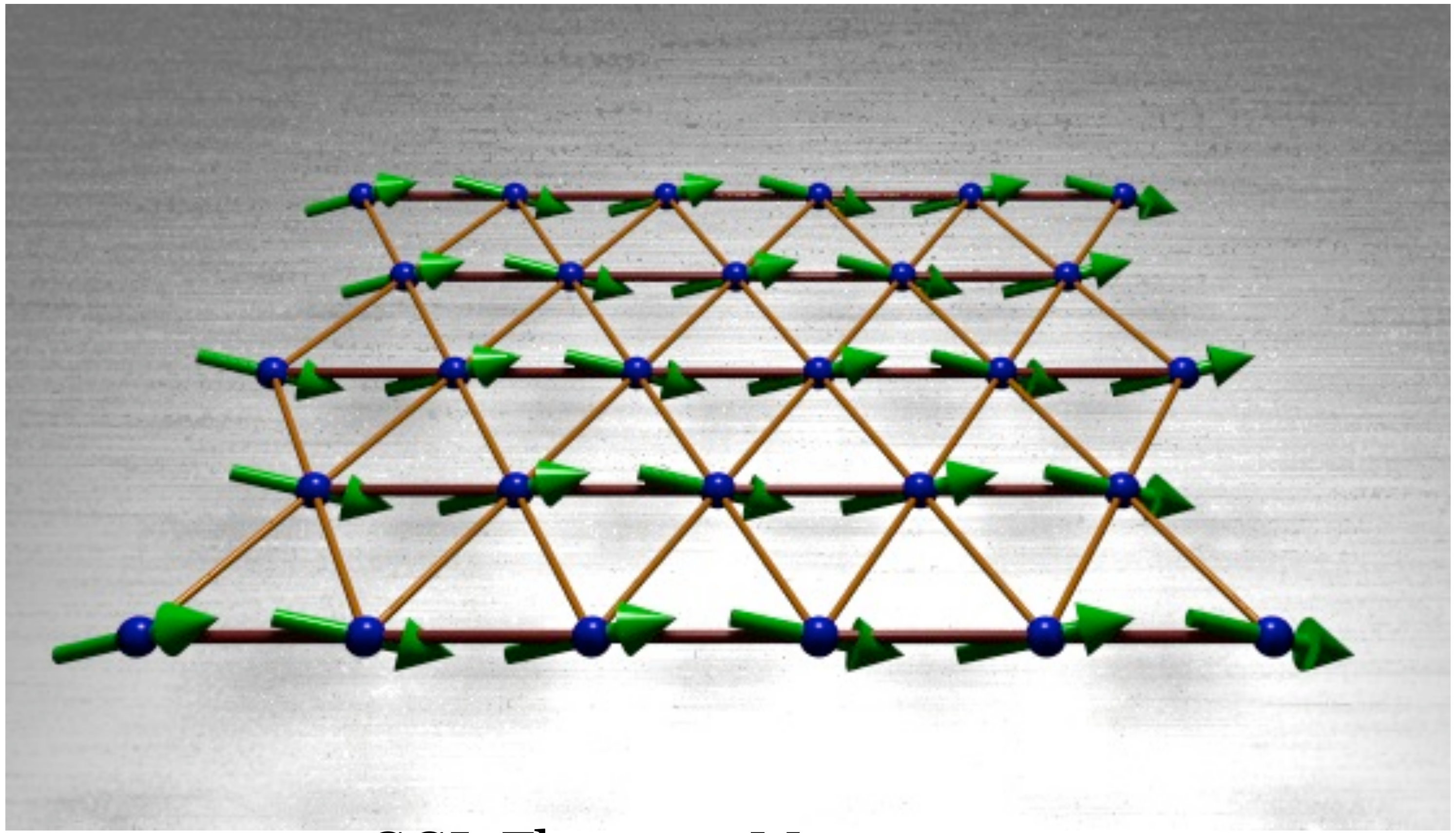
Magnetization plateau and other unusual phases of a spatially anisotropic quantum antiferromagnet on triangular lattice

Oleg Starykh, University of Utah



GGI, Florence, May 25, 2012

Collaborators

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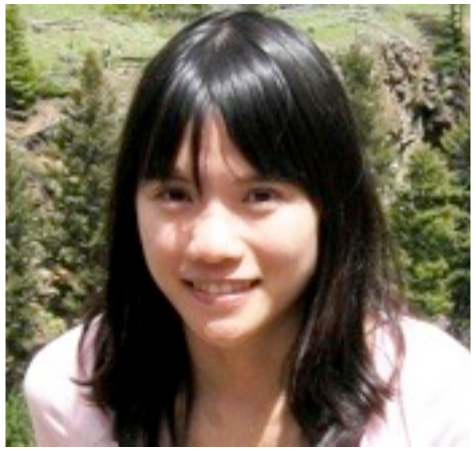

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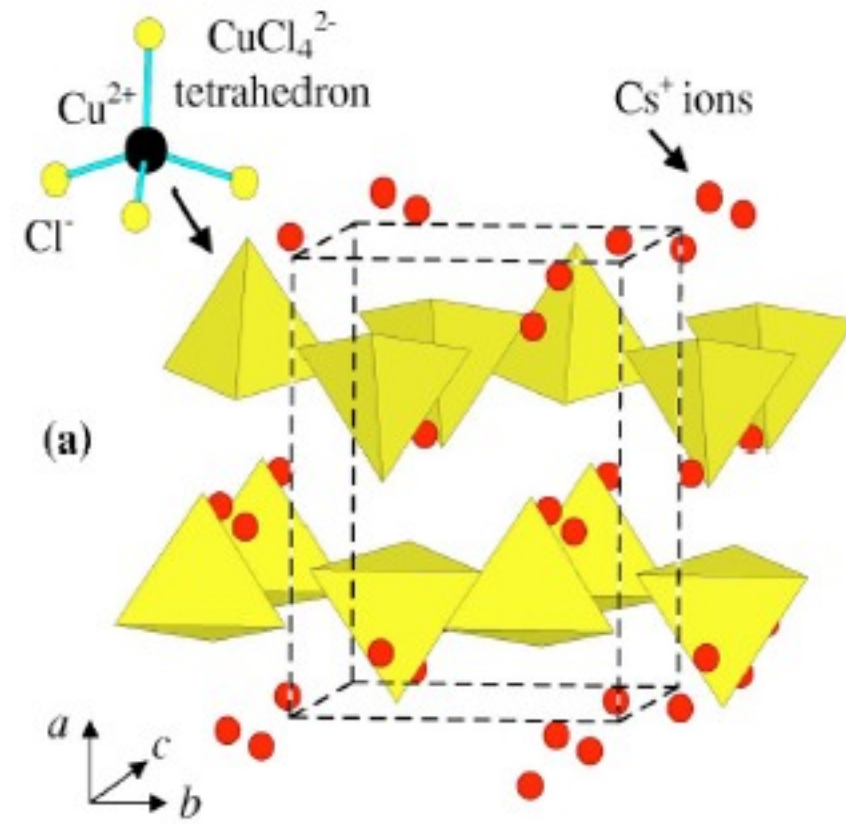
Outline

- motivation: Cs_2CuBr_4 , Cs_2CuCs_4
- classical antiferromagnet in a field: entropic selection
  - spatial anisotropy - high-T stabilization of the plateau
- Quantum model in magnetic field
  - DMRG (3-leg ladder)
  - Various analytical limits
    - large-S analysis of interacting spin waves
    - weakly coupled spin chains
    - magnetization plateau(s) and selection rules
- Summary

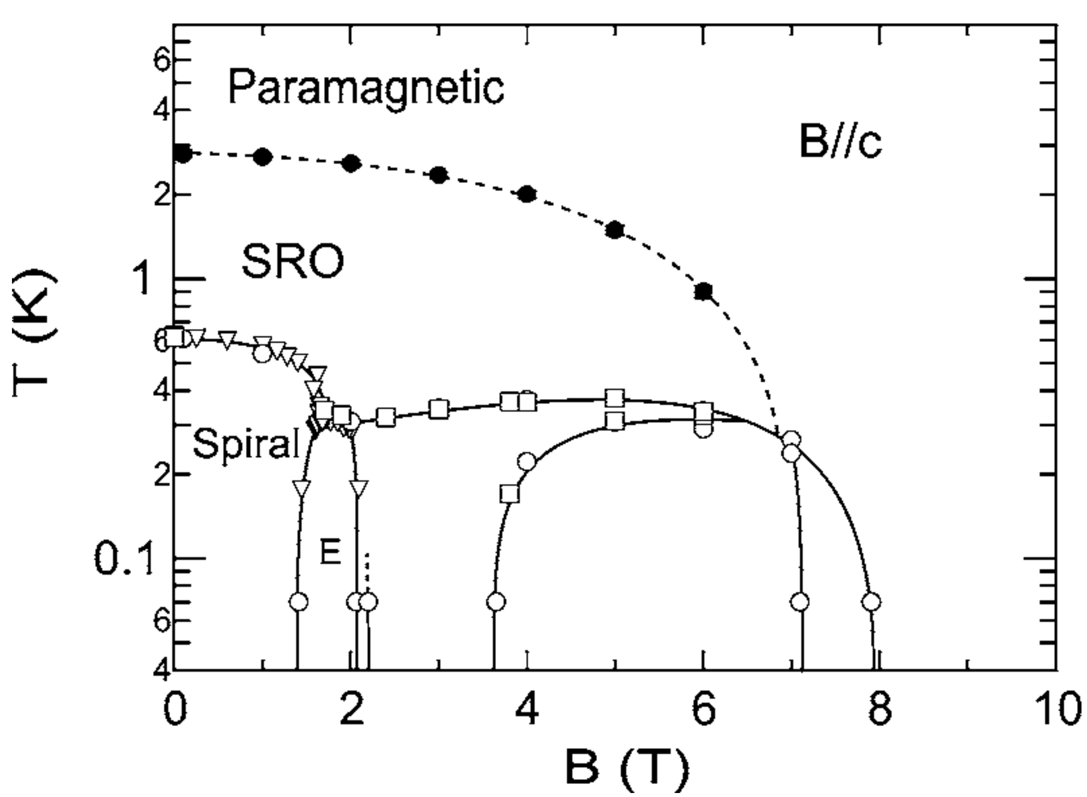
Cs_2CuBr_4, Cs_2CuCl_4



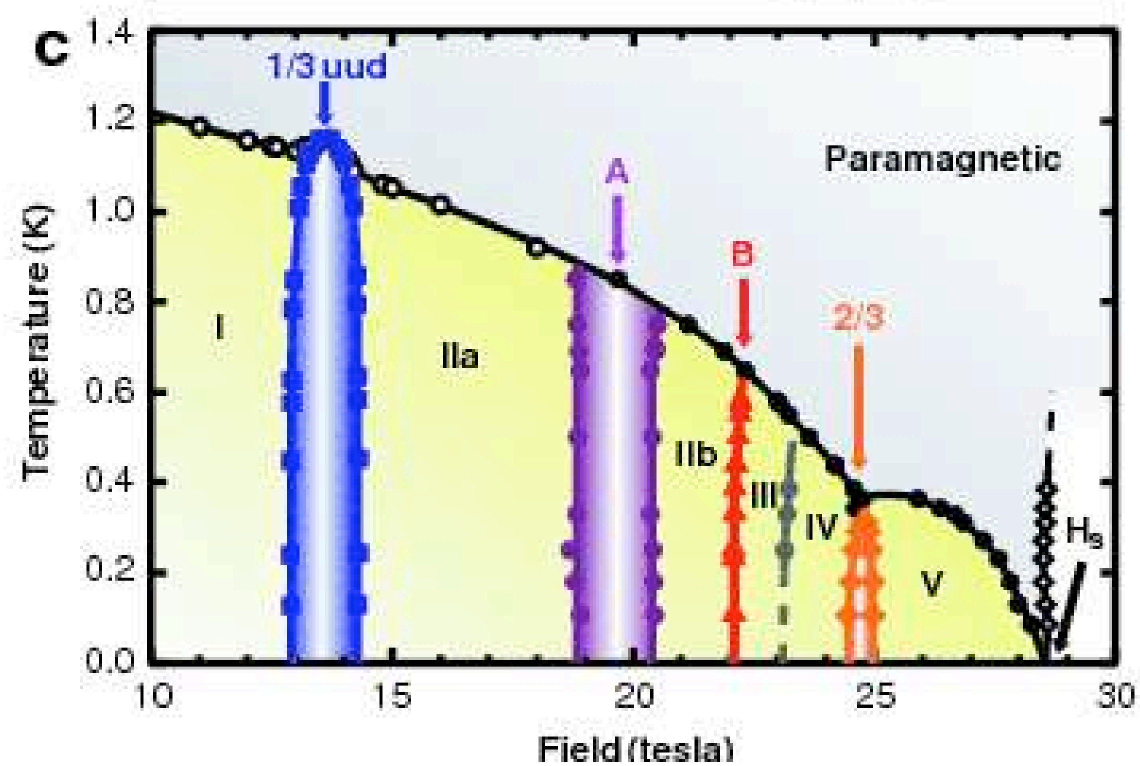
- S=1/2 quantum triangular antiferromagnets with small exchange
  - complex evolution in field!

Cs_2CuCl_4

Y. Tokiwa et al, 2006



Cs_2CuBr_4

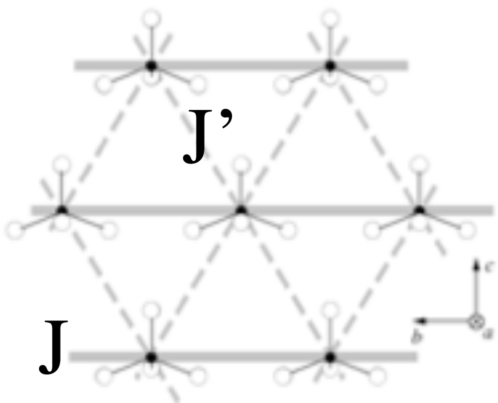


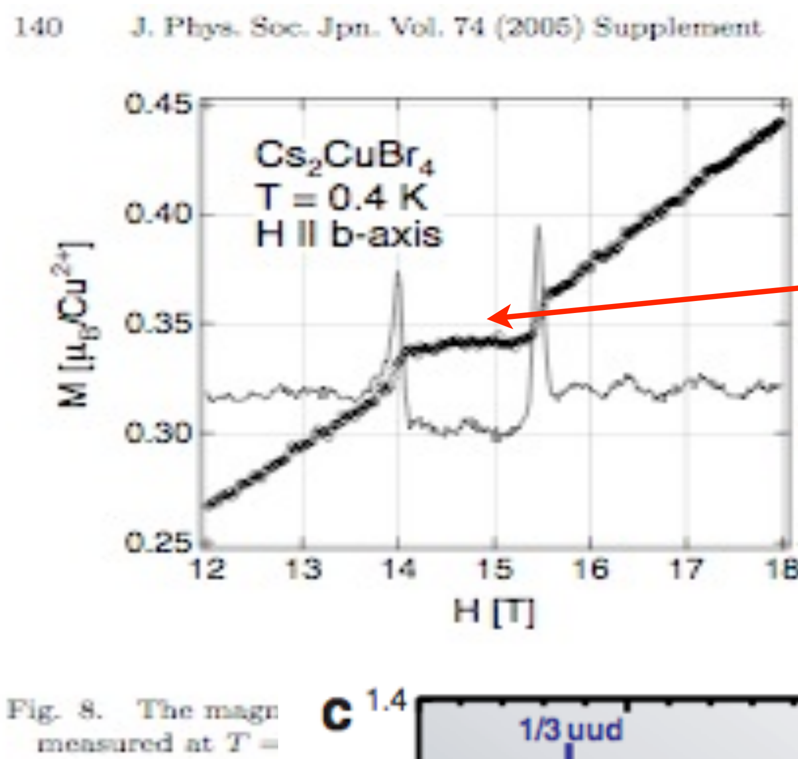
Fortune *et al*, 2009

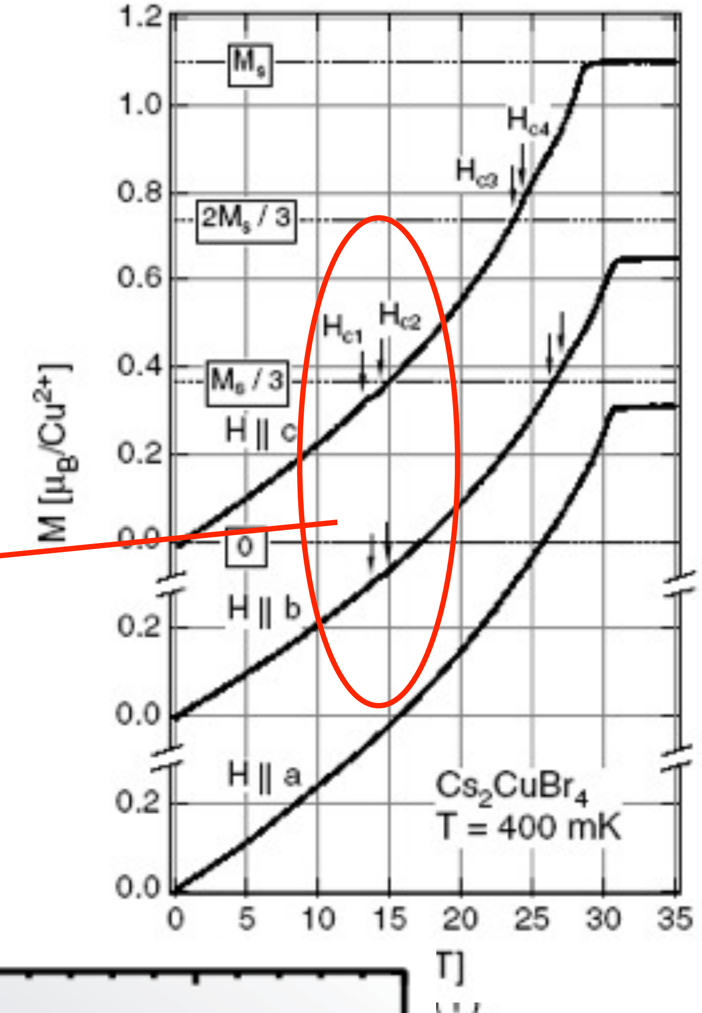
M=1/3 magnetization plateau in Cs_2CuBr_4

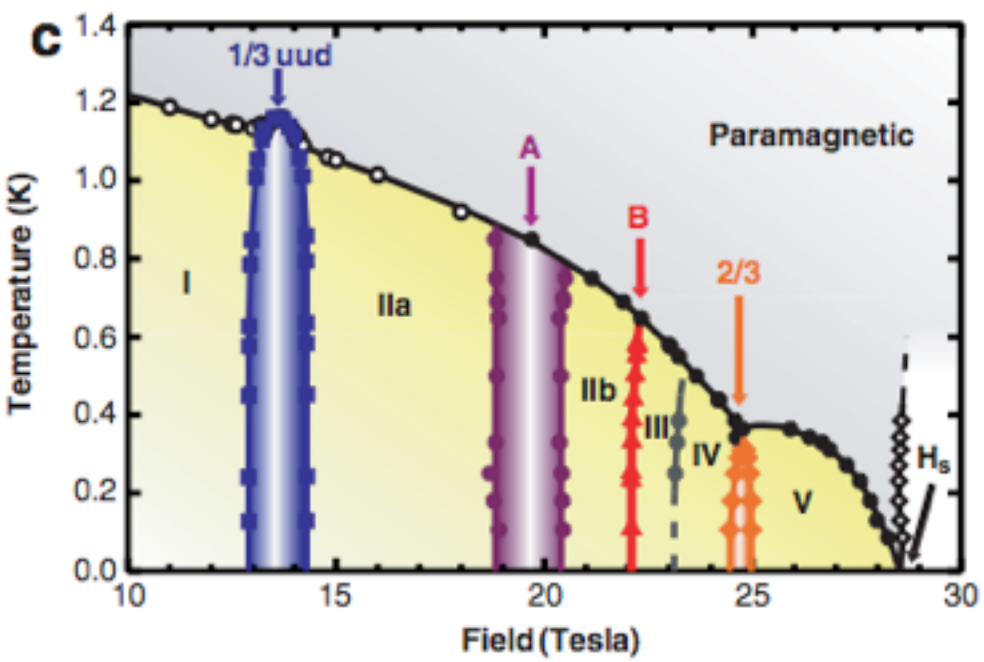
★ Observed in Cs_2CuBr_4 (Ono 2004, Tsuji 2007) **J'/J = 0.75**
but not Cs_2CuCl_4 **[J'/J = 0.34]**

S=1/2









★ first observation of "**up-up-down**" state
in spin-1/2 triangular lattice antiferromagnet

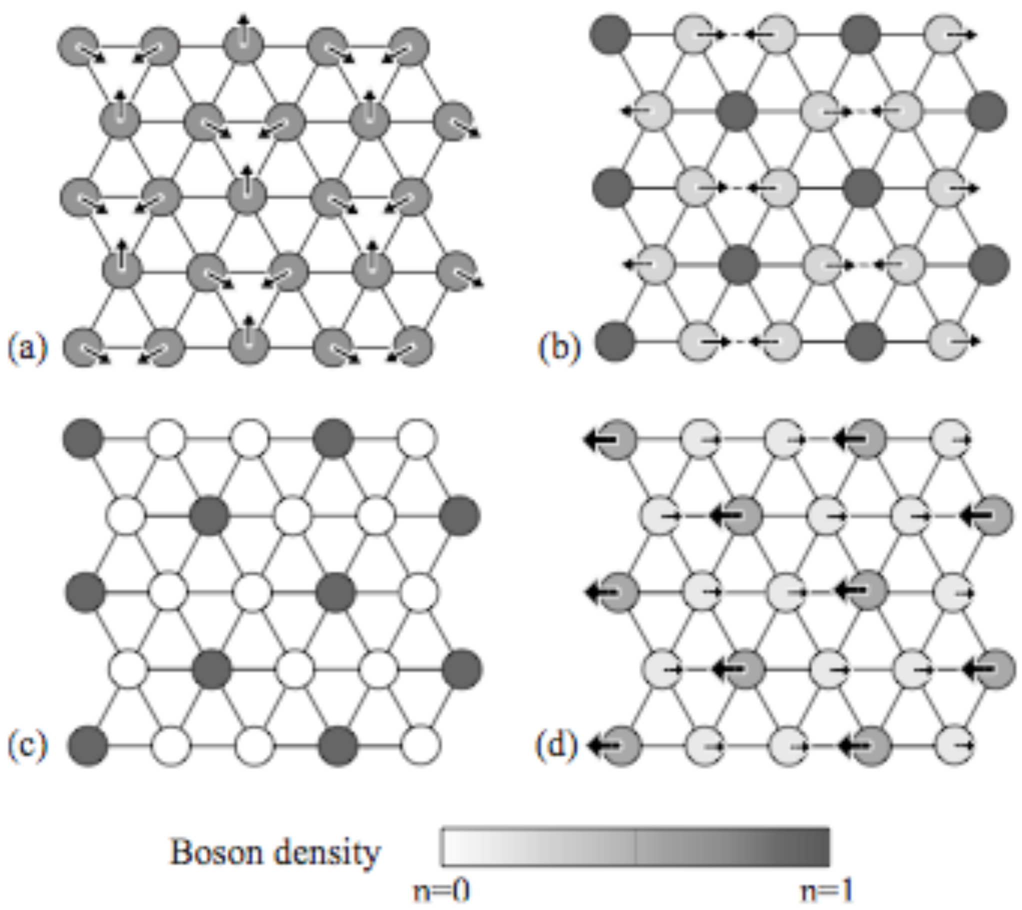
★ and **8** more **phases** (instead of 2 expected)!

Both materials are spatially anisotropic triangular antiferromagnets

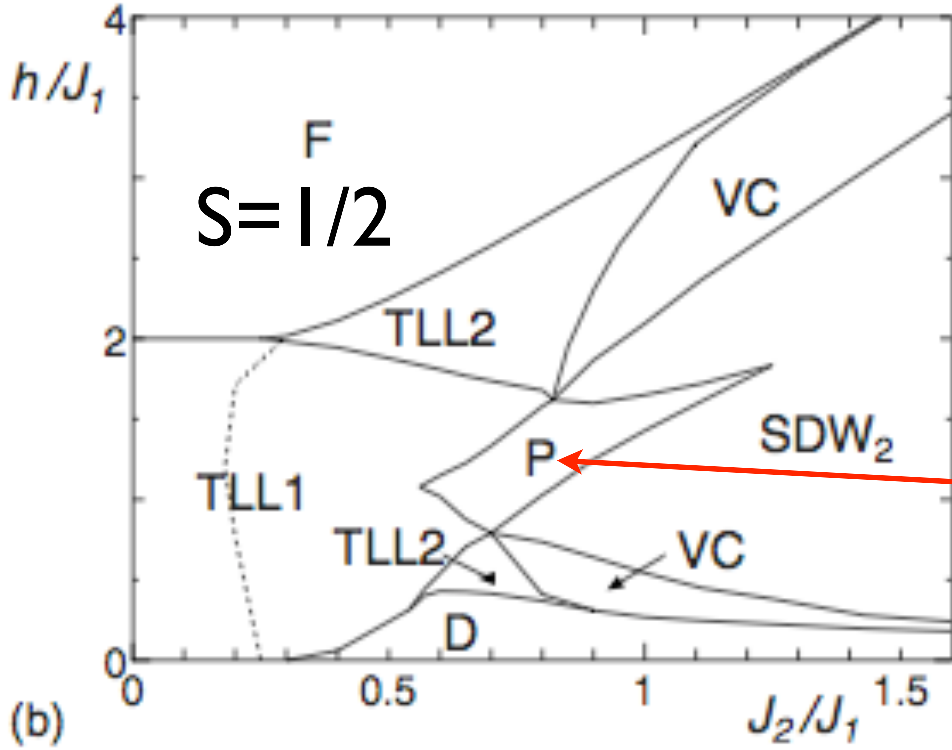
2D bosons on the lattice

- connections with interacting boson system
  - Superfluids (XY order)
  - Mott insulators
  - Supersolids

Andreev, Lifshitz 1969
Nikuni, Shiba 1995
Heidarian, Damle 2005
Wang et al 2009
Jiang et al 2009
Tay, Motrunich 2010

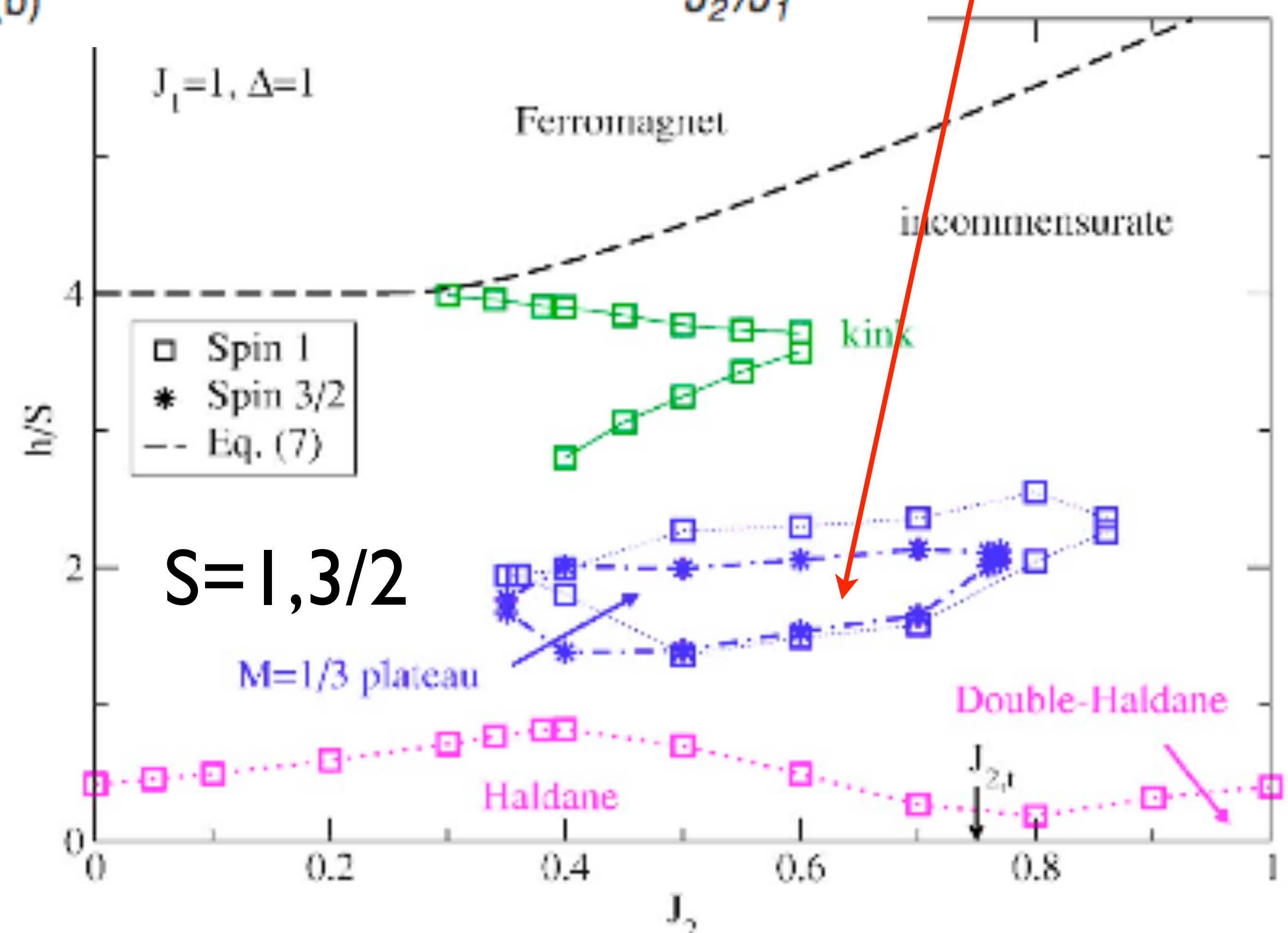


Magnetization plateau in one dimensional J_1-J_2 chain (zig-zag ladder)



Okunishi, Tonegawa JPSJ (2003)
Hikihara et al PRB (2010)

M=1/3 plateau



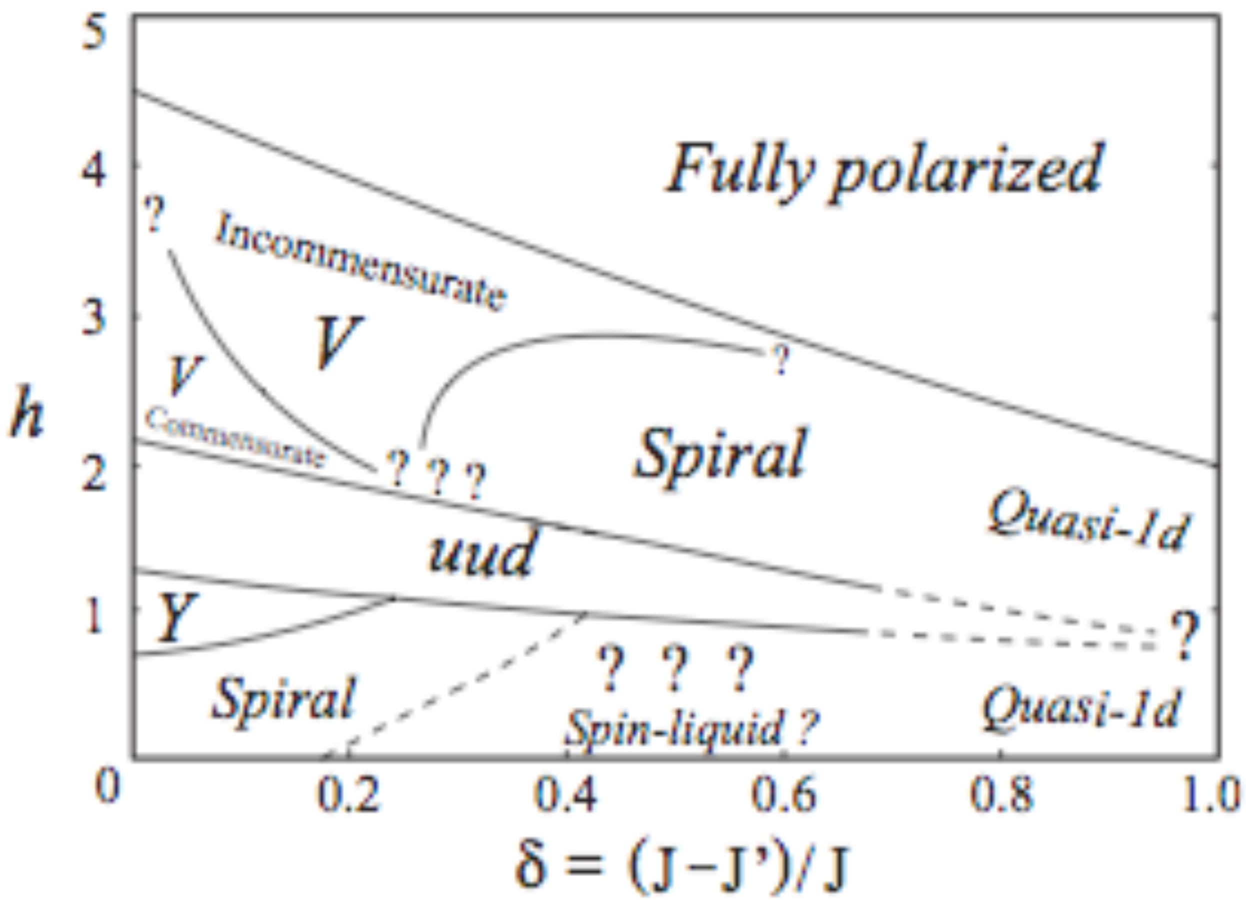
Heirich-Meisner et al PRB (2007)

agrees with Oshikawa, Yamanaka, Affleck argument (PRL 2007):

$$p\,S\,(1 - M) = \text{integer}$$

p = period, S = spin,
M = magnetization:
M=1/3, p=3
possible for all S

Studies in 2D



We will see many similarities with this study

Variational Monte Carlo on 2D triangular lattice
Tay, Motrunich (2010)

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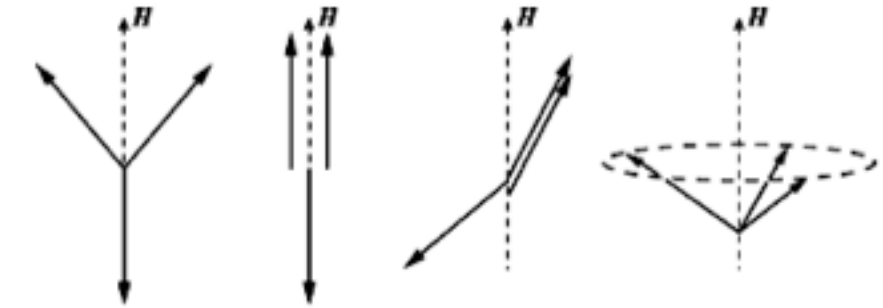
Classical isotropic Δ AFM in magnetic field T=0

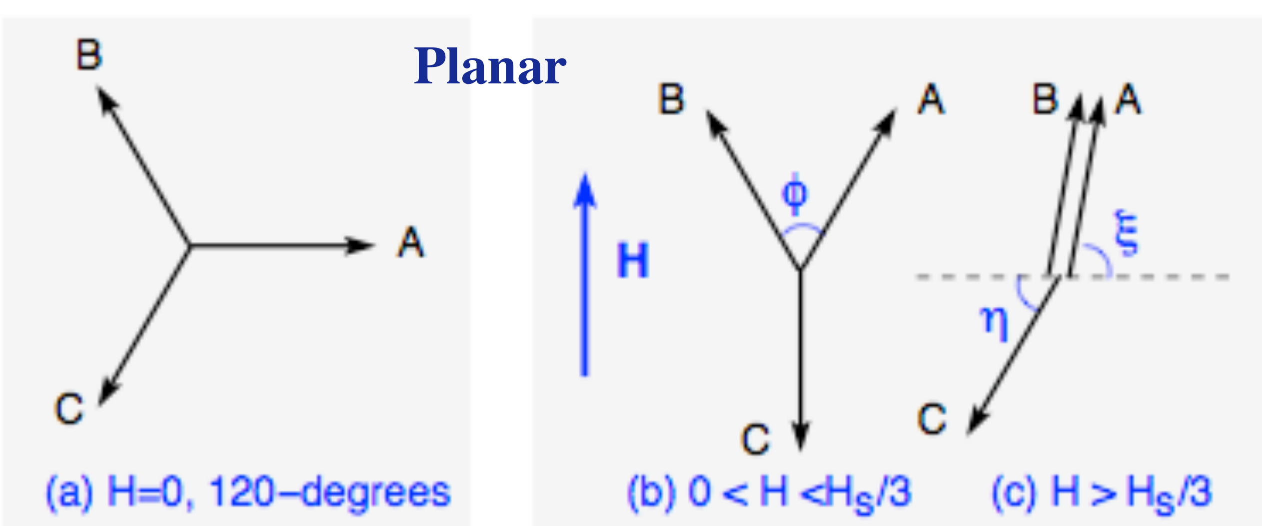


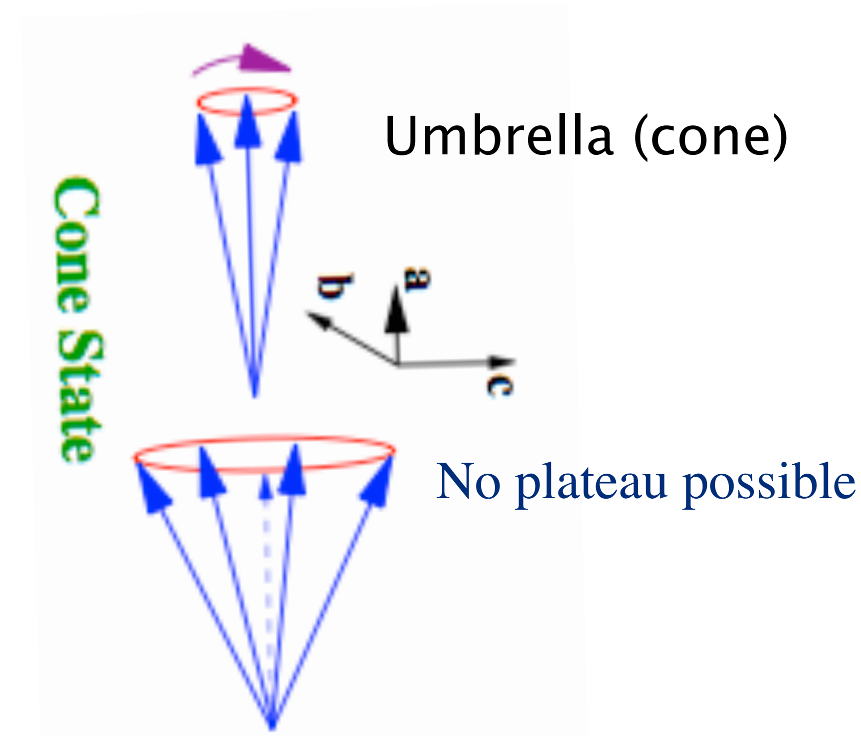
- Zero magnetic field: spiral (120 degree) state
- Magnetic field: **accidental degeneracy**

$$H = J\sum_{i,j}\vec{S}_i\cdot\vec{S}_j - \sum_i \vec{h}\cdot\vec{S}_i$$

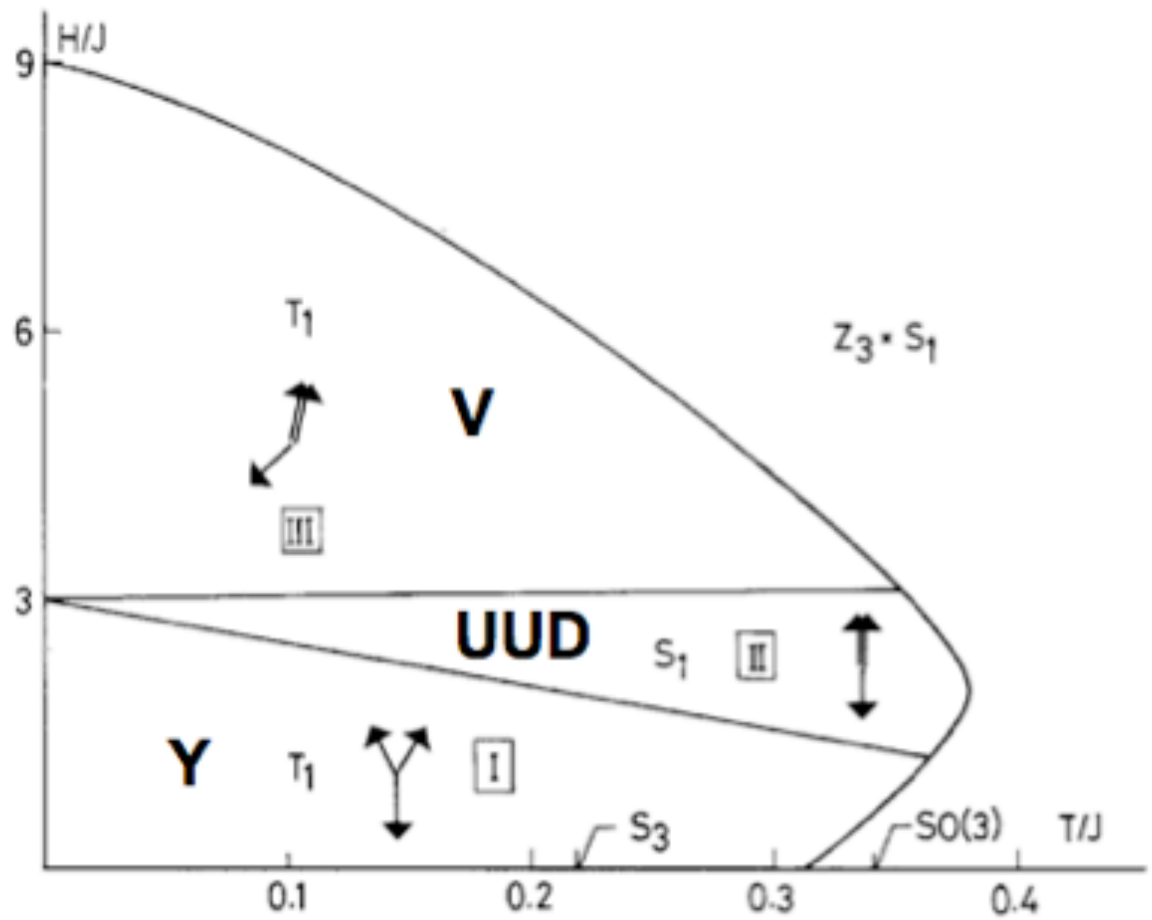
$$H = \frac{1}{2}J\sum_{\triangle}\Big(\sum_{i\in\triangle}\vec{S}_i - \frac{\vec{h}}{3J}\Big)^2$$

- all states with $\vec{S}_{i1}+\vec{S}_{i2}+\vec{S}_{i3} = \frac{\vec{h}}{3J}$ form the lowest-energy manifold
  - 6 angles, 3 equations => **2 continuous angles** (upto global U(1) rotation about **h**)

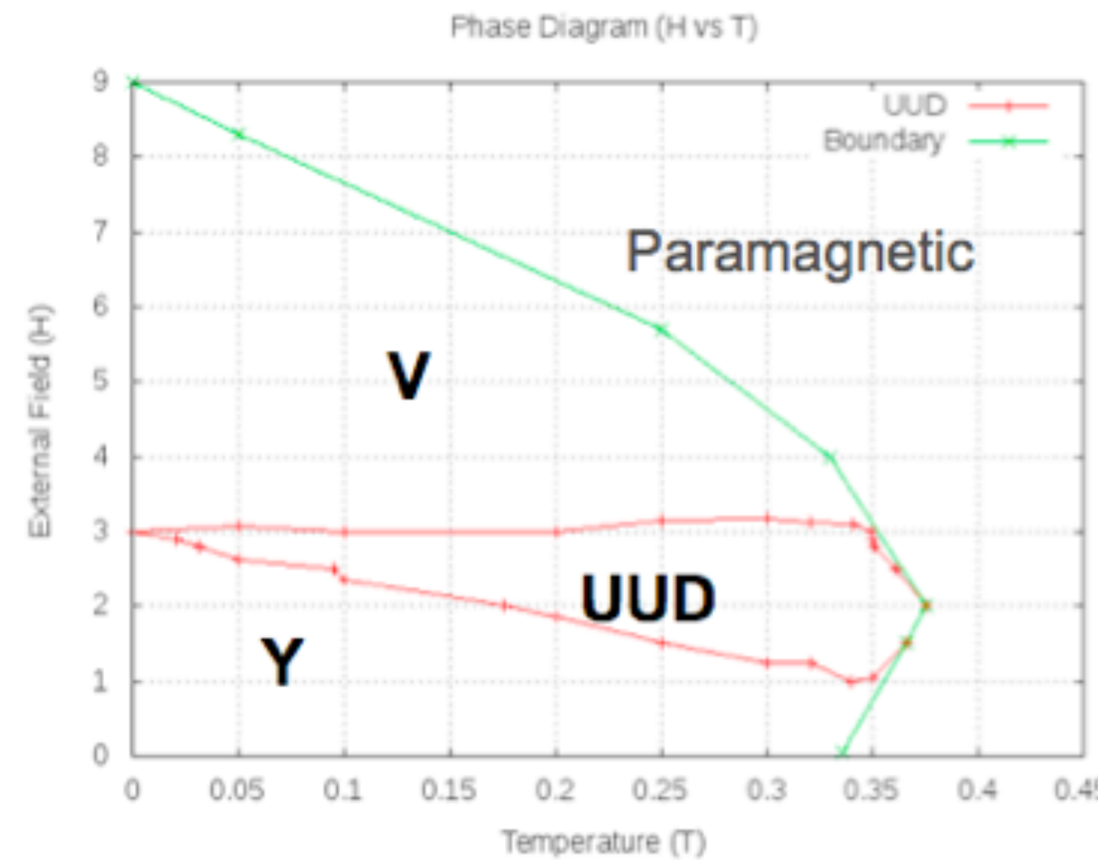




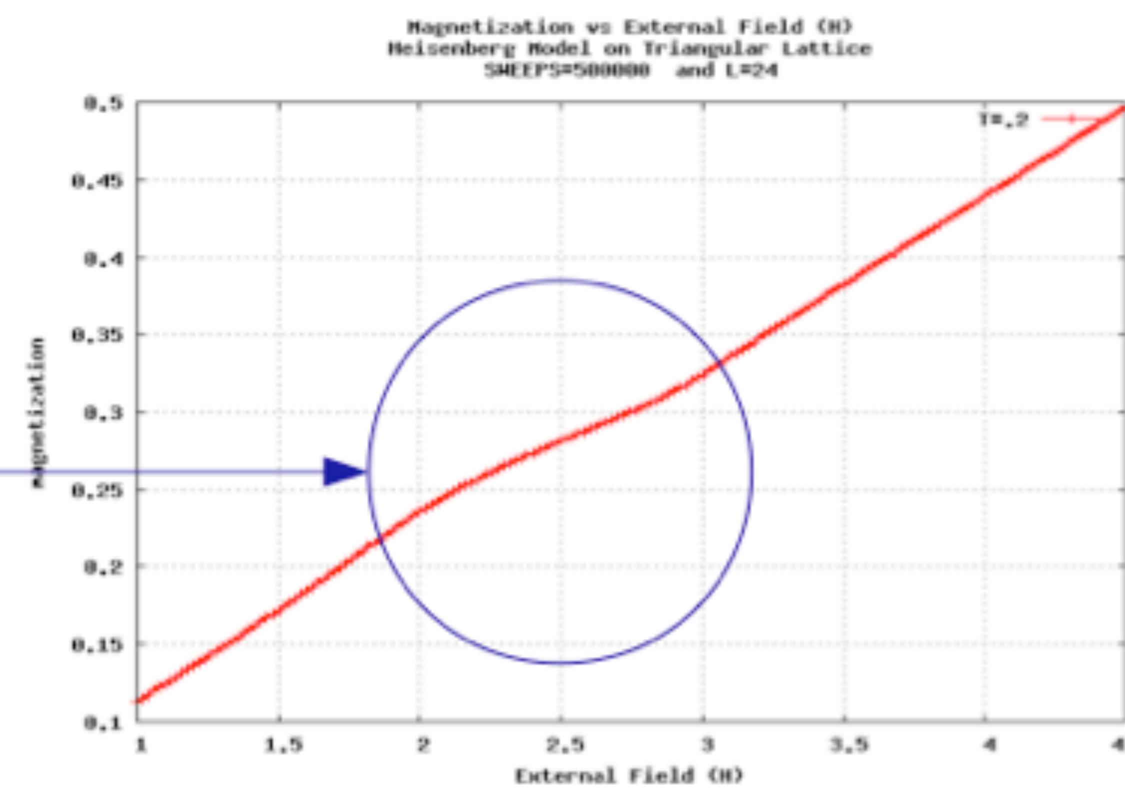
Phase diagram at finite T



H. Kawamura, S. Miyashita:
JPSJ (1985)



Head, Griset, Alicea, OS 2010



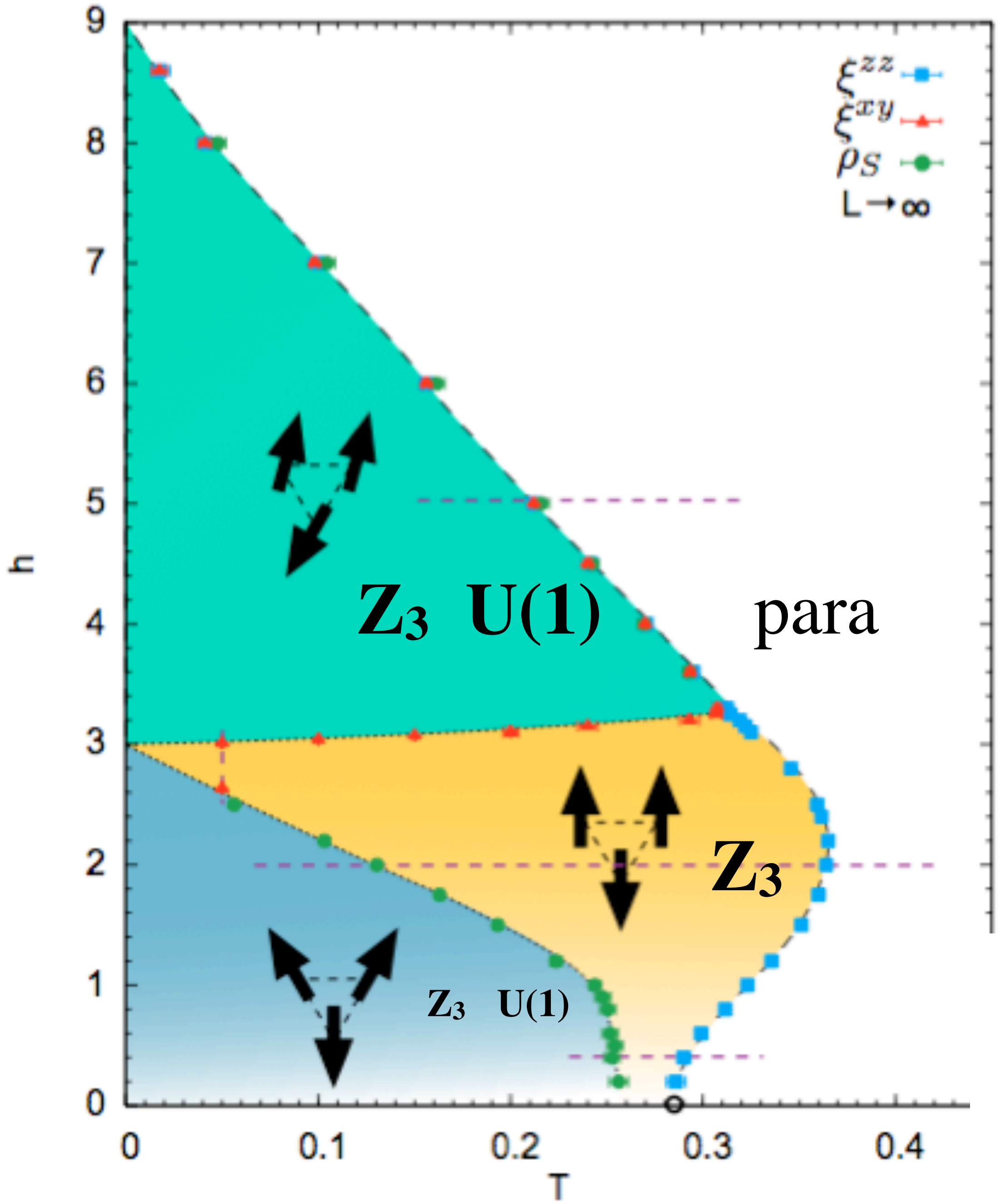
Entropic Selection:

- Planar states favored by thermal fluctuations
- UUD state around m=1/3 resulting in quasi-plateau

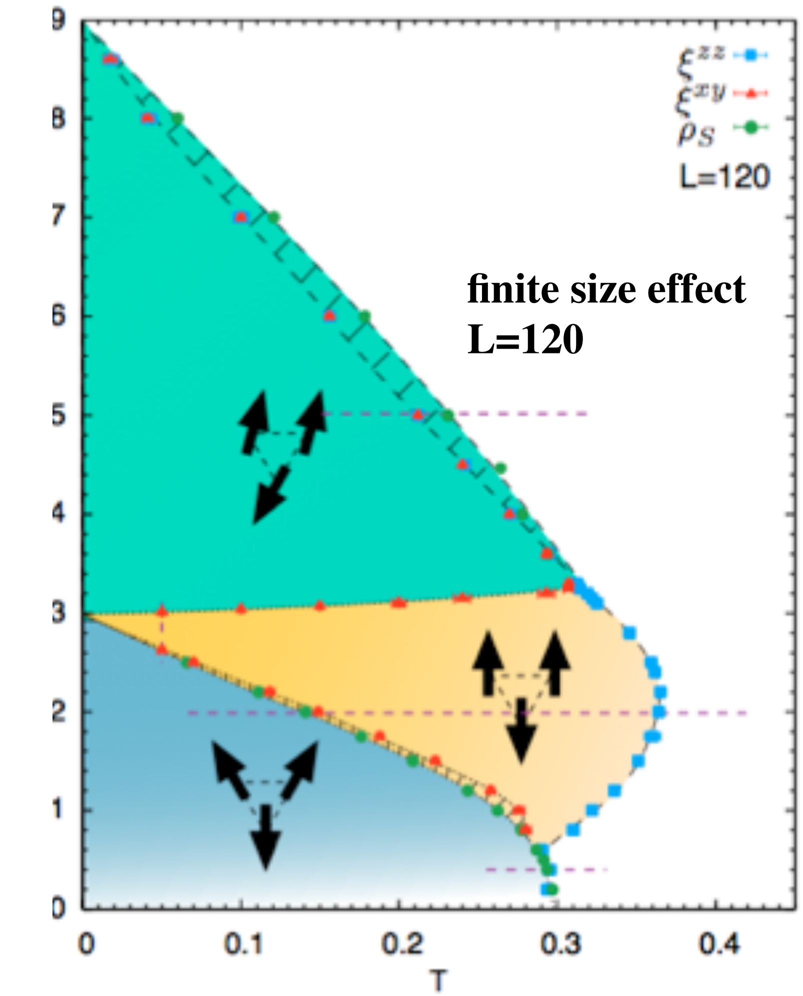
Finite T: minimize F = E - T S

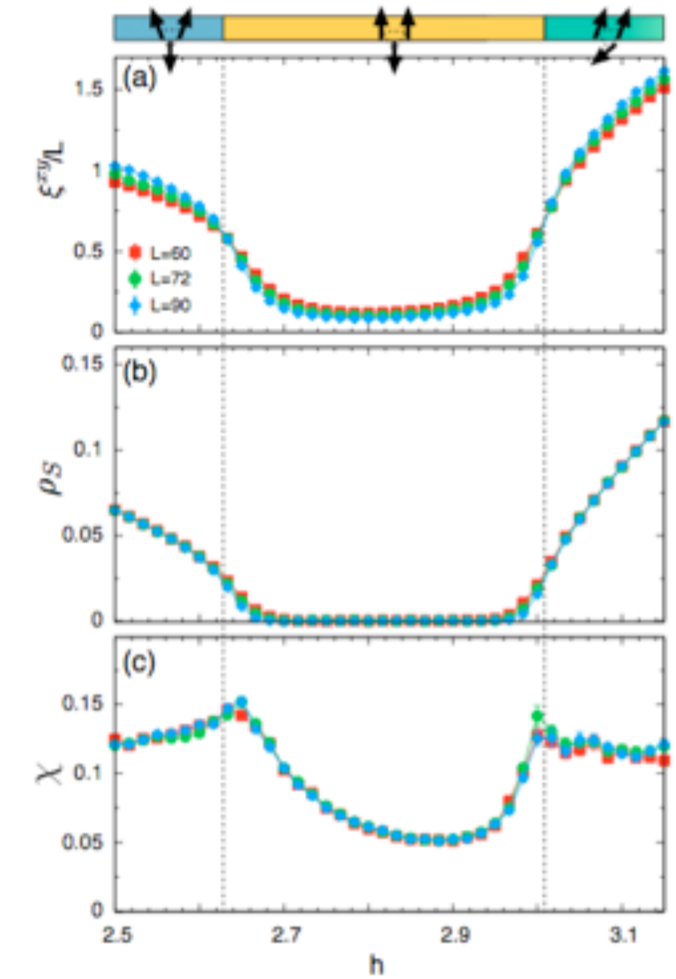
Planar states have higher entropy!

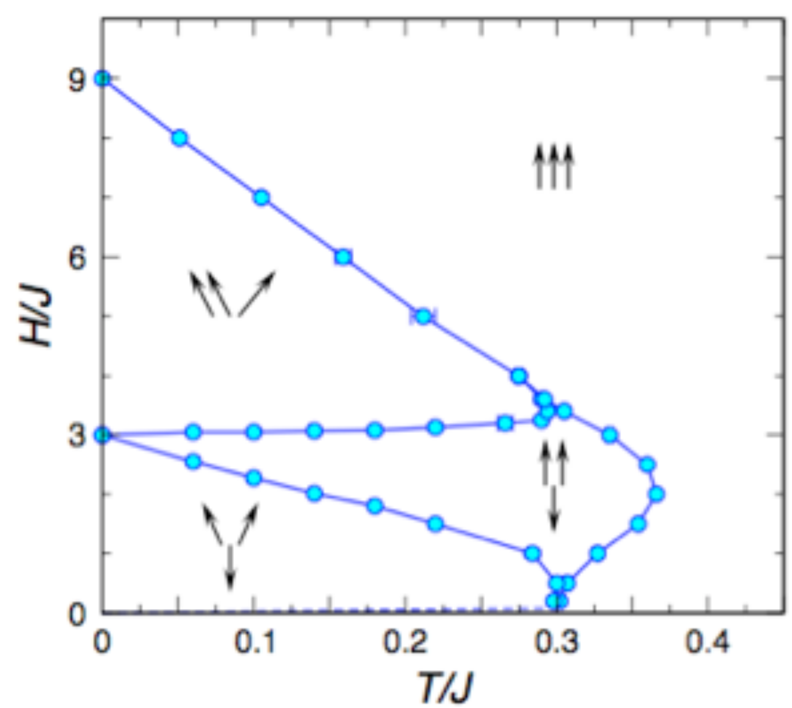
Phase diagram of the classical model: Monte Carlo



Seabra, Momoi, Sindzingre, Shannon 2011

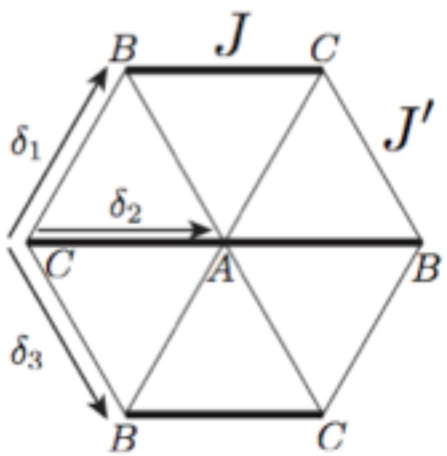


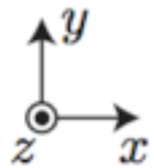




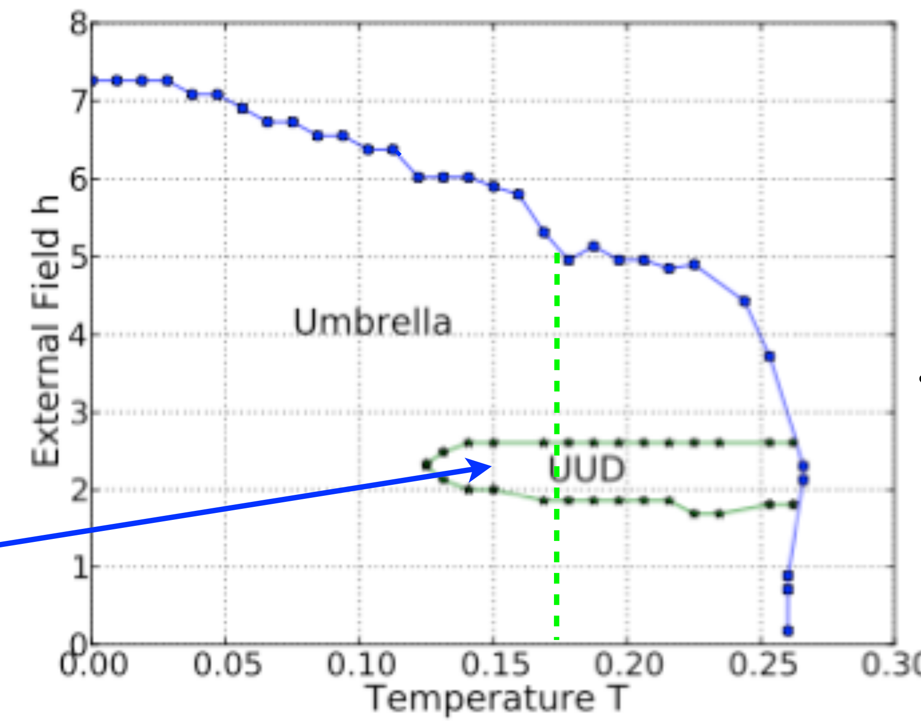
Gvozdikova, Melchy, Zhitomirsky 2010

Effect of spatial anisotropy J’ < J: energy vs entropy



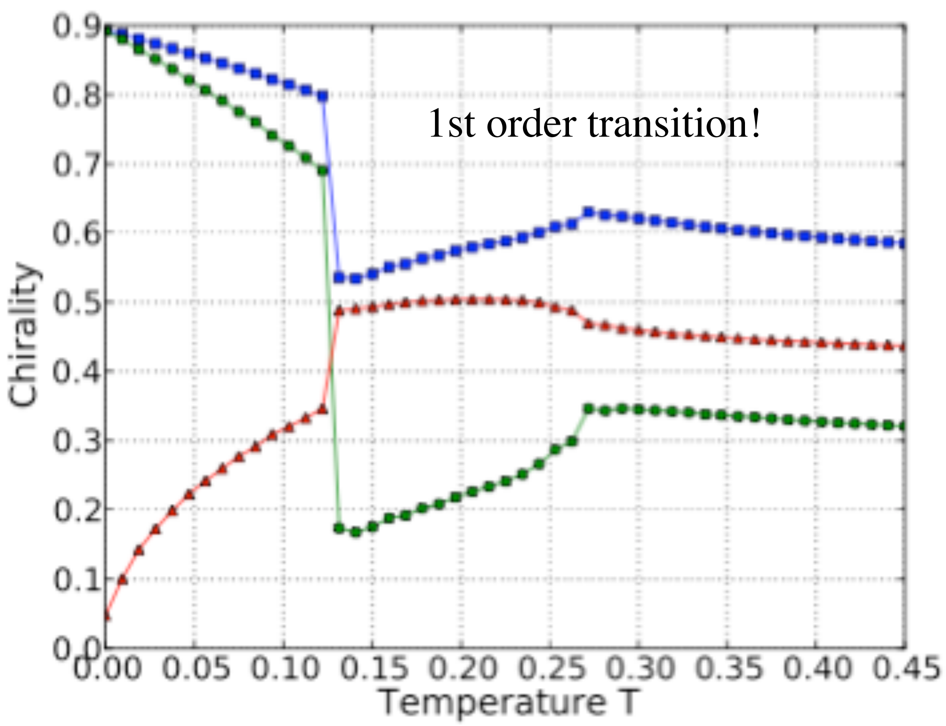


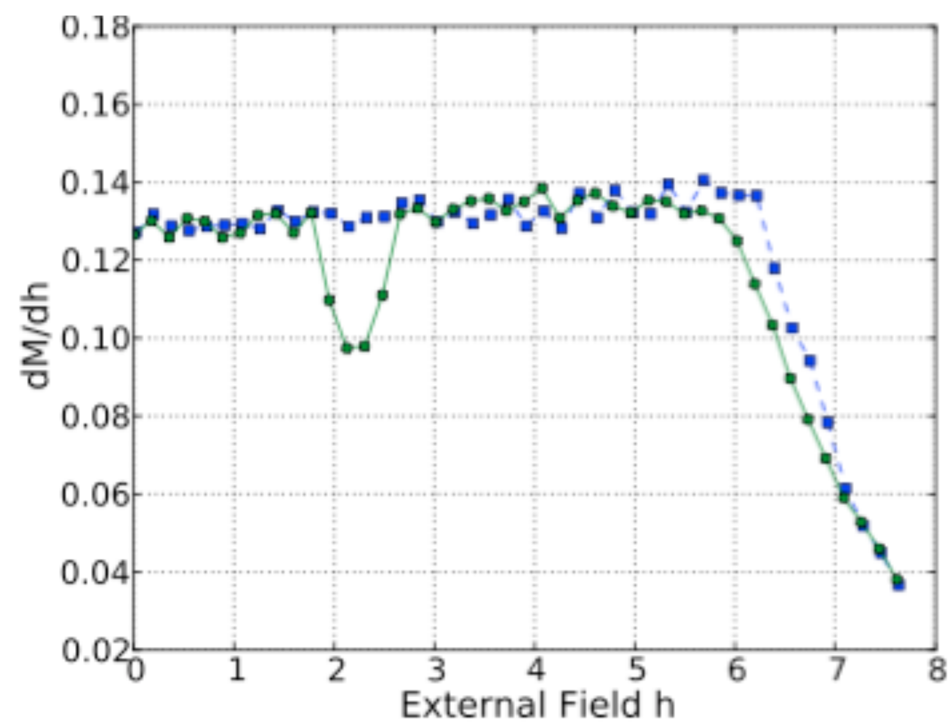
Umbrella state:
favored classically,
energy gain (J-J’)²/J



J’ = 0.765 J

similar with Pomeranchuk effect (He3):
crystal-like UUD is stabilized at high temperature





$$\boldsymbol{\kappa} = \frac{2}{3\sqrt{3}}\frac{1}{N}\sum_{\mathbf{r}}\Big(\mathbf{S}_{\mathbf{r}} \times \mathbf{S}_{\mathbf{r}+\boldsymbol{\delta}_1} + \\ +\mathbf{S}_{\mathbf{r}+\boldsymbol{\delta}_1} \times \mathbf{S}_{\mathbf{r}+\boldsymbol{\delta}_2} + \mathbf{S}_{\mathbf{r}+\boldsymbol{\delta}_2} \times \mathbf{S}_{\mathbf{r}}\Big)$$

Low T: energetically preferred umbrella
High T: entropically preferred UUD
Y and V are less stable.

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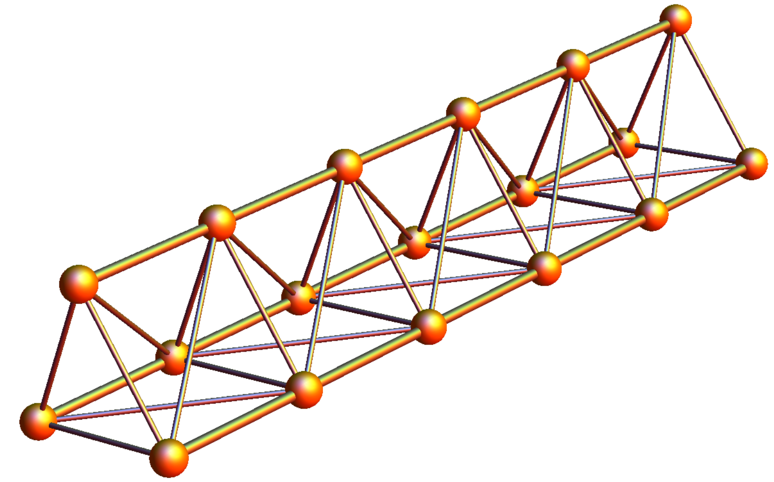
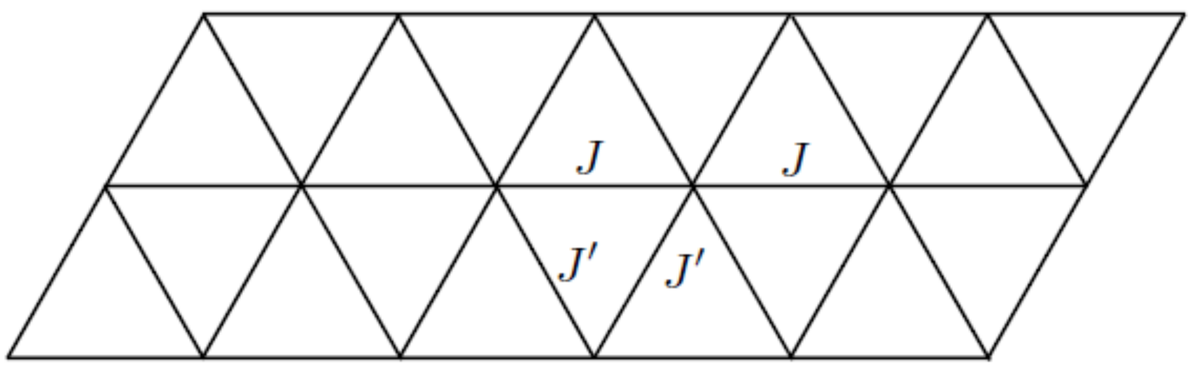
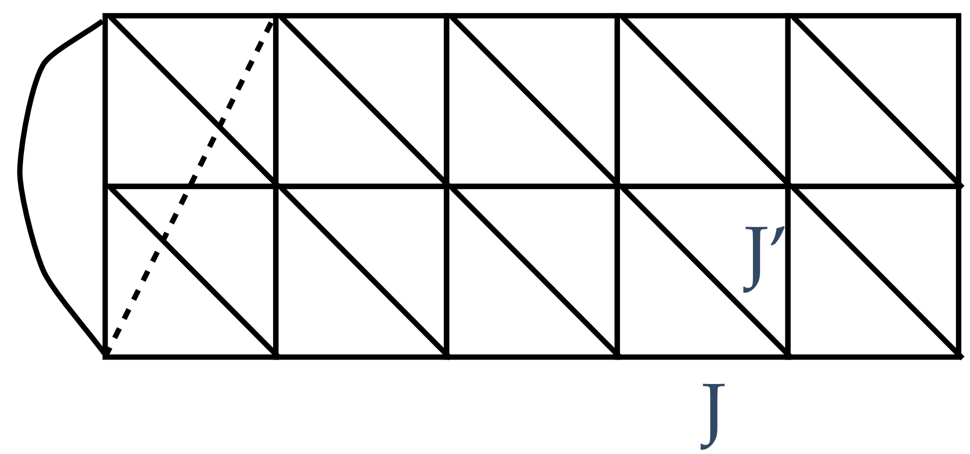
Model



Periodic boundary conditions along y in numerical studies





- Hamiltonian

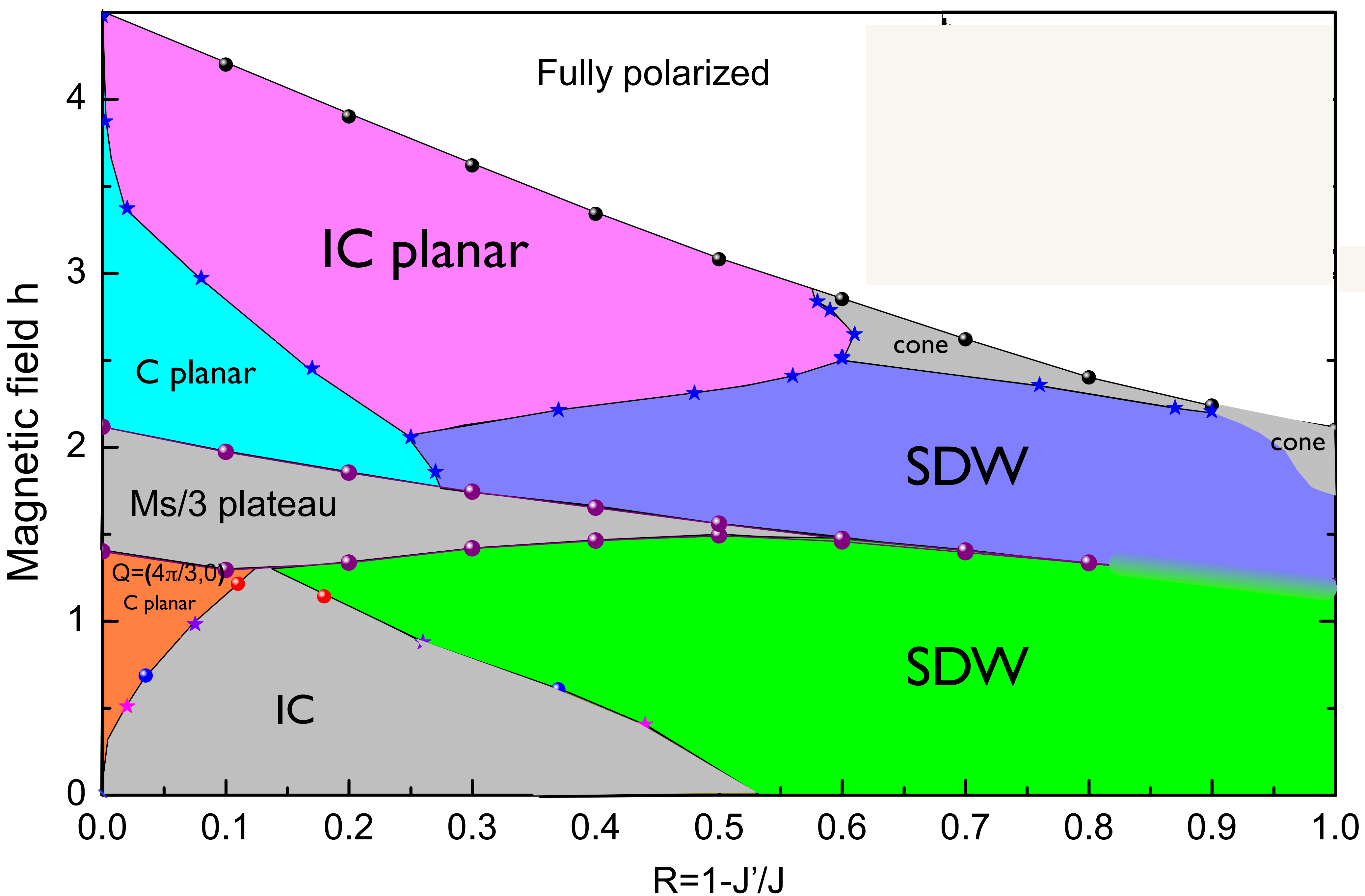
$$H = \sum_{x,y} J S_{x,y} \cdot S_{x+1,y} + J' \left(S_{x,y} \cdot S_{x,y+1} + S_{x,y} \cdot S_{x-1,y+1} \right)$$

$$-h \sum_{x,y} S^z_{x,y}$$

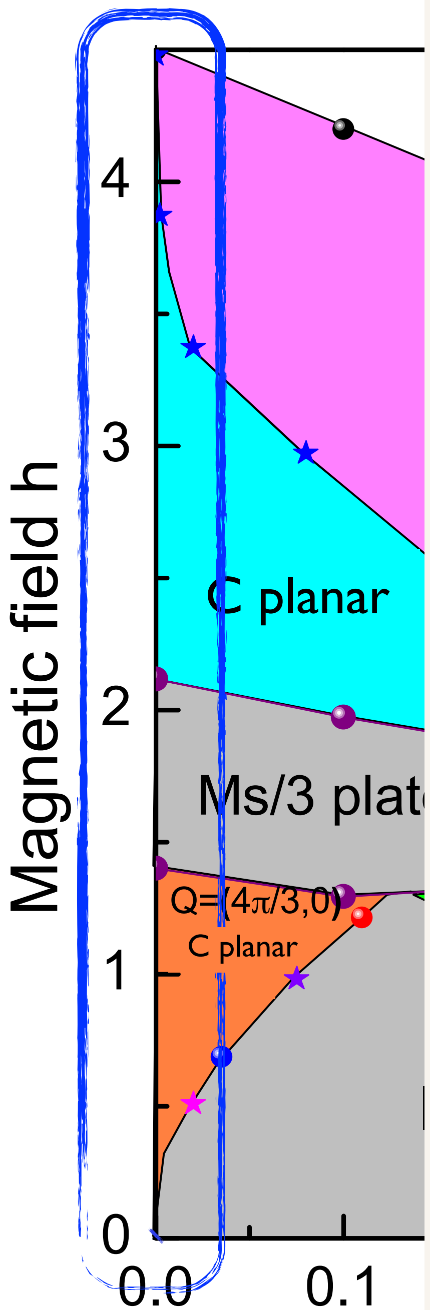
Cs_2CuCl_4: J′ / J = 0.34

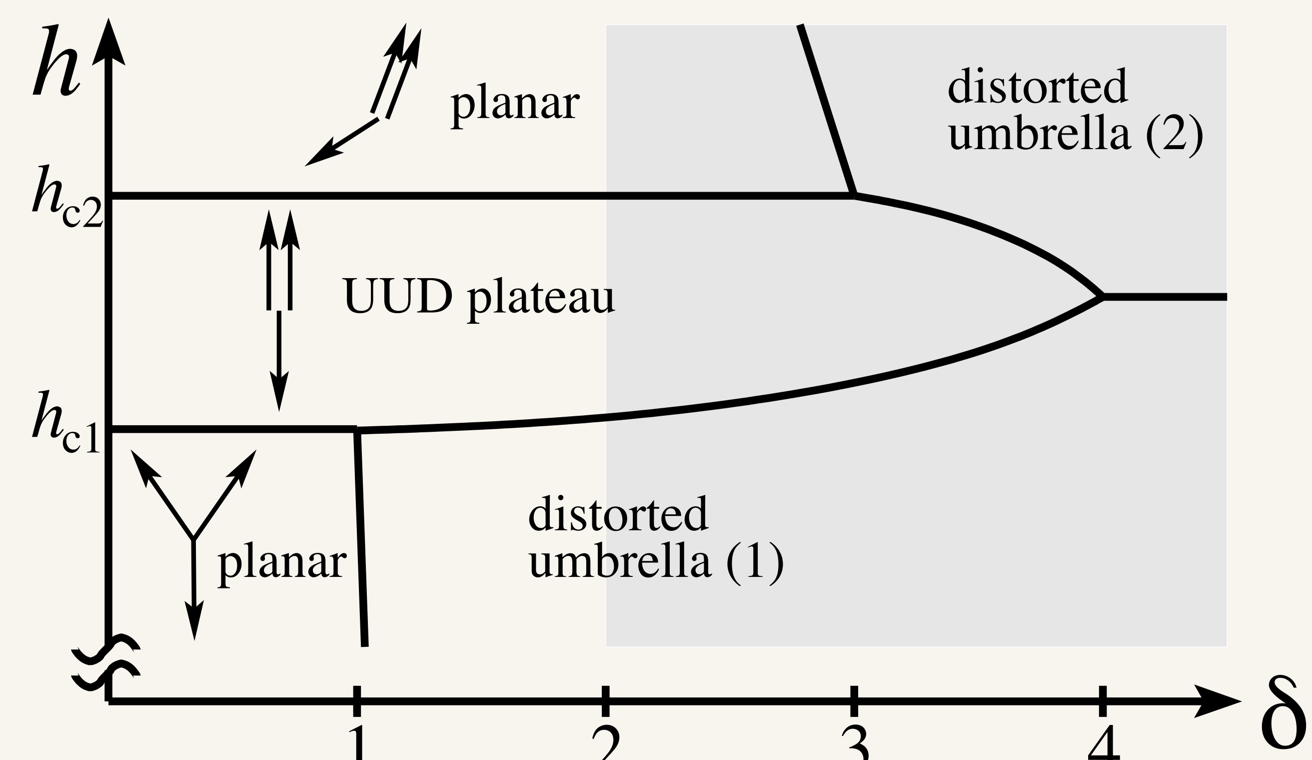
Cs_2CuBr_4: J′ / J = 0.5-0.7

Phase diagram (3 leg ladder)



Isotropic case: quantum fluctuations select co-planar states



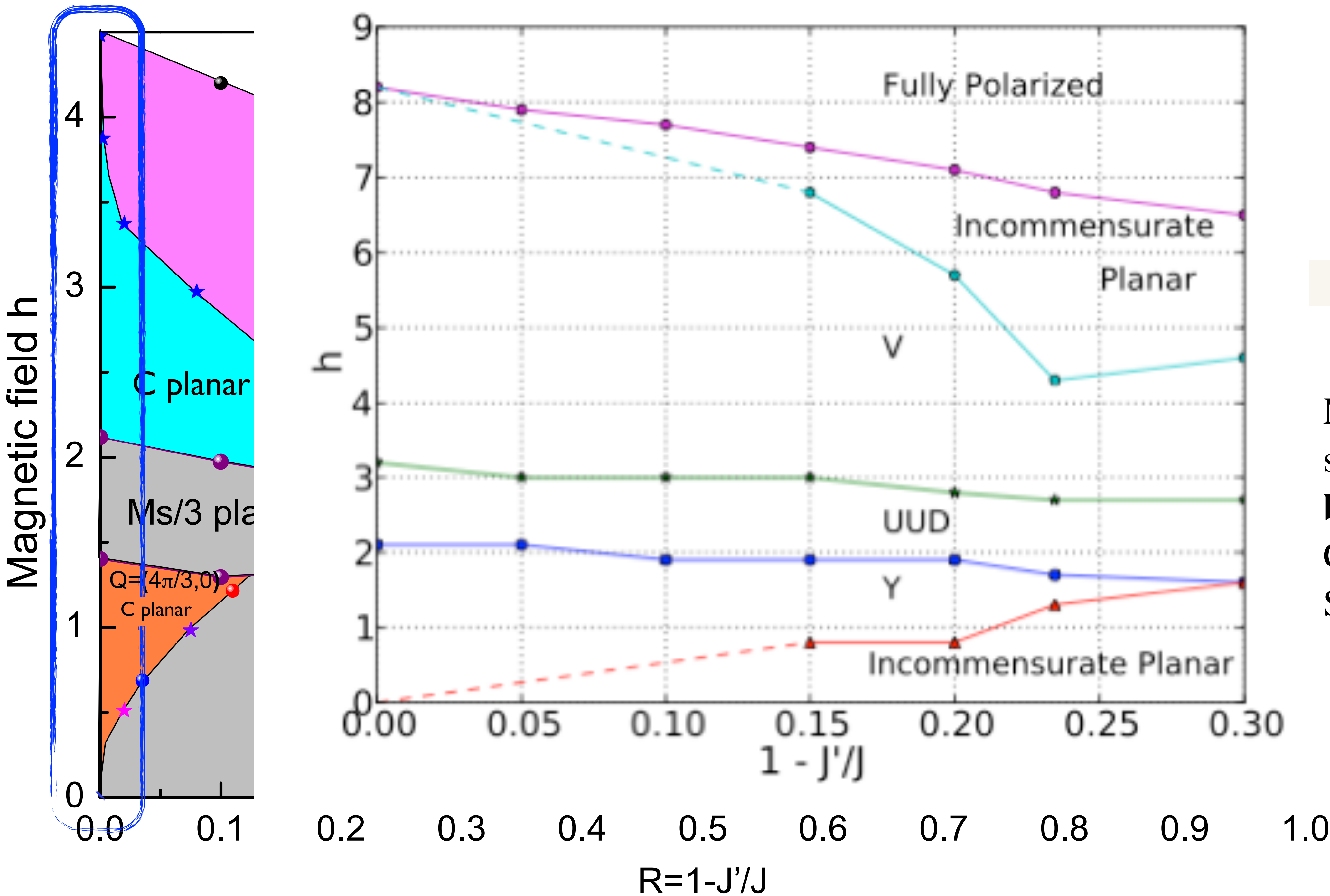


Interacting magnons perturbed by spatial anisotropy

isotropic case: Chubukov+Golosov, 1991; Alicea, Chubukov, Starykh, 2008

UUD preserves U(1) symmetry. Gapped spin waves

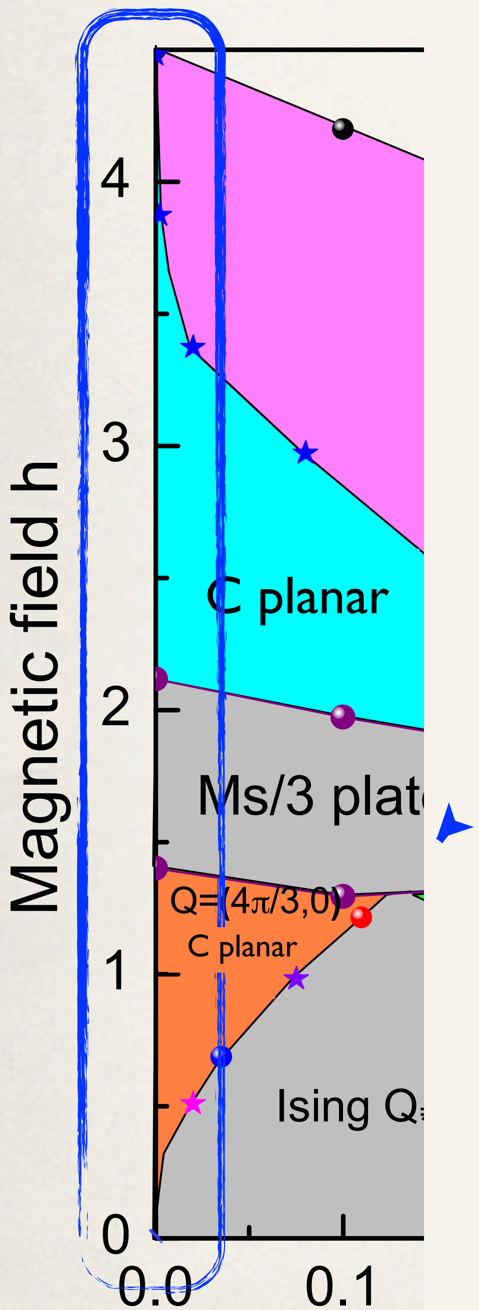
Isotropic case



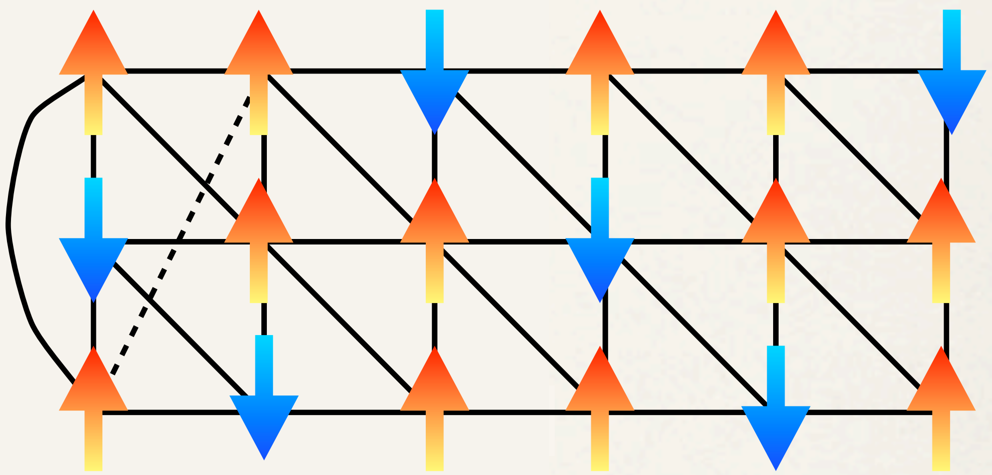
Slightly different take: Monte Carlo on generalized classical model

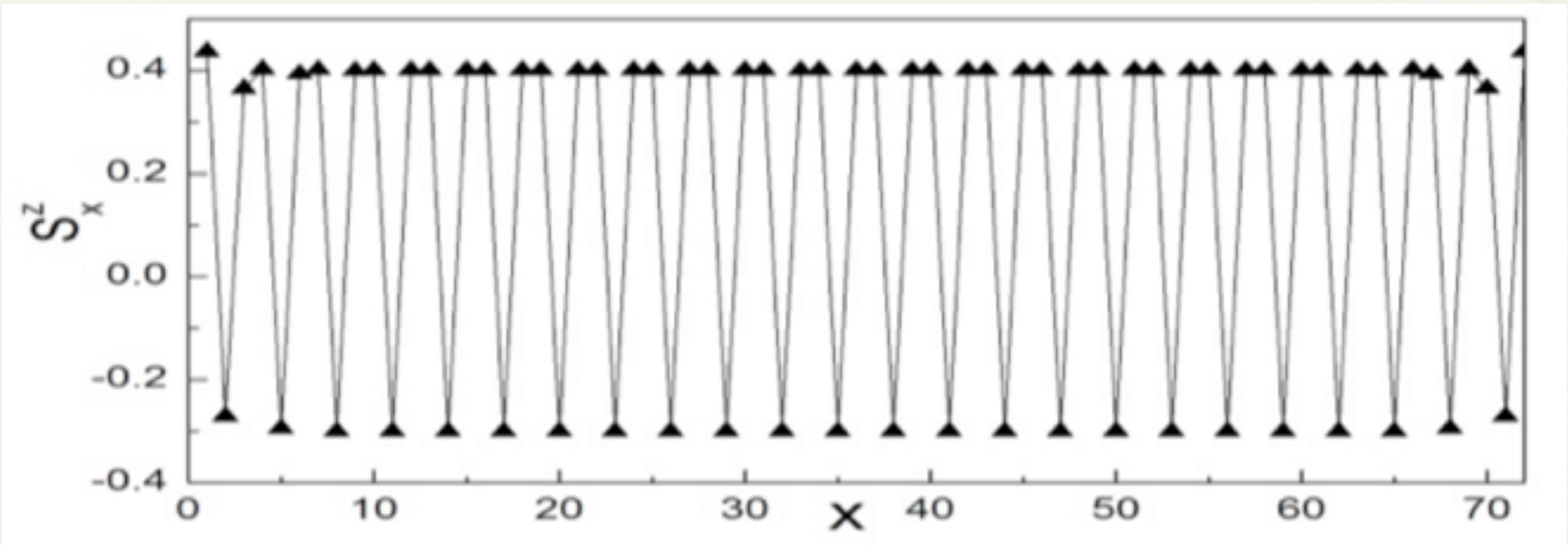
Modeling quantum spins by classical with **biquadratic** interaction Griset, Head, Alicea, Starykh (2011)

Plateau phase

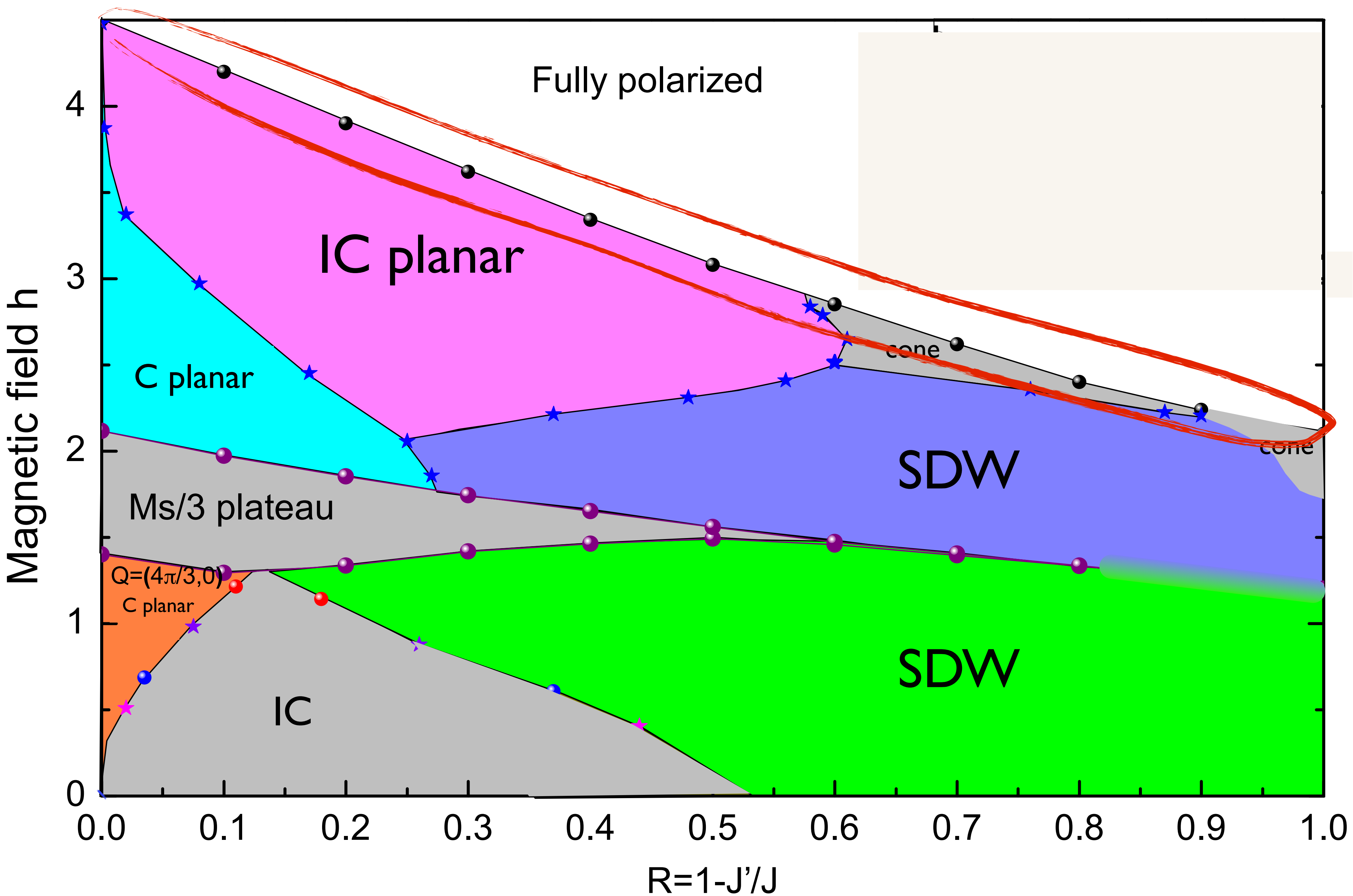


M/Ms=1/3 plateau:
uud state

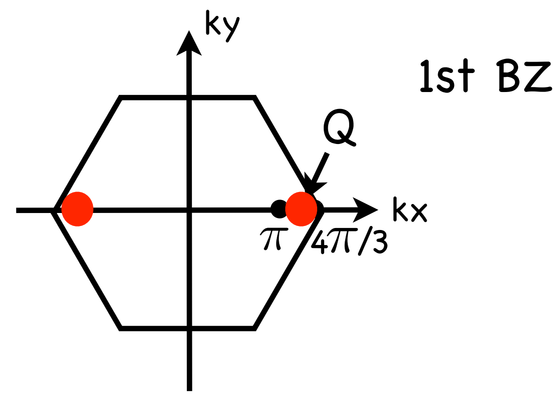




High field: condensation of spin flips



Spin-flip bosons



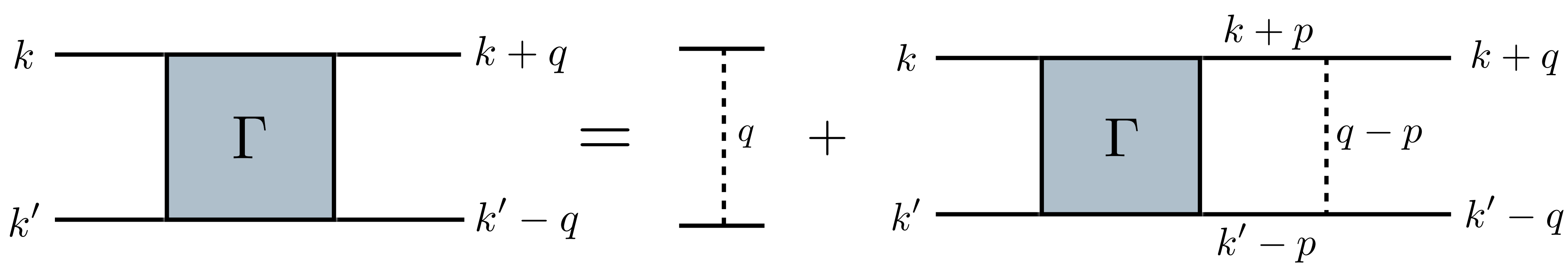
- Magnons at k=Q and k=-Q are degeneracy by inversion symmetry, but Q varies smoothly with R=1-J'/J
  - Two "Bose condensates" $\langle S^+ \rangle = \psi_1 e^{iQx} + \psi_2 e^{-iQx}$
  - Free energy

$$F = -\mu(|\psi_1|^2 + |\psi_2|^2) + \frac{1}{2}\Gamma_1(|\psi_1|^4 + |\psi_2|^4) + \Gamma_2|\psi_1|^2|\psi_2|^2$$

  - $\Gamma_1 > \Gamma_2 : |\psi_1| = |\psi_2|$ $\qquad$ $\Gamma_1 < \Gamma_2 : \psi_1\psi_2 = 0$

Spin-flip bosons

- Quadratic parameters can be computed from single magnon spectra and quartic ones from exact solution of Bethe-Saltpeter equation

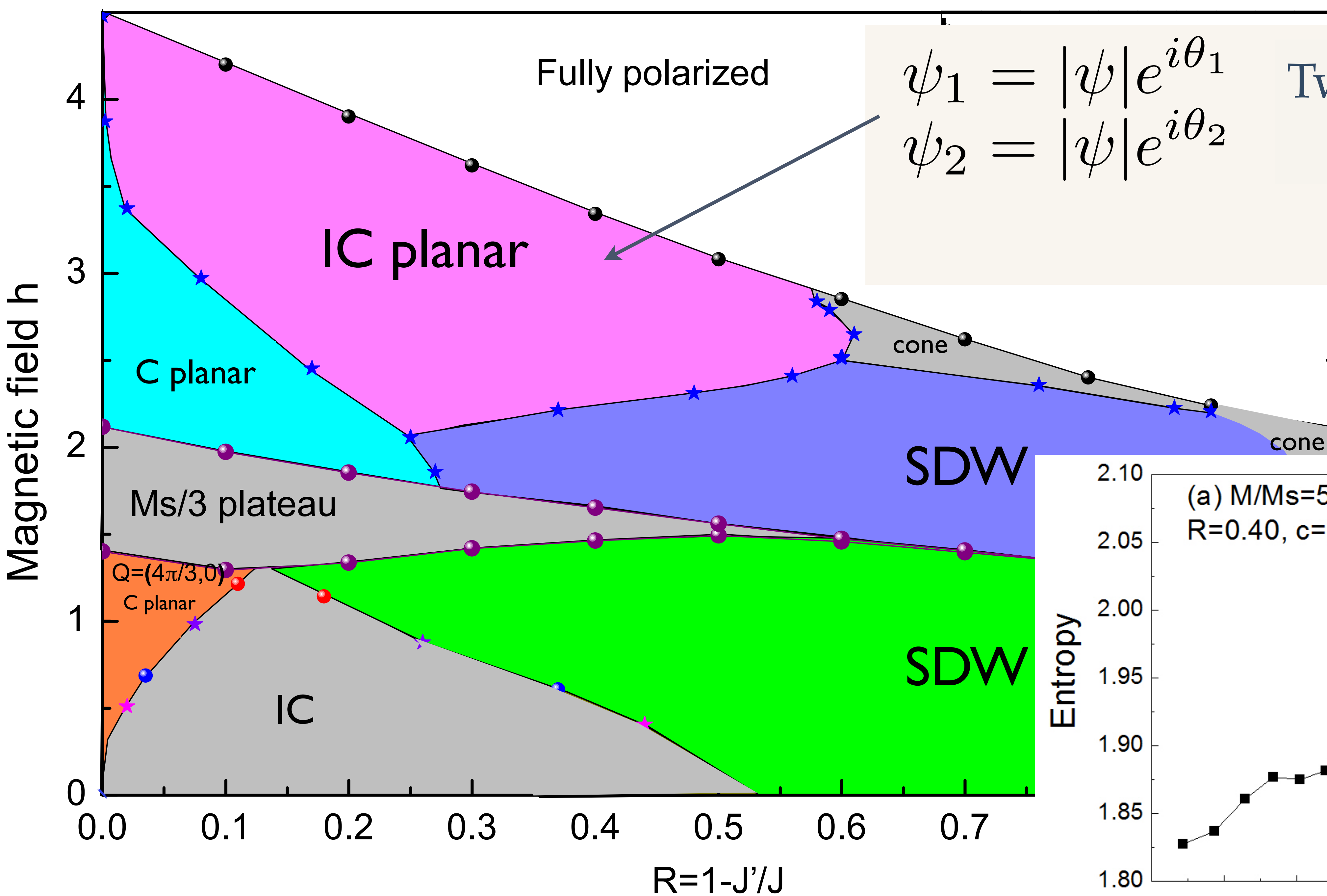


- Results:

  - In 2d, $\Gamma_1 > \Gamma_2$ for all R: *incommensurate planar state* near saturation for S=1/2 [planar-cone transition does appear for S > 1/2]

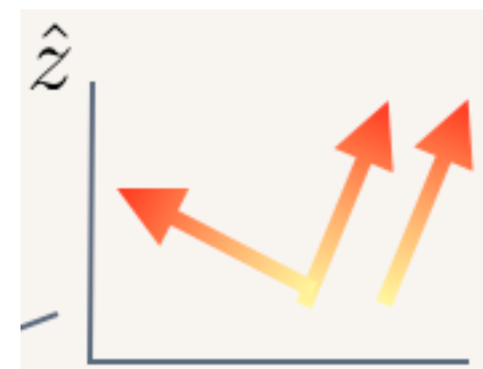
  - For 3-leg ladder, $\Gamma_1 = \Gamma_2$ for R = 0.57: transition between cone and planar states (S=1/2)

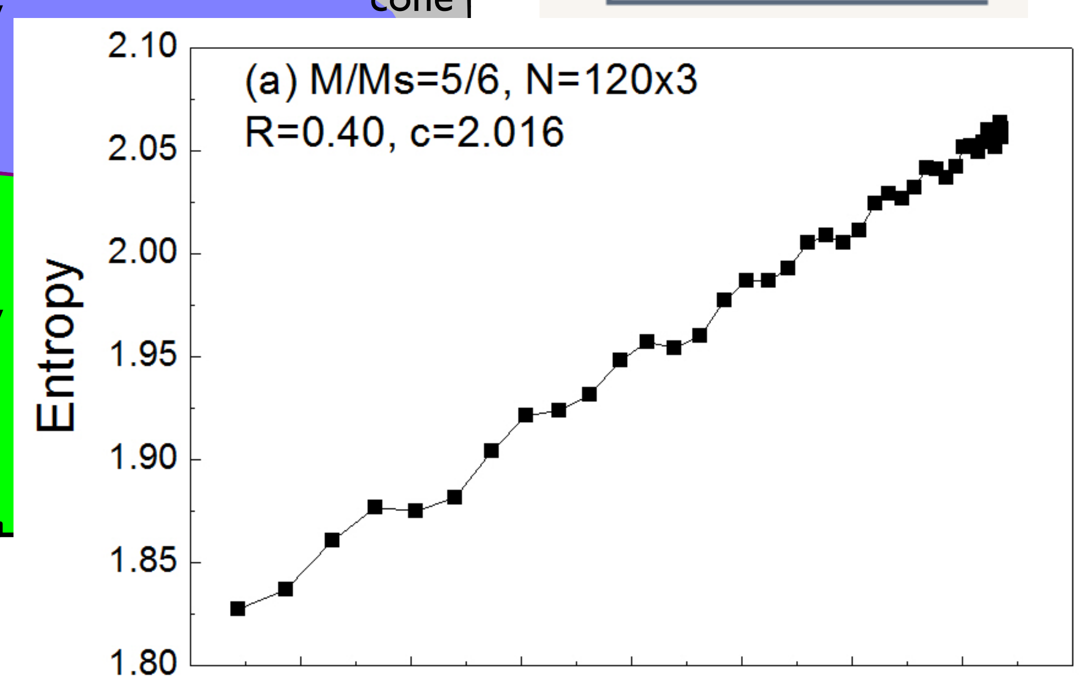
High field: spin flip bosons



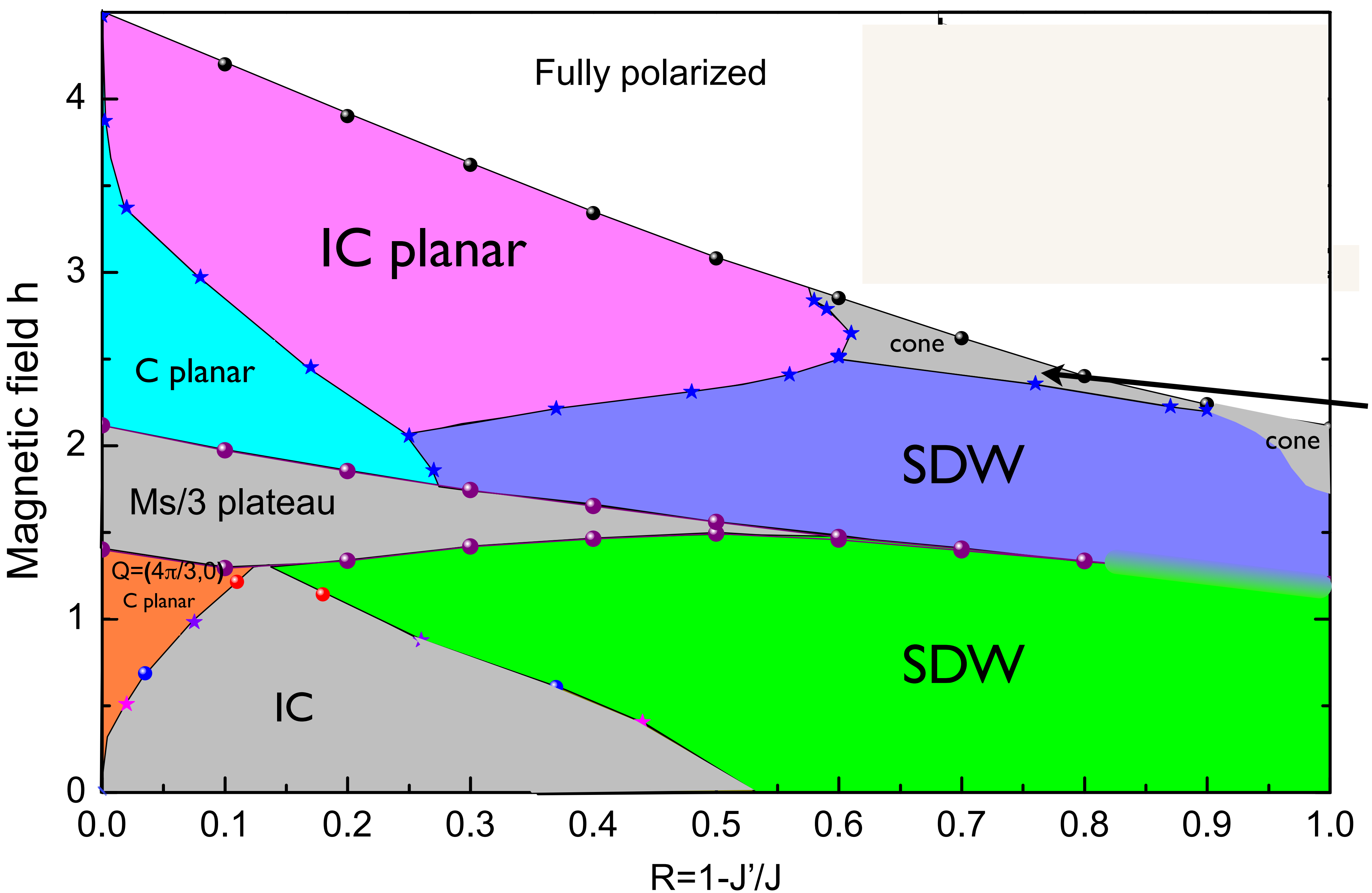
$$\psi_1 = |\psi| e^{i\theta_1}$$
$$\psi_2 = |\psi| e^{i\theta_2}$$

Two boson (c=2) theory





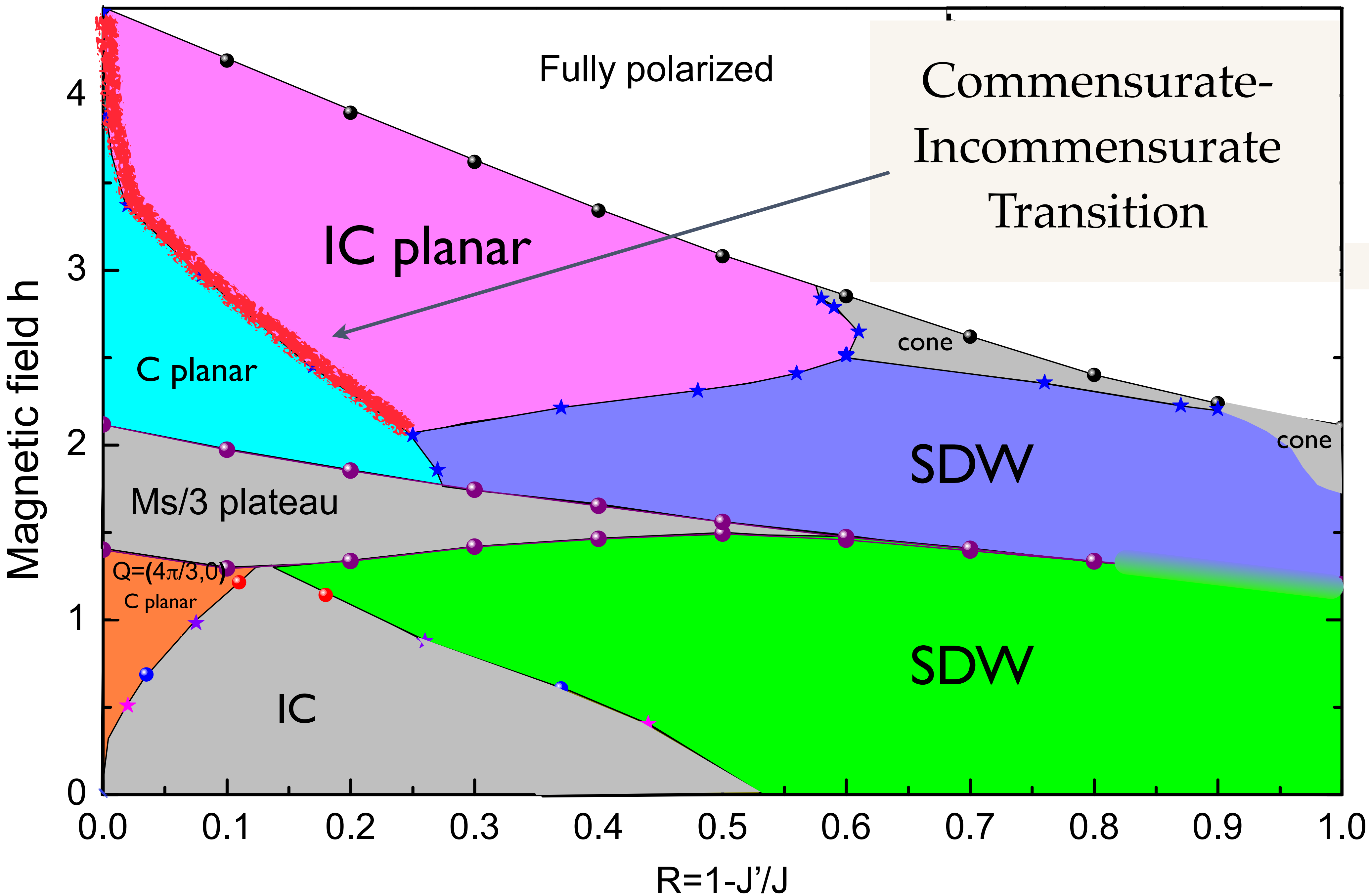
High field: spin flip bosons



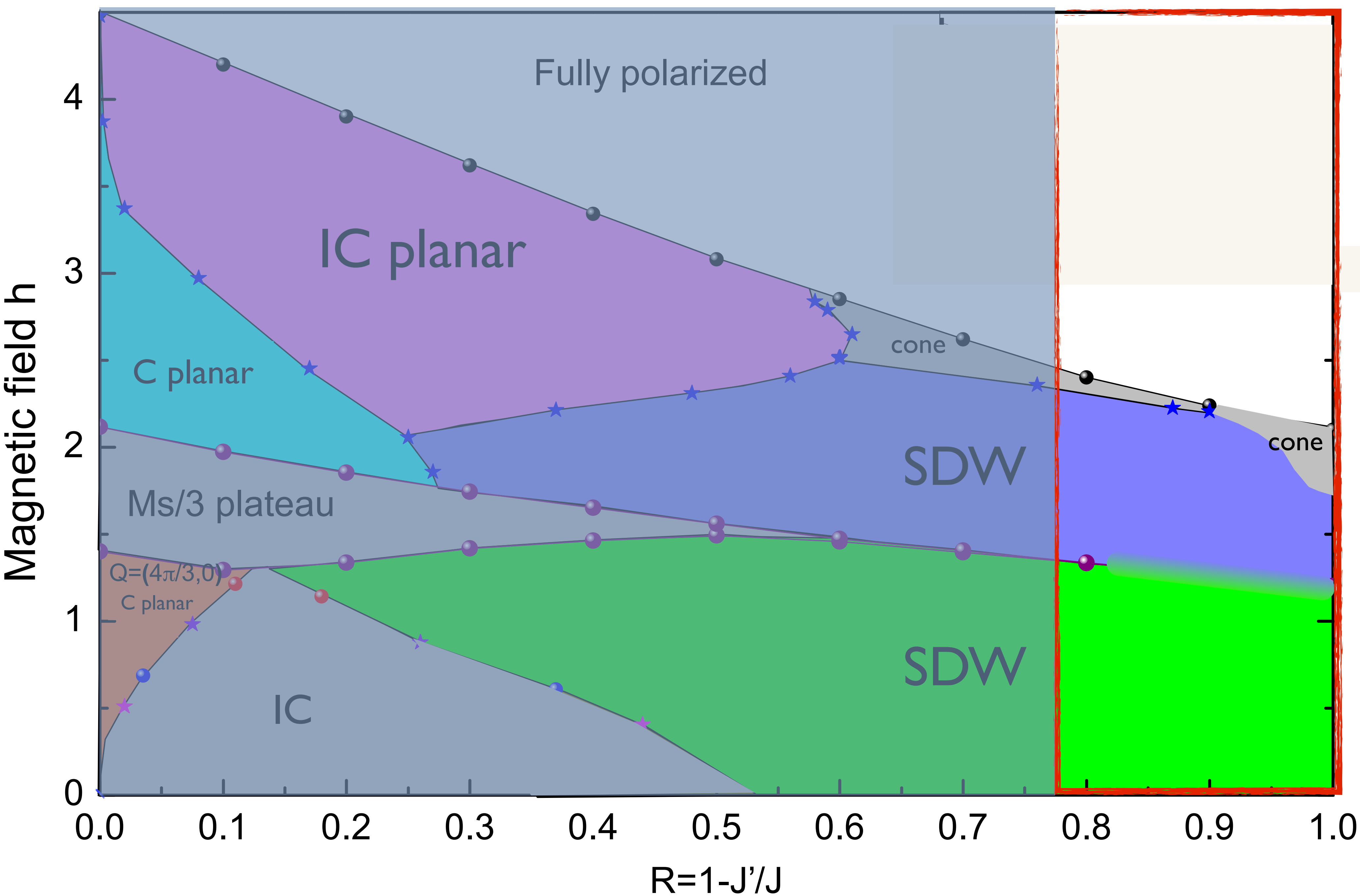
$$\psi_1 = |\psi| e^{i\theta}$$
$$\psi_2 = 0$$

c=1 phase: incommensurate cone

High field: spin flip bosons

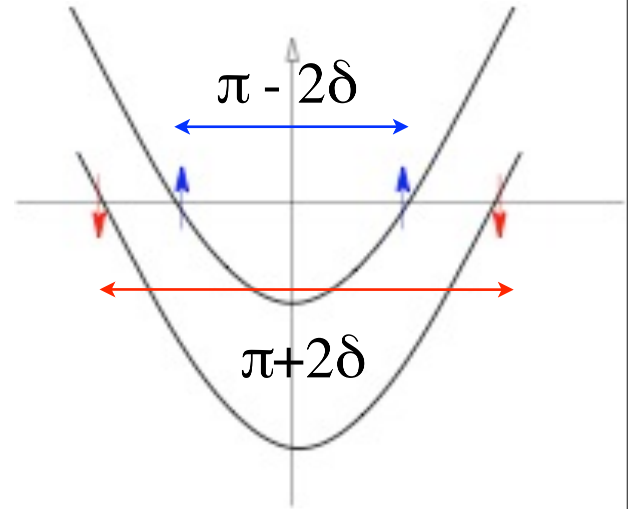


Weakly coupled chains



weakly coupled chains

S=1/2 AFM Chain in a Field



$$\mathcal{H} = J\sum_x \vec{S}(x)\cdot\vec{S}(x+1) - h\sum_x S^z(x)$$

- Field-split Fermi momenta:

$$k_{F\uparrow} = \frac{\pi}{2} - \delta\ ,\ k_{F\downarrow} = \frac{\pi}{2} + \delta$$

  ✓ Uniform magnetization

$$M = \frac{k_{F\uparrow} - k_{F\downarrow}}{2\pi} \rightarrow \delta = \pi M$$

  ✓ Half-filled condition

$$n = \frac{1}{2} = \frac{k_{F\uparrow} + k_{F\downarrow}}{2\pi} \rightarrow k_{F\uparrow} + k_{F\downarrow} = \pi$$

- S^z component ($\Delta S=0$) peaked at $\pi \pm 2\delta$
  scaling dimension
  increases

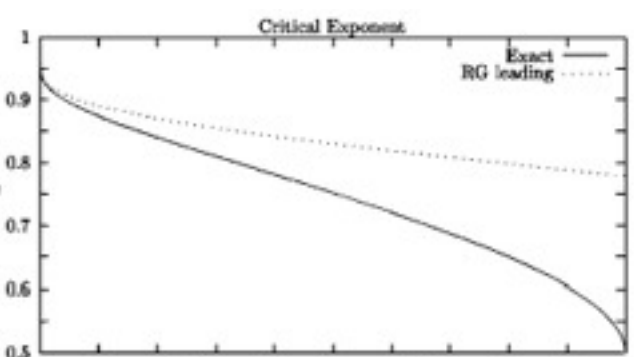
$1/4\pi R^2$

$S^z_{\pi\pm2\delta}$

Affleck and Oshikawa, 1999

1

$2\pi R^2$



1/2 0 1

h/h_{sat}

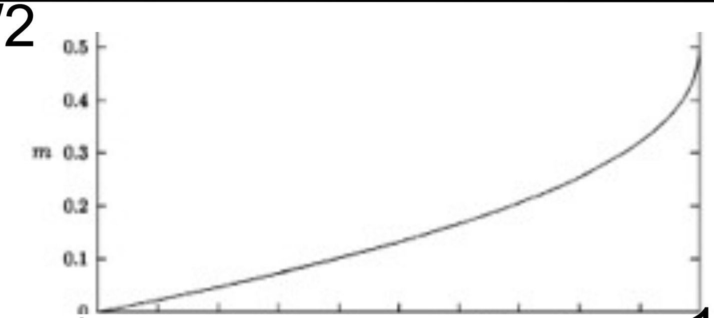
- $S^{x,y}$ components ($\Delta S=1$) remain at π
  scaling dimension
  decreases

$S^{\pm}_{\pi}$

- Derived for free el πR^2 ıt correct always - *Luttinger Theorem*

1/2

M



0 1

h/h_{sat}

1

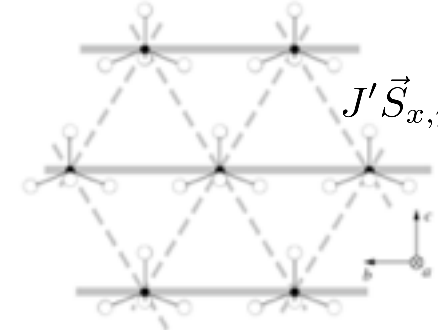
h_{sat}=2J

$2k_F$ spin density fluctuations

- XY AF correlations grow with h *and remain commensurate*
- Ising "SDW" correlations decrease with h and shift from π

Ideal J-J' model in magnetic field

OS, Balents 2007



$J'\vec{S}_{x,y} \cdot (\vec{S}_{x,y+1} + \vec{S}_{x+1,y+1})$

- Two important couplings for h>0
- Quantum phase transition between SDW and Cone states

Magnetic field relieves frustration!

$$\mathcal{H}_{\text{eff}} \sim \sum_{y \in 2\mathcal{Z}} \left[J' \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1) + J' \left(S^+_\pi(y) \partial_x S^-_\pi(y+1) + \text{h.c.} \right) \right]$$

dim $1/2\pi R^2$: 1 -> 2

dim $1+2\pi R^2$: 2 -> 3/2

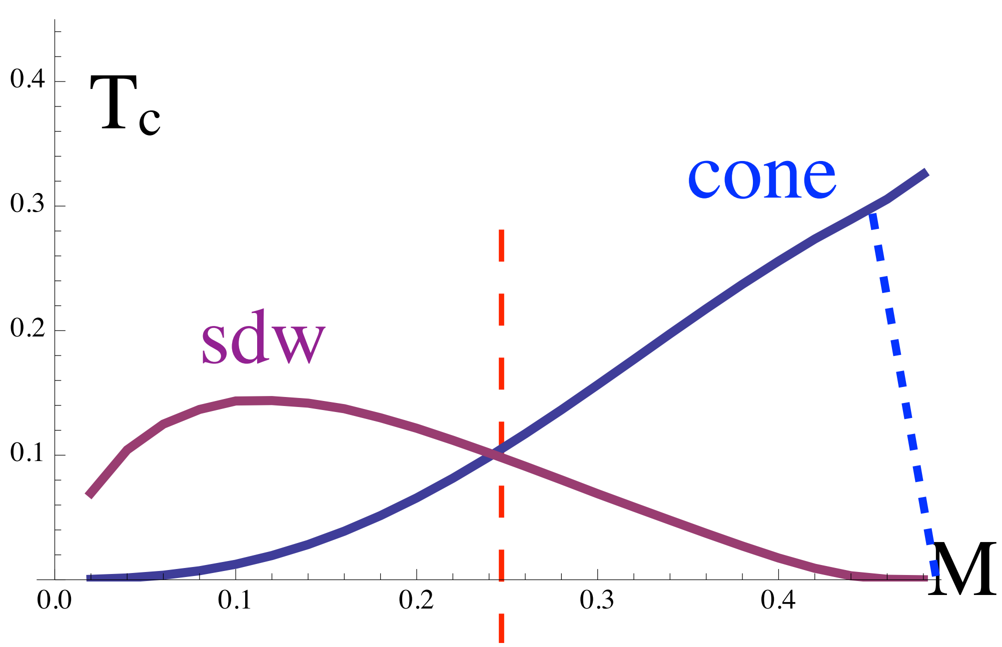
spiral “cone” state

$$k_{F\downarrow} - k_{F\uparrow} = 2\delta = 2\pi M$$

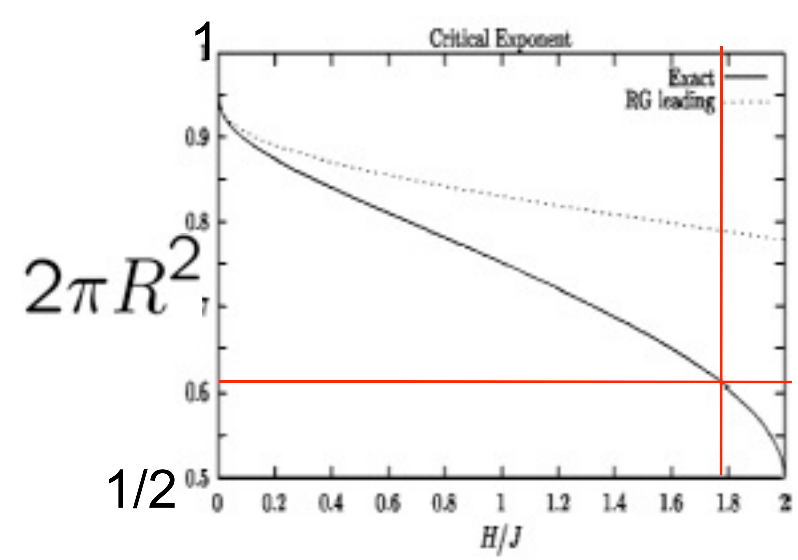
“collinear” SDW

- “Critical point”: $1+2\pi R^2 = 1/2\pi R^2$ gives at M = 0.3

$$2\pi R^2 = (\sqrt{5}-1)/2 \approx 0.62$$



also: Kolezhuk, Vekua 2005



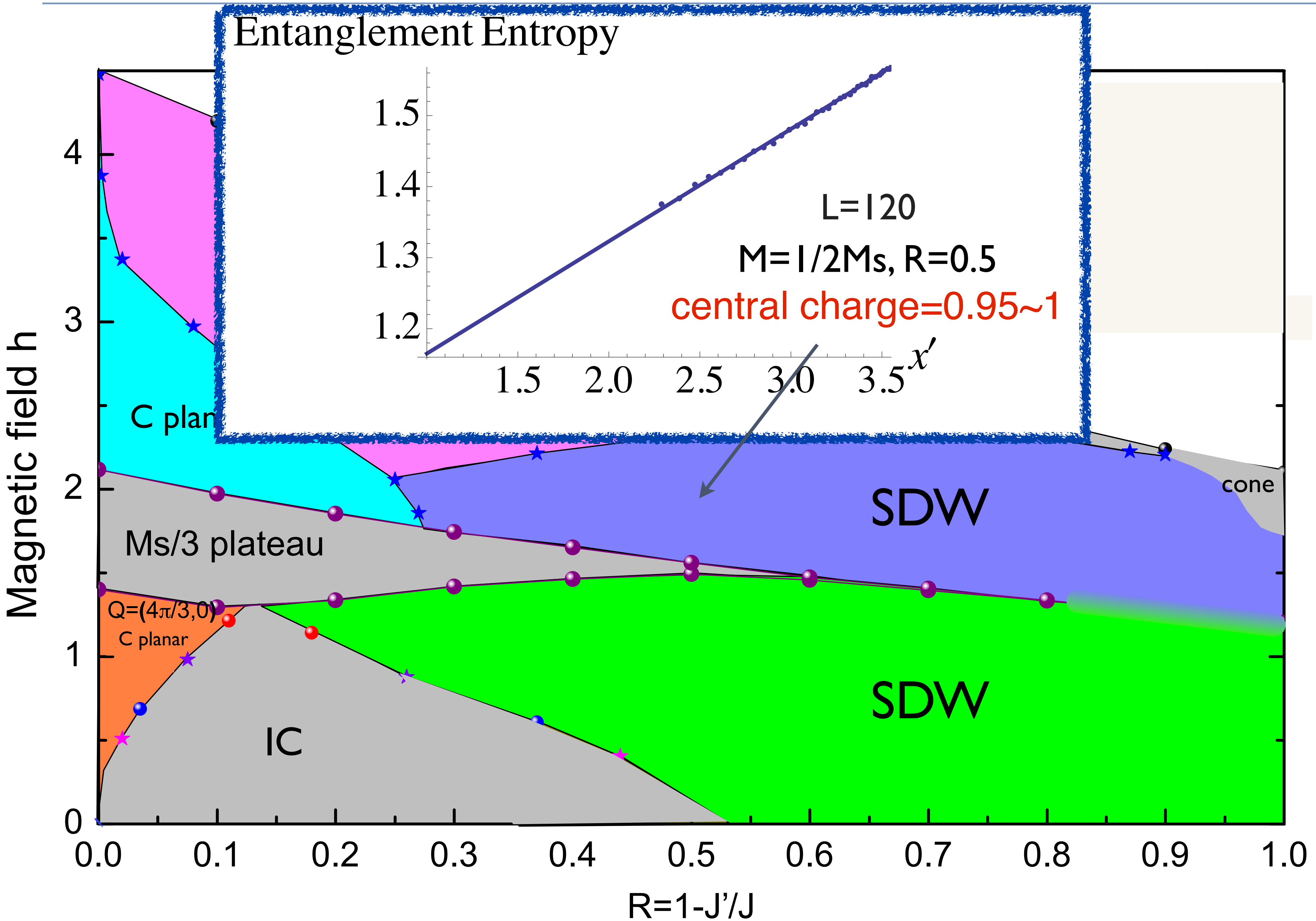
h/h_{sat}

“SDW” and “cone” states

- In 1d, there is no long-range SDW or cone order
- Both these states are *Luttinger liquids*, with one gapless mode (c=1)
- But SDW has very distinct correlations
  - Gap for S=1, 2 $\langle S^+_{x,y} S^-_{x',y'} \rangle \sim A e^{-\frac{|x-x'|}{\xi_{sdw}}}$
  - Multipolar correlations $\langle \prod_{y=1}^{3} (S^+_{x,y} S^-_{x',y}) \rangle \sim \frac{\cos q(x-x')}{|x-x'|^{1/\eta}}$
  - Slow SDW correlations

$$\eta = 1/6\pi R^2 \leq 2/3 \qquad \langle S^z_{x,y} S^z_{x',y'} \rangle \sim \frac{\cos Q(x-x'+y-y')}{|x-x'|^{\eta}}$$

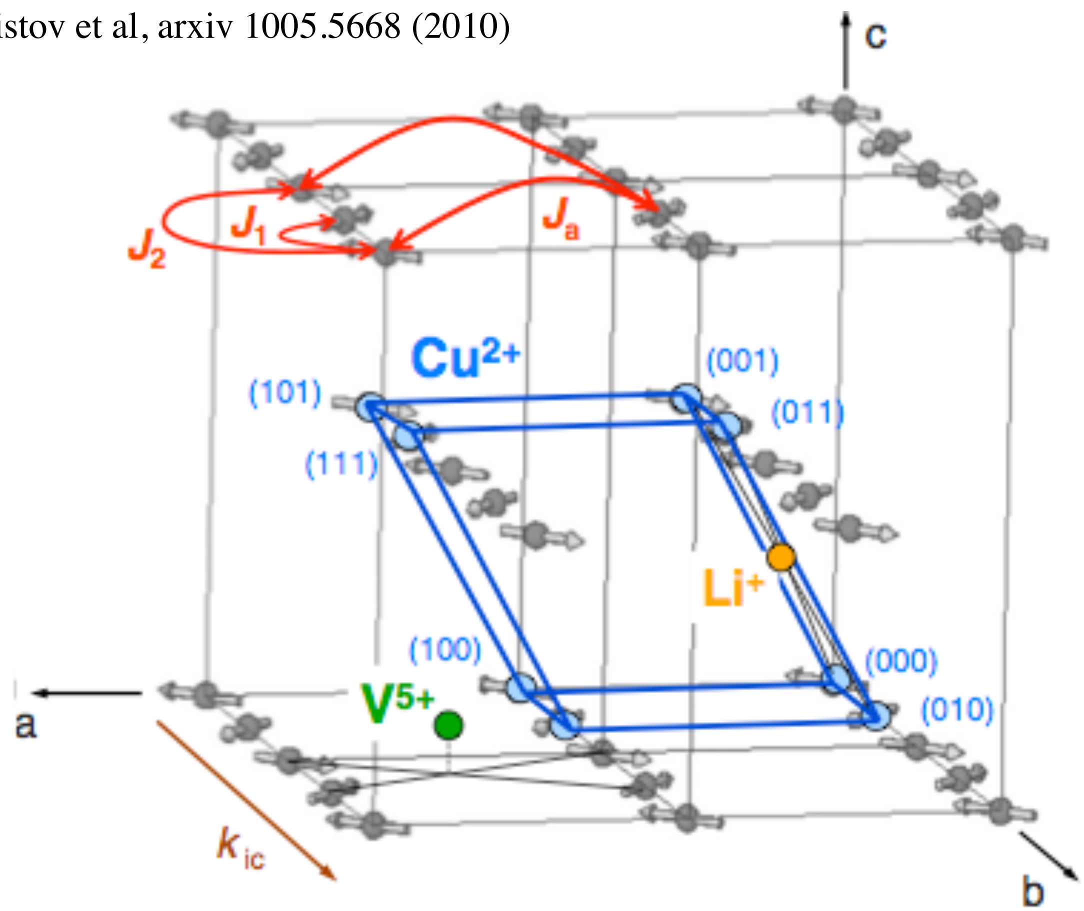
SDW

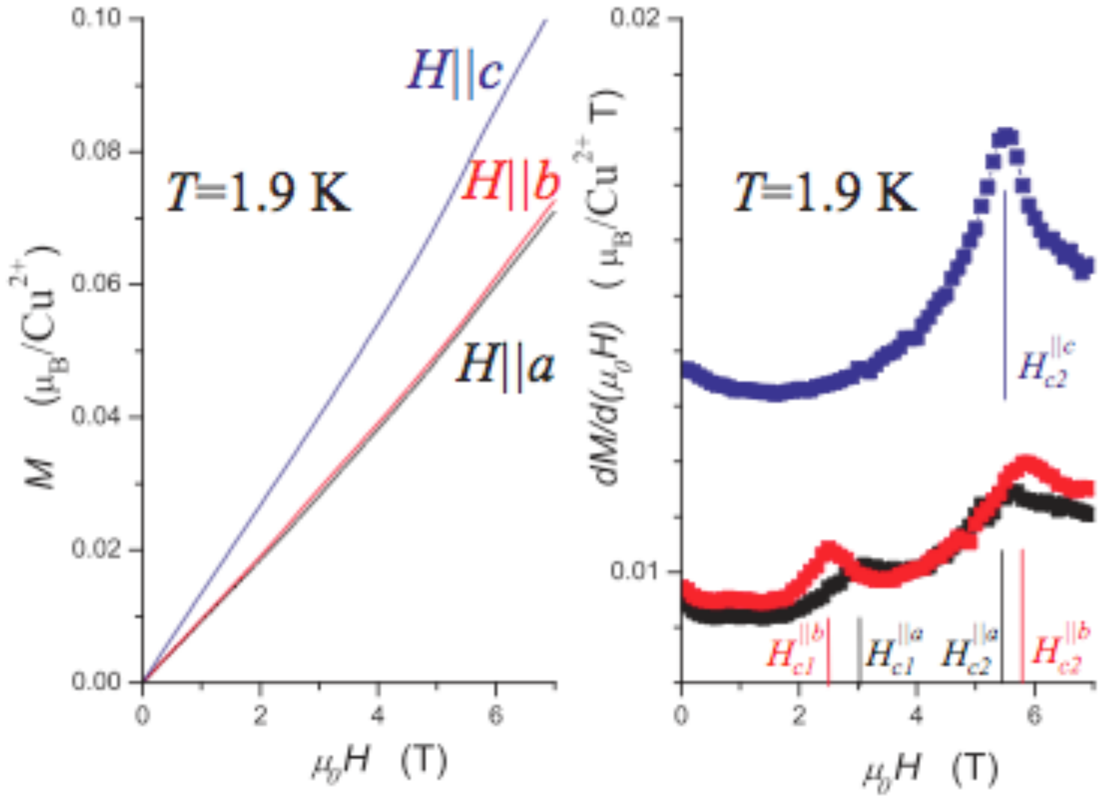


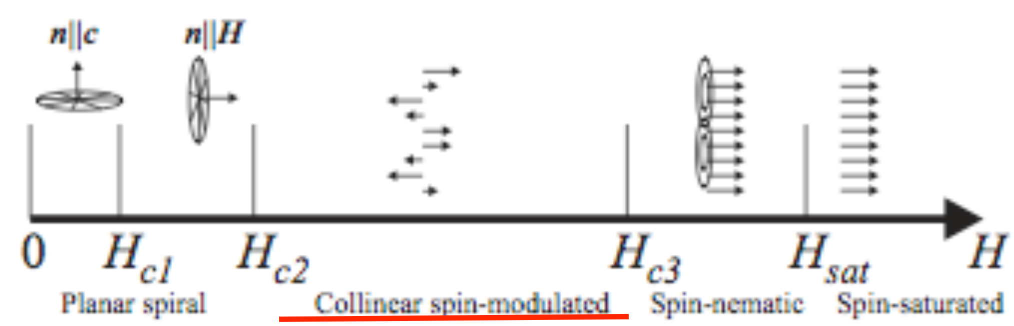
SDW seems remarkably robust

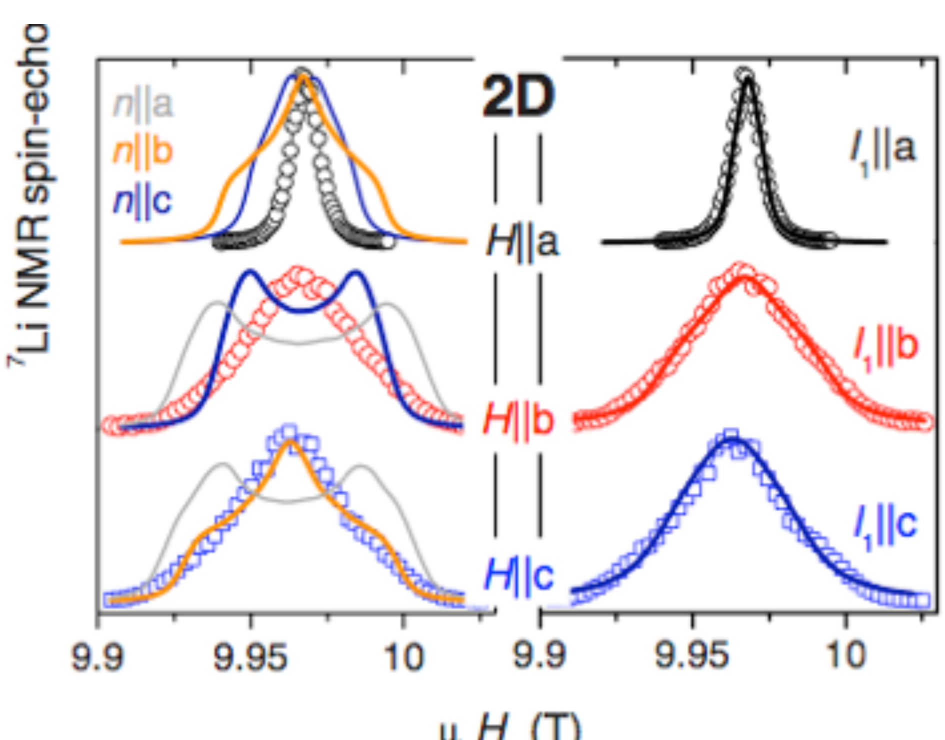
SDW in $LiCuVO_4$: J_1=-18K, J_2=49 K, J_a=-4.3K

Buttgen et al, PRB 81, 052403 (2010);
Svistov et al, arxiv 1005.5668 (2010)









In conclusion, the magnetic structure of the high-field magnetic phase of the quasi-1D antiferromagnet $LiCuVO_4$ was studied by NMR experiments. We determined that the spin-modulated magnetic structure ($\mathbf{l_1} \| \mathbf{H}$) with long-range magnetic order within the **ab** plane and a random phase relation between the spins of neighboring **ab** planes is realized in $LiCuVO_4$ at $H > H_{c2}$ and low temperatures $T < T_N$. The observed NMR spectra can be satisfactorily described by the following structure:

$$\mu(x,y,z) = \mu_{Cu} \cdot \mathbf{l} \cdot \cos[k_{ic} \cdot y + \phi(z)], \qquad (2)$$

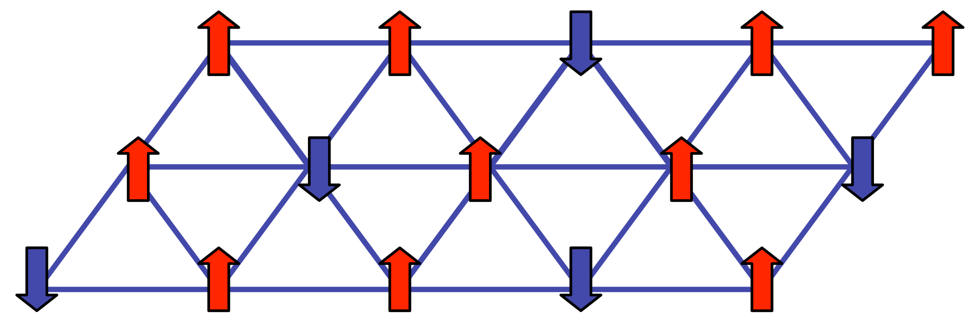
where $\mathbf{l}$ is the unit vector parallel to the applied magnetic field $\mathbf{H}$ and the phase $\phi(z)$ between adjacent spins in $\mathbf{c}$ di-

Plateau from SDW

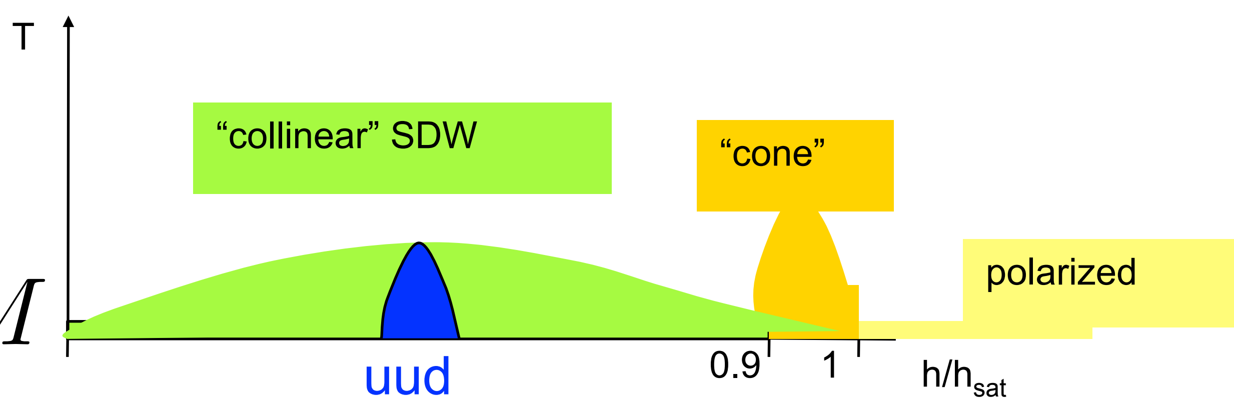
- "Collinear" SDW state *locks* to the lattice

  -"irrelevant" (1d) umklapp terms become relevant once SDW order is present (when *commensurate*): multiparticle umklapp scattering

  -strongest locking is at $M = 1/3\ M_{sat}$



$$\delta = \pi M$$



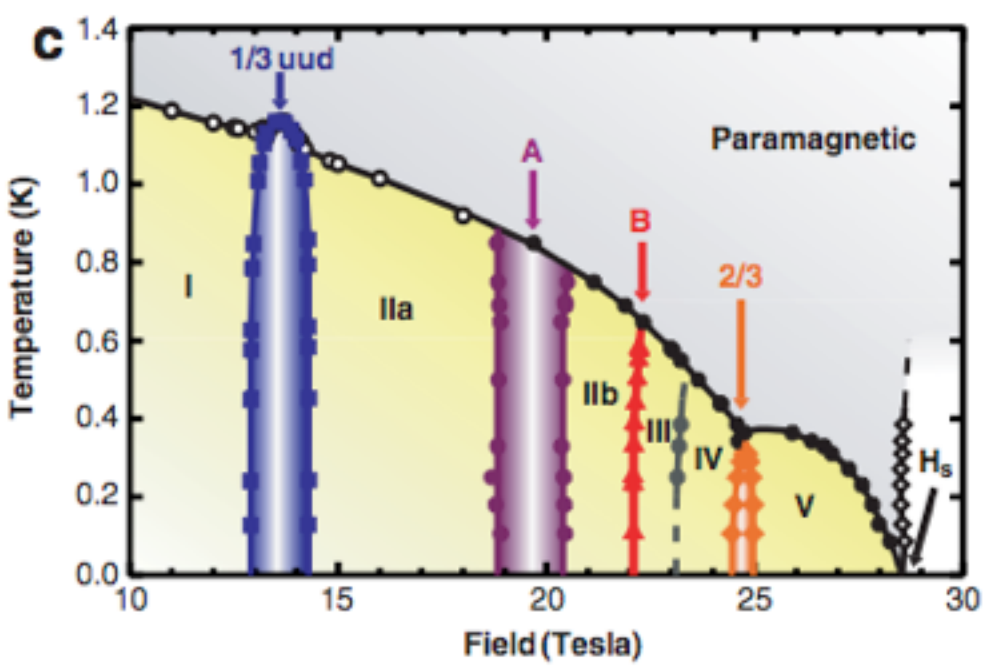
$$\left(\Psi_R^\dagger \Psi_L\right)^n \rightarrow (\pi - 2\delta)n = 2\pi m \rightarrow \boxed{2M = 1 - 2m/n}$$

n	3	4	5	5	6
m	1	1	1	2	1
2M	1/3	1/2	3/5	1/5	2/3

naively thinking

Cs_2CuBr_4 Fortune et al 2009

1/3 2/3



Plateau more carefully

OS, Katsura, Balents PRB 2010

$$M^{(n,m)} = \frac{1}{2}\left(1 - \frac{2m}{n}\right)$$

- Umklapp must respect triangular lattice symmetries
  - translation along chain direction $\phi_y(x) \to \phi_y(x+1) - R(\pi - 2\delta)$
  - translation along diagonal $\phi_y(x) \to \phi_{y+1}(x+1/2) - R(\pi - 2\delta)/2$
  - spatial inversion $\phi_y(x) \to \pi R - \phi_y(-x)$

$$H_{umk}^{(n)} = \sum_y \int dx\, t_n \cos[\frac{n}{R}\phi_y]$$ **and** $$n = m \ (mod\ 2)$$ ***same parity condition***

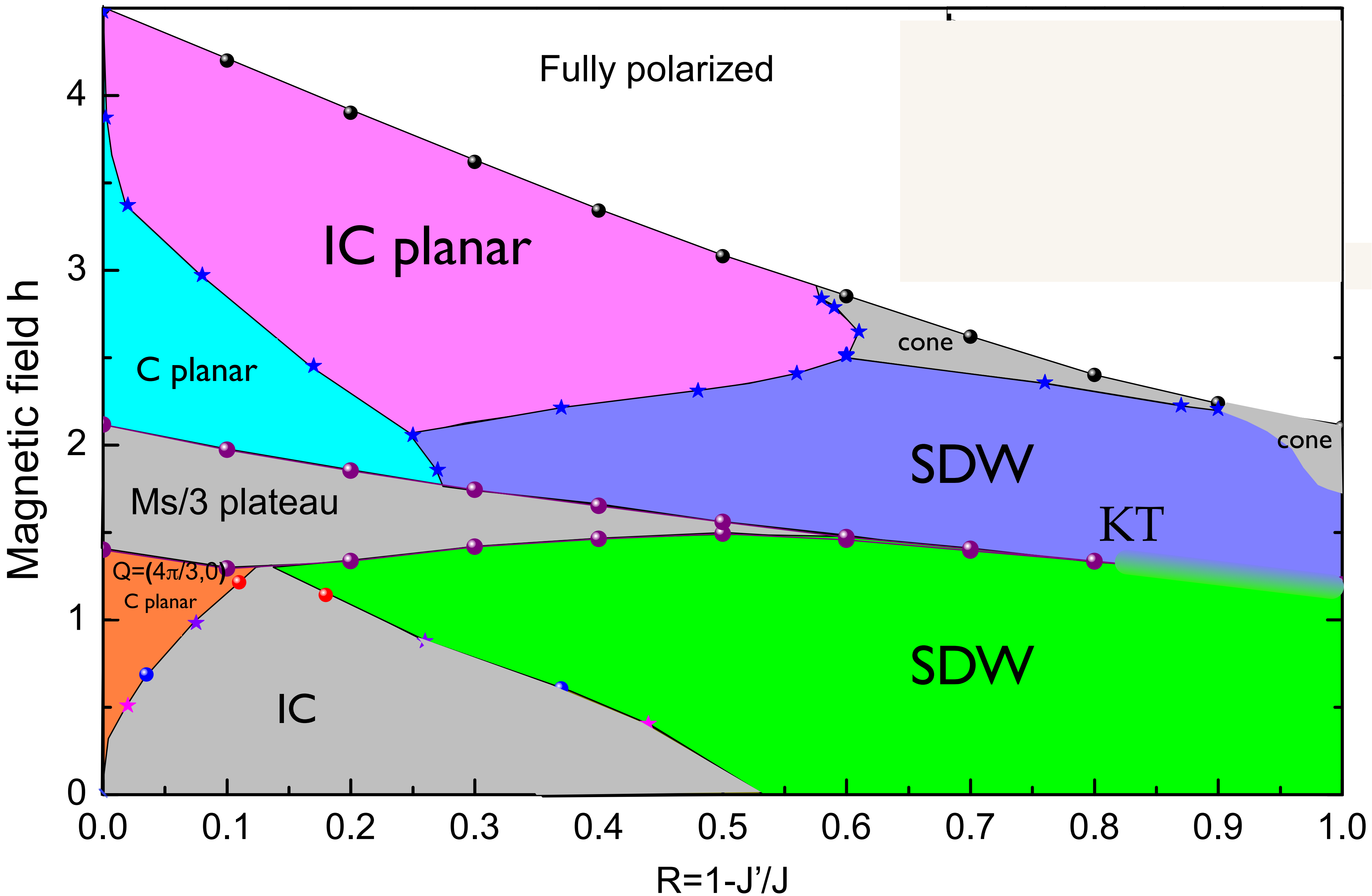
- n-th plateau width (in field) $width \sim \left(J'/J\right)^{n^2/(4(4\pi R^2 - 1))}$

n	**3**	**8**	**5**	**10**	**12**
m	**1**	**2**	**1**	**4**	**2**
2M	**1/3**	1/2	**3/5**	1/5	2/3

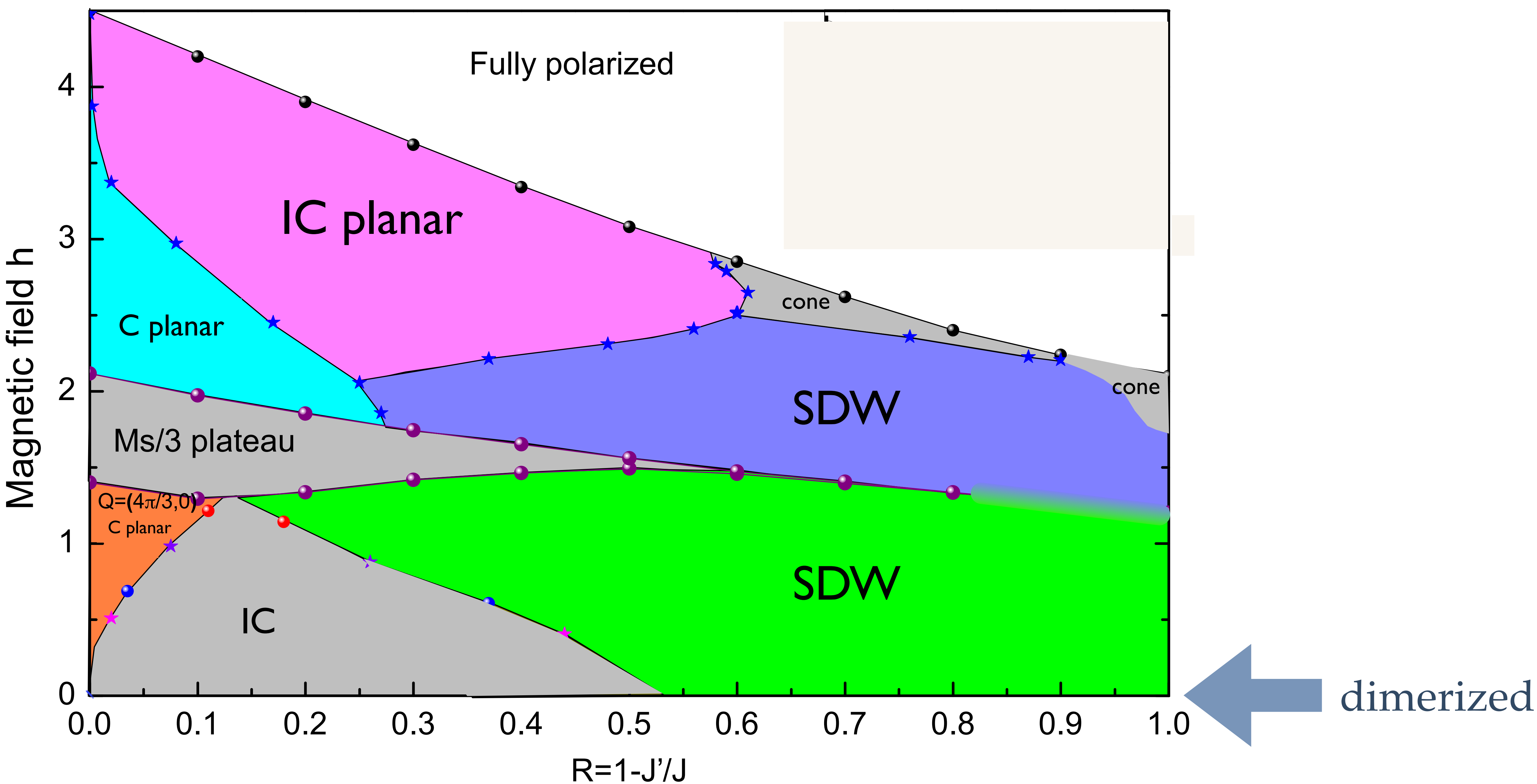
large **n** leads to exponential suppression

- Ladder: Kosterlitz-Thouless transition to the plateau state @ R=0.7±0.1 (J'/J = 0.3)

Plateau endpoint (ladder)

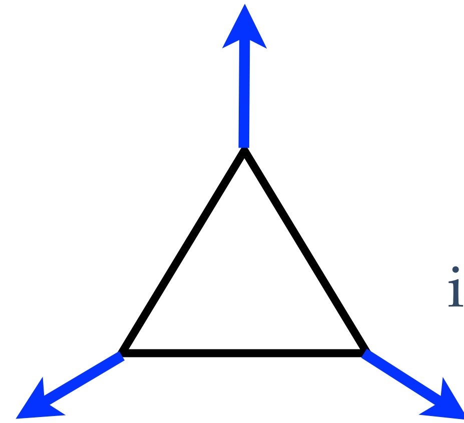


Zero field

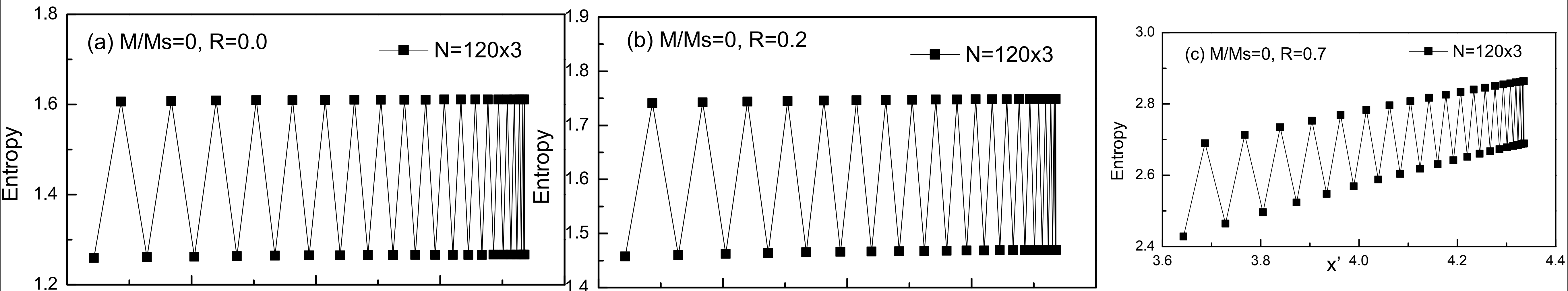


Fully polarized
IC planar
C planar
cone
cone
SDW
Ms/3 plateau
Q=(4π/3,0)
C planar
SDW
IC
Magnetic field h
R=1-J'/J
0
1
2
3
4
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
dimerized

Zero field



non-collinear short range
spin correlations
induced by periodic boundary conditions



- Numerics shows dimerization for 0<J′<J (and larger!)

c.f. Fouet *et al*, 2005

  - Theory: persists for J′ << J

Schulz 1996, Kawano and Takahashi 1997

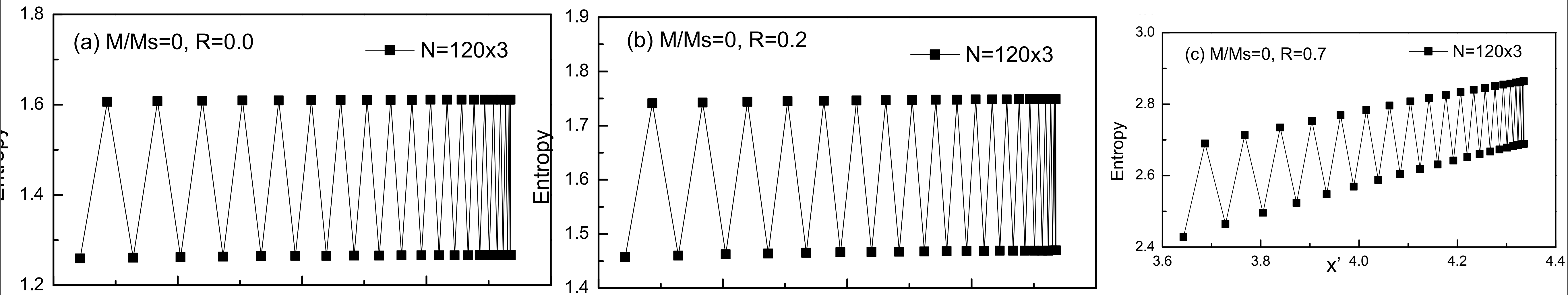
- Possible physical picture: *effective spin-orbital model for any **odd*** L_y

$$H_{\text{eff}} \propto (J')^3 \sum_{x,y} \vec{S}_y \cdot \vec{S}_{y+2} \rightarrow g' \sum_x \mathbf{s}_x \cdot \mathbf{s}_{x+1} [1 + \tau_x^+ \tau_{x+1}^- + \text{h.c.}]$$

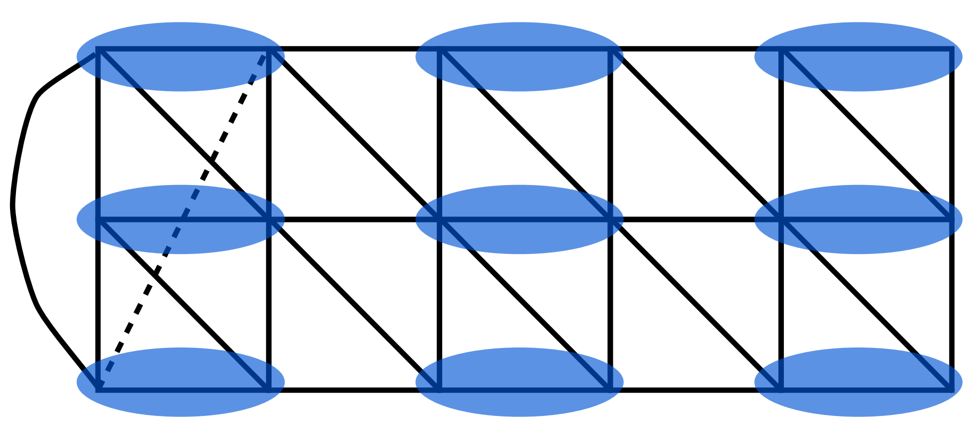
eff. spin 1/2 chirality

RG flow to strong coupling ⟶ eff.Hamiltonian in 4-dim. ground state manifold

Zero field

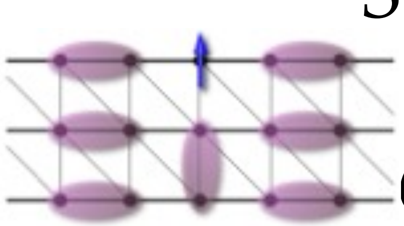


- Numerics shows dimerization for 0<J'<J (and larger!)
  - Theory: persists for J' << J
- Cartoon



Soliton liquids above dimerized state

$S^z = 1/2$ solitons

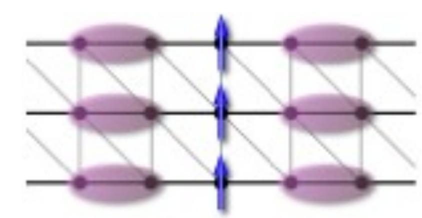


$$Q = 6\pi M/M_s$$

gapless "soliton pair" excitations carry $S^z = 0, \pm 1, \pm 2, \ldots$

$$\langle S^+_{x,y} S^-_{x',y'} \rangle \sim A/|x - x'|^\eta$$

$S^z = 3/2$ solitons



$$Q = 2\pi M/M_s$$

gapless "soliton pair" excitations carry $S^z = 0, \pm 3, \pm 6, \ldots$

$$\langle S^+_{x,y} S^-_{x',y'} \rangle \sim A e^{-\frac{|x - x'|}{\xi}}$$

Soliton liquids above dimerized state

SDW phase!

S^z =1/2 solitons

$$Q = 6\pi M/M_s$$

gapless "soliton pair" excitations carry S^z=0,±1,±2,...

$$\langle S^+_{x,y} S^-_{x',y'} \rangle \sim A/|x-x'|^\eta$$

S^z =3/2 solitons

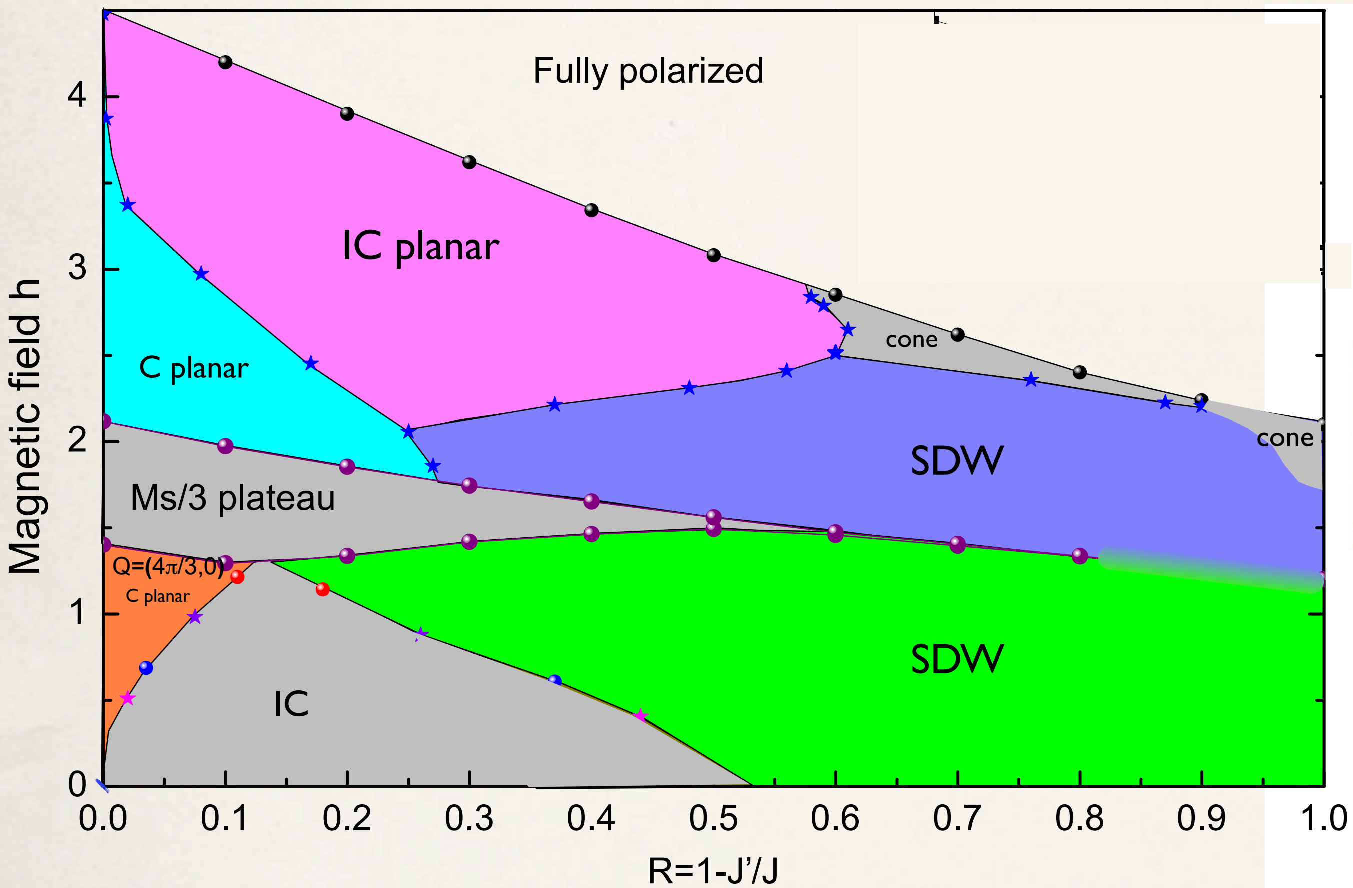
$$Q = 2\pi M/M_s$$

gapless "soliton pair" excitations carry S^z=0,±3,±6,...

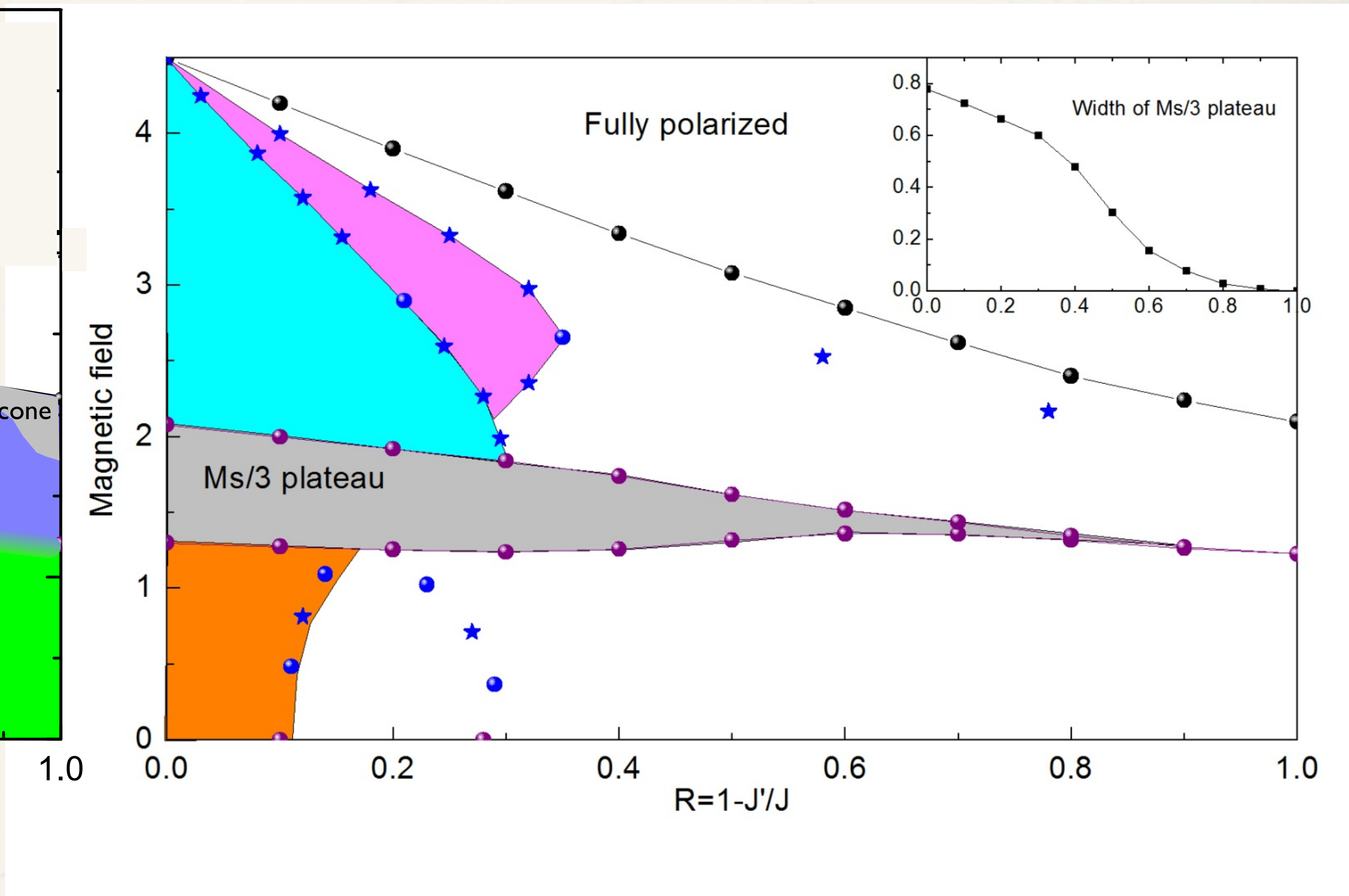
$$\langle S^+_{x,y} S^-_{x',y'} \rangle \sim A e^{-\frac{|x-x'|}{\xi}}$$

Summary

ladder

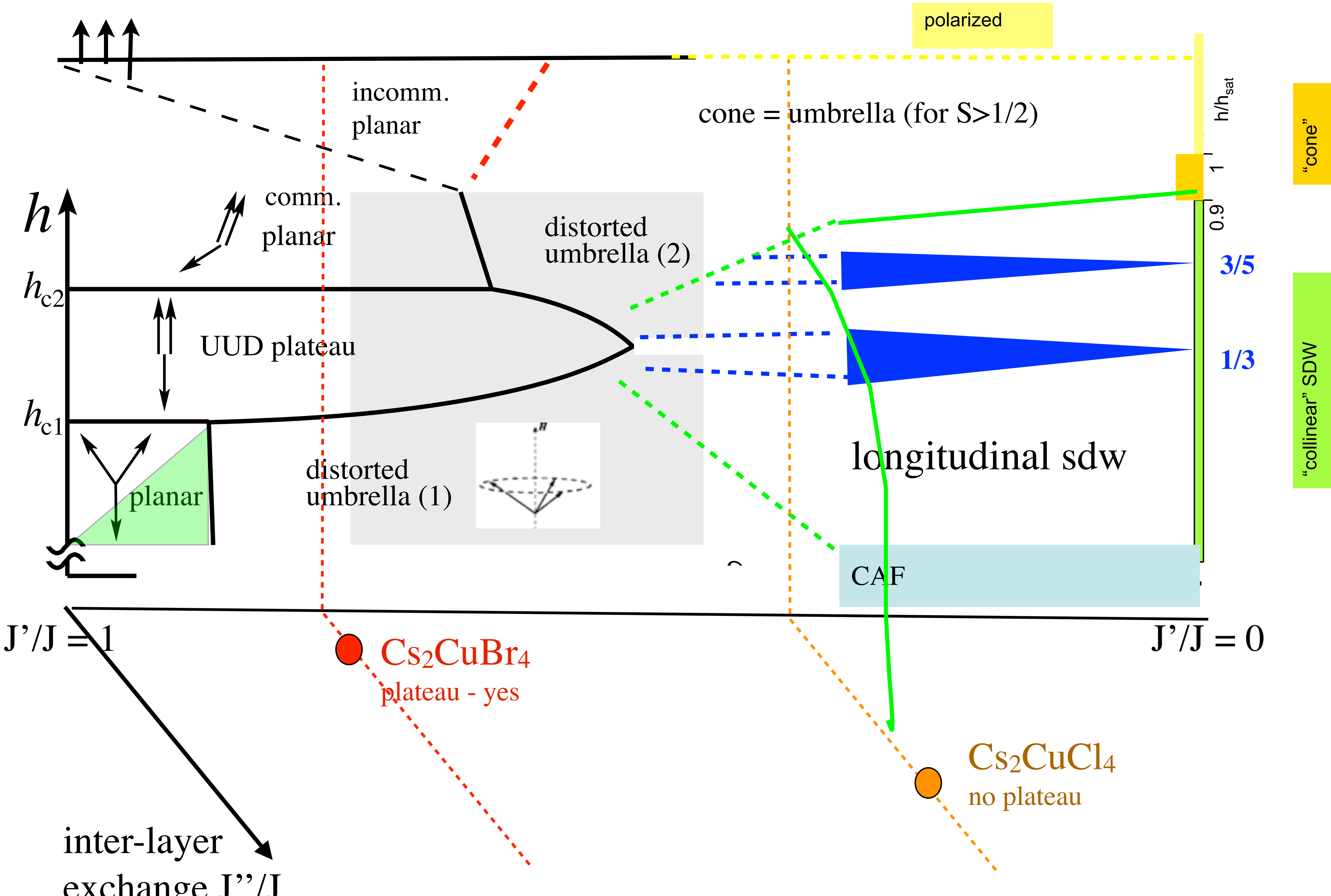


2d DMRG



- plateau and co-planar phases are surprisingly stable
- ✓ 7 out of 8 phases are of quantum origin

Bird’s eye view



large J”/J favors classical cone order !