Universality in the equilibration of isolated systems after a small quench

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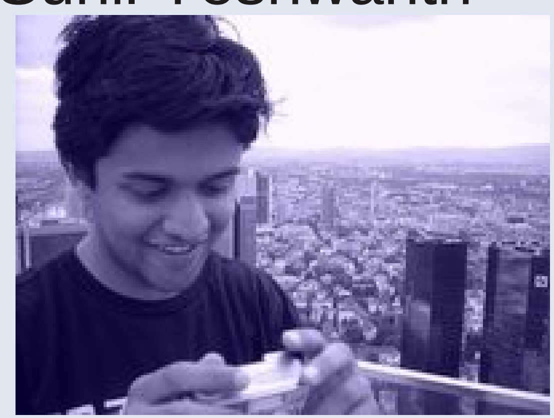


GGI, Firenze, May 25 2012

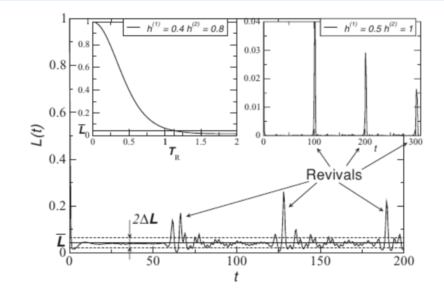
Preliminaries

1. Isolated, ***finite*** system
2. Prepare initial state ρ_0
3. Evolve with *H:* $\rho(t) = e^{-iHt} \rho_0 e^{iHt}$
4. Monitor observable A:

$$a(t) = \langle A(t) \rangle = Tr\, A \rho(t)$$

Equilibration? $\rho(t) \Rightarrow \rho_{eq}$

1. No strong convergence $\| \rho(t) - \rho_{eq} \| = cnst$
2. For finite systems no weak convergence
3. Stochastic convergence



•Observation window $[0, T]$

•Time average $\bar{f} = \int_0^T f(t) \frac{dt}{T}$

$$P_A(a)\, da = \mathtt{Prob}\left(\langle A(t) \rangle \in [a, a + da],\ t \in [0, T]\right)$$

$$P_A(a) = \overline{\delta(a - a(t))}$$

Equiliration = concentration of *P*$_A$*(a)*

Equilibration

$$\overline{a(t)} = \overline{Tr\, A\rho(t)} = Tr\, A\bar{\rho} \quad \Rightarrow \quad \rho_{eq} = \bar{\rho}$$

$$\Delta^2 a = \overline{a(t)^2} - \bar{a}^2 \leq \mathrm{Ran}_A^2\, Tr\, \bar{\rho}^2$$

+ "gap" non-degeneracy

$$E_i - E_j = E_l - E_m$$
$$\Rightarrow i = j, l = m \vee i = l,\ j = m$$

Chebyshev's inequality

$$\mathrm{Prob}(|a(t) - \bar{a}| \geq \epsilon) \leq \frac{\Delta^2 a}{\epsilon^2}$$

Reimann, PRL (2008)

$$Tr\, \bar{\rho}^2 = \overline{L(t)}$$

$$L(t) = \left| \langle \Psi | e^{-itH} | \Psi \rangle \right|^2 = \exp 2 \sum_{n=1} \langle H^n \rangle_c \frac{(-t^2)^n}{2n!} = e^{Vf(t)}$$

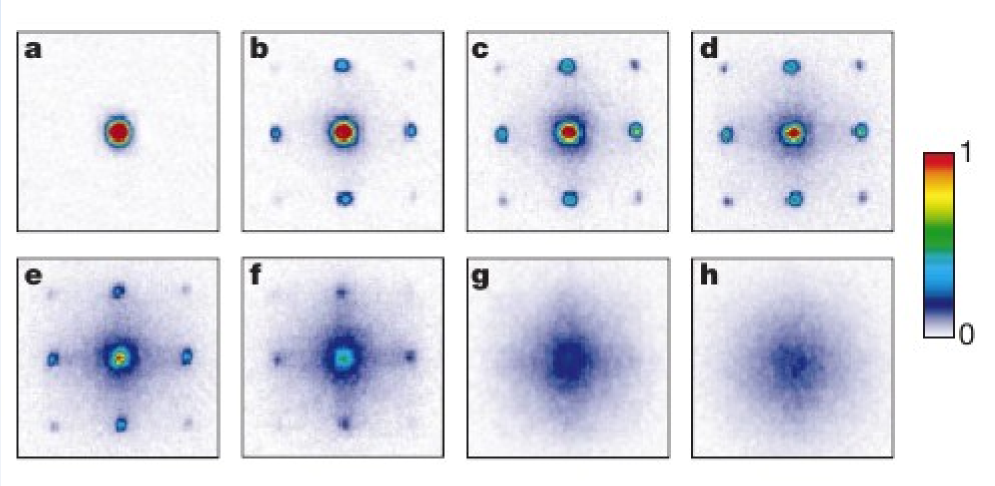
$$\Rightarrow \quad Tr\, \bar{\rho}^2 \propto e^{-\alpha V}$$

Free systems?

Quench: dynamical detection of QPT's

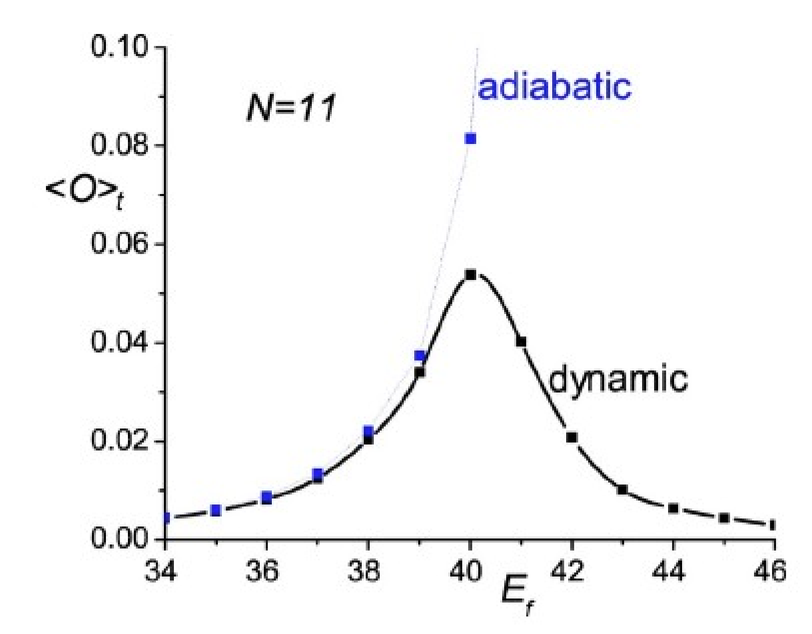
$$H \;\to\; |\Psi_0\rangle$$
$$H' = H + \delta\lambda V$$

superfluid-Mott transition: experiment



Greiner et al., Nature (2002)

Theoretical description



Sengupta, Powell, Sachdev, PRA (2004)

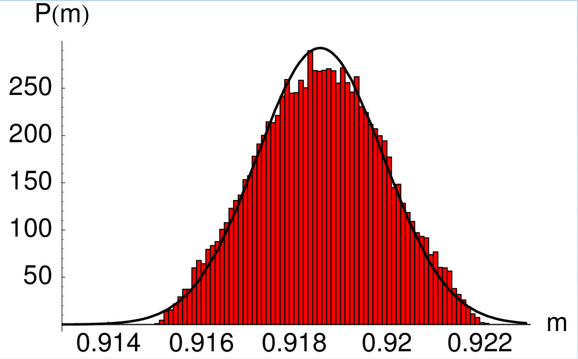
Small* quench: full statistics

$\delta\lambda = 0.04$

- Small quench, off-critical

  $L \gg \xi$

L = 12

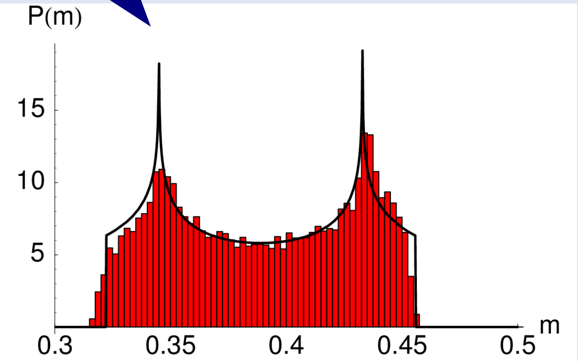


Bi-modal: phase transition

- Small quench, quasi-critical

  $\xi \gg L$

L = 16



(*) Small: $\delta\lambda^2 \chi_F \ll 1 \;\Rightarrow\; \delta\lambda \ll \frac{1}{L^{1/\nu}}$

Small quench: CLT

$$\langle A(t)\rangle = \sum_{n,m} A_{n,m}\overline{c(E_n)}c(E_m)e^{-it(E_m-E_n)} \qquad c(E_n) = \langle E_n, \Psi_0\rangle$$

$$\approx \overline{A} + \sum_{n>0} F_n \cos(t(E_n - E_0)) \qquad F_n \approx 2A_{n,0}c(E_n)$$

rational independence → *a(t)* sum of independent variables

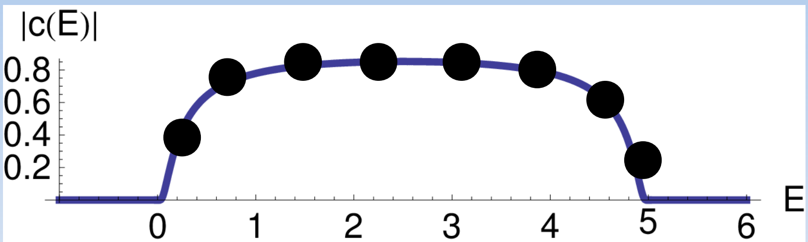
$$\Delta^2 a = \frac{1}{2}\sum_{n>0} F_n^2 \leqslant 2[\langle 0|A^2|0\rangle - \langle 0|A|0\rangle^2] \propto L$$

→ *"Generally" CLT*

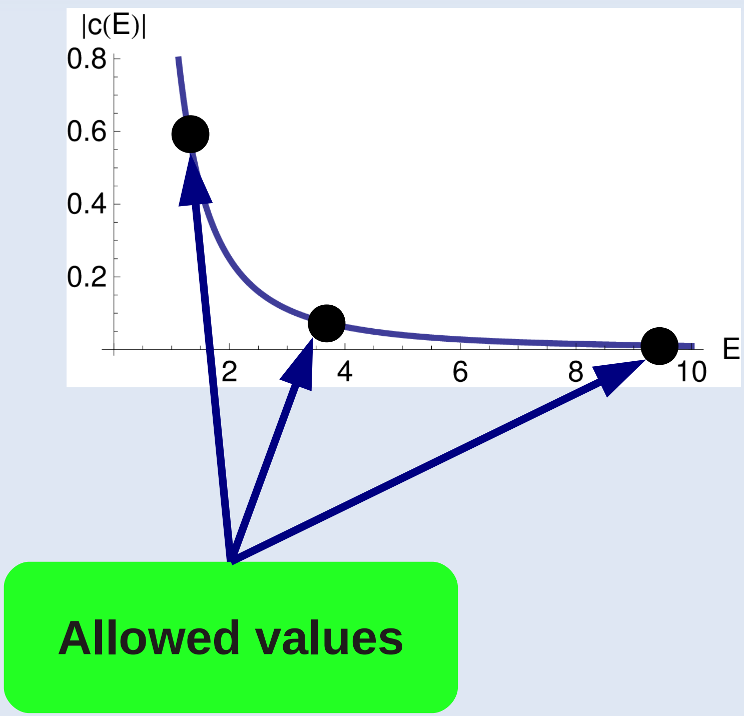
Look at F_n, $c(E_n)$

Small quench: explanation

Off-critical distribution



Critical case distribution



Scaling prediction at criticality:

$$c(E) \sim E^{-1/\nu}$$

sum rule

$$\sum_E |c(E)|^2 = 1$$

Only few $|E\rangle$ states contribute:
Bi-modal distribution

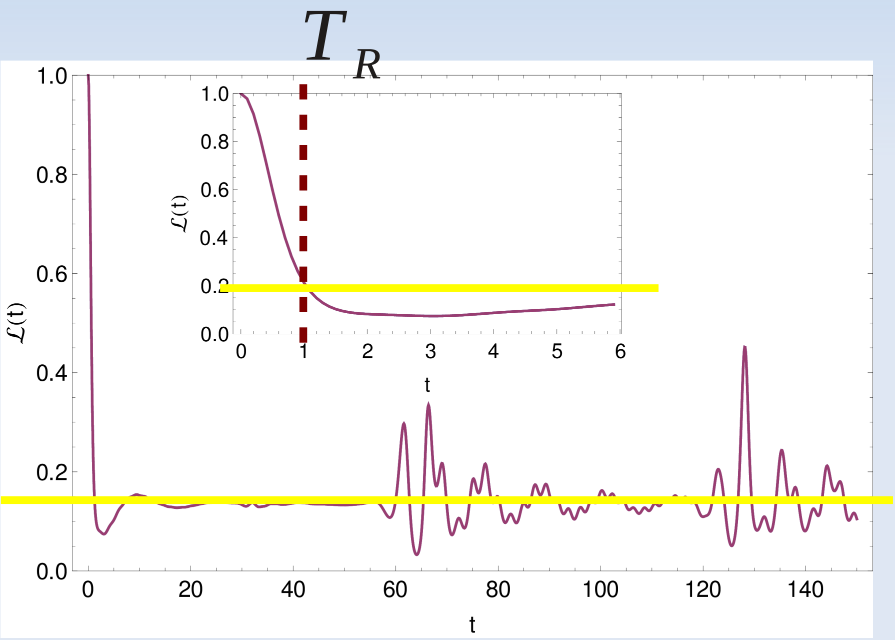
Small quench: critical case

- Q: How to break CLT?
- A: most $F_n \to 0$

(Quantum) critical points are
more stable against perturbations

Relaxation time

(Talk by Michael Pustilnik)



$$\langle A(T_R)\rangle := \overline{A}$$

Loschmidt echo

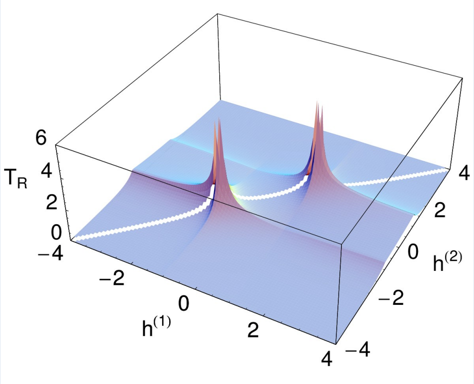
$$L(T_R) := \overline{L}$$

Relaxation time II

Loschmidt echo: short time - cumulant expansion

Generally
(+small quench off-critical)

$$L(t) = e^{-\sigma^2 t^2} \qquad \overline{L} = e^{-\alpha L^d}\, , \sigma^2 \sim L^d \;\Rightarrow\; T_R = O(L^0)$$



small quench criticality

$$\overline{L} \simeq e^{-2\delta\lambda^2 \chi_F}\, , \sigma^2 \sim L^{2(d-\Delta)} \, , \chi_F \sim L^{2(d+\zeta-\Delta)} \;\Rightarrow\; T_R = O(L^{\zeta})$$

$$T_R \sim |\lambda - \lambda_c|^{-\zeta\nu}$$

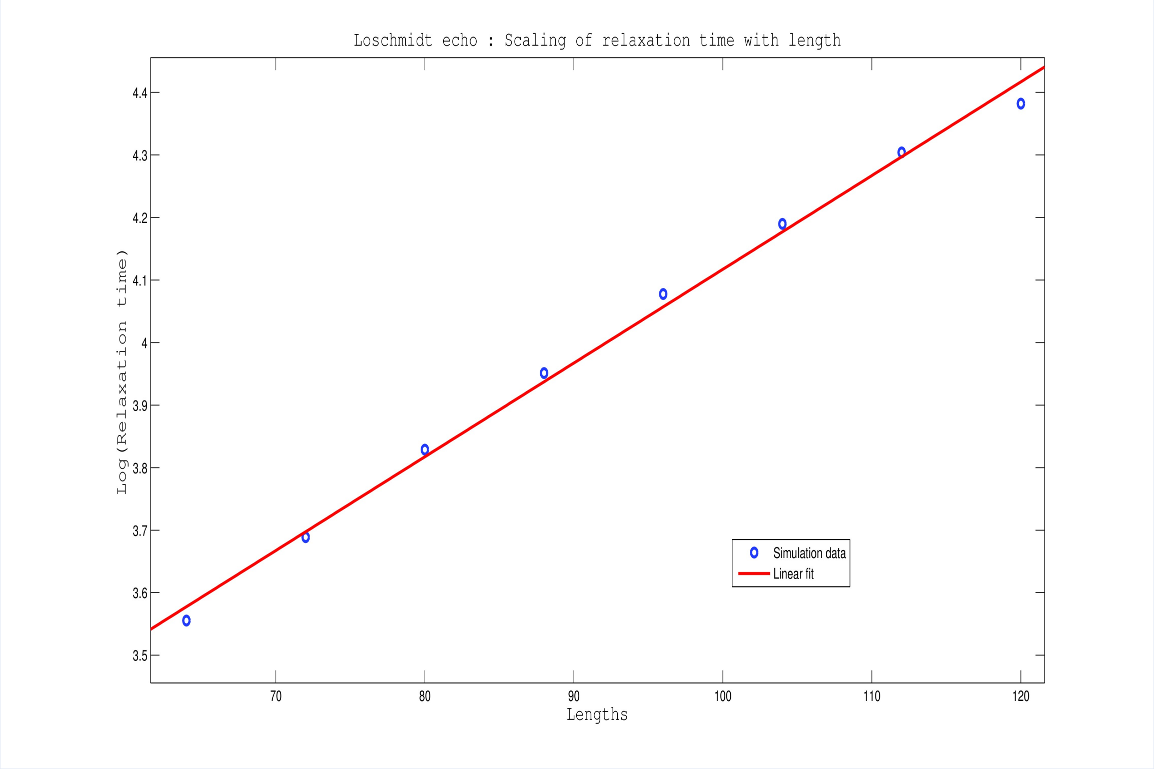
Relaxation time: Random Systems

~ Inguscio, Modugno, LENS

$$H=\sum_x \left(c_x^\dagger c_{x+1}+c_{x+1}^\dagger c_x\right)-\mu_x c_x^\dagger c_x$$

Random field

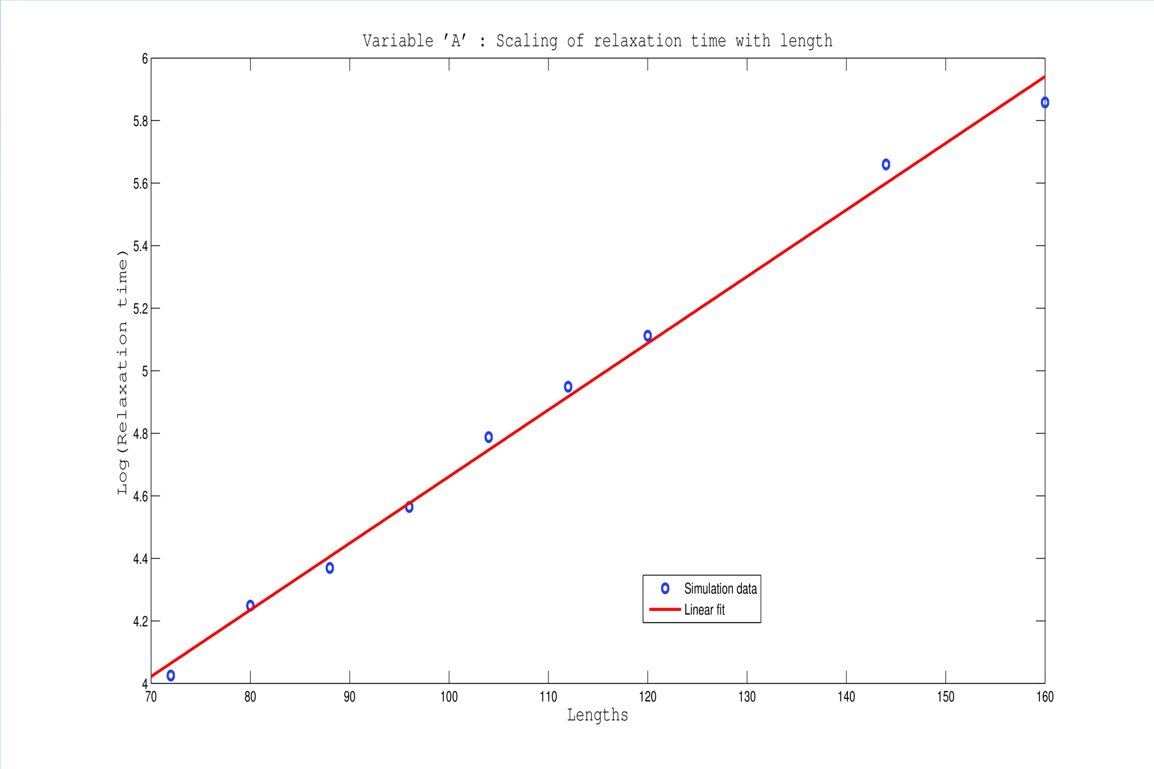
Loschmidt echo



$$\mathrm{E}\left[L(t)\right]$$

Relaxation time: Random Systems

Number operator $\mathrm{E}\left[\left\langle N_l(t)\right\rangle\right]$



$$T_{\mathrm{Relax}} \sim e^{\alpha L}$$

Equilibration & Integrability

$$a(t) = \langle A(t) \rangle \quad \rightarrow \quad P_A(a) = \overline{\delta(a - a(t))}$$

Generally, for *A* extensive:

$$\overline{a(t)} \propto V \qquad\qquad \Delta^2 a \leq O\left(e^{-\alpha V}\right)$$

Integrable systems (free Fermions)

$$H = \sum c_x^\dagger M_{x,y} c_y$$
$$A = \sum c_x^\dagger A_{x,y} c_y$$
$$R_{y,x} = \langle c_x^\dagger c_y \rangle$$

$$\begin{aligned} a(t) &= Tr\left(A e^{-itM} R e^{itM}\right) \\ &= \sum_{k,q} \underbrace{A_{q,k} R_{k,q}}_{F_{k,q}/2} e^{-it(\epsilon_k - \epsilon_q)} \end{aligned}$$

Stat-mech parallel

Rational independence

$$\overline{e^{\lambda a(t)}} = \sum_{\theta' s} e^{\lambda E(\theta' s)} = e^{f(\lambda) V}$$

$$E(\theta' s) = \sum_{k,q} F_{k,q} \cos(\theta_k - \theta_q)$$

$$F_{k,q} = F(|k-q|)$$

Classical *XY* model
on lattice $F_{i,j}$
(infinite temperature)

All cumulants
extensive:
CLT

$$Z = \frac{(a(t) - \bar{a})}{\sqrt{V}}$$

Gaussian

Equilibration & Integrability

Generally, for *A* extensive:

$$\overline{a(t)} \propto V \qquad \Delta^2 a \leq O\left(e^{-\alpha V}\right)$$

Integrable systems (free Fermions)

$$\overline{a(t)} \propto V \qquad \Delta^2 a = O(V)$$

Gaussian(poor) equilibration

Loschmidt echo

$$[R, M] = 0$$

$$L(t) = \prod_k \left(1 - \alpha_k \sin^2(t\epsilon_k/2)\right)$$

- Applies to *XY* model
- Generalizes to thermal quenches
- Generalizes to Ulman Fidelity

$$Z = \frac{\log L(t) - \overline{\log L(t)}}{\sqrt{L}}$$

Gaussian, => *L(t)* Log-Normal

For general models (RI spectrum), work in progress

$$\mu_n(L(t)) = f\left(Tr\left(\bar{\rho}^{2k}\right)\right)$$

Curiosity: Riemman zeta

$$\zeta(\sigma+it) = Tr\left(e^{-itH}\rho_{\sigma}\right), \quad \rho_{\sigma}=e^{-\sigma H}$$

H primon gas:
free bosons

$$Z := \log\left|\zeta(\sigma_0+it)\right|^2$$

Satisfies
CLT

Very similar
to Loschmidt Echo)

Conclusions

- Finite systems
- Look at full time statistics

} Ingredients

- Small quench: a tool to detect criticality, engineer ”*new quantum states of matter*”
- Relaxation time can be defined
- Integrability & equilibration:

  integrable systems concentrate less

Thank you