Quantum Quench in Conformal Field Theory from a General Short-Ranged State

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(Global) Quantum Quench

- prepare an extended system at time $t = 0$ in a (translationally invariant) pure state $|\psi_0\rangle$ – e.g. the ground state of some hamiltonian H_0
- evolve unitarily with a hamiltonian H for which $|\psi_0\rangle$ is not an eigenstate and has extensive energy above the ground state of H
- how do correlation functions and entanglement evolve as a function of t?
- for a compact subsystem do they become stationary?
- if so, what is the stationary state?
- is the reduced density matrix thermal?

Quantum quench in a 1+1-dimensional CFT

- P. Calabrese + JC [2006] studied this problem in 1+1 dimensions when $H = H_{\rm CFT}$ and $|\psi_0\rangle$ is a state with short-range correlations and entanglement
- H_{CFT} describes the low-energy, large-distance properties of many gapless 1d systems
- 1+1-dimensional CFT is exactly solvable

Results

- one-point functions in general decay towards their ground state values

$$\langle \Phi(x,t) \rangle \sim e^{-\pi \Delta_\Phi t/2\tau_0}$$

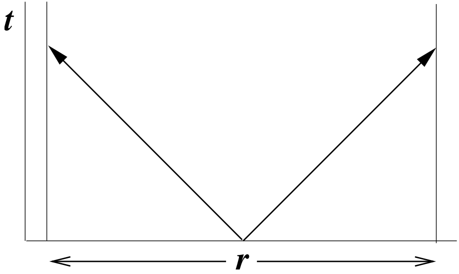
- for times $t > |x_1 - x_2|/2\mathrm{v}$, the correlation functions become stationary

$$\langle \Phi(x_1,t_1)\Phi(x_2,t_2) \rangle \sim e^{-\pi \Delta_\Phi |x_1 - x_2|/2\mathrm{v}\tau_0}$$

  for $t_1 = t_2$ and $\sim e^{-\pi \Delta_\Phi |t_1 - t_2|/2\tau_0}$ for $x_1 = x_2$
- the (conserved) energy density is $\pi c/6(4\tau_0)^2$
- the von Neumann entropy of a region of length ℓ saturates for $t > \ell/2\mathrm{v}$ at

$$S \sim (\pi c/3(4\tau_0))\ell$$

- all these results are precisely those expected for the CFT at temperature $T = (4\tau_0)^{-1}$
- they accord with a simple physical picture of entangled pairs of quasiparticles emitted from correlated regions



Quantum quenches in integrable models

- however studies of quenches in integrable models [(Rigol,Dunjko,Yurovsky,Olshanii),...,(Calabrese,Essler,Fagotti)] have led to the conclusion that the steady state should be a 'generalised Gibbs ensemble' (GGE) with a separate 'temperature' conjugate to each local conserved quantity
- 1+1-dimensional CFT is super-integrable: e.g. all powers $T(z)^p$ and $\overline{T}(\bar{z})^{\bar{p}}$ of the stress tensor correspond to local conserved currents, leading to conserved charges
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- so why did CC find a simple Gibbs ensemble?
- this can be traced to a simplifying assumption about the form of the initial state
- what is the effect of relaxing this assumption?

Review of CC [2006,2007]

- we want to compute

$$\langle\psi_0|e^{itH_{\text{CFT}}}\,\mathcal{O}\,e^{-itH_{\text{CFT}}}|\psi_0\rangle$$

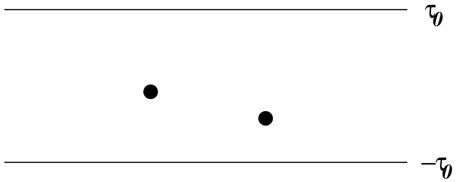
- we could get this from imaginary time by considering

$$\langle\psi_0|e^{-\tau_2 H_{\text{CFT}}}\,\mathcal{O}\,e^{-\tau_1 H_{\text{CFT}}}|\psi_0\rangle$$

  and continuing $\tau_1 \to it$, $\tau_2 \to -it$
- ‘slab’ geometry with boundary condition $\equiv \psi_0$, but thickness $\tau_1 + \tau_2 = 0$ ☹

Resolution: ‘Moving the goalposts’

- resolution: replace boundary condition at $\tau = \pm 0$ by ‘idealised’ bc at $\tau = \pm\tau_0$
- idea of ‘extrapolation length’ in boundary critical behaviour: idealised bc $\equiv$ boundary RG fixed point



- we then need to compute

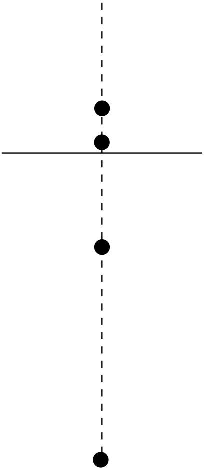
$$\langle \mathcal{O}(\tau) \rangle_{\text{slab}}$$

  and continue the result to $\tau \to \tau_0 + it$

- in CFT, the correlations in the slab are related to those in the upper half z-plane by $z = e^{\pi w/2\tau_0}$
- power-law behaviour in the z-plane $\Rightarrow$ exponential behaviour in t and x

- in particular, $x + i(\tau_0 + it)$ is mapped to

$$z = i\, e^{\pi(x-t)/2\tau_0} \qquad \bar{z} = -i\, e^{\pi(x+t)/2\tau_0} \neq z^*\,(!)$$



- except for narrow regions $O(\tau_0)$ near the light cone, points are exponentially ordered along imaginary z-axis: correlators can be computed by successive OPEs
- for $t \to \infty$ the $\bar{z}$'s move off to $-i\infty$ and the boundary effectively plays no role $\Rightarrow$ we have periodicity in $w \to w + 4i\tau_0$: finite temperature!

Relaxing CC's assumption

- CC's prescription is equivalent to assuming

$$|\psi_0\rangle \propto e^{-\tau_0 H_{\rm CFT}}|B\rangle$$

  where $|B\rangle$ is a conformally invariant boundary state
- in general we expect any translationally invariant state sufficiently close to $|B\rangle$ to have the form

$$|\psi_0\rangle \propto e^{-\sum_j \lambda_j \int \phi_j^{(b)}(x)dx}|B\rangle$$

  where $\phi_j^{(b)}$ are all possible irrelevant boundary operators
- one of the most important is the stress tensor $T_{\tau\tau}$ with RG eigenvalue $1-2=-1$: note that $\int T_{\tau\tau}(x)dx = H_{\rm CFT}$, so CC's assumption is that this is the most important one: if it is the *only* one all the conclusions of CC follow *exactly*
- a similar argument has been made in explaining the entanglement spectrum of quantum Hall states [Dubail,Read,Rezayi]

- so let us suppose

$$|\psi_0\rangle \propto e^{-\tau_0 H_{\rm CFT}}\, e^{-\sum_j{}' \lambda_j \int \phi_j^{(b)}(x)dx}|B\rangle \qquad \text{where } \Delta_j > 1$$

- since the $\phi_j^{(b)}$ are irrelevant, we might expect to be able to do perturbation theory in the λ_j: in the ground state this would lead to corrections to scaling
- for most simple models the only operators $\phi^{(b)}$ which do not explicitly break the symmetry are descendants of the stress tensor, e.g. $T\overline{T}$
- as an example, first order correction to $\langle\Phi(\tau)\rangle_{\rm slab}$ is

$$-\lambda \int_{\rm boundary} \langle\Phi(\tau)T\overline{T}(x)\rangle_{\rm slab}\, dx$$

- this can be computed by mapping to the UHP

- after continuing $\tau \to \tau_0 + it$ we find a first-order correction

$$e^{-\pi\Delta_\Phi t/2\tau_0}\left(1 + \lambda\Delta_\Phi^2\tau_0^{-4}\,t + \cdots\right)$$

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- higher orders in λ exponentiate up to leading order, so we get an inverse relaxation time

$$\frac{\pi\Delta_\Phi}{2\tau_0} - \lambda\frac{\Delta_\Phi^2}{(2\tau_0)^4} + O(\lambda^2)$$

- note that effective temperature now depends on which operator Φ we measure!
- we get the same effective temperature shift in the spatial decay of $\langle\Phi(x_1,t)\Phi(x_2,t)\rangle$ for $2\mathrm{v}t > |x_1 - x_2| \gg \mathrm{v}\tau_0$

Is this a Generalised Gibbs Ensemble?

- in GGE an equal-time correlation function should have the form

$$\langle\Phi(x_1,t)\Phi(x_2,t)\rangle = \mathrm{tr}\left[e^{-\beta H}\,e^{-\sum_p \beta_p H_p}\,\Phi(x_1)\Phi(x_2)\right]$$

where $\{H, H_p\}$ are an infinite set of commuting conserved charges.

- in CFT a minimal set are $H_p = \int [:T(x,t)^p: + :\overline{T}(x,t)^p:]dx$ for $p = 2, 3, \ldots$
- in terms of Virasoro operators

$$H_p \propto \sum_{n_1+\cdots+n_p=0} :L_{n_1}L_{n_2}\cdots L_{n_p}: + \text{c.c.}$$

- the normal ordering implies that $n_1 \leq n_2 \leq \cdots \leq n_p$, so

$$H_p \propto L_0^p + \text{terms with } n_p \geq 1 + \text{c.c.}$$

so acting on a primary operator $H_p \propto \Delta_\Phi^p$

- so for a *primary* operator $\langle\Phi(x_1,t)\Phi(x_2,t)\rangle_{\text{GGE}} \sim e^{-|x_1-x_2|/\xi}$ where

$$\xi^{-1} = \frac{2\pi}{\beta}\Delta_\Phi - \sum_p \beta_p \left(\frac{2\pi\Delta_\Phi}{\beta^2}\right)^p$$

  Compare with result from a perturbed boundary state

$$\xi^{-1} = \frac{\pi\Delta_\Phi}{2\tau_0} - \lambda\frac{\Delta_\Phi^2}{(2\tau_0)^4} + O(\lambda^2)$$

- this has exactly the same form, with $\beta = 4\tau_0$ and $\beta_{2p} \propto \lambda^p$
- acting with other irrelevant descendants of T on the initial state gives similar results, all consistent with GGE ☺

More general boundary perturbations

- more general irrelevant boundary perturbations $\phi_j^{(b)}$ with scaling dimensions $\Delta_j \neq$ integer are consistent with a GGE only if we posit the existence of bulk *parafermionic* holomorphic currents $\phi_j(z)$ with dimension Δ_j and include the corresponding non-local conserved charges $H_j = \int \phi_j(x,t)dx$ in the GGE ?☹?
- the stationary state becomes more like pure Gibbs as $T_{\text{eff}} \downarrow 0$, i.e. a shallow quench

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- the stationary state becomes more like pure Gibbs as $T_{\text{eff}} \downarrow 0$, i.e. a shallow quench
- one can also add irrelevant terms like to H_{CFT} : e.g.
  - $T\overline{T}$, corresponding to left-right scattering
  - $T^p + \overline{T}^p$, corresponding to curvature of dispersion relation
- however perturbatively they don't appear to change the overall picture ?☺? ?☹?

Conclusions

- a quantum quench in 1+1-dimensional CFT from a more general state leads to results consistent with a GGE, so the conclusions of CC [2006] as predicting strict thermalisation should not be interpreted too literally!