Fingerprinting dark energy: distinctive marks of viscosity

Elisabetta Majerotto

Work done in collaboration with
Domenico Sapone
accepted by PRD [arXiv:1203.2157]

“What is ν?” workshop at GGI
Florence, 15th of June 2012

summary

1. motivation

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2. cosmological perturbations

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3. analytical solutions

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4. observable effects?

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3. analytical solutions
4. observable effects?
5. conclusions

motivation

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- Cosmological constant? $\rightarrow$ fine tuning

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In any (4D projection of) modified gravity model

$$X_{\mu\nu} = -8\pi G T_{\mu\nu} \qquad X_{\mu\nu} = G_{\mu\nu} + Y_{\mu\nu}$$

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hence I can write it as an effective fluid with

$$G_{\mu\nu} = -8\pi G \left(T_{\mu\nu} + \frac{Y_{\mu\nu}}{8\pi G} \right)$$

viscous dark energy

Effective fluid description: all parameters are seen as effective functions describing an ***effective dark energy fluid***.

Standard parameters describing dark energy:

- **equation of state** $w = p/\rho$. $w_\Lambda = -1$, $w_\phi = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$
- **speed of sound** c_s^2: $\delta p = c_s^2 \delta\rho + \frac{3aH\left(c_s^2 - c_a^2\right)}{k^2}\rho V$.
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We add one **extra parameter: the viscosity of the fluid** c_v^2 W. Hu, Astrophys. J. 506 (1998) 485-494.
As an effective parameter, may describe more exotic models: extra dimensions, non minimally coupled scalar fields, modified 4D gravity...
Equation for the anisotropy σ:

$$\sigma' + \frac{3}{a}\sigma = \frac{8}{3}\frac{c_v^2}{(1+w)^2}\frac{V}{a^2 H}$$

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- motivation: recovers the free streaming equations of motion for radiation (neutrinos + photons) up to the quadrupole
- for classic scalar fields $c_{v,\phi}^2 = 0$

first order perturbation equations for dark energy

CMB $\rightarrow$ homogeneous and isotropic Universe at large scales.
At $z = 1090$, during radiation domination, the inhomogeneities are as small as 10^{-5}.
Later, when matter becomes dominant, they grow: $\delta_m \gtrsim 1$.
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\begin{aligned}
\delta' &= -\frac{V}{Ha^2}\left[1 + \frac{9a^2H^2\left(c_s^2 - w\right)}{k^2}\right] - \frac{3}{a}\left(c_s^2 - w\right)\delta + 3\left(1+w\right)\phi' \\
V' &= -(1-3c_s^2)\frac{V}{a} + \frac{k^2 c_s^2 \delta}{a^2 H} + \frac{(1+w)k^2}{a^2 H}\left[\psi - \sigma\right] \\
\sigma' &= -\frac{3}{a}\sigma + \frac{8}{3}\frac{c_v^2}{(1+w)^2}\frac{V}{a^2 H}
\end{aligned}
$$

$+$ Einstein equations.
perturbed metric: $ds^2 = a^2\left[-(1+2\psi)d\tau^2 + (1-2\phi)dx_i dx^i\right]$

our aims

Past work was mainly numerical:

- Constraints from CMB, LSS and SNIa T. Koivisto and D. F. Mota, Phys. Rev. D 73, 083502 (2006).
- Forecasts on how well future CMB experiments will constrain an early, cold and stressed dark energy. E. Calabrese, R. de Putter, D. Huterer, E. V. Linder and A. Melchiorri, Phys. Rev. D 83 (2011) 023011 [arXiv:1010.5612
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Our goals:

1. find ***analytical solutions*** in simple assumptions (matter domination, fluid description)
2. use them to **understand general behaviours** of viscous dark energy fluid
3. and to **predict observable effects**: matter power spectrum, growth of matter perturbations, ISW (integrated Sachs-Wolfe) effect.

analytical solutions

$$\begin{aligned}
\delta &= \frac{3(1+w)^2}{3c_s^2(1+w)+8\left(c_s^2-w\right)c_v^2}\frac{\phi_0}{k^2} \\
V &= -3aH\left(c_s^2-w\right)\delta \\
\sigma &= -\frac{8c_v^2\left(c_s^2-w\right)}{3c_s^2(1+w)+8(c_s^2-w)c_v^2}\frac{\phi_0}{k^2}
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Remind that

$$aH = H_0\sqrt{\Omega_m}a^{-1/2}$$

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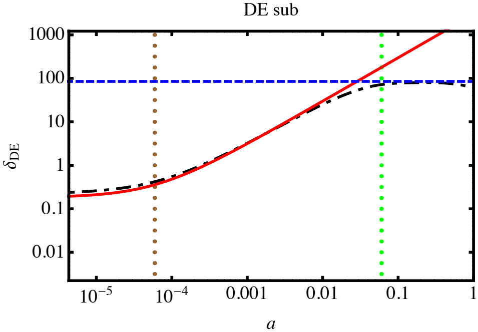

numerical solution computed with CAMB for a model with $c_v^2 = 10^{-4}$, $c_s^2 = 0$ and $w = -0.8$ for the mode $k = 200H_0$

approximated solution for $c_v^2 = 0$

approximated solution for $c_v^2 \neq 0$

a at which the mode enters the causal horizon

radiation omitted for visualisation purposes

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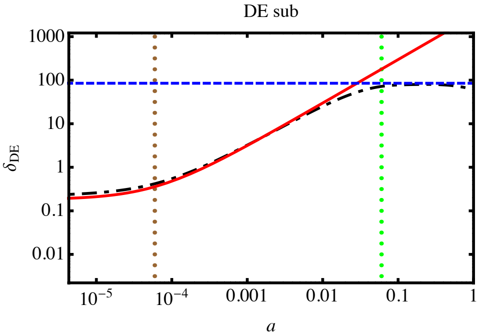

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a at which the mode enters the *anisotropic horizon:*

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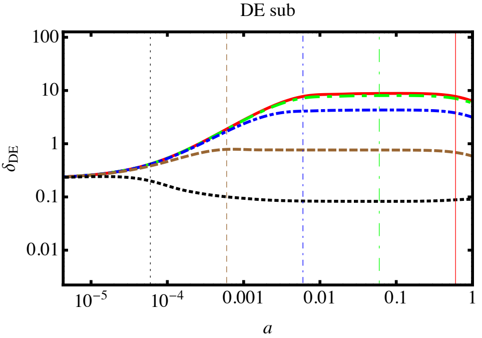
effective sound speed:

$$c_{\rm eff}^2 = c_s^2 + \frac{8}{3}\frac{\left(c_s^2-w\right)}{(1+w)}c_v^2$$

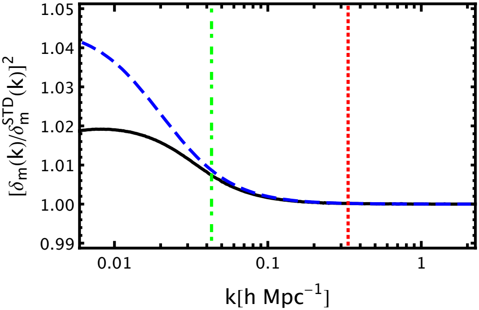


numerical solution with $k = 200H_0$ for different values of c_v^2:
$c_v^2 = 10^{-1}$, 10^{-2}, 10^{-3}, 10^{-4} and 10^{-5}
vertical lines: a at which each mode enters the *anisotropic horizon*
$c_s^2 = 10^{-2}$

radiation omitted for visualisation purposes

observable effects: matter power spectrum

$c_v^2 = 5 \times 10^{-5}$ and $c_s^2 = 10^{-6}$



numerical solution using CAMB
our analytical solution
anisotropic horizon
sound horizon

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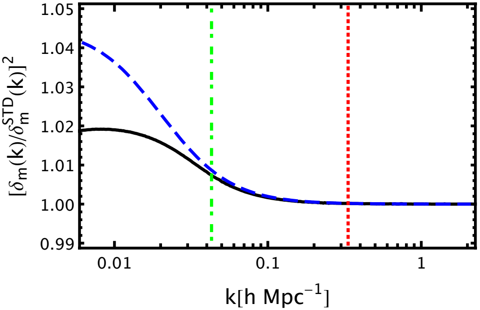

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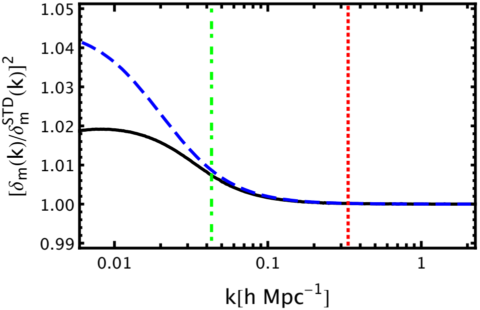

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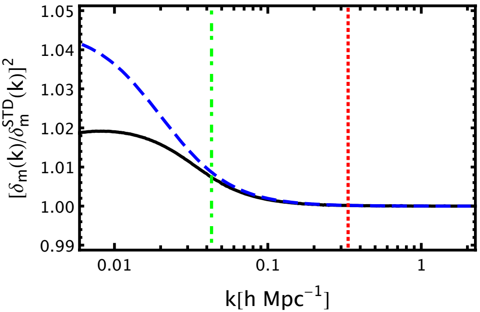

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- once matter perturbations enter the anisotropic horizon, the solution tends to the unclustered dark energy solution.
- very small effect

observable effects: growth factor

In LCDM:

- radiation domination: dark matter perturbations grow logarithmically with a
- matter domination: dark matter perturbations grow linearly
- dark energy domination: growth of dark matter perturbations is suppressed.

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Growth function: $G\left(a\right) \equiv \frac{\delta_m(a)}{\delta_m(a_0)}$ can be written as $G\left(a\right) = \exp\left\{ \int_{a_0}^{a} \frac{\Omega_m\left(a'\right)^{\gamma}}{a'} \mathrm{d}a' \right\}$

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Define clustering parameter Q and anisotropic stress parameter η: D. Sapone, M. Kunz Phys. Rev. D 80 (2009) 083519

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Q - 1 &\equiv \frac{\delta\rho}{\delta\rho_m} = \frac{1-\Omega_{m_0}}{\Omega_{m_0}}(1+w)\frac{a^{-3w}}{1-3w+\frac{2k^2 a}{3H_0^2\Omega_{m_0}}c_{\mathrm{eff}}^2} \\
\eta &\equiv \frac{\psi}{\phi} - 1 = -\frac{9}{2}H_0^2(1-\Omega_{m_0})(1+w)\frac{a^{-1-3w}}{k^2 Q}\left(1-\frac{c_s^2}{c_{\mathrm{eff}}^2}\right)
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Express γ as a function of Q and η E.V. Linder and R.N. Cahn, Astropart. Phys. 28, 481 (2007)

$$\gamma = \frac{3\left(1-w-A\left(Q,\eta\right)\right)}{5-6w} \qquad A\left(Q,\eta\right) = \frac{\left(1+\eta\right)Q-1}{1-\Omega_m\left(a\right)}$$

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- in some modified gravity model, e.g. DGP, $\gamma > \gamma_{LCDM}$
- in our model it can happen that $\gamma > \gamma_{LCDM}$, but even if we assume the viscosity term to be $c_v^2 = 1$ then $A(Q, \eta) \simeq -1.5 \times 10^{-5}$ for scales $k \simeq 200 H_0$

observable effects: ISW

$$\zeta = \frac{\Delta T\left(\hat{n}\right)}{T_0} = \int \left(\frac{\partial \phi}{\partial \tau} + \frac{\partial \psi}{\partial \tau}\right) d\tau = \int_0^{\chi_H} d\chi W_\zeta\left(\chi\right) \Delta_{m,0}\left(k\right)$$

$$W_\zeta\left(\chi\right) = \frac{3}{c^3} \frac{H_0^2 \Omega_{m_0}}{k^2} a^2 H \frac{\partial}{\partial a} \left\{G\left(a,k\right) \Sigma\left(a,k\right)\right\} \qquad \Sigma = Q\left(1 + \frac{1}{2}\eta\right)$$

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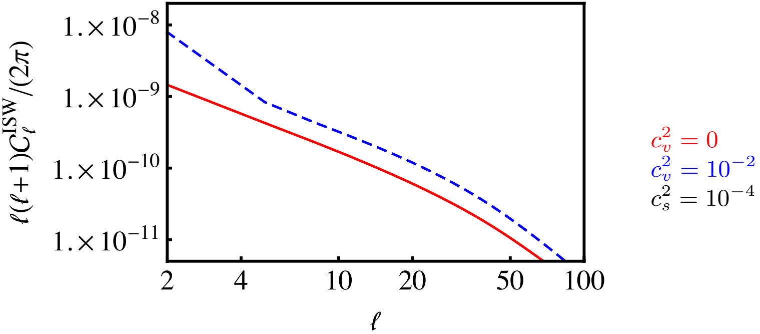
$C_\ell \equiv C_{\zeta\zeta}$ = ISW-auto correlation spectrum



observable effects: ISW

What determines the (small) effect? How does it depend on the model parameters?
$\rightarrow$ Use our analytical solution!

$$\frac{\partial(\Sigma G)}{\partial a} \equiv G'\Sigma + G\Sigma' \simeq G'(1 + \frac{G}{G'}\Sigma)$$

- $G' < 0$ when dark energy starts dominating because it slows down the growth of perturbations
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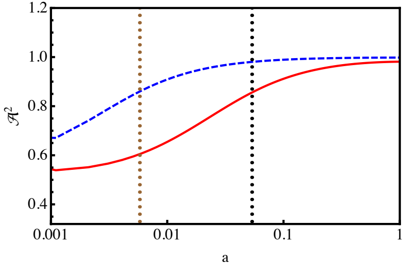

$c_v^2 = 10^{-4}$
$c_v^2 = 10^{-3}$
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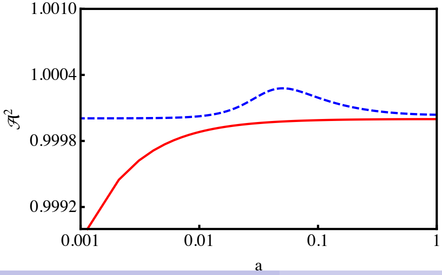

$c_v^2 = 0$
$c_v^2 = 1$
$c_s^2 = 1$

- we have studied an **imperfect fluid dark energy** with non-vanishing **viscous anisotropic stress**. parameters: w, c_s^2, c_v^2.

summary

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- we have found ***analytical solutions*** for dark energy density and velocity perturbations which match very well numerical results.

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- work in progress: compute **forecasts** on how well it will be possible to measure c_s^2, c_v^2 from the Euclid galaxy survey.