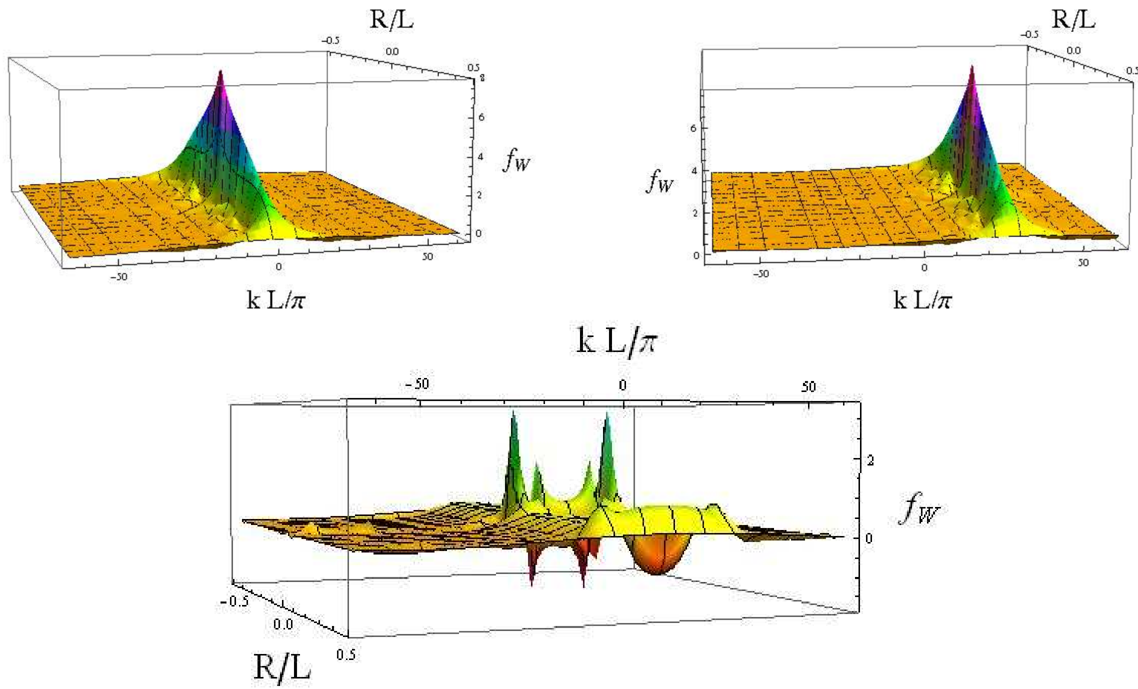
with $\{i = -(N-1)/2, .., (N-1)/2, k = 1..N\}$, and barrier velocity $v = 2\pi n/mL$

- *Particle correlations* and Bose symmetry under particle exchange
- *Superposition* in each single particle state
- Is it a nonclassical state?

Wigner function

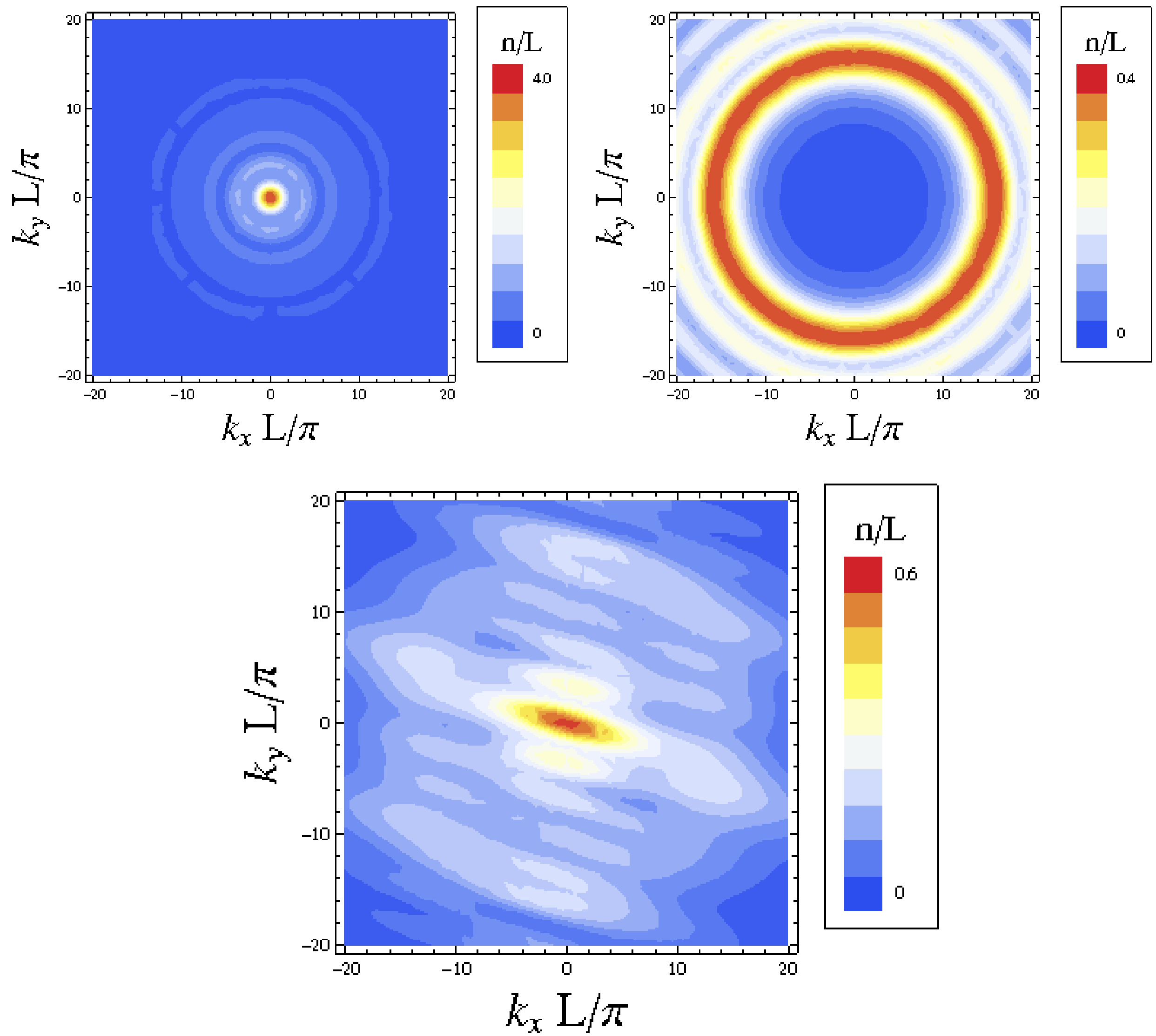
demonstrating nonclassicality of the superposition...

- Wigner function



[C Schenke, AM and FWJ Hekking, PRA 84, 053636 (2011)]

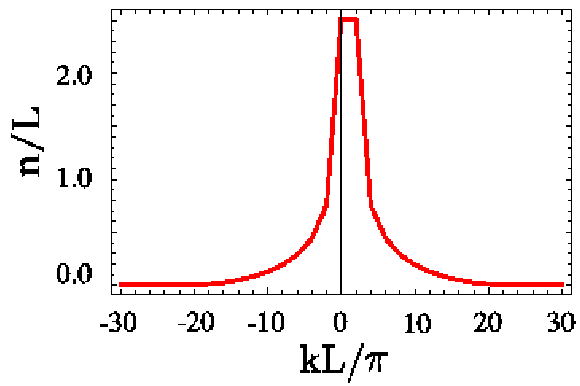
Time-of-flight images

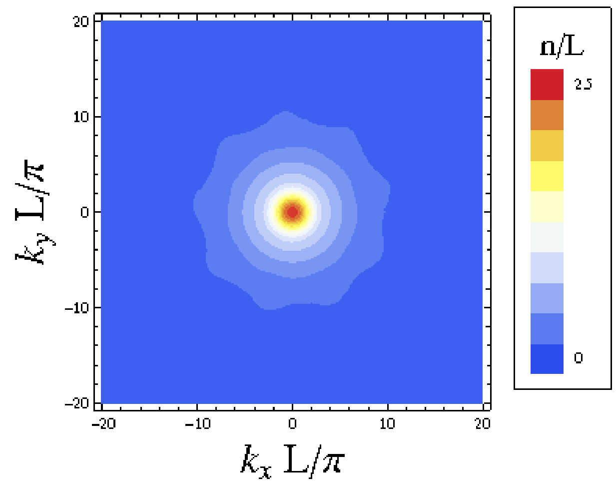


superposition state: interferences in TOF

Resolving the components

- momentum distribution and TOF images for a small velocity $v = \pi\hbar/mL$



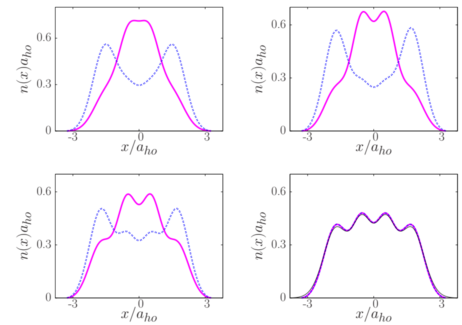


the components are not well resolved at $v \ll v_F$
(the Fermi spheres largely overlap)

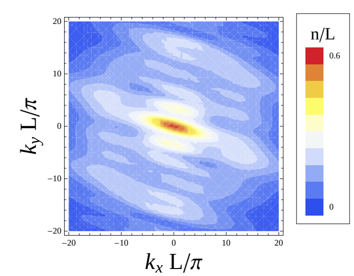
- Same results are obtained for adiabatic stirring at large velocities: *importance of the quench*

Conclusions

- *Progress on solvable models:* wavefunction of the inhomogeneous Bose-Fermi mixture at large interactions



- *Exact dynamical solution for a quench problem:* macroscopic superpositions of correlated bosons on a ring trap



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