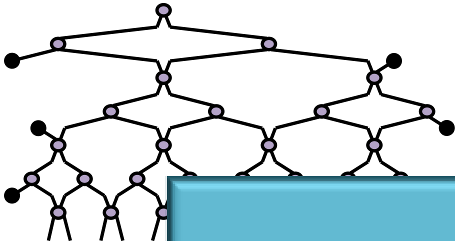
The **Galileo Galilei Institute** for Theoretical Physics Arcetri, Florence, May 2012

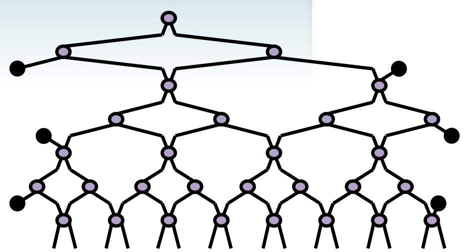
Workshop: "New states of matter in and out of equilibrium"

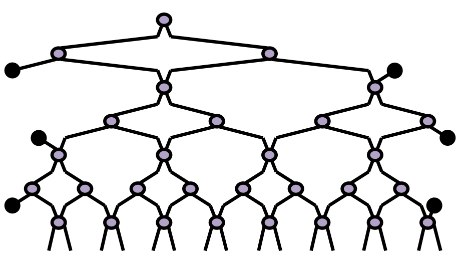


Tensor network states
that go beyond the boundary law
for entanglement entropy

Guifre Vidal, Perimeter Institute

collaboration with
Glen Evenbly, Caltech





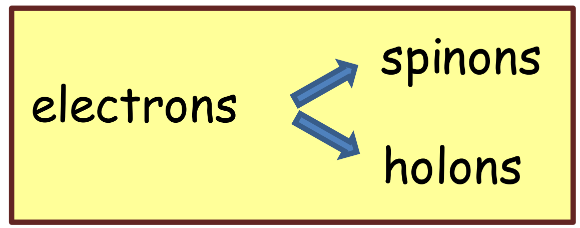
Evenbly, Vidal, arxiv1205.0639
Evenbly, Vidal, arxiv120x.yyyy

MOTIVATION:

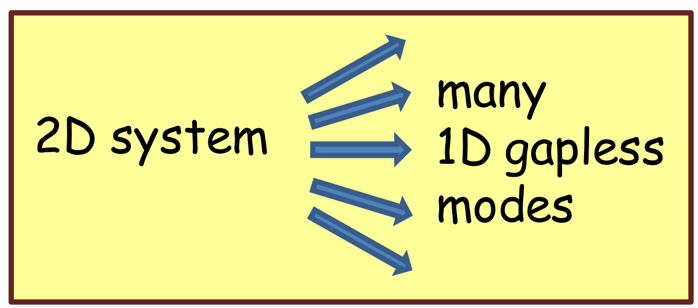
At low energies, "a many-body system may decouple into two (or several) sets of independent degrees of freedom"

Examples:

- 1D system: spin-charge separation



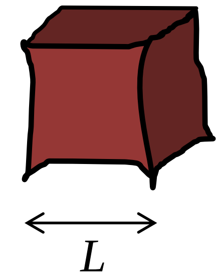
- 2D systems with 1D Fermi surface (or 1D Bose surface)
  - Fermi liquids
  - spin Bose metal



Haldane, Shankar, Swingle, Fisher, …

MOTIVATION:

Boundary law for entanglement entropy: $S_L \approx L^{D-1}$

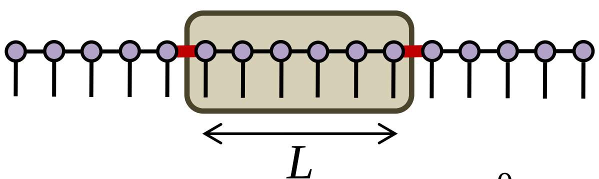


> Systems with a Fermi/Bose surface are among the **most entangled phases** of quantum matter:

- Logarithmic violation: $S_L \approx L^{D-1}\log(L)$
- Beyond reach of tensor network states in D>1 dimensions:

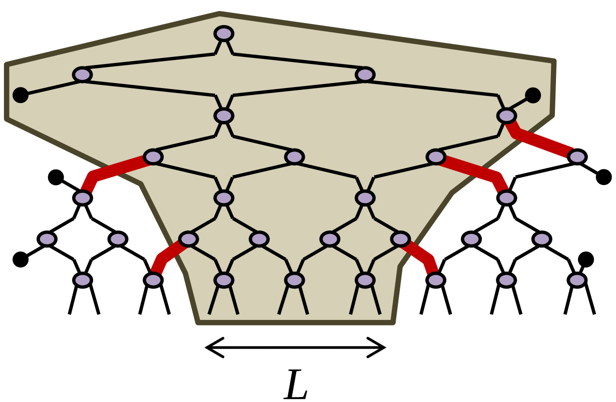
1D

MPS



$S_L \approx L^0 = \text{const.}$

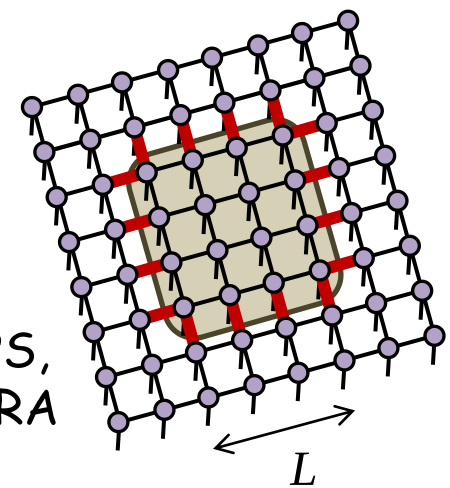
MERA



$S(L) \approx L^0 \log L = \log L$

D>1

PEPS, MERA



$S_L \approx L^{D-1}$

Outline

Glen Evenbly

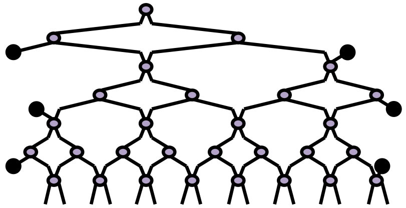
- Introduction

  Quantum circuits, simulatability and entanglement

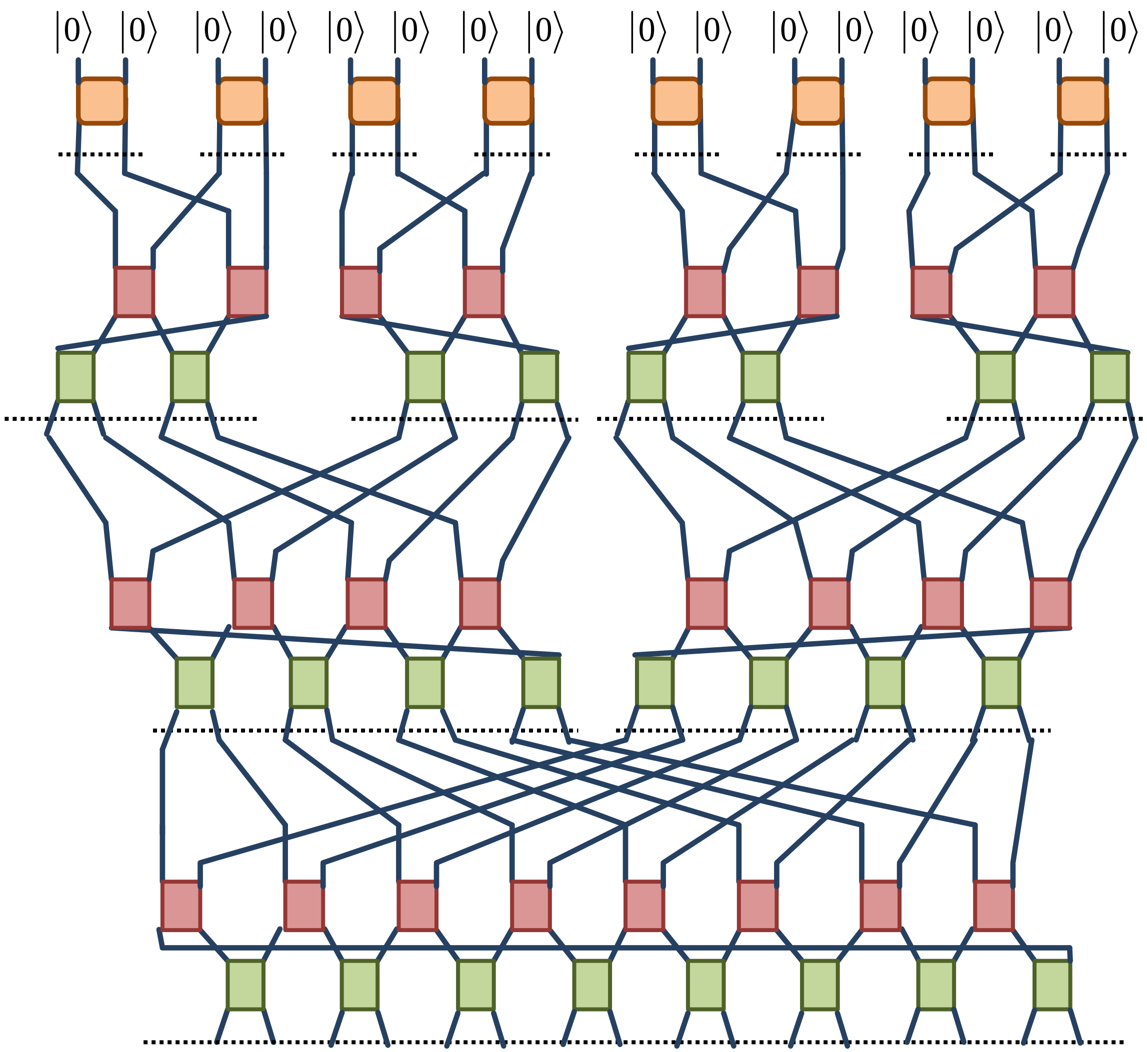
- MPS and TTN

- MERA

- branching MERA



branching MERA



$$S_L \approx L^{D-1}\log(L)$$

(actually, even $S_L \approx L^D$)

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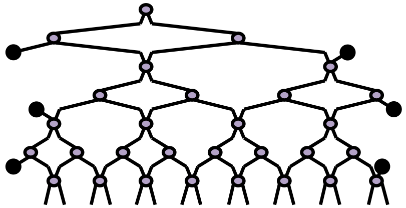
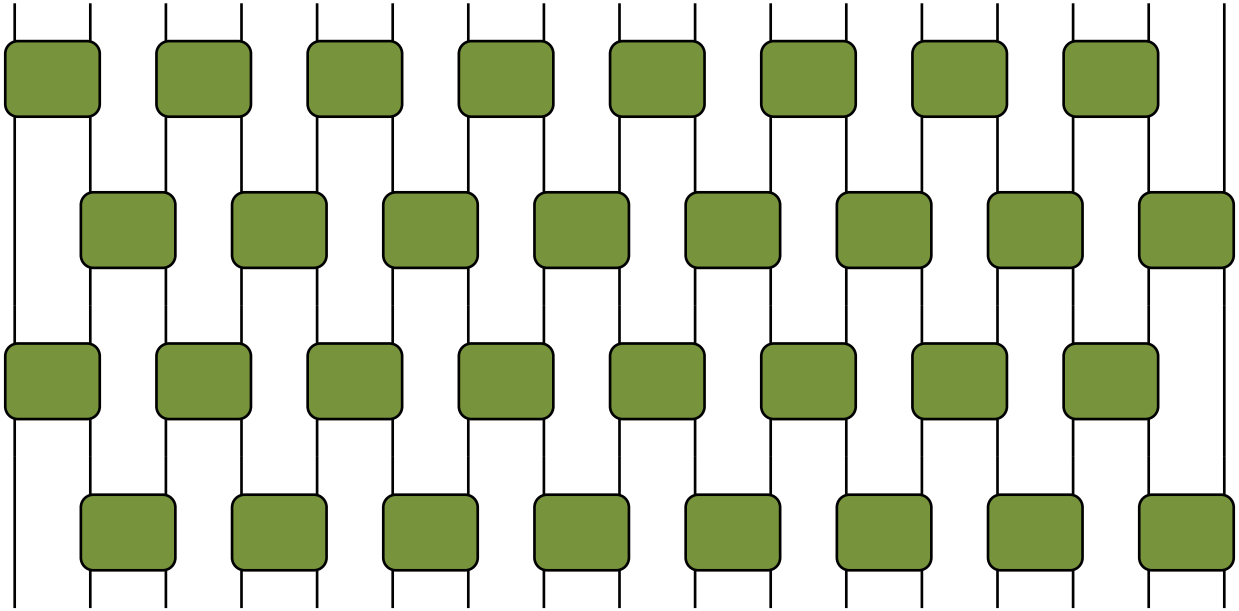
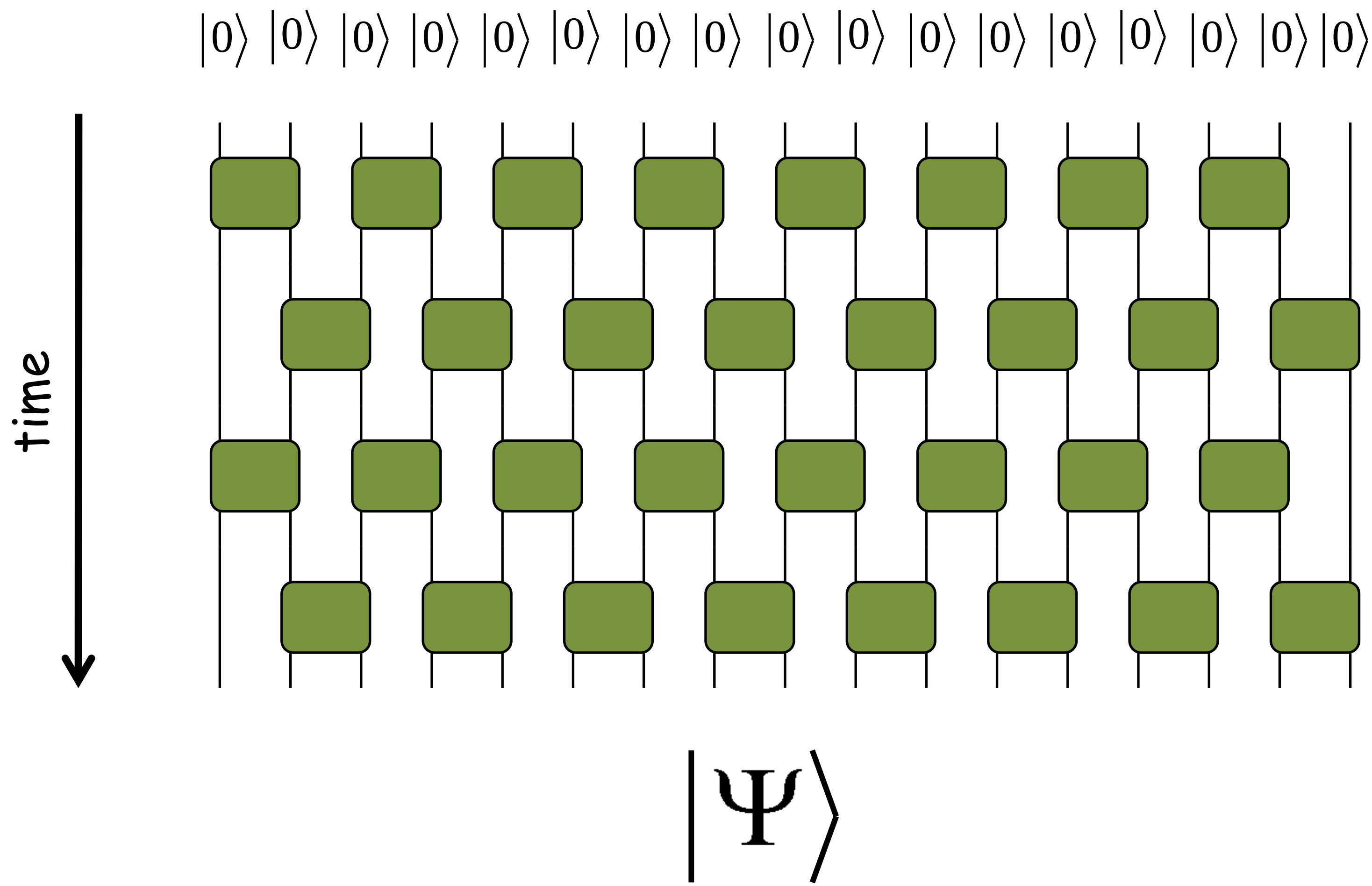


Quantum Circuit



Quantum Circuit

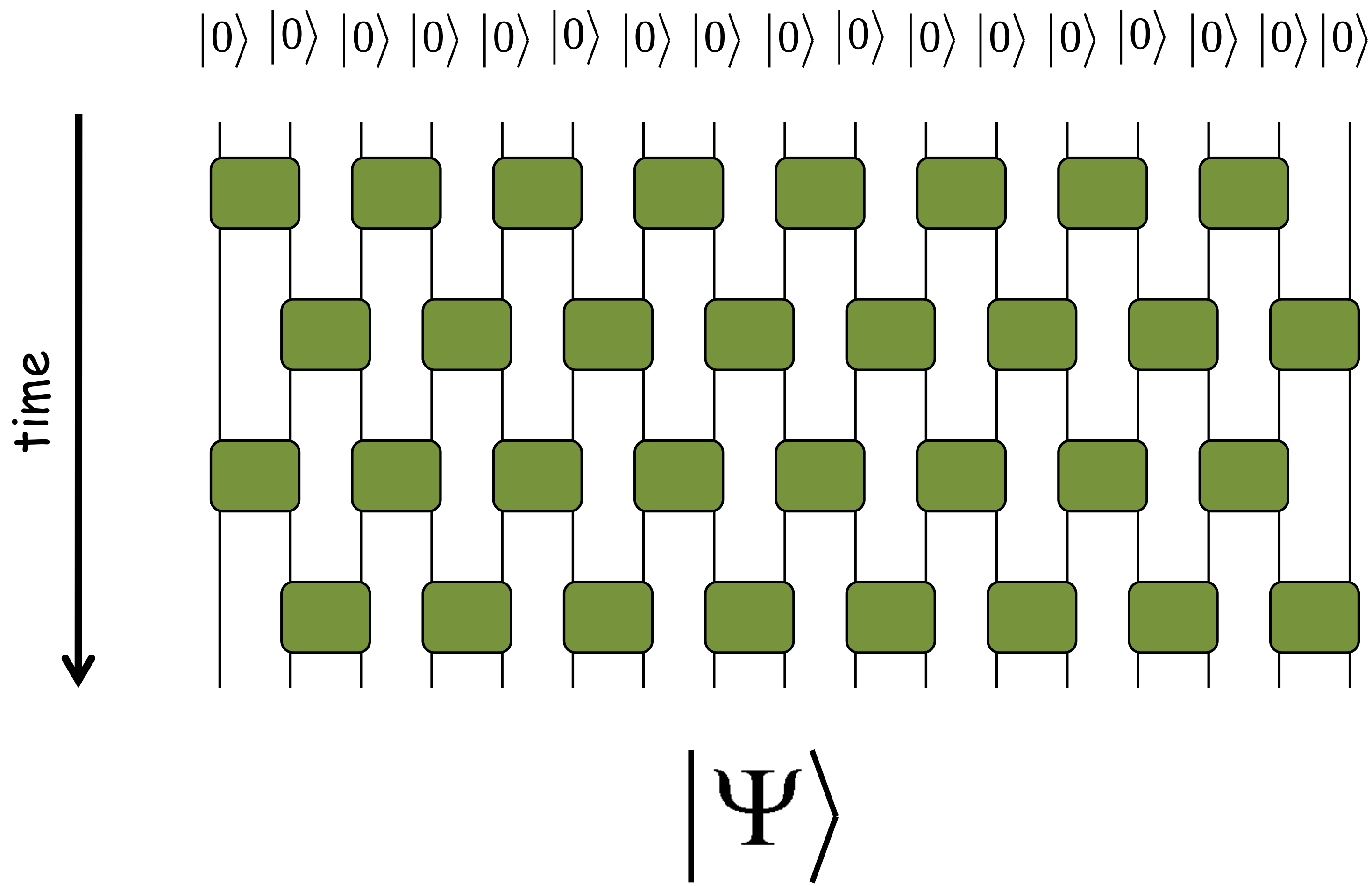
Can be used to *efficiently* encode many-body states:

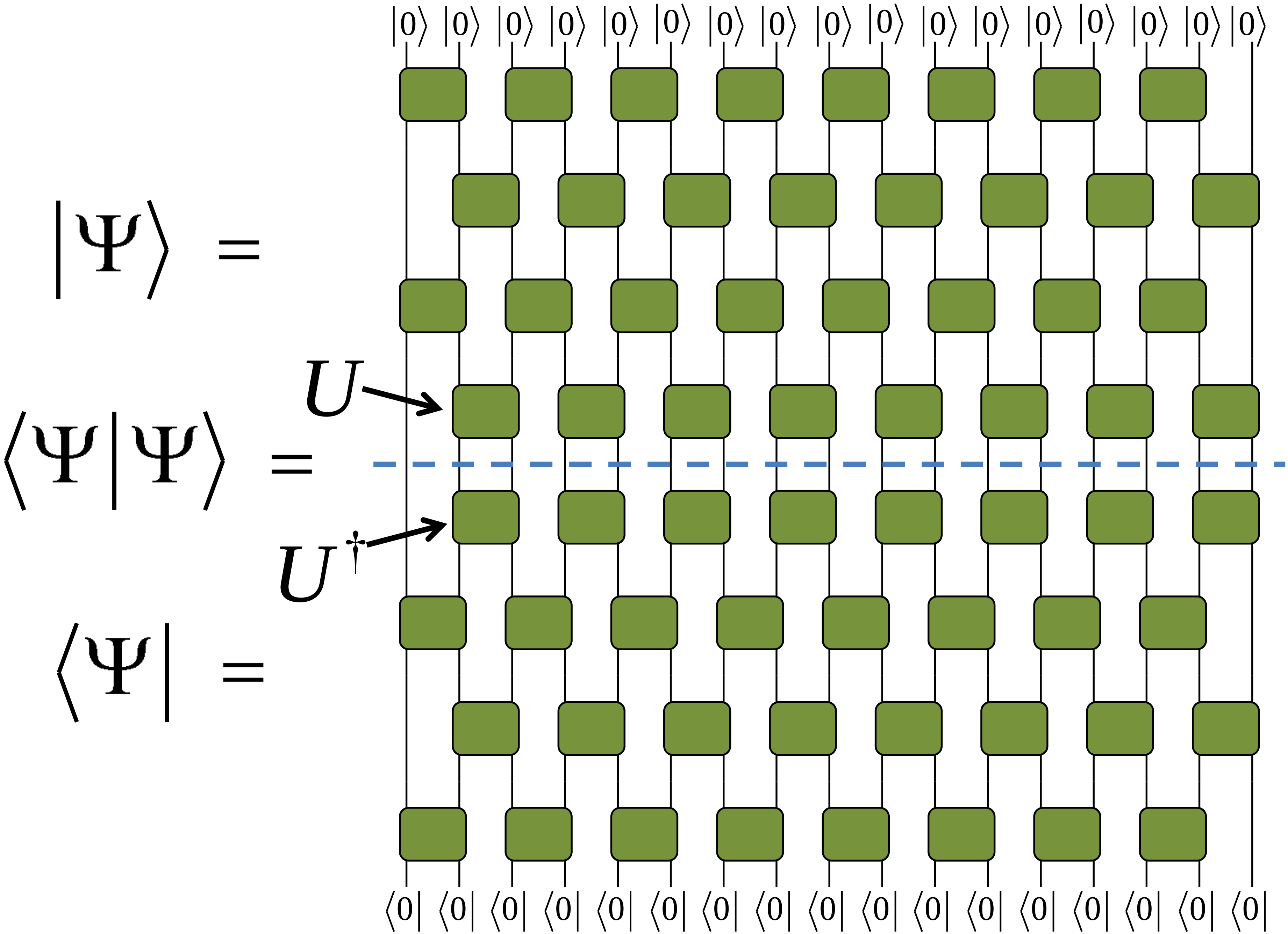


Quantum Circuit as a many-body variational ansatz

Questions:

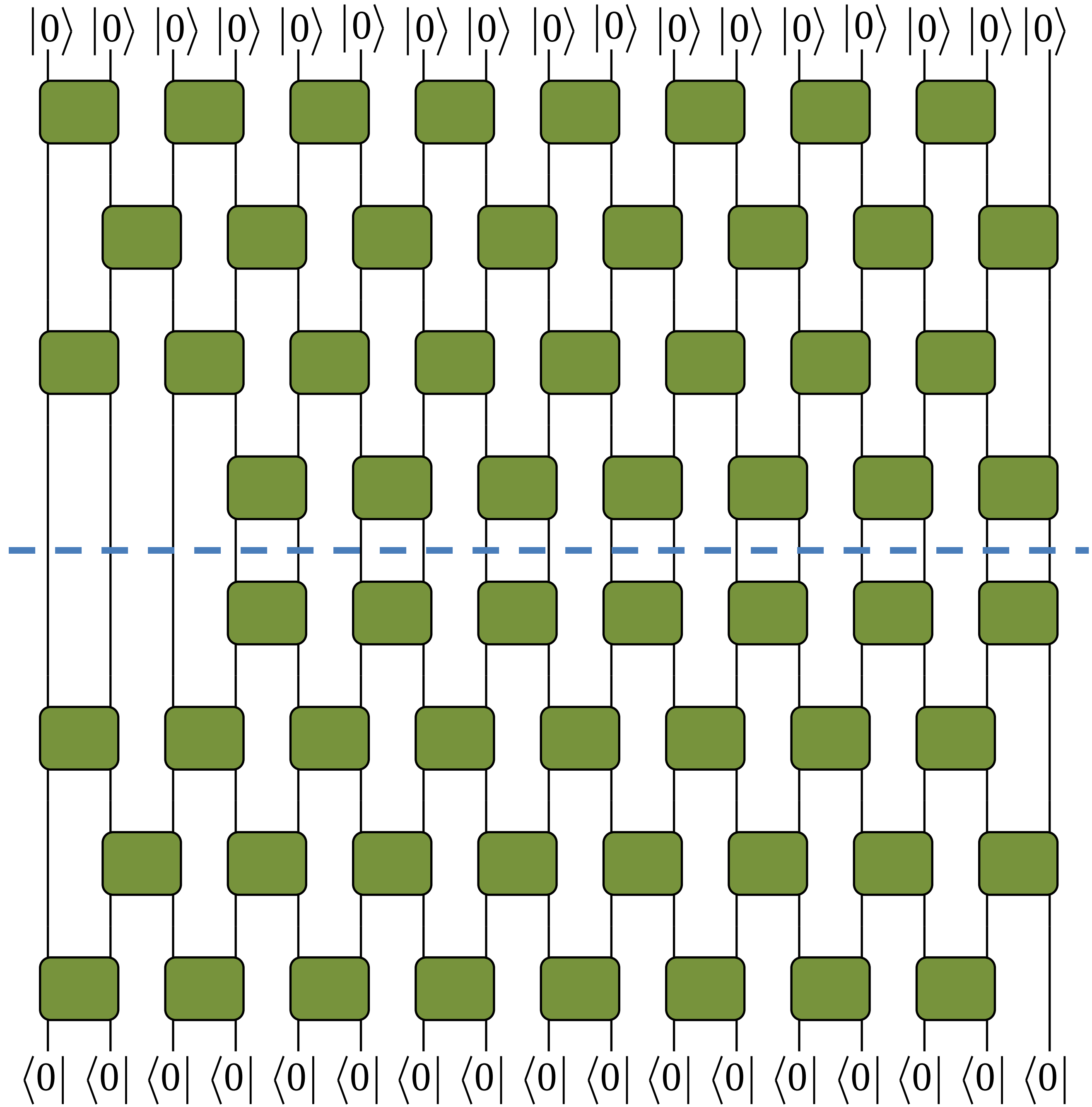
1) Cost of computing a local reduced density matrix
2) Entropy of a block of contiguous sites



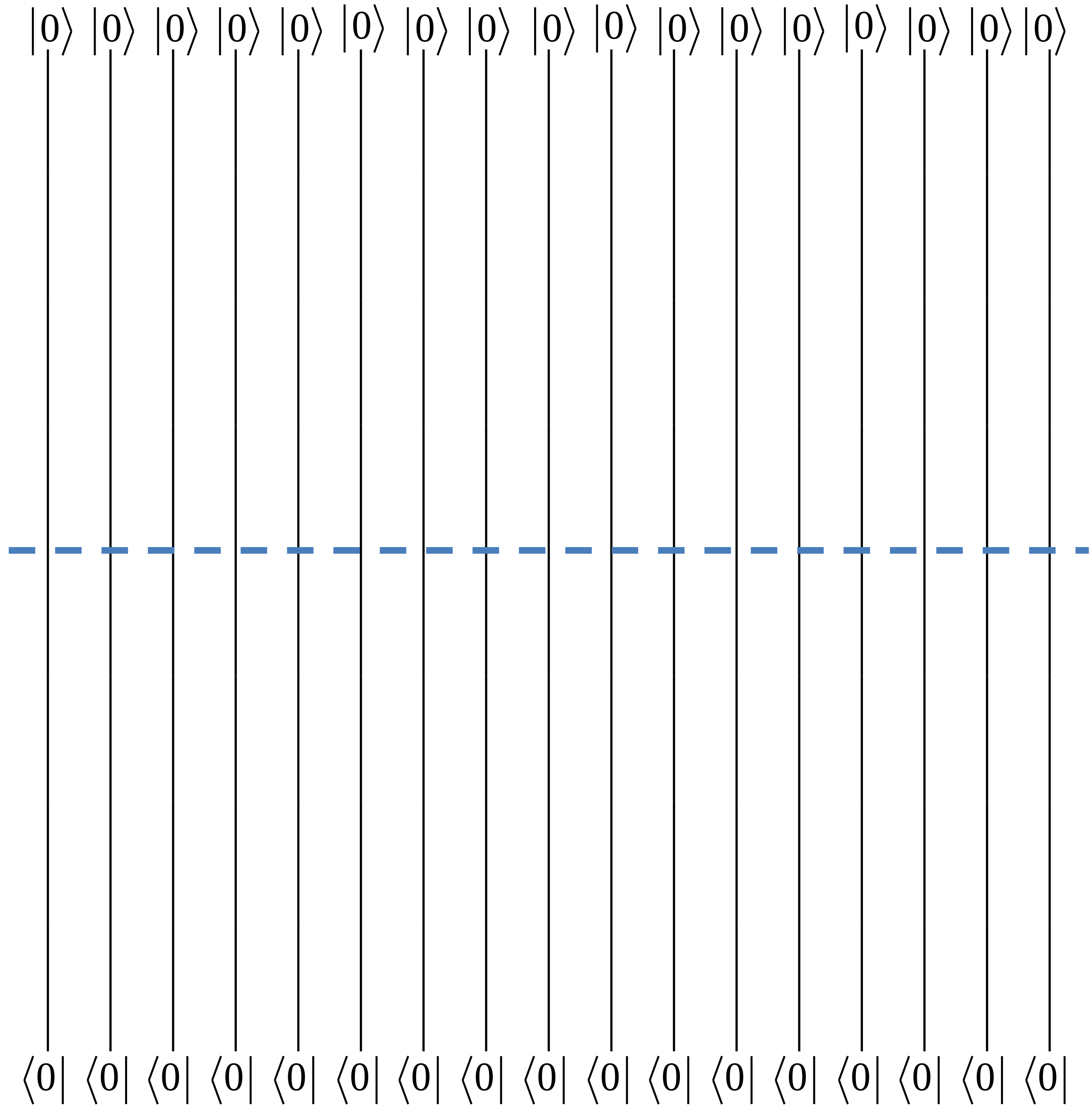


$|\Psi\rangle =$
$\langle\Psi|\Psi\rangle =$
$\langle\Psi| =$
U
$U^\dagger$
$|0\rangle$
$\langle 0|$

$$\langle \Psi | \Psi \rangle =$$

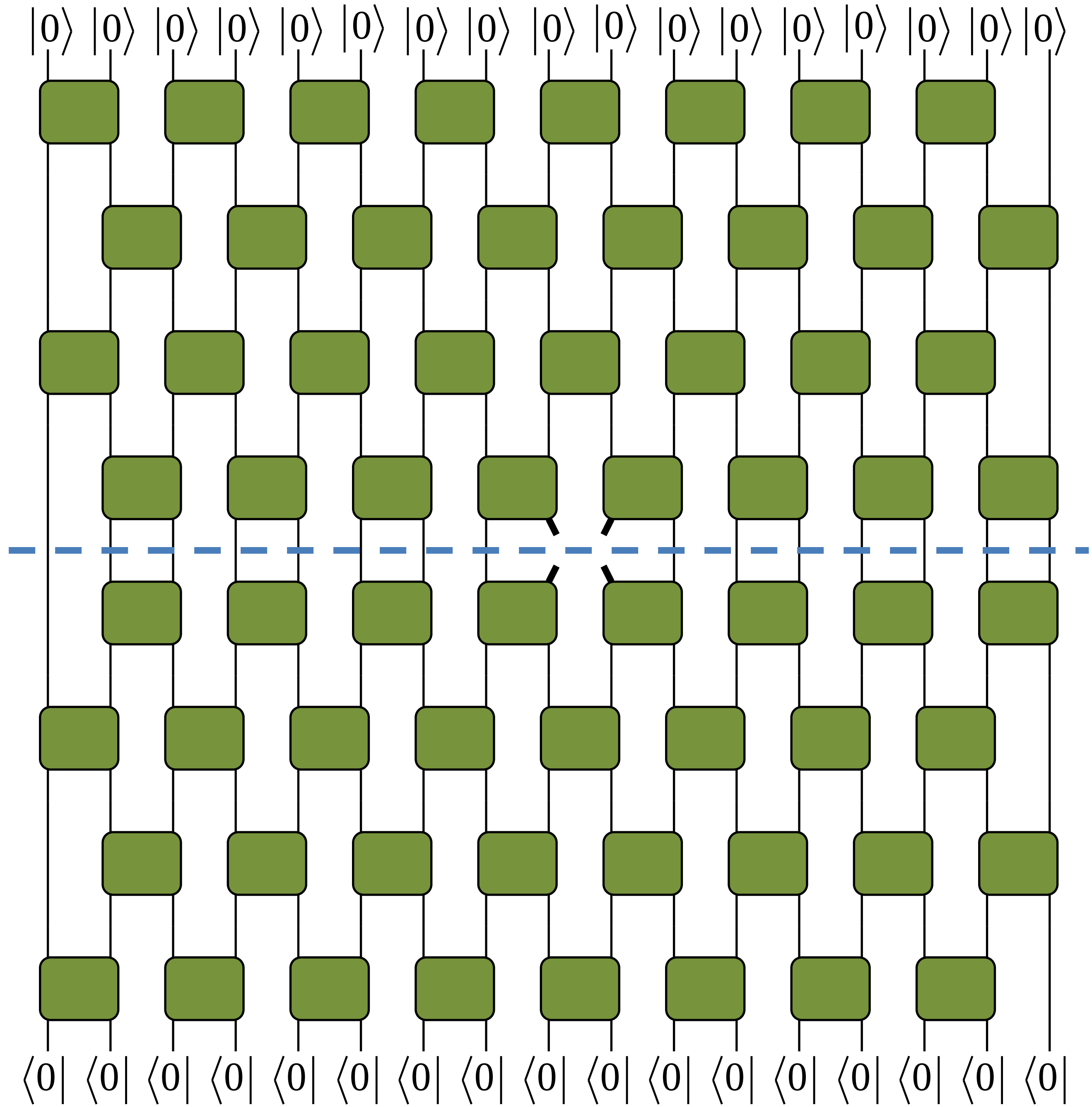


$$\langle \Psi | \Psi \rangle =$$



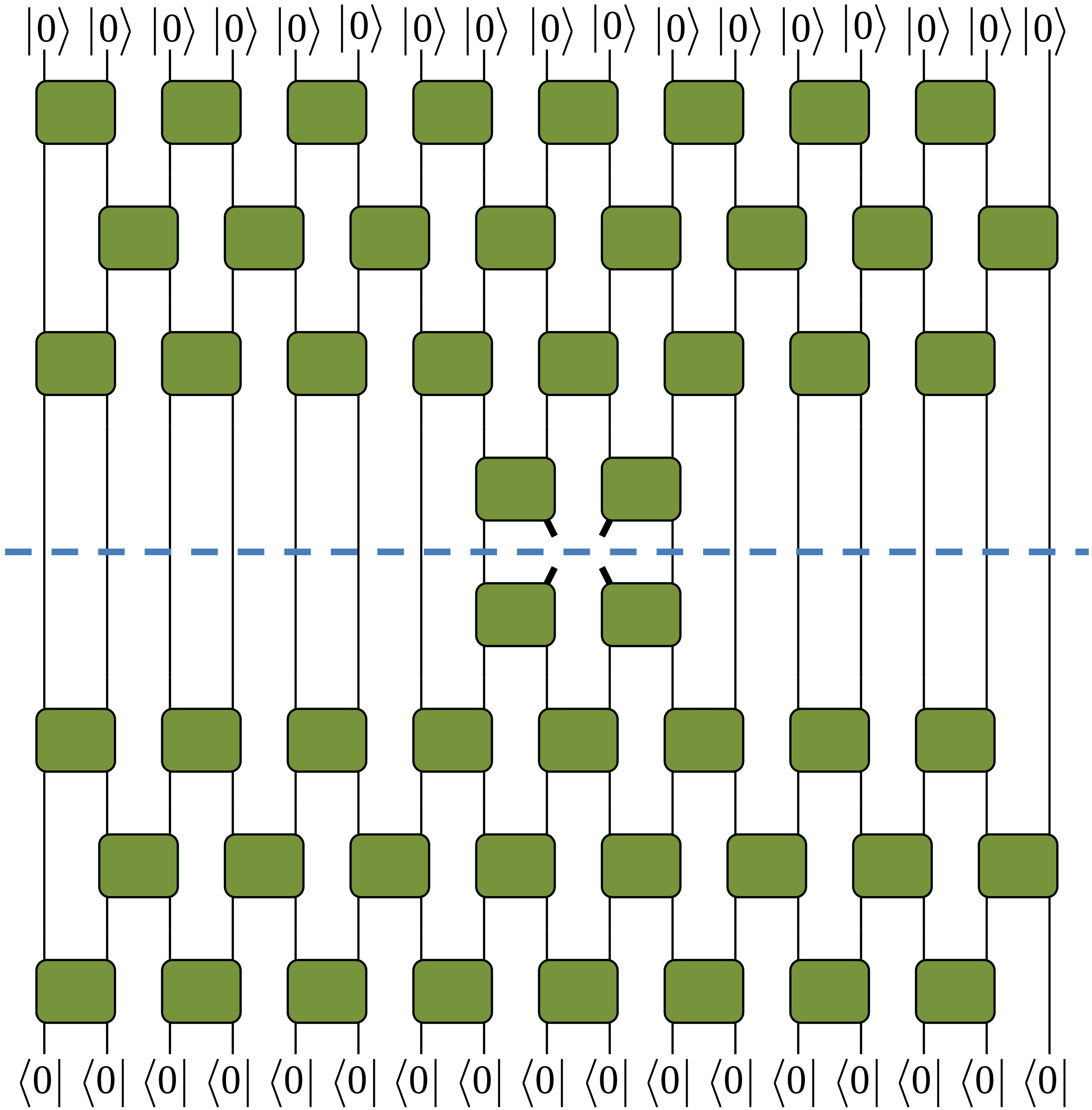
Cost of computing a local reduced density matrix

$$\rho(A) =$$



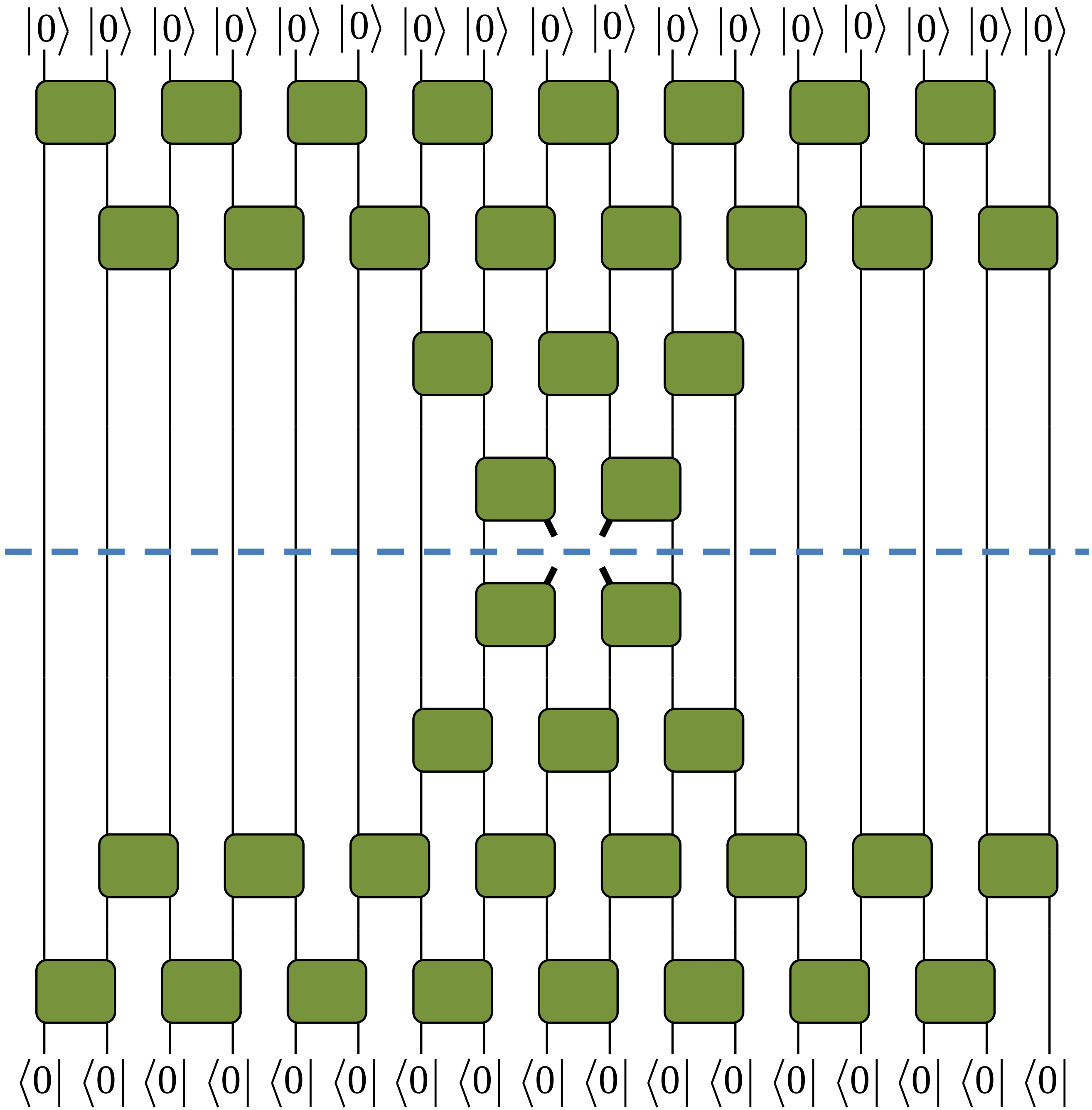
Cost of computing a local reduced density matrix

$$\rho(A) =$$



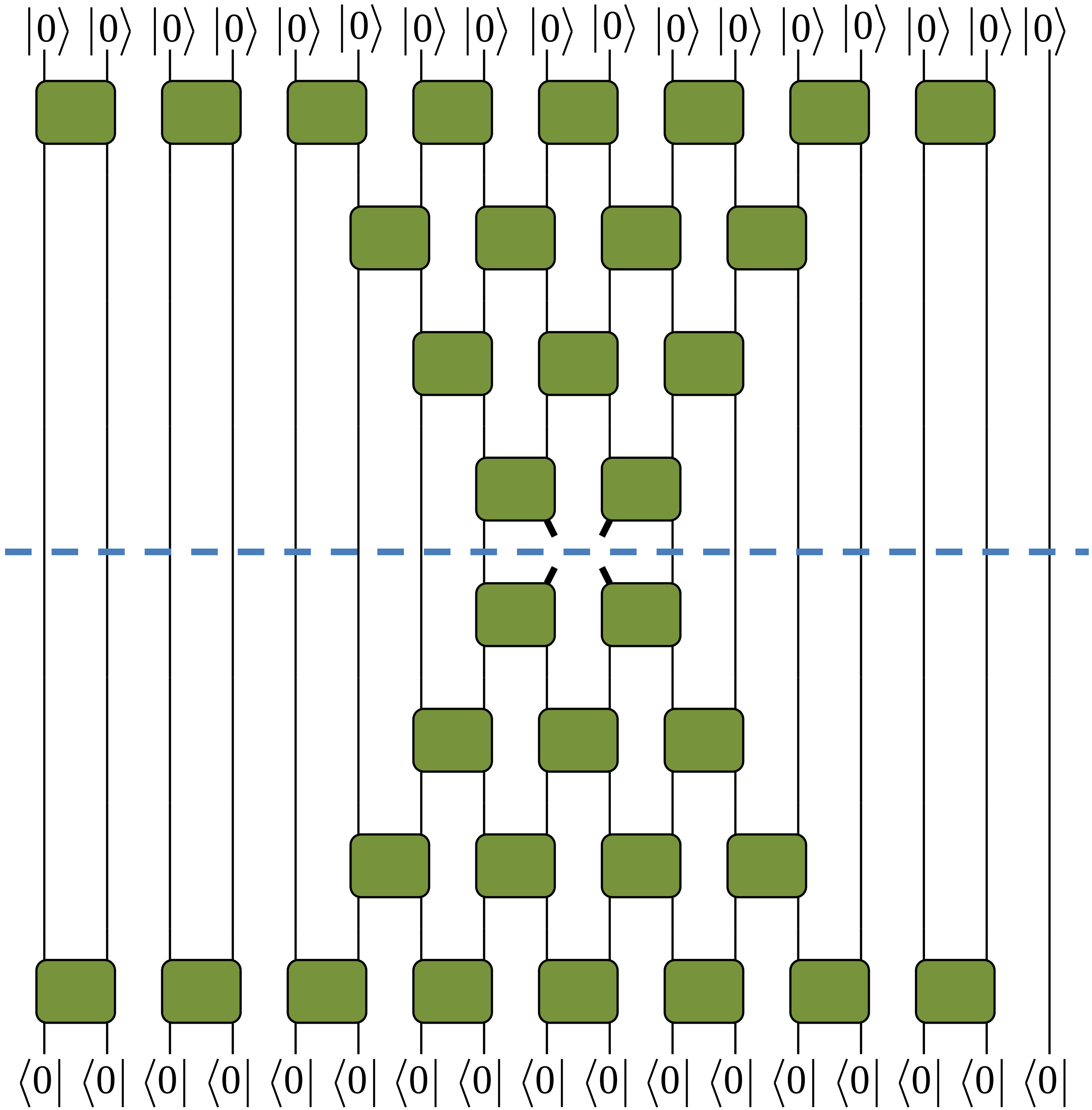
Cost of computing a local reduced density matrix

$$\rho(A) =$$



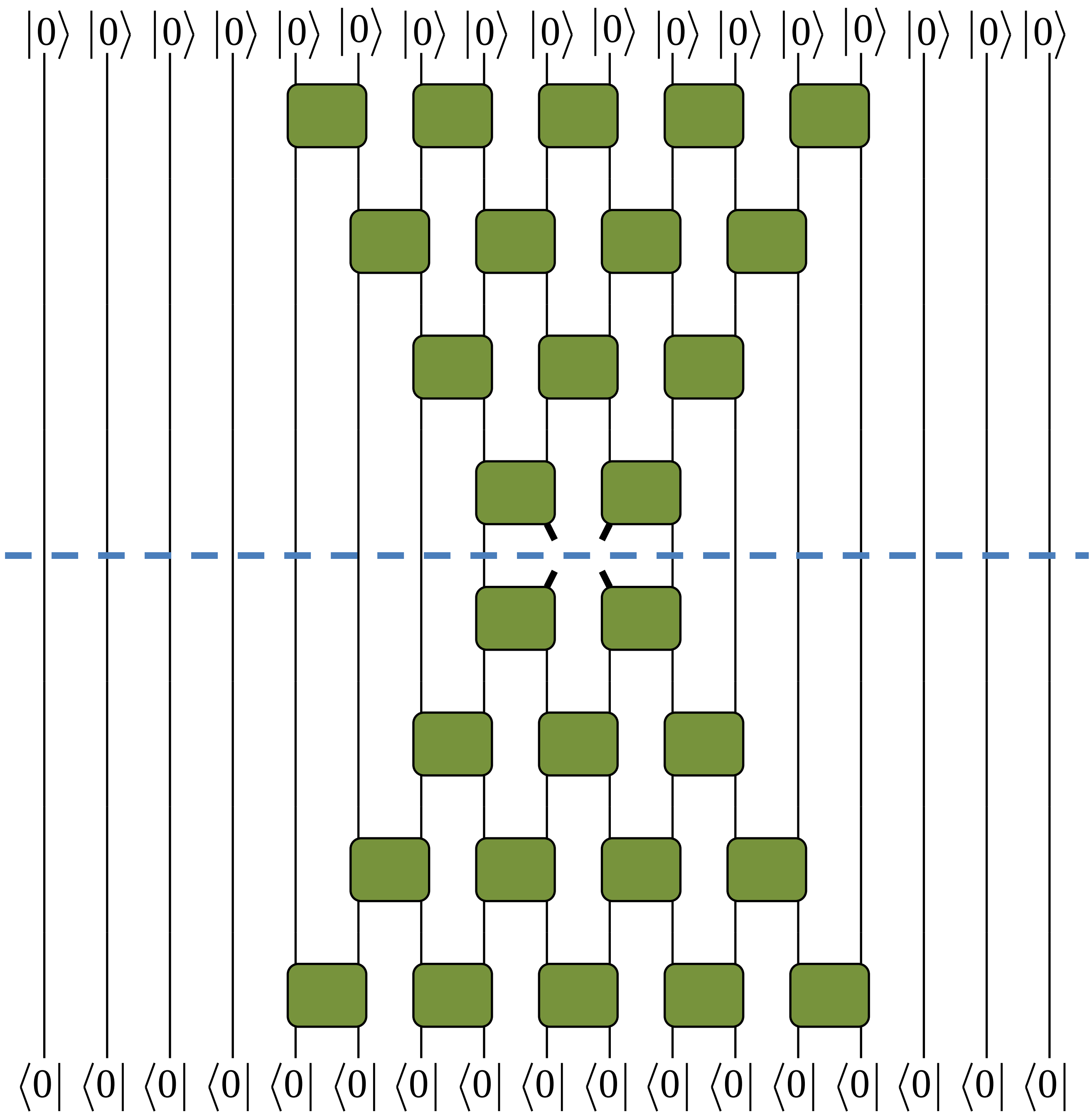
Cost of computing a local reduced density matrix

$$\rho(A) =$$



Cost of computing a local reduced density matrix

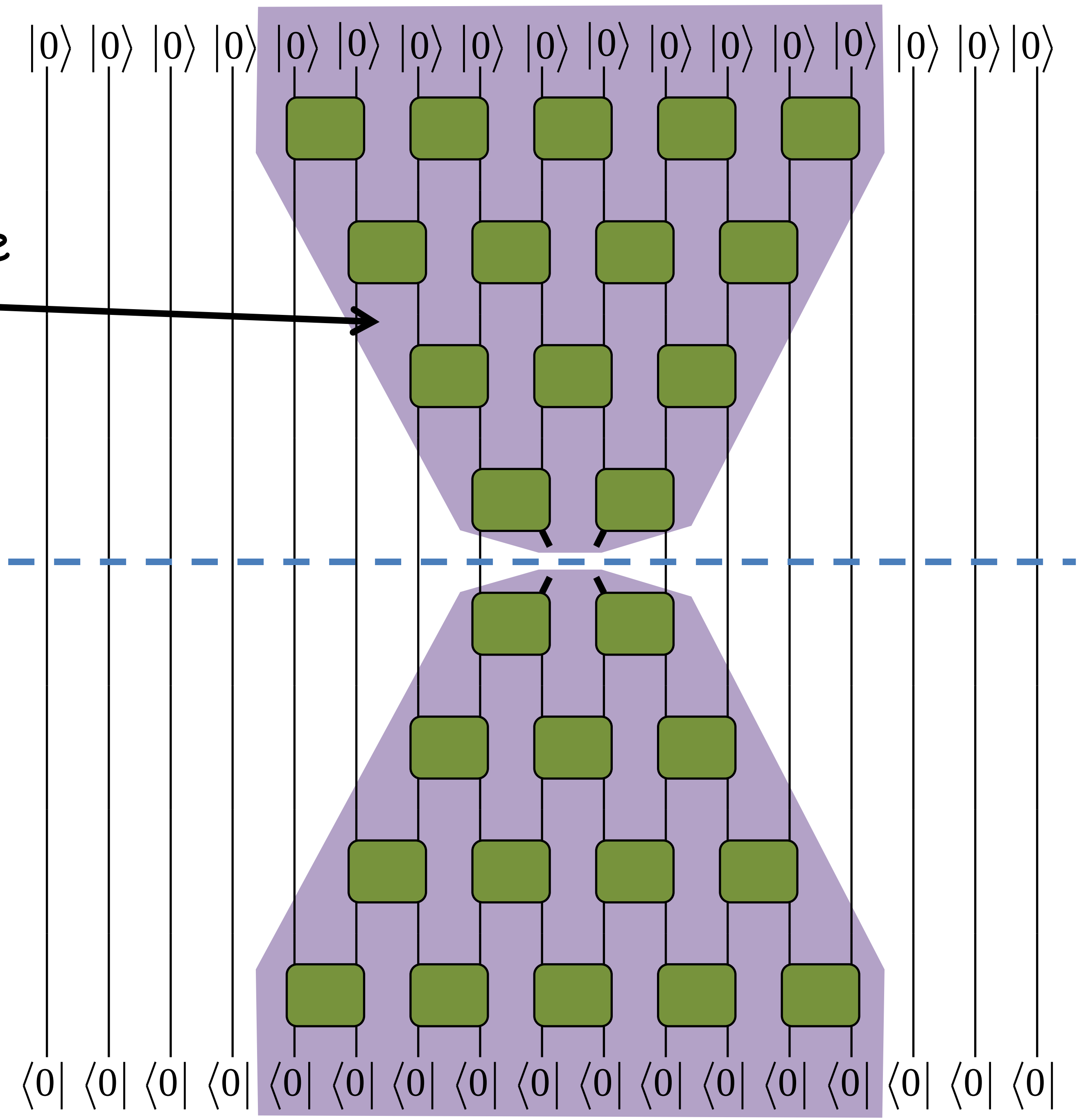
$$\rho(A) =$$

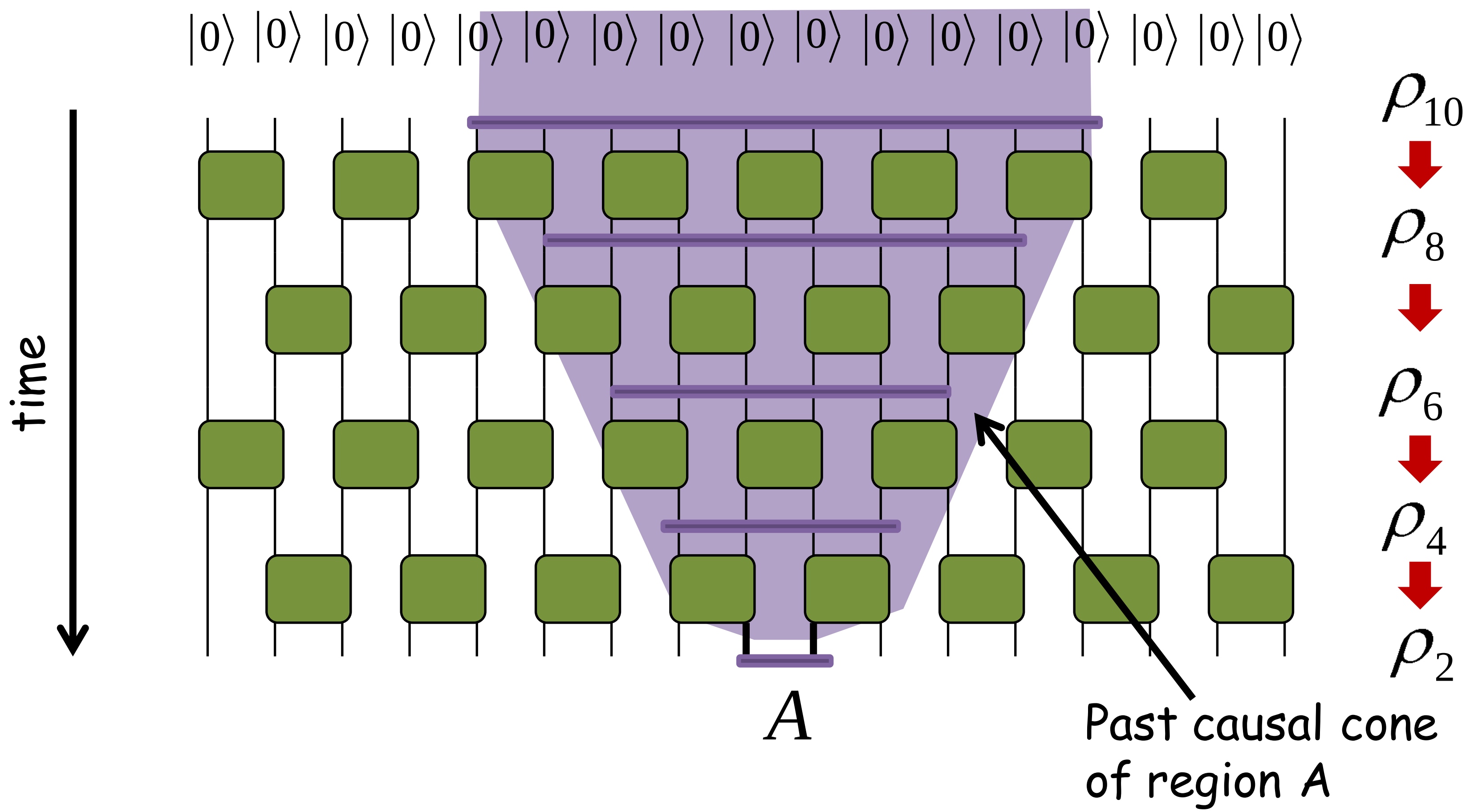


Cost of computing a local reduced density matrix

Past causal cone of region A

$$\rho(A) =$$

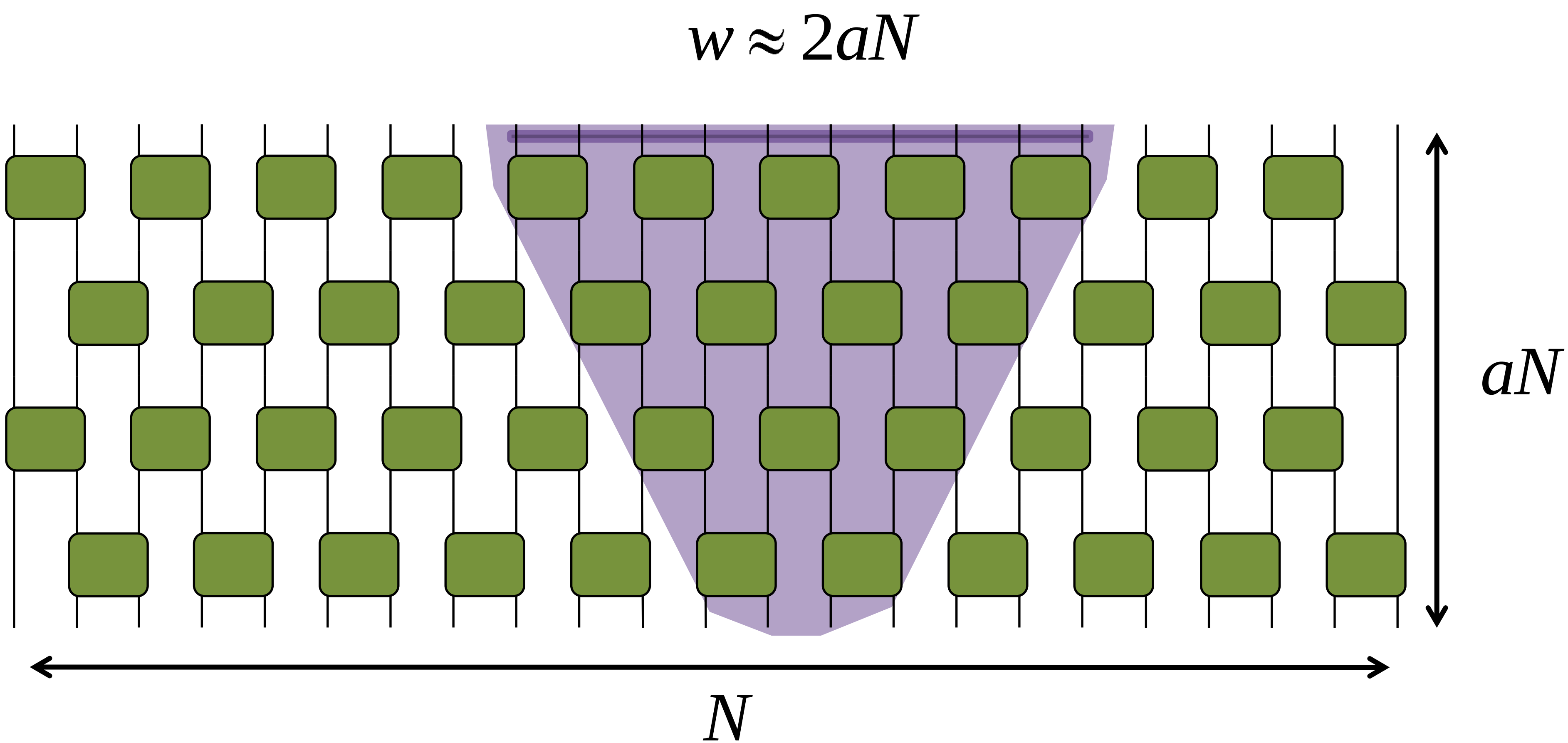




width of causal cone: $w(t)$ $\qquad$ $w \equiv \max_t w(t)$

cost of computing $\rho(A)$: $c \approx \exp(w)$

Example I:

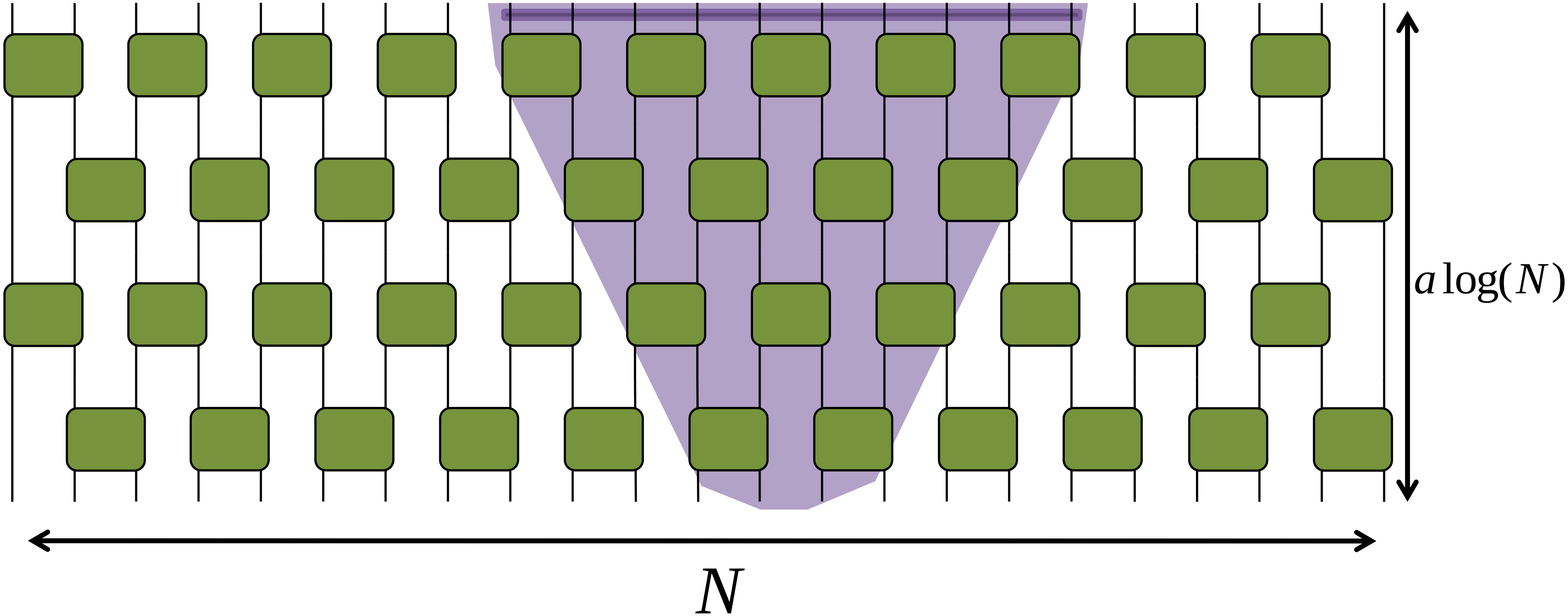


cost of computing $\rho(A)$: $c \approx \exp(2aN)$

inefficient

Example II:

$$w \approx 2a \log(N)$$

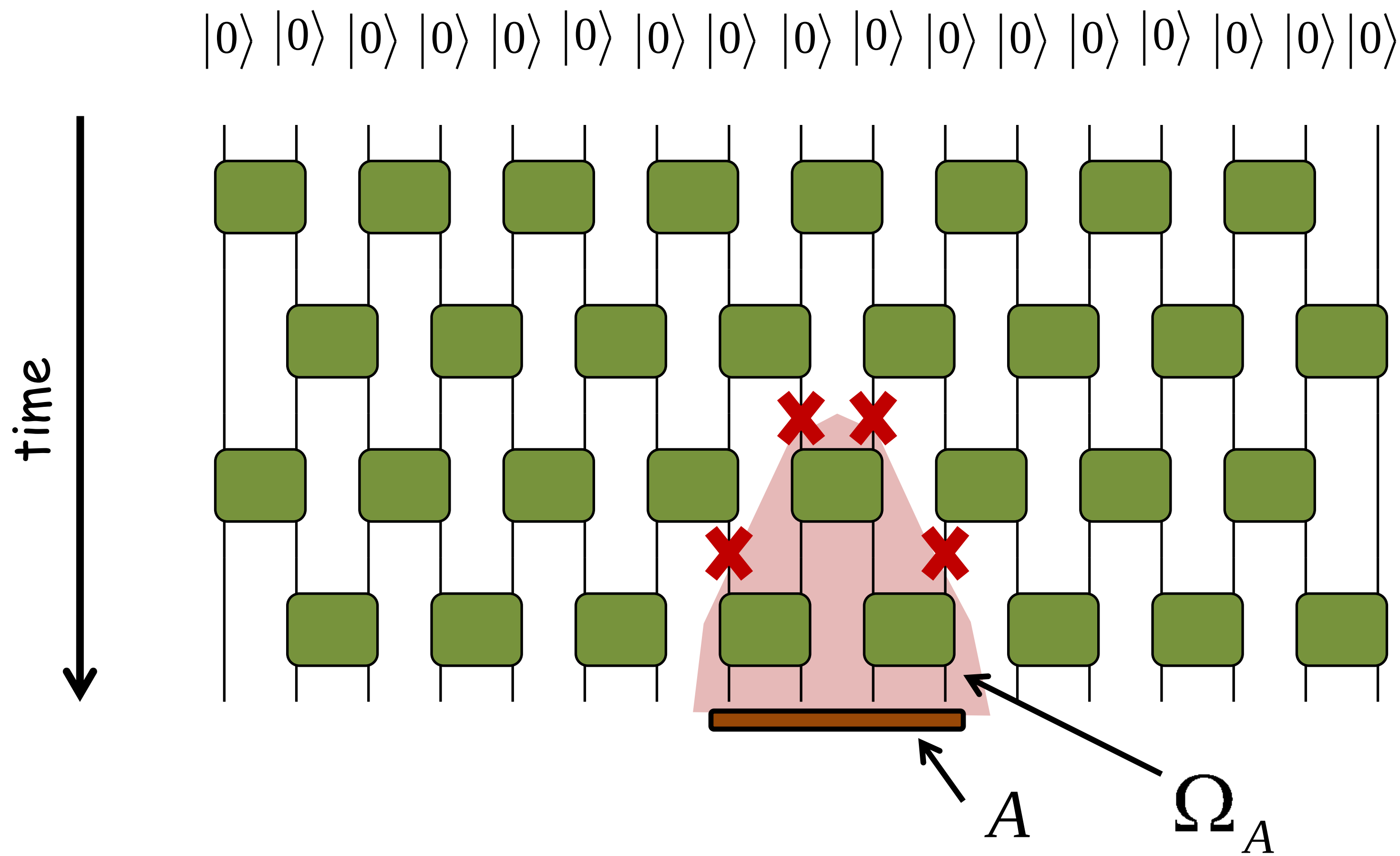


cost of computing $\rho(A)$: $c \approx \exp(2a\log(N)) \approx N^{2a}$

efficient

How entangled is the resulting state?

- Entanglement entropy of a block of contiguous sites



Upper bound on entanglement entropy

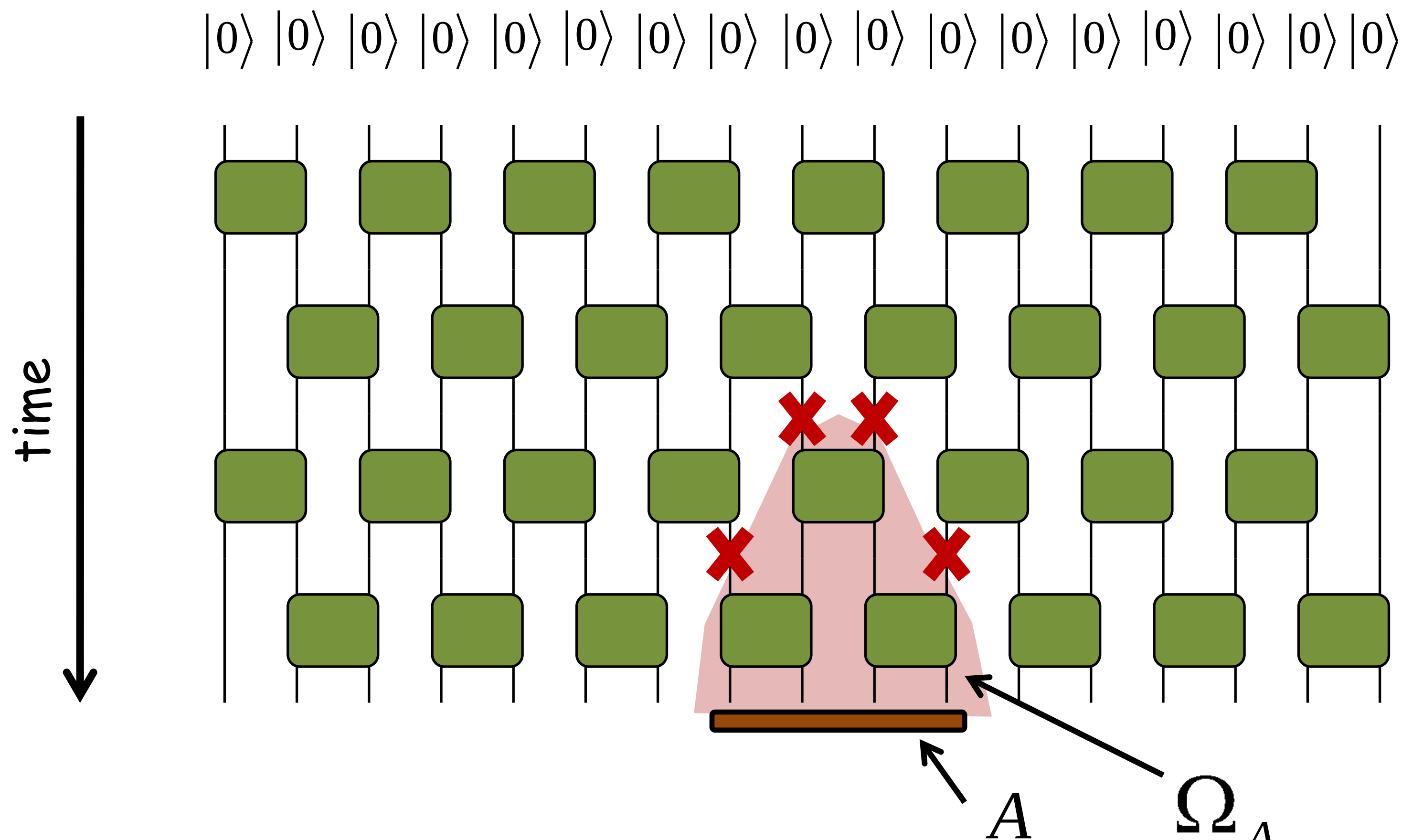
minimal connectivity of region A

# of bond indices

$$n(A) = 4$$

How entangled is the resulting state?

- Entanglement entropy of a block of contiguous sites



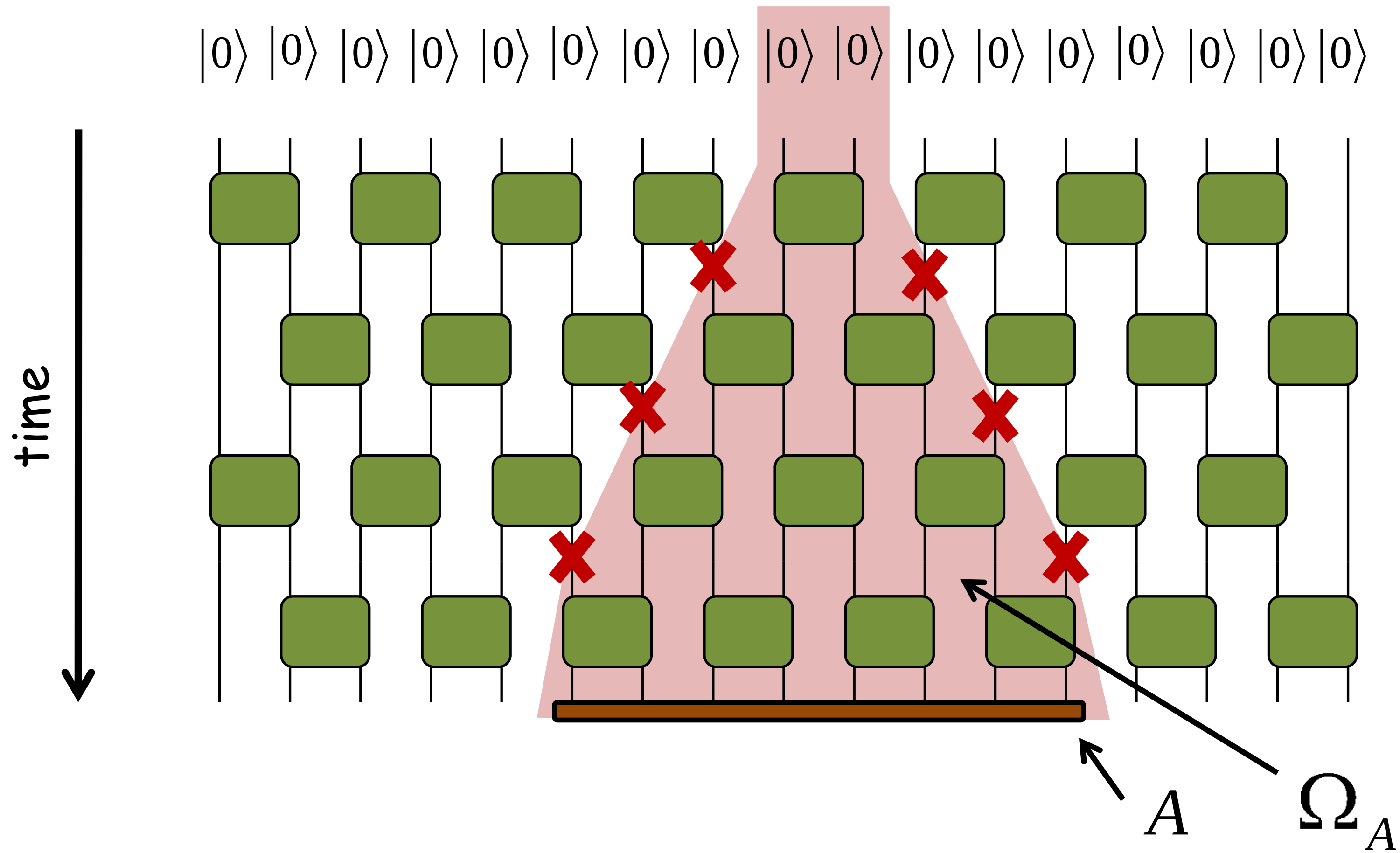
Upper bound on entanglement entropy

$$S(A) \leq \gamma n(A)$$

# of bond indices $n(A) = 4$

How entangled is the resulting state?

- Entanglement entropy of a block of contiguous sites



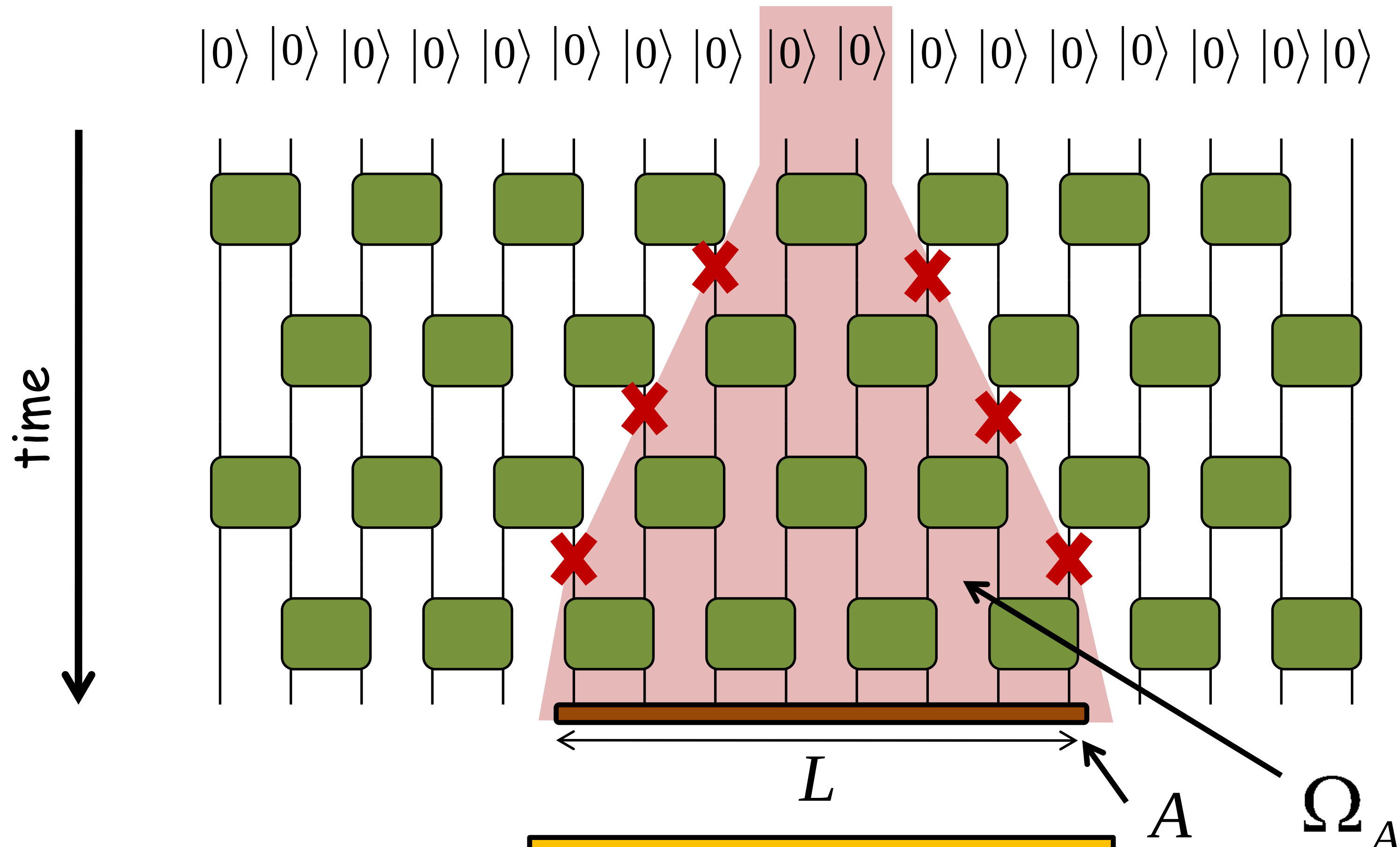
Upper bound on entanglement entropy

$$S(A) \leq \gamma n(A)$$

# of bond indices $n(A) = 6$

How entangled is the resulting state?

- Entanglement entropy of a block of contiguous sites

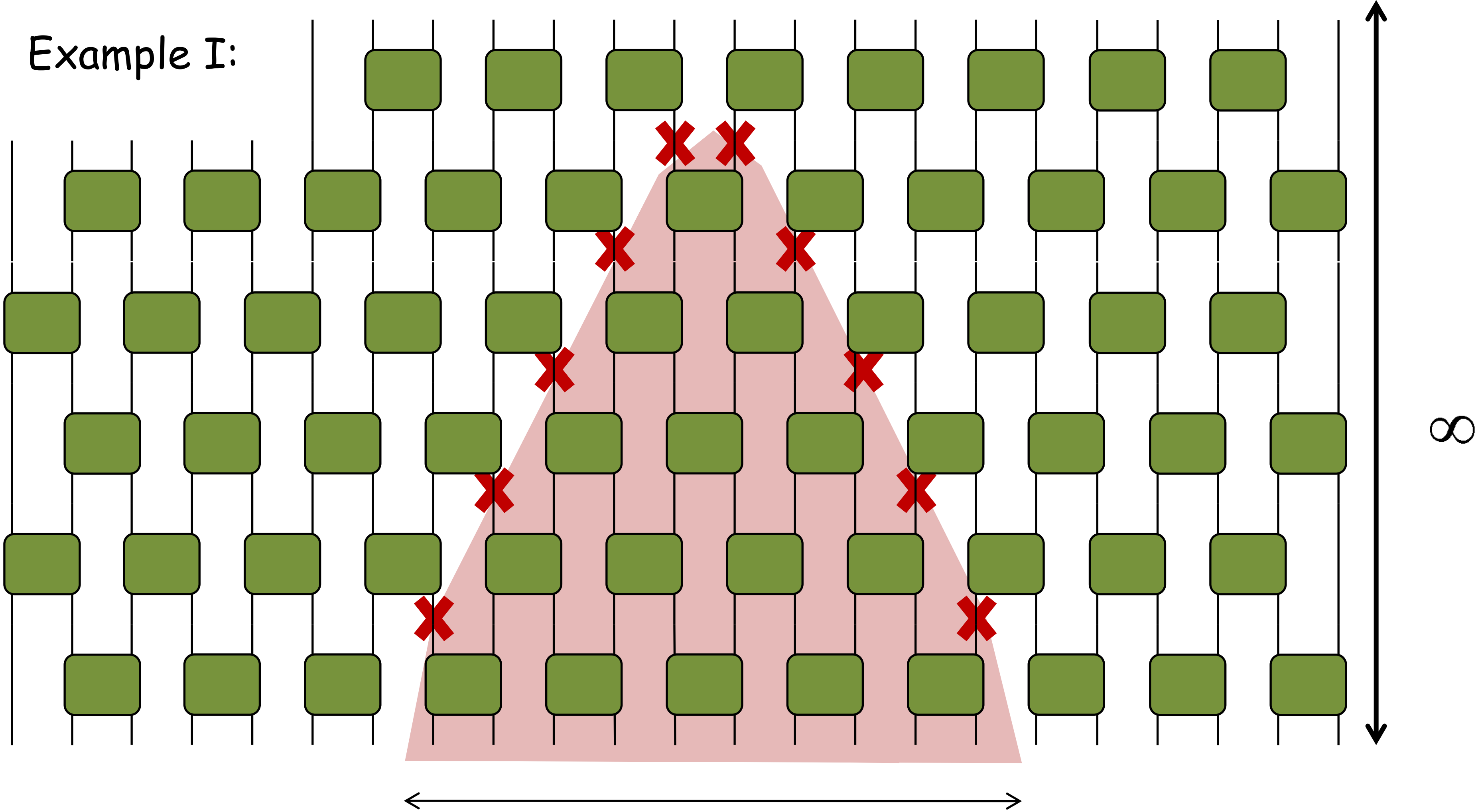


Scaling of entropy with size L of region A

$$S(A) \approx \begin{cases} const \\ \log(L) \\ L \end{cases} \quad ?$$

for simplicity, $N \to \infty$

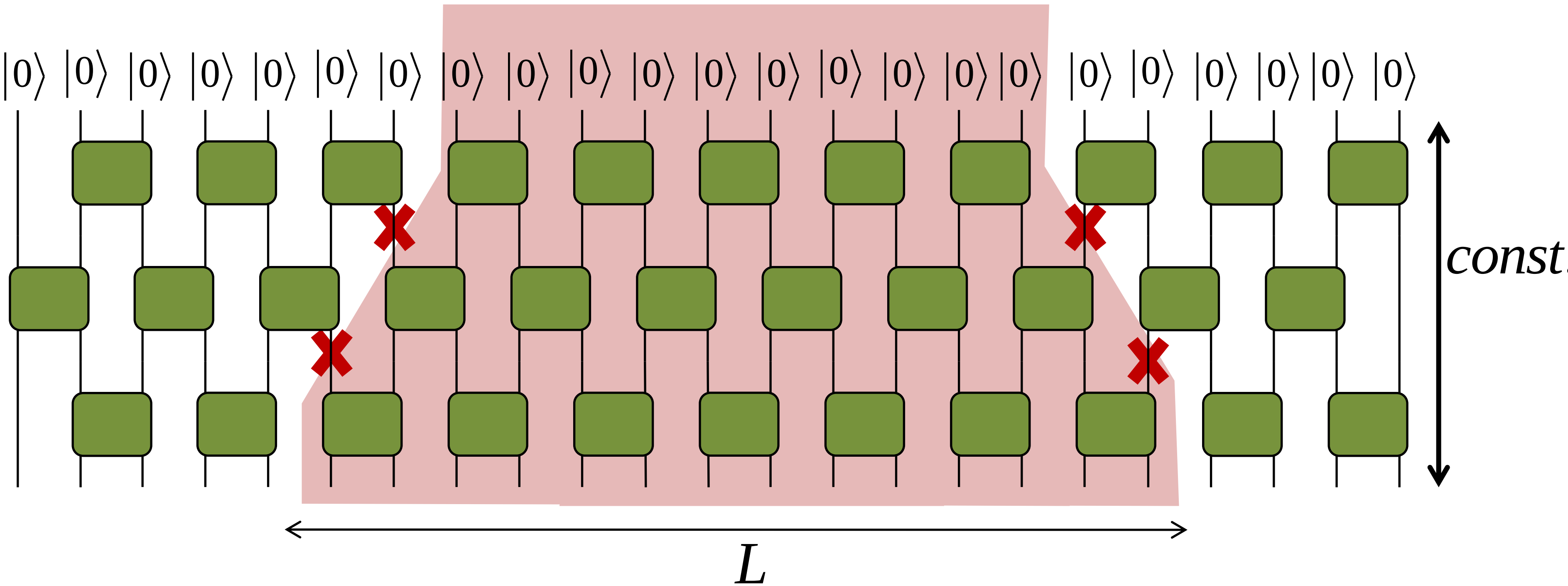
Example I:



$$n(A) \approx L$$

scaling of entropy: $S(A) \approx L$

Example II:



$$n(A) \approx const$$

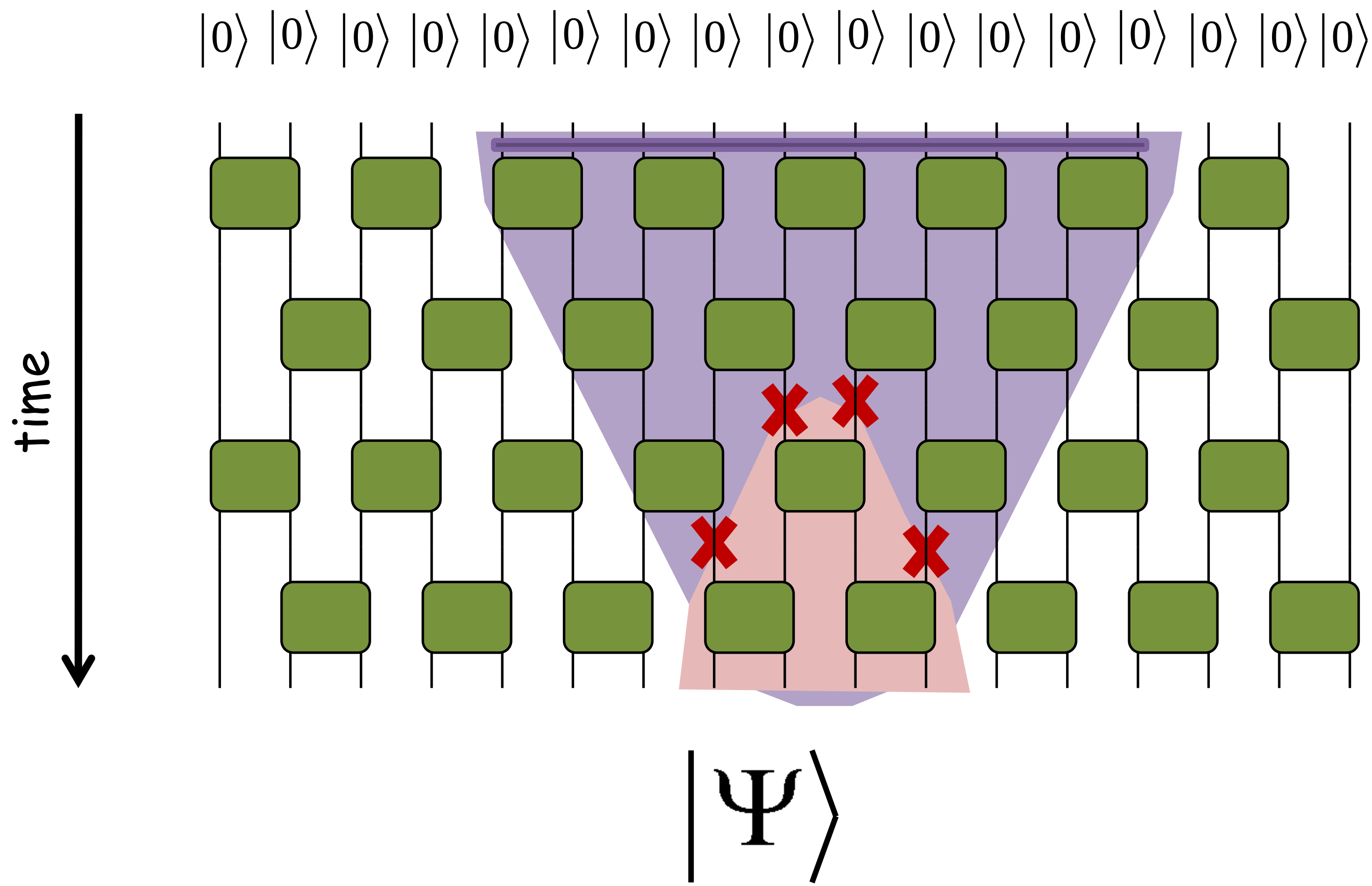
scaling of entropy: $S(A) \approx const$

Summary:

Quantum Circuit as a many-body variational ansatz

Questions:

- Cost of computing a local reduced density matrix
- Entropy of a block of contiguous sites



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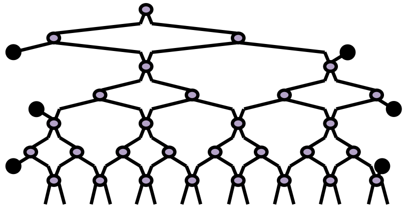
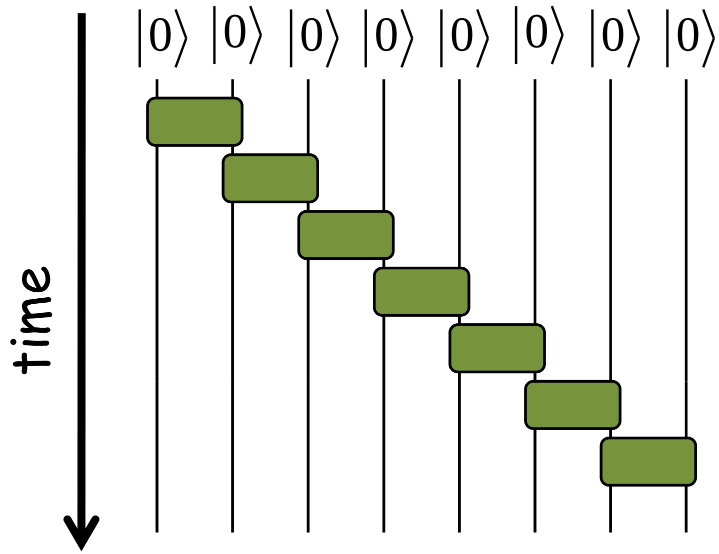
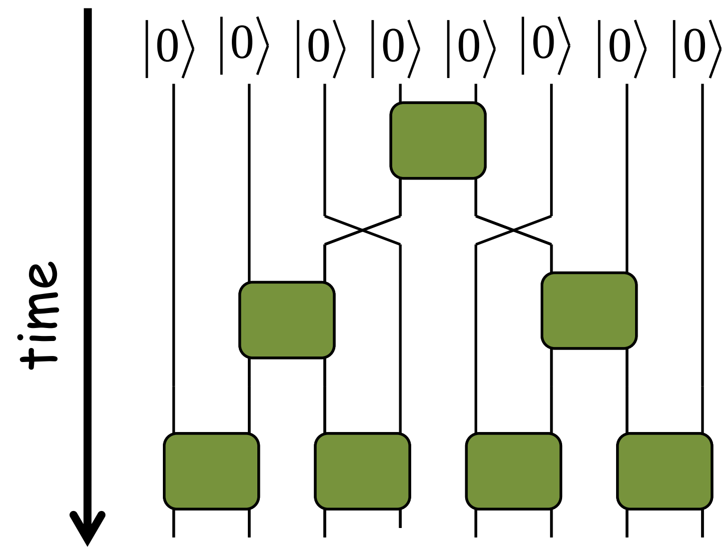


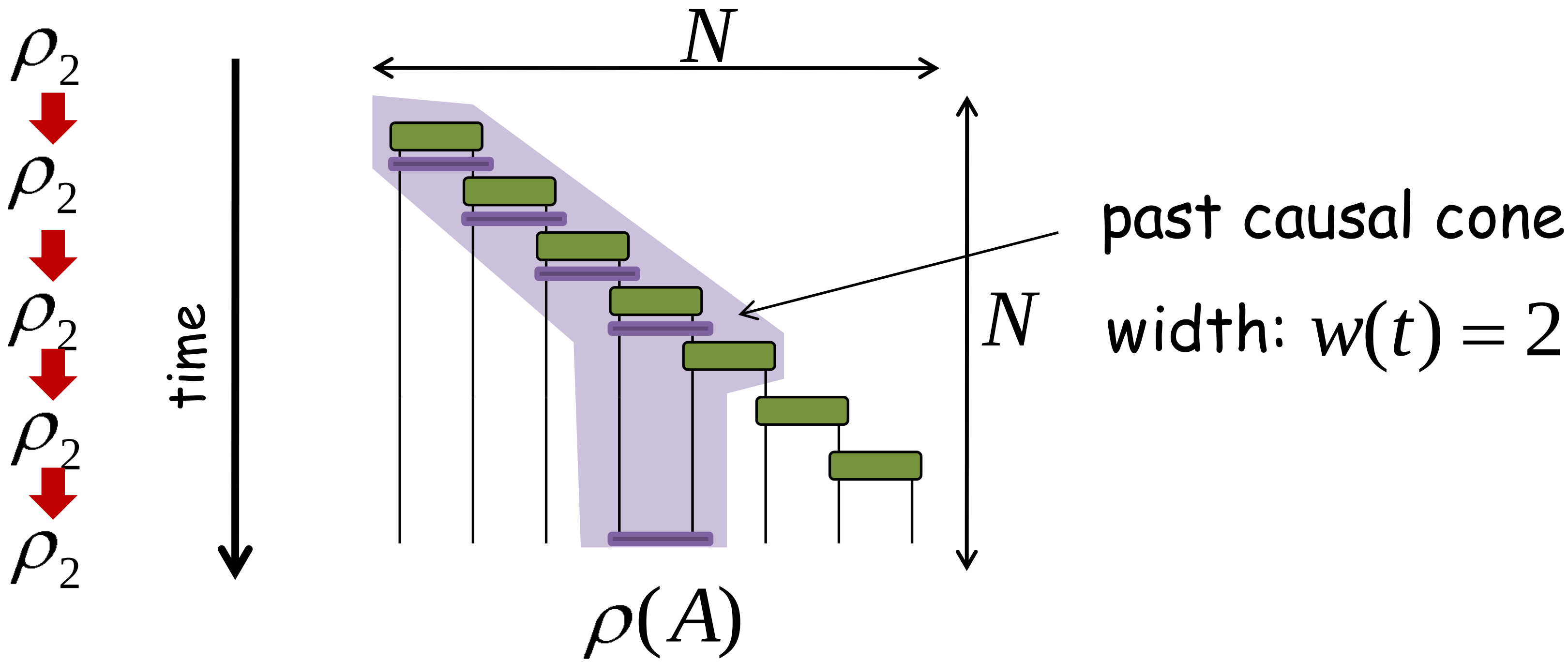


matrix product state
MPS



tree tensor network
TTN

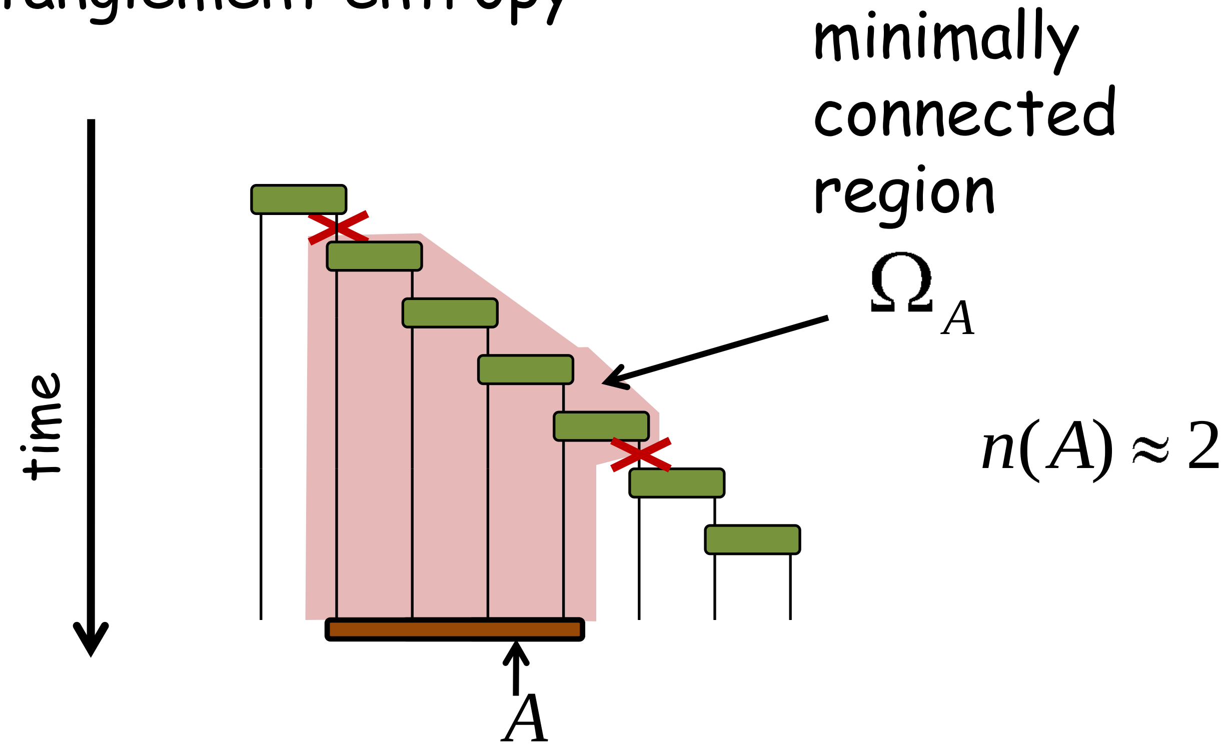
MPS: computational cost



cost of computing $\rho(A)$: $c \approx \exp(w) = const$

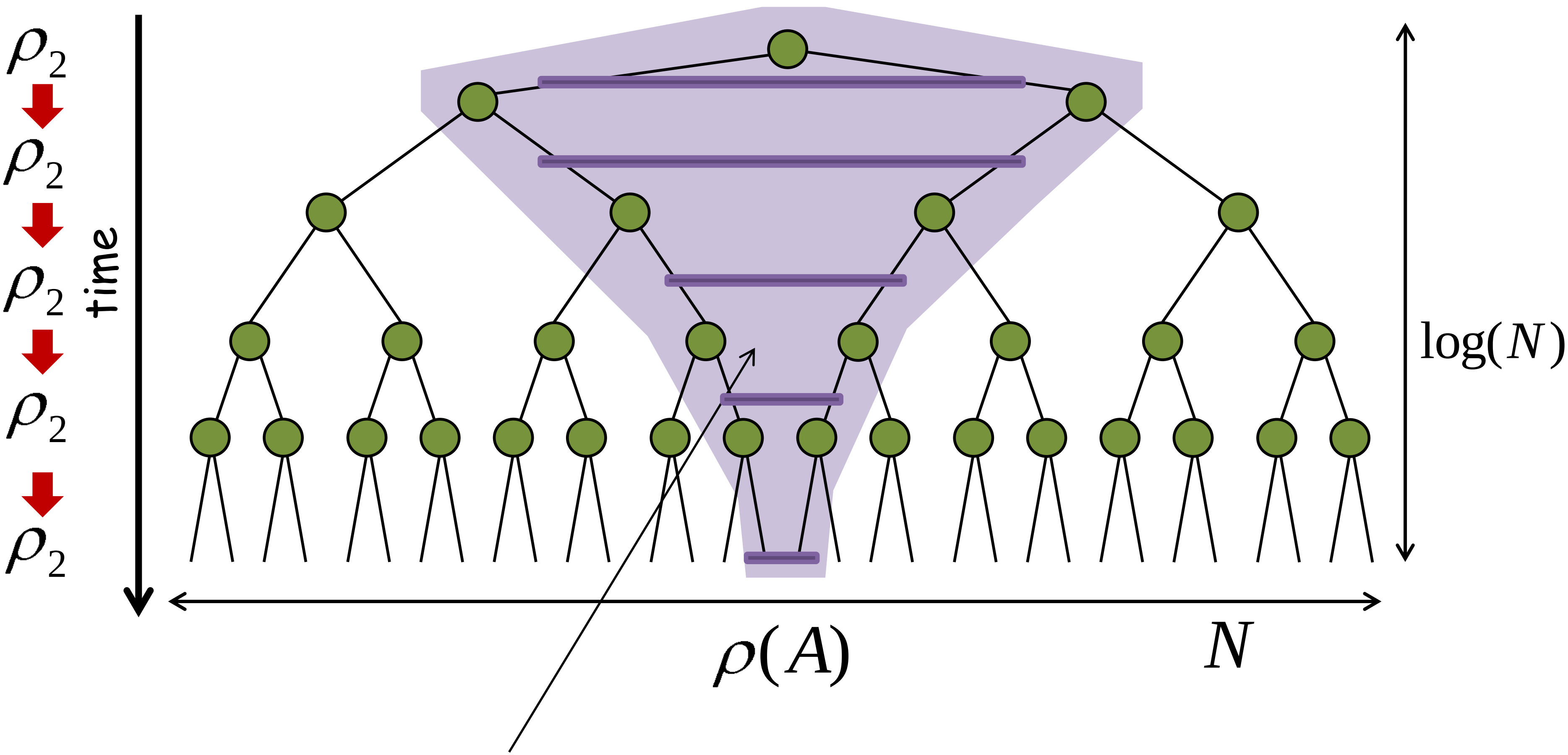
$$c \approx O(N)$$

MPS: entanglement entropy



scaling of entropy: $S(A) \approx const$

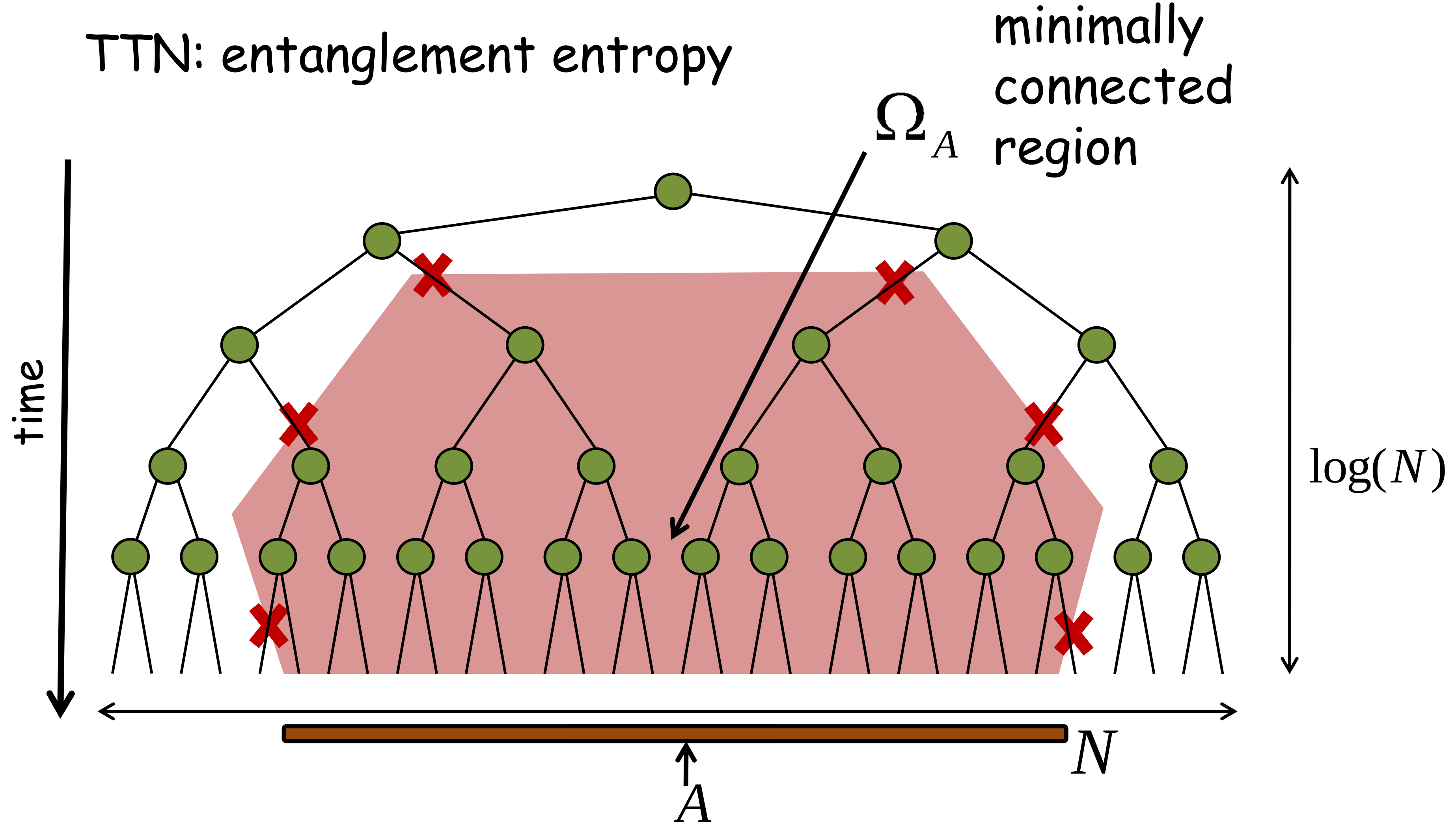
TTN: computational cost



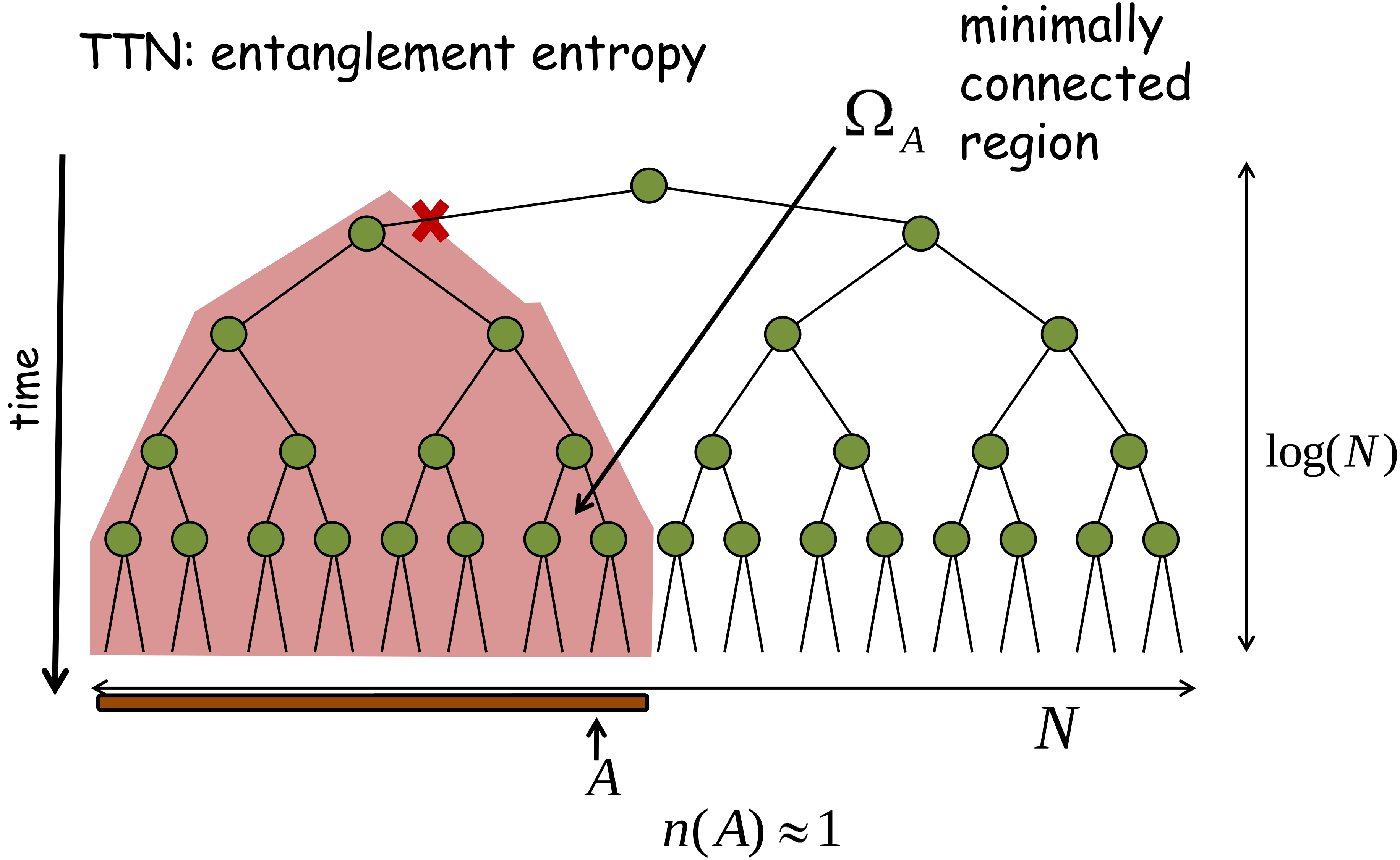
past causal cone   width: $w(t) = 2$

cost of computing $\rho(A)$: $c \approx \exp(w) = const$ $c \approx \log(N)$

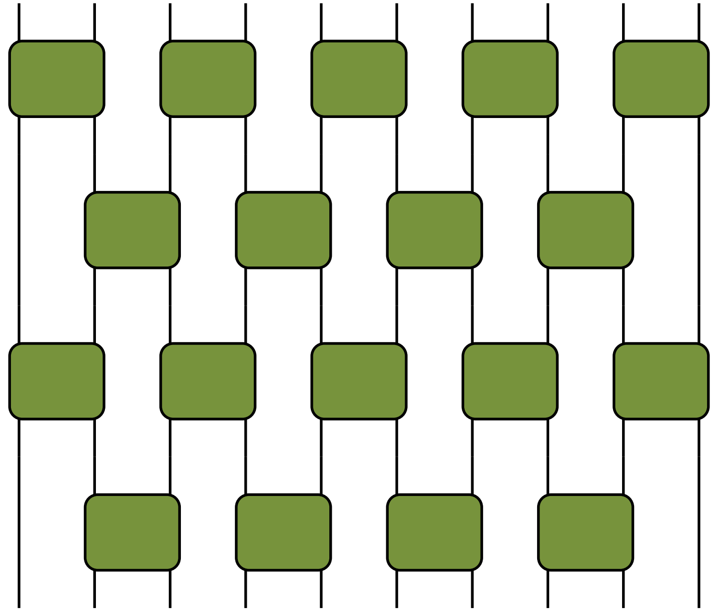
TTN: entanglement entropy



TTN: entanglement entropy



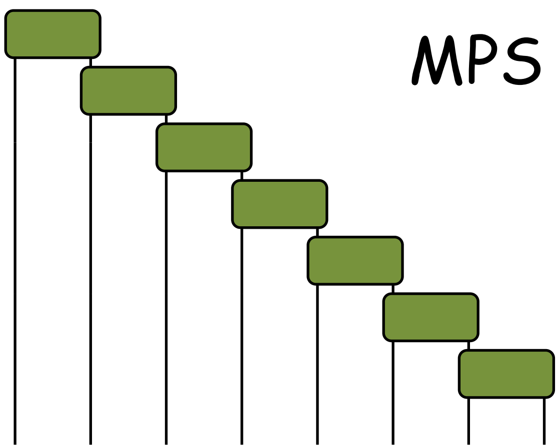
scaling of entropy: $S(A) \approx const$



$c \approx \exp(N)$

$S(A) \approx L$

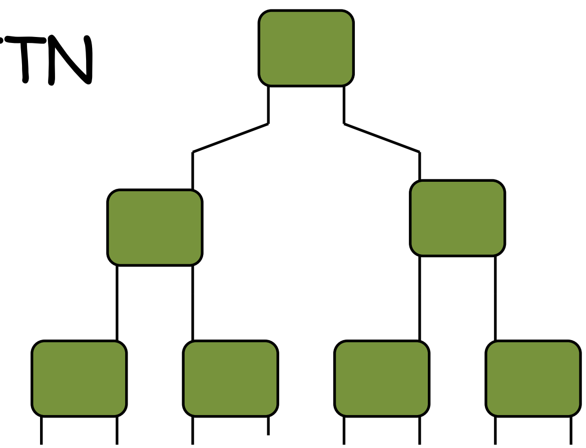
MPS



$c \approx N$

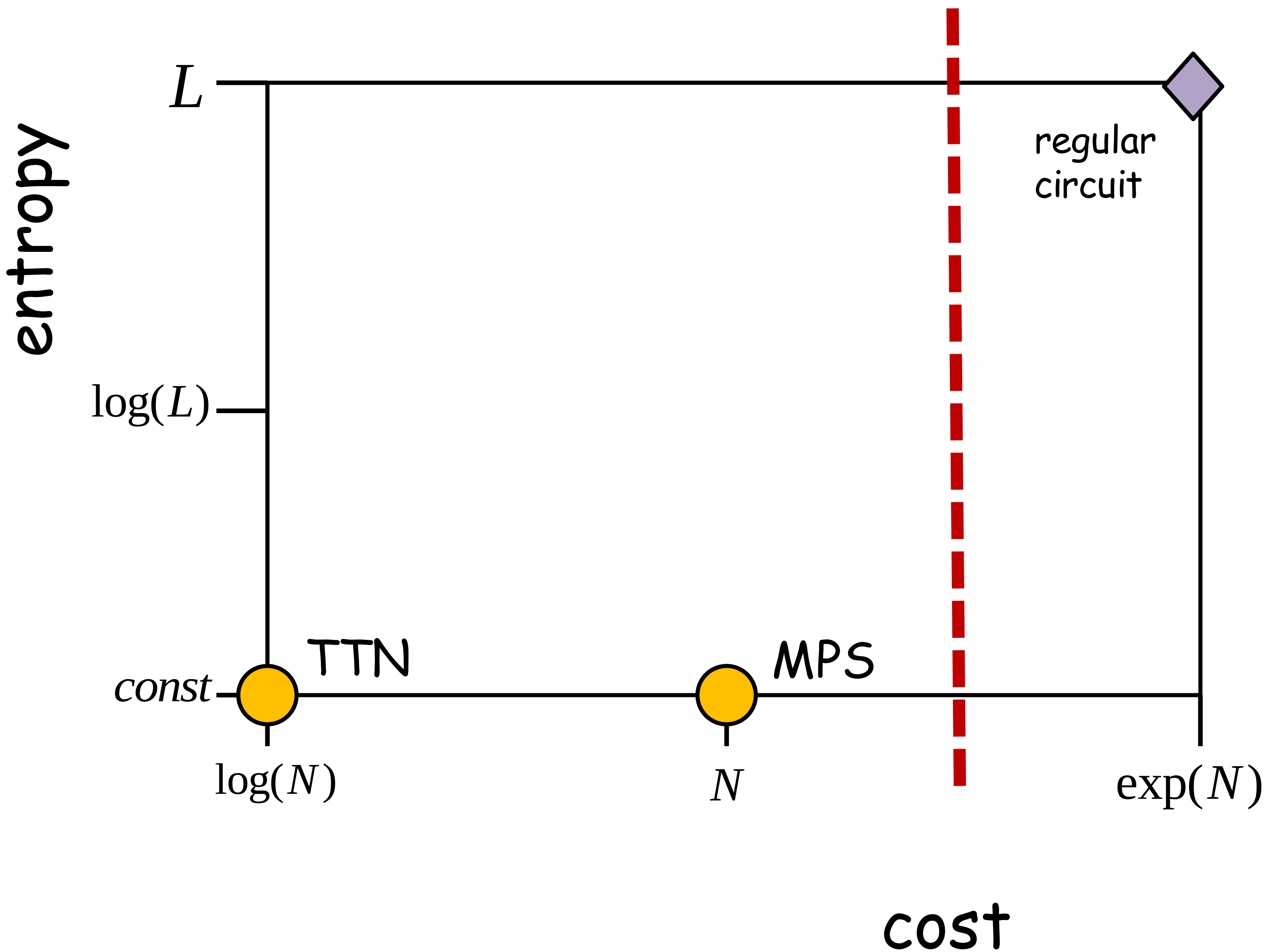
$S(A) \approx const$

TTN



$c \approx \log(N)$

$S(A) \approx const$


entropy
L
log(L)
const
regular
circuit
TTN
MPS
log(N)
N
exp(N)
cost

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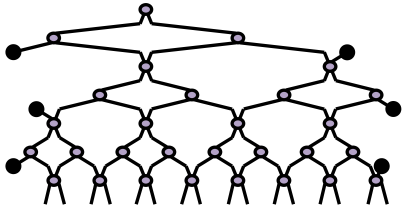
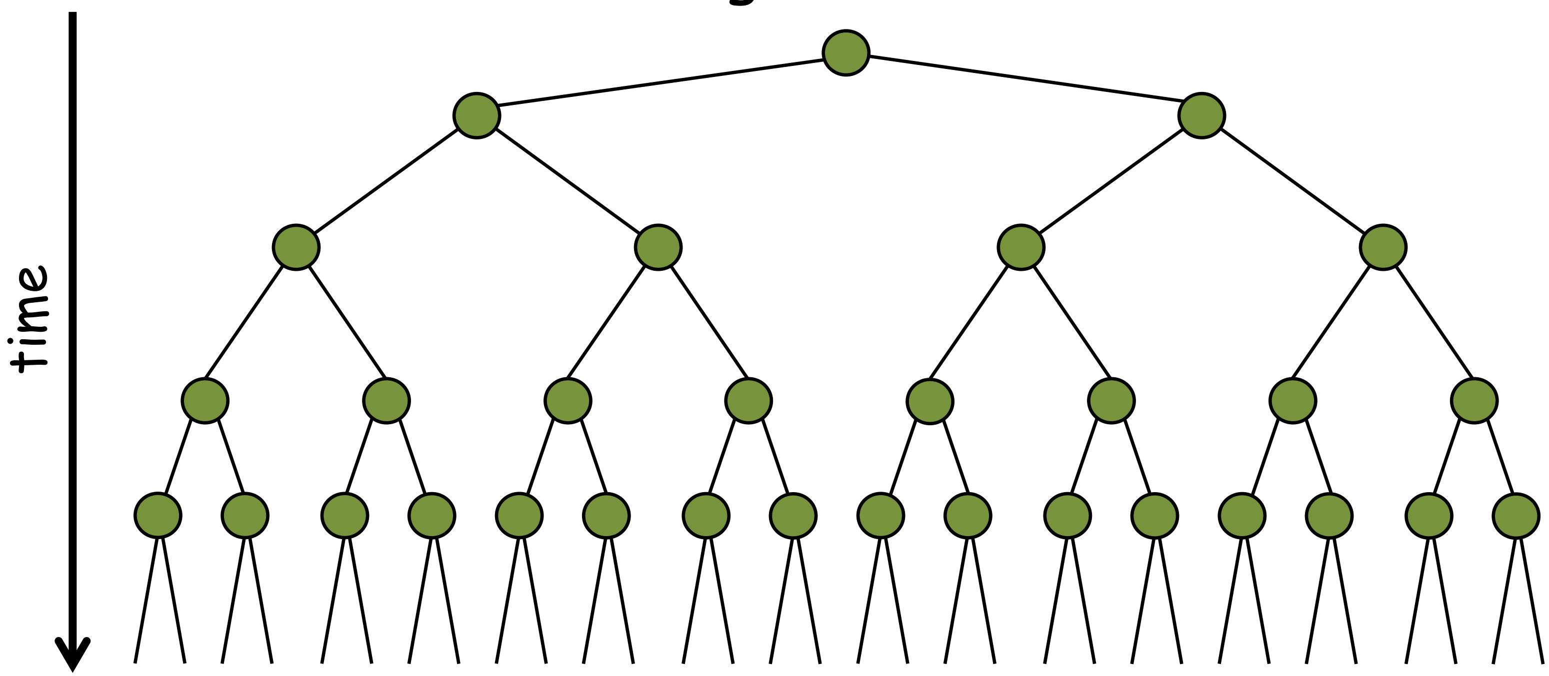
- branching MERA



MERA (multi-scale entanglement renormalization ansatz)

TTN + disentanglers



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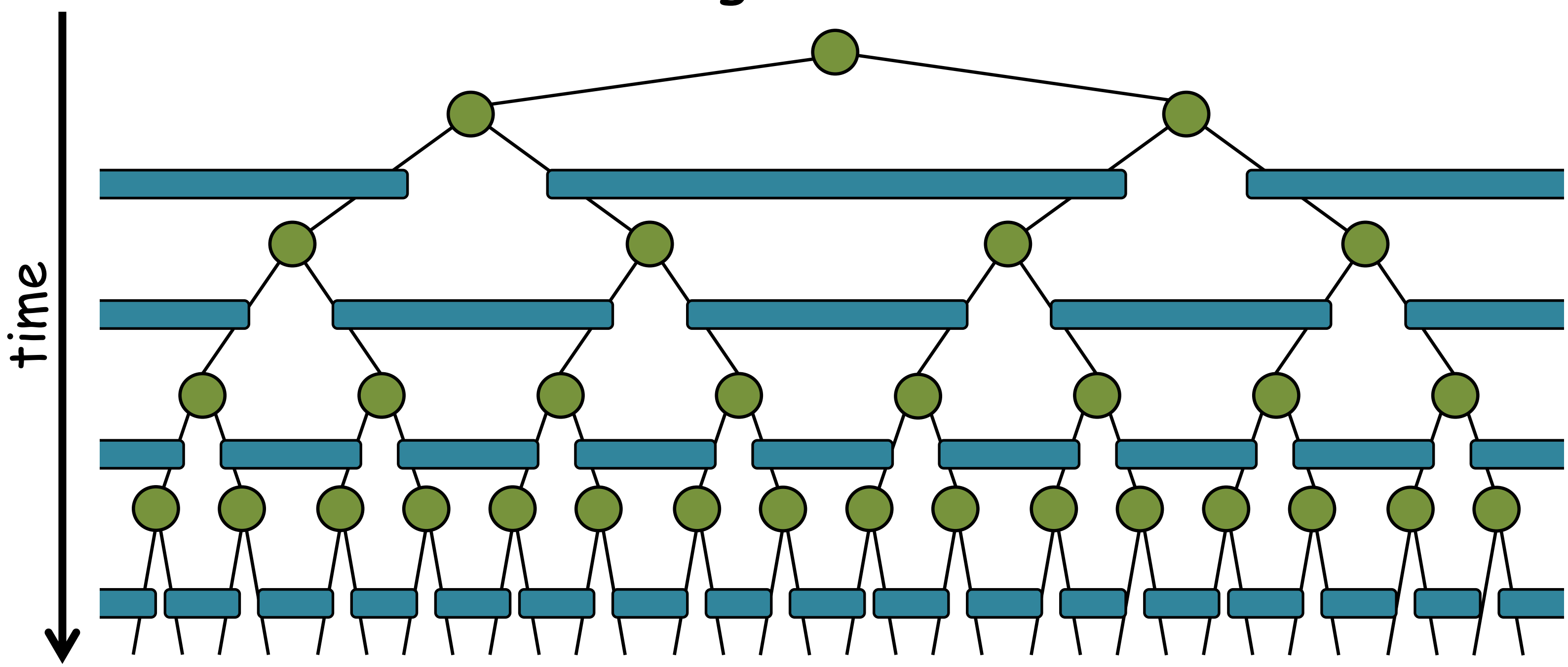

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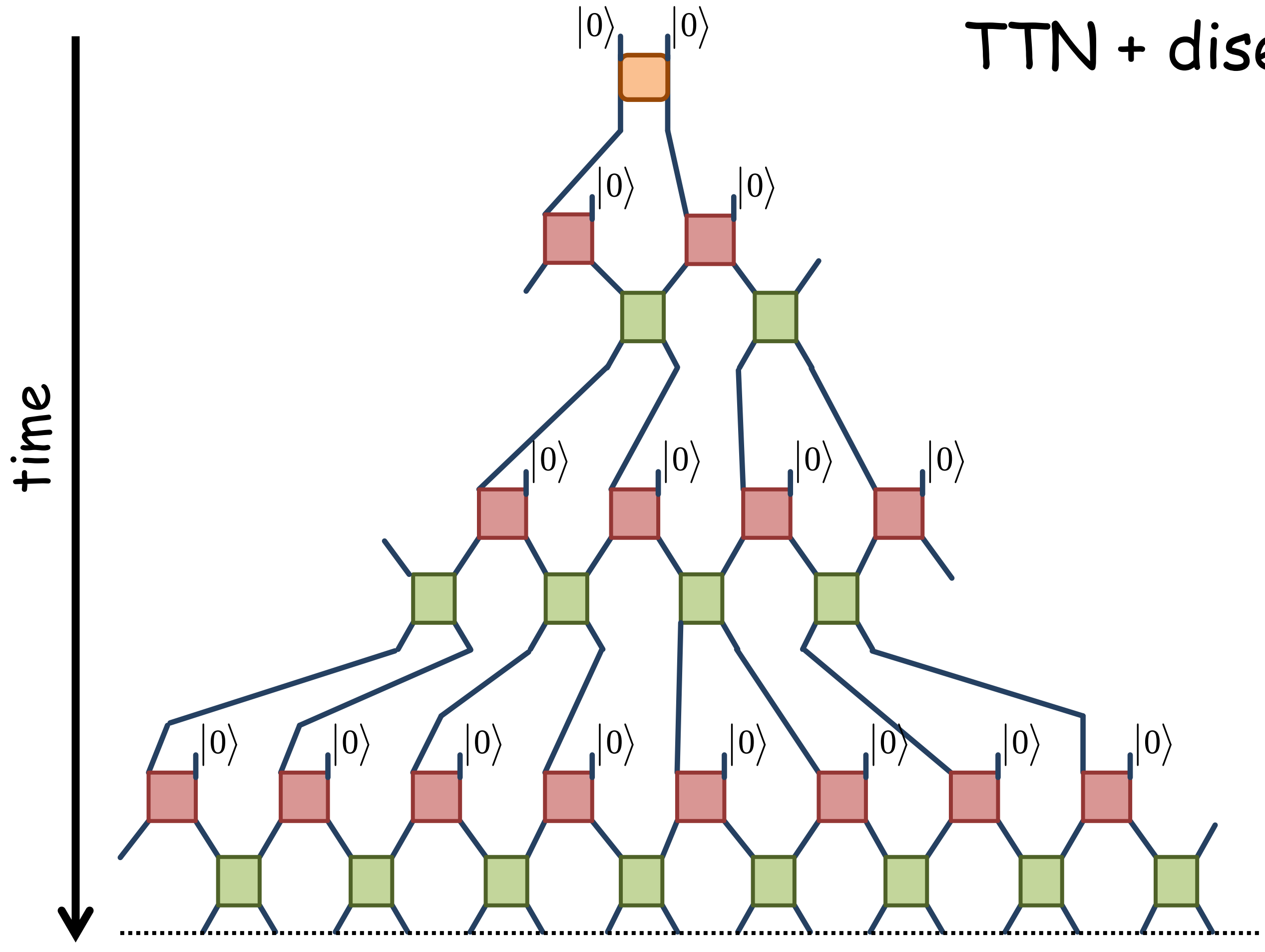
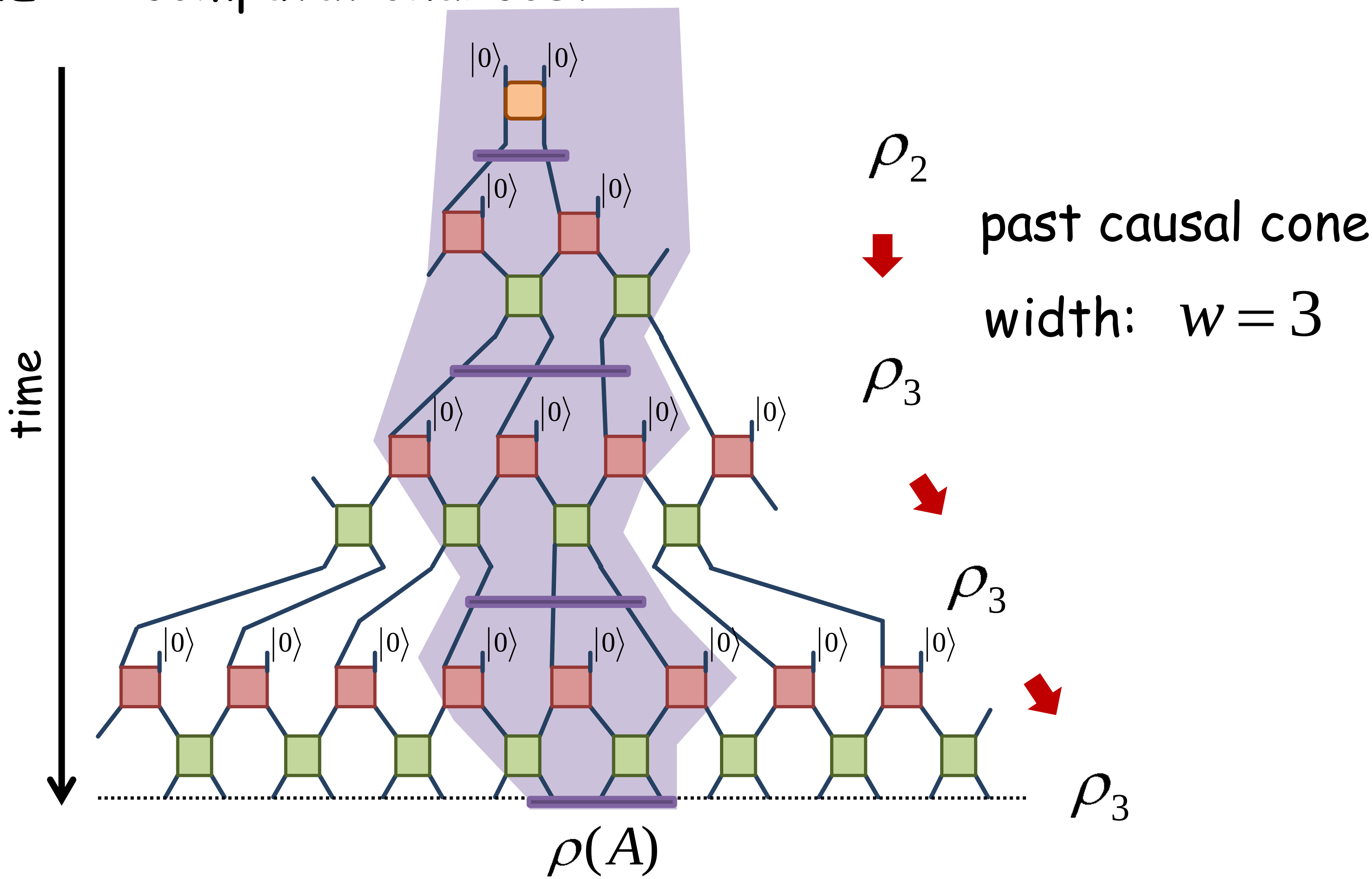
TTN + disentanglers

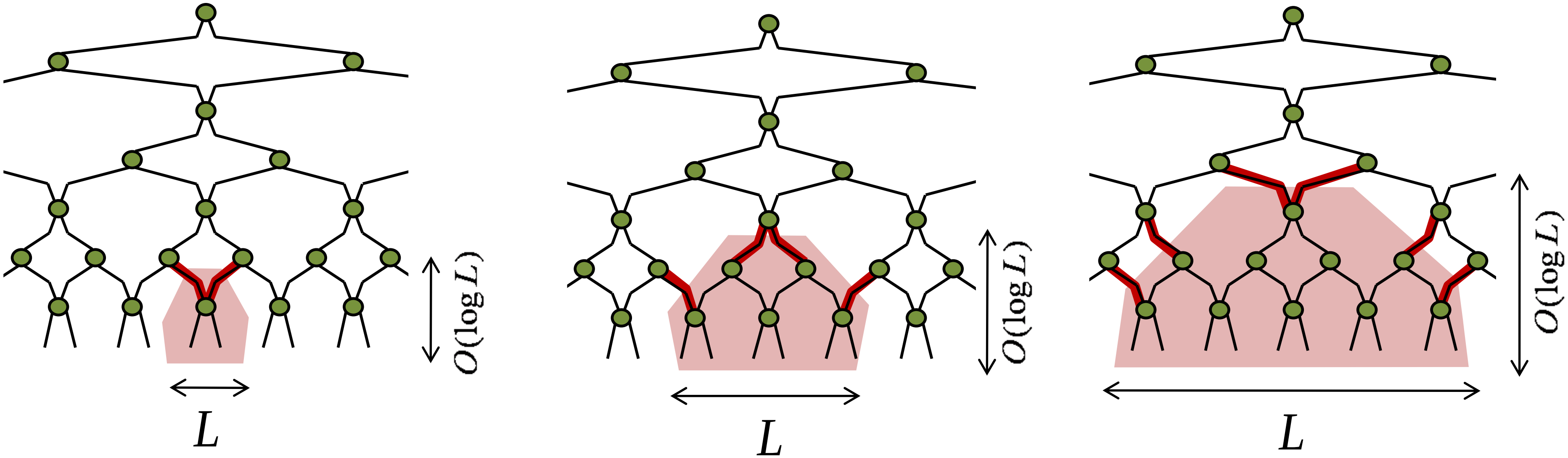


MERA: computational cost



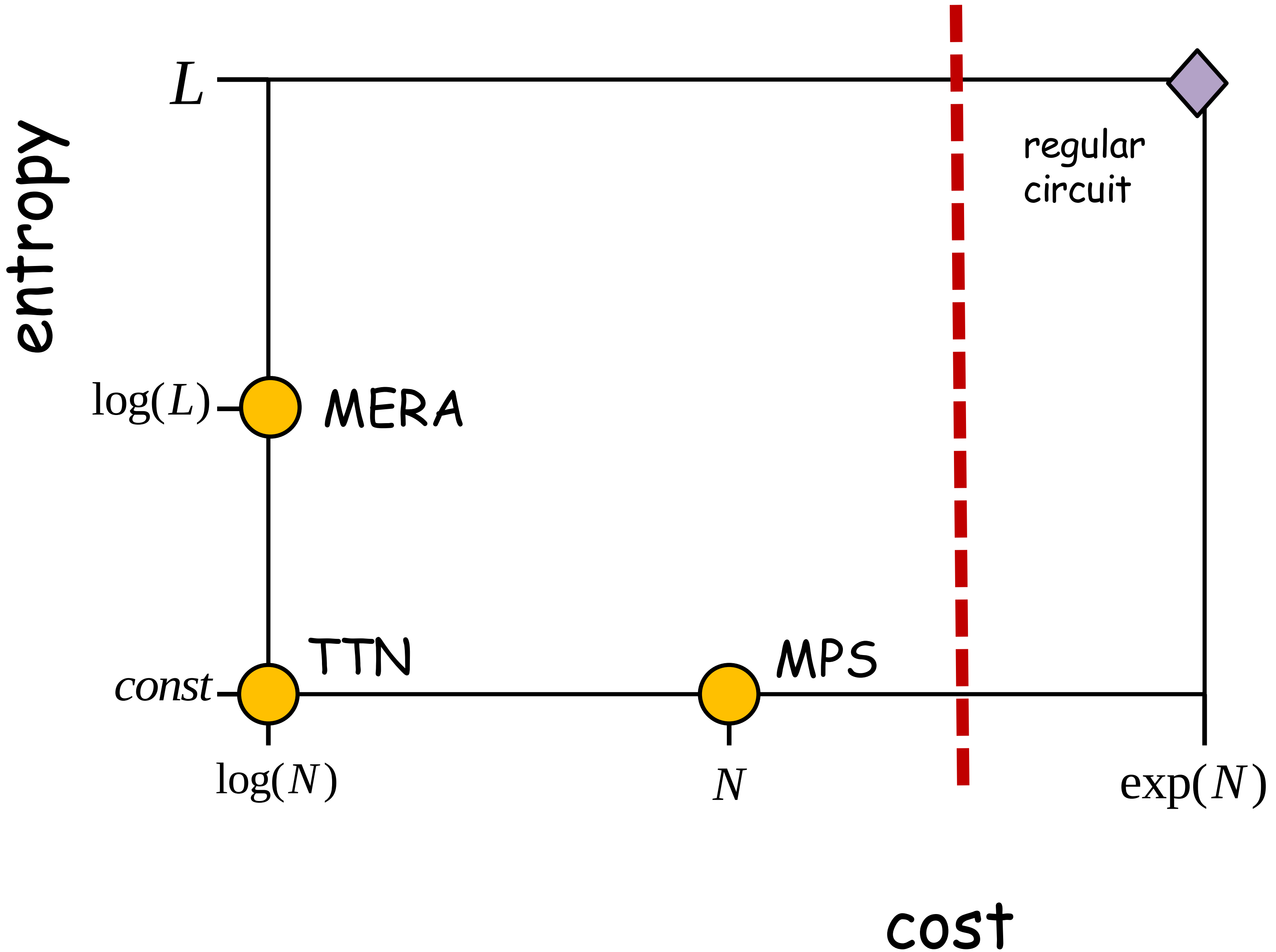
cost of computing $\rho(A)$: $c \approx \exp(w) = const$ $c \approx \log(N)$

MERA: entanglement entropy



$$n(A) \approx \log(L)$$

scaling of entropy: $S(A) \approx \log(L)$



entropy
L
log(L)
MERA
const
TTN
MPS
regular
circuit
log(N)
N
exp(N)
cost

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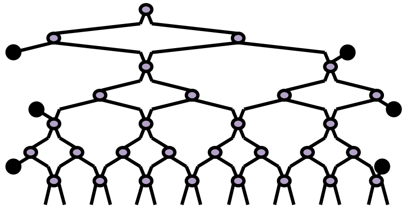
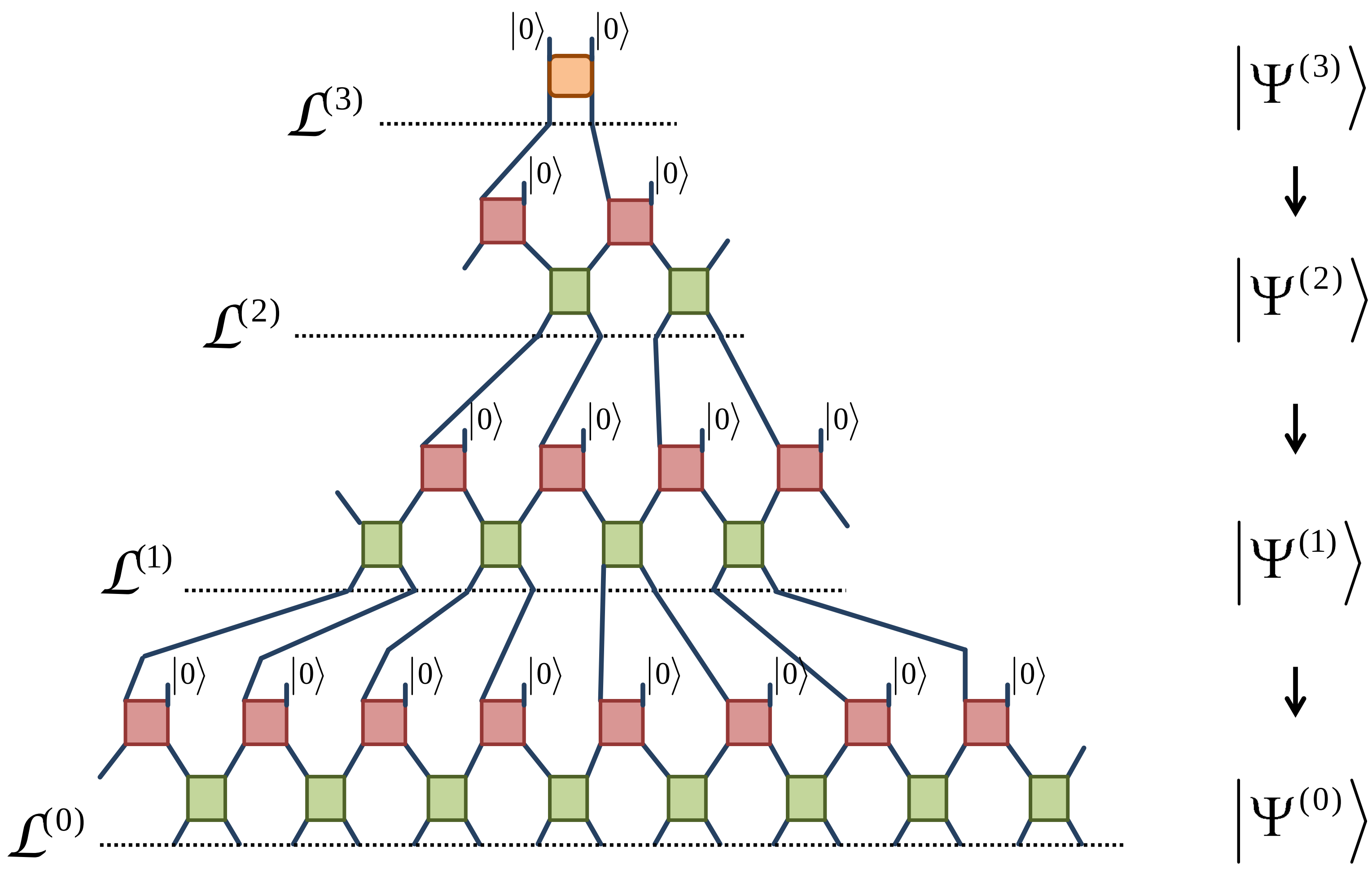
- branching MERA

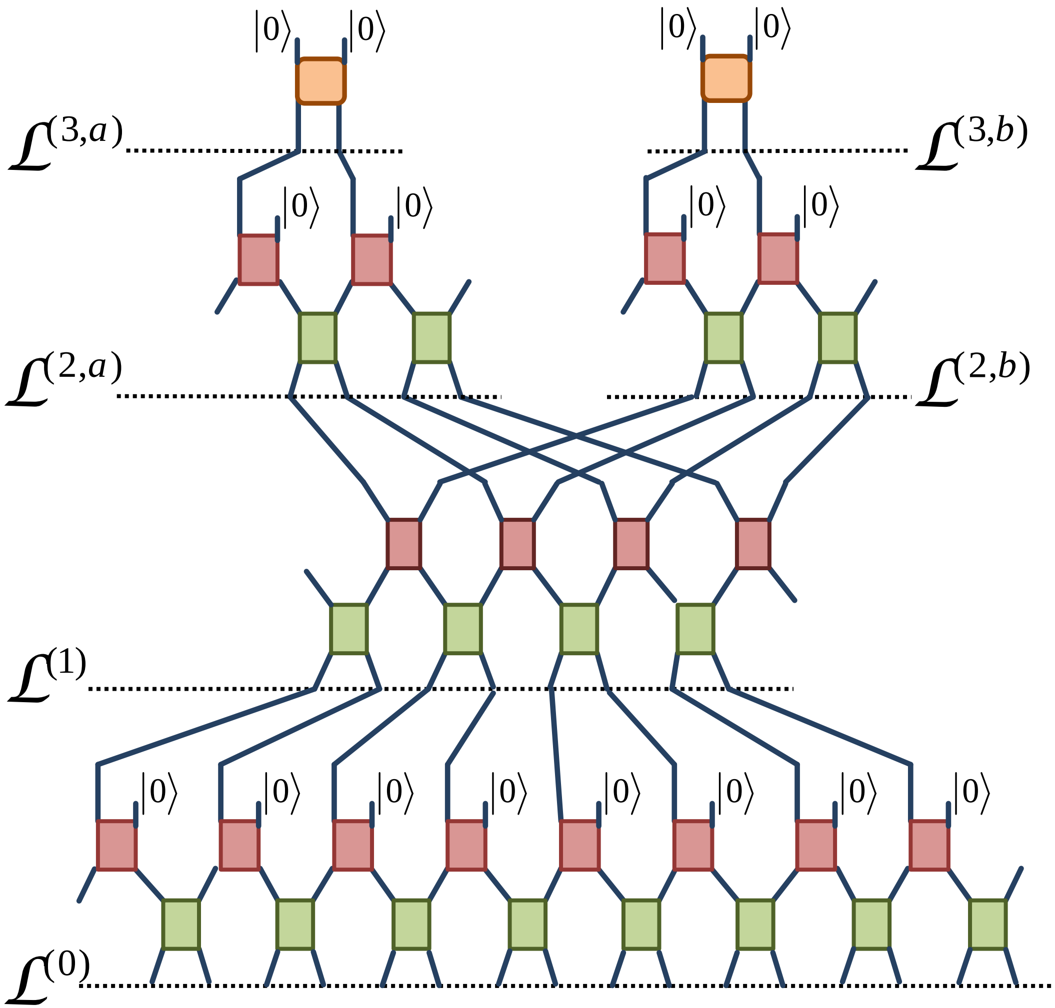


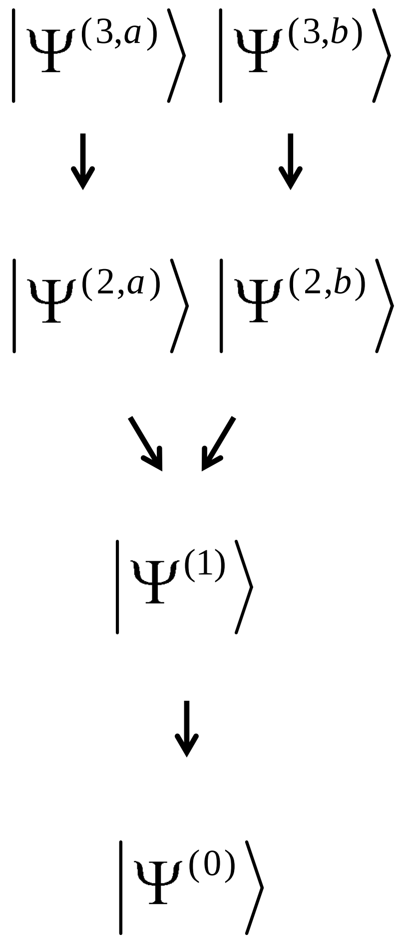
MERA



$|0\rangle$
$\mathcal{L}^{(3)}$
$\mathcal{L}^{(2)}$
$\mathcal{L}^{(1)}$
$\mathcal{L}^{(0)}$
$|\Psi^{(3)}\rangle$
$|\Psi^{(2)}\rangle$
$|\Psi^{(1)}\rangle$
$|\Psi^{(0)}\rangle$

branching MERA

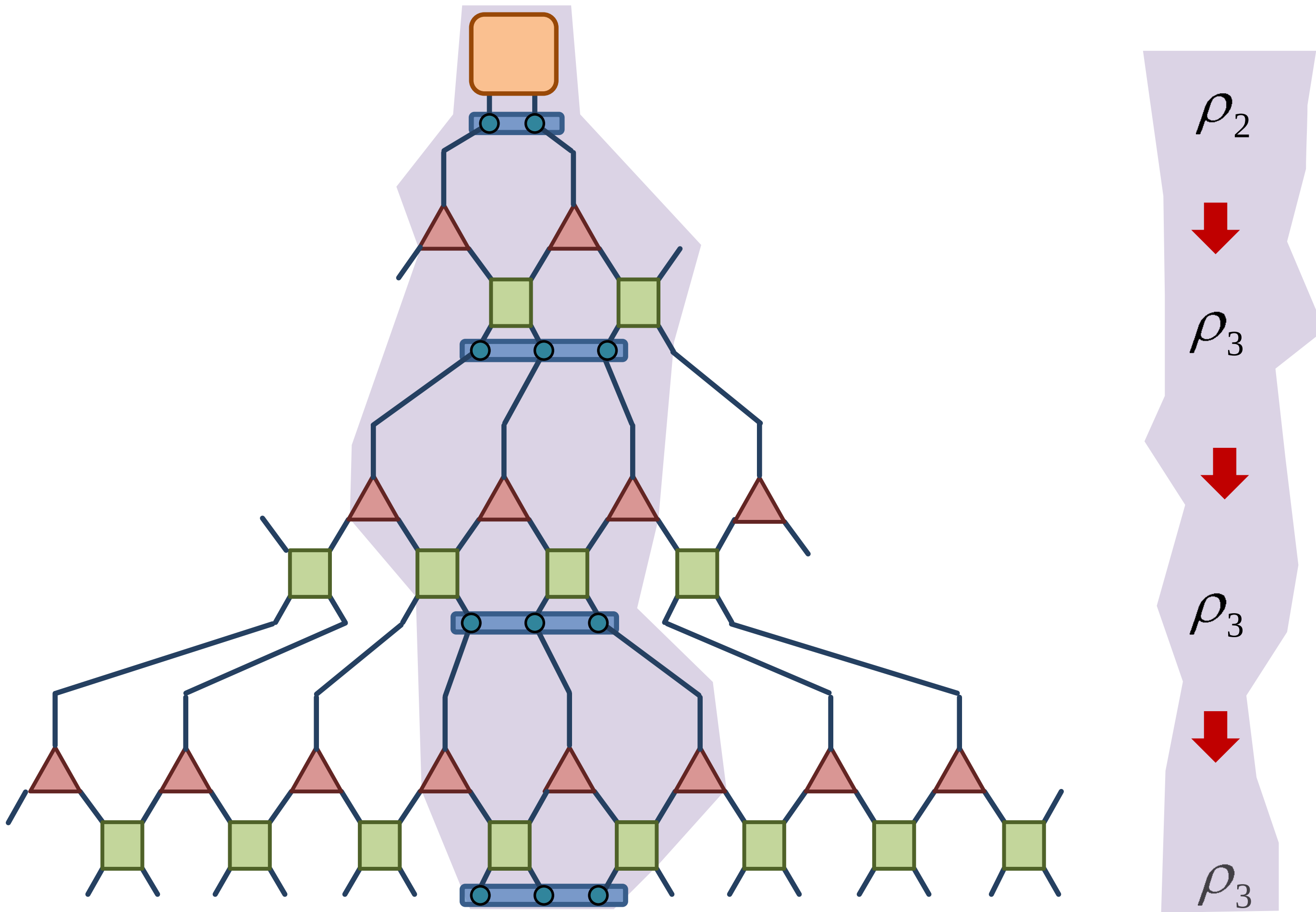




MERA: computational cost

past causal cone

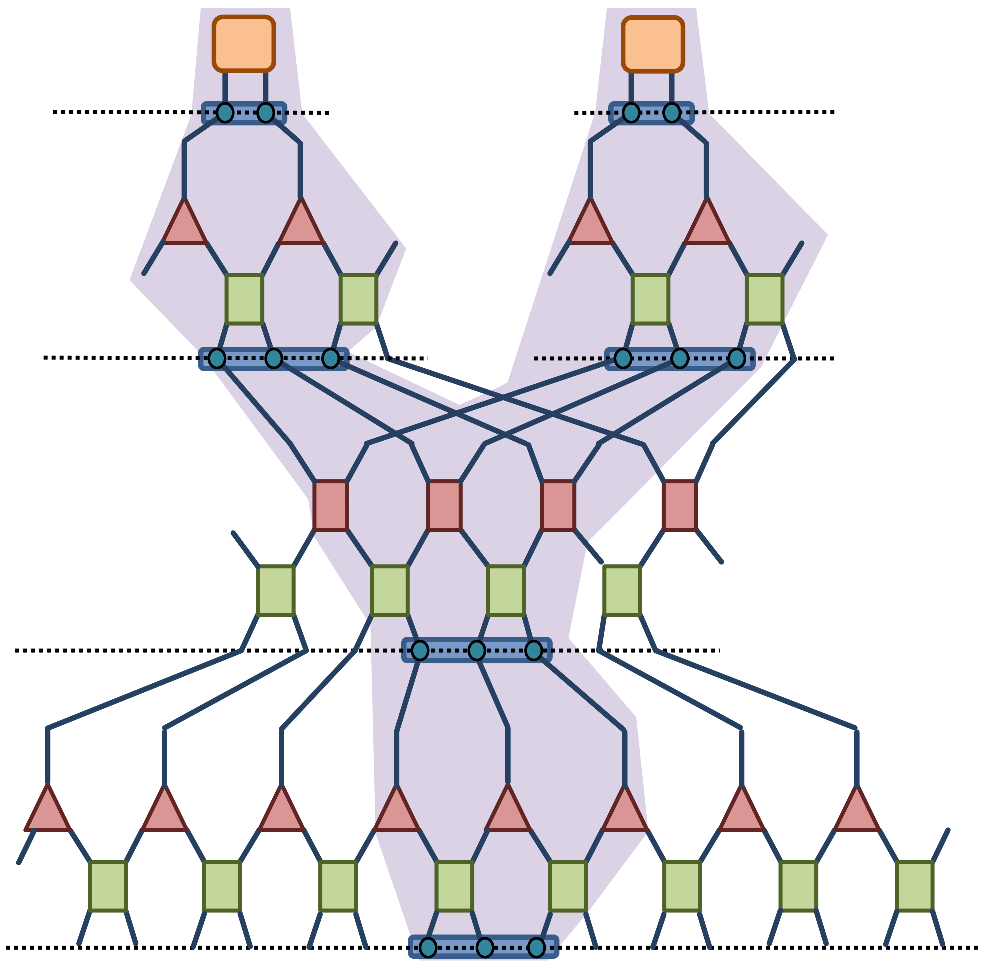
width: $w = 3$

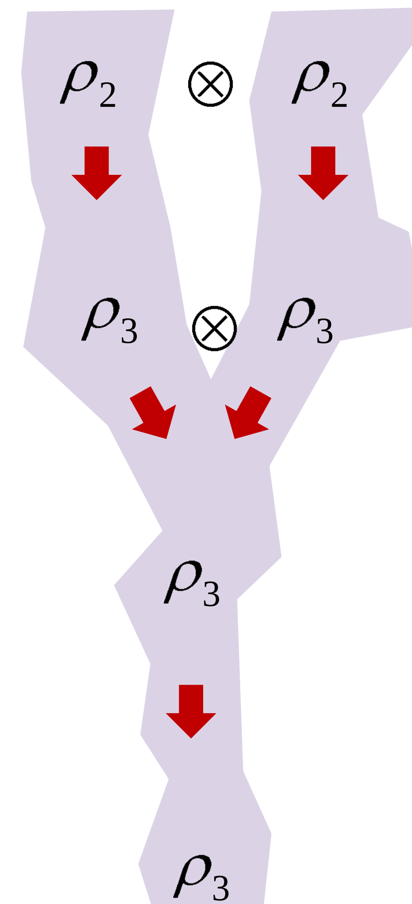


cost of computing $\rho(A)$: $c \approx \exp(w) = const$ $\quad c \approx \log(N)$

branching MERA: computational cost

past causal cone width: $w' = 2w$

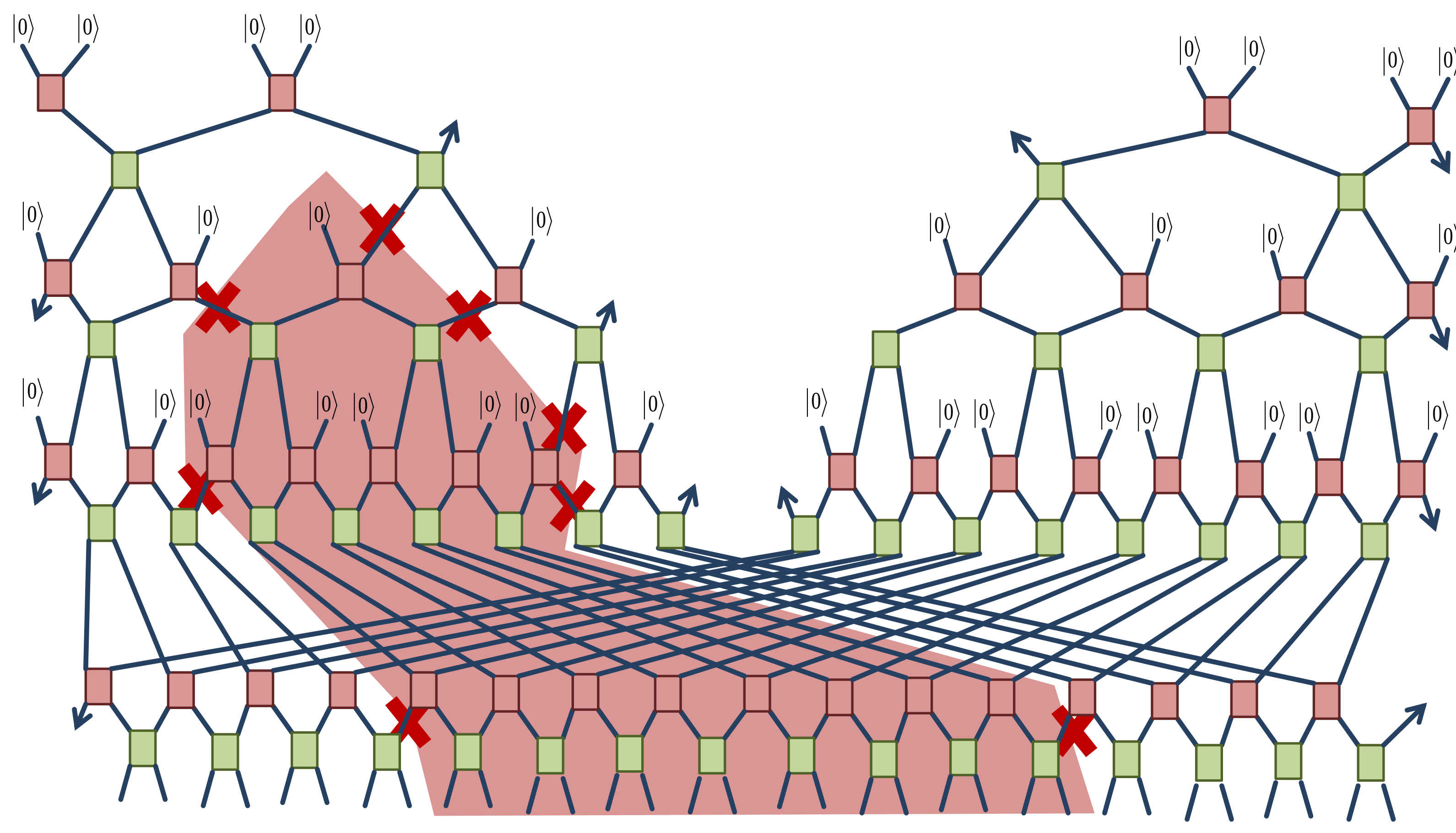




cost of computing $\rho(A)$: $c \approx 2\exp(w)$

$c \approx 2\log(N)$

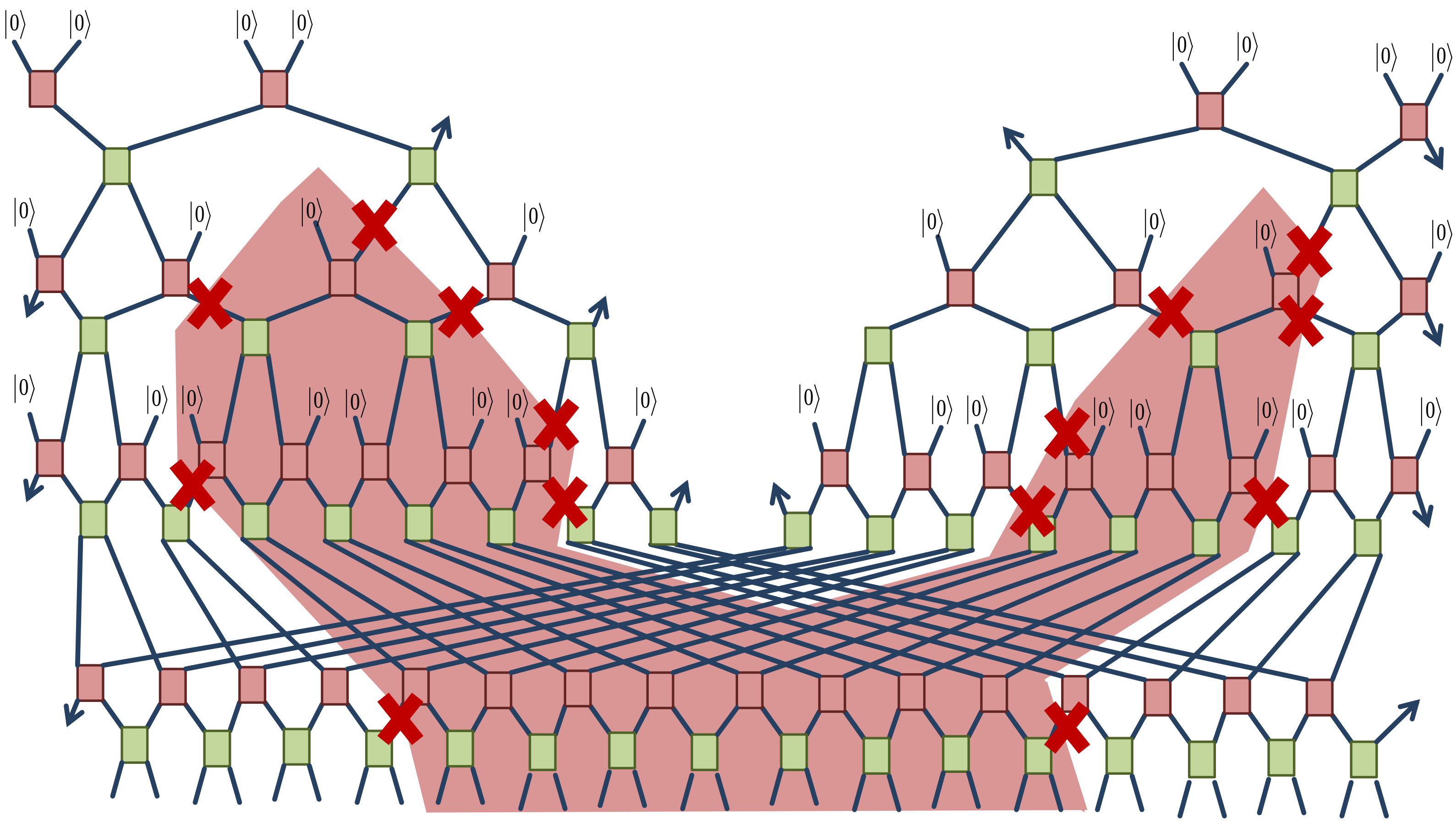
MERA: entanglement entropy



$$n(A) \approx \log(L)$$

scaling of entropy: $S(A) \approx \log(L)$

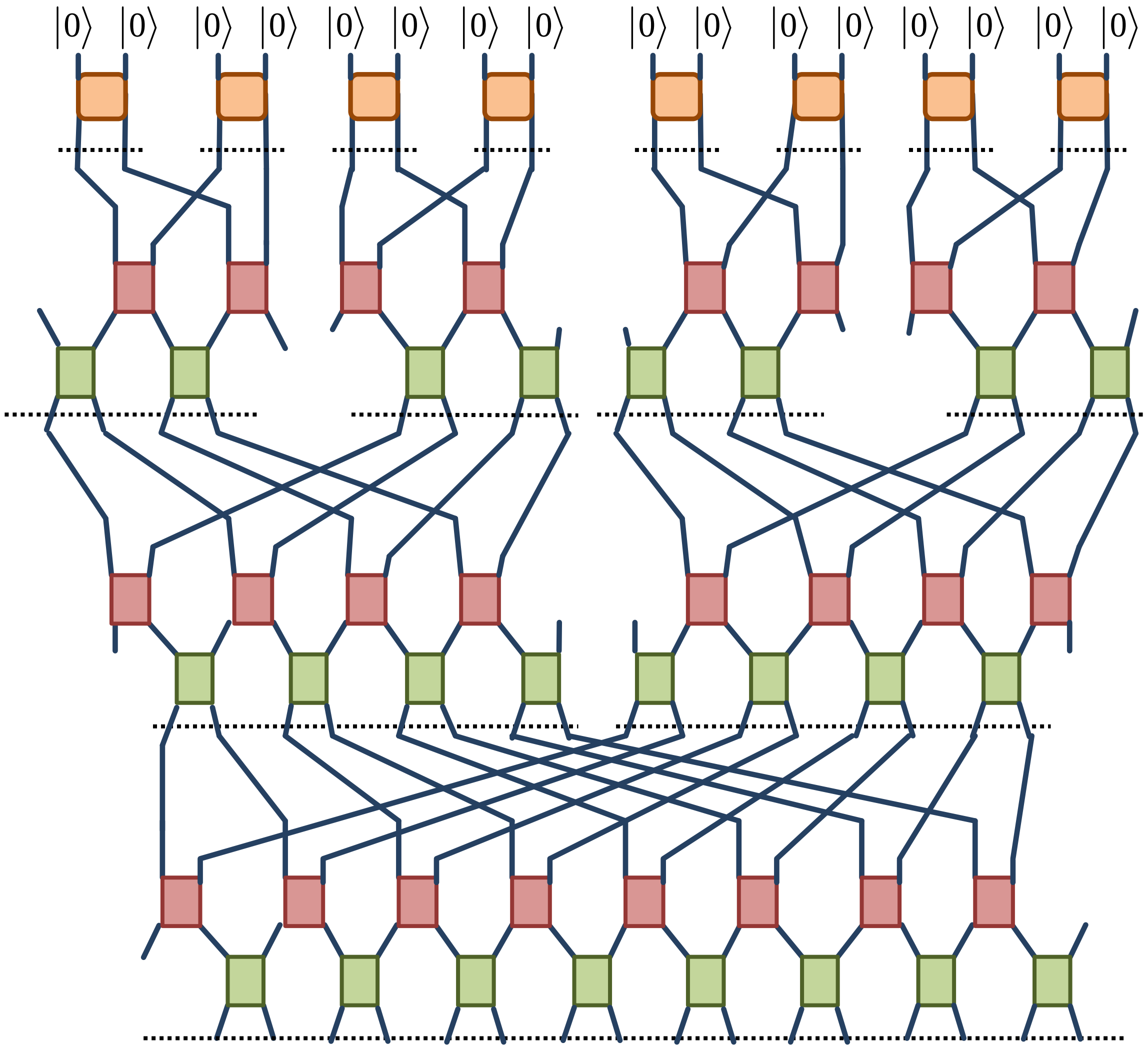
ranching MERA: entanglement entropy



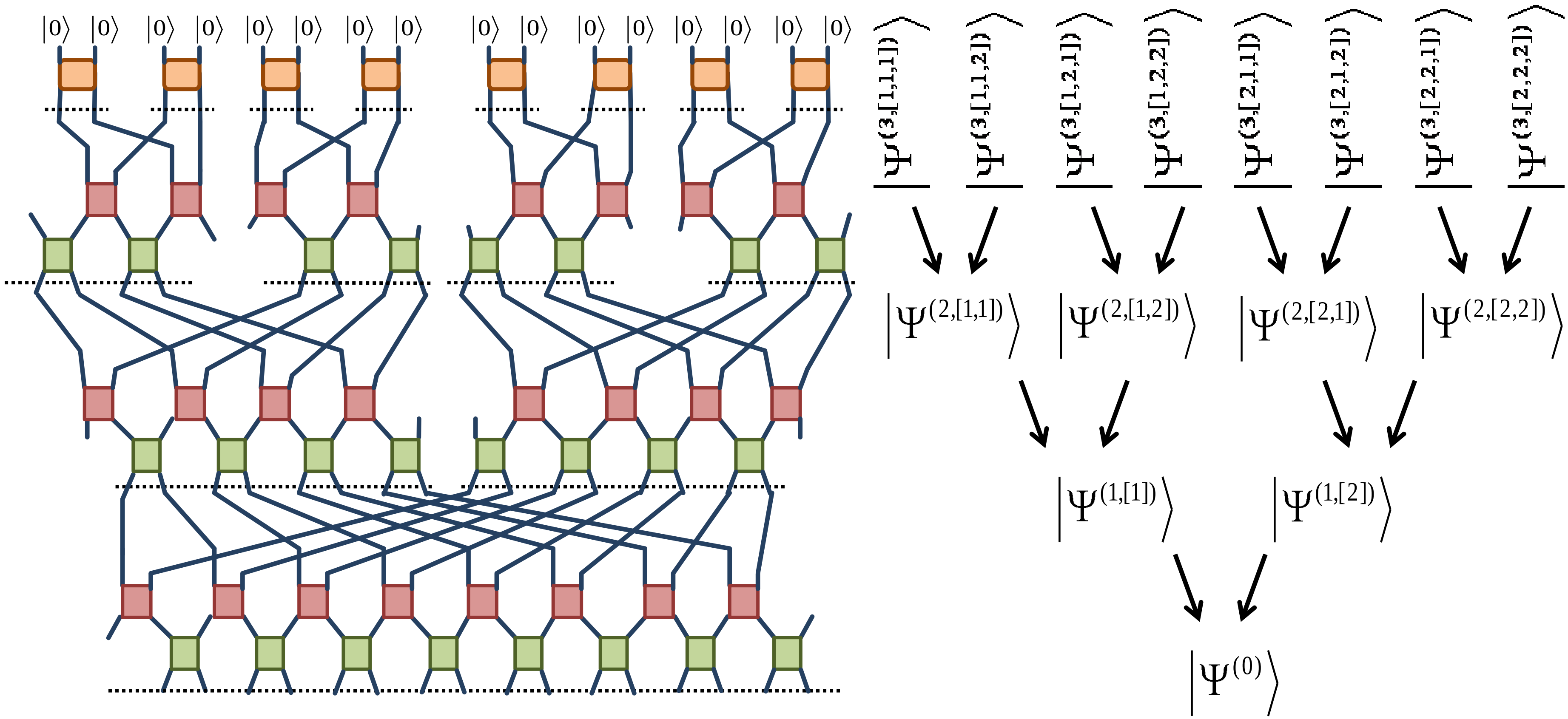
$$n(A) \approx 2\log(L)$$

scaling of entropy: $S(A) \approx 2\log(L)$

branching MERA



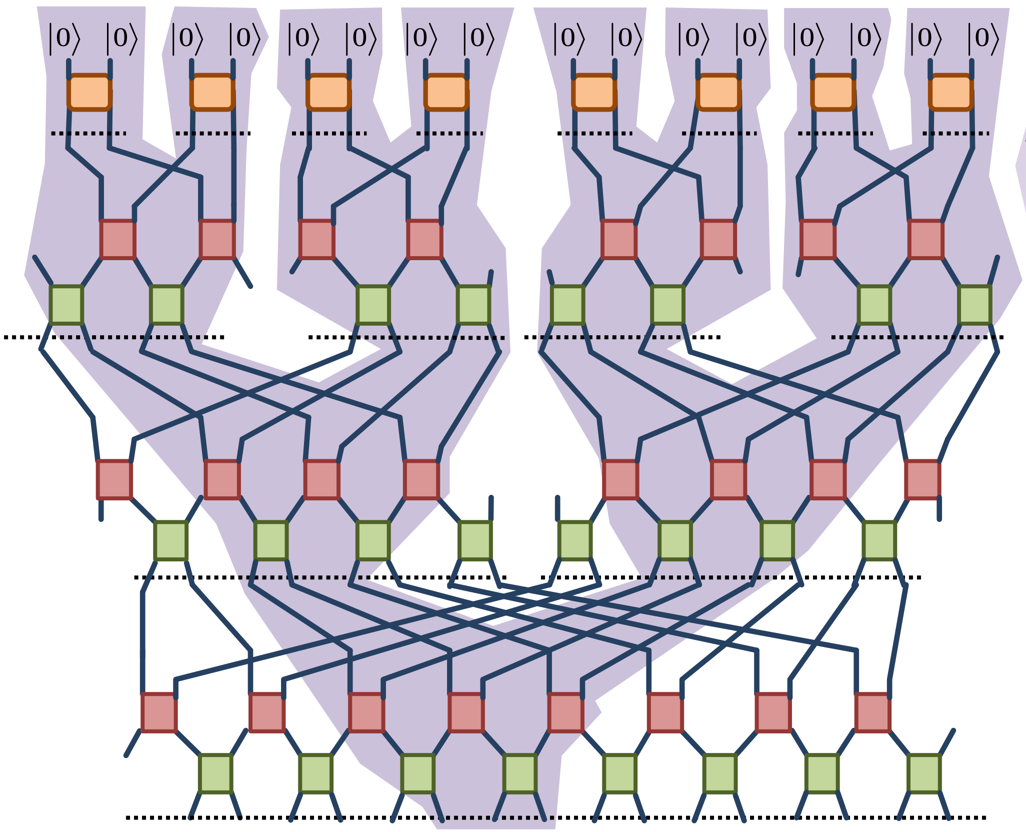
branching MERA

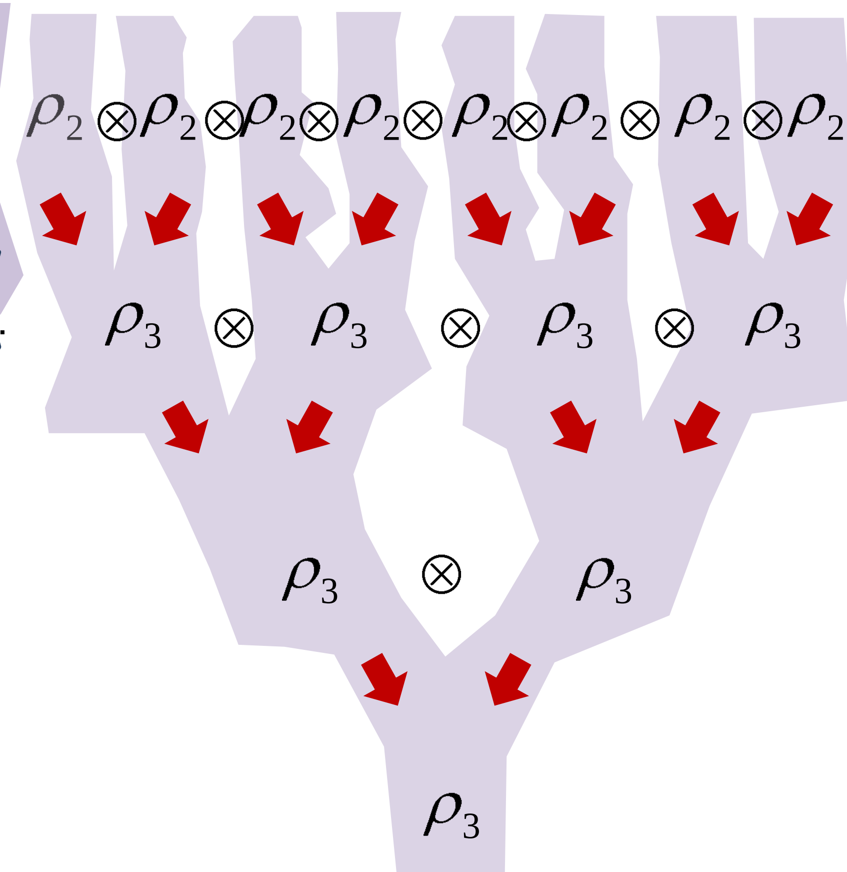


branching MERA: computational cost

past causal cone

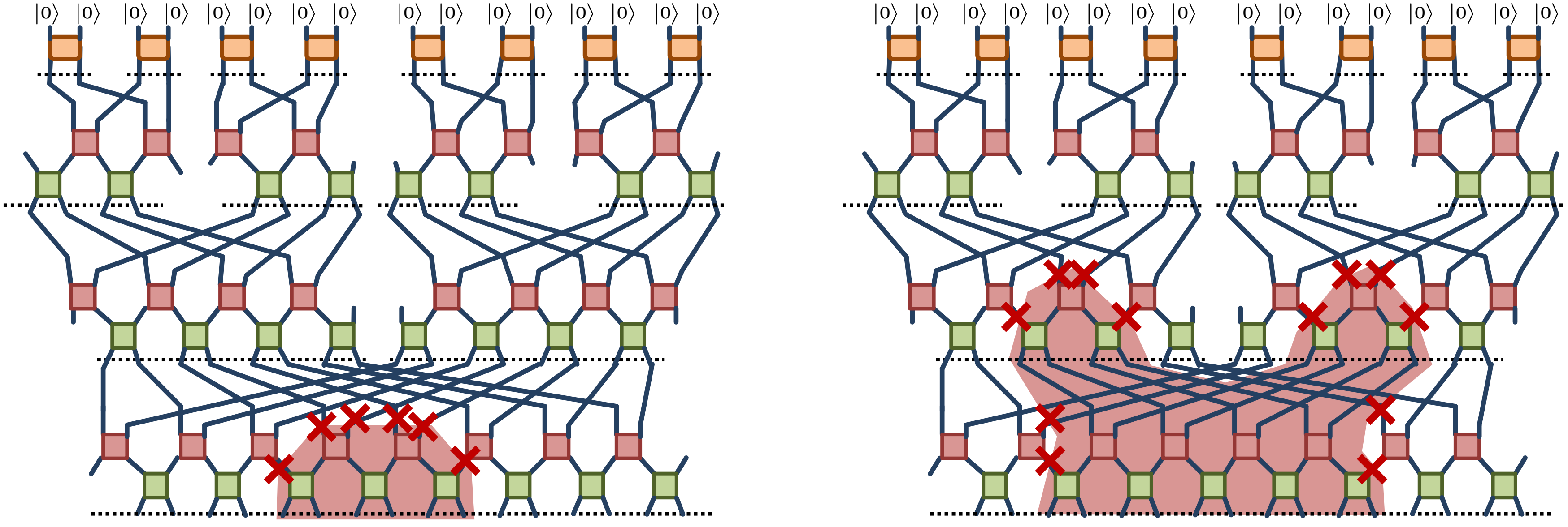
width: $w' = qw$





cost of computing $\rho(A)$: $c \approx q \exp(w)$ $c \approx O(N)$

branching MERA: entanglement entropy

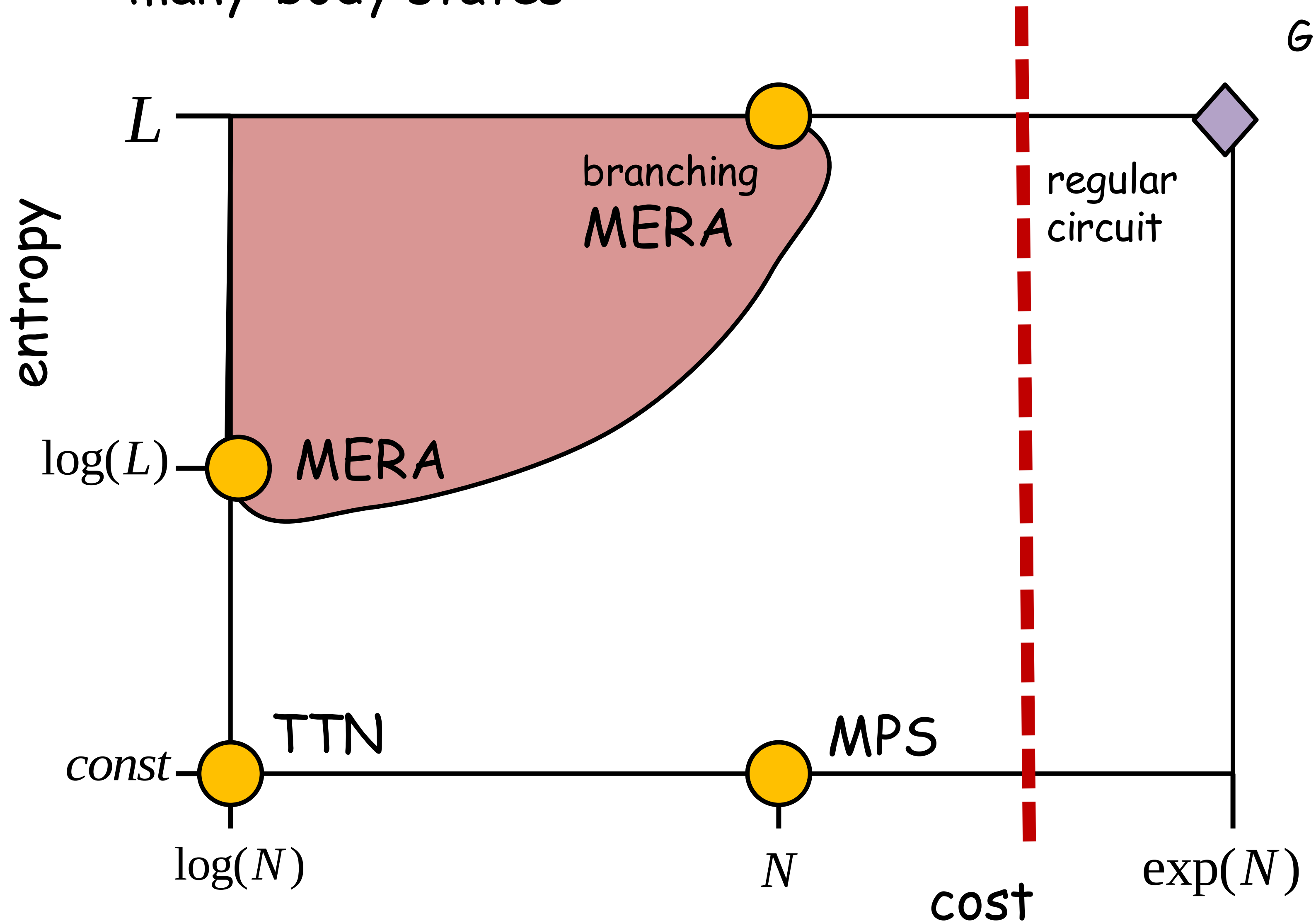


$$n(A) \approx O(L)$$

scaling of entropy: $S(A) \approx L$

Conclusions

- quantum circuits can be used to encode many-body states



Conclusions

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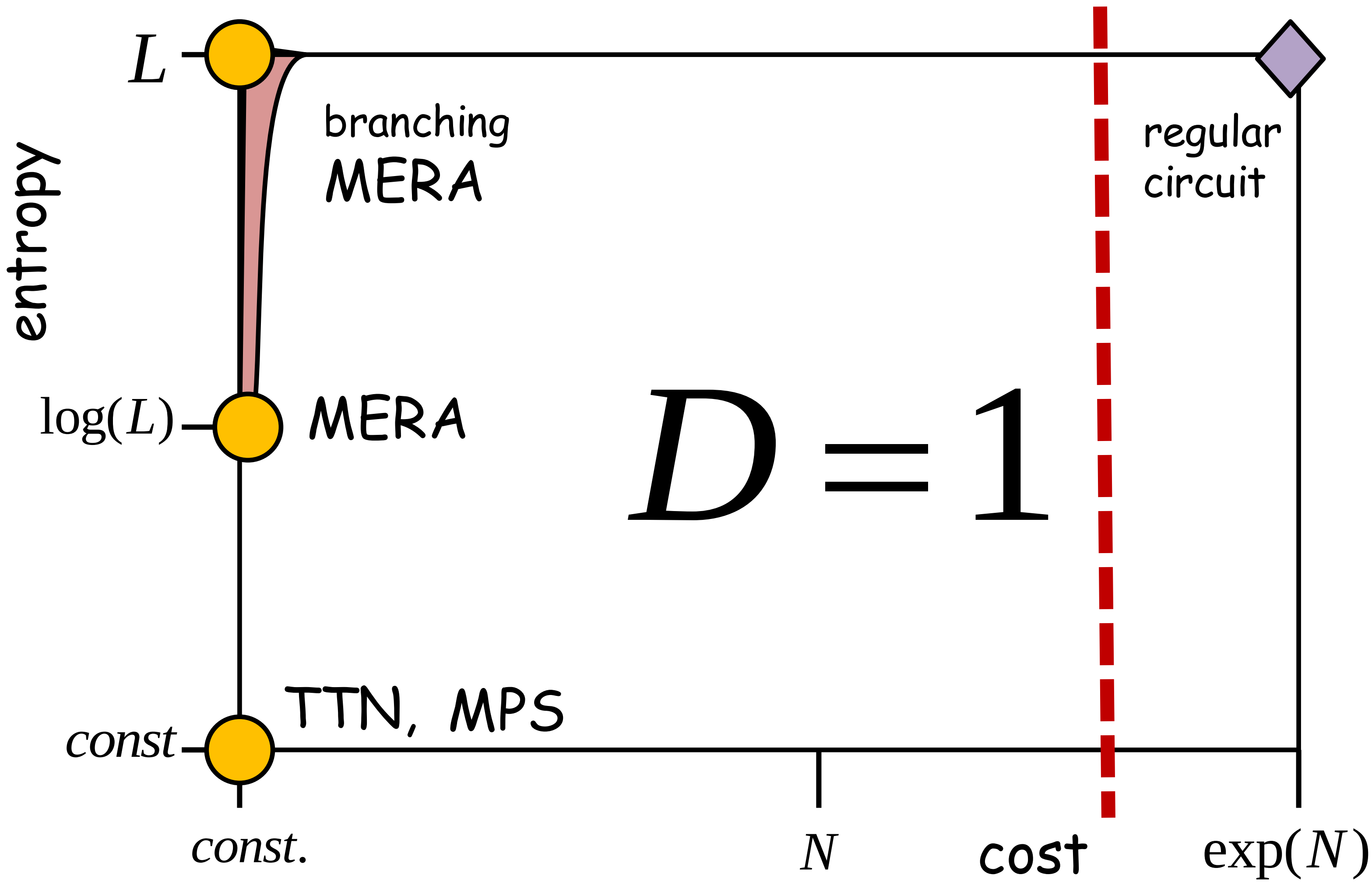
let us add translation (+scale) invariance



Conclusions

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