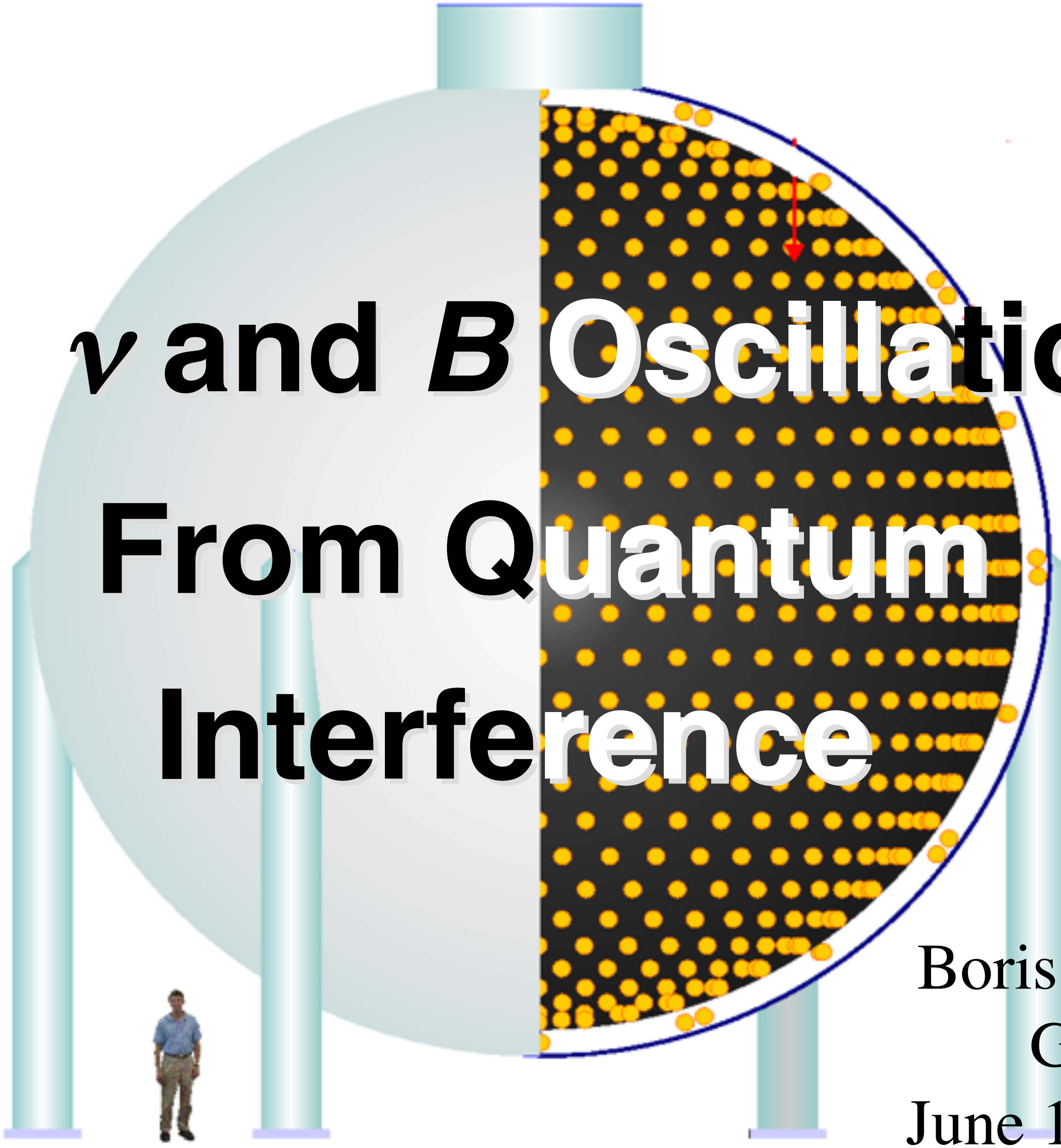
ν and *B* Oscillation From Quantum Interference



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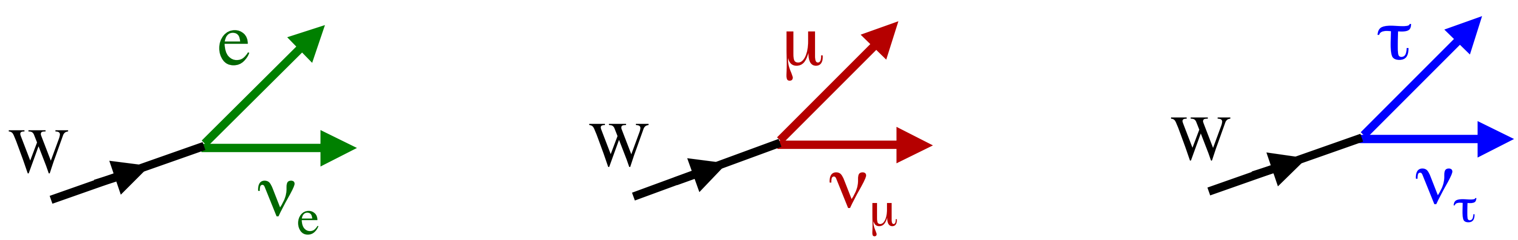
GGI
June 18, 2012

What Is Neutrino Oscillation?

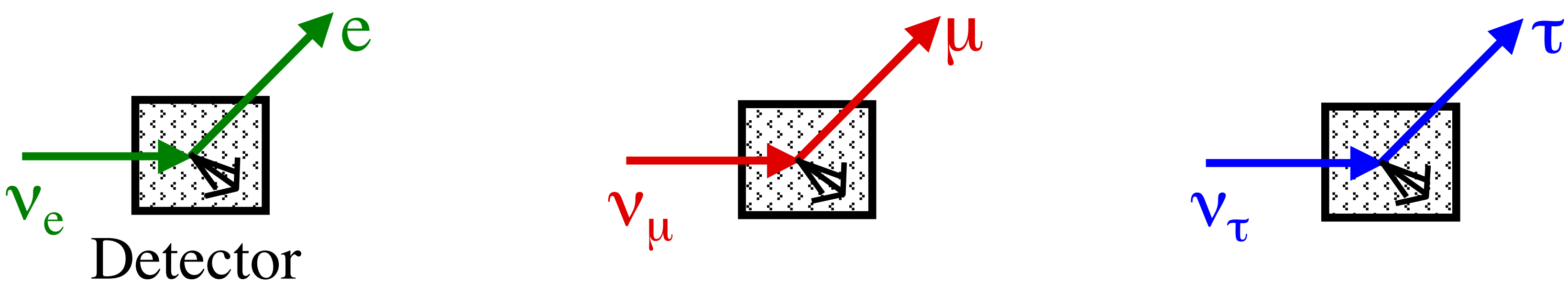
There are three flavors of charged leptons: e , μ , τ

There are three known flavors of neutrinos: ν_e, ν_μ, ν_τ

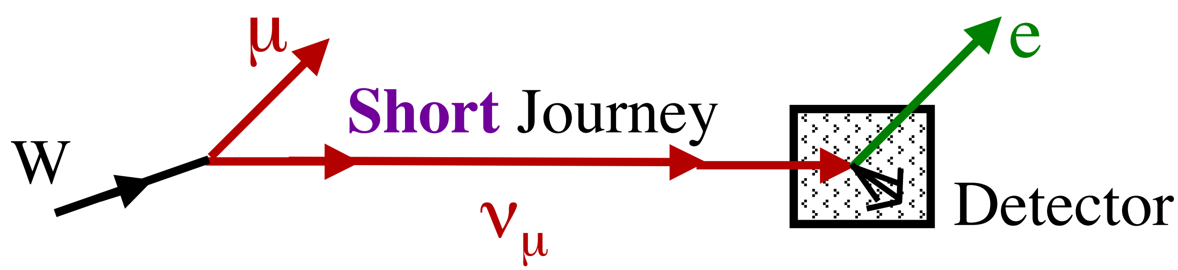
We ***define*** the neutrinos of specific flavor, ν_e, ν_μ, ν_τ, by W boson decays:



As far as we know, when interacting,
a neutrino of given flavor creates
only the charged lepton of the same flavor.

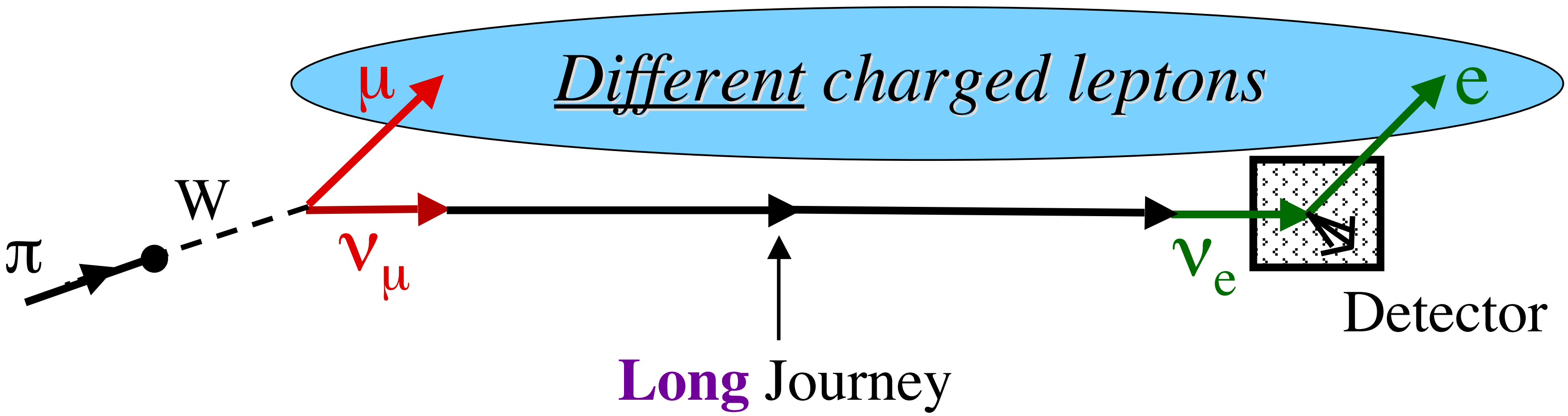


As far as we know, neither



nor any other change of flavor in the $\nu \rightarrow \ell$ ***interaction***
ever occurs.

Neutrino Flavor Change (Oscillation)



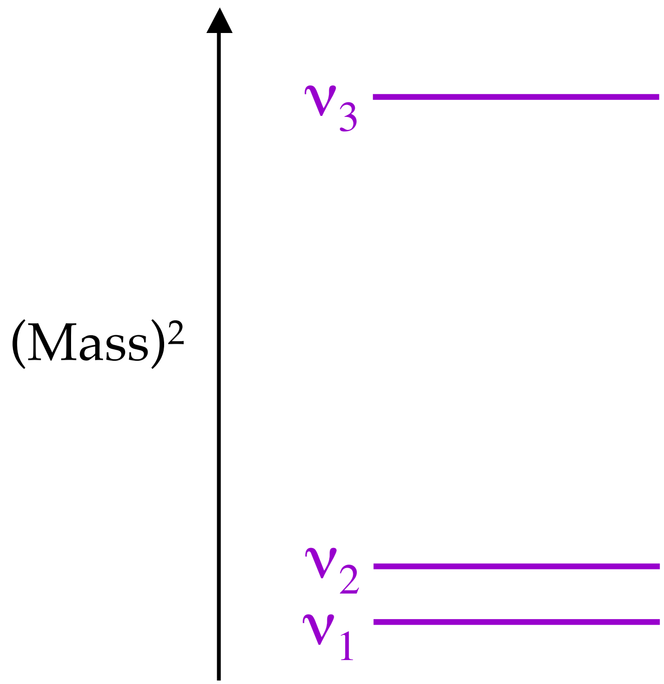
Given time, a ν can change its flavor.

$$\nu_\mu \longrightarrow \nu_e$$

The last 14 years have brought us compelling evidence that flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum of neutrino mass eigenstates ν_i:



Mass $(\nu_i) \equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

($W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau$)

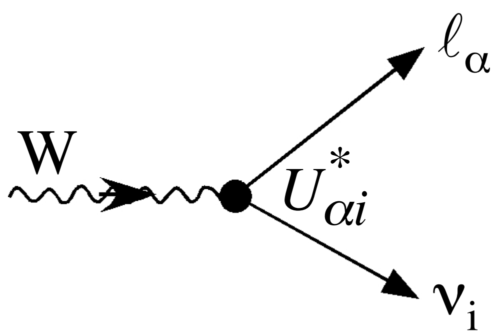
are superpositions of the neutrinos of definite mass:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} \, |\nu_i> \quad .$$

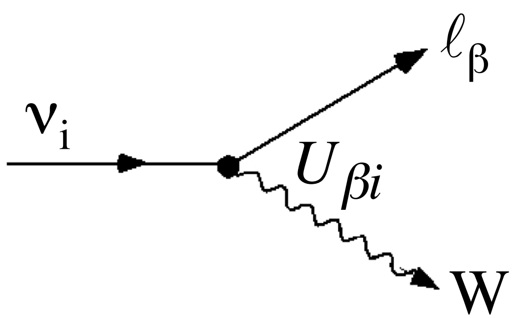
Neutrino of flavor $\alpha = e, \mu$, or τ

Unitary (?) Leptonic Mixing Matrix

Neutrino of definite mass m_i



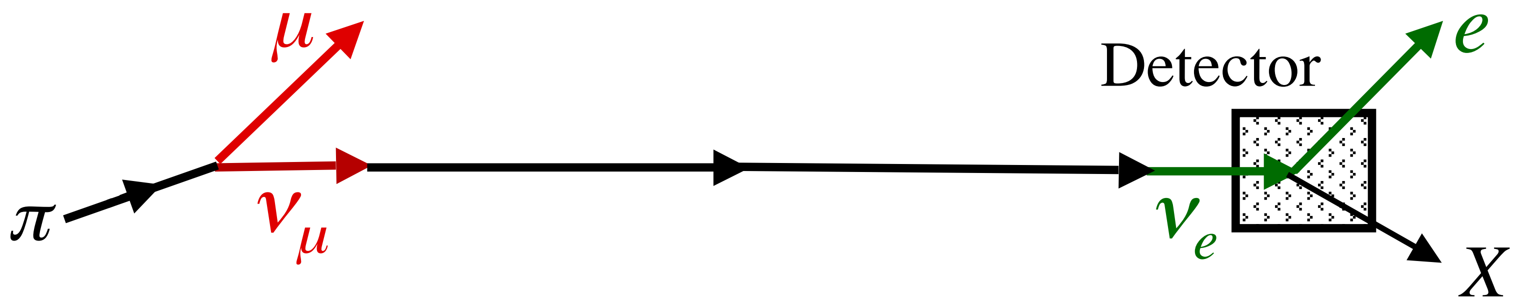
and



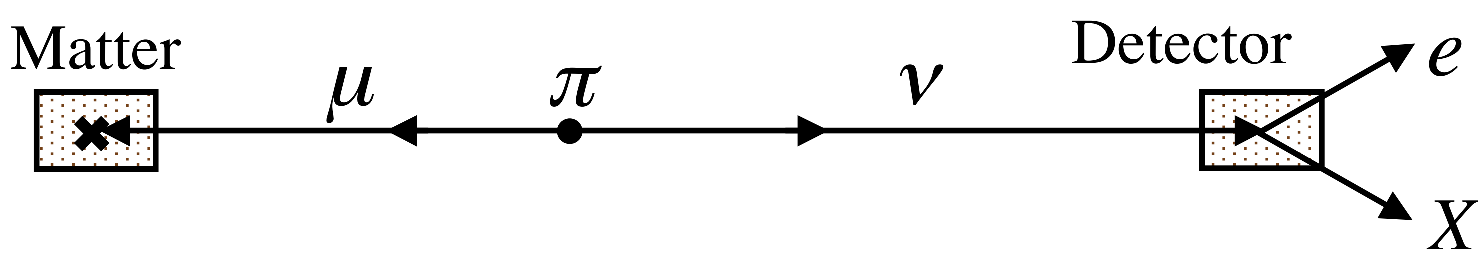
ℓ_α is a charged lepton ($\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$).

The Probability of Neutrino Oscillation, $P(\nu_\mu \rightarrow \nu_e)$

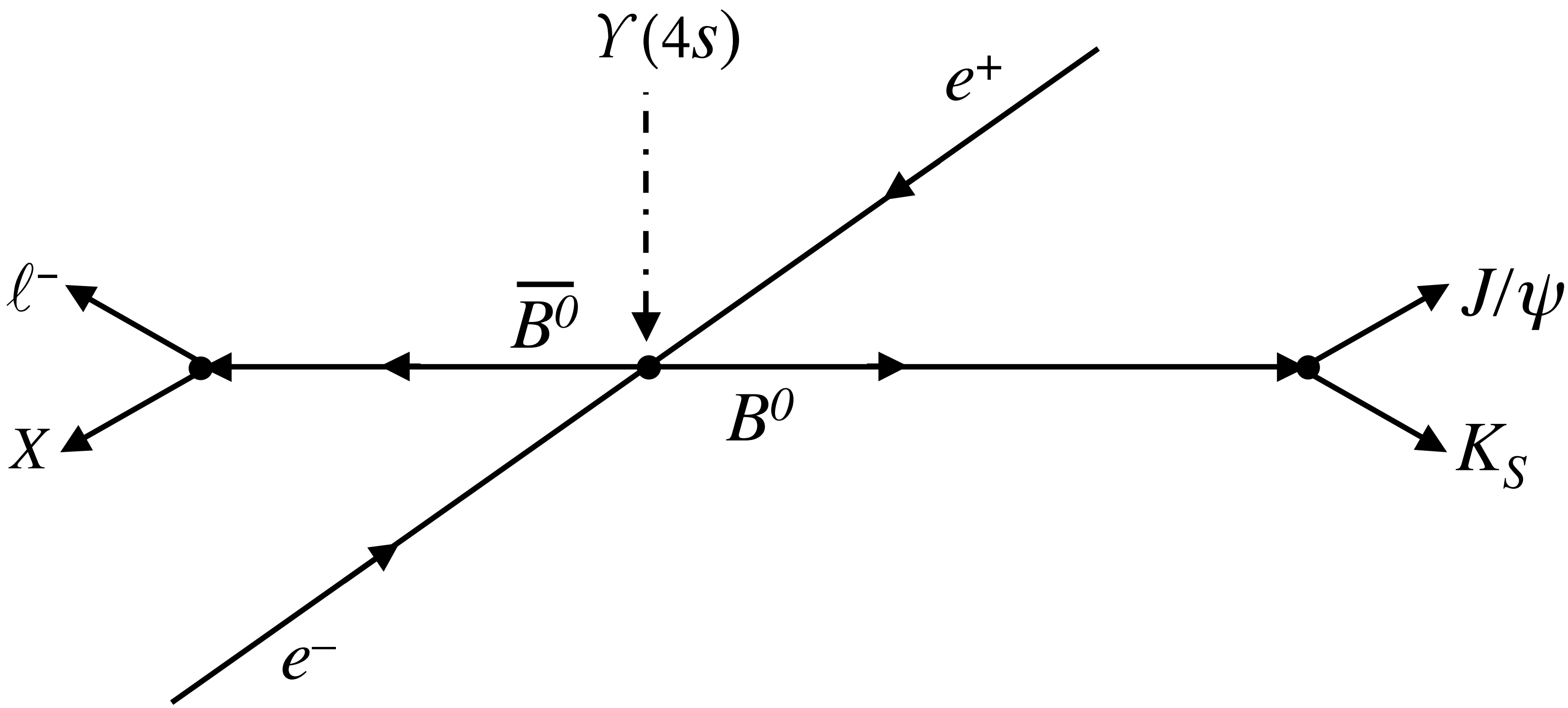
We will view —



from the pion rest frame:



This view calls to mind the B-factory experiments —



A neutral B will oscillate
back and forth between B^0 and $\overline{B^0}$,
like a ν oscillates between flavors.

$$\left|\left\langle B^0 \middle| B^0(\tau)\right\rangle\right|^2 = e^{-\Gamma\tau}\cos^2\left(\frac{\Delta m}{2}\tau\right), \quad \left|\left\langle \overline{B}^0 \middle| B^0(\tau)\right\rangle\right|^2 = e^{-\Gamma\tau}\sin^2\left(\frac{\Delta m}{2}\tau\right)$$

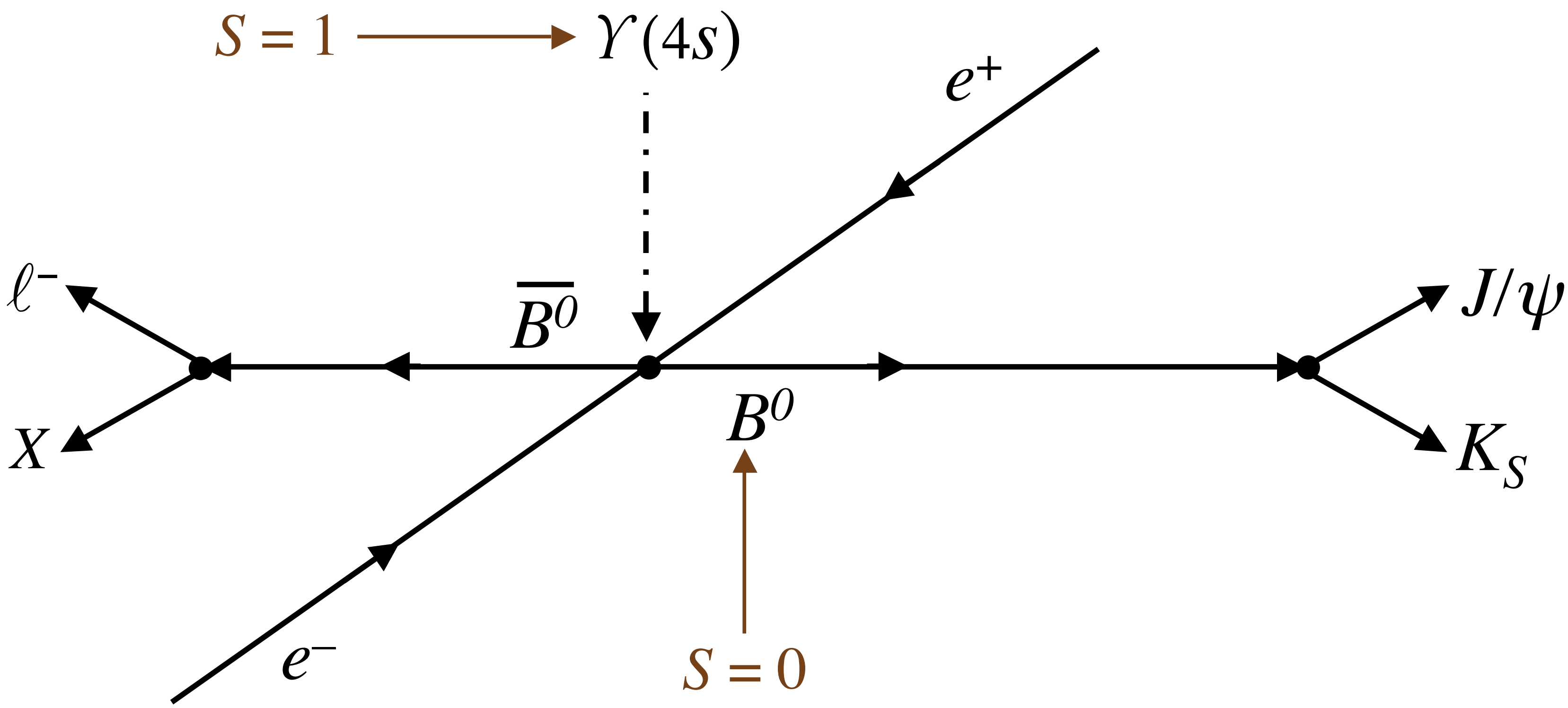
↑ Proper time since birth as a pure B^0

There are two neutral B mass eigenstates, $B_{H(eavy)}$ and $B_{L(ight)}$, which are linear combinations of B^0 and $\overline{B^0}$.

B_H and B_L have approximately the same width Γ.

They have a mass splitting $\Delta m = 3.3 \times 10^{-4}\,\mathrm{eV}$.

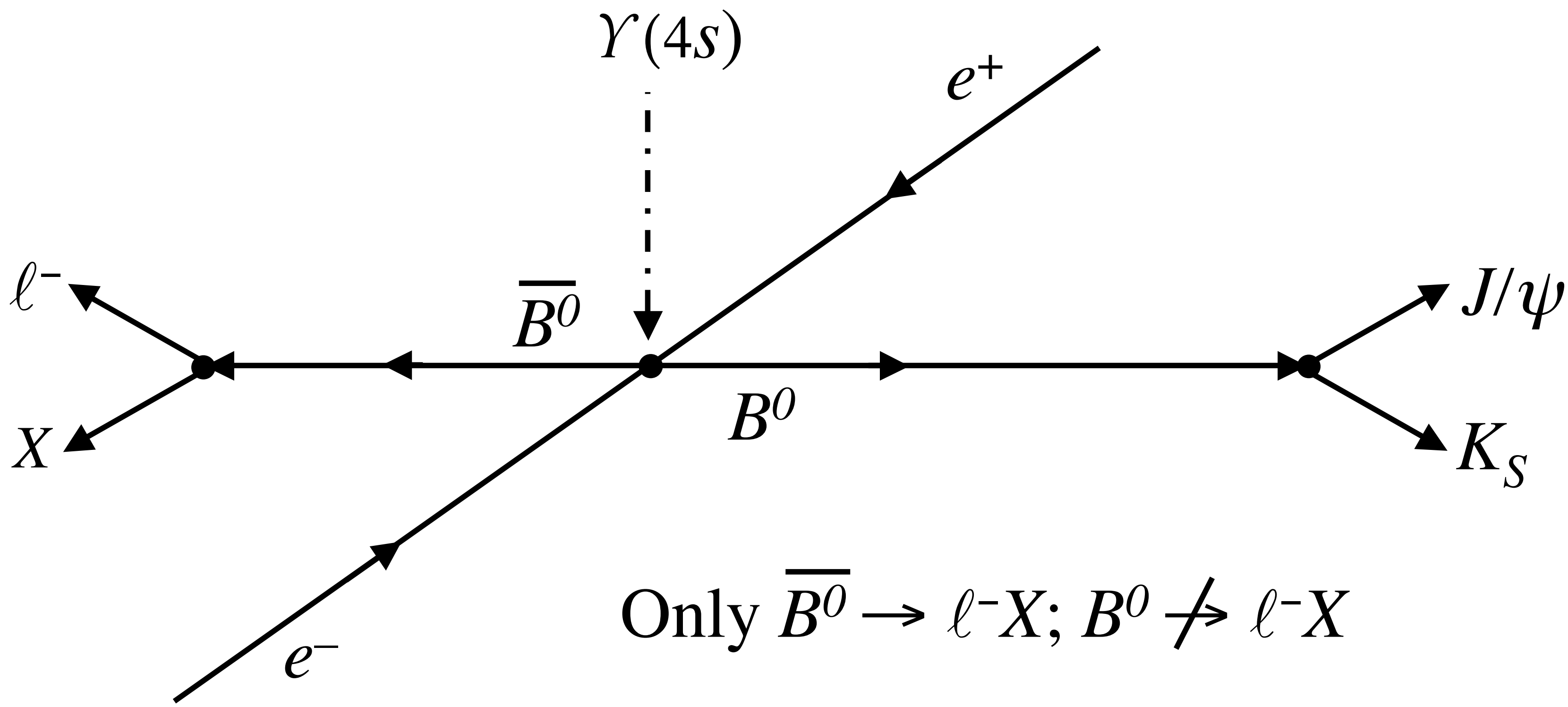
Of course, there is no oscillation between B_H and B_L.



$$\Rightarrow \left|\Upsilon(4s)\right\rangle \rightarrow \left|B^0\overline{B}^0;\ \text{p wave}\right\rangle$$

Despite the oscillation, at any time t in the $\Upsilon(4s)$ rest frame, if one B is a $\overline{B^0}$, the other is a B^0.

The members of the B pair are ***entangled*** —
they are ***Einstein-Podolsky-Rosen correlated***.



The decay $\overline{B^0} \rightarrow \ell^- X$ at time t_ℓ in the $\Upsilon(4s)$ rest frame collapses the BB wave function.

At time t_ℓ, the remaining B must be a pure B^0.

Allowing the remaining B to evolve from time t_ℓ,
one finds —

$$\Gamma\left(\text{One } B \to \ell^- X \text{ after } t_\ell;\ \text{Other } B \to \psi K \text{ after } t_{\psi K}\right)$$

$$\propto e^{-\Gamma\left(t_{\psi K}+t_\ell\right)}\left\{1+\sin\phi_{CP}\sin\left[\Delta m\left(t_{\psi K}-t_\ell\right)\right]\right\}$$

B_H and B_L common width

A CP violating phase

$m(B_H) - m(B_L)$

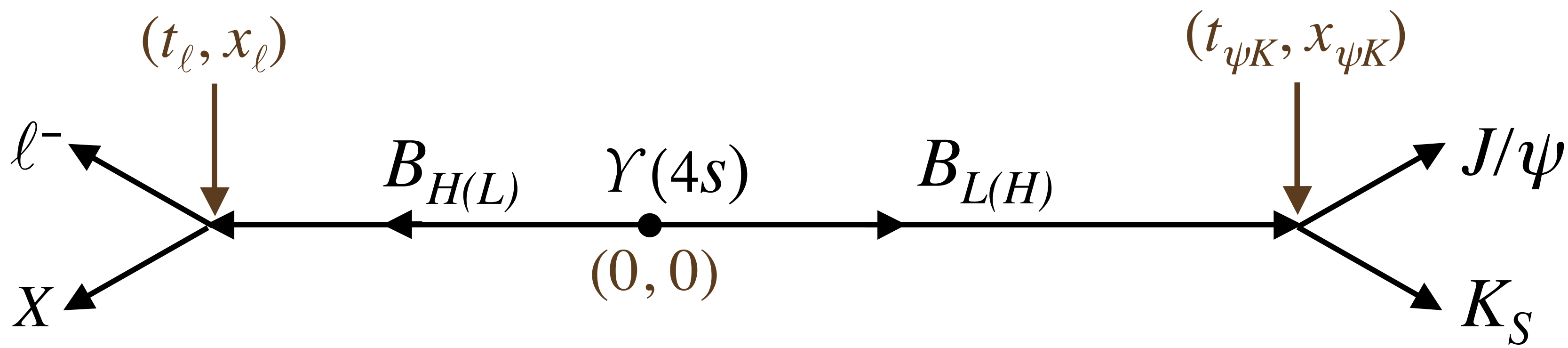
This is a perfectly valid analysis.

But how does the surviving B know how the first B decayed, and when it did so?

A less puzzling approach

(B. K., Stodolsky)

We calculate the ***amplitude*** for the whole process —

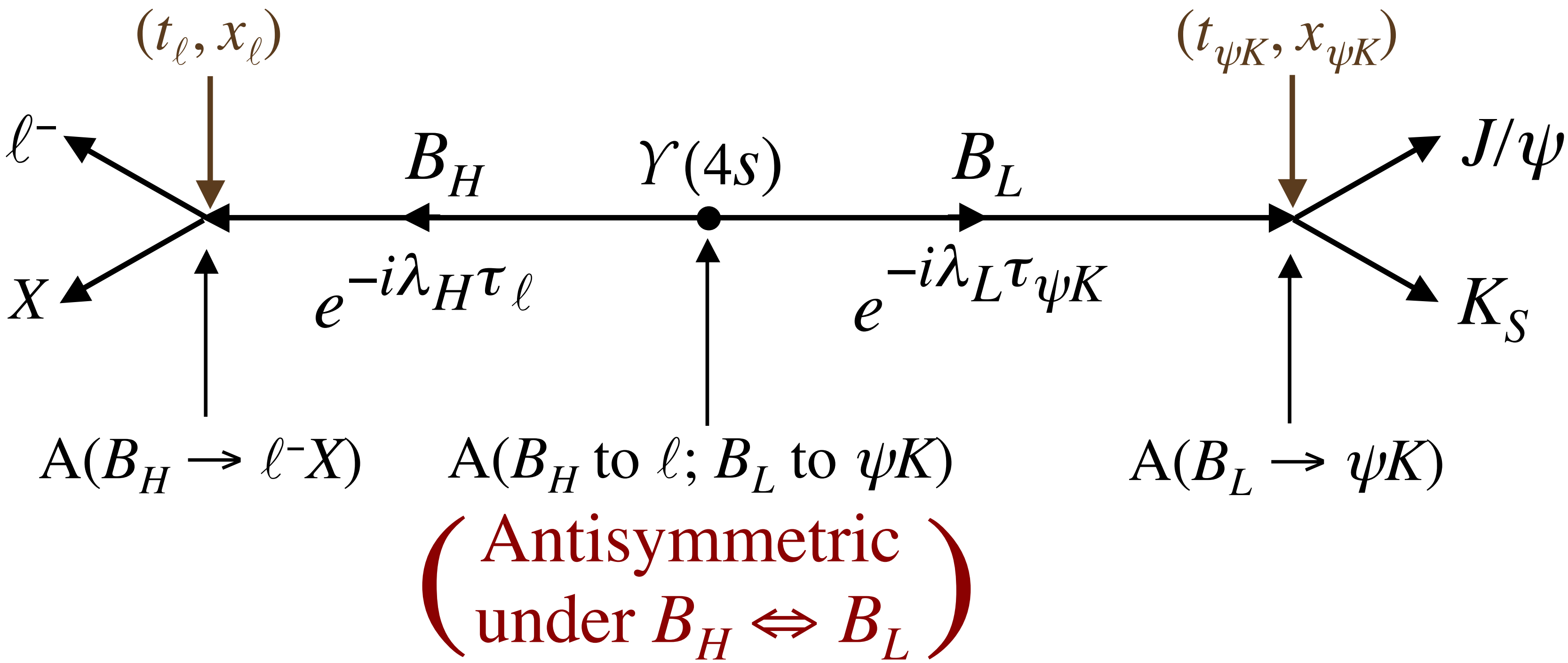


We use —

Amplitude (Particle of mass $\lambda = m - i\frac{\Gamma}{2}$

propagates for a proper time τ) = $\exp(-i\lambda\tau)$

$$\{ \ \exp\left[i\left(px - Et\right)\right] = \exp\left(-im\tau\right) \ \}$$



$$\mathrm{Amp} = e^{-i\lambda_H \tau_\ell}\, e^{-i\lambda_L \tau_{\psi K}}\, \mathrm{A}(B_H \to \ell^- X)\, \mathrm{A}(B_L \to \psi K)$$

$$- B_H \Leftrightarrow B_L$$

(Lorentz invariant)

Using —

$$\lambda_{H,L} = m \pm \frac{\Delta m}{2} - i\frac{\Gamma}{2} \; ,$$

B_H and B_L have ~ the same width

and the Standard-Model B_H and B_L decay amplitudes,

one finds that —

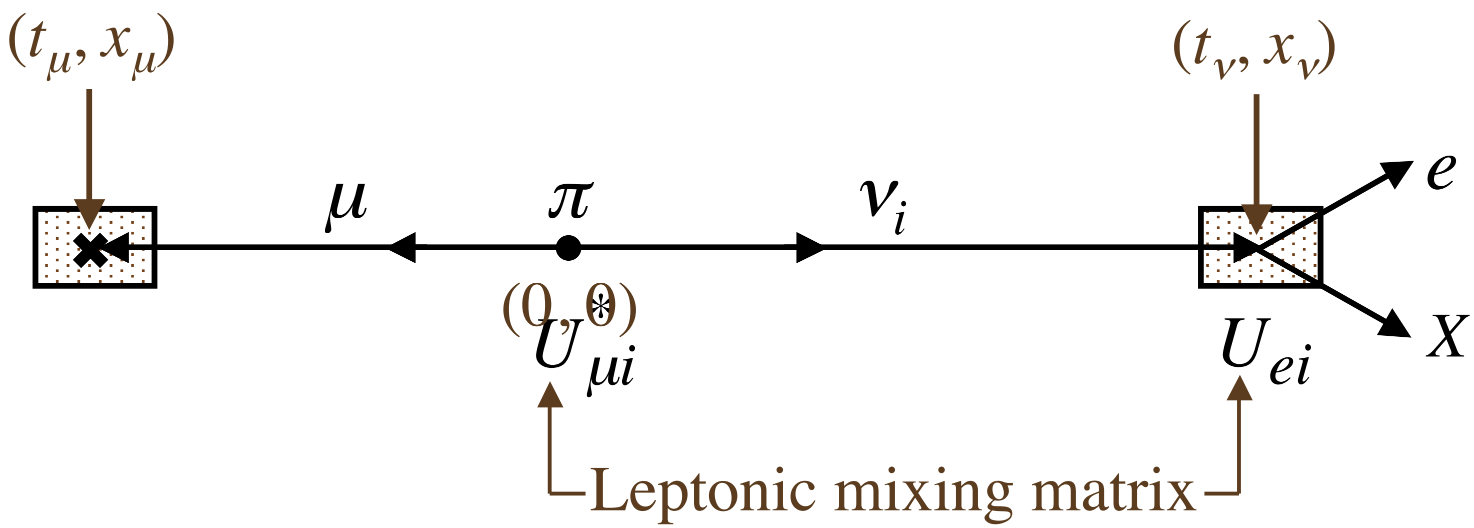
$$\Gamma\left(\text{One } B \to \ell^- X \text{ after } \tau_\ell;\ \text{Other } B \to \psi K \text{ after } \tau_{\psi K}\right) = \left|\text{Amp}\right|^2$$

$$\propto e^{-\Gamma\left(\tau_{\psi K} + \tau_\ell\right)}\left\{1 + \sin\phi_{CP} \sin\left[\Delta m\left(\tau_{\psi K} - \tau_\ell\right)\right]\right\}$$

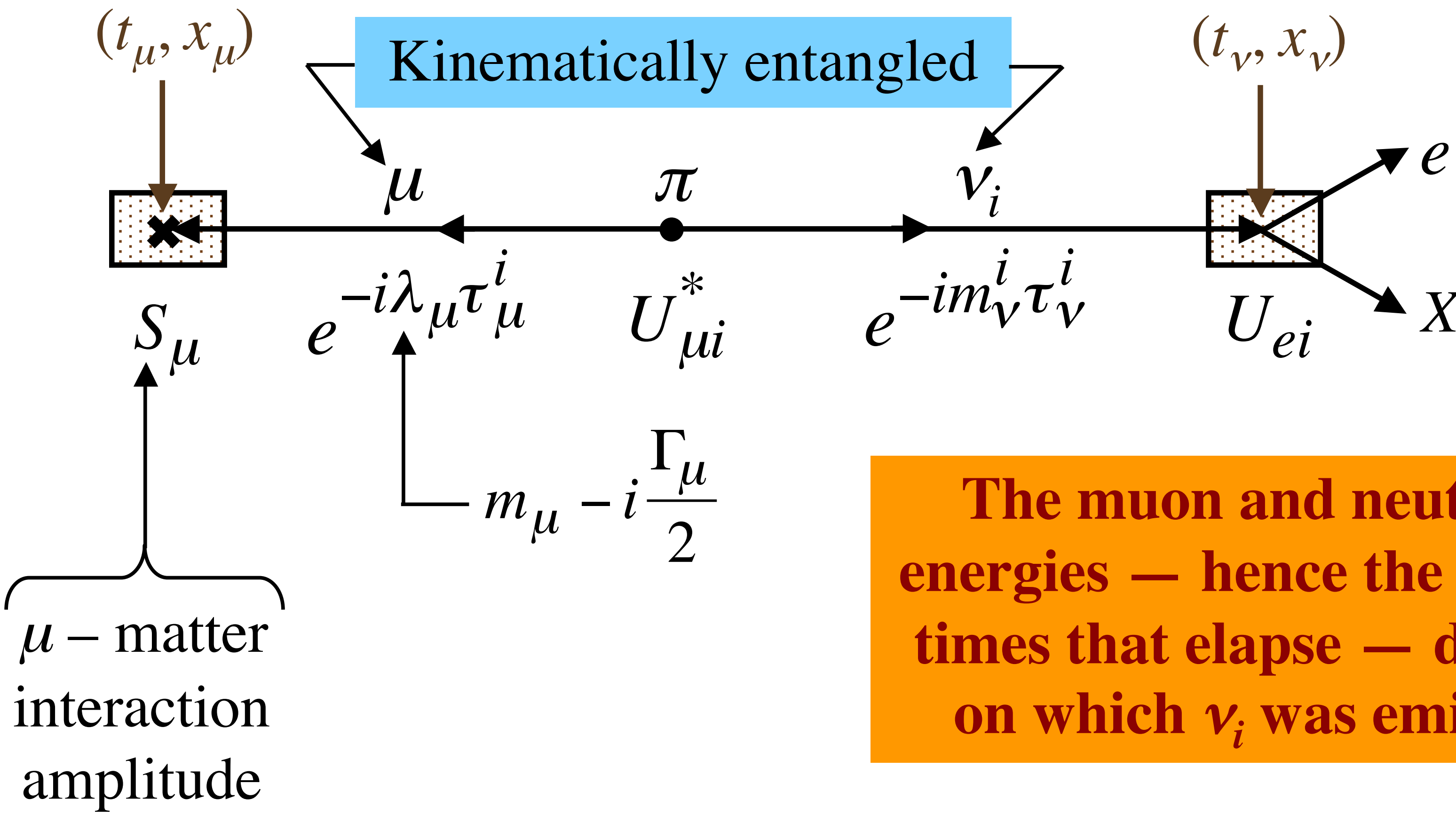
This is the usual result, except that times in the $\Upsilon(4s)$ rest frame are replaced by proper times in the *B* rest frames.

No need to think in terms of a collapsing wave function.

Neutrino Oscillation Via the Same Approach



The goal: To eliminate the non-intuitive assumpion that all the interfering neutrino mass eigenstates in a beam have the same energy, or else the same momentum.



The muon and neutrino energies — hence the proper times that elapse — depend on which ν_i was emitted.

$$\text{Amp} = \sum_{i=1,2,3} S_\mu \, e^{-i\left(m_\mu - i\frac{\Gamma_\mu}{2}\right)\tau_\mu^i} U_{\mu i}^* \, e^{-im_\nu^i \tau_\nu^i} U_{ei}$$

(Lorentz invariant)

$$\text{Amp} = \sum_{i=1,2,3} S_\mu \, e^{-i\left(m_\mu - i\frac{\Gamma_\mu}{2}\right)\tau_\mu^i} U_{\mu i}^* \, e^{-i m_\nu^i \tau_\nu^i} U_{ei}$$

How do the kinematical phase factors depend on *i*?

To lowest (first) order in the $\Delta m_{ij}^2 \equiv \left(m_\nu^i\right)^2 - \left(m_\nu^j\right)^2$,
the *muon* phase factor

$$e^{-i\left(m_\mu - i\frac{\Gamma_\mu}{2}\right)\tau_\mu^i}$$

does not depend on i, so it will not influence the |Amp|^2, except in overall normalization, and can be dropped.

(First noticed by Akhmedov and Smirnov)

In the phase factor for the *neutrino*, $e^{-im_\nu^i \tau_\nu^i}$,

$$m_\nu^i \tau_\nu^i = E_\nu^i t_\nu - p_\nu^i x_\nu \, .$$

Energy and momentum of neutrino ν_i in π-rest-frame

Since in practice neutrinos are ultra relativistic, we choose $t_\nu = x_\nu \equiv L^0$ to avoid (Event rate) = 0.

Using —

$$E_\nu^i = \frac{m_\pi^2 + \left(m_\nu^i\right)^2 - m_\mu^2}{2m_\pi} \quad \text{and} \quad \left(p_\nu^i\right)^2 = \left(E_\nu^i\right)^2 - \left(m_\nu^i\right)^2 ,$$

we find that to lowest (first) order in $\Delta m_{ij}^2 \equiv \left(m_\nu^i\right)^2 - \left(m_\nu^j\right)^2$,

$$m_\nu^i \tau_\nu^i - m_\nu^j \tau_\nu^j = \Delta m_{ij}^2 \frac{L^0}{2E^0}$$

Distance ν travels in the π rest frame (L^0)

Energy ν would have in the π rest frame if it were massless (E^0)

Thus, we may take the neutrino phase factor, $e^{-im_\nu^i \tau_\nu^i}$, to be —

$$e^{-i\left(m_\nu^i\right)^2 \frac{L^0}{2E^0}}$$

.

Using this result, and dropping the *i*-independent muon interaction and propagation amplitudes, we have —

$$\text{Amp} = \sum_{i=1,2,3} U_{\mu i}^{*}\, e^{-i\left(m_{\nu}^{i}\right)^{2}\frac{L^{0}}{2E^{0}}}\, U_{ei}$$

[Recall that L^0 and E^0 are the neutrino travel distance and energy (neglecting its mass) *in the π rest frame*.]

From $\Delta p \Delta x \geq \hbar$, we cannot observe ν oscillation vs. travel distance in the lab unless there is a spread in lab-frame π momenta, so that the π is somewhat localized.

Because neutrinos are ultra-relativistic, when the parent π is moving in the lab, the ν travel distance and energy in the lab frame, L and E, are related to their π-rest-frame counterparts, L^0 and E^0, by —

$$\frac{L}{E} = \frac{L^0}{E^0}$$

Thus, in terms of lab-frame variables,

$$\text{Amp} = \sum_{i=1,2,3} U_{\mu i}^* \, e^{-i\left(m_\nu^i\right)^2 \frac{L}{2E}} \, U_{ei}$$

This leads to —

$$P\left(\nu_\mu \to \nu_e\right) = \left|\mathrm{Amp}\right|^2 = -4 \sum_{i>j} \mathrm{Re}\left(U_{\mu i}^* U_{ei} U_{\mu j} U_{ej}^*\right) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right)$$

$$+ 2 \sum_{i>j} \mathrm{Im}\left(U_{\mu i}^* U_{ei} U_{\mu j} U_{ej}^*\right) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)$$

This is the usual result.

We derived it now in the same way as we treat B-factory experiments.

We allowed for the $\nu - \mu$ kinematical entanglement, which proved to be irrelevant.

We didn't need to make any assumption about how the energies of the different neutrino mass eigenstates are related.

Previous consideration of entanglement in processes with oscillation

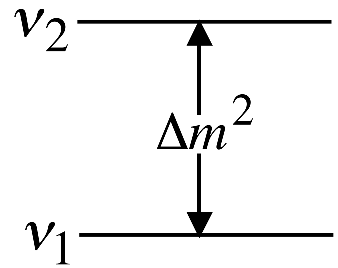
B. K., Stodolsky
Goldman
Nauenberg
Dolgov, Morozov, Okun, Schepkin
Burkhardt, Lowe, Stephenson, Goldman
Lowe, Bassalleck, Burkhardt, Rusek, Stephenson
Cohen, Glashow, Ligeti
B. K., Kopp, Robertson, Vogel
Akhmedov, Smirnov

For arbitrary initial flavor α and final one β —

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*\right) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right)$$

$$+ 2\sum_{i>j} \mathrm{Im}\left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*\right) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)$$

When Only Two Mass Eigenstates, and Two Flavors, Matter



$$U = \begin{bmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e^{i\xi} & 0 \\ 0 & 1 \end{bmatrix}$$

Majorana ~~CP~~ phase ($e^{i\xi}$)

Mixing angle (θ)

For $\beta \neq \alpha$, $\quad P\left(\nu_\alpha \leftrightarrow \nu_\beta\right) = \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4E}\right)$

For no flavor change, $\quad P\left(\nu_\alpha \rightarrow \nu_\alpha\right) = 1 - \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4E}\right)$

Comparison Between Neutrino and B-Meson Oscillation

Laboratory neutrinos are ultra-relativistic, with $E \approx p$.

Thus —

$$P\left(\nu_\alpha \to \nu_\beta\right) = \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4p}\right)$$

We had —

$$P\left(B^0 \to \overline{B}^0\right) = \left|\left\langle \overline{B}^0 \middle| B^0(\tau)\right\rangle\right|^2 = e^{-\Gamma\tau} \sin^2\left(\frac{\Delta m}{2}\tau\right)$$

B_H and B_L are 50 – 50 mixtures of B^0 and $\overline{B^0}$.

That is, $B^0 - \overline{B^0}$ mixing is maximal; $\sin^2 2\theta = 1$.

Furthermore, if a B travels a distance L in the lab with momentum p, the proper time τ that evolves in its own rest frame during the journey is given by —

$$\tau = \frac{L}{\beta}\frac{1}{\gamma} = \frac{m}{m}\frac{L}{\beta}\frac{1}{\gamma} = \frac{m_H + m_L}{2}\frac{L}{p} .$$

Thus —

$$\frac{\Delta m}{2}\tau = \frac{m_H - m_L}{2}\frac{m_H + m_L}{2}\frac{L}{p} = \Delta m^2 \frac{L}{4p}$$

Hence, in the limit that we neglect the decay of the B,

$$P\left(B^0 \to \overline{B}^0\right) = \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4p}\right)$$

By comparison, when only two neutrinos matter,

$$P\left(\nu_\alpha \to \nu_\beta\right) = \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4p}\right)$$

Do you notice any similarities?