Transport in quantum wires and conservation laws

Jesko Sirker

Department of Physics, TU Kaiserslautern

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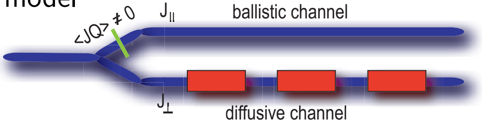
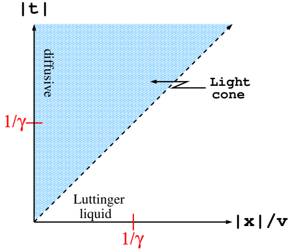
Outline

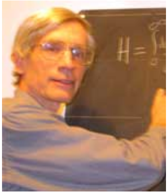
- Diffusive and ballistic spin transport in the anisotropic Heisenberg model



- Violation of the Luttinger liquid paradigm



Collaborations



Ian Affleck (UBC Vancouver)

Rodrigo Pereira (U Sao Paulo)

- JS, R. G. Pereira, I. Affleck
  PRL **103**, 216602 (2009), PRB **83**, 035115 (2011)

The role of conservation laws in 1D quantum systems

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Is transport always ballistic? Thermalization in closed systems?

Current-current correlations and the Drude weight

- Conductivity and Drude weight:

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$$D(T) = \lim_{t\to\infty}\lim_{L\to\infty}\frac{1}{2LT}\langle\mathcal{J}(t)\mathcal{J}(0)\rangle \geq \lim_{L\to\infty}\frac{1}{2LT}\sum_n \frac{\langle\mathcal{J}Q_n\rangle^2}{\langle Q_n^2\rangle}$$

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Ballistic versus diffusive based on current-current correlation:

Diffusive: $D(T>0)=0$

Ballistic: $D(T>0)\neq 0$

Local versus non-local conservation laws

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  - $\langle\mathcal{J}Q_n\rangle \sim L$
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- Non-local conservation law:
  - Possibly still $\langle \mathcal{J} Q_n\rangle \sim L$
  - but $\langle Q_n Q_n\rangle \sim L^2$ or even e^L
  - Contribution vanishes in the thermodynamic limit

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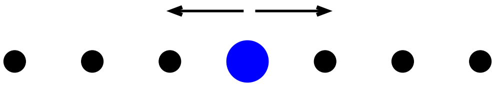

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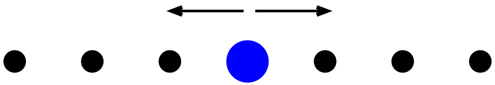

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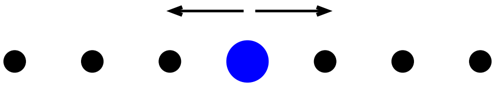

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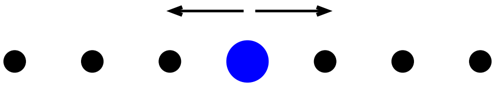

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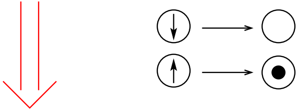
Diffusion in the autocorrelation function of the conserved quantity does not exclude the possibility of ballistic transport
$\leftrightarrow$ different correlation functions!

The anisotropic Heisenberg (XXZ) chain

- Spin-1/2 Heisenberg model with additional anisotropy Δ:

$$H = J \sum_j \left\{ \frac{1}{2} \left(S_j^+ S_{j+1}^- + S_j^- S_{j+1}^+ \right) + \Delta S_j^z S_{j+1}^z \right\}$$

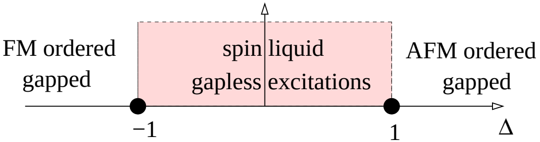
Jordan−Wigner transformation



- Equivalent to spinless fermion model:

$$H = J \sum_j \left\{ -\frac{1}{2} \left(c_j^\dagger c_{j+1} + c_j c_{j+1}^\dagger \right) + \Delta (n_j - 1/2)(n_{j+1} - 1/2) \right\}$$

Phase diagram:



Drude weight in the anisotropic Heisenberg model

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- Drude weight: $\sigma'(q=0,\omega) = 2\pi D\delta(\omega) + \sigma_{reg}(\omega)$
- Current operator: $\mathcal{J} = \frac{i}{2}\sum_l \left(S_l^+ S_{l+1}^- - S_{l+1}^+ S_l^-\right)$

The *XXZ* model is integrable, the energy current

$$\mathcal{J}_E = \sum_l j_l^E \quad \text{with} \quad j_l^E = -\mathrm{i}[h_{l-1,l}, h_{l,l+1}]$$

is conserved.

Finite magnetic field (broken particle-hole symmetry)

- $h \neq 0$: $\langle \mathcal{J}\mathcal{J}_E \rangle \neq 0$ then $\mathcal{J} = \mathcal{J}_\parallel + \mathcal{J}_\perp$, and $\mathcal{J}_\parallel = \frac{\langle \mathcal{J}\mathcal{J}_E \rangle}{\langle \mathcal{J}_E^2 \rangle}\mathcal{J}_E$ cannot decay

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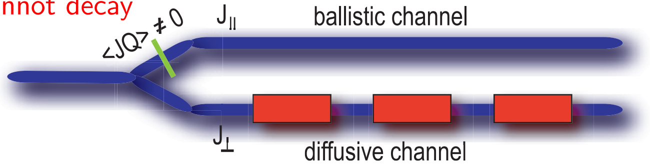

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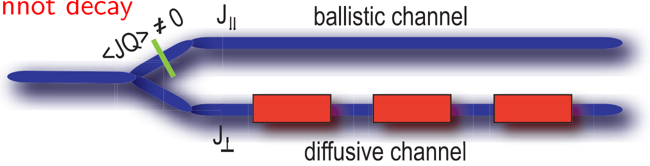
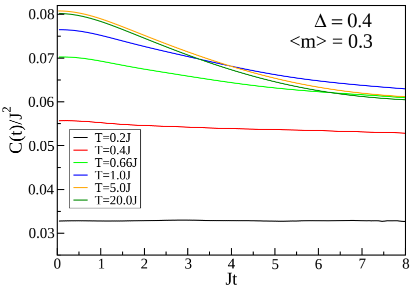


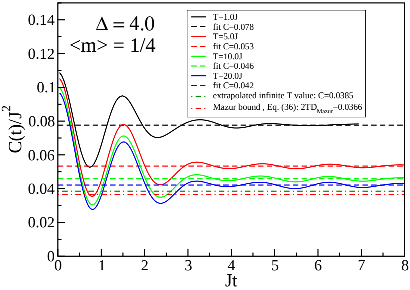
In general, ballistic and diffusive channels coexist

- Ballistic channel controls long-time asymptotics of the current-current correlation function
- Diffusive channel dominates long-time asymptotics of the low-energy, long-wavelength contribution of $\langle S^z_{l+x}(t) S^z_l(0) \rangle$

Transfer-matrix DMRG at finite magnetic field

- TMRG: Finite temperature, infinite system size — Bursill *et al.* (96)
- Current CF: $C(t) = \lim_{L\to\infty} \langle \mathcal{J}(t)\mathcal{J}(0)\rangle / L$ — JS, Klümper (05)





If a local conservation law protects a substantial part of the current then $C(t)$ converges relatively fast towards the asymptotic value

Linear response and bosonization

- Bosonization of the XXZ model at $h = 0$ leads to:

$$H = H_0 + H_{\mathrm{u}} + H_{\mathrm{bc}}$$

$$H_0 = \frac{v}{2}\int dx\,\left[\Pi^2 + (\partial_x\phi)^2\right], \quad H_{\mathrm{u}} = \lambda\int dx\cos(\sqrt{8\pi K}\phi)$$

$$H_{\mathrm{bc}} = -2\pi v\lambda_+\int dx(\partial_x\phi_R)^2(\partial_x\phi_L)^2 - 2\pi v\lambda_-\int dx\,\left[(\partial_x\phi_R)^4 + (\partial_x\phi_L)^4\right]$$

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- In linear response we have to calculate the retarded spin-spin corr. fct. $\chi_{\rm ret}(q,\omega)$, related to the boson propagator:

$$\frac{\chi_{\rm ret}(q,\omega)}{Kq^2/2\pi} = \langle\phi\phi\rangle^{\rm ret}(q,\omega) = \frac{v}{\omega^2 - v^2q^2 - \Pi^{\rm ret}(q,\omega)}$$

related approach for Hubbard: [Giamarchi PRB **44**, 2905 (91)]

Linear response and bosonization (II)

- We have obtained a parameter-free result for $\Pi^{\mathrm{ret}}(q,\omega)$ in second order Umklapp, and first order in band curvature

$$\Pi^{\mathrm{ret}}(q,\omega) \approx -2i\gamma\omega - b\omega^2 + cv^2q^2$$

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Spin-lattice relaxation: $\frac{1}{T_1 T} \sim \sqrt{\frac{\gamma(T)}{\omega_e}} \sim \sqrt{\frac{T/\ln^2(J/T)}{\omega_e}}$

Violations of the LL paradigm for time-like CF

- At $T = 0$ long-time asymptotics of $G(0,t) = \langle S^z_l(t) S^z_l(0)\rangle$ is dominated by high-energy excitations: (band curvature)

$$G(0,t) \sim \frac{e^{-ivt}}{t^\eta}, \quad \eta = (K+1)/2$$

Pereira *et al* PRL **100**, 027206 (08)

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- At $T\gg|h|$ a low-energy term dominates: (Umklapp)

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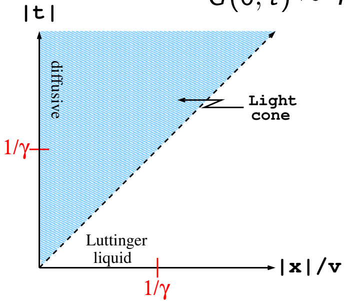

JS, Pereira, Affleck PRL **103**, 216602 (09)

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Conductivity and spin-lattice relaxation rate

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$$\frac{1}{T_1} = \frac{1}{2N}\sum_{r,r'}\int \frac{dq}{2\pi}|A(q)|^2 S^{+-}(q,\omega_N)\Big|_h$$

$$\approx -\frac{2T}{\omega_e}\int \frac{dq}{2\pi}|A(q)|^2 \chi''_{ret}(q,\omega_e)\Big|_{h=0} \quad (T \gg \omega_e)$$

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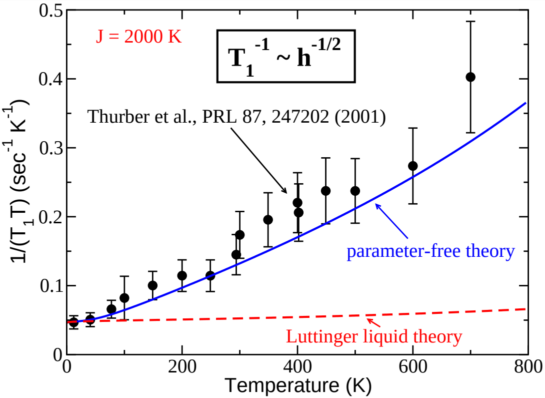
Diffusive part directly measured by $1/T_1$ if $A(q)$ picks $q \sim 0$ mode

^{17}O nuclear magnetic resonance for Sr_2CuO_3

$$\frac{1}{T_1} = \int \frac{dq}{2\pi} \underbrace{|A(q)|^2}_{\sim\cos(q/2)} S^{zz}(q, -\omega_e)\Big|_{h=0}$$

picks only $q \sim 0$ mode



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- Can be implemented by a memory-matrix approach [Rosch, Andrei, PRL (00)], reduces to self-energy result if $\langle JQ\rangle \to 0$:

$$\sigma'(\omega) = \frac{Kv(1-b_1)}{2\pi(1+y)}\left[\pi y\delta(\omega) + \frac{2(1+y)\gamma}{\omega^2 + [2(1+y)\gamma]^2}\right]$$

$$y \equiv \langle\mathcal{J}Q\rangle^2/(\langle\mathcal{J}^2\rangle\langle Q^2\rangle - \langle\mathcal{J}Q\rangle^2) \sim (h/T)^2 \quad (0 < |h| \ll T)$$

Weight shifts from diffusive into ballistic part

Relaxation rate: $\gamma \to (1+y)\gamma$

Drude weight in the half-filled case

$h = 0$: $\langle \mathcal{J} Q_n \rangle \equiv 0$ for all Q_n needed to construct the BA solution (particle-hole symmetry) Zotos, Naef, Prelovsek, PRB **55**, 11029 (97)

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Numerical results and BA seemed to point to $D \neq 0$ but were partly contradictory:

- BA: Zotos PRL **82**, 1764 (99), Benz *et al* JPSJ **74**, 181 (05)
- ED: F. Heidrich-Meisner *et al*, PRB **68**, 134436 (03), J. Herbrych *et al*, PRB **84**, 155125 (11)
- Plus many more

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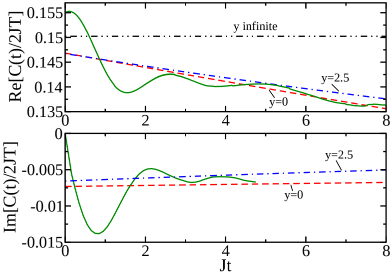
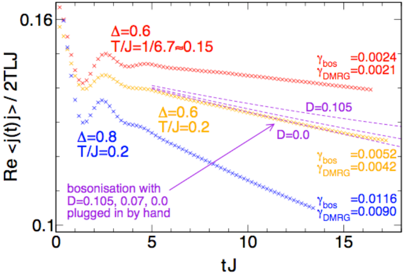
Is there another local conservation law not obtained from the family of commuting transfer matrices (BA)?

DMRG studies of the current correlation function

$$C(t) = \langle \mathcal{J}(t)\mathcal{J}(0)\rangle / L \sim e^{-2\gamma t} \approx 1 - 2\gamma t \quad (t \ll 1/\gamma)$$



$\Delta = 0.6$, $T/J = 0.2$; JS *et al* PRB (11)



C. Karrasch *et al* arXiv (11)

- Relaxation rate γ at small T agrees well with bosonization
- No clear evidence for finite Drude weight at low T
- Some numerical evidence for $D > 0$ at large T

A pseudo-local conservation law

- Construction of a pseudo-local conservation law $\mathcal{Q}$ for open boundary conditions: T. Prosen, PRL **106**, 217206 (11)

$$\mathcal{Q} = \sum_{d=1}^{L} \mathrm{e}^{-d} \sum_j q_{d,j}$$

$$[H, \mathcal{Q}] \sim -S_1^z + S_L^z$$

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- $\mathcal{Q}$ has part which is odd under p-h transformations
- $\langle \mathcal{J}\mathcal{Q} \rangle \sim L$ and $\langle \mathcal{Q}\mathcal{Q} \rangle \sim L$ for $|\Delta| < 1$
- Exponential localization and conservation up to boundary terms are sufficient to give bound $D(T > 0, |\Delta| < 1) > 0$
- At $\Delta = 1$: $\mathcal{Q}$ becomes non-local, $\langle \mathcal{Q}\mathcal{Q} \rangle \sim L^2$
  $\rightarrow$ Bound vanishes at $\Delta = 1$

Outlook

- Attractive Hubbard model at half filling:

$$H = \sum_j \left[(-c^\dagger_{j,\alpha} c_{j+1,\alpha} + h.c.) - U(n_j - 1/2)^2 \right]$$

  - Spin sector gapped
  - Charge sector: Gaussian model with Umklapp term, $\gamma \sim \lambda^2 T$
  - Is there a finite Drude weight?

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- Thermalization (GGE) in integrable models:
  - One common belief: Include only local conservation laws, non-local not important in TD limit
  - What about quasi-local conservation laws?

Conclusions

Spin conductivity in the *XXZ* model

- ballistic and diffusive transport channels coexist
- Memory matrix: δ-fct. peak on top of a Lorentzian with linewidth determined by Umklapp scattering ($|h| \ll T$)
- Diffusive contribution explains NMR experiment on Sr_2CuO_3
- Quasi-local conservation law at $h = 0$ important, not part of the usual set of conserved operators

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Violations of the Luttinger liquid paradigm

- Violations of LL paradigm for time-like CF: band curvature (non-linear LL, $T = 0$) and Umklapp scattering ($T > 0$)

Spin diffusion: $\langle S_l^z(t) S_l^z(0) \rangle \sim T \sqrt{\frac{\gamma(T)}{t}}, \quad (1/t \ll \lambda^2 T^{4K-3})$