Relaxation in driven Integer quantum Hall edge states

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Phys. Rev. B 84, 085105 (2011) and arXiv:1111.3914

Motivation

- Experiments on evolution of nonequilibrium electron distribution

Description of edge states

- Model
- Electron fractionalization

Theoretical approaches to equilibration

- Quantum quench as an idealization
- Exact solution via bosonization + refermionization

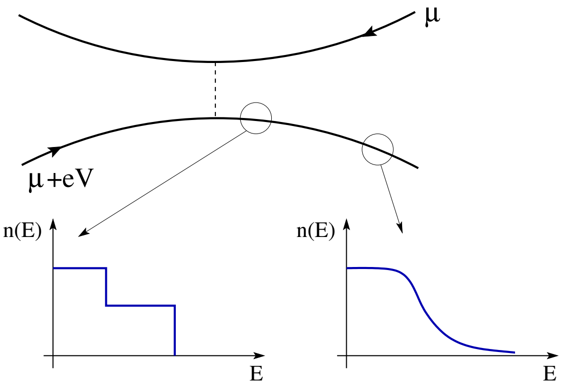
Results

- Relaxation in an integrable system
- Non-thermal steady states

Summary

Schematic view of experiment

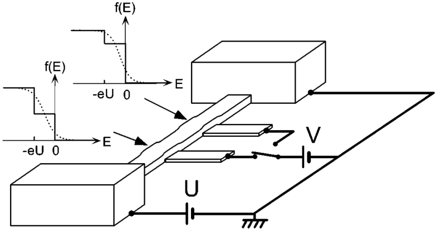
Two edge-states coupled via quantum point contact

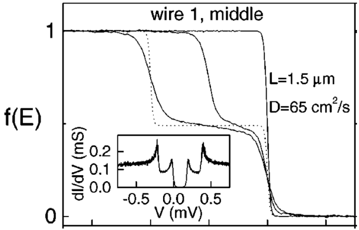


H. le Sueur, C. Altimiras, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre

Phys. Rev. Lett 105, 056803 (2010)

Previous experiments in quantum wires

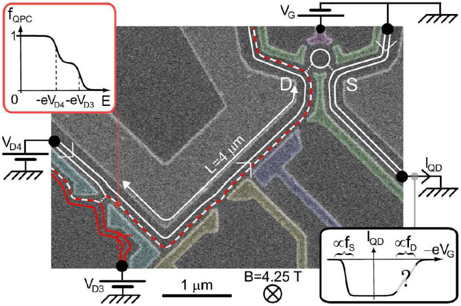




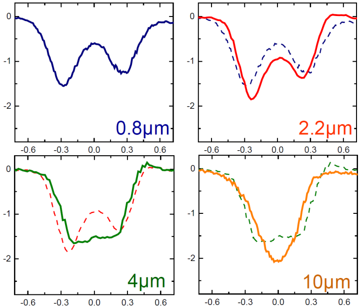
H. Pothier, S. Gueron, Norman O. Birge, D. Esteve, M.H. Devoret

Phys. Rev. Lett. 79, 3490 (1997)

Actual experiment with QHE edge states



Evolution of the distribution



$V_D \sim 40\mu\mathrm{V}, \quad T \sim 40\mathrm{mK}$

$$I_{QD}(E) \sim dn(E)/dE$$

H. le Sueur, C. Altimiras, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre
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At long distances from the QPC the system reaches a steady state

Theoretical description of edge states

As fermions

$$\hat{H} = -i\hbar v_f \int \hat{\psi}^\dagger(x)\partial_x\hat{\psi}(x)dx + \frac{1}{2}\int U(x-x')\hat{\rho}(x)\hat{\rho}(x')dxdx'$$

$$\hat{\rho}(x) = \hat{\psi}^\dagger(x)\hat{\psi}(x)$$

Bosonic representation

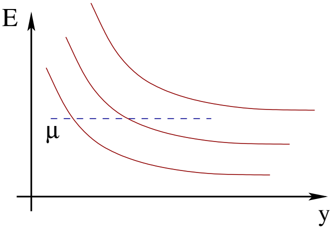
$$\hat{H} = \frac{\hbar v_f}{2}\int (\partial_x\hat{\phi}(x))^2\frac{dx}{2\pi} + \frac{1}{2}\int U(x-x')\partial_x\hat{\phi}(x)\partial_{x'}\hat{\phi}(x')\frac{dx}{2\pi}\frac{dx'}{2\pi}$$

In terms of collective modes (plasmons)

$$\hat{H} = \sum_q \hbar\omega_q\hat{b}_q^\dagger\hat{b}_q, \quad \omega_q = q[u+v(q)], \quad v(q) = \frac{1}{2\pi\hbar}\int dx e^{iqx}U(x)$$

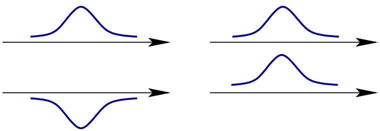
Fractionalization at $\nu = 2$

Landau levels



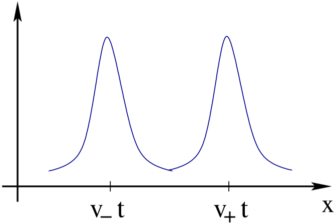
Modes mixed by interactions

$$v_{\pm} = v \pm g$$



Injected electron fractionalizes

$$|\langle \hat{\psi}^{\dagger}(x,t)\hat{\psi}(0,0)\rangle|^2 \rightarrow$$



Electron wave-packet spits into two, propagating with different velocities

Edge states far from equilibrium

Difficulties

- Interactions treated most simply via bosonization
- Tunneling at QPC simplest in fermionic language

Previous work on theory for relaxation experiments

- Boltzmann equation: Lunde et. al. (2010)
- QPC as source of plasmon noise: Degiovanni et. al. (2010)
- Weak tunnelling at QPC: Levkivskyi and Sukhorukov (2012)

Approaches here

- Quantum quench as idealisation
- Exact treatment via bosonization + refermionisation

Theoretical Idealisation: Quantum quench

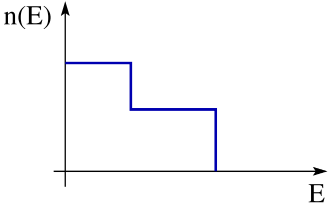
Evade treatment of point contact

Study the time evolution in translationally invariant edge

Initial state $|\hat{\psi}\rangle$

Time evolution

$|\hat{\psi}(t)\rangle = e^{-i\hat{H}t}|\hat{\psi}\rangle$



Initial distribution

Properties of $|\hat{\psi}(t)\rangle$?

Energies of collective modes conserved $\rightarrow$ consequences for equilibration ?

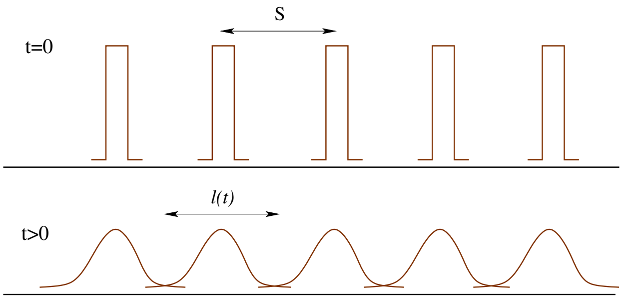
Physical picture of equilibration

Hamiltonian in terms of collective modes $\hat{H} = \sum_{qn} \hbar\omega_{qn} \hat{b}^\dagger_{qn} \hat{b}_{qn}$

Edge magnetoplasmon dispersion $\rightarrow$ *electron* equilibration ?

Initial distance between wave-packets $S = \hbar v_f / eV$

Wave-packet spread due to dispersion $l(t)$



Equilibration when $l(t) \gtrsim S$

Equilibration from two mode velocities

Two edge states coupled by short-range interactions (assume screening of Coulomb interactions)

Two plasmon modes with linear dispersion and different velocities $\omega_+(q) = v_+ q$ and $\omega_-(q) = v_- q$

Initial wave-packets separation $S = \hbar v / eV$

Wave-packets size $l(t) = t \times (v_+ - v_-)$

Equilibration when wave-packets spread $l(t) \gtrsim S$

Equilibration time $t_{eq} = \frac{\hbar}{eV} \frac{v_+ + v_-}{v_+ - v_-}$

What are the properties of equilibrium states

Characterize using single-particle (electron) correlation functions

Need to calculate $G(x,t) = \langle \hat{\psi}^\dagger(x,t)\hat{\psi}(0,0)\rangle$
In a thermal state: $G(x,t) = [-2i\beta\hbar v \sinh(\pi[x+i0]/\beta\hbar v)]^{-1}$

Approach to calculations:
Alternate between bosonic and fermionic description

Initial state is simple in fermions $\hat{\psi}(x) \sim e^{-i\hat{\phi}(x)}$

Time evolution is described using bosonic operators

$\hat{\phi}(x,t) \sim \sum_q [\frac{2\pi}{qL}]^{1/2} [\hat{b}_q e^{iqx - i\omega_q t} + h.c]$

Back to fermions to calculate observables $\hat{b}_q^\dagger \sim [\frac{2\pi}{qL}]^{1/2} \sum_k \hat{c}_{k+q}^\dagger \hat{c}_k$

What are the properties of equilibrium states

Characterize using single-particle (electron) correlation functions
Need to calculate $G(x,t) = \langle \hat{\psi}^\dagger(x,t)\hat{\psi}(0,0)\rangle$

We obtain $G(x,t) \sim \langle \exp[i \int K(x,t;y)\hat{\rho}(y)dy]\rangle$

At long times $K(x,t;x/2 - vt + \xi) \sim F(x/l(t), \xi/l(t))$

where $l(t)$ is the size of the wave-packet

For $\nu = 1$ in the long-time limit

$G(x,t) \sim \exp[-\int C(x,t)\langle\hat{\rho}(0)\hat{\rho}(y)\rangle dy]$

with $C(x,y) = \pi^2[|x-y| + |x+y| - 2|y|]$, independent of $U(x)$

Generalization of Dzyaloshinskii-Larkin theorem

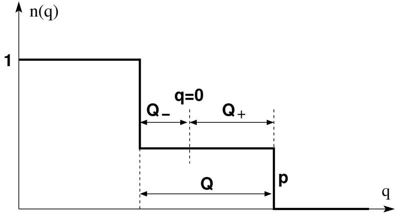
Comparison with a thermal state

Short distance correlations are the same as in the thermal state

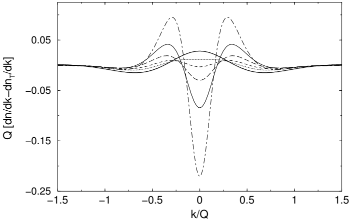
Long distance correlations $G(x,t) \sim \exp(-\alpha|x|)$ where α is not fixed by the energy density

$$\mathcal{G}(x) \approx \frac{i}{2\pi} \frac{|e^{1+\gamma_E} Qx|^{2p(1-p)}}{x+ia} e^{-\pi p(1-p)|Qx|}$$

$n(k,t) = \frac{1}{2} - \frac{1}{\pi} e^{\alpha(1+\gamma_E)} \Gamma(\alpha) \mathrm{Im}[(-ik/Q + \pi\alpha/2)^{-\alpha}]$, $\alpha \equiv 2p(1-p)$.



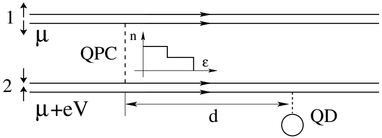
Initial state



Difference with a thermal state

Model of the experiment

Assume short-range interactions



$$\hat{H}_0 = -i\hbar v \sum_{\eta,s} \int \hat{\psi}^{\dagger}_{\eta s}(x)\partial_x \hat{\psi}_{\eta s}(x)\, \mathrm{d}x + 2\pi\hbar g \sum_{\eta} \int \hat{\rho}_{\eta\uparrow}(x)\hat{\rho}_{\eta\downarrow}(x)\, \mathrm{d}x$$

Tunneling between down channels

$$\hat{H}_{\text{tun}} = t_{\text{QPC}} \hat{\psi}^{\dagger}_{1\downarrow}(0)\hat{\psi}_{2\downarrow}(0) + \text{h.c.}$$

Need to calculate the current throught the QD

$$I_{QD}(E) = \frac{e|t_{\text{D}}|^2}{\hbar^2} \int G_s(d,\tau) e^{-iE\tau/\hbar}\, d\tau$$

with the Green function at distance d from the QPC

$$G_s(d,\tau) = \langle e^{i\hat{H}\tau/\hbar} \hat{\psi}^{\dagger}_{2s}(d) e^{-i\hat{H}\tau/\hbar} \hat{\psi}_{2s}(d) \rangle$$

Bosonization

Introduce bosonic fields $\hat{\phi}_{\eta s}(x)$ and Klein-factors $\hat{F}_{\eta s}$

$$\hat{\psi}_{\eta s}(x) = (2\pi a)^{-1/2} \hat{F}_{\eta s} e^{i \frac{2\pi}{L} \hat{N}_{\eta s}} e^{-i \hat{\phi}_{\eta s}(x)}$$

Rotate the bosonized Hamiltonian using a matrix

$$U = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

so that $(\hat{\chi}_{S_+} \; \hat{\chi}_{A_-} \; \hat{\chi}_{A_+} \; \hat{\chi}_{S_-})^T = U(\hat{\phi}_{1\uparrow} \; \hat{\phi}_{1\downarrow} \; \hat{\phi}_{2\downarrow} \; \hat{\phi}_{2\uparrow})^T$

$$\hat{H} = \frac{\hbar v_+}{2} \int (\partial_x \hat{\chi_+}(x))^2 \frac{dx}{2\pi} + \frac{\hbar v_-}{2} \int (\partial_x \hat{\chi_-}(x))^2 \frac{dx}{2\pi}$$

Two Fermi velocities $v_\pm = v \pm g$ for the corresponding modes

Refermionization of the bosonic Hamiltonian

The tunneling Hamiltonian in bosonized form

$$\hat{H}_{\mathrm{tun}} = t_{\mathrm{QPC}} \hat{F}^{\dagger}_{1\downarrow} \hat{F}_{2\downarrow} e^{i[\hat{\phi}_{1\downarrow}(0) - \hat{\phi}_{2\downarrow}(0)]} + \mathrm{h.c.}$$

and $\hat{\phi}_{1\downarrow}(0) - \hat{\phi}_{2\downarrow}(0) = \hat{\chi}_{A_+}(0) - \hat{\chi}_{A_-}(0)$

Need to introduce new Klein-factors

$$\begin{aligned} \hat{F}^{\dagger}_{S_-} \hat{F}^{\dagger}_{A_-} &= \hat{F}^{\dagger}_{1\uparrow} \hat{F}_{1\downarrow}, \quad \hat{F}_{S_-} \hat{F}^{\dagger}_{A_-} = \hat{F}^{\dagger}_{2\downarrow} \hat{F}_{2\uparrow}, \\ \hat{F}^{\dagger}_{S_-} \hat{F}^{\dagger}_{A_+} &= \hat{F}^{\dagger}_{1\uparrow} \hat{F}_{2\downarrow}, \quad \hat{F}^{\dagger}_{S_+} \hat{F}^{\dagger}_{A_-} = \hat{F}^{\dagger}_{1\uparrow} \hat{F}^{\dagger}_{2\downarrow}. \end{aligned}$$

The refermionized Hamiltonian has a free electron form

$$\hat{H} = -i\hbar \sum_{\alpha} v_{\alpha} \int \hat{\Psi}^{\dagger}_{\alpha} \partial_x \hat{\Psi}_{\alpha} dx - [t_{\mathrm{QPC}} \; \hat{\Psi}^{\dagger}_{A_+}(0) \hat{\Psi}_{A_-}(0) + \mathrm{h.c.}]$$

also $\hat{F}^{\dagger}_{1\uparrow} \hat{F}^{\dagger}_{1\downarrow} = \hat{F}^{\dagger}_{A_+} \hat{F}^{\dagger}_{S_+}$ and $\hat{F}^{\dagger}_{2\downarrow} \hat{F}^{\dagger}_{2\uparrow} = \hat{F}_{A_+} \hat{F}^{\dagger}_{S_+}$, bias voltage

Results

Exact expression for the correlator

$$G_s(d,\tau) = G_0(\tau)\langle e^{-i\pi[\mathcal{N}_{A+}(d,\tau)\pm\mathcal{N}_{A-}(d,\tau)]}\rangle_{\rm norm}$$

where we introduce electron counting operator

$$\hat{\mathcal{N}}_\alpha(d,\tau) = \int_d^{d+v_\pm\tau} \hat{\Psi}^\dagger_\alpha(y)\hat{\Psi}_\alpha(y)dy$$

and $G_0(\tau) = \frac{i}{2\beta\hbar(v_+v_-)^{\frac{1}{2}}}\frac{1}{\sinh^{\frac{1}{2}}[\frac{\pi}{\beta\hbar v_+}(-v_+\tau+ia)]}\frac{1}{\sinh^{\frac{1}{2}}[\frac{\pi}{\beta\hbar v_-}(-v_-\tau+ia)]}$

Long-distance behavior of the GF

$\langle e^{-i\pi[\mathcal{N}_{A+}(d,\tau)\pm\mathcal{N}_{A-}(d,\tau)]}\rangle_{\rm norm} = \chi_-(\mp\pi,\tau)\chi_+(-\pi,\tau)$
with $\chi_\pm(\delta,\tau) = \langle e^{i\delta\hat{\mathcal{N}}_\pm(d,\tau)}\rangle_{\rm norm}$

For asympt. at long times, see Mirlin et.al.

Numerical evaluation of the correlators

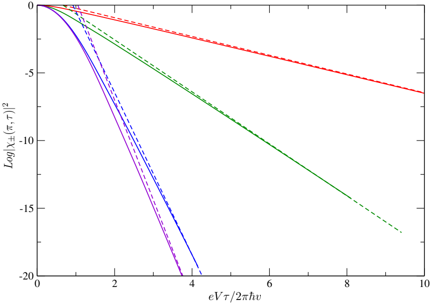
Approach by Mirlin et.al.: only for equilibrium states

write χ as a determinant

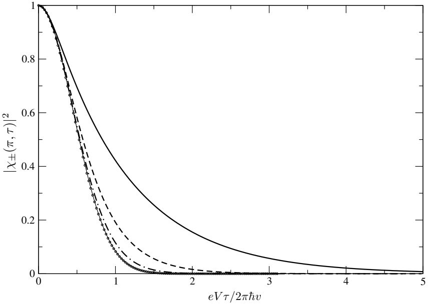
$$\langle e^{i\delta\hat{\mathcal{N}}_{\pm}(d,\tau)}\rangle = \det\{[1-\hat{P}(e^{-i\delta}-1)\hat{n}_{\pm}(\varepsilon)\hat{P}]\}$$

can be represented as $\langle e^{i\delta\hat{N}_{\pm}(\tau)}\rangle = \det[f(t_i - t_j)]$

$\tilde{f}(\varepsilon) = [1 - n_{\pm}(\varepsilon)(e^{-i\delta}-1)]e^{-i\frac{\delta}{2}\frac{\varepsilon}{\Lambda}}$



for different values of δ



for different values of βeV

Intermediate distances from QPC

Levitov's FCS formula. Use scattering states.

$$\langle [\hat{S}^I(\infty)]^\dagger e^{-i\pi[\mathcal{N}_{A+}(d,\tau)\pm\mathcal{N}_{A-}(d,\tau)]}\hat{S}^I(\infty)\rangle_0$$
$$= \det[1 - n(\varepsilon)[[\hat{S}(\infty)]^\dagger e^{-i\pi[\pm\hat{\mathcal{N}}_{A-}(d,\tau)+\hat{\mathcal{N}}_{A+}(d,\tau)]}\hat{S}^I(\infty) - 1]$$

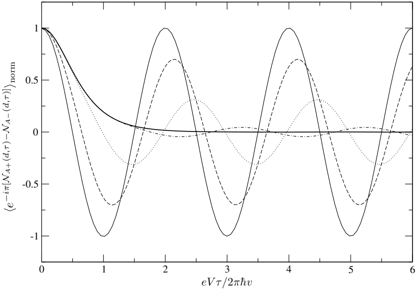
The scattering matrices can be calculated from a noninteracting Hamiltonian with two Fermi-velocities

$$[\hat{S}^I(\infty)]^\dagger \hat{\Psi}_{A+}(x)\hat{S}^I(\infty) =$$
$$\cos\theta\,\hat{\Psi}_{A+}(x) - i\sin\theta\,[v_-/v_+]^{1/2}\,\hat{\Psi}^\dagger_{A_-}(v_- x/v_+)$$

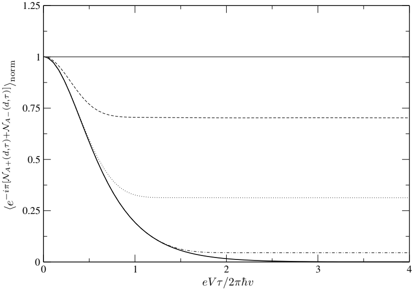
$$[\hat{S}^I(\infty)]^\dagger \hat{\Psi}_{A-}(x)\hat{S}^I(\infty) =$$
$$\cos\theta\,\hat{\Psi}_{A-}(x) - i\sin\theta\,[v_+/v_-]^{1/2}\,\hat{\Psi}^\dagger_{A_+}(v_+ x/v_-)$$

Normalized Green functions

Channel coupled to QPC



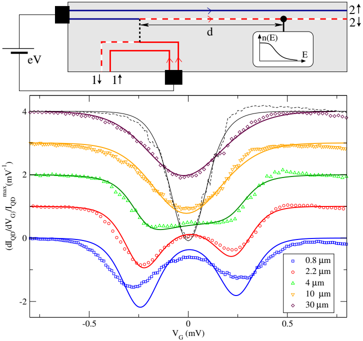
Channel not coupled to QPC

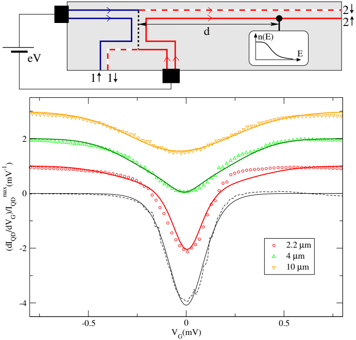


As a function of voltage at $T = 0$ for different distances from QPC, with initial GF corresponding to a double-step

At large distances from QPC the GFs in the inner and outer channels converge to the same function (in the strong coupling limit)

Comparison with the experimental data





Single fitting parameter, interaction strength

$v_f = (v_f^2 + g^2)/2g = 6.5 \times 10^4 \mathrm{m/s}$

D. L. Kovrizhin and J. T. Chalker, arXiv:1111.3914

Summary

Quantum quench approach on isolated edge is useful toy model of experiment with two edges coupled at QPC

- Interactions bring system into non-thermal steady state
- At $\nu = 1$ steady state is independent of interactions

Full problem with QPC is solvable at $\nu = 2$

- Asymptotic steady state is same as for quench, and is non-thermal
- Calculated evolution of tunneling density of state with distance matches experiment well

Exact approach can be used to describe electronic Mach-Zehnder interferometers at $\nu = 2$