Non-linear Theory of FQH Edge:

Fractionally Charged Solitons, Emergent Topology in Non-linear Waves, Quantum Hydrodynamics of FQH liquid

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(Discussions with friends: Abanov, Bettelheim, Cappelli)

Phys. Rev. Lett. 108, 206810 (2012)

Florence

May 25, 2012

Messages

- Waves on the Edge of FQH are essentially non-linear;
- Emergence of quantization in non-linear dynamics;
- FQH - hydrodynamics "Hall-viscosity" in the bulk propels to the boundary (a universal corrections to Chern-Simon "theory")
- Relation between FQHE and CFT - revised

Only Laughlin's states (for now).

FQHE - Laughlin's state(s)

Particles on a plane in a quantized magnetic field (with a strong Coulomb Interaction)

$$\Psi_0(z_1,\ldots,z_N) = \left[\Delta(z_1,\ldots,z_N)\right]^{\beta} e^{-\sum_i |z_i|^2/4\ell_B^2}$$

- $\Delta = \prod_{i\neq j}(z_i - z_j)$- VanDerMonde determinant
- ℓ_B -magnetic length;
- $\nu = 1/\beta$ - is a filling fraction;
- $\beta = 1$ - IQHE; $\beta = 3$- FQHE.

Important features:

- Wave-function is holomorphic;
- Degree of zero at $z_i \to z_j$ is larger than 1;

Spectrum in the bulk:

- The ground state is $\beta = \nu^{-1}$ - degenerate;
- All excitations are gapped: $\Delta_{1/3} \sim 10 - 30K, \quad kT \ll \Delta_{1/3} \ll \hbar\omega_c$
- Coherent States: deformation of Laughlin's state by a holomorphic function

$$\Psi_0(z_1, \ldots, z_N) = \left[\Delta(z_1, \ldots, z_N)\right]^\beta e^{-\frac{1}{2}\sum_i |z_i|^2/2\ell_B^2}$$

$$\Psi_V(z_1, \ldots, z_N) = \Psi_0 e^{\sum_i V(z_i)},$$

- Singularities of V are vortices (or "quasi-holes")

$$\sigma = -\frac{1}{4\pi}\Delta V = \text{Real}$$

Edge states: FQHE in a confining potential

- Potential well lifts a degeneracy:

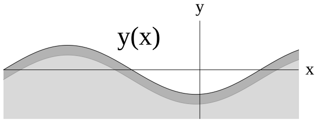
$$H_0 \to H = H_0 + \sum_i U(|r_i|)$$

- Low energy states emerge. They are localized on an edge → Edge States;

- Smooth potential: Curvature of the potential is small compared to the gap but a slope is larger than electric field

$$\ell_B^2 \nabla_y^2 U \ll \Delta_\nu, \quad \ell_B^2 \nabla_y U \gg e^2$$

Figure: Boundary waves: the boundary layer is highlighted



LINEAR EDGE STATES THEORY (WEN, 1991)

- Density is a chiral field:

$$\rho(x) = \sum_k e^{ikx}\rho_k, \quad \rho_k = \rho^\dagger_{-k},$$
$$[\rho_k, \rho_l] = \nu k \delta_{k+l},$$

$$(\partial_t - c_0 \nabla_x)\rho = 0,$$

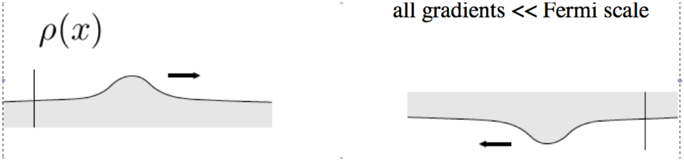
- $c_0 = \hbar^{-1}\ell_B^2|\nabla_y U|$ is a slope of the potential well (non-universal);

- factor ν proliferates to the exponent in edge tunneling

- Common believe (I disagree with):

  $c = 1$ - CFT of free bosons with a compactification radius $\nu = \beta^{-1}$.

Propagation of a wave-packet



The linear theory does not answer a question:
how does a smooth non-equilibrium state (a wave packet) propagate?

$$\dot{\rho} - c_0 \nabla \rho = 0, \quad \text{wave equation,}$$
$$\rho(x,t) = \rho(x - c_0 t, t = 0)$$

The shape does not change !?

$$\boxed{\text{A new scale must be included in the theory;}}$$

New scale: $\Delta_{1/3} \ll \hbar\omega_c$ - energy of a hole or a vortex (non-universal scale);

Most phenomena do not depend on the scale and are universal;

NON-LINEAR THEORY OF EDGE STATES

- Linearized version (fails in $10ps$): $(\partial_t - c_0\nabla_x)\rho = 0$
- Universal description of non-linear chiral boson at FQHE edge

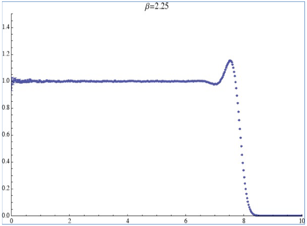
$$\boxed{(\partial_t - c_0\nabla_x)\rho - \kappa\nabla\left(\frac{1}{2}\rho^2 - \frac{1-\nu}{4\pi}\nabla\rho_H\right) = 0}$$

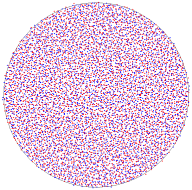
$$[\rho(x),\, \rho(x')] = \nu\nabla\delta(x-x'),$$

$$f_H(x) = \frac{1}{\pi}P.V.\int \frac{f(x')}{x-x'}dx'$$

- New scale: $\kappa \sim \Delta_\nu \ell_B^2/\hbar$ - energy of a quasi-hole less cyclotron energy (non-universal scale) but the form of equation is universal;
- The universal coefficient (important!)

Overshoot: dipole moment of FQHE droplet



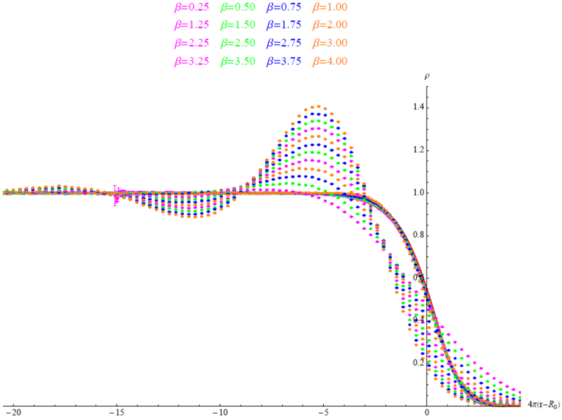


$$\Psi_0 = \left[\prod_{i>j}(z_i - z_j)\right]^{\beta} e^{-\frac{1}{2}\sum_i |z_i|^2/4\ell_B^2},$$

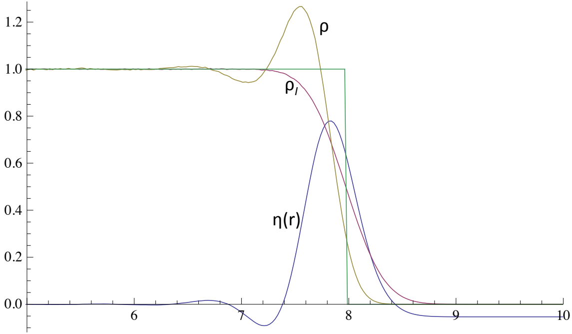
$$\mathbf{d} = \int_0^R (r-R)\rho(r)dr = \frac{1-2\nu}{8\pi}$$

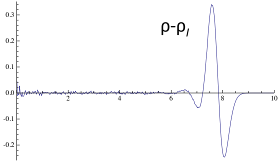
Dipole moment of a spherical droplet

$$\langle \rho \rangle = \lim_{N\to\infty} \int |z - z_i|^{2\beta} |\Delta|^{2\beta} e^{-\sum_i |z_i|^2/2\ell_B^2} \prod_i d^2 z_i$$



No overshoot at $\beta = \frac{1}{\nu} \leq 1$.





$$\boxed{\rho(y) \approx \rho_I(y) + \eta\delta'(y)}$$

$$\boxed{\eta = \int y(\rho - \rho_I)dy = \frac{1-\nu}{4\pi}}$$

Benjamin-Ono Equation: Properties

$$\boxed{(\partial_t - c_0 \nabla)\rho - \kappa \nabla \left(\frac{1}{2}\rho^2 - \frac{1-\nu}{4\pi} \nabla \rho_H \right) = 0}$$

- Classical Benjamin-Ono equation describes surface waves of interface of stratified fluids;
- Integrable (despite being non-local);

BENJAMIN-ONO EQUATION: FRACTIONALLY QUANTIZED SOLITONS

$$\boxed{(\partial_t - c_0\nabla)\rho - \kappa\nabla\left(\tfrac{1}{2}\rho^2 - \tfrac{1-\nu}{4\pi}\nabla\rho_H\right) = 0}$$

- Two branches of solitons:
  - **subsonic**: *holes* propagating to the left;

    Charge $\nu = 1/\beta$: $\int \rho_h dx = \text{integer} \times \nu$

  - **ultrasonic**: *particles* propagating to the right:

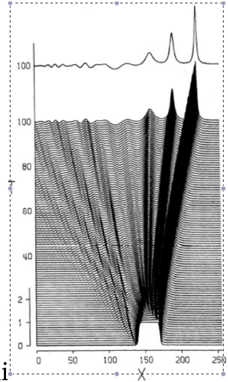
    Charge 1: $\int \rho_p dx = \text{integer}$

$$\boxed{\rho = \frac{q}{\pi}\frac{A}{(x - V_q t)^2 + A^2}} \quad q = 1,\ -\nu, \quad V_q = q\kappa A$$

- Classical Benjamin-Ono equation has only one branch - *particles*

$$\boxed{\text{Benjamin-Ono is the only integrable equation with a quantized charge of solitons}}$$

Two branches of excitations: Separation between holes (moving right) and particles (moving left)



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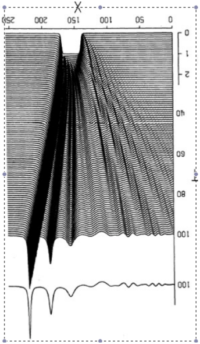
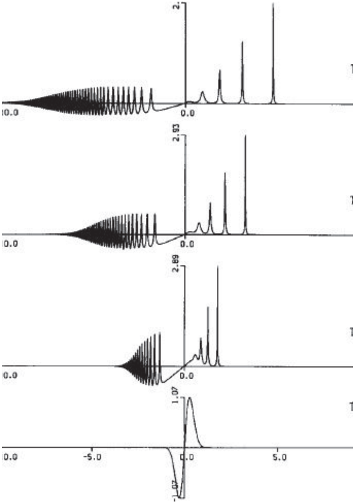


Quantization through evolution



Input: Quantum Hydrodynamics

- Laughlin's coherent states:

  (i) Analyticity, (ii) Degree of zeros $\beta = 3$

$$\Psi_V = \prod_{i<j} (z_i - z_j)^\beta e^{-\sum_i |z_i|^4/2\ell_B^2 + \sum_i V(z_i)}$$

- Galilean Invariance

$$H = \frac{m}{2}\mathbf{v}^\dagger \rho \mathbf{v}, \quad \Delta_{1/3} \sim \frac{\hbar}{m\ell_B^2}$$

- Velocity $\mathbf{v} = \mathbf{v}_x - i\mathbf{v}_\mathbf{y}$

$$\mathbf{v}\,|\Psi_0\rangle = 0,$$

$$\frac{i}{2\hbar} m_\nu \mathbf{v}_i = \partial_{z_i} - \frac{e}{2c} A(z_i) - \sum_{j\neq i} \frac{\beta}{z_i - z_j}, \quad \beta = \frac{1}{\nu}.$$

Incompressible Chiral Quantum Fluid

- Laughlin's coherent states:

$$\Psi_V = \prod_{i<j}(z_i - z_j)^\beta e^{-\sum_i |z_i|^2/2\ell_B^2 + \sum_i V(z_i)}$$

- Velocity

$$\frac{i}{2\hbar} m_\nu \mathbf{v}_i = \partial_{z_i} - \frac{e}{2c} A(z_i) - \sum_{j\neq i} \frac{\beta}{z_i - z_j}, \quad \beta = \frac{1}{\nu}.$$

- Velocity matrix elements

$$m\mathbf{v}\,|\Psi_V\rangle = -2i\partial_z V|\Psi_V\rangle$$

- Incompressibility

$$\nabla \cdot \mathbf{v} = 0, \quad \mathbf{v} = \nabla \times \Psi$$

- Edge states dynamics - irrotational flow (no vortices)

$$\nabla \times \mathbf{v} = 0, \quad \Delta\Psi = 0$$

Subtleties and main steps

Chiral Constraint (Property of Laughlin's states)

Relation between velocity and density

$$\boxed{\nu(\nabla \times \mathbf{v}) = \rho - \rho_I + \frac{1-\nu}{4\pi}\Delta \log \rho}$$

(Wiegmann, Zabrodin, 2006)

Potential flow and the Boundary Waves

$$\Delta \mathbf{v} = 0, \quad im\mathbf{v} = \int \frac{\rho_\nu - \rho_I}{z - z'} \frac{d^2 z'}{2\pi\nu} + \frac{1-\nu}{4\pi\nu} \partial \log \rho$$

Boundary value of velocities

$$m(\mathbf{v}_x - c_0) = -\nu^{-1} \bar{\rho} y(x), \quad m\mathbf{v}_y = \frac{1-\nu}{4\pi\nu} y^H_{xx}$$

Kinematic Boundary Condition

$$\boxed{\dot{y} + \mathbf{v}_x \nabla_x y + \mathbf{v}_y = 0}$$

leads to Quantum Benjamin Ono Equation.

Quantum Hydrodynamics in the bulk

- Laughlin's state reformulated as a hydrodynamics if the bulk:

$[\mathbf{v}(r), \rho(r')] = -i\nabla\delta(r-r');$	canonical hydro-variables,
$\dot{\rho} + \nabla(\rho\mathbf{v}) = 0,$	continuity equations
$\nabla \cdot \mathbf{v} = 0;$	incompressibility,
$[\mathbf{v}(r) \times \mathbf{v}(r')] = 4\pi\nu^{-1} i\delta(r-r'),$	Heisenberg algebra
$\boxed{\nu(\nabla \times \mathbf{v}) = \rho - \rho_I + \frac{1-\nu}{4\pi}\Delta \log \rho}$	Chiral constraint

Subtleties

- Short-distance anomaly or OPE

$$\langle \rho \mathbf{v} \rangle = \langle \rho \rangle \langle \mathbf{v} \rangle - \frac{1}{4\nu} \nabla^* \langle \rho \rangle$$

- Dipole moment and singularity on the boundary

$$\mathbf{d} = \int_0^R (r - R)\rho(r) dr = \frac{1 - 2\nu}{8\pi}$$

Subtleties: Stress energy tensor

- Input: Gallilean invariance $E = \frac{m\rho\mathbf{v}^2}{2}$
- Outcome: Stress energy tensor - "Hall-viscositiy"

$$T_{xx} = m\rho\mathbf{v}_x\mathbf{v}_x + \frac{\hbar}{4\nu}\rho\left(\nabla_x\mathbf{v}_y + \nabla_y\mathbf{v}_x\right)$$

Benjamin-Ono Equation as a Deformed Boundary CFT

- Boundary CFT exterior of the droplet;
- Boundary stress energy tensor component

$$T_{nn} = \frac{1}{2}(\nabla_n \varphi)^2 + \frac{\nu - 1}{4\nu} \nabla_n \nabla_s \varphi, \quad -\nabla \varphi = \rho$$

- Benjamin-Ono Equation is a deformation of CFT:

$$\boxed{\dot{\rho} = \nabla T_{nn}}$$

- Deformation of a boundary is generated by the normal components of the stress-energy tensor.

CFT AND FQHE

- FQHE Edge hydrodynamics is a deformation of **Boundary** CFT with

$$\boxed{c = 1 - 6\left(\sqrt{\nu} - 1/\sqrt{\nu}\right)^2 < 1}$$

- CFT lives outside of a droplet;

Contrary to a common believe that FQHE $c = 1$ bulk CFT.

SUMMARY

- Boundary waves in FQHE are essentially nonlinear;
- Two branches of solitons with charges 1 and -ν;
- Deformation of the boundary are generated by a stress energy tensor of CFT situated outside of the dropletwith

$$c = 1 - 6\nu^{-1}(\nu - 1)^2 < 1$$

- An origin of shifting the central charge is a dipole moment located on the boundary of the droplet