Dynamics in strongly correlated quantum gases

Corinna Kollath

MaNEP SWITZERLAND

UNIVERSITÉ DE GENÈVE

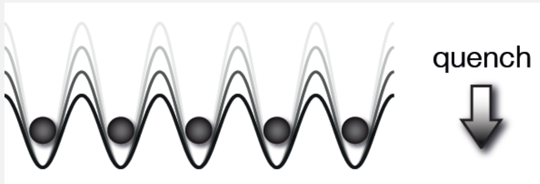
FNSNF

Fonds national suisse
Schweizerischer Nationalfonds
Fondo nazionale svizzero
Swiss National Science Foundation

Fundamental questions in interacting systems

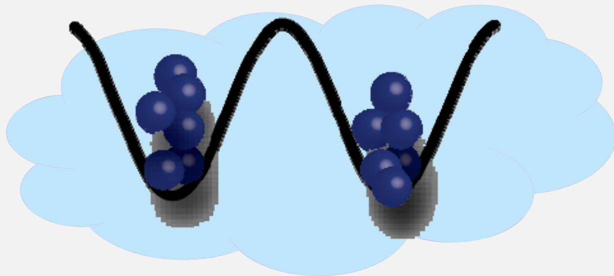
➢ ***spreading of correlations***

coherent dynamics in a Mott-insulator



- preparation of complex states, e.g. BEC formation
- limit of 'speed of information'

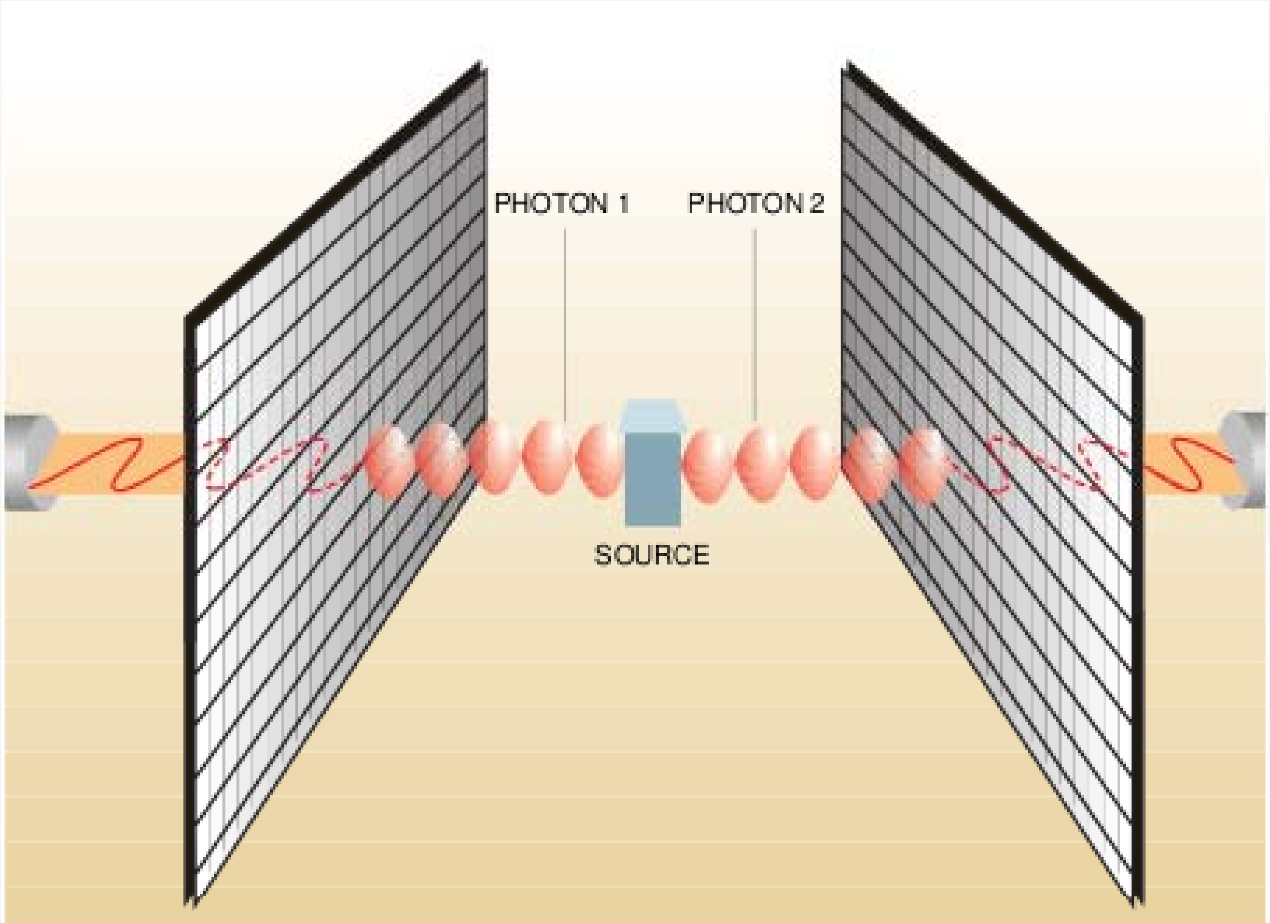
➢ **destruction of coherence by dissipation:**
interplay between dissipation and interaction



- heating in optical lattices
- interaction can fight dissipation?
- can dissipation be used to prepare complex quantum states?

many papers dealing with similar questions....

Relativistic quantum mechanics



De Lahaye, Science (1998)

relativistic: correlation propagation maximally with speed of light

non-relativistic: no such maximal velocity build in

Lieb and Robinson Bound

The Finite Group Velocity
of Quantum Spin Systems

Elliott H. Lieb*
Dept. of Mathematics, Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

Derek W. Robinson**
Dept. of Physics, Univ. Aix-Marseille II, Marseille-Luminy, France

Received May 15, 1972

- **finite group velocity for spreading of correlations**

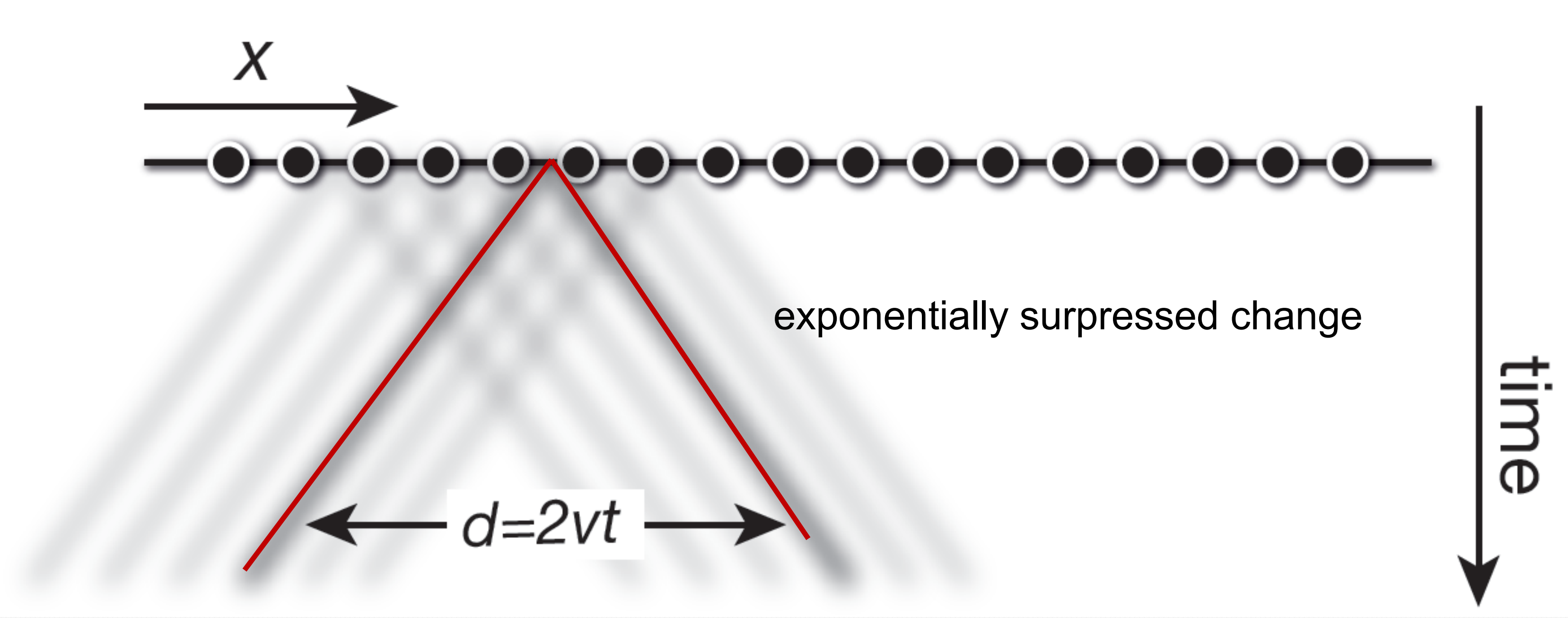
outside of a 'light-cone' only exponentially small changes

not yet proven in generality for bosonic unbounded systems
counter example has been constructed

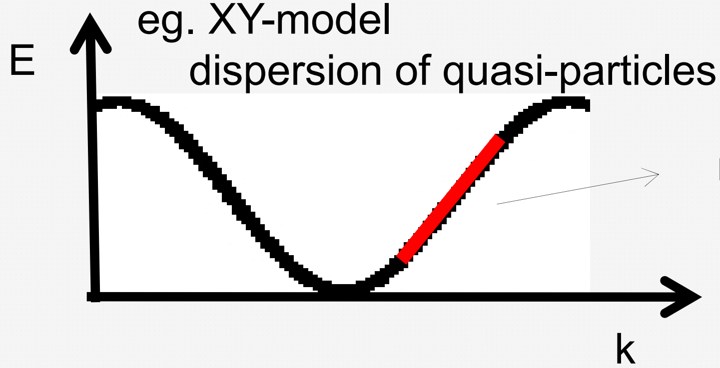
Bravy , Hastings,
Nachtergaele, Sims, Eisert …

Quasi-particle interpretation

pairs of entangled quasi-particles carry correlations (CFT)



what is the velocity of quasi-particles?



maximal velocity

$$\mathrm{v}(k) = 2\frac{d}{dk}\omega(k)\,,$$

Calabrese and Cardy (2006)

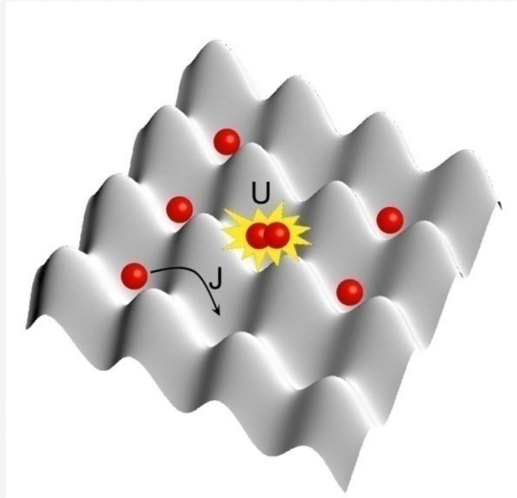
& other specific models

(Igloi, Rieger, Laeuchli, Cazalilla, Muramatsu, Manmana, Mathey, Schollwöck, Eisert,...)

How fast can correlations spread in an optical lattice?

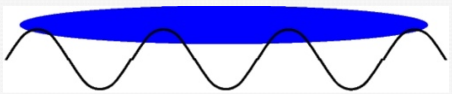
kinetic energy | interaction energy

$$H = -J\sum_{<ij>}(b_i^+ b_j + h.c.) + U/2\sum_j n_j(n_j - 1)$$

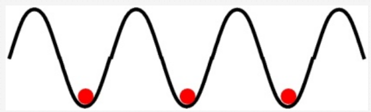


Jaksch et al. (1998)

Superfluid Phase



Mott-Insulating Phase



U/J

sound velocity

insulating behaviour

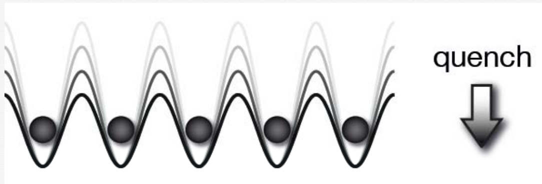
Fisher et al. (1989)

Very different 'characteristic' velocities

How do correlations spread?

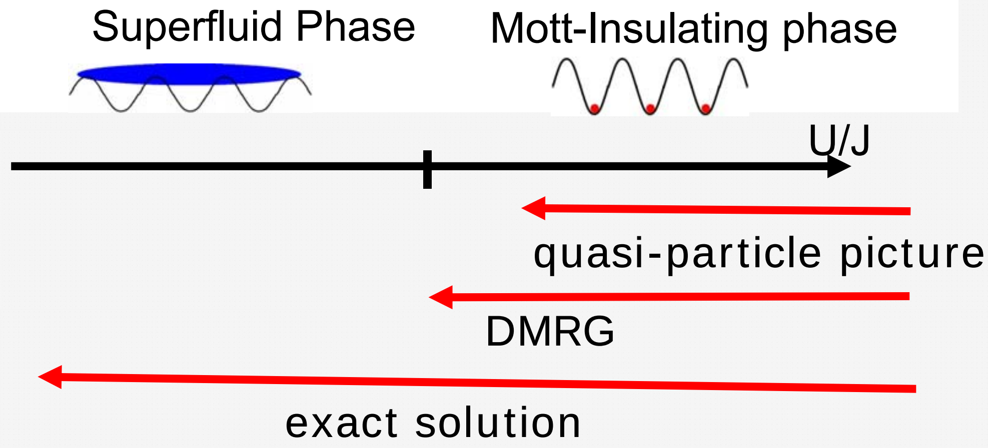
Experiments:

sudden decrease or increase of the optical lattice height



Theory:

decrease or increase of ratio U/J

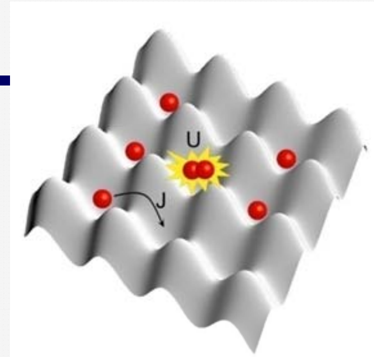


M. Chenau (MPQ Munich), M. Endres, T. Fukuhara, C. Weitenberg, P. Schauß, C. Gross, I. Bloch (MPQ Munich, LMU Munich) S. Kuhr (Strathclyde, Glasgow)

P. Barmettler, D. Poletti, C. Kollath

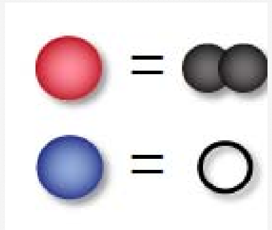
M. Cheneau et al., Nature (2012)
P. Barmettler et al, PRA (2012)

Strongly interacting bosons



- mapping onto three local states
- introduce fermionic operators for holes and doubly occupied sites

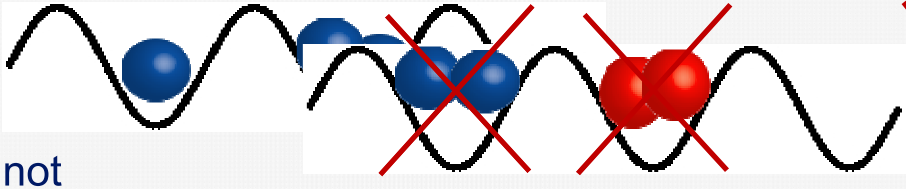
-> quadratic Hamiltonian
(solve by Bogolyubov transformation)



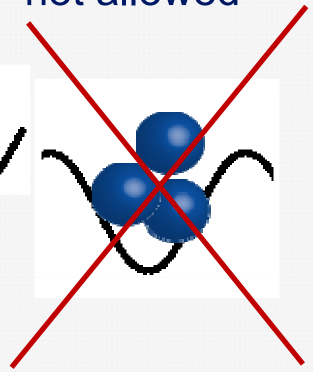
fermionic nature suppresses unphysical states (important!)

(must be enforced in bosonic approach)

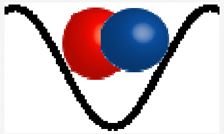
allowed



not allowed



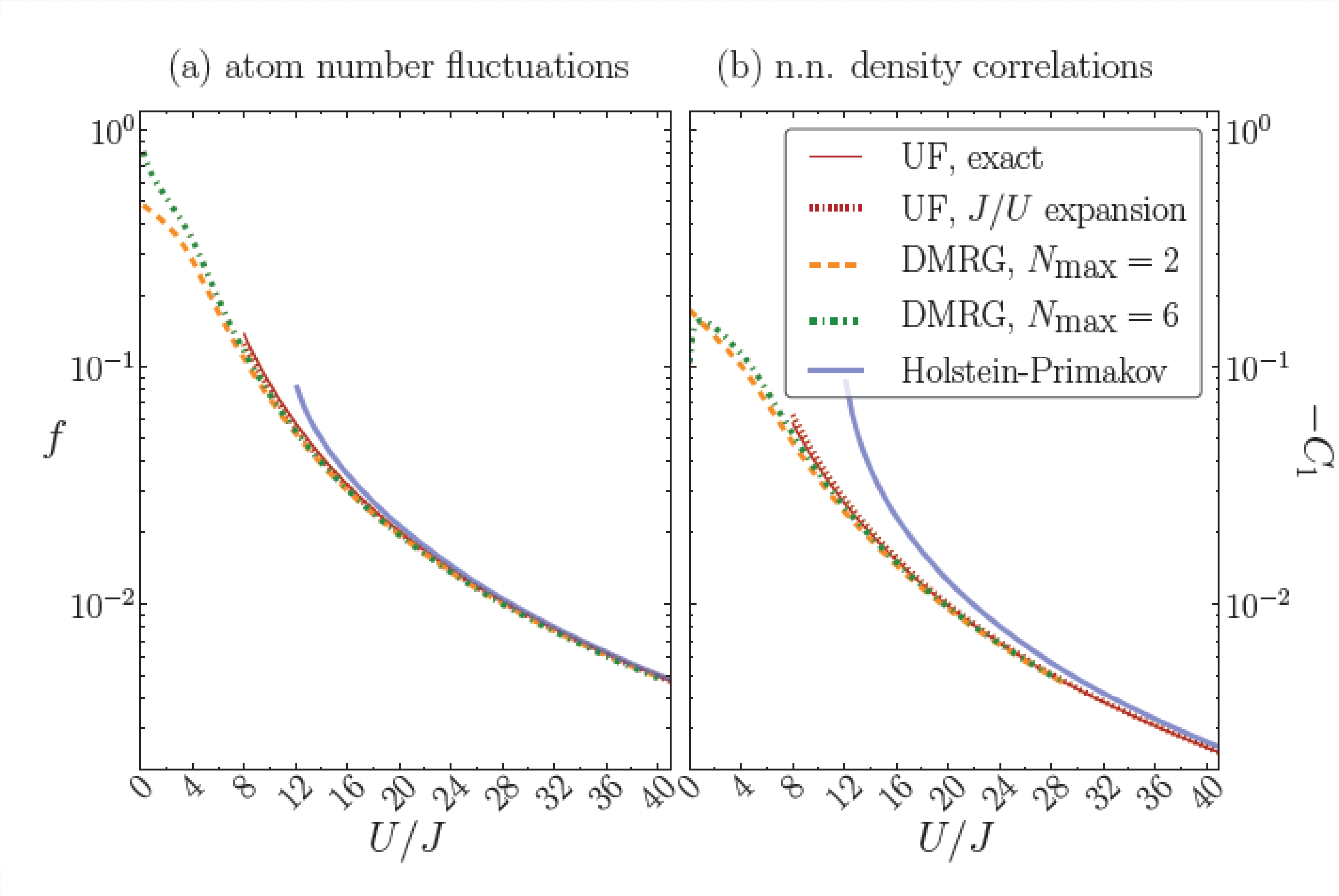
but not



only important for low final interation strength

P. Barmettler et al. in preparation
Batista and Ortiz PRL 2001 for spin1 -chains
bosonic approach see E. Altman and Auerbach and
S. Huber et al.

Equilibrium local observables



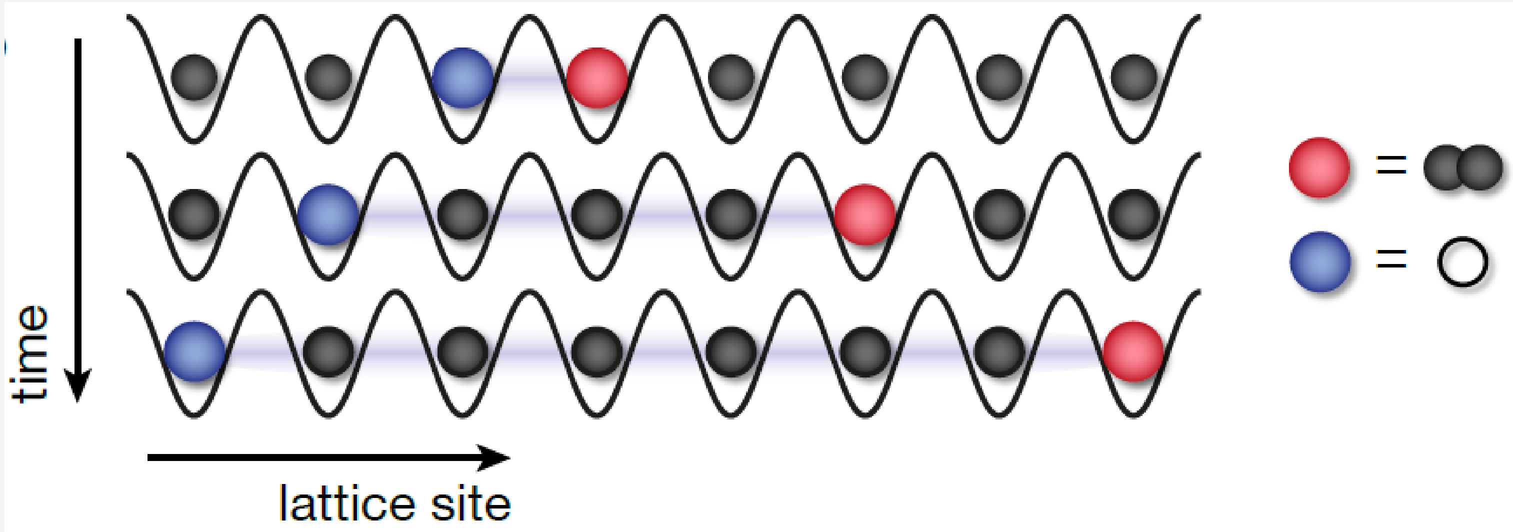
$$C_d(t) \propto \langle n_j n_{j+d} \rangle - \langle n_j \rangle \langle n_{j+d} \rangle$$

short range quantities well described
both by Fermionic quasi-particles and perturbation theory in J/U

Dynamic of Fermionic quasi-particles

large interaction limit:

symmetric coherent superposition (no change of density!)



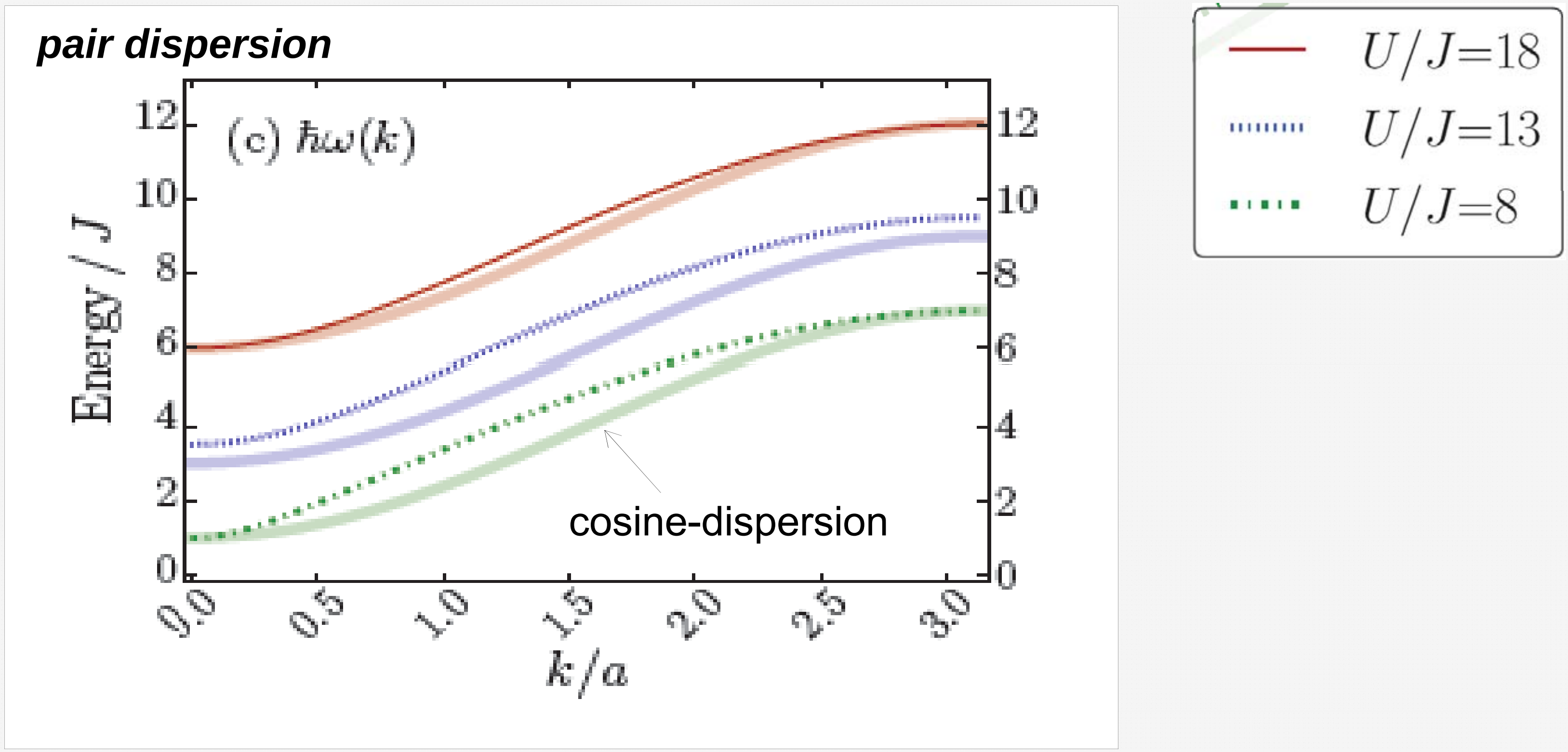
first order perturbation in J/U (more complicated expression for quasiparticles):

$$|\psi(t)\rangle = |\bar{n}\rangle + i\frac{2\sqrt{2}J}{U}\sum_k \sin(k) c^\dagger_{k,+} c^\dagger_{-k,-}|\bar{n}\rangle$$

$$- i\frac{2\sqrt{2}J}{U}\sum_k \sin(k) e^{i6J\cos(k)t/\hbar} c^\dagger_{k,+} c^\dagger_{-k,-}|\bar{n}\rangle .$$

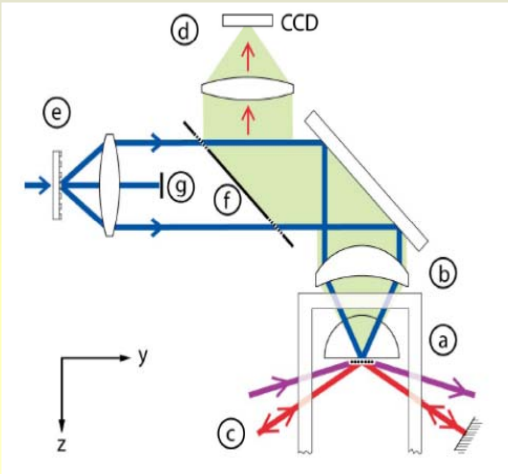
$$= \sum_{j,d} \frac{(-i)^d \hbar d}{3Jt} \mathcal{J}_d(6Jt/\hbar) c^\dagger_{j,+} c^\dagger_{j+d,-}|\bar{n}\rangle$$

Dispersion of quasi-particles

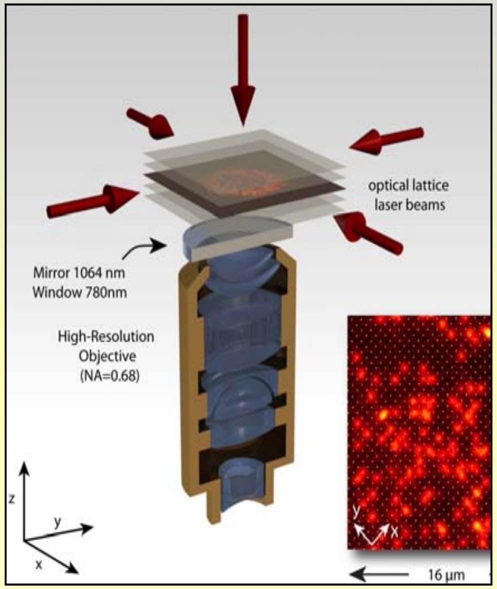


Real space imaging

Local flourescence imaging



Bakr et al. 2009



Sherson et al. Nature 2010

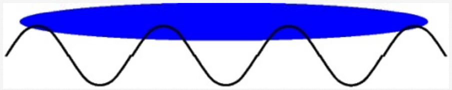
parity distribution in real space

parity correlations

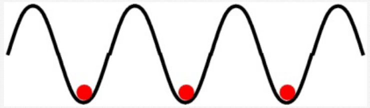
$$C_d(t) \propto \langle n_j^{even} n_{j+d}^{even} \rangle - \langle n_j^{even} \rangle \langle n_{j+d}^{even} \rangle$$

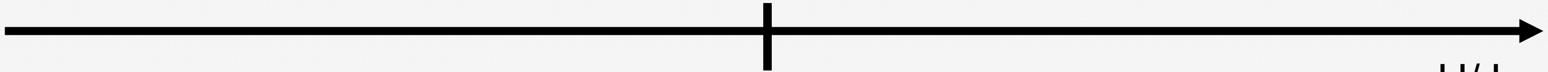
Parity distribution: Superfluid to Mott-insulator

Superfluid Phase



Mott-Insulating Phase

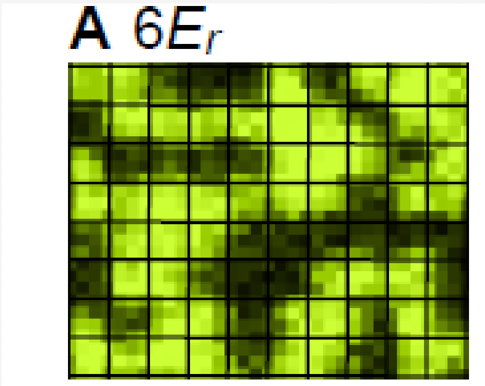


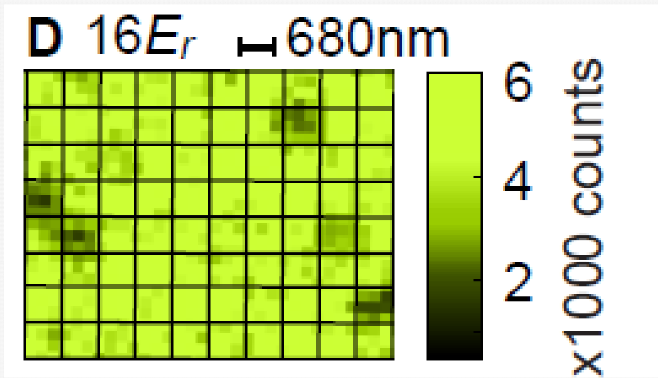


U/J

Fisher et al. (1989)

parity distribution in real space
flourescence imaging:



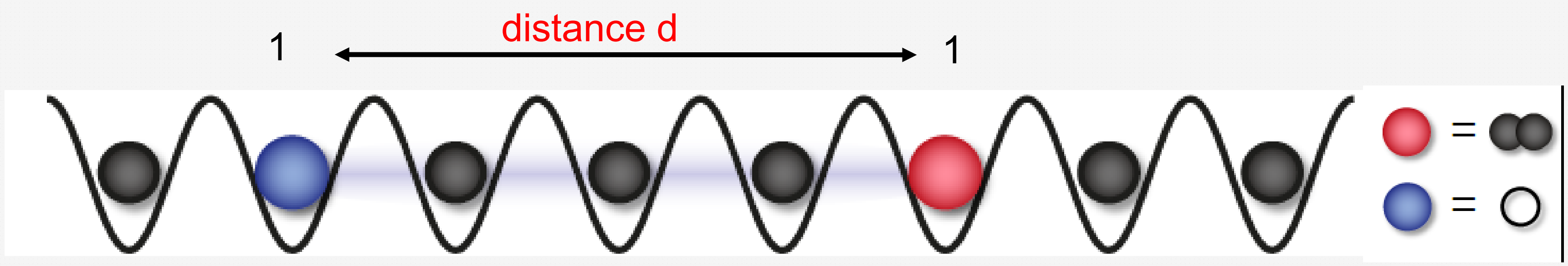


Bakr et al. (2009)

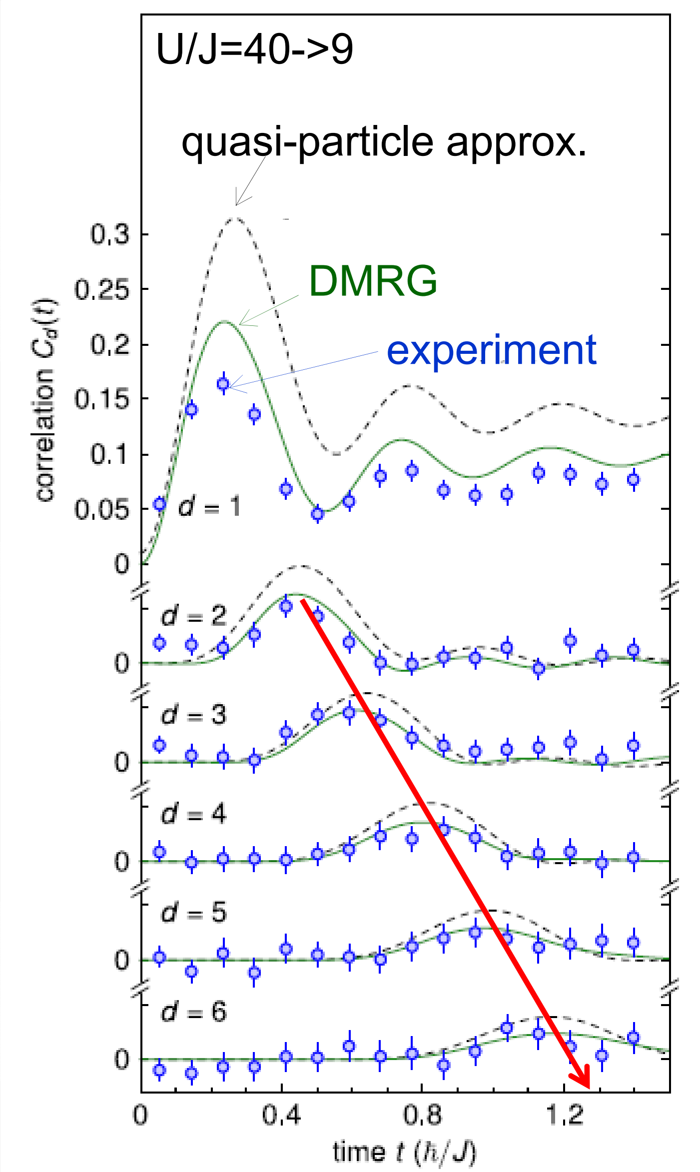
Real space imaging

equal time parity correlation
between sites of distance d

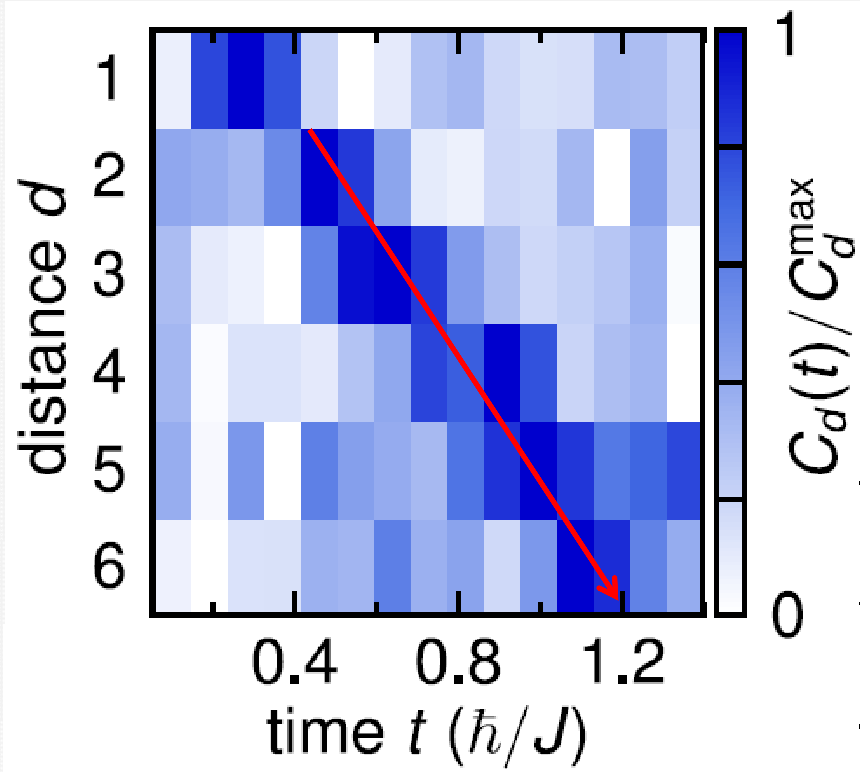
$$C_d(t) \propto \langle n_j^{even} n_{j+d}^{even} \rangle - \langle n_j^{even} \rangle \langle n_{j+d}^{even} \rangle$$



Propagation of correlations in Mott-insulator

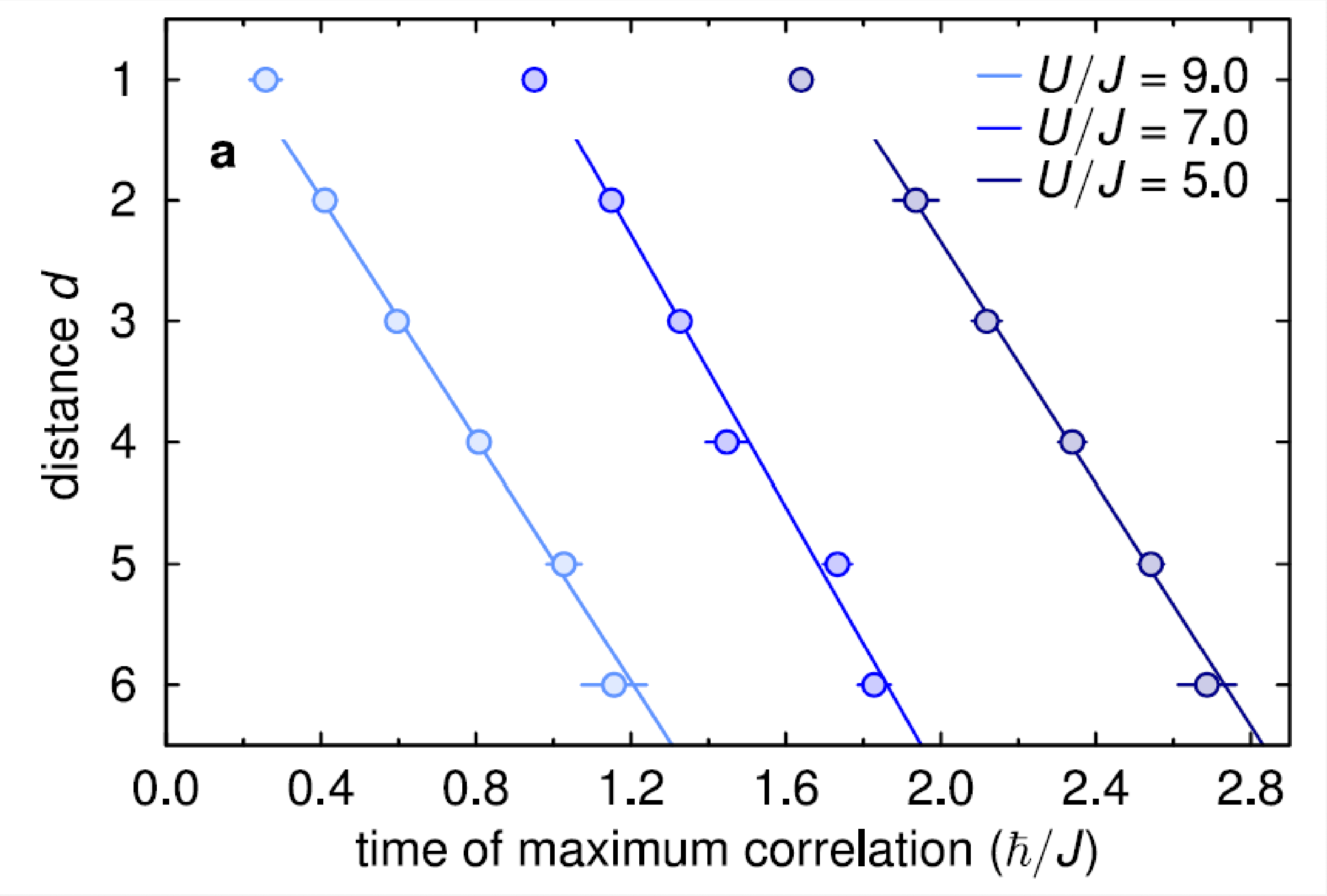


density remains constant!



M. Cheneau et al, Nature (2012)

Cone-like spreading?

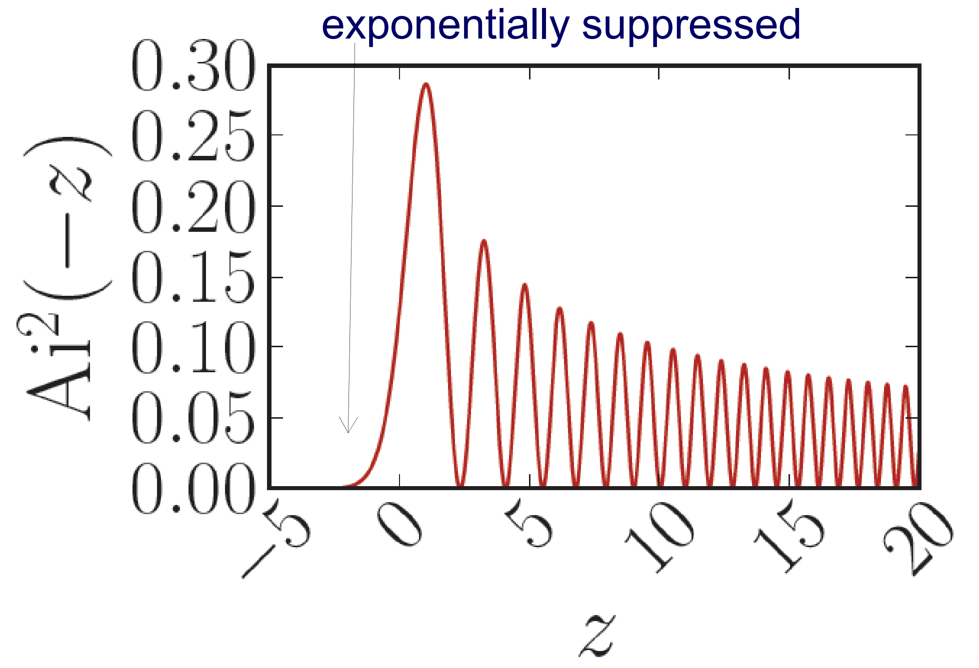


linear spreading

only exponentially small corrections before (Besselfunction)

Deviations from light cone propagation

$$C_{d\gg 1} \approx -\left(\frac{2d^{2/3}2^{1/3}J\hbar}{3Ut}\right)^2 \mathrm{Ai}^2\left(-(2/d)^{1/3}(6Jt/\hbar - d)\right)$$

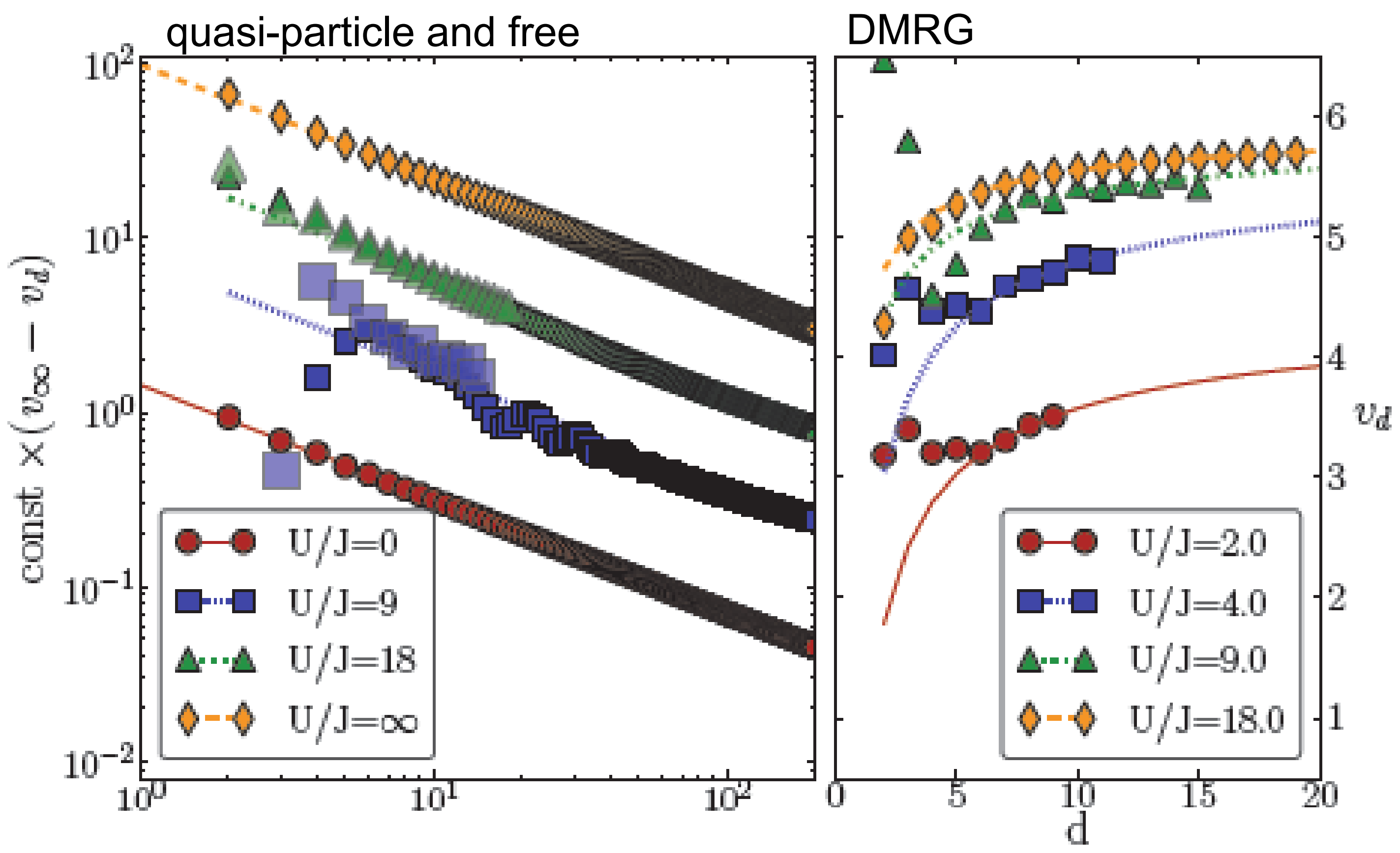


$$\mathrm{v}_d = \frac{\hbar}{(t_{\mathrm{peak}}(d+1) - t_{\mathrm{peak}}(d))} = \mathrm{v}_\infty\left(1 - \frac{z_0}{2^{1/3}3}d^{-2/3}\right) + \mathcal{O}(d^{-5/3})$$

algebraic corrections to light cone (perturbation theory)

width ~d^1/3

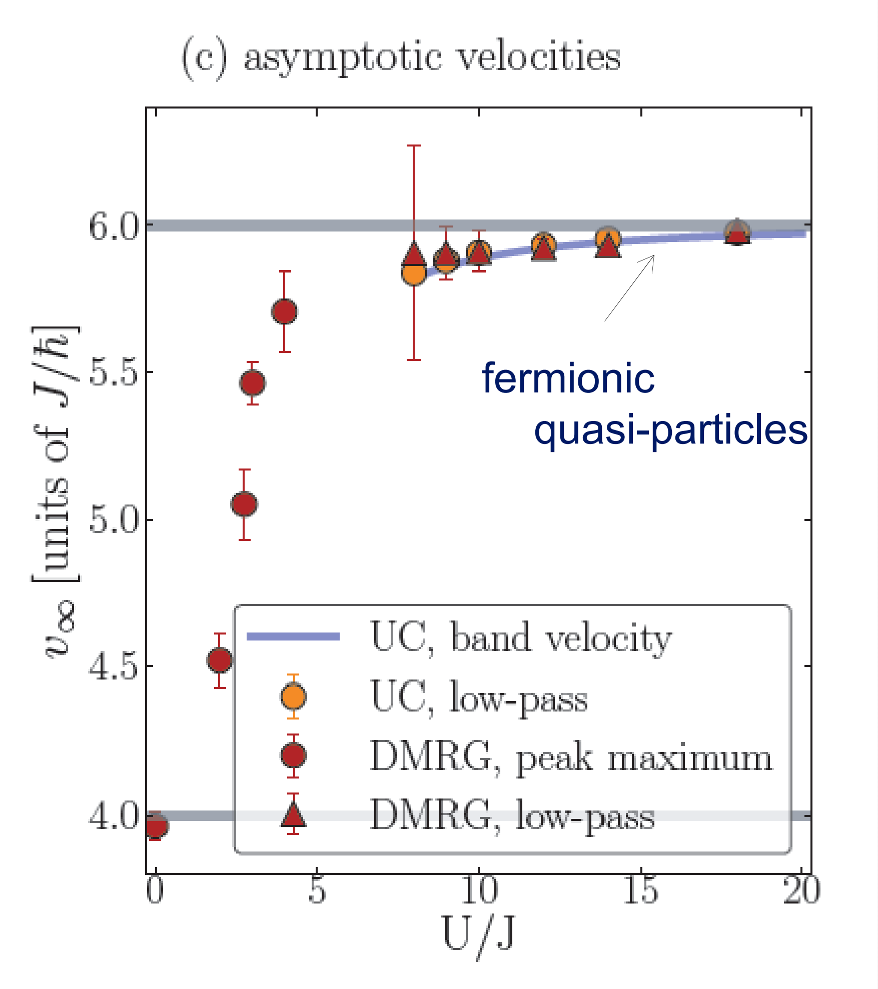
Velocity at long times



algebraic corrections to linear spreading

corrections for small distances t~d+d^1/3

Velocity of correlation spreading



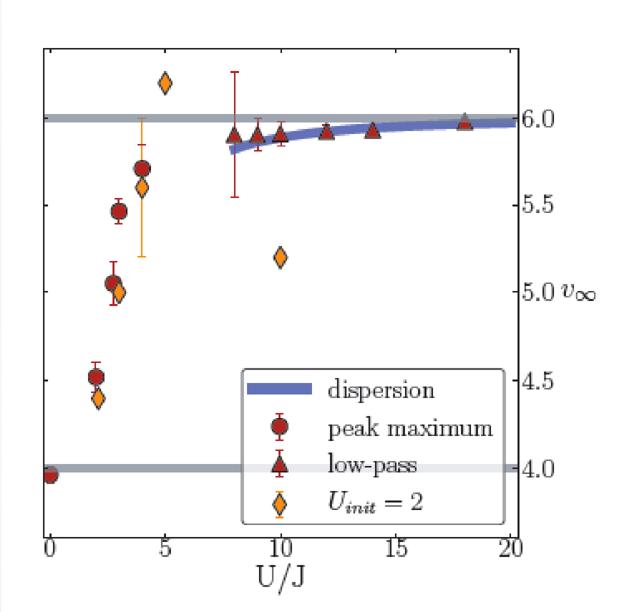
velocity:

generic maximal velocity in bounded model

continuous across equilibrium phase transition

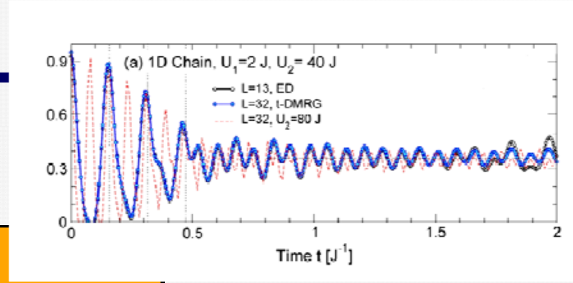
(at phase transition difficult to determine)

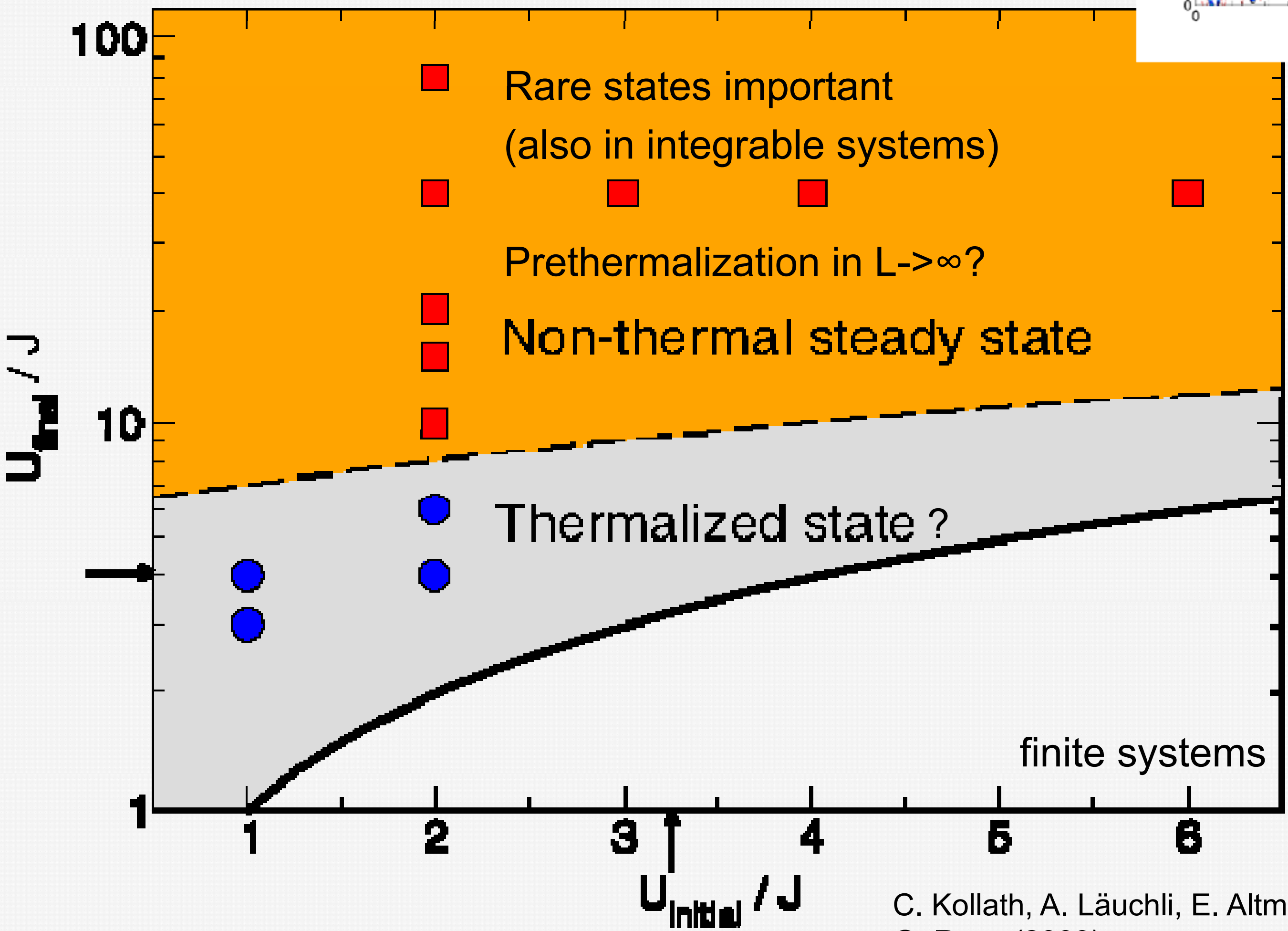
sf->MI larger velocities found
(origin higher bands?)



Thermalization inside the light cone?

local onsite observables seem thermal, but non-local correlations



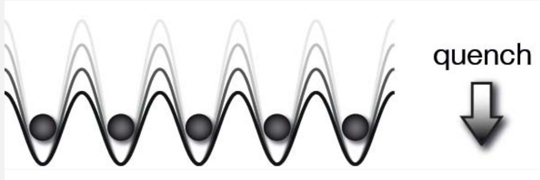


C. Kollath, A. Läuchli, E. Altman, PRL (2007)
G. Roux (2009)
G. Biroli, C. Kollath, and A. Läuchli , PRL (2010)

many others in different models:
cannot cite all names here!

Fundamental questions in interacting systems

➢ coherent spreading of correlations

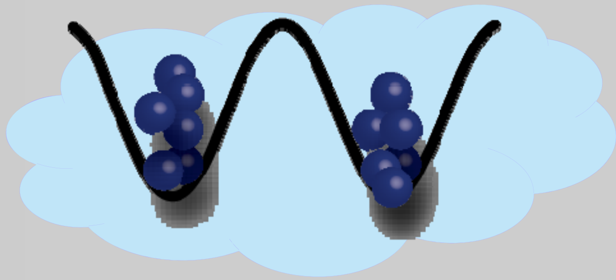


- light-cone like dynamics
- well explained by quasi-particle picture
- generic max. velocity in system with bounded spectra

open questions:

- influence of higher bands
- faster quantum channels?
- approach for higher dimensional systems

➢ interplay between dissipation and interaction



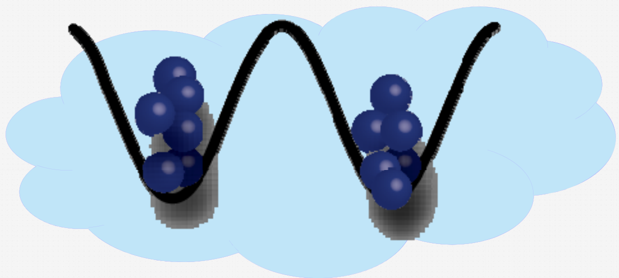
- heating in optical lattices
- interaction can fight dissipation?
- can dissipation be used to prepare complex quantum states?

Theoretical model for dissipative coupling

dissipative effects:

scattering with thermal atoms

flourescence scattering with light fields



described by Markovian master equation:

closed system dynamics

$$i\hbar\partial_t\hat{\rho} = [\hat{H},\hat{\rho}] + i\mathcal{L}(\hat{\rho})$$

dissipative dynamics

$$\mathcal{L}(\hat{\rho}) = \Gamma\sum_{l=1}^{L}\left(\hat{n}_l\hat{\rho}\hat{n}_l - \frac{1}{2}\hat{n}_l^2\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{n}_l^2\right)$$

atom detection

no-detection event

ρ density matrix

Γ dissipative coupling

H system Hamiltonian

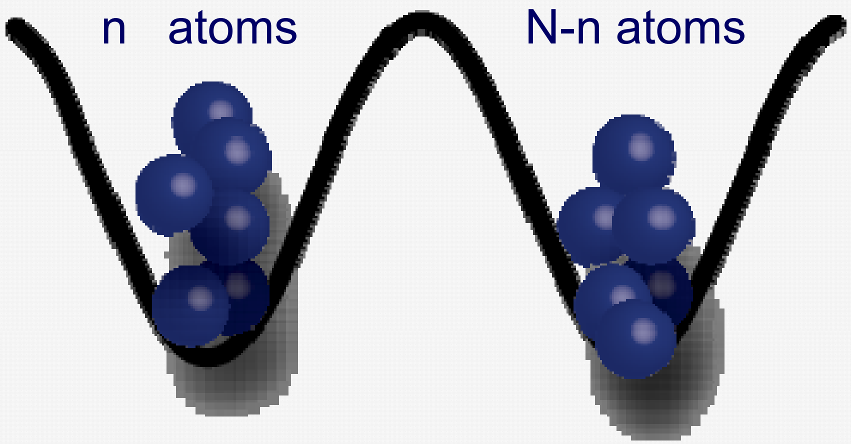
F. Gerbier and Y. Castin (2010)

S. Pichler et al (2010)

D. Poletti, JS. Bernier, A. Georges, C. Kollath, arXiv (2012)

Double well: lowest band approximation

two-site Bose-Hubbard model
(lowest band only)
N atoms



start with ground state and apply dissipation at time t=0

how long does the initial coherence survive?

$$C = \langle \hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1 \rangle$$

Localization effects

large interaction

&

dissipation:
local measurement
of particle number

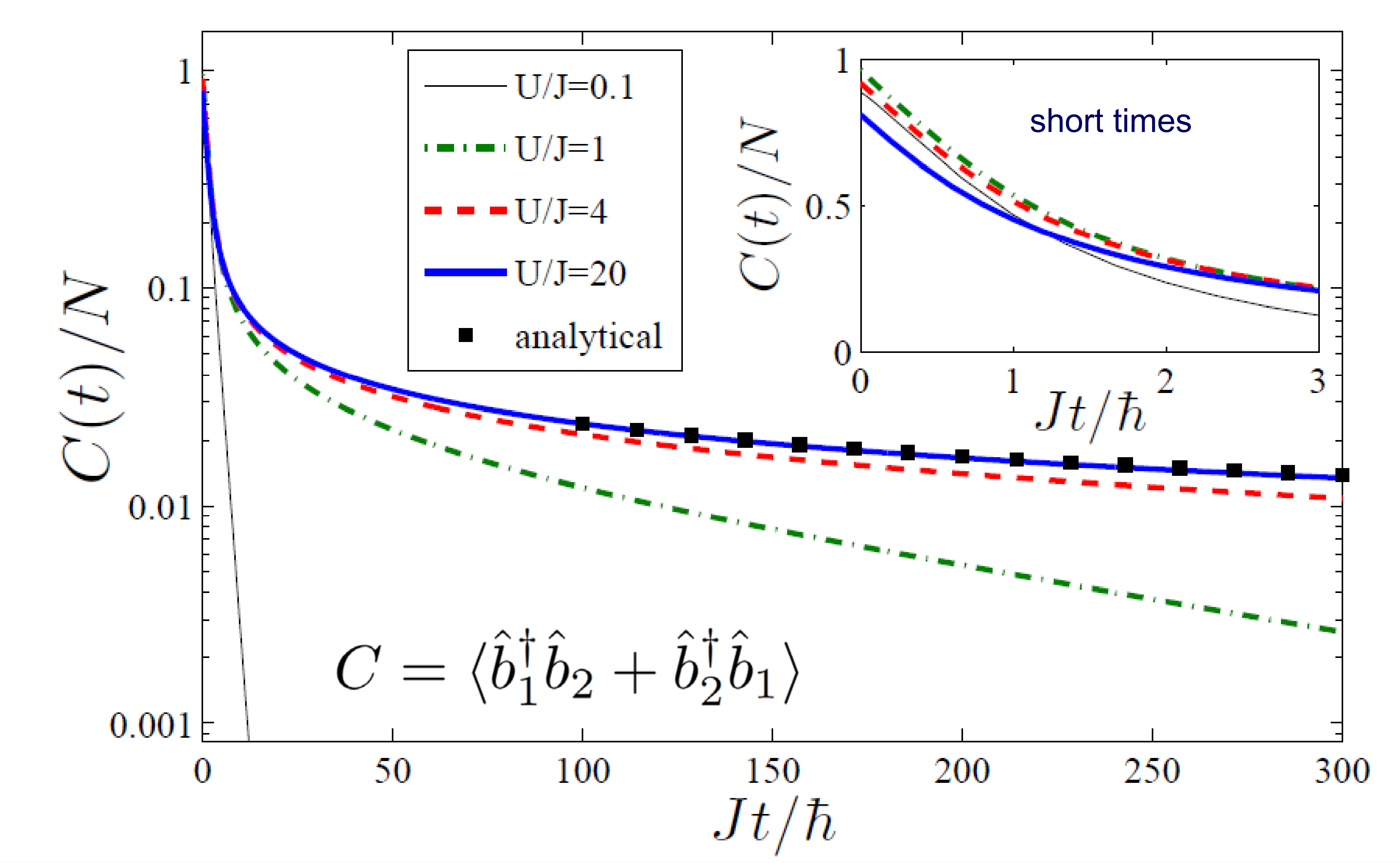
localization of atoms
destruction of coherence

interaction fights dissipation
slower destruction of coherence

exponentially fast
destruction of coherence
long time limit:
totally mixed state

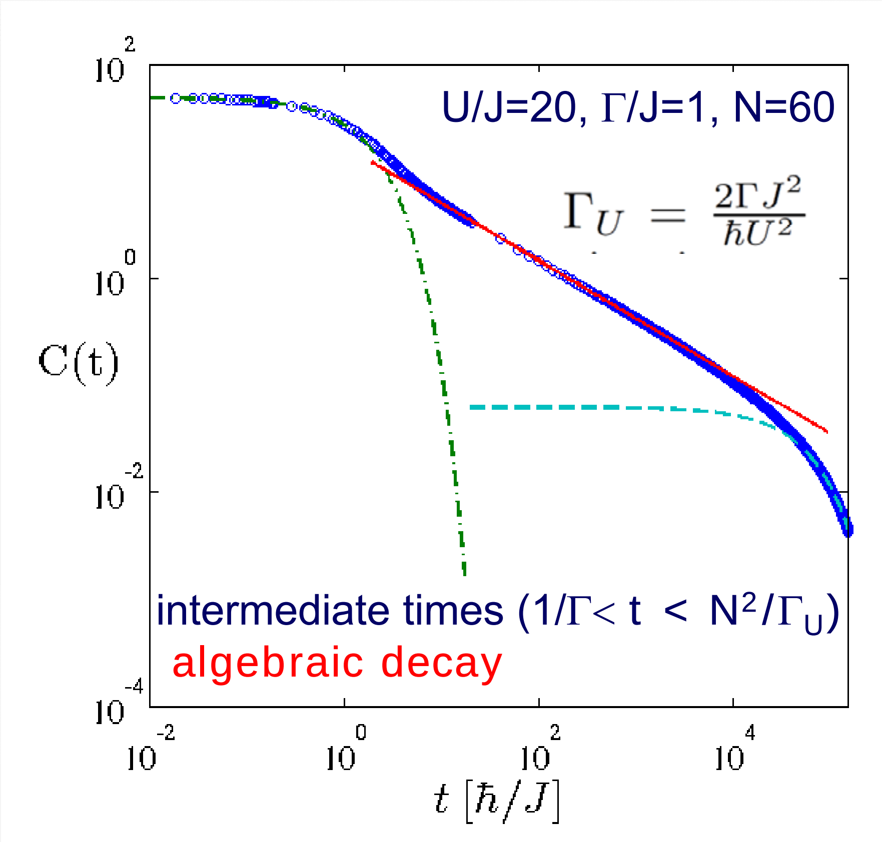
Interaction saves coherence

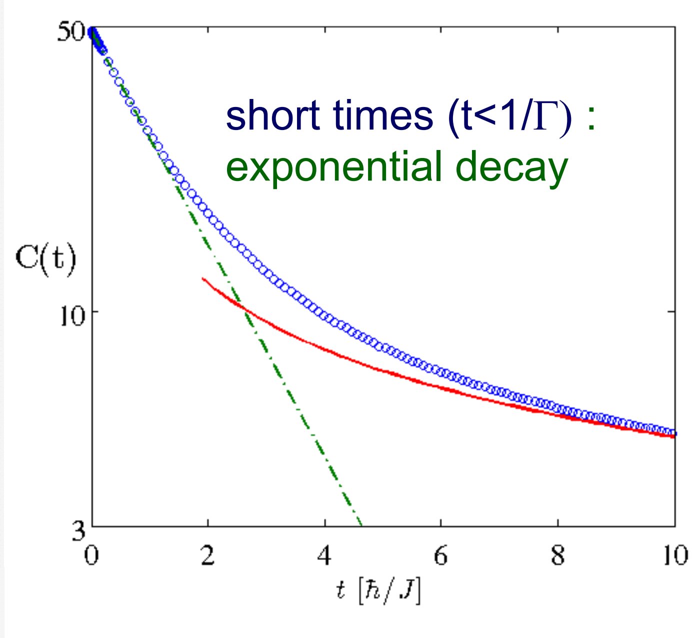
$\Gamma/J=1$, N=60

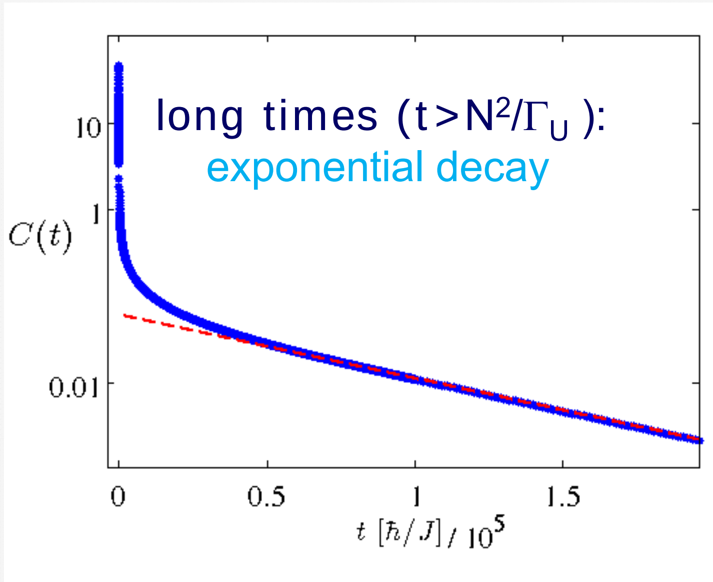


Evolution of coherence for large interaction

3 regimes:







Mapping to classical diffusion equation (large U, N)

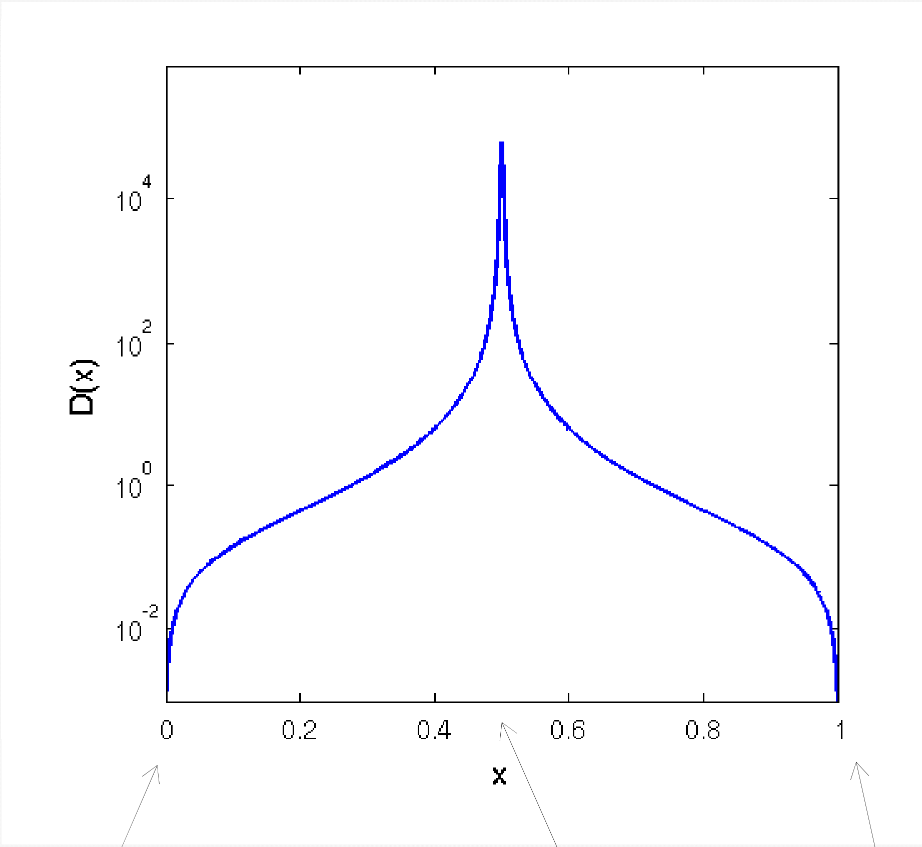
neglecting fast modes ($t > 1/\Gamma$) (adiabatic elimination)
and performing large N limit

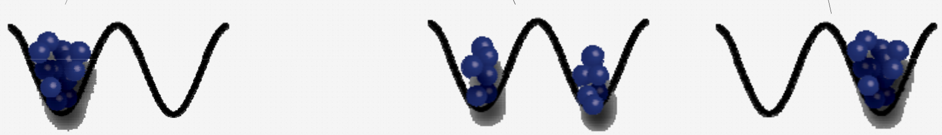
$$\frac{d}{dx}\left(D(x)\frac{d}{dx}\phi(x,\tilde{t})\right)=\frac{\hbar U^2}{2J^2\Gamma}\frac{\partial}{\partial\tilde{t}}\phi(x,\tilde{t})$$

$$D(x)=\frac{x(1-x)}{(2x-1)^2},\qquad \tilde{t}=t/N^2$$

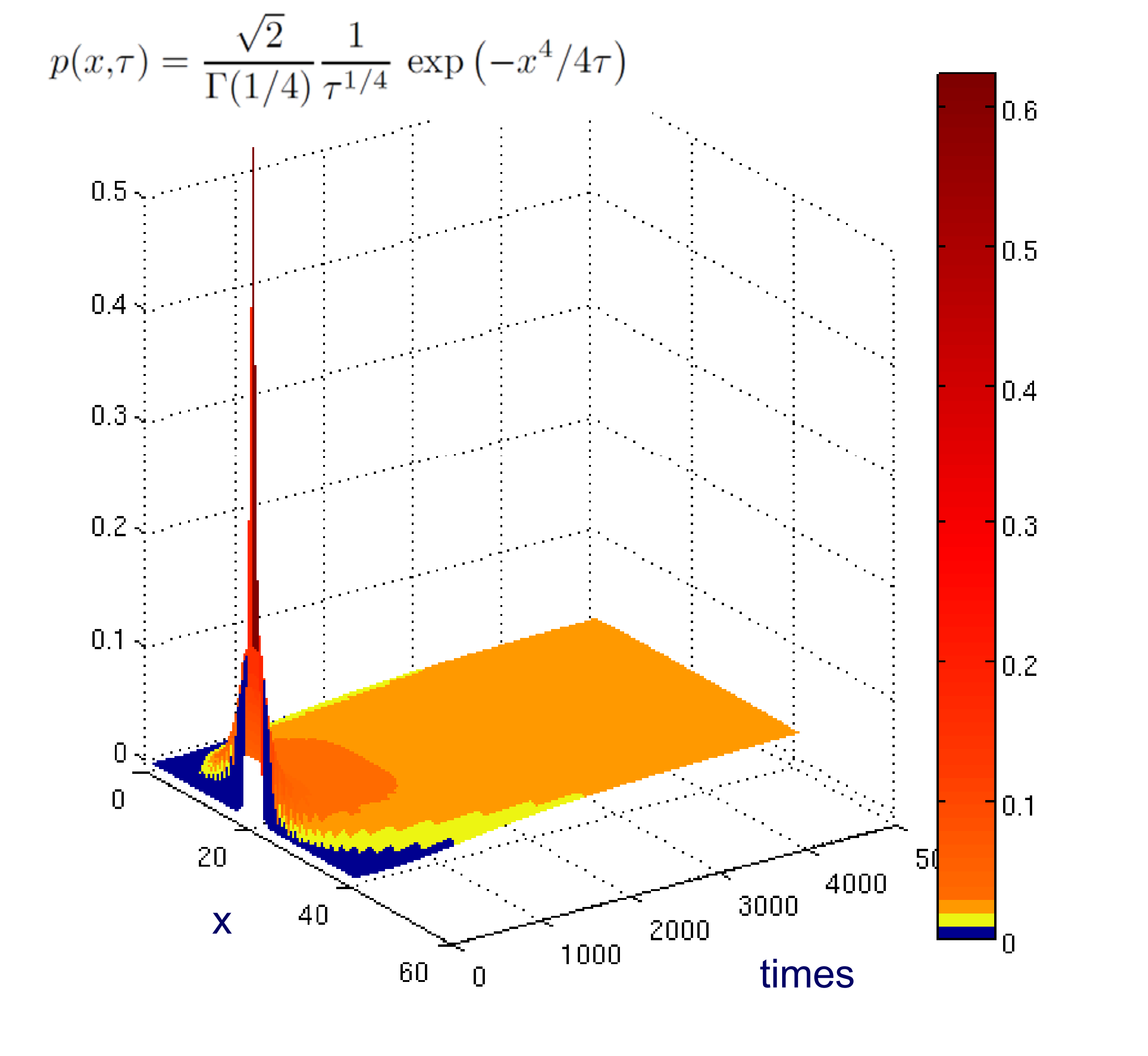
corresponds to
diagonal density matrix elements

slowly diffusing states at the boundary
correspond to large imbalance

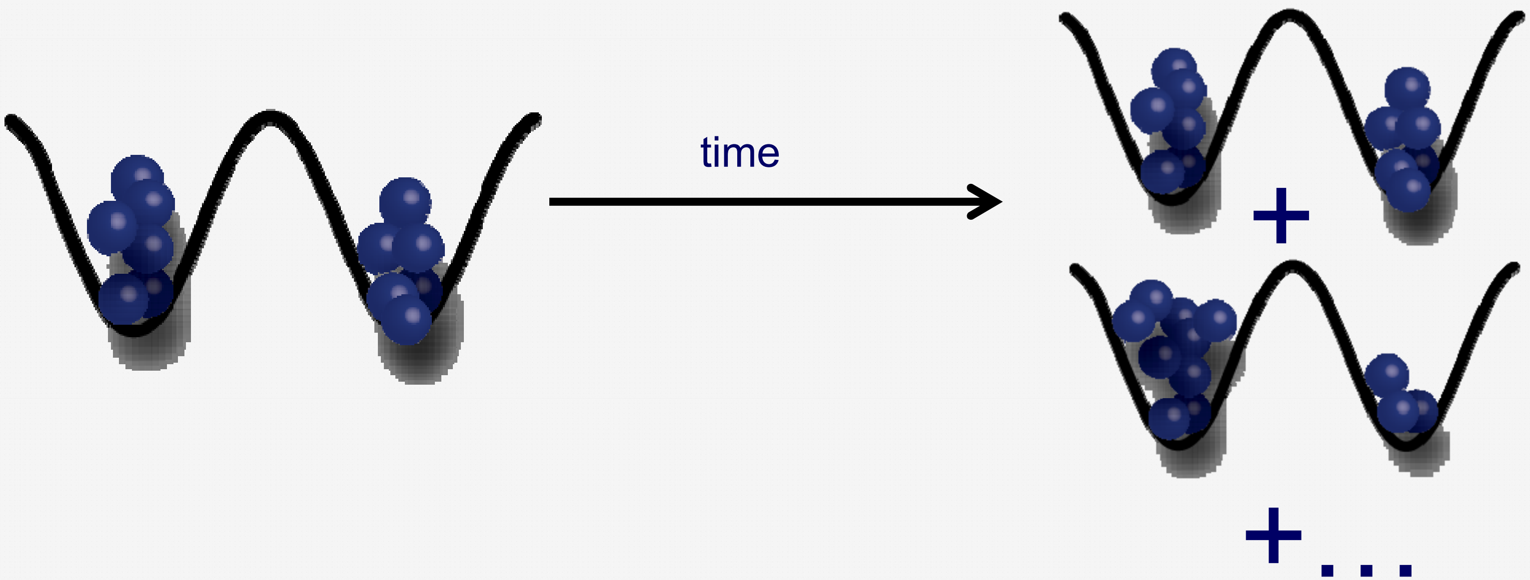




Diffusion of initial state (non-Brownian)



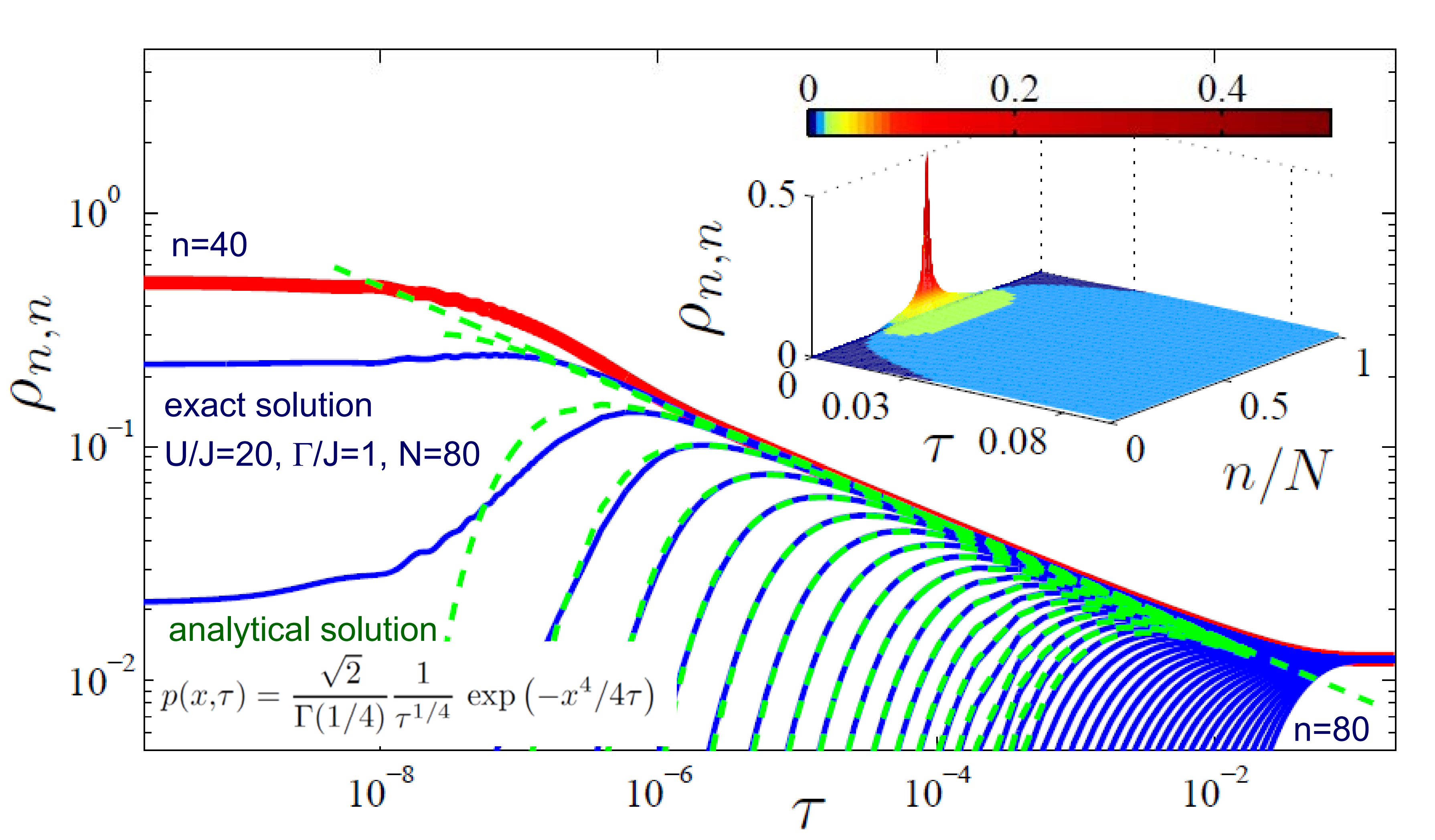
Slow evolution towards totally mixed state



ground state close to balanced

build up of strongly imbalanced states
energetically very costly

Diffusion of initial state (non-Brownian)



Scaling of coherence

Continuum limes: $\tilde{t} = t / N^2$

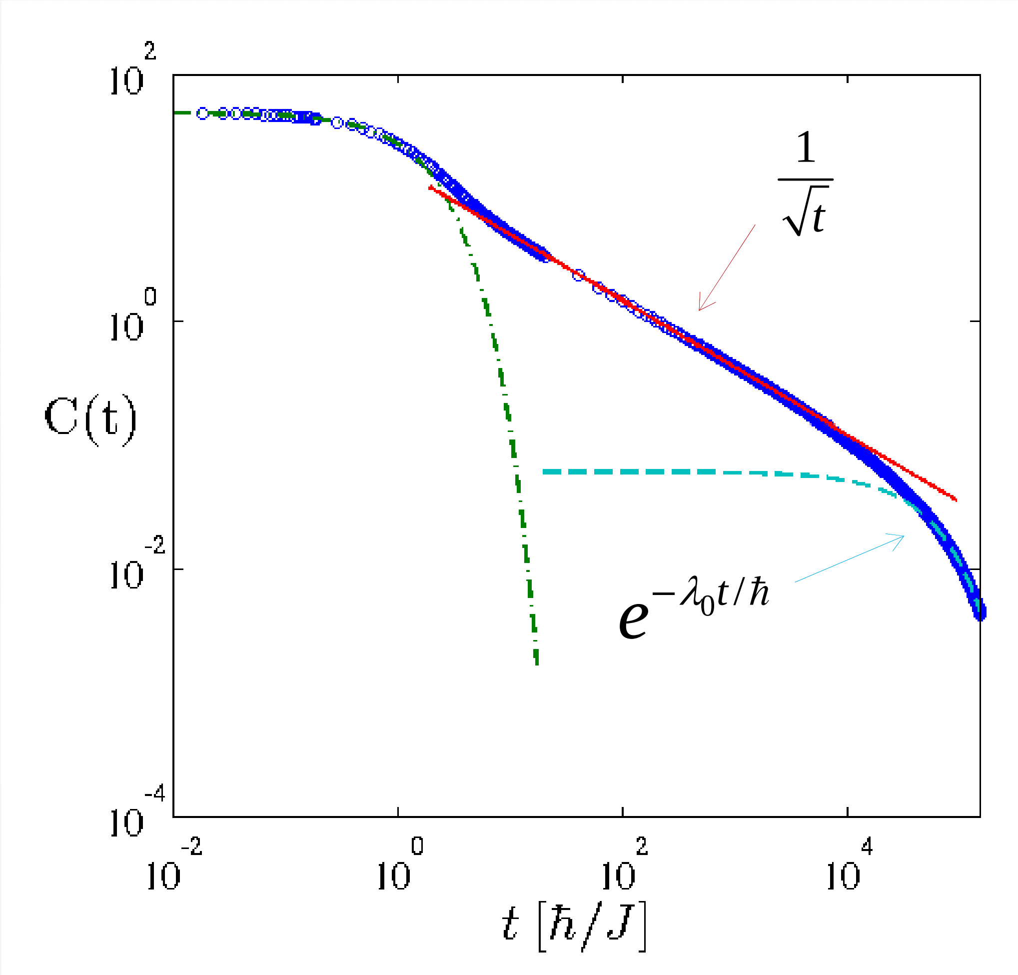
$$C(t) = f(t / N^2)$$

hand-waving:
C(t)/N should not depend on N

$$C(t) / N \sim \frac{1}{\sqrt{t}}$$

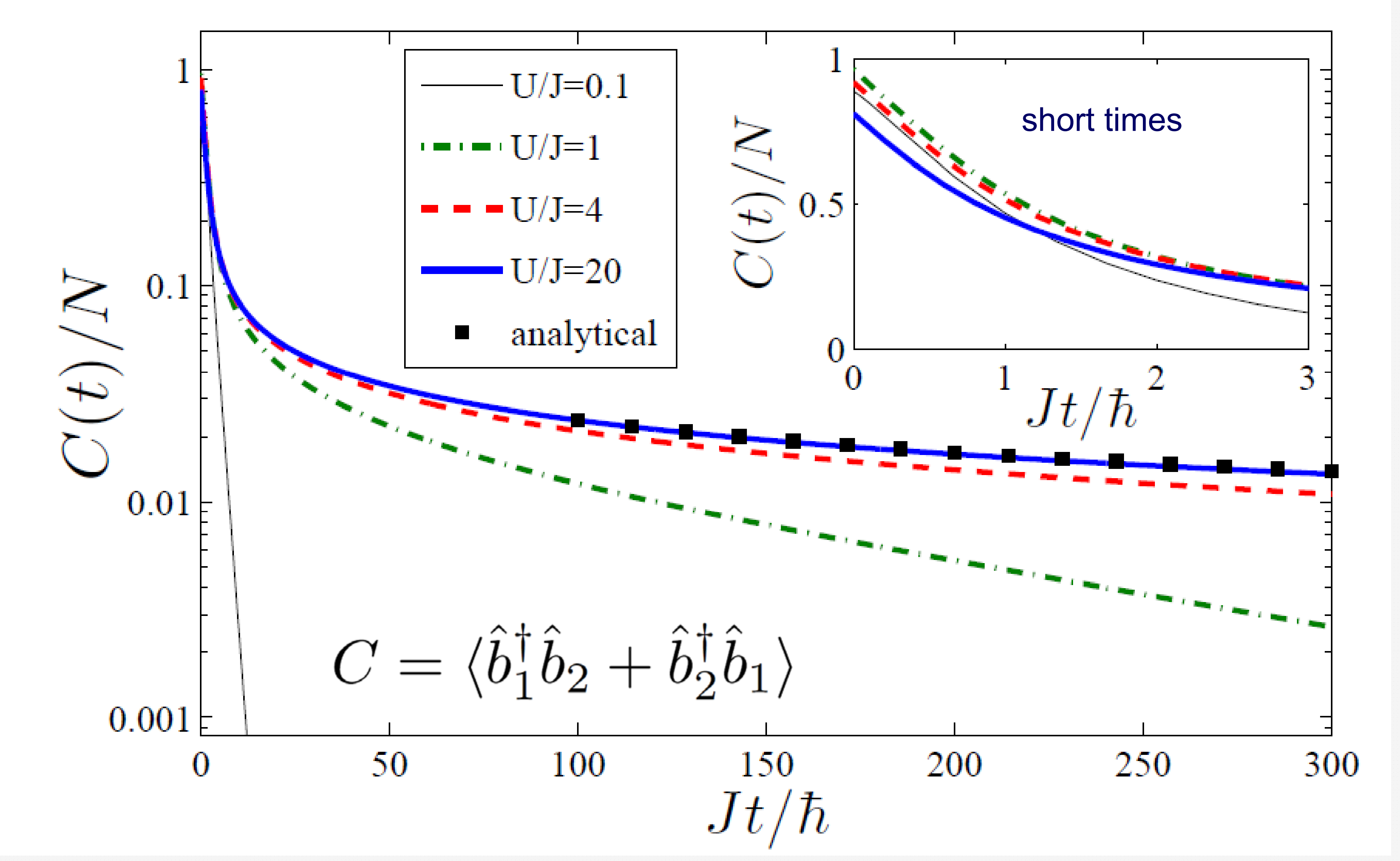
for large times $t >> \hbar / \lambda_0$
can feel discreteness

$$C(t) / N \propto e^{-\lambda_0 t / \hbar}$$



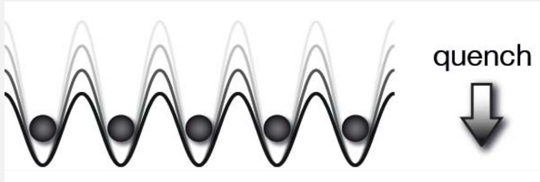
Interaction saves coherence

Γ/J=1, N=60



Fundamental questions in interacting systems

spreading of correlations

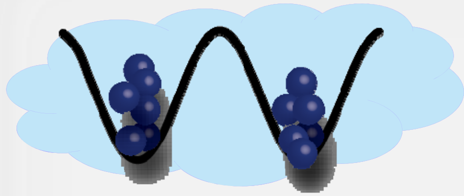


- light-cone like dynamics
- well explained by quasi-particle picture
- generic max. velocity in system with bounded spectra

open questions:

- faster quantum channels?
- implications for preparation of complex states in cold gases?
- approach for higher dimensional systems?

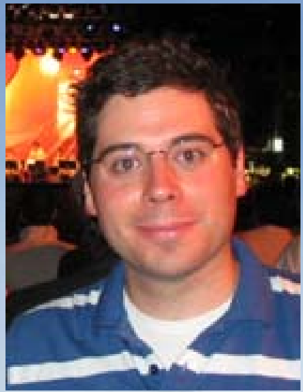
interplay: dissipation and interaction



- 3 regimes for decay of coherence
- slow algebraic decay due to interaction blocking
- slow diffusive states in configuration space

open questions:

- can interaction be used to prevent dissipation?
- implications for heating of complex states in cold gases?



Jean-Sebastien Bernier
(CIFAR Junior fellow
at UBC)

Peter Barmettler

Dario Poletti
-> faculty position at
Singapore University
of Technology and Design

Experimental groups:

M. Chenau (MPQ Munich)
M. Endres,
T. Fukuhara,
C. Weitenberg,
P. Schauß,
C. Gross,
I. Bloch (MPQ Munich, LMU Munich)
S. Kuhr (Strathclyde, Glasgow)

Theory groups:

A. Georges (Ecole Polytechnique)
A. Laeuchli (MPI Dresden)
E. Altman (Weizman Institute)
G. Biroli (CEA Saclay)

….

…