Competing phases in frustrated ferromagnetic spin chain

Only ground-state phase diagram
No quantum quench, ….

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Shigeki Onoda (RIKEN)
Masahiro Sato (Aoyama Gakuin U.)

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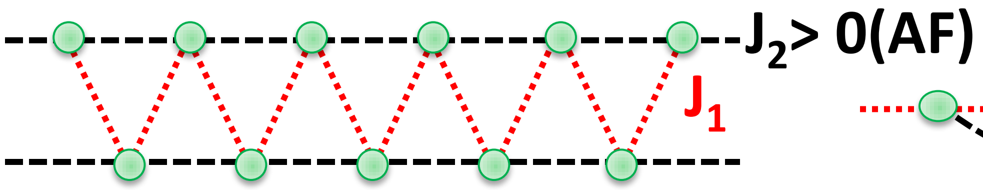
Outline

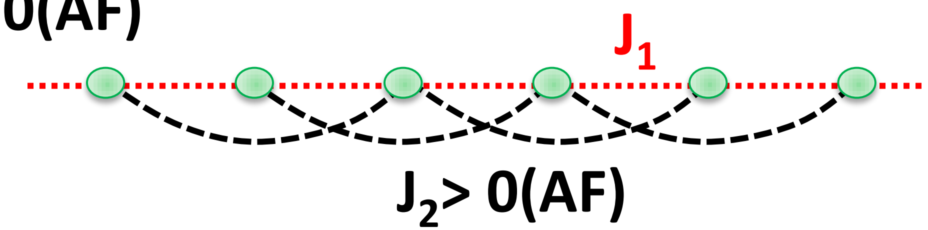
1. Introduction: frustrated spin-1/2 J_1-J_2 XXZ chain
2. Phase diagram of J_1-J_2 XXZ spin chain (J_2 is antiferromagnetic)
   a. $J_1>0$ (antiferromagnetic) review
   b. $J_1<0$ (ferromagnetic) new results

frustrated spin-1/2 XXZ chain

$$H=\sum_{n=1,2}\sum_{j}J_n\left(S_j^xS_{j+n}^x+S_j^yS_{j+n}^y+\Delta S_j^zS_{j+n}^z\right)$$

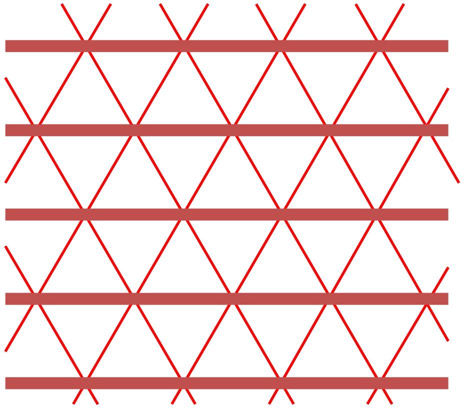
easy-plane anisotropy $0\le\Delta\le1$





If J_2 is antiferromagnetic,
spins are frustrated regardless of the sign of J_1.

J_1-J_2 spin chain is the simplest spin model
with frustration.



Quasi-1D spin-1/2 frustrated magnets with ferro J_1

Li_2ZrCuO_4 — Drechsler et al. PRL,2007

$LiCu_2O_2$ — Masuda et al. PRL,2004; PRB,2005

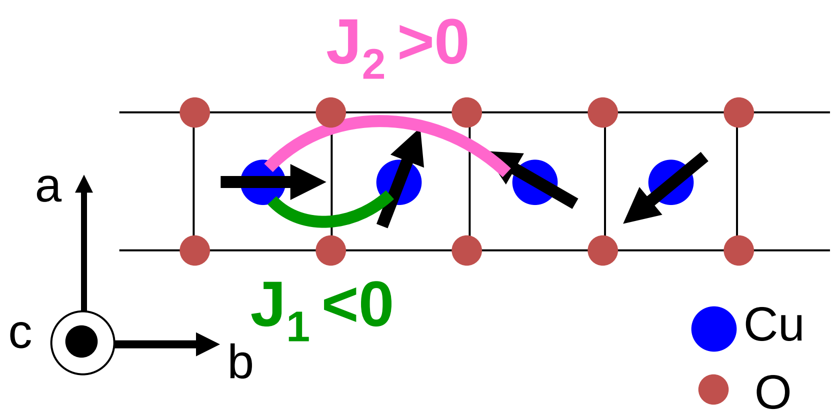
$LiCuVO_4$ — Enderle et al. Europhys.Lett.,2005

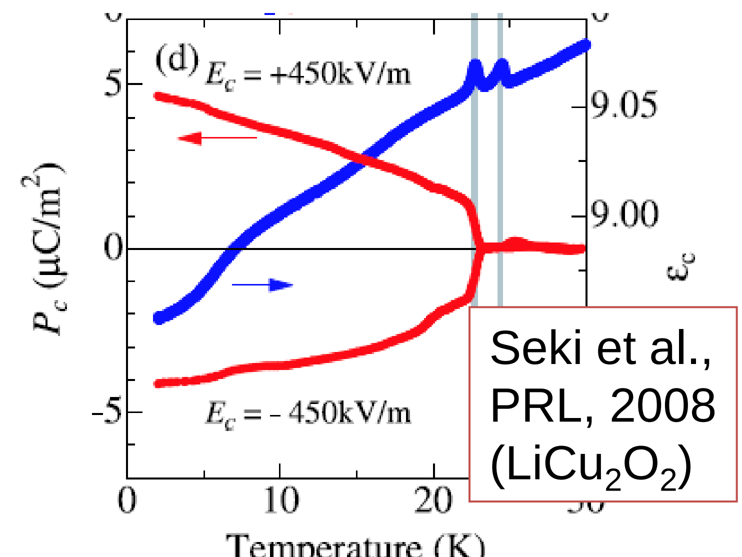
- CuO_2 chain: edge-sharing network

  → ferromagnetic J_1 (Kanamori-Goodenough rule)

- Multiferroicity $\vec{P}_j \propto \vec{e}_b \times (\vec{S}_j \times \vec{S}_{j+1})$

  Observation of chiral ordering through electric polarization P





Seki et al., PRL, 2008 ($LiCu_2O_2$)

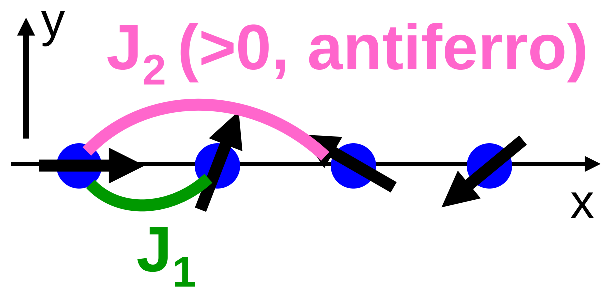
Model

Frustrated spin-1/2 J_1-J_2 XXZ chain

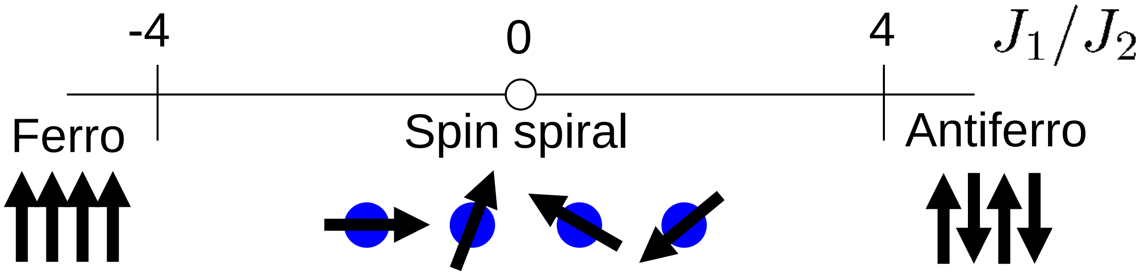
$$H = \sum_{m=1,2} \sum_{j} J_m (S_j^x S_{j+m}^x + S_j^y S_{j+m}^y + \Delta S_j^z S_{j+m}^z)$$

easy-plane anisotropy $\quad 0 \leq \Delta \leq 1$

Spins are frustrated when $J_2 > 0$,
irrespective of the sign of J_1.



Classical ground state $(0 \leq \Delta \leq 1)$



pitch angle $\quad Q = \arccos\left(-\frac{J_1}{4J_2}\right)$

finite chirality $\quad (\vec{S}_j \times \vec{S}_{j+1})^z \neq 0$

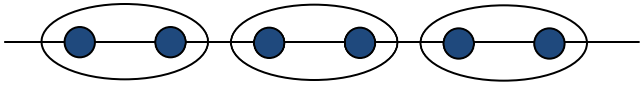
Quantum case S=1/2

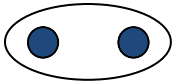
- Classical spiral (chiral) order is destroyed by strong quantum fluctuations in 1D.

- Antiferromagnetic case (J_1>0,J_2>0) is well understood.

  - Singlet dimer order is stabilized (J_2/J_1>0.24).

    Haldane, PRB1982
    Nomura & Okamoto, J.Phys.A 1994
    White & Affleck, PRB 1996
    Eggert, PRB 1996



 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

  - Vector chiral order (quantum remnant of the spiral phase) is found for small $J_1/J_2 < 0.8, \quad \Delta < 0.2$

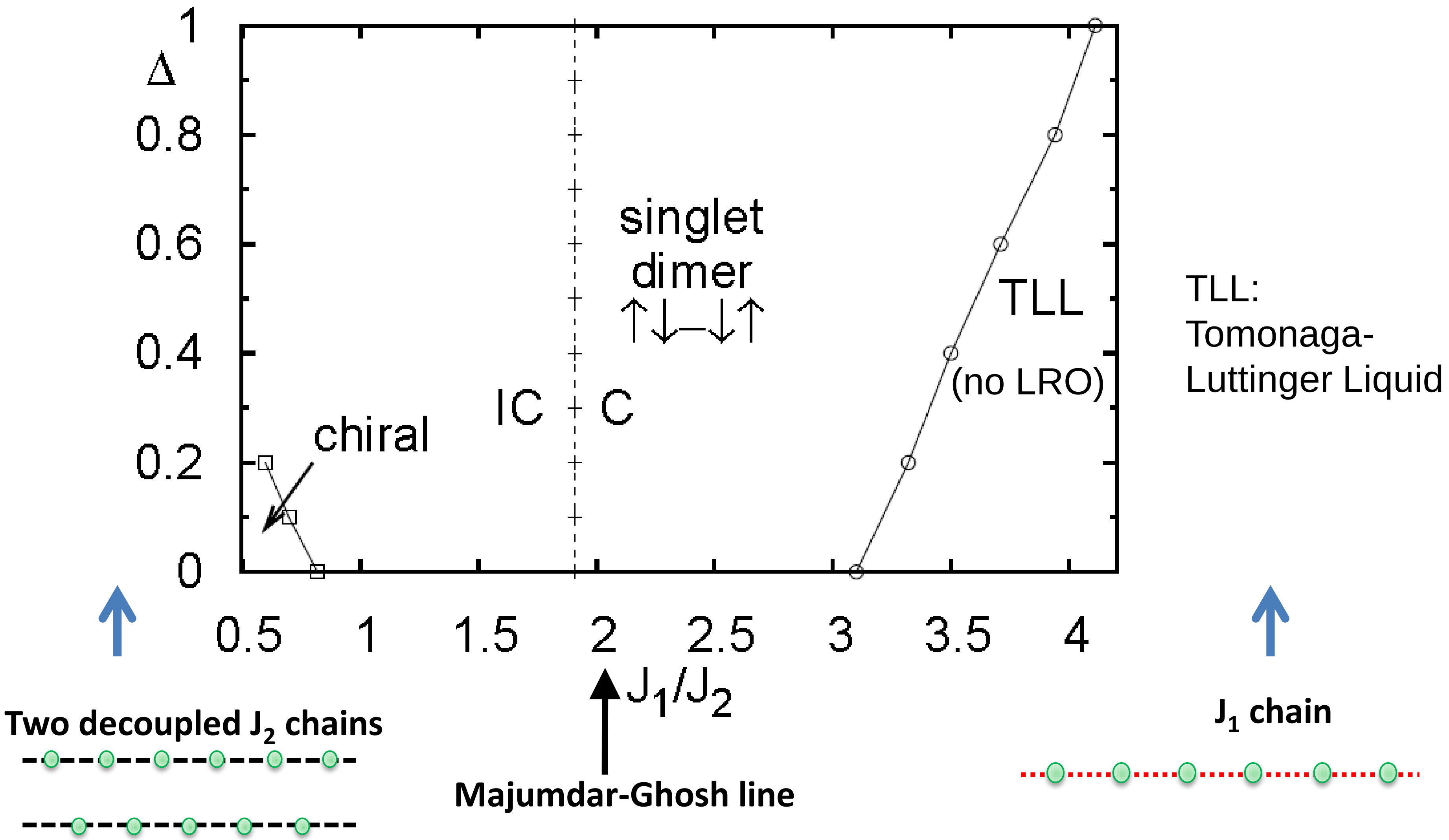
    Nersesyan,Gogolin,& Essler, PRL 1998
    Hikihara,Kaburagi,& Kawamura, PRB 2001

$\left(\vec{S}_j \times \vec{S}_{j+1}\right)_z$ LRO

Quantum analogue of spiral state

Phase diagram of spin-1/2 J_1-J_2 chain

Ground-state phase diagram for AF-J_1 case



Quantum case S=1/2

- The ferromagnetic-J_1 case ($J_1<0, J_2>0$) not well understood

  -Stability of the vector chiral order ?
  (crucial in understanding the emergence of multiferroicity)

  - Any other novel quantum phases arising from frustration?

Our work: the ground-state phase diagram for $J_1<0$.

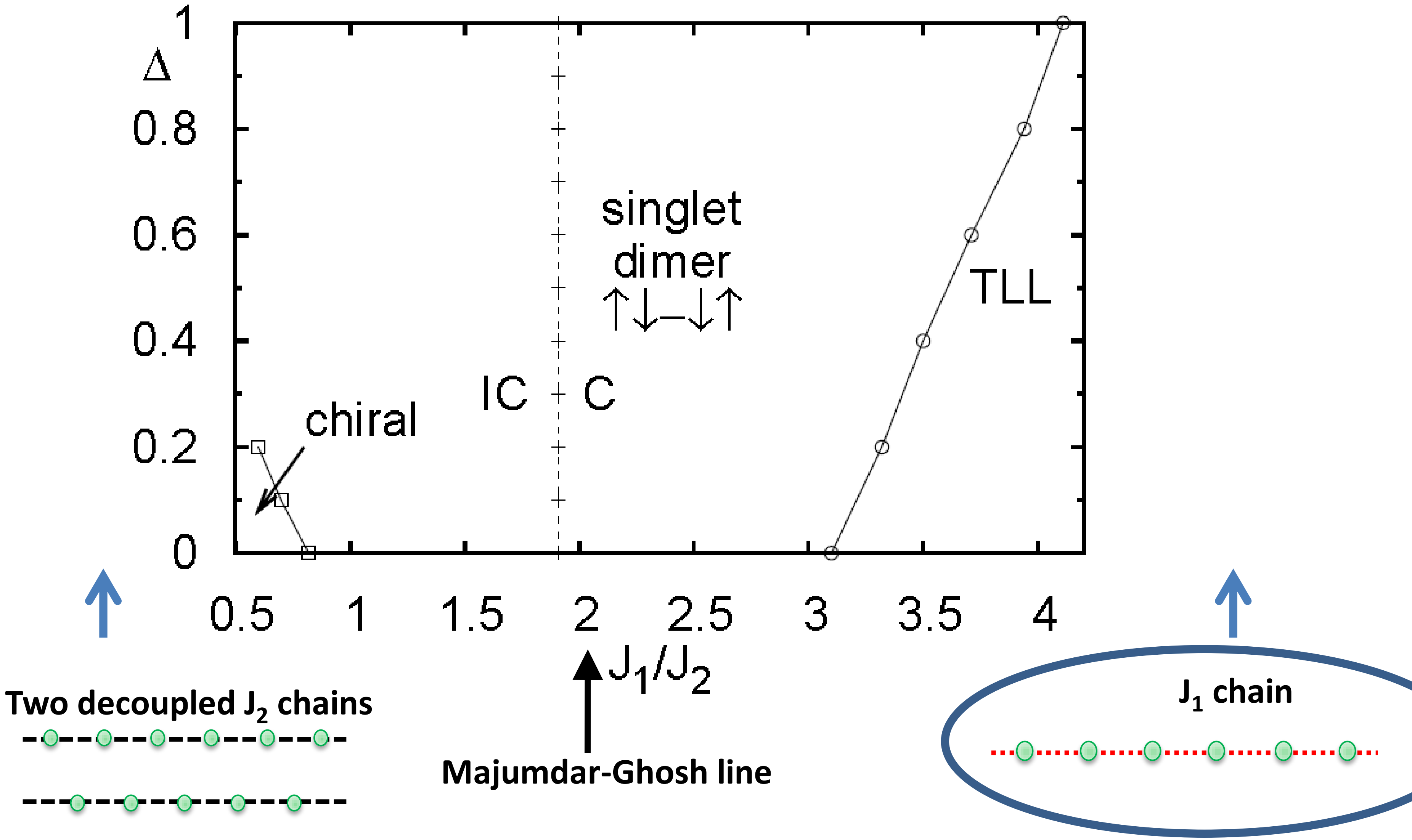
Method:

- numerics:
  - ✓ time evolving block decimation for infinite system (iTEBD)
  - ✓ exact diagonalization
- perturbative RG analysis around $J_1=0$ or $J_2=0$.

Symmetry broken state can be studied.
Order parameter can be measured.

antiferromagnetic J_1: brief review

Ground-state phase diagram for AF-J_1 case

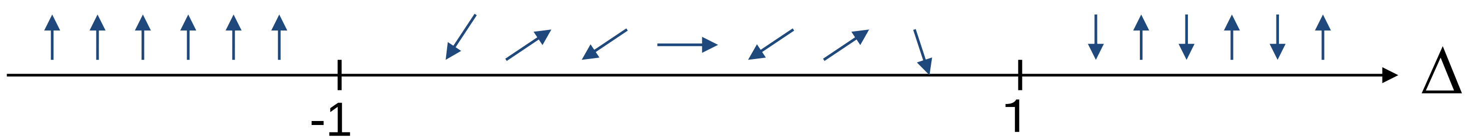


XXZ spin chain

$$H = J_1 \sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right)$$

- Exactly solvable: Bethe ansatz

gapless phase $-1 < \Delta \le 1$



ferromagnetic Ising order	Tomonaga-Luttinger liquid	antiferromagnetic Ising order
LRO	gapless excitations	LRO
energy gap	power-law correlations	energy gap

- Effective field theory: bosonization

$$H_{\text{eff}} = \frac{v}{2}\int dx\left[K\left(\partial_x\theta\right)^2 + \frac{1}{K}\left(\partial_x\phi\right)^2 - \lambda\cos\left(\sqrt{8\pi}\phi\right)\right]$$

$\phi(x),\ \theta(x)$: bosonic field $\qquad \left[\phi(x),\partial_y\theta(y)\right] = i\delta(x-y)$

$\cos\left(\sqrt{8\pi}\phi\right)$ is

- relevant for $|\Delta| > 1$
- irrelevant for $|\Delta| < 1$
- marginally irrelevant for $\Delta = 1$

For $|\Delta| \le 1$

$$\Delta = -\cos\left(\frac{\pi}{K}\right): \quad K = 2 \text{ at } \Delta = 0, \quad K = 1 \text{ at } \Delta = 1$$

$$v = \frac{\sin(\pi\eta)}{2(1-\eta)} \qquad \eta = \frac{1}{K} = 1 - \frac{1}{\pi}\cos^{-1}\Delta$$

In the critical phase $-1 < \Delta \le 1$

The cosine term is irrelevant in the low-energy limit

$$\tilde{H} = \frac{v}{2}\int dx\left[K\left(\partial_x\theta\right)^2 + \frac{1}{K}\left(\partial_x\phi\right)^2\right]$$: Gaussian model

$\phi(x),\ \theta(x)$: bosonic scalar field

$$\left[\phi(x),\partial_y\theta(y)\right] = i\delta(x-y)$$

$$\eta = \frac{1}{K}$$

- Spin operators

$$S_l^+ = e^{i\sqrt{\pi}\theta(l)}\left[b_0(-1)^l + b_1\cos\left(\sqrt{2\pi}\phi(l)\right) + \cdots\right]$$

$$S_l^z = \frac{1}{\sqrt{2\pi}}\partial_x\phi(l) - a_1(-1)^l\cos\left(\sqrt{2\pi}\phi(l)\right) + \cdots$$

$$\Rightarrow$$

$$\left\langle S_0^x S_r^x\right\rangle = A_0^x\frac{(-1)^r}{r^\eta} - A_1^x\frac{1}{r^{\eta+1/\eta}} + \cdots$$

$$\left\langle S_0^z S_r^z\right\rangle = -\frac{1}{4\pi^2\eta r^2} + A_1^z\frac{(-1)^r}{r^{1/\eta}} + \cdots$$

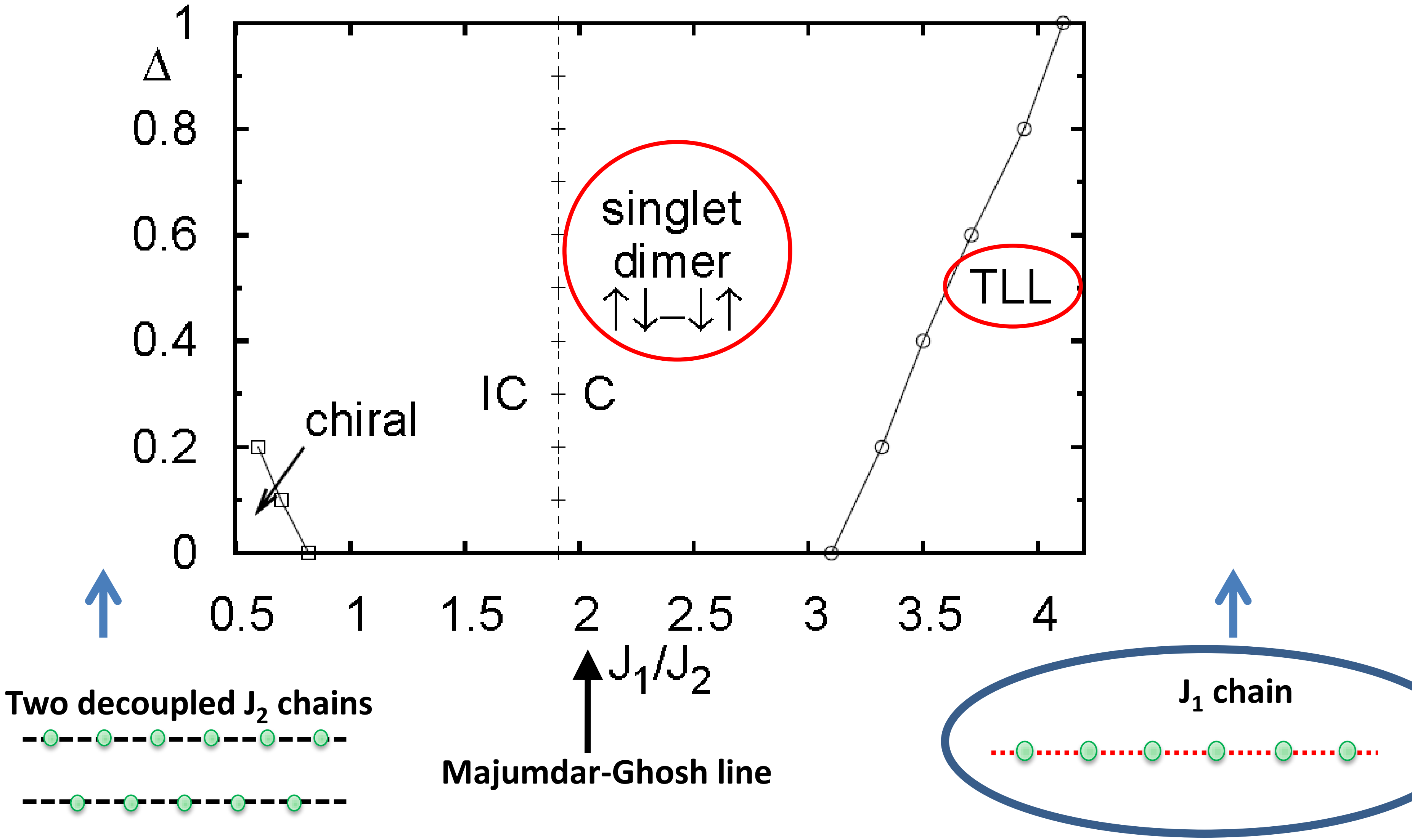
$\eta = 1$ at $\Delta = 1$

- NN bond (energy) operators

$$O_e^\pm(l) = \frac{1}{4}\left(S_l^+S_{l+1}^- + S_l^-S_{l+1}^+\right), \qquad O_e^z(l) = S_l^zS_{l+1}^z$$

$$O_e^\alpha(l) = c_0^\alpha + c_1^\alpha(-1)^l\sin\left[\sqrt{2\pi}\phi(x_l)\right] + c_\varphi^\alpha\left(\partial_x\phi(x_l)\right)^2 + c_\theta^\alpha\left(\partial_x\theta(x_l)\right)^2 + \cdots$$

Ground-state phase diagram for AF-J_1 case



dimer phase $\left\langle \vec{S}_j \cdot \vec{S}_{j+1} - \vec{S}_{j+1} \cdot \vec{S}_{j+2} \right\rangle \neq 0$

$$\vec{S}_j \cdot \vec{S}_{j+1} = c_0^{\alpha} + c_1^{\alpha} (-1)^l \sin\left[\sqrt{2\pi}\phi(x_l)\right] + \cdots$$

Haldane '82
White & Affleck '96
……..

$$H_{\mathrm{eff}} = \frac{v}{2}\int dx \left[K(\partial_x \theta)^2 + \frac{1}{K}(\partial_x \phi)^2 - \lambda \cos\left(\sqrt{8\pi}\phi\right) \right]$$

J_2>0 changes λ (and scaling dimension of $\cos\left(\sqrt{8\pi}\phi\right)$) $\lambda \propto J_1 - cJ_2$

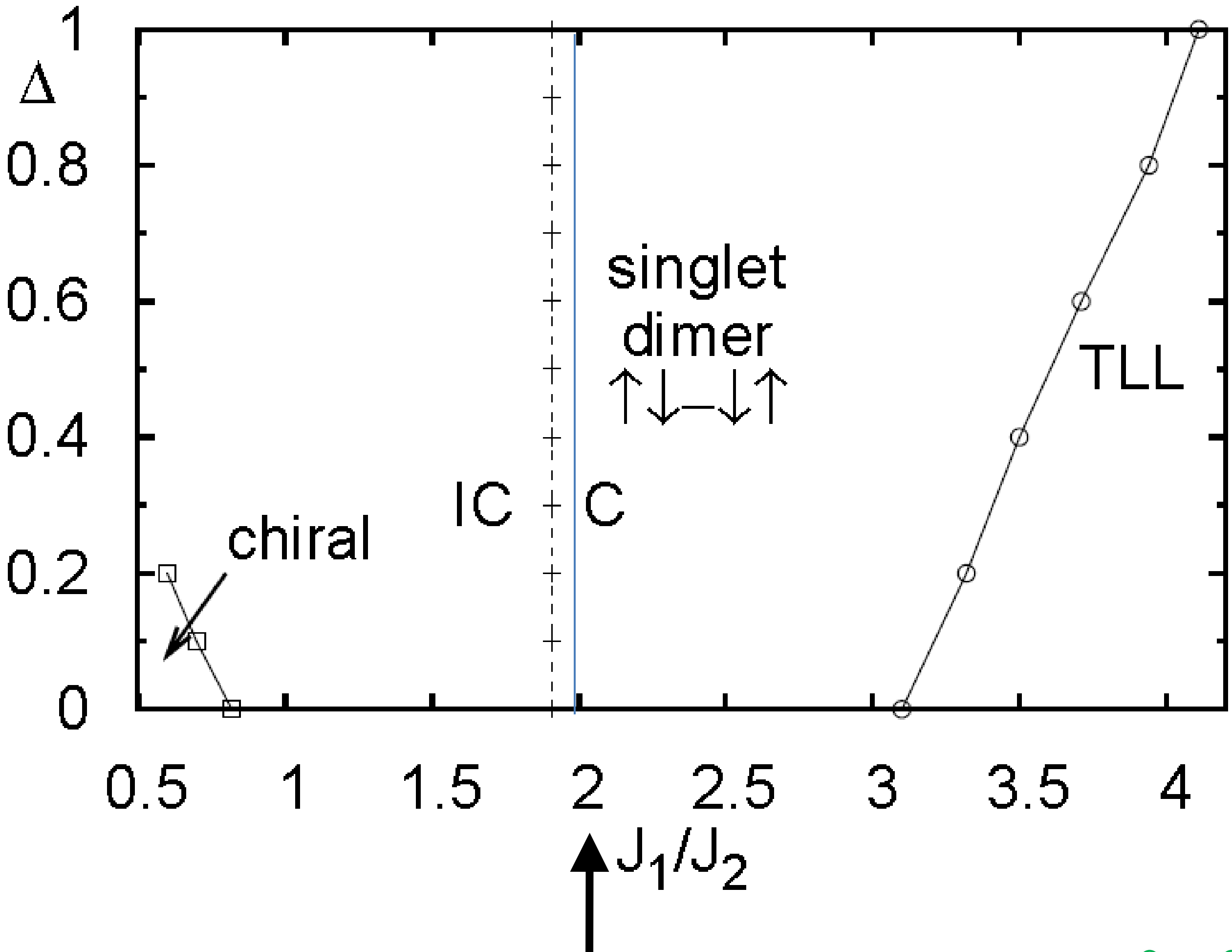
If $\cos\left(\sqrt{8\pi}\phi\right)$ is relevant and $\lambda < 0$,
then $\phi(x)$ is pinned at $\phi = +\sqrt{\pi/8}$ or $-\sqrt{\pi/8}$. $\Rightarrow$ $\left\langle \sin\left(\sqrt{2\pi}\phi\right) \right\rangle \neq 0$

dimer LRO

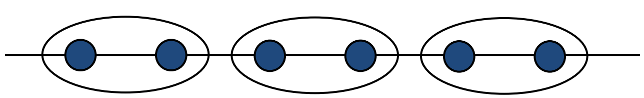
If $\cos\left(\sqrt{8\pi}\phi\right)$ is relevant and $\lambda > 0$,
then $\phi(x)$ is pinned at $\phi = 0$ or $\sqrt{\pi/2}$. $\Rightarrow$ $\left\langle \cos\left(\sqrt{2\pi}\phi\right) \right\rangle \neq 0$

Neel LRO

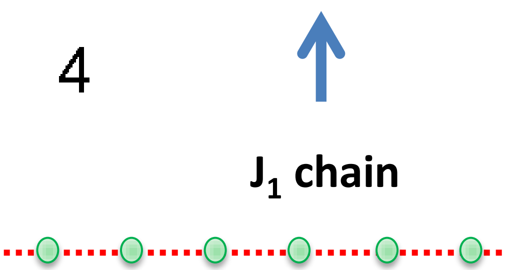
Ground-state phase diagram for AF-J_1 case



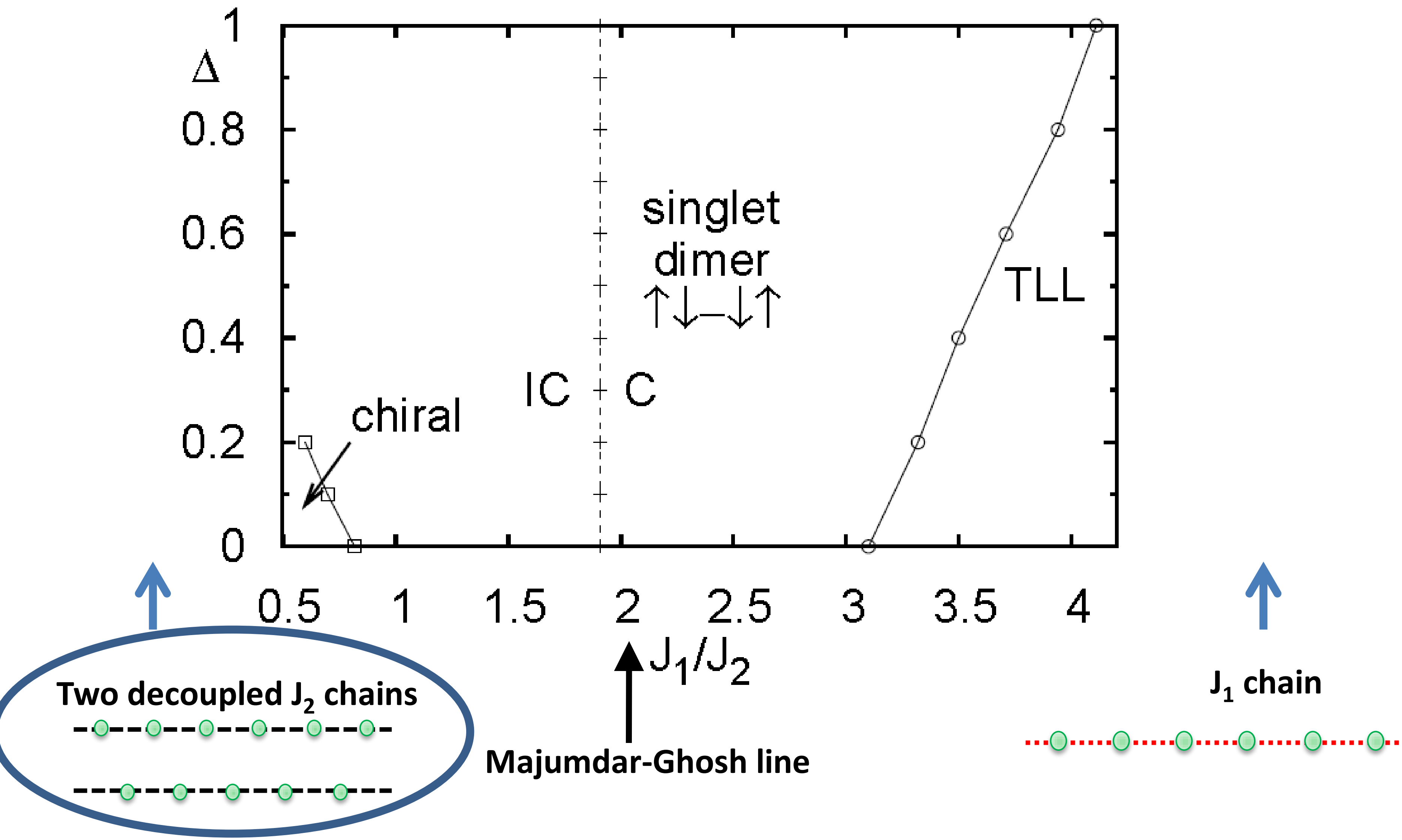
Majumdar-Ghosh line



J_1 chain



Ground-state phase diagram for AF-J_1 case



Perturbative RG at J_1<<J_2

SU(2) symmetric case

Non-Abelian bosonization SU(2)$_1$ WZW White & Affleck, PRB (1996)

$$\vec{S}_{1,j} = \vec{M}_1(x) + (-1)^j \vec{N}_1(x) \qquad \vec{S}_{2,j} = \vec{M}_2(x) + (-1)^j \vec{N}_2(x)$$

$$J_1 \sum_j \vec{S}_{1,j} \cdot \left(\vec{S}_{2,j} + \vec{S}_{2,j+1}\right) \to g_1 \int dx \left(\vec{M}_{1L} \cdot \vec{M}_{2R} + \vec{M}_{1R} \cdot \vec{M}_{2L}\right)$$

1-loop RG $\dfrac{dg_1}{dl} = g_1^2$

$J_1 > 0$ marginally relevant ⇨ dimer order

$J_1 < 0$ marginally irrelevant ⇨ ???

Perturbative RG at $J_1 << J_2$

XXZ case Nersesyan, Gogolin, & Essler, PRL (1998)

$$S^+_{\mu,l} = e^{i\sqrt{2\pi}\theta_\mu(l)}\left[b_0(-1)^l + b_1 \cos\left(\sqrt{2\pi}\phi_\mu(l)\right) + \cdots \right]$$

$$S^z_{\mu,l} = \frac{1}{\sqrt{2\pi}}\partial_x \phi_\mu(l) - a_1(-1)^l \cos\left(\sqrt{2\pi}\phi_\mu(l)\right) + \cdots$$

$$J_1\left(S^+_{1,j} + S^+_{1,j+1}\right)S^-_{2,j} + \mathrm{h.c.} \sim g_1 \cos\left(\sqrt{4\pi}\phi_+\right)\cos\left(\sqrt{4\pi}\theta_-\right) + g_2 \partial_x \theta_+ \sin\left(\sqrt{4\pi}\theta_-\right)$$

$$\phi_\pm = \frac{1}{\sqrt{2}}(\phi_1 \pm \phi_2), \quad \theta_\pm = \frac{1}{\sqrt{2}}(\theta_1 \pm \theta_2)$$

dimer order

vector chiral order

$$J_1\left(S^z_{1,j} + S^z_{1,j+1}\right)S^z_{2,j} \sim J_1\left[\left(\partial_x \phi_+\right)^2 - \left(\partial_x \phi_-\right)^2\right]$$

$$\Rightarrow \quad K_\pm \approx K \mp K^2 \frac{J_1}{\pi v}$$

scaling dimensions

$$g_1 : \boxed{K_+} + \frac{1}{K_-}$$

$$g_2 : \ 1 + \frac{1}{K_-}$$

$\underline{K = 2 \ \text{ at } \Delta = 0,}$ $\quad K = 1 \text{ at } \Delta = 1$

Vector chiral order is favored near Δ= 0

Vector chiral phase

p-type nematic Andreev-Grishchuk (1984)

When $g_2 \frac{d\theta_+}{dx}\sin\left(\sqrt{2\pi}\theta_-\right)$ is relevant $\rightarrow$ $\left\langle \sin\left(\sqrt{2\pi}\theta_-\right)\right\rangle \neq 0,\ \left\langle \frac{d\theta_+}{dx}\right\rangle \neq 0$

Nersesyan-Gogolin-Essler (1998)

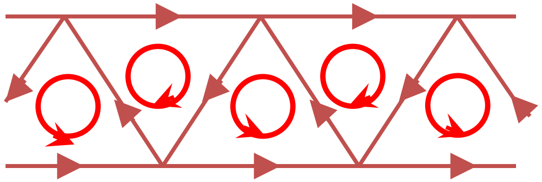
- Characteristics of the vector chiral state

  - Vector chiral order

$$\chi_l^{(1)} = \left\langle \left(\vec{s}_l \times \vec{s}_{l+1}\right)^z \right\rangle \sim -\left\langle \sin\left(\sqrt{2\pi}\theta_-\right)\right\rangle$$

$$\chi_l^{(2)} = \left\langle \left(\vec{s}_l \times \vec{s}_{l+2}\right)^z \right\rangle \sim -\left\langle \frac{d\theta_+}{dx}\right\rangle$$

Vector chiral order ◀ $\chi < 0$



opposite sign $\left(\chi^{(1)},\chi^{(2)}\right)=(+,-)$ or $(-,+)$

doubly degenerate ground state

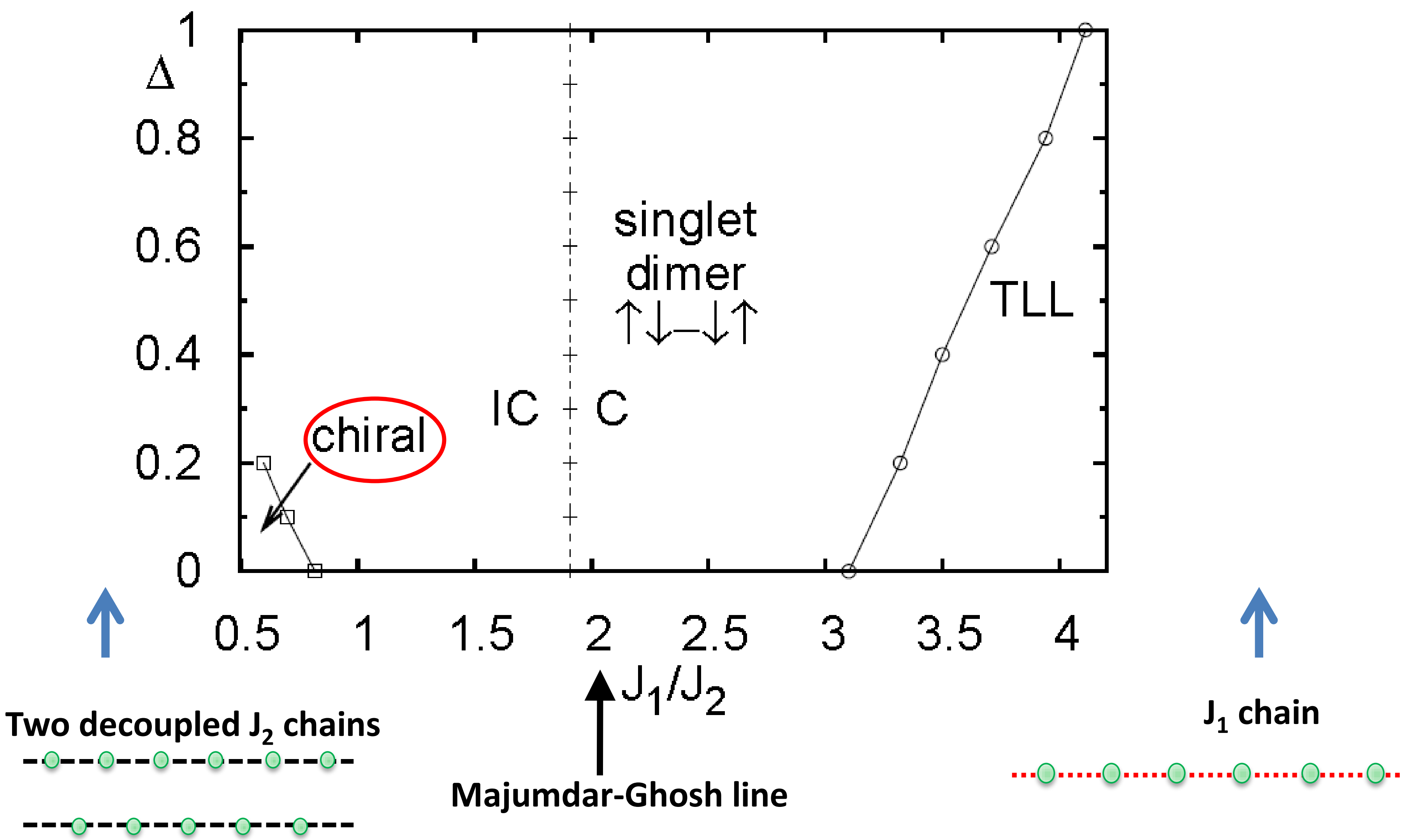
  - S^xS^x & S^xS^y spin correlation

$$\left\langle s_0^x s_r^x \right\rangle \sim r^{-1/4K_+}\cos(qr) \qquad \left\langle s_0^x s_r^y \right\rangle \sim \pm r^{-1/4K_+}\sin(qr)$$

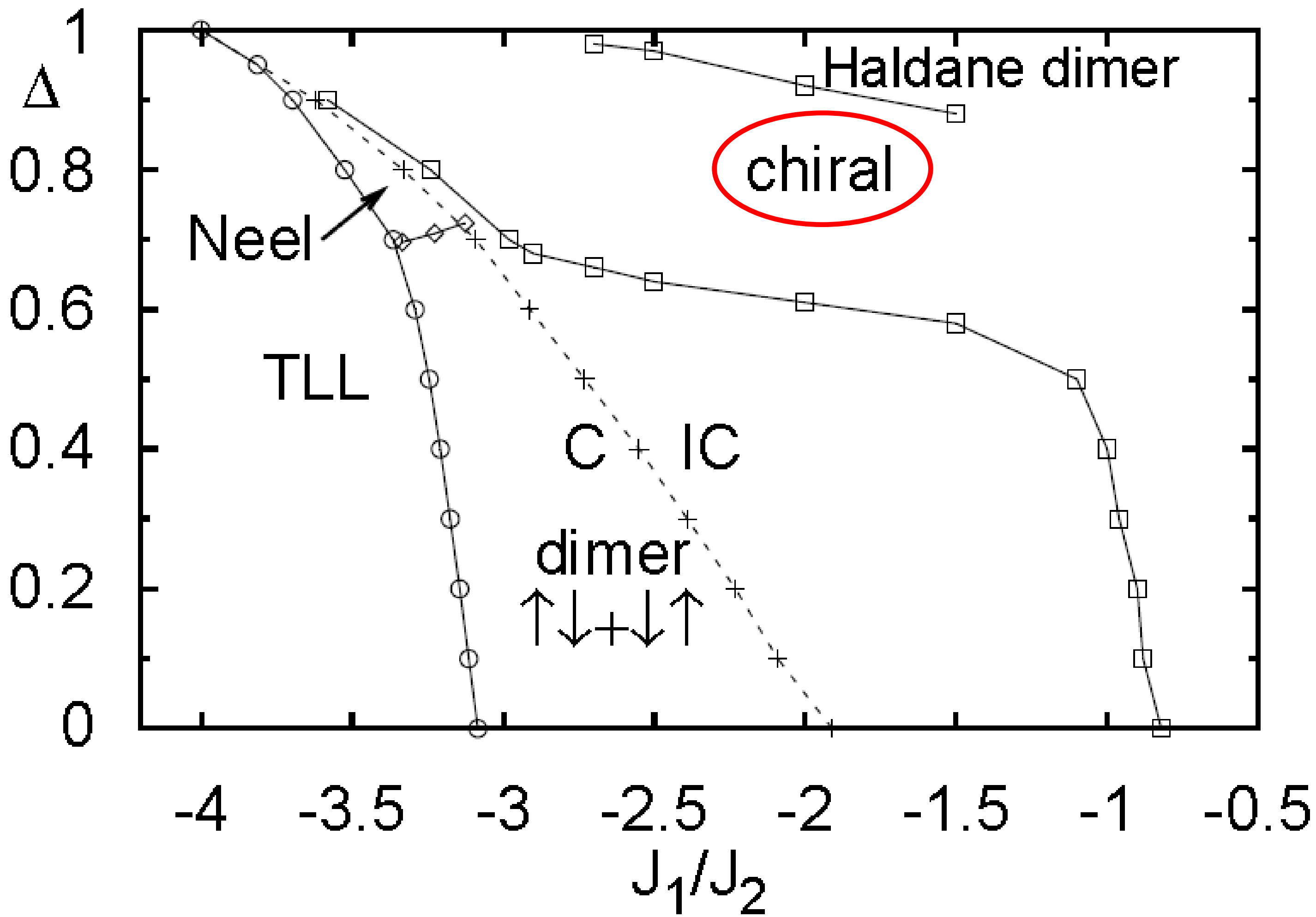
power-law decay, incommensurate

A quantum counterpart of the classical helical state

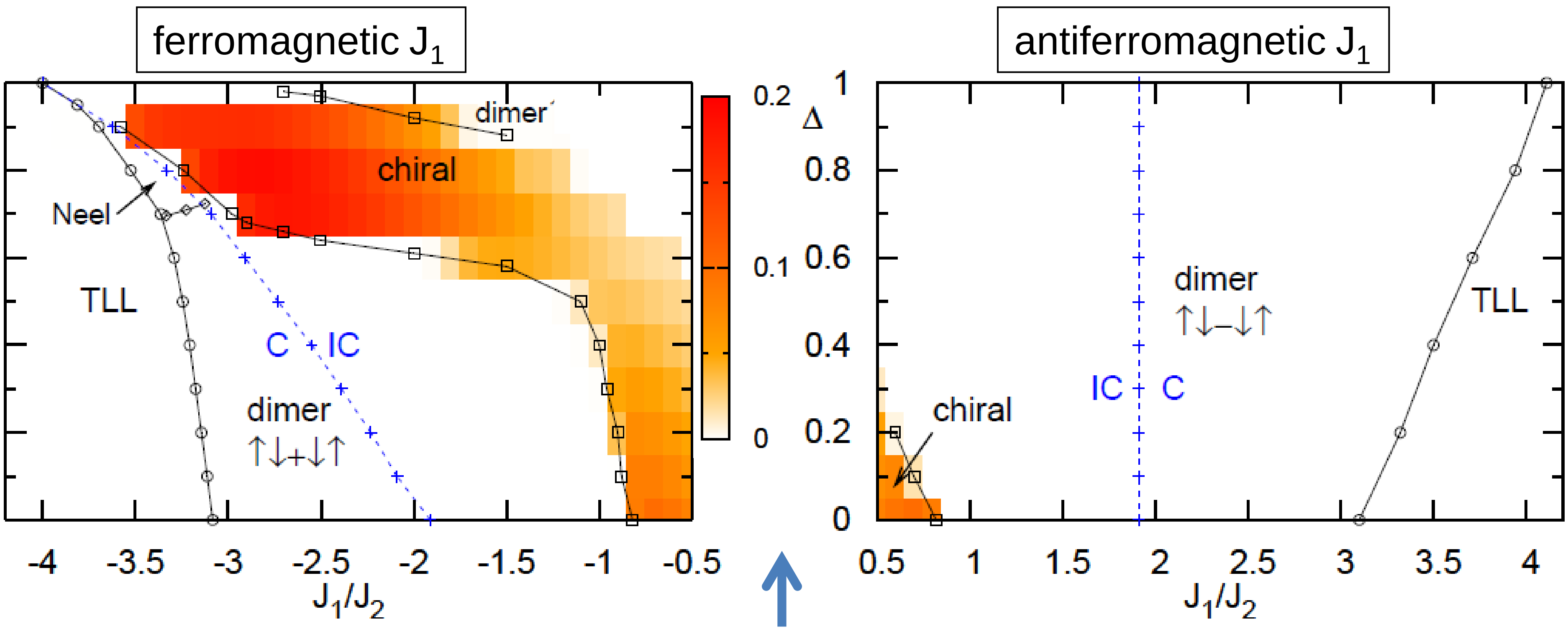
Ground-state phase diagram for AF-J_1 case



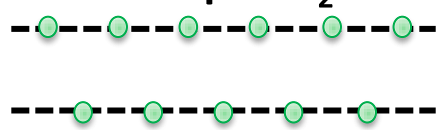
New Results for the J_1-J_2 spin chain with ferromagnetic J_1



Phase diagram & chiral order parameter $\langle(\vec{S}_1 \times \vec{S}_2)^z\rangle$



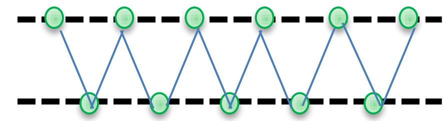
Two decoupled J_2 chains



The vector chiral phase is larger in the ferromagnetic J_1 case and extends up to the vicinity of the isotropic case $\Delta \approx 1$.

Perturbation around J_1=0

Two decoupled J_2 chains



$$H_{\rm eff} = \sum_{\mu=1,2} \frac{v}{2} \int dx \left[K \left(\partial_x \theta_\mu \right)^2 + \frac{1}{K} \left(\partial_x \phi_\mu \right)^2 + \lambda \cos\left(\sqrt{8\pi} \phi_\mu \right) \right]$$

$$S^+_{\mu,l} = e^{i\sqrt{2\pi}\theta_\mu(l)} \left[b_0 (-1)^l + b_1 \cos\left(\sqrt{2\pi} \phi_\mu(l) \right) + \cdots \right]$$

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$$J_1 \left(S^+_{1,j} + S^+_{1,j+1} \right) S^-_{2,j} + \text{h.c.} \sim g_1 \cos\left(\sqrt{4\pi} \phi_+ \right) \cos\left(\sqrt{4\pi} \theta_- \right) + g_2 \partial_x \theta_+ \sin\left(\sqrt{4\pi} \theta_- \right)$$

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dimer order

vector chiral order

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$$\Rightarrow \quad K_\pm \approx K \mp K^2 \frac{J_1}{\pi v}$$

$$K = 2 \ \text{at} \ \Delta = 0, \quad K = 1 \ \text{at} \ \Delta = 1$$

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- Characteristics of the vector chiral state
  - Vector chiral order

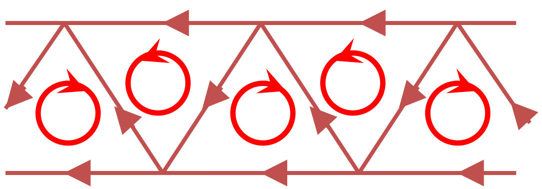
$$\chi_l^{(1)} = \left\langle \left(\vec{s}_l \times \vec{s}_{l+1}\right)^z \right\rangle \sim -\left\langle \sin\left(\sqrt{2\pi}\theta_-\right)\right\rangle$$

$$\chi_l^{(2)} = \left\langle \left(\vec{s}_l \times \vec{s}_{l+2}\right)^z \right\rangle \sim -\left\langle \frac{d\theta_+}{dx}\right\rangle$$

Same sign $\left(\chi^{(1)}, \chi^{(2)}\right) = \underline{(+,+) \text{ or } (-,-)}$

doubly degenerate ground state

Vector chiral order ◀ $\chi < 0$



no net spin current flow

$$J_1\chi_l^{(1)} + 2J_2\chi_l^{(2)} = 0$$

  - S^xS^x & S^xS^y spin correlation

$$\left\langle s_0^x s_r^x \right\rangle \sim r^{-1/4K_+}\cos(qr) \quad \left\langle s_0^x s_r^y \right\rangle \sim \pm r^{-1/4K_+}\sin(qr)$$

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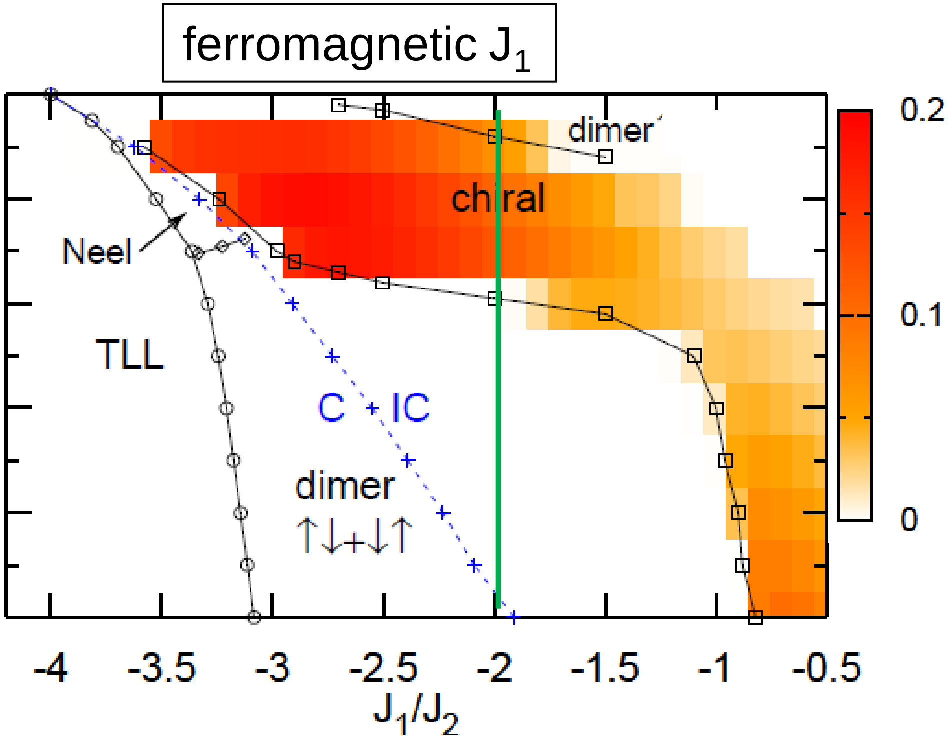
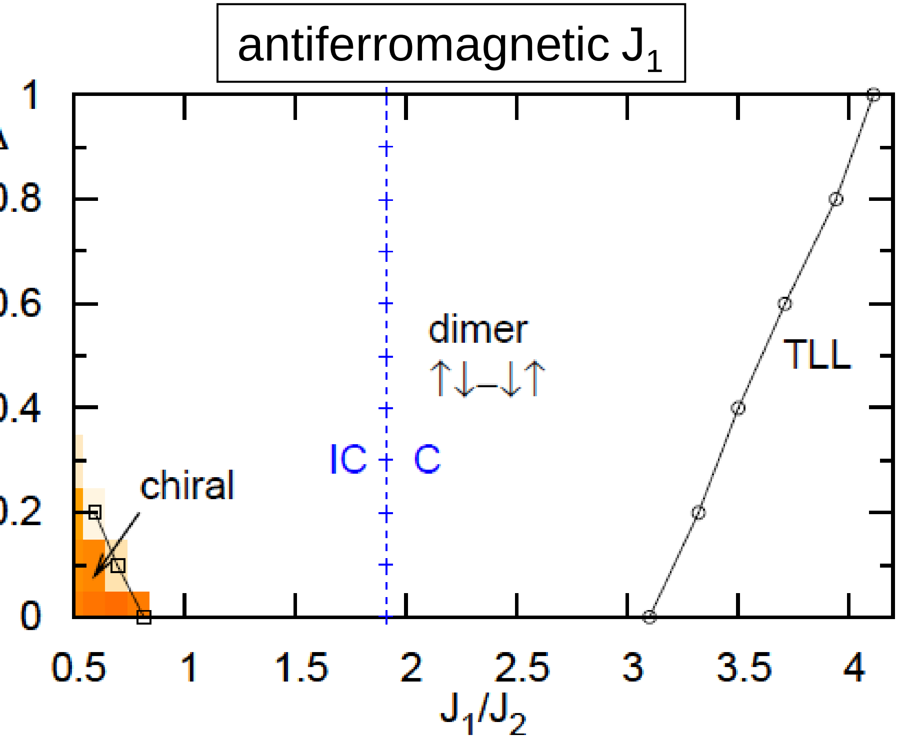
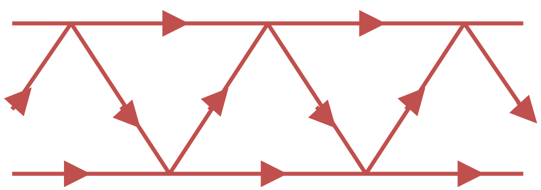
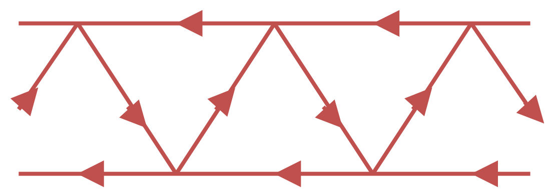




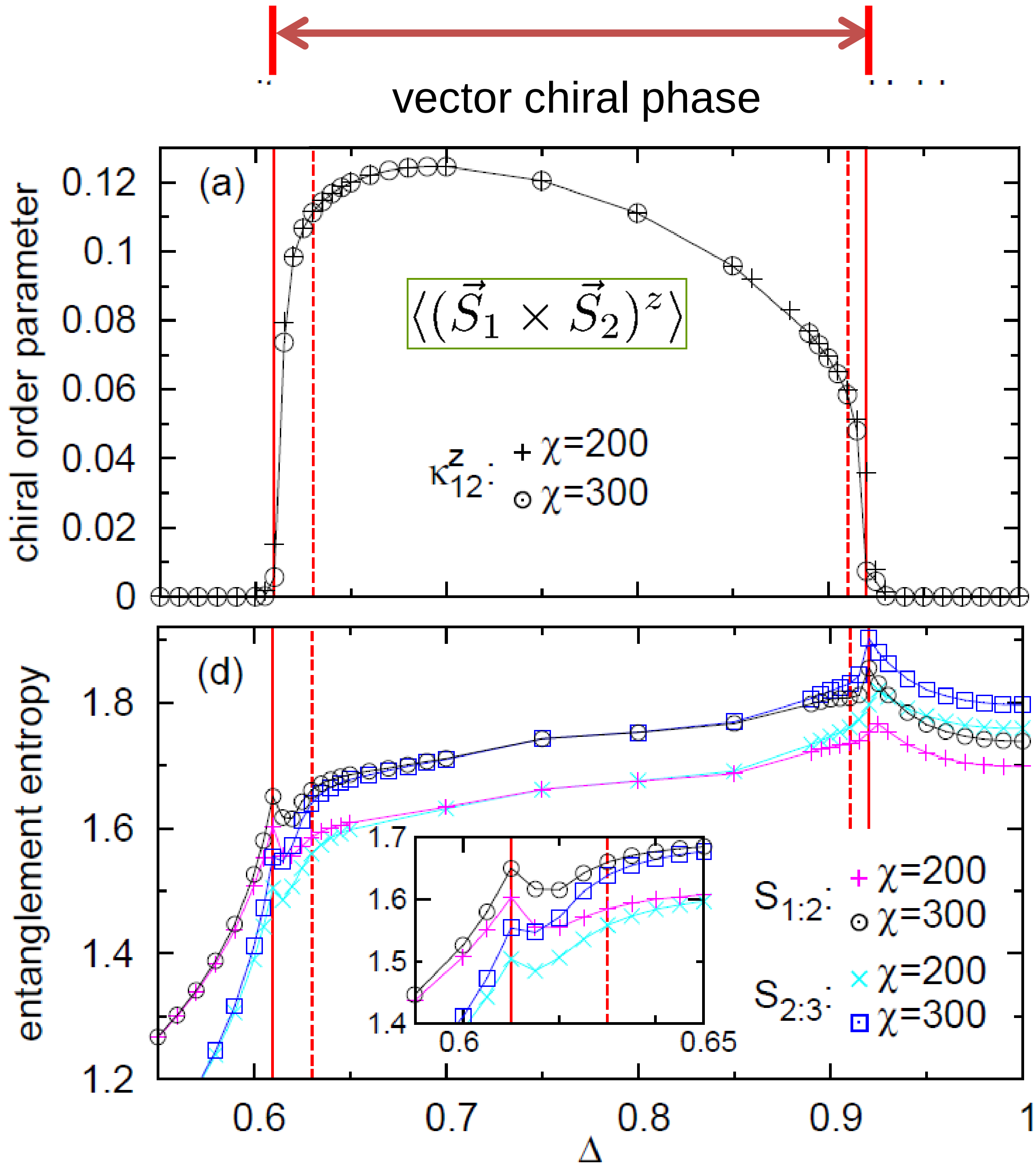




$$J_1 \frac{d\theta_+}{dx} \sin\left(\sqrt{2\pi}\theta_-\right) \Rightarrow \left\langle \left(\vec{s}_l \times \vec{s}_{l+1}\right)^z \right\rangle \sim -\left\langle \sin\left(\sqrt{2\pi}\theta_-\right) \right\rangle, \quad \left\langle \left(\vec{s}_l \times \vec{s}_{l+2}\right)^z \right\rangle \sim -\left\langle \frac{d\theta_+}{dx} \right\rangle$$

Chiral order parameter & entanglement entropy

$\boxed{J_1/J_2 = -2}$



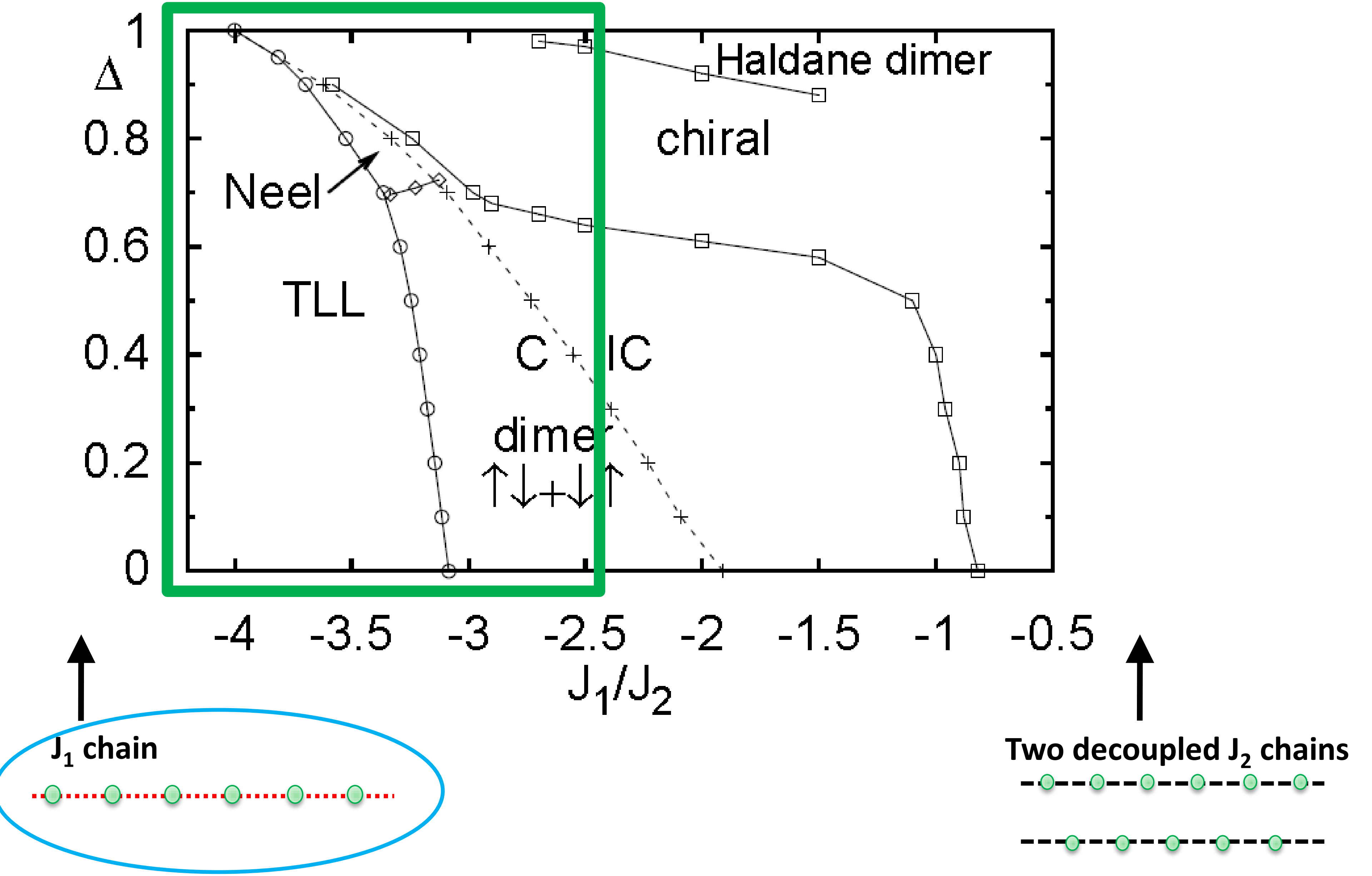
- Rapid growth of the chiral order parameter
- Peak of half-chain entanglement entropy

(coming from Ising nature of the transitions)

Determinatin of
the phase boundaries
with reasonable accuracy

iTEBD

Ground-state phase diagram for Ferro-J_1 case



Sine-Gordon model for spin-1/2 J_1-J_2 XXZ chain with ferromagnetic coupling J_1

We begin with the J_2=0 limit.

J_1 chain

$J_1 < 0$ **Ferromagnetic**

$0 < \Delta < 1$ **Easy-plane**

$\Delta = 1$ **Ferromagnetic SU(2) Heisenberg**

$$\mathcal{H}_{J_1} = \sum_{j=1} J_1 \left[\frac{1}{2} \left(S_j^+ S_{j+1}^- + \text{h.c.} \right) + \Delta S_j^z S_{j+1}^z \right]$$

Effective Hamiltonian (sine-Gordon model)

$$H_{\text{eff}} = \frac{v}{2} \int dx \left[K \left(\partial_x \theta \right)^2 + \frac{1}{K} \left(\partial_x \phi \right)^2 - \lambda \cos\left(\sqrt{8\pi} \phi \right) \right]$$

TL-liquid (free-boson) part **irrelevant perturbation**

$$\Delta = \cos\left(\pi\eta \right), \quad 0 < \eta \le \frac{1}{2}$$

velocity $v = \left| J_1 \right| \dfrac{\sin\left(\pi\eta \right)}{2\left(1 - \eta \right)}$

TL-liquid parameter $K = \dfrac{1}{\eta}$

Spin and dimer operators

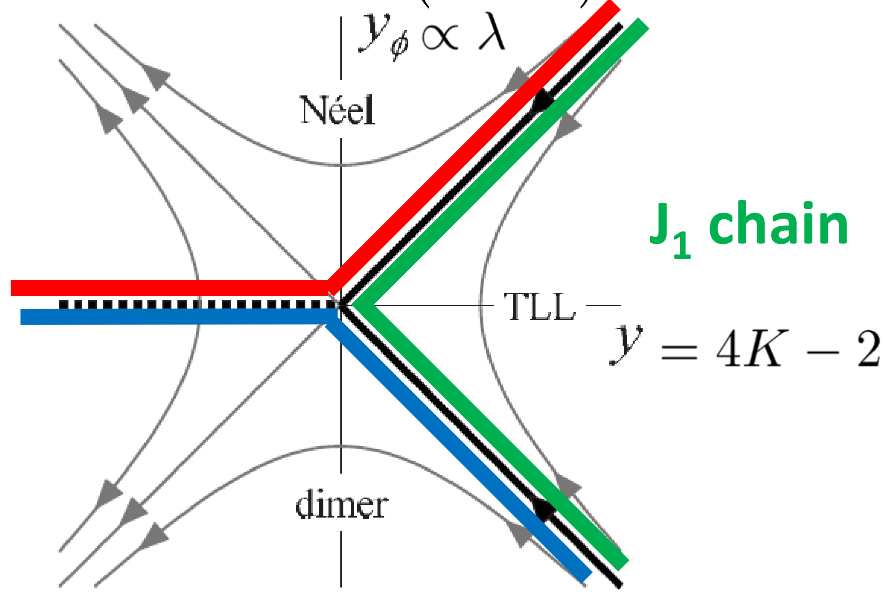
$$\begin{cases} S_j^z = \dfrac{1}{\sqrt{2\pi}}\partial_x\phi + (-1)^j \cos\left(\sqrt{2\pi}\phi\right) + \dots \\ S_j^{\pm} = e^{i\sqrt{2\pi}\theta}\left[b_0 + (-1)^j b_1 \cos\left(\sqrt{2\pi}\phi\right) + \dots\right] \\ \vec{S}_j \cdot \vec{S}_{j+1} - \vec{S}_j \cdot \vec{S}_{j-1} = c(-1)^j \sin\left(\sqrt{2\pi}\phi\right) + \dots \end{cases}$$

If the cosine term $-\lambda\cos\left(\sqrt{8\pi}\phi\right)$ **becomes relevant, then**

$$\begin{cases} \lambda > 0 \Rightarrow \phi = 0, \sqrt{\pi/2} \Rightarrow \cos\left(\sqrt{2\pi}\phi\right) \neq 0 & \textbf{Neel order} \\ \lambda < 0 \Rightarrow \phi = \pm\sqrt{\pi/8} \Rightarrow \sin\left(\sqrt{2\pi}\phi\right) \neq 0 & \textbf{dimer order} \end{cases}$$

BKT-type RG equation

$$\begin{cases} \dfrac{dy(l)}{dl} = -y_\phi^2(l), \\ \dfrac{dy_\phi(l)}{dl} = -y_\phi(l)y(l). \end{cases}$$



Exact coupling constant for the XXZ chain (J_2=0)

$$\lambda_{J_2=0} = -\frac{4}{\pi}\sin\left(\frac{\pi}{\eta}\right)\left[\Gamma\left(\frac{1}{\eta}\right)\right]^2\left[\frac{\Gamma\left(1+\frac{\eta}{2-2\eta}\right)}{\sqrt{4\pi}\,\Gamma\left(1+\frac{1}{2-2\eta}\right)}\right]^{\frac{2}{\eta}-2}$$

S. Lukyanov, Nucl. Phys. B (1998)

It vanishes and changes sign at

$\eta = 1/n$ **i.e.,** $\Delta = \cos(\pi/n) \qquad n = 3, 4, 5, \cdots.$

1st order correction (in λ) to excitation gaps of finite systems

$$\frac{L}{v}(E_{\mathrm{D}} - E_{\mathrm{N}}) = 2\pi\lambda\left(\frac{L}{2\pi}\right)^{2-2/\eta}.$$

exact diagonalization

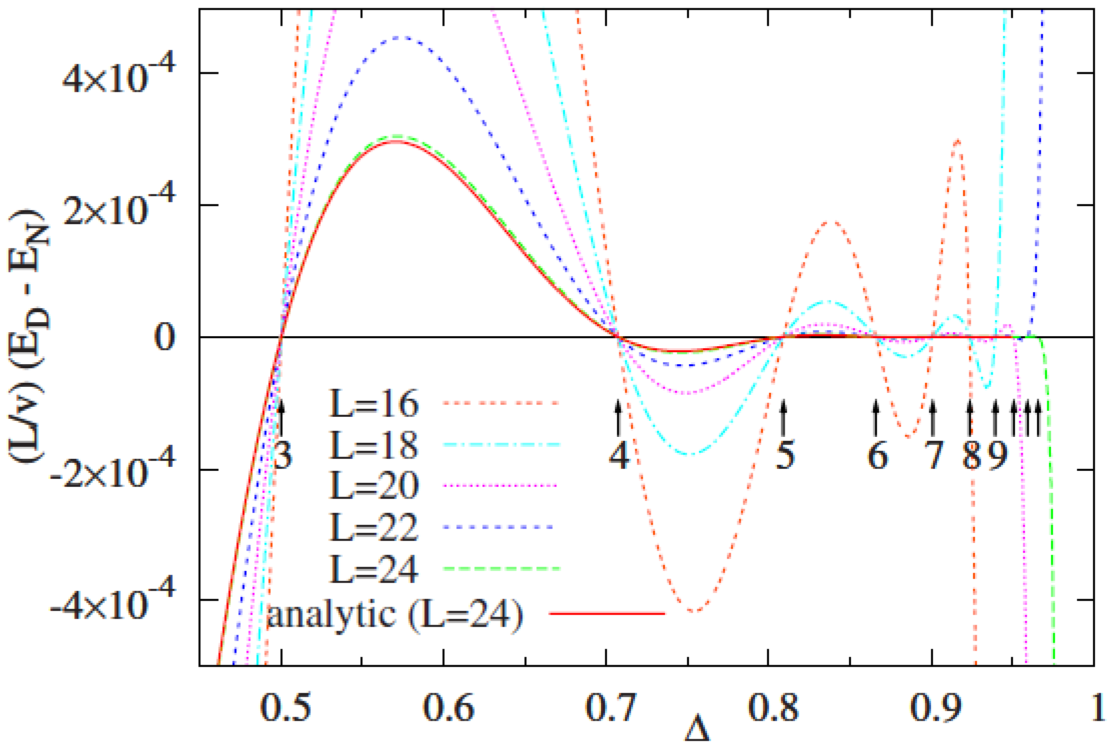
$$\begin{cases} E_D - E_{gs} & \text{“Dimer” excitation} \\ E_N - E_{gs} & \text{“Neel” excitation energy} \end{cases}$$

Level spectroscopy
K. Okamoto & Kiyohide Nomura

We can find the position of λ=0 using numerical exact-diagonalization method.

$$\frac{L}{v}(E_{\mathrm{D}} - E_{\mathrm{N}}) = 2\pi\lambda\left(\frac{L}{2\pi}\right)^{2-2/\eta}.$$

J_2=0



$\lambda = 0$ at $\Delta = \cos(\pi/n)$

$$\frac{L}{v}(E_{\mathrm{D}} - E_{\mathrm{N}}) = 2\pi\lambda \left(\frac{L}{2\pi}\right)^{2-2/\eta}$$

Generally the exact value of λ is not known
in the presence of additional perturbations (J_2).

We can find the position of λ=0 numerically by the condition
$E_{\mathrm{D}} - E_{\mathrm{N}} = 0$.

⇩

If J_2 perturbation makes the λ term relevant ,
Neel and dimer phases will emerge.

$\lambda > 0$ ⇒ $\phi = 0, \sqrt{\pi/2}$ ⇒ $\cos\left(\sqrt{2\pi}\phi\right) \neq 0$ Neel order

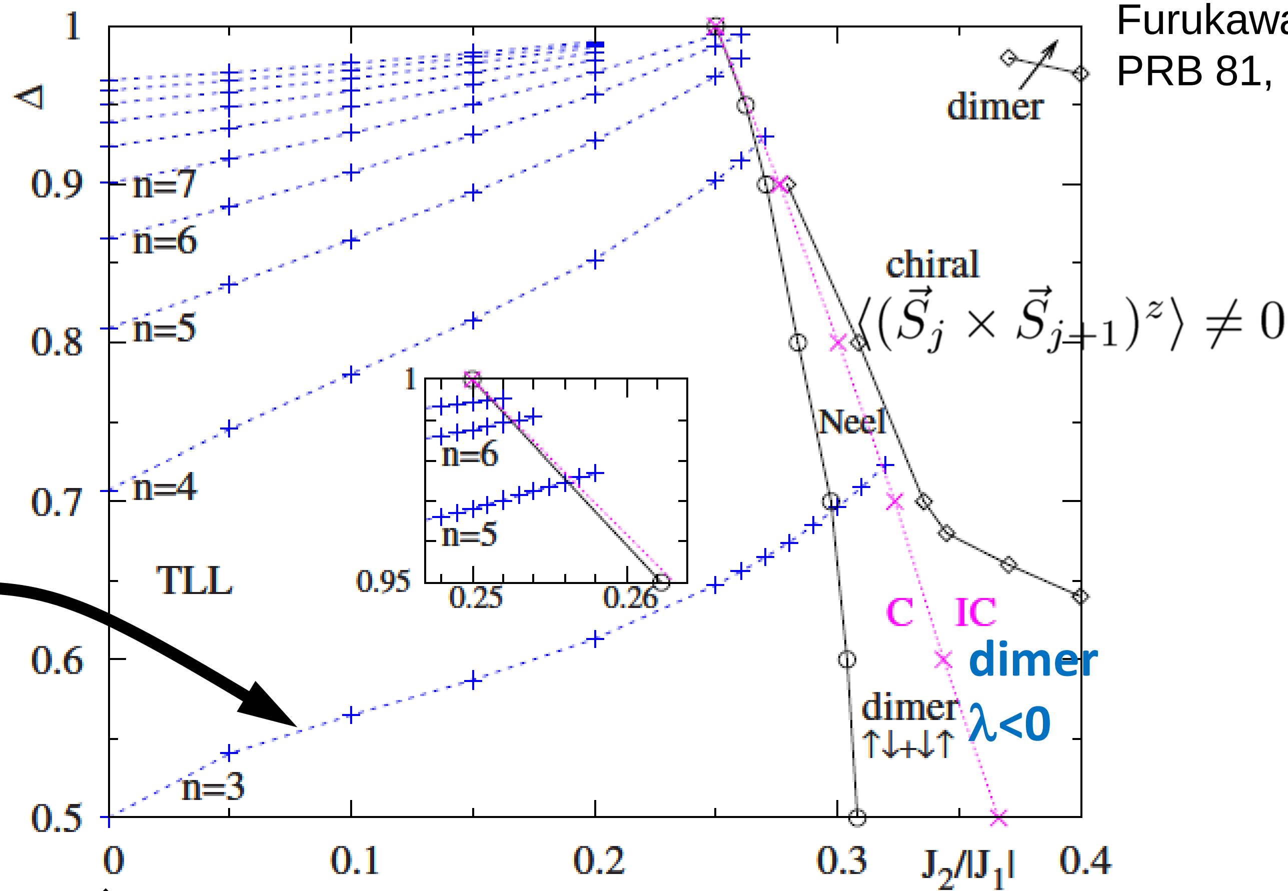
$\lambda = 0$ ⇒ Gaussian phase transition point (c=1)

$\lambda < 0$ ⇒ $\phi = \pm\sqrt{\pi/8}$ ⇒ $\sin\left(\sqrt{2\pi}\phi\right) \neq 0$ dimer order

Phase diagram and Neel/dimer order parameters

Ground-state phase diagram of easy-plane anisotropic J_1-J_2 chain

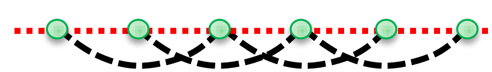
Furukawa, Sato & AF
PRB 81, 094410 (2010)



Curves of λ=0

J_1 chain

λ irrelevant **λ relevant**



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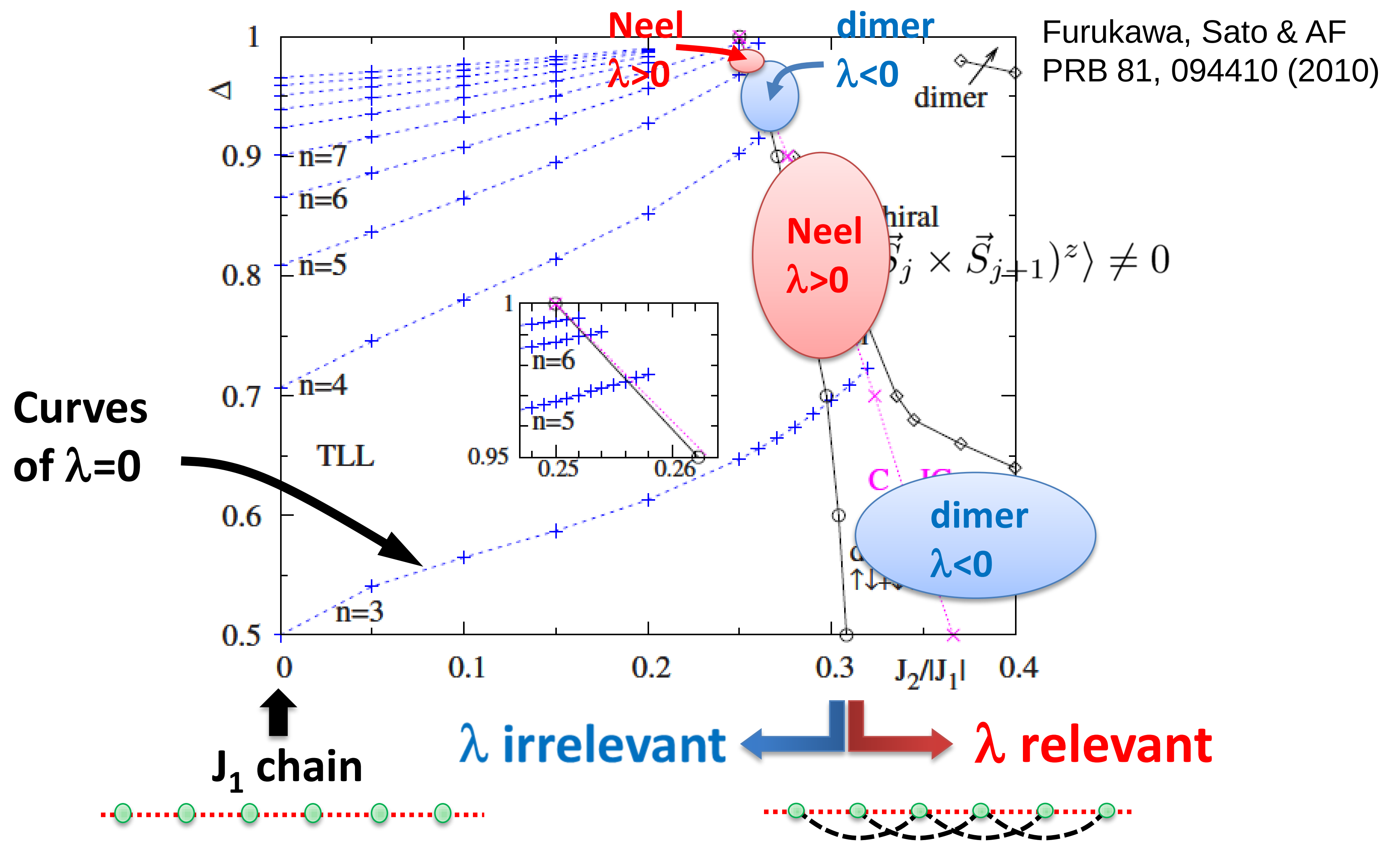
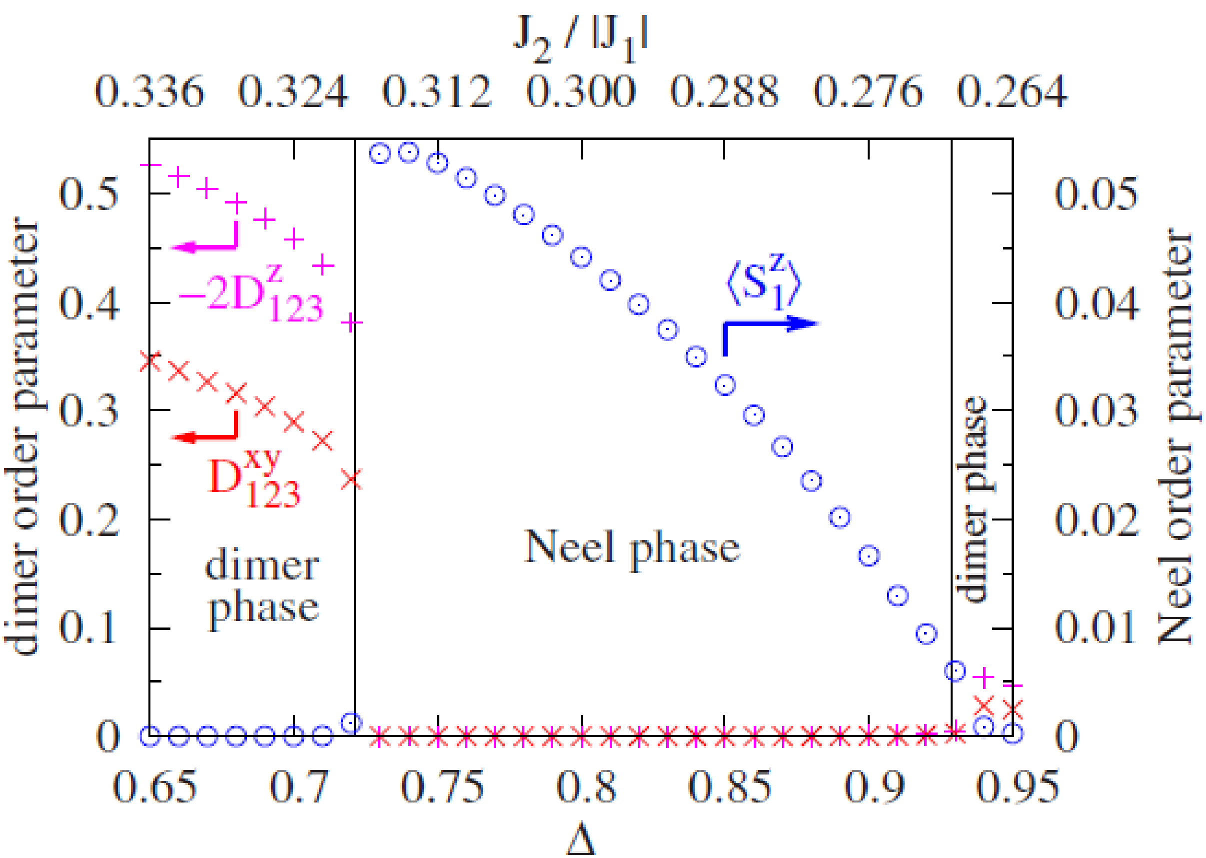
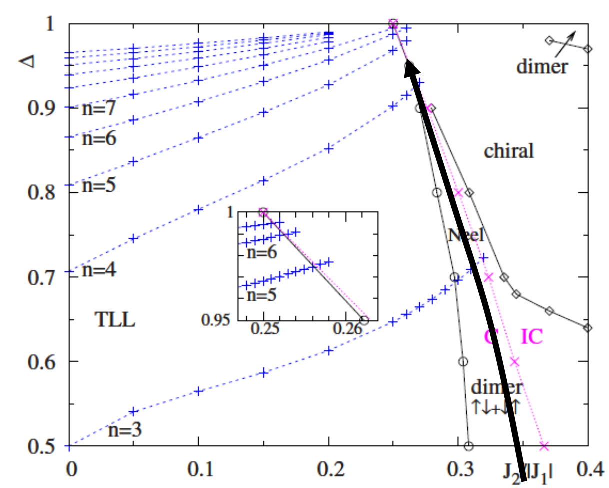


Direct calculation of order parameters using iTEBD

$$
\left\{
\begin{array}{lll}
\textbf{XY component of dimer} & D^{xy}_{123} = & (S^x_1 S^x_2 + S^y_1 S^y_2) - (S^x_2 S^x_3 + S^y_2 S^y_3), \\
\textbf{Z component of dimer} & D^{z}_{123} = & S^z_1 S^z_2 - S^z_2 S^z_3. \\
\textbf{Neel operator (z component of spin)} & S^z_1 & \text{staggered magnetization}
\end{array}
\right.
$$



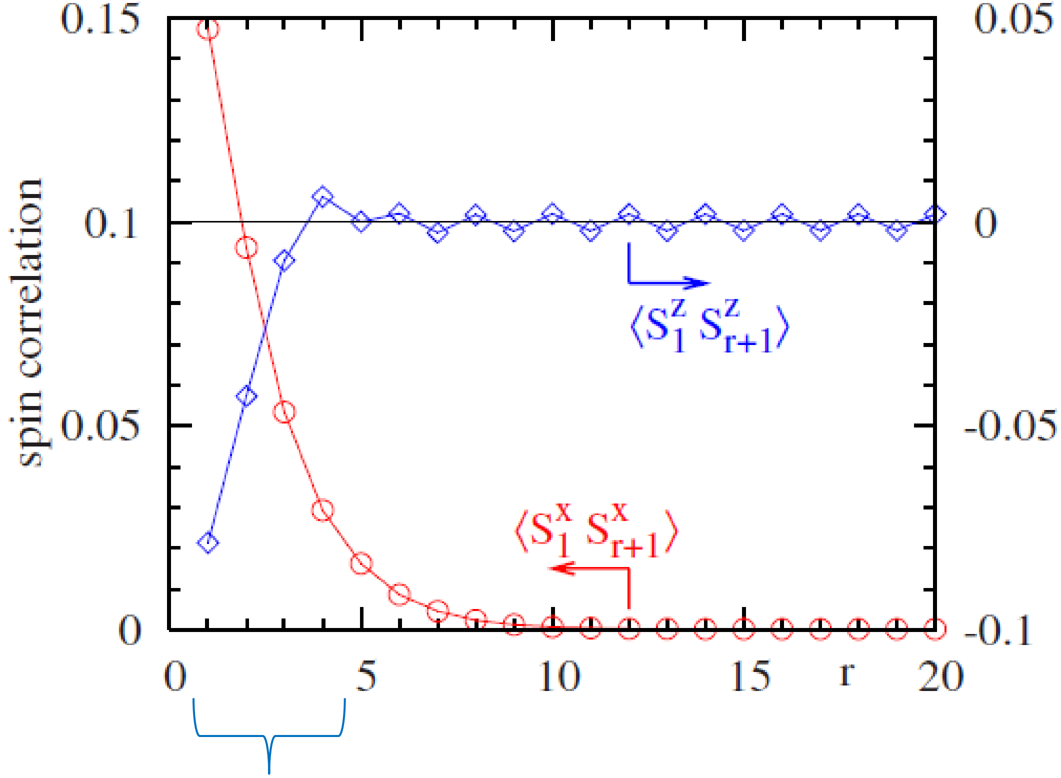


λ<0 **λ>0** **λ<0**

Neel phase

The existence of the Neel phase is against our intuition: ferromagnetic $J_1 < 0$ & easy-plane anisotropy $\Delta < 1$.

Spin-spin correlation functions in the Neel phase

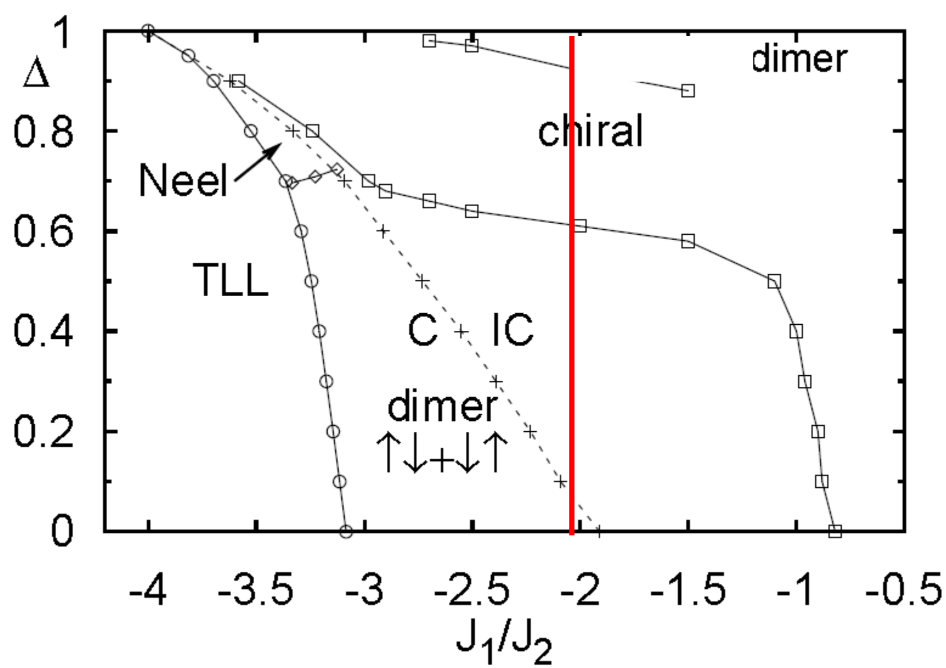


Short-range behavior is different from that of the standard Neel order.

Dimer order parameter

$$D_{123}^{xy} := \langle\ [(S_1^x S_2^x + S_1^y S_2^y) - (S_2^x S_3^x + S_2^y S_3^y)]\ \rangle$$

$$D_{123}^{z} := \langle (S_1^z S_2^z - S_2^z S_3^z) \rangle$$



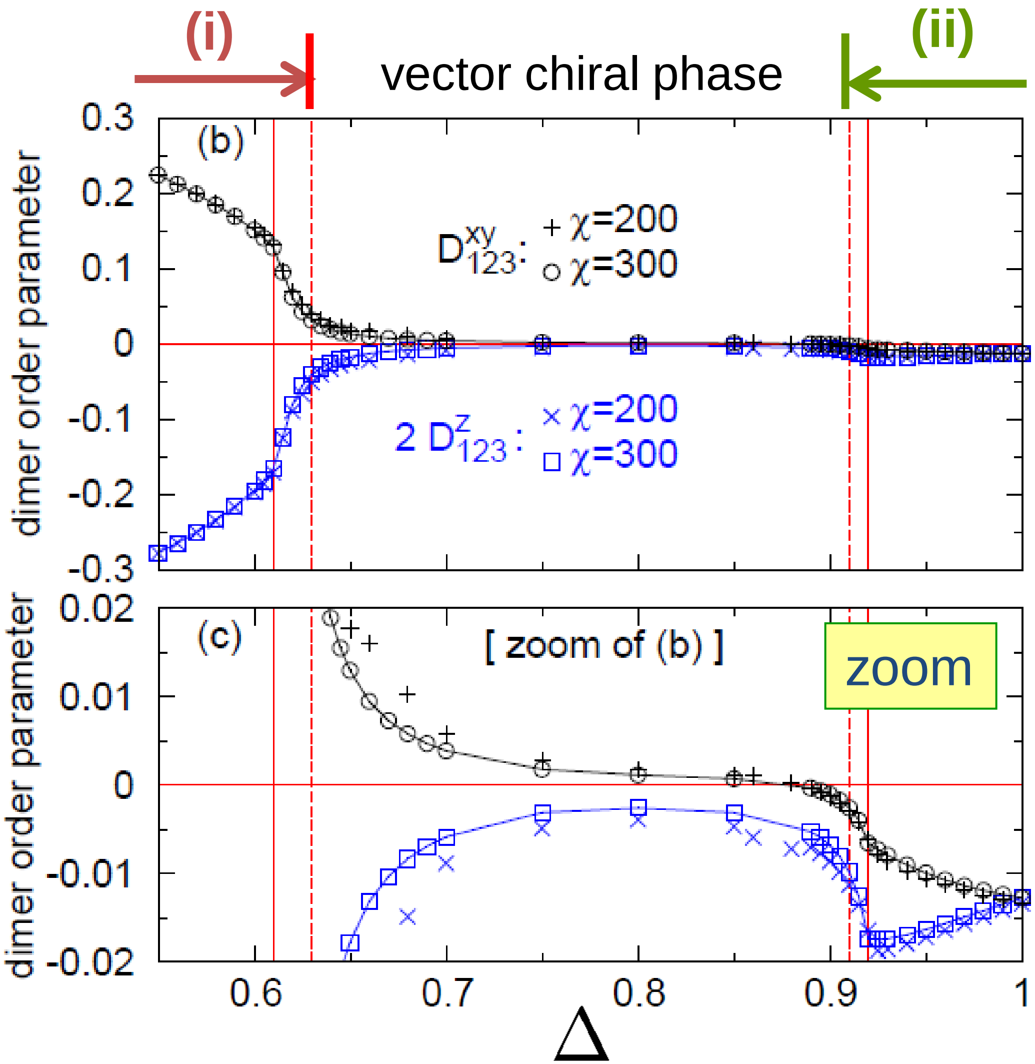
$$\boxed{J_1/J_2 = -2}$$

Dimer order parameters - I

$$D^{xy}_{123} := \langle\ [(S^x_1 S^x_2 + S^y_1 S^y_2) - (S^x_2 S^x_3 + S^y_2 S^y_3)]\ \rangle$$

$$D^z_{123} := \langle (S^z_1 S^z_2 - S^z_2 S^z_3) \rangle$$

$$\boxed{J_1/J_2 = -2}$$



(i) $D^{xy}_{123} > 0,\ D^z_{123} < 0$

``Even-parity dimer phase''

Exact GS @ $(J_1/J_2, \Delta) = (-2, 0)$

= Product of even-parity dimers

$\uparrow\downarrow + \downarrow\uparrow$

Chubukov, PRB, 1991

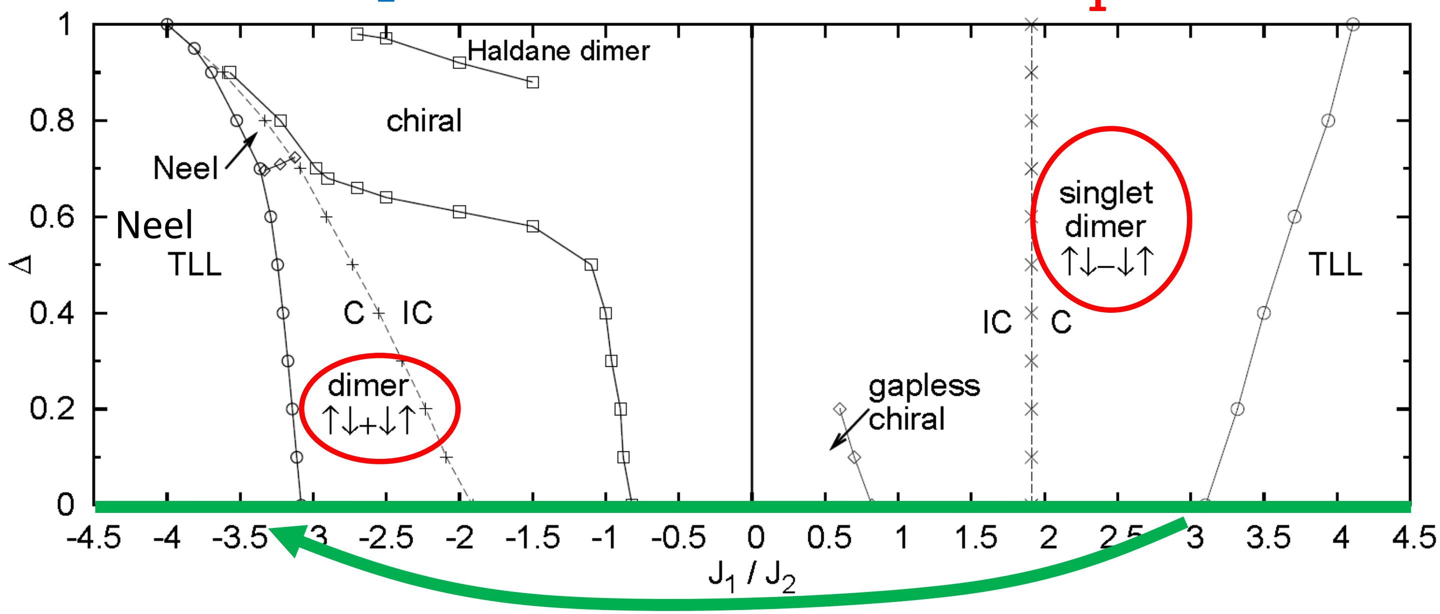
Obtained by applying gauge transformation to the singlet dimer state

$\uparrow\downarrow - \downarrow\uparrow$

Dimer phases

$$D_{123}^{xy} = (S_1^x S_2^x + S_1^y S_2^y) - (S_2^x S_3^x + S_2^y S_3^y),$$
$$D_{123}^{z} = S_1^z S_2^z - S_2^z S_3^z.$$

FM-J$_1$ case **AF-J$_1$ case**



- **dimer phase in the AF-J$_1$ region** $\mathrm{sign}(D_{123}^{xy}) = \mathrm{sign}(D_{123}^{z})$
- **dimer phase in the FM-J$_1$ region** $\mathrm{sign}(D_{123}^{xy}) = -\mathrm{sign}(D_{123}^{z})$

On the XY line (Δ=0)

J$_1$-J$_2$ XY chain with FM J$_1$ $S_{j=\mathrm{even}}^{x,y} \to -S_{j=\mathrm{even}}^{x,y}$ J$_1$-J$_2$ XY chain with AF J$_1$

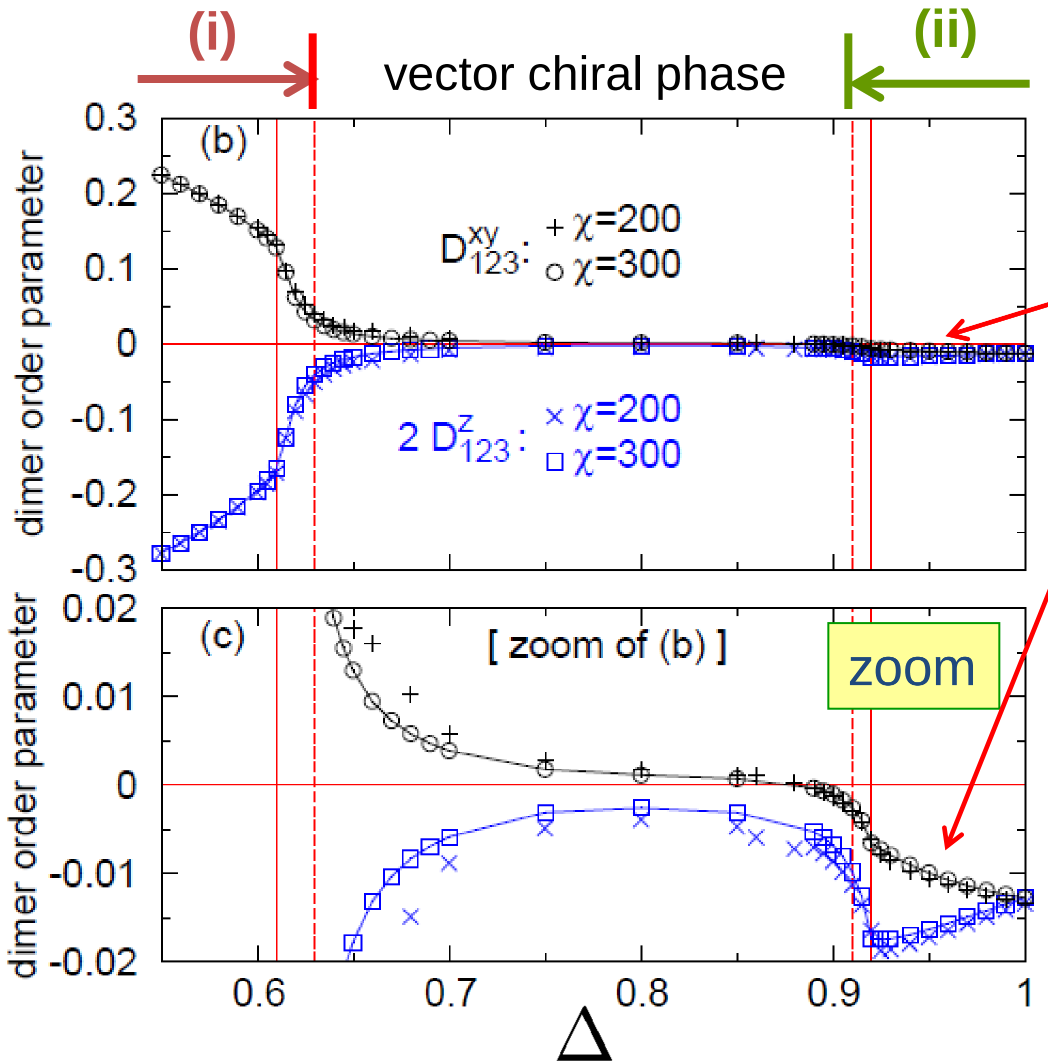
"even-parity" dimer **π rotation at even sites** "singlet" dimer

$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Dimer order parameters - II

$$D^{xy}_{123} := \langle\ [(S^x_1 S^x_2 + S^y_1 S^y_2) - (S^x_2 S^x_3 + S^y_2 S^y_3)]\ \rangle$$
$$D^z_{123} := \langle (S^z_1 S^z_2 - S^z_2 S^z_3) \rangle$$

$\boxed{J_1/J_2 = -2}$



(ii) $D^{xy}_{123} < 0,\ D^z_{123} < 0$

weak dimer order

Distinct from the even-parity dimer phase in the relative sign of the order parameters

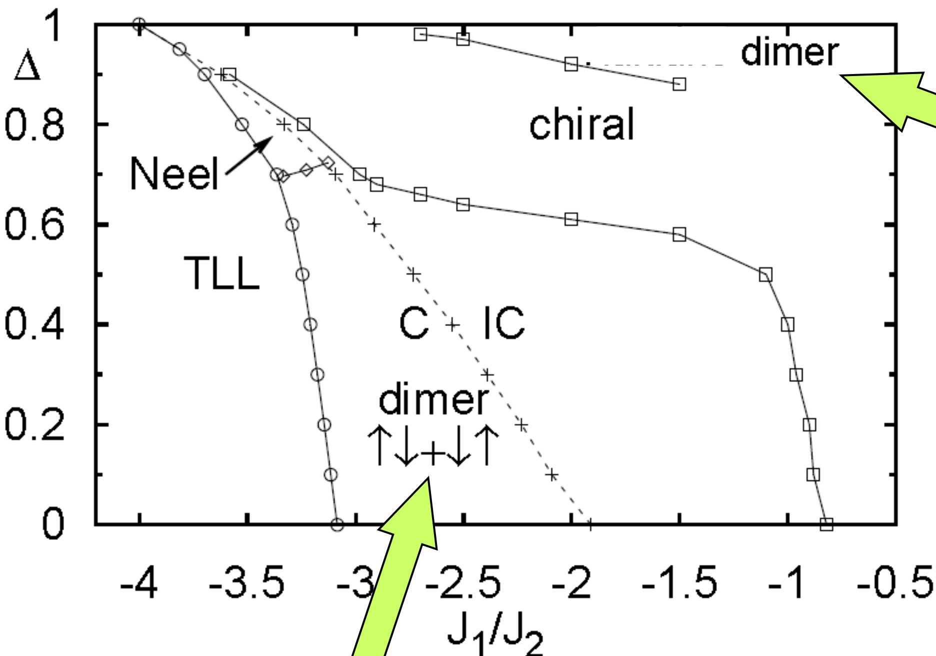
The singlet dimer state is unlikely for $J_1<0$.

What is this state?

Dimer order parameters

$$D^{xy}_{123} := \langle\ [(S^x_1 S^x_2 + S^y_1 S^y_2) - (S^x_2 S^x_3 + S^y_2 S^y_3)]\ \rangle$$

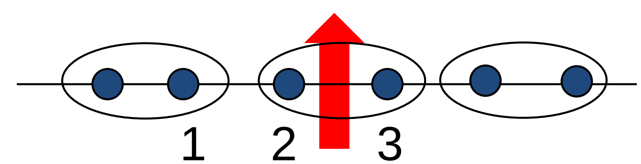
$$D^{z}_{123} := \langle (S^z_1 S^z_2 - S^z_2 S^z_3) \rangle$$



different type of dimer order

$$D^{xy}_{123} < 0,\ D^{z}_{123} < 0$$

$$\left\langle \vec{S}_2 \cdot \vec{S}_3 \right\rangle > \left\langle \vec{S}_1 \cdot \vec{S}_2 \right\rangle > 0$$



Haldane dimer phase

valence bond solid

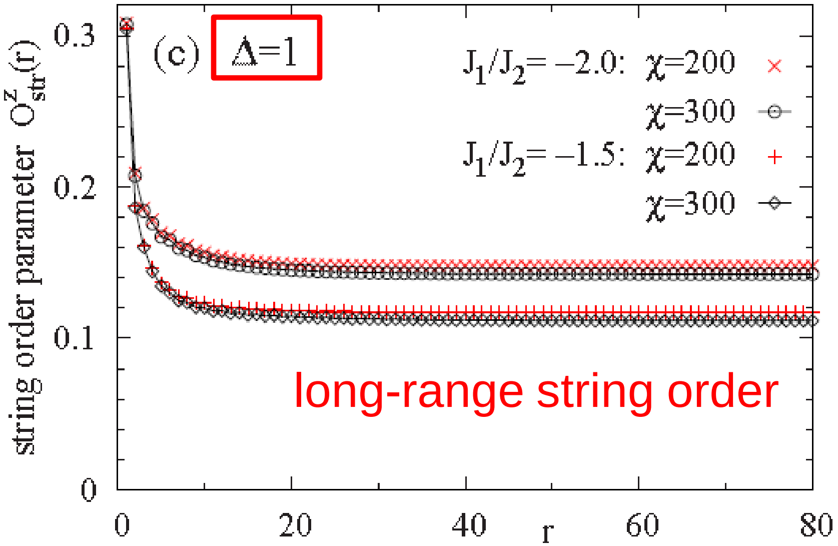
$$D^{xy}_{123} > 0,\ D^{z}_{123} < 0$$

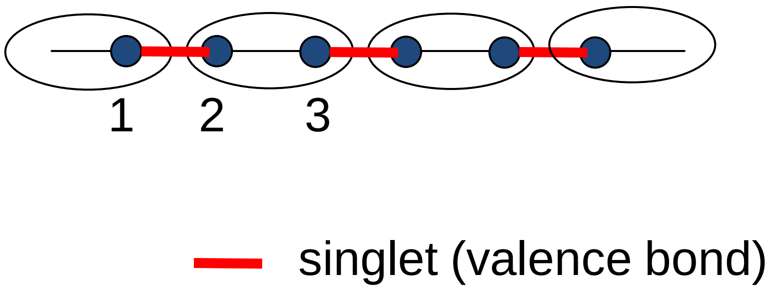
dimer $\uparrow\downarrow + \downarrow\uparrow$

string correlation function

$$\left\langle \left(S_{2j}^{\alpha}+S_{2j+1}^{\alpha}\right)\exp\left[i\pi\sum_{l=j+1}^{k-1}\left(S_{2l}^{\alpha}+S_{2l+1}^{\alpha}\right)\right]\left(S_{2k}^{\alpha}+S_{2k+1}^{\alpha}\right)\right\rangle$$

$(2j, 2j+1)$: dimerized bond

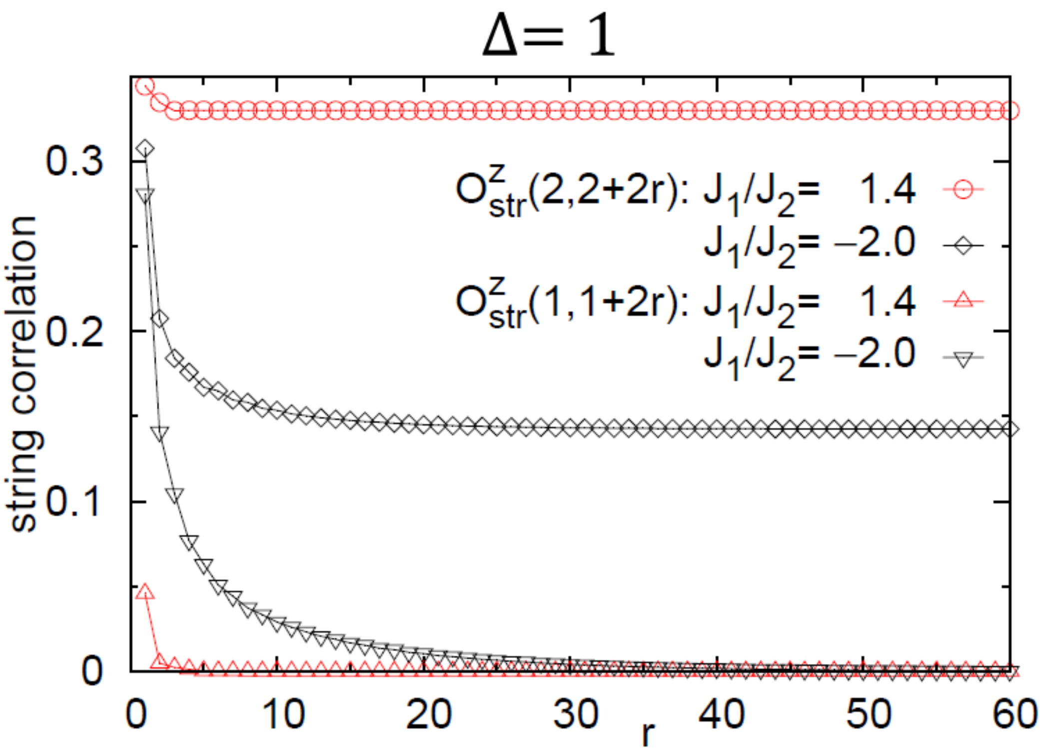


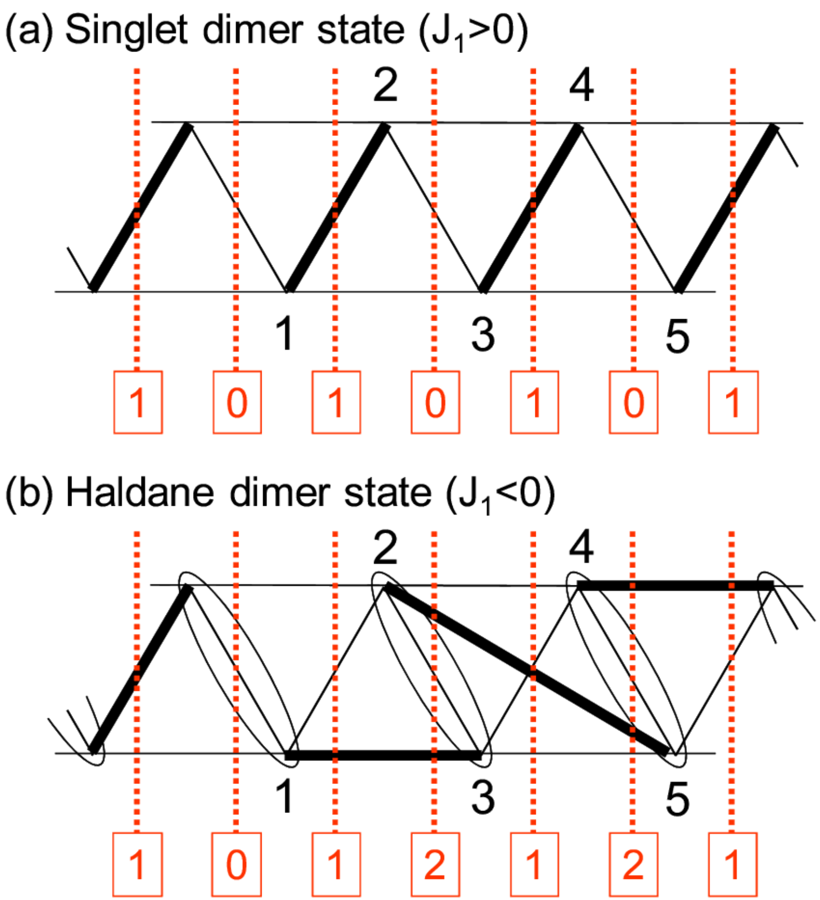


Sato, Furukawa, Onoda & AF
Mod. Phys. Lett. 25, 901 (2011)

String correlation functions

$$O^z_{\rm str}(\ell,\ell+2r) := -\left\langle (S^z_\ell + S^z_{\ell+1}) \exp\left[i\pi \sum_{m=\ell+2}^{\ell+2r-1} S^z_m \right] (S^z_{\ell+2r} + S^z_{\ell+2r+1}) \right\rangle$$





1-loop RG revisited

SU(2) symmetric case $\vec{S}_{\mu,j} = \vec{M}_\mu(x) + (-1)^j \vec{N}_\mu(x)$

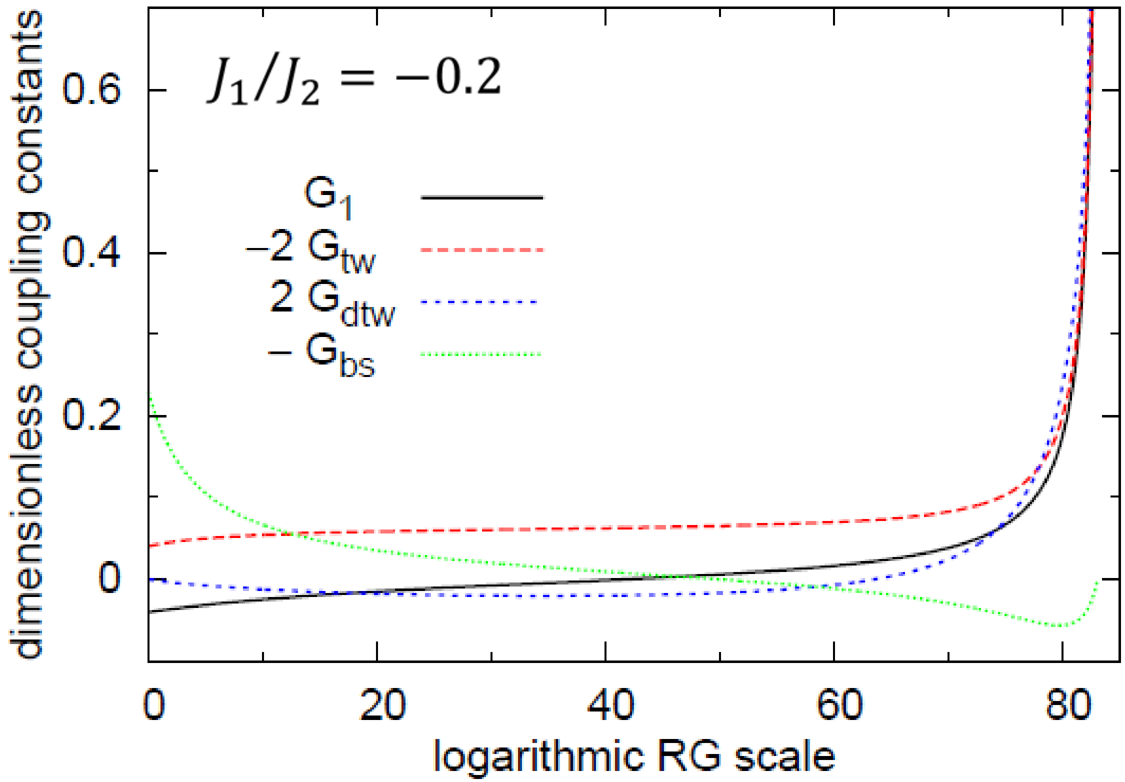
$$\begin{aligned}
\mathcal{O}_{\rm bs} &= \boldsymbol{M}_{1R}\cdot\boldsymbol{M}_{1L} + \boldsymbol{M}_{2R}\cdot\boldsymbol{M}_{2L},\\
\mathcal{O}_1 &= \boldsymbol{M}_{1R}\cdot\boldsymbol{M}_{2L} + \boldsymbol{M}_{1L}\cdot\boldsymbol{M}_{2R},\\
\mathcal{O}_2 &= \boldsymbol{M}_{1R}\cdot\boldsymbol{M}_{2R} + \boldsymbol{M}_{1L}\cdot\boldsymbol{M}_{2L},\\
\mathcal{O}_{\rm tw} &= \frac{a}{2}(\boldsymbol{N}_1\cdot\partial_x\boldsymbol{N}_2 - \boldsymbol{N}_2\cdot\partial_x\boldsymbol{N}_1),\\
\mathcal{O}_{\rm dtw} &= \frac{a}{2}(\epsilon_1\partial_x\epsilon_2 - \epsilon_2\partial_x\epsilon_1).
\end{aligned}$$

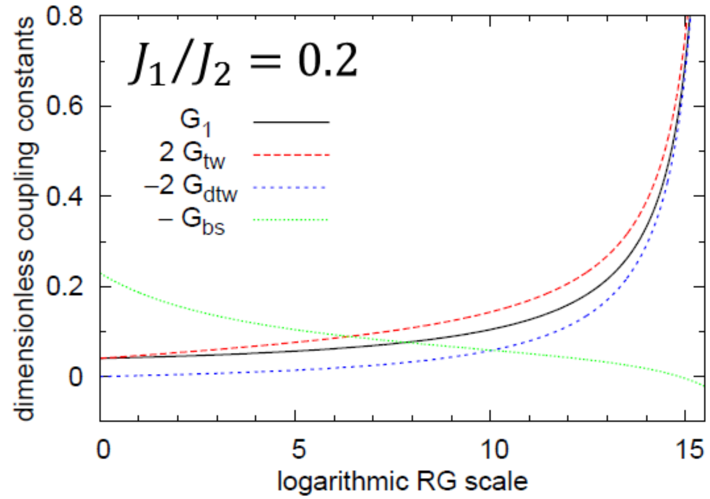
$$g_{\rm bs}(0) = -0.23(2\pi v), \quad g_1(0) = g_2(0) = 2J_1 a,$$
$$g_{\rm tw}(0) = J_1 a, \quad g_{\rm dtw}(0) = 0,$$

$$\begin{aligned}
\dot{G}_{\rm bs} &= G_{\rm bs}^2 + G_{\rm tw}^2 - G_{\rm dtw}^2,\\
\dot{G}_1 &= G_1^2 + G_{\rm tw}^2 - G_{\rm tw}G_{\rm dtw},\\
\dot{G}_{\rm tw} &= -\frac{1}{2}G_{\rm bs}G_{\rm tw} + G_1G_{\rm tw} - \frac{1}{2}G_1G_{\rm dtw},\\
\dot{G}_{\rm dtw} &= \frac{3}{2}G_{\rm bs}G_{\rm dtw} - \frac{3}{2}G_1G_{\rm tw},
\end{aligned}$$

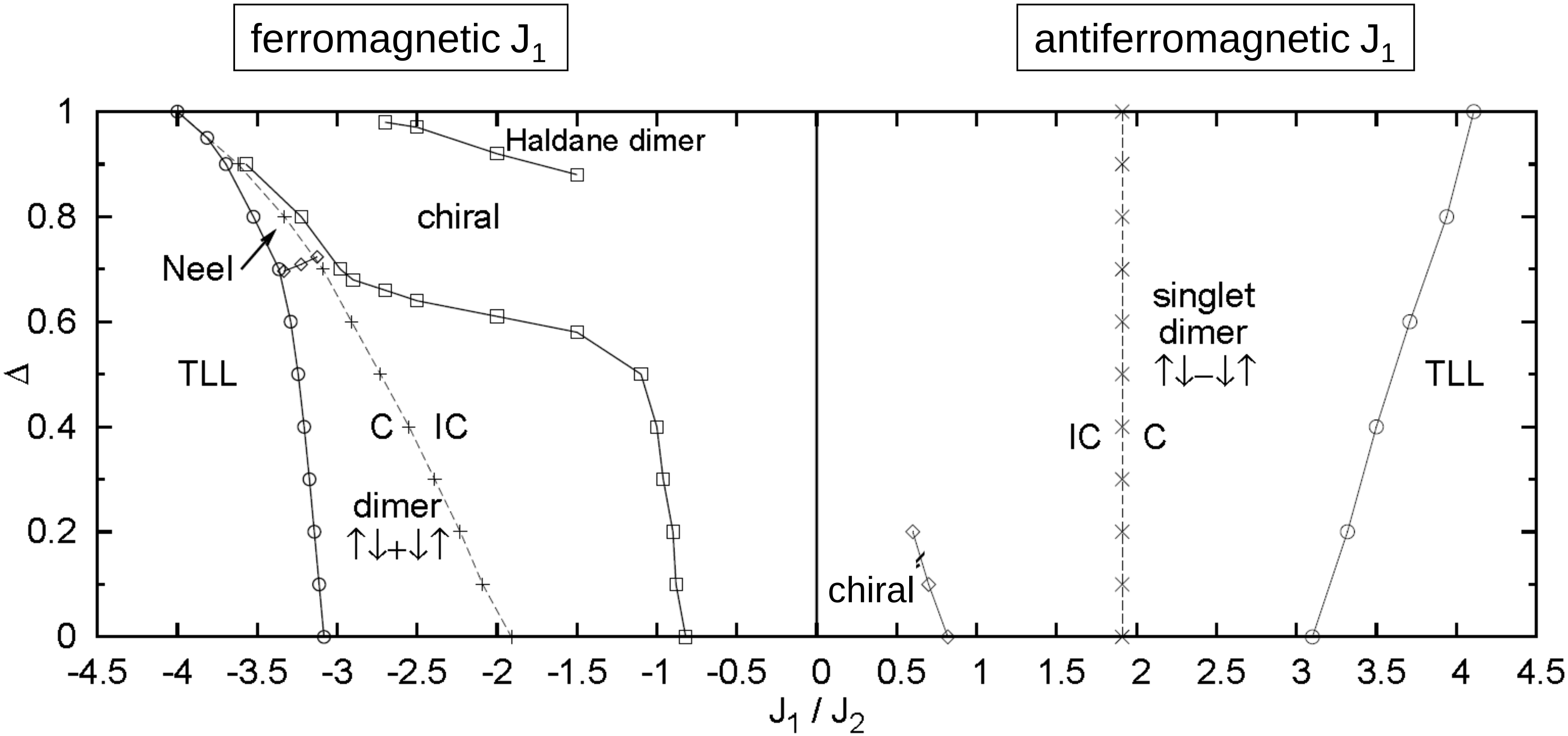
Nersesyan, Gogolin, Essler (1998)
Itoi, Qin (2001)
Starykh, Balents (2004)

Numerical integration of 1-loop RG eq.





Summary



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In preparation (to be submitted to PRB)