Physics of neutrino oscillations & flavor conversion

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Invisibles network INT Training lectures
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Content:

1. Oscillations without Paradoxes
2. Matter effects: oscillations, flavor conversion
3. Neutrino propagation in the Earth

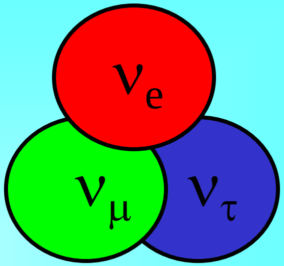
Modern version

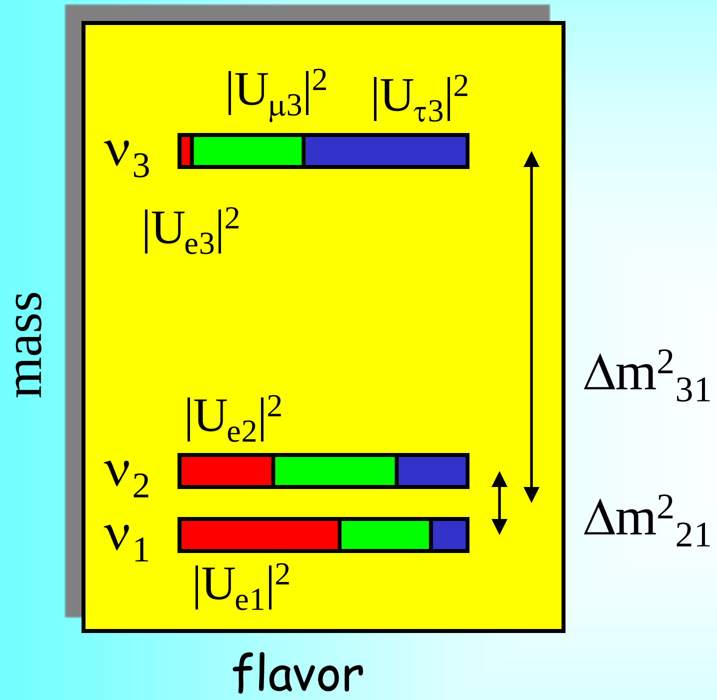
Oscillation and flavor conversion are consequence of
- the lepton mixing and
- production of mixed (flavor) states

Masses & mixing

also to set notations

Mixing





Normal mass hierarchy

$\Delta m^2_{31} = m^2_3 - m^2_1$

$\Delta m^2_{21} = m^2_2 - m^2_1$

Mixing parameters

$\tan^2\theta_{12} = |U_{e2}|^2 \; / \; |U_{e1}|^2$

$\sin^2\theta_{13} = |U_{e3}|^2$

$\tan^2\theta_{23} = |U_{\mu 3}|^2 \; / \; |U_{\tau 3}|^2$

Mass states can be enumerated by amount of electron flavor

Mixing matrix:

$\nu_f = U_{PMNS}\, \nu_{mass}$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$U_{PMNS} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$

Who mixes neutrinos?

Mixing in CC → mixing in produced states

Non-trivial interplay of

Charged current weak interactions

Kinematics of specific reactions

Difference of the charged lepton masses

Breaking of coherence

ν_e → $\beta-$ decays, energy conservation

ν_μ → $\pi-$ decays, chirality suppression

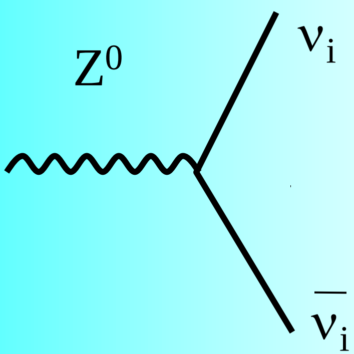
ν_τ → Beam dump, D - decay

What about neutral currents?

Can NC interactions prepare mixed state?

Z is flavor blind

What is the neutrino state produced in the Z-decay in the presence of mixing?

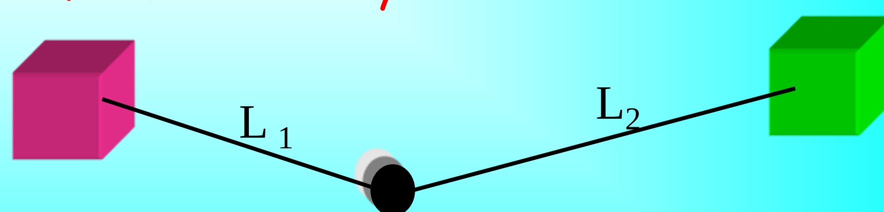


$$|f\rangle = \frac{1}{\sqrt{3}}\left[\, |\bar{\nu}_1 \nu_1\rangle + |\bar{\nu}_2 \nu_2\rangle + |\bar{\nu}_3 \nu_3\rangle \,\right]$$

$$|\langle f|H|Z\rangle|^2 = 3\,|\langle \bar{\nu}_1 \nu_1|H|Z\rangle|^2$$

Do neutrinos from Z^0- decay oscillate?

Two detectors experiment: detection of both neutrinos



If the flavor of one of the neutrino is fixed, another neutrino oscillates

$$P = \sin^2 2\theta \; \sin^2 [\, \pi (L_1 + L_2)/ l_\nu]$$

Oscillations

Without paradoxes

compromize

Simple and straightforward and still correct derivation

Challenging theory of neutrino oscillations

New aspects

New experimental setups
- LBL long tunnels
- Long leaved parents

Based on

[1] Neutrino production coherence and oscillation experiments.
E. Akhmedov, D. Hernandez, A. Smirnov, JHEP 1204 (2012) 052,
arXiv:1201.4128 [hep-ph]

[2] Neutrino oscillations: Entanglement, energy-momentum conservation and QFT.
E.Kh. Akhmedov, A.Yu. Smirnov, Found. Phys. 41 (2011) 1279-1306
arXiv:1008.2077 [hep-ph]

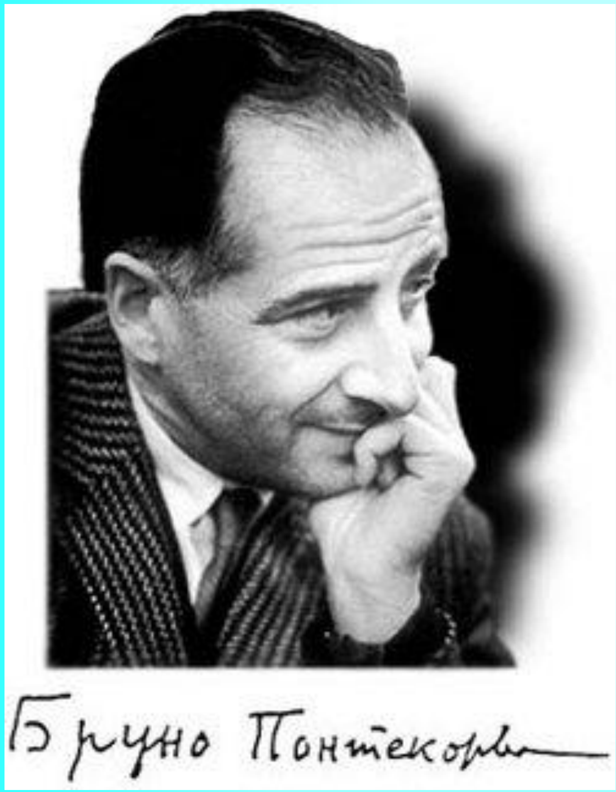
[3] Paradoxes of neutrino oscillations.
E. Kh. Akhmedov, A. Yu. Smirnov Phys. Atom. Nucl. 72 (2009) 1363-1381
arXiv:0905.1903 [hep-ph]

[4] Active to sterile neutrino oscillations: Coherence and MINOS results.
D. Hernandez, A.Yu. Smirnov, Phys.Lett. B706 (2012) 360-366
arXiv:1105.5946 [hep-ph]

[5] Neutrino oscillations: Quantum mechanics vs. quantum field theory.
E. Kh. Akhmedov, J. Kopp, JHEP 1004 (2010) 008
arXiv:1001.4815 [hep-ph]

55 years ago...

Pisa, 1913



B. Pontecorvo

``Mesonium and antimesonium"

Zh. Eksp.Teor. Fiz. 33, 549 (1957)
[Sov. Phys. JETP 6, 429 (1957)] translation

mentioned a possibility of neutrino mixing and oscillations

Oscillations imply non-zero masses (mass squared differences) and mixing

Proposal of neutrino oscillations was motivated
by rumor that Davis sees effect in Cl-Ar detector from atomic reactor

Computing oscillation effects

Lagrangian

$$\frac{g}{2\sqrt{2}}\,\bar{l}\,\gamma^{\mu}(1-\gamma_5)\nu_l\,W^{+}_{\mu}$$

$$-\ \tfrac{1}{2}\,m_L\ \nu_L^{T} C \nu_L$$

$$-\ \bar{l}_L\, m_l\, l_R\ \ +\ h.c.$$

Starting from
the first principles

QFT

QM

Amplitudes,
probabilities
of processes

Observables,
Number of
events, etc..

What is the problem?

Set-up

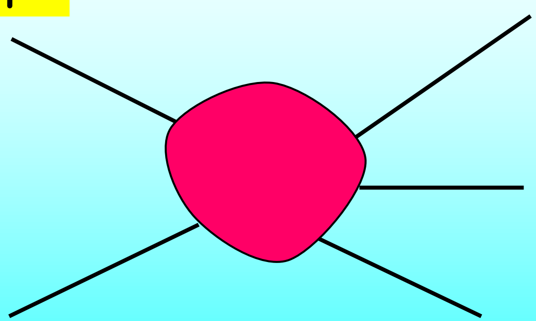
Formalism should be adjusted to specific physics situation

Initial conditions

Recall, the usual set-up

asymptotic states described by plane waves

- enormous simplification



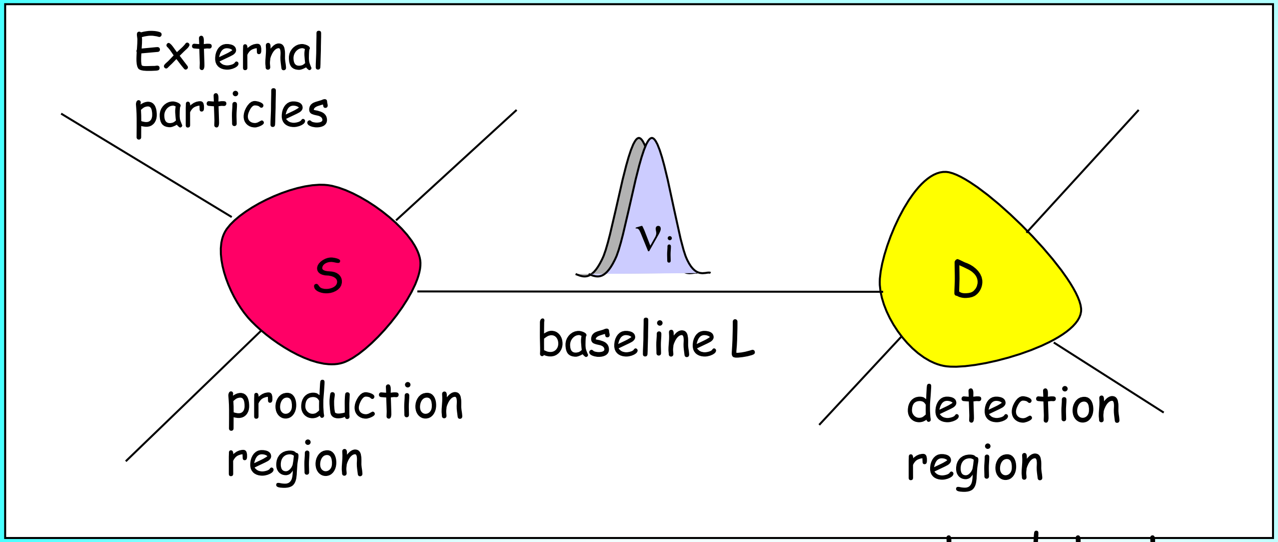
single interaction region

Approximations

Approximations, if one does not want to consider whole history of the Universe to compute signal in Daya Bay

Truncating the process

Oscillation set-up



not a detector

E. Akhmedov, A.S.

QFT but

formalism
should be
adjusted to
these condition

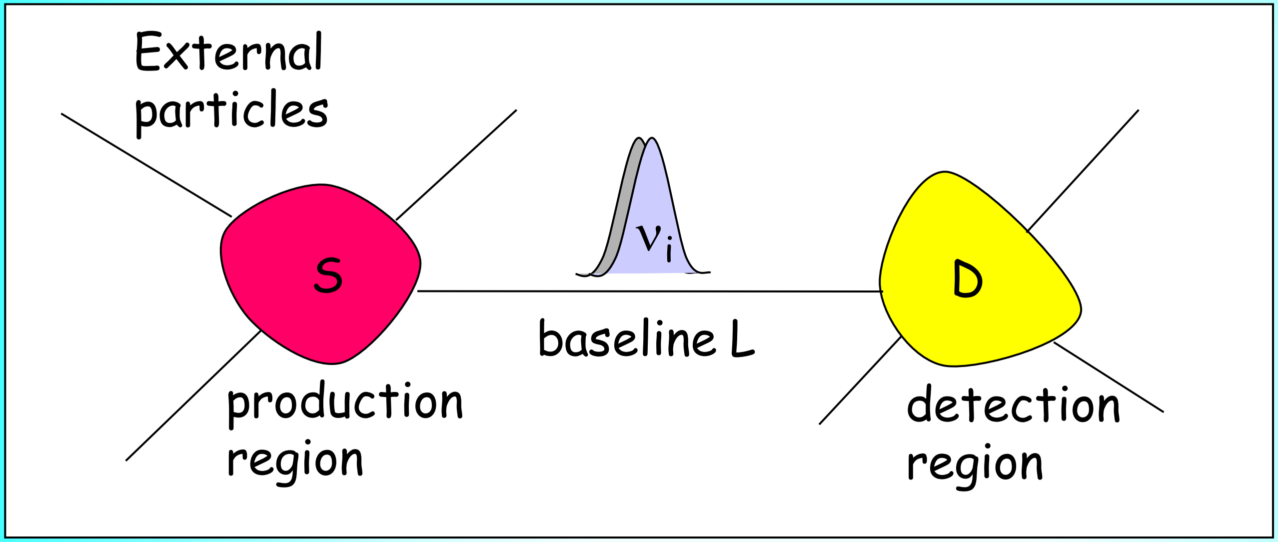
Finite space and time phenomenon

Two interaction regions
in contrast to usual scattering problem

Neutrinos: propagator

Where to truncate?

How external particles
Should be described?



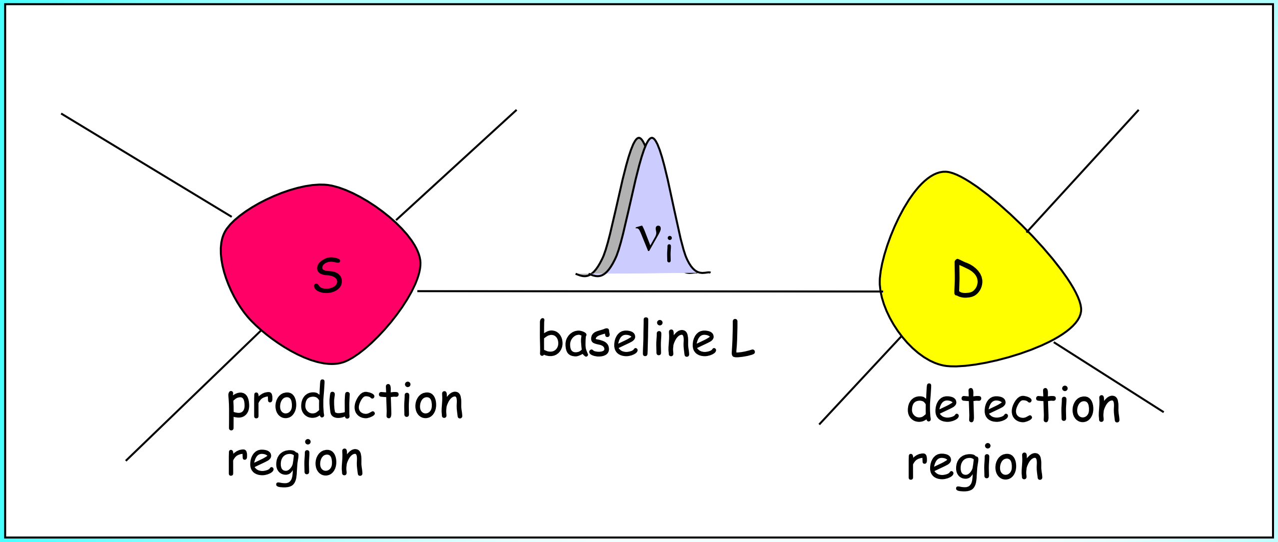
detection/production areas
are determined by localization
of particles involved in neutrino
production and detection
not source/detector volume
(still to integrate over)

wave packets for
external particles

Finite space-time
integration limits

Describe by plane waves
but introduce finite integration

How to treat neutrinos?

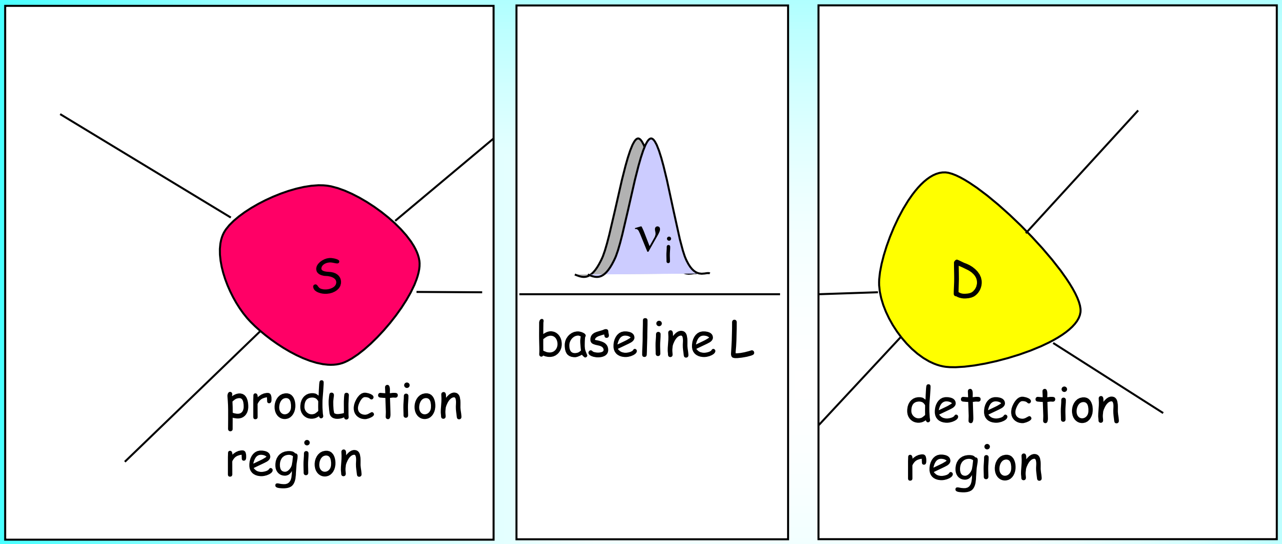


Unique process,
neutrinos with definite masses are described by propagators,
Oscillation pattern – result of interference of
amplitudes due to exchange of different mass eigenstates

Very quickly converge to mass shell

Real particles – described by wave packets

Factorization



If oscillation effect in Production/detection regions can be neglected

$r_D\ ,\ r_S \ll l_\nu$

factorization

Production propagation and Detection can be considered as three independent processes

Wave packets & oscillations

B. Kayser, Phys. Rev D 48 (1981) 110

Wave packet formalism.
Consistent description of oscillations requires consideration of wave packets of neutrino mass states.

31 years later, GGI lectures:

The highest level of sophistication:
to use proper time for neutrino mass and get correct result!

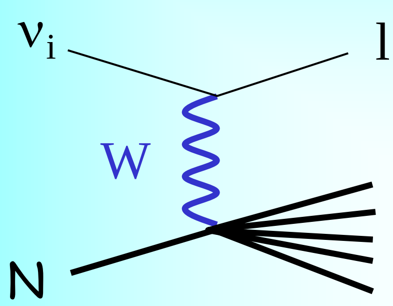
Key point: phases of mass eigenstates should be compared in the same space-time point

If not - factor of 2 in the oscillations phase

In terms of mass eigenstates

Without flavor states

Scattering

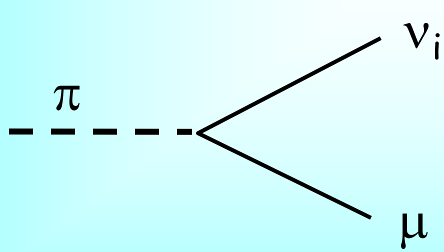


$$\frac{g}{2\sqrt{2}} U_{li} \, \bar{l} \, \gamma^{\mu} (1 - \gamma_5) \, \nu_i \, W^{+}_{\mu} + \text{h.c.}$$

interaction constant

Eigenstates of the Hamiltonian in vacuum

$\pi \rightarrow \mu \, \nu_i$



Lagrangian of interactions

wave functions of accompanying particles

compute the wave function of neutrino mass eigenstates

Wave packet

Wave packets and oscillations

Suppose ν_α be produced in the source centered at x = 0, t = 0

After formation of the wave packet (outside the production region)

In general

$$|\nu_\alpha(x,t)\rangle = \Sigma_k\, U_{\alpha k}^*\, \Psi_k(x,t)|\nu_k\rangle$$

$$\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$$

$$E_k(p) = \sqrt{p^2 + m_k^2}$$ - dispersion relation

$f_k(p - p_k)$ - the momentum distribution function peaked at

p_k - the mean momentum

Expanding around mean momentun

describes spread of the wave packets

$$E_k(p) = E_k(p_k) + (dE_k/dp)\Big|_{p_k}(p - p_k) + (dE_k^2/dp^2)\Big|_{p_k}(p - p_k)^2 + \ldots$$

$v_k = (dE_k/dp)\Big|_{p_k} = (p/E_k)\Big|_{p_k}$ - *group velocity of* ν_k

Shape factor and phase factor

$$E_k(p) = E_k(p_k) + v_k(p - p_k)$$

(neglecting spread of the wave packets)

Inserting into $\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$

$$\Psi_k \sim e^{ip_k x - iE_k(p_k)t}\, g_k(x - v_k t)$$

Phase factor

$$e^{i\phi_k}$$

$$\phi_k = p_k x - E_k t$$

Depends on mean characteristics p_k and corresponding energy:

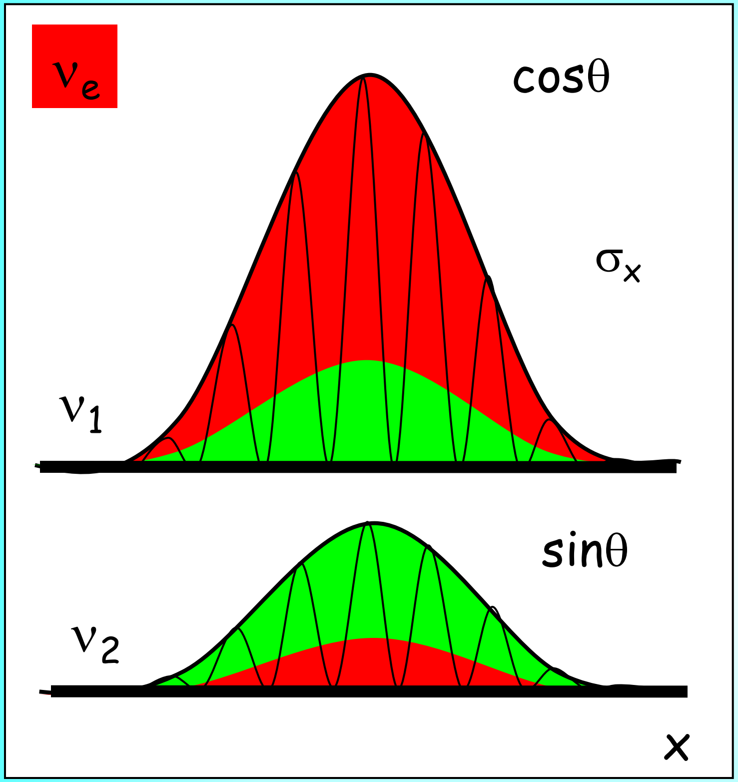
$$E_k(p_k) = \sqrt{p_k^2 + m_k^2}$$

Shape factor

$$g_k(x - v_k t) = \int dp\, f_k(p)\, e^{ip(x - v_k t)}$$

Depends on x and t only in combination $(x - v_k t)$ and therefore Describes propagation of the wave packet with group velocity v_k without change of the shape

Wave packet picture



$$\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$$
$$\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$$

opposite phase

$$\nu_1 = \cos\theta\ \nu_e - \sin\theta\ \nu_\mu$$
$$\nu_2 = \cos\theta\ \nu_\mu + \sin\theta\ \nu_\varepsilon$$

Interference of the same flavor parts

$\phi = 0$

Main, effective frequency

$$|\nu(x,t)\rangle = \cos\theta\ g_1(x - v_1 t)e^{i\phi_1}|\nu_1\rangle + \sin\theta\ g_2(x - v_2 t)e^{i\phi_2}|\nu_2\rangle$$

$\phi = \phi_2 - \phi_1$

Oscillation phase

Mixing & mixed states

One needs to compute the state which is produced
i.e. compute

the shape factors

$g_k(x - v_k t)$

mean momenta p_k

- Fundamental interactions
- Kinematics
- characteristics of parent and accompanying particles

Process dependent

If heavy neutrinos are present but can not be produced for kinematical reasons, flavor states in Lagrangian differe from the produced states, etc..

Propagation of wave packets

What happens?

Phase difference change

Due to different masses (dispersion relations) → phase velocities

Oscillations

Separation of wave packets

Due to different group velocities

Loss of coherence

Spread of individual wave packets

Due to presence of waves with different momenta and energy in the packet

...homework

Oscillation phase

$\phi = \phi_2 - \phi_1$

$\phi_i = - E_i t + p_i x$

$p_i = \sqrt{E_i^2 - m_i^2}$

Dispersion relation enters here

$\phi = \Delta E t - \Delta p x$

$\Delta p = (dp/dE)\, \Delta E + (dp/dm^2)\, \Delta m^2 \quad = \quad 1/v_g\, \Delta E + (1/2p)\, \Delta m^2$

group velocity

$$\phi \;=\; \Delta E/v_g\, (v_g t - x) + \frac{\Delta m^2}{2E}\, x$$

standard oscillation phase

$\Delta E^0 \sim \Delta m^2/2E$

$< \sigma_x$

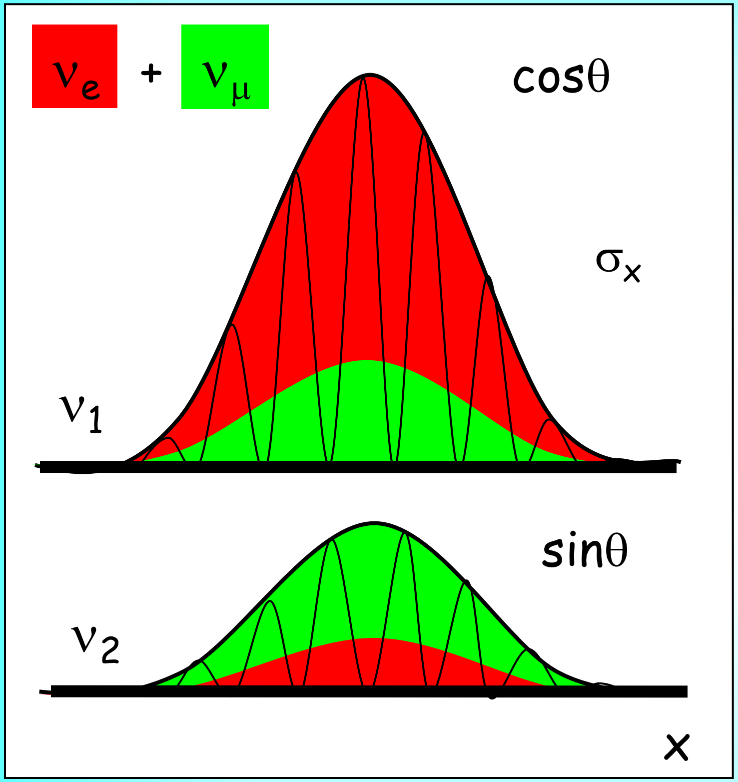
$\Downarrow$

$\sigma_x \Delta m^2/2E$

Effect of averaging over size of the wave packet

usually- small

Oscillations



$$\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$$
$$\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$$

opposite phase

Interference pattern depends on relative (oscillation) phase

$\phi = \pi$

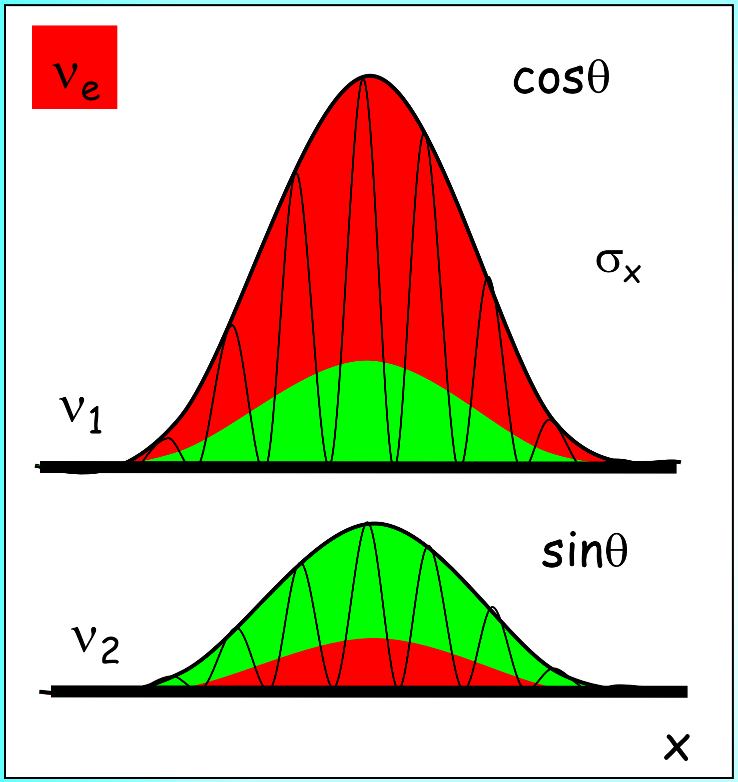
Main, effective frequency

$$|\nu(x,t)\rangle = \cos\theta\, g_1(x - v_1 t)|\nu_1\rangle + \sin\theta\, g_2(x - v_2 t)e^{i\phi}\ |\nu_2\rangle$$

$\phi = \phi_2 - \phi_1$

Oscillation phase

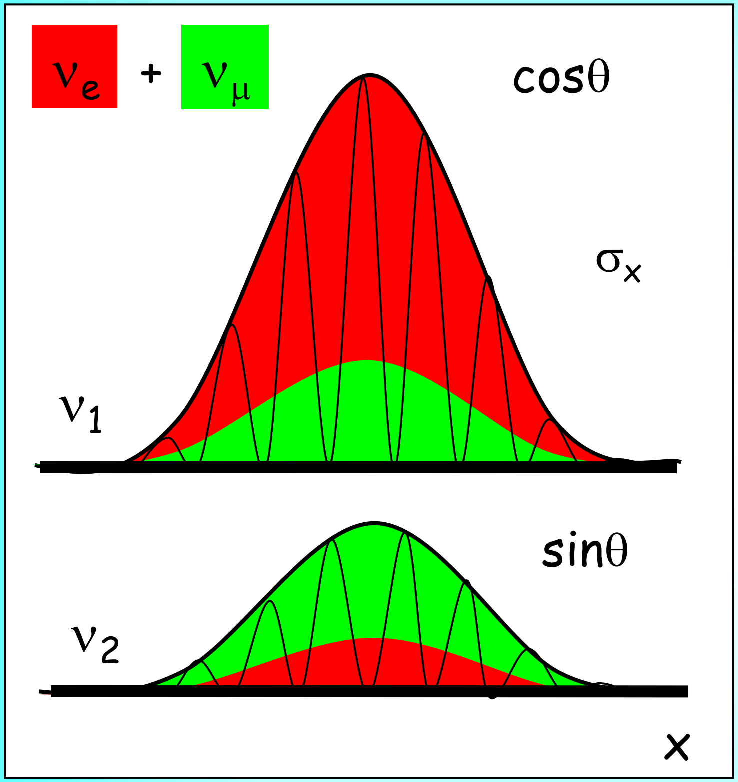
Oscillations



$\phi = 0$

- Destructive interference of the muon parts
- Constructive interference of electron parts

Oscillations



$\phi = \pi$

- Destructive interference of the electron parts
- Constructive interference of muon parts

Detection:

As important as production
Should be considered symmetrically with production

Detection effect can be included in
the generalized shape factors

$$g_k(x - v_k t) \rightarrow G_k(L - v_k t)$$

$x \rightarrow L$ - distance between central points of the production and detection regions

HOMEWORK…

Oscillation probability

Amplitude of (survival) probability

$$A(\nu_e) = \langle \nu_e | \nu(x,t) \rangle = \cos^2\theta \, g_1(x - v_1 t) + \sin^2\theta \, g_2(x - v_2 t) \, e^{i\phi}$$

Probability in the moment of time t

interference

$$P(\nu_e) = \int_{-\infty}^{+\infty} dx \, |\langle \nu_e | \nu(x,t) \rangle|^2 =$$

$$= \cos^4\theta + \sin^4\theta + 2\sin^2\theta \cos^2\theta \cos\phi \int dx \, g_1(x - v_1 t) \, g_2(x - v_2 t)$$

!

If $\int dx |g_k|^2 = 1$

!

If $g_1 = g_2$

$$P(\nu_e) = 1 - 2\sin^2\theta \cos^2\theta \, (1 - \cos\phi) = 1 - \sin^2 2\theta \, \sin^2 \tfrac{1}{2}\phi$$

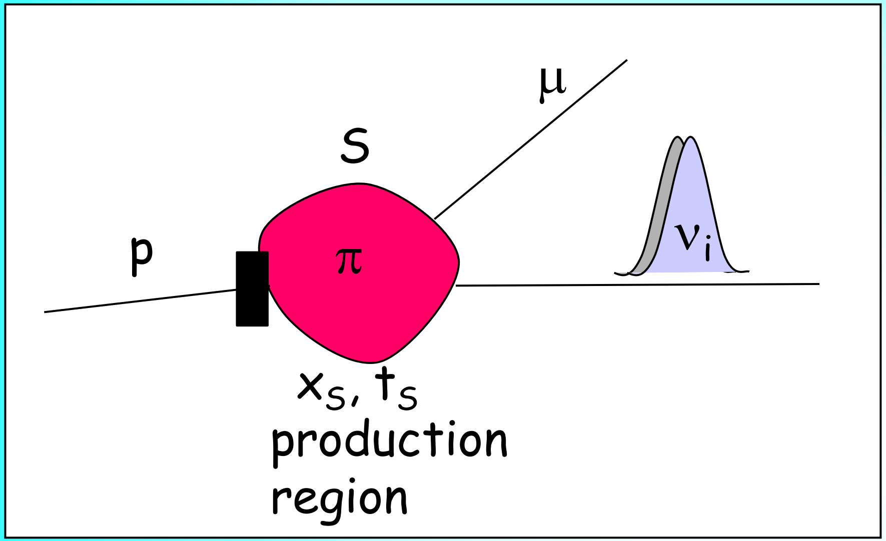
depth of oscillations

$$\phi = \frac{\Delta m^2 \, x}{2E} = \frac{2\pi x}{l_\nu}$$

$$l_\nu = \frac{4\pi E}{\Delta m^2}$$

Oscillation length

Formation of the wave packet



In most of the cases precise form of the shape factor and therefore details of its formation are not important

It is important in the cases of

of partial separation of wave packets

production region is comparable with oscillation length

$r_S \sim l_\nu$

Solving the wave problem: pion moves and emits neutrino waves

Integration of the neutrino waves emitted from space-time points where pion lives

Formation of the wave packet

Pion decay: **!**

E. Akhmedov, D. Hernandez, A.S.

$$g_i(x,t)\, e^{i\phi_i} = \int dp \int dx_S\, dt_S\; M\, \psi_\pi(x_S, t_S)\, \overline{\psi_\mu}(x_S, t_S)\, \exp[ip\,(x - x_S) - iE_i\,(t - t_S)]$$

integration over production region

part of matrix element

plane wave for neuutrino

Pion wave function:

$$\psi_\pi(x_S, t_S) = \exp[-\tfrac{1}{2}\Gamma t_S]\, g_\pi(x_S, t_S)\, \exp[-i\phi_\pi(x_S, t_S)]$$

usually: $g_\pi(x_S, t_S) \sim \delta\,(x_S - v_\pi t_S)$

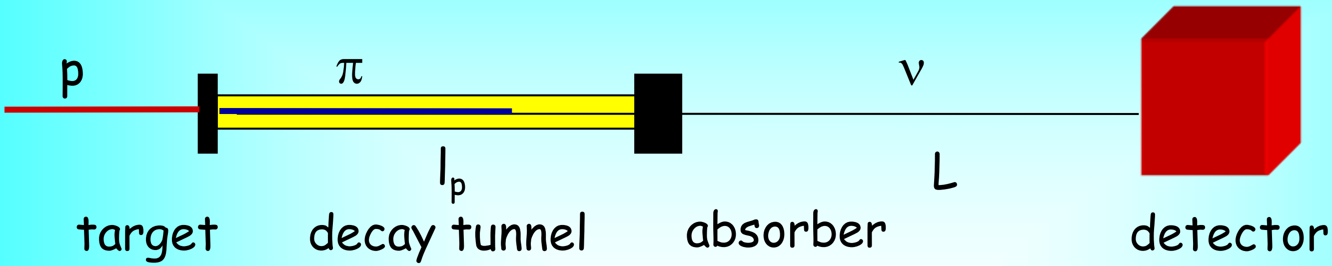
Muon wave function:

$$\psi_\mu(x_S, t_S) = g_\mu(x' - x_S, t' - t_S)\, \exp[\,i\phi_\mu(x' - x_S\,, t' - t_S)]$$

determined by detection of muon

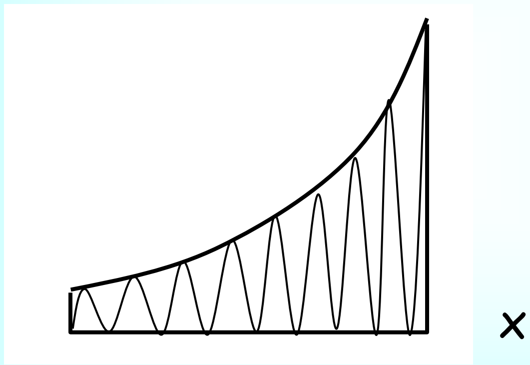
If muon is not detected: plane wave → phase factor → disappears from probability

Neutrino wave packets



D. Hernandez, AS
E. Kh Akhmedov,
D. Hernandez, AS
arXiv:1110.5453

ν wave packet



Doppler effect

The length of the ν wave packet
emitted in the forward direction

$$\sigma = l_p \frac{v - v_\pi}{v_\pi}$$

-shorten

Shape factor

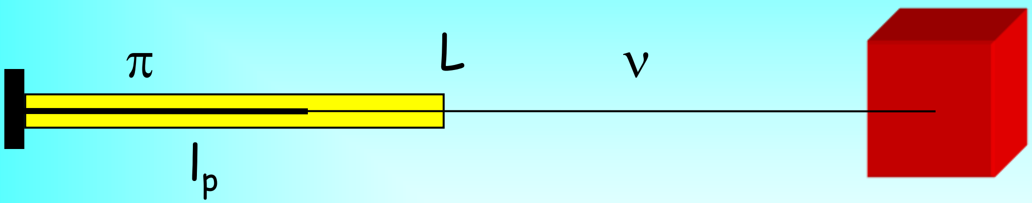
$$g = g_0 \exp\left(\frac{\Gamma}{2(v - v_\pi)}(x - \sigma)\right) \Pi(x, [0, \sigma])$$

box function

frequency increases

Decoherence at production

D. Hernandez, AS



$\Delta E_{ij} \sim \Gamma$

$$P = \overline{P} + \frac{\sin^2 2\theta}{2(1+\xi^2)} \frac{1}{1 - e^{-\Gamma l_p}} [\cos \phi_L + K]$$

$\xi = \Delta m^2 / 2E\Gamma$

decoherence parameter

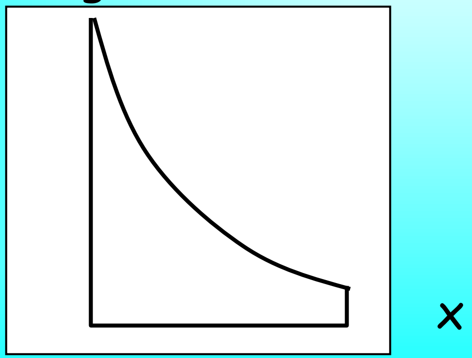
$$K = \xi \sin \phi_L - e^{-\Gamma l_p} [\cos(\phi_L - \phi_p) - \xi \sin(\phi_L - \phi_p)]$$

$\phi_L = \Delta m^2 L/2E \quad \phi_p = \Delta m^2 l_p/2E$

MINOS: $\xi \sim 1$

β–beam ?

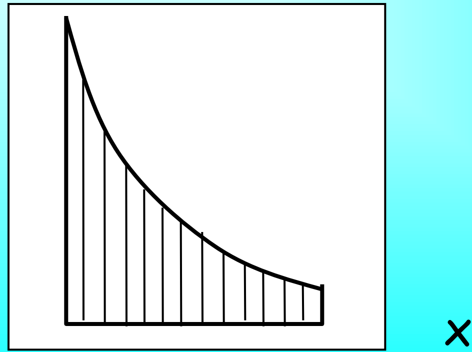
Coherent ν-emission
- long WP



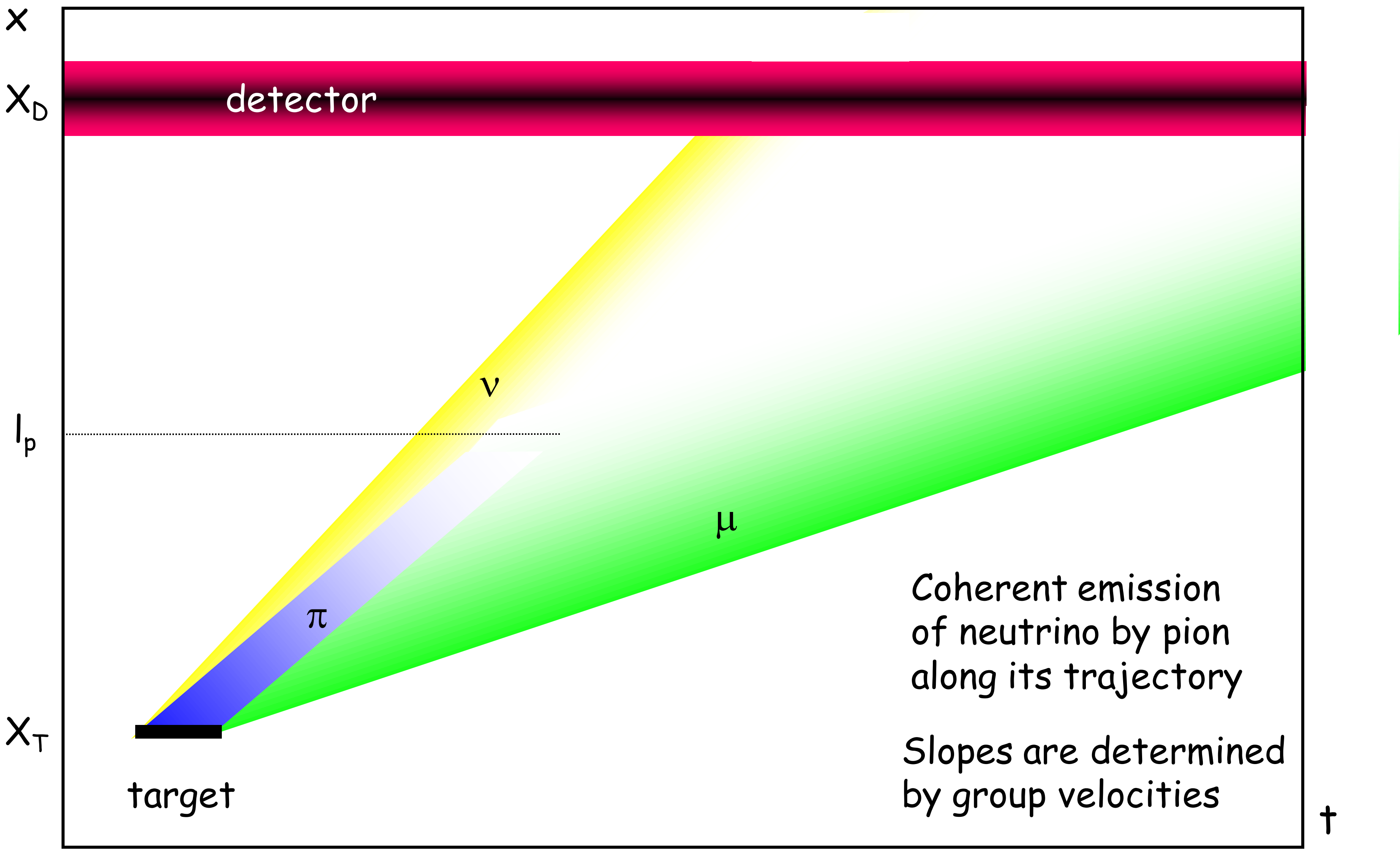
Equivalence

for point-like pion

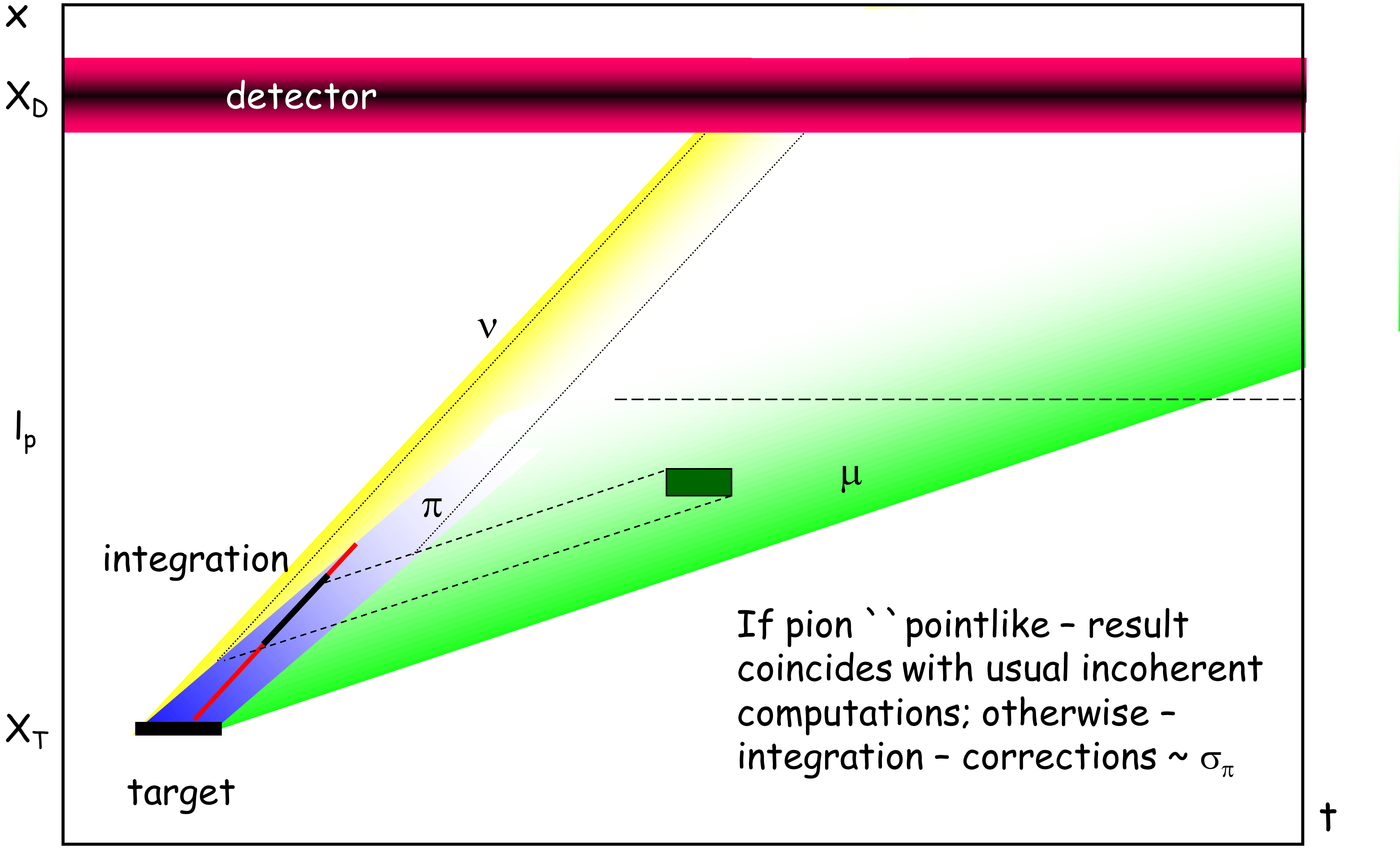
Incoherent ν-emission
- short WP



Space-time diagrams



Coherent and incoherent



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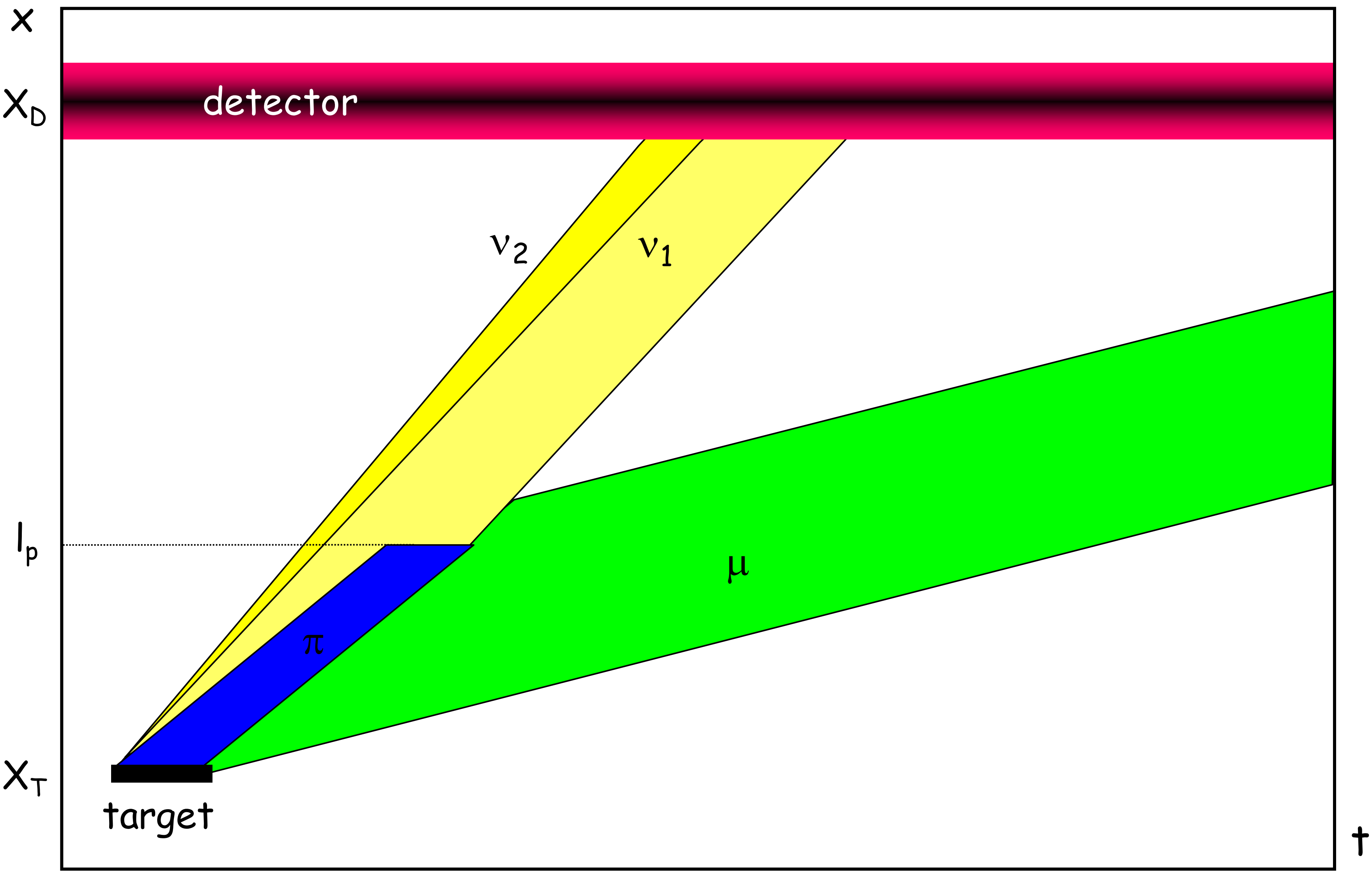


Master equation

If loss of coherence and other complications related to WP picture are irrelevant – `` point-like" picture

$$i \frac{d\Psi}{dt} = H\Psi$$

$$\Psi = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix}$$

with mater effects

$$H = \frac{M M^+}{2E} + V(t)$$

generalization

$$E \sim p + \frac{m^2}{2E}$$

M is the mass matrix

$V = \mathrm{diag}\,(V_e\,,\ 0,\ 0)$ – effective potential

Mixing matrix in vacuum

$$M M^+ = U M_{diag}^2 U^+$$

$$M_{diag}^2 = \mathrm{diag}\,(m_1^2,\ m_2^2,\ m_3^2)$$

Neutrino polarization vectors

$$\psi = \begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix} \Rightarrow$$

Polarization vector:

$$\boxed{\mathbf{P} = \psi^+ \sigma/2\, \psi}$$

$$\mathbf{P} = \begin{pmatrix} \mathrm{Re}\, \nu_e^+ \nu_\tau, \\ \mathrm{Im}\, \nu_e^+ \nu_\tau, \\ \nu_e^+ \nu_e - 1/2 \end{pmatrix}$$

Evolution equation:

$$i \frac{d\Psi}{dt} = H\Psi \Rightarrow \boxed{i \frac{d\Psi}{dt} = (\mathbf{B}\sigma)\Psi}$$

$$\mathbf{B} = \frac{2\pi}{l_m}(\sin 2\theta_m,\ 0,\ \cos 2\theta_m)$$

Differentiating **P** and using equation of motion

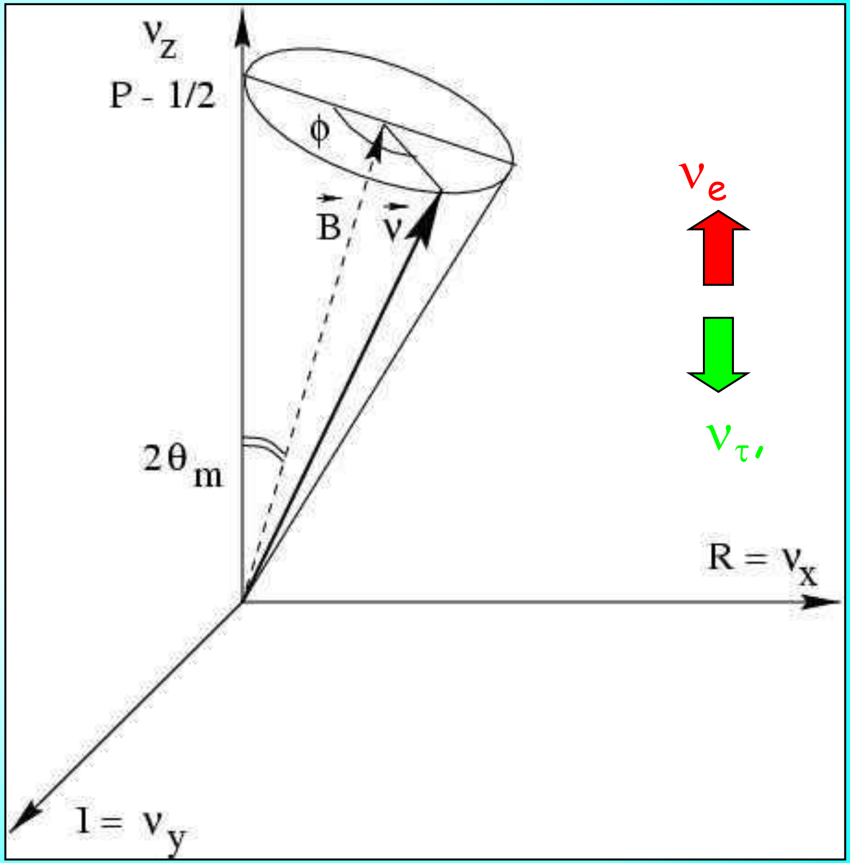
$$\boxed{\frac{d\mathbf{P}}{dt} = (\mathbf{B} \times \mathbf{P})}$$

Coincides with equation for the electron spin precession in the magnetic field

Graphical representation

$\vec{\nu} = \mathbf{P} =$
$(\mathrm{Re}\, \nu_e^+ \nu_\tau, \;\; \mathrm{Im}\, \nu_e^+ \nu_\tau, \;\; \nu_e^+ \nu_e - 1/2)$

$$\mathbf{B} = \frac{2\pi}{l_m}(\sin 2\theta_m, \; 0, \; \cos 2\theta_m)$$



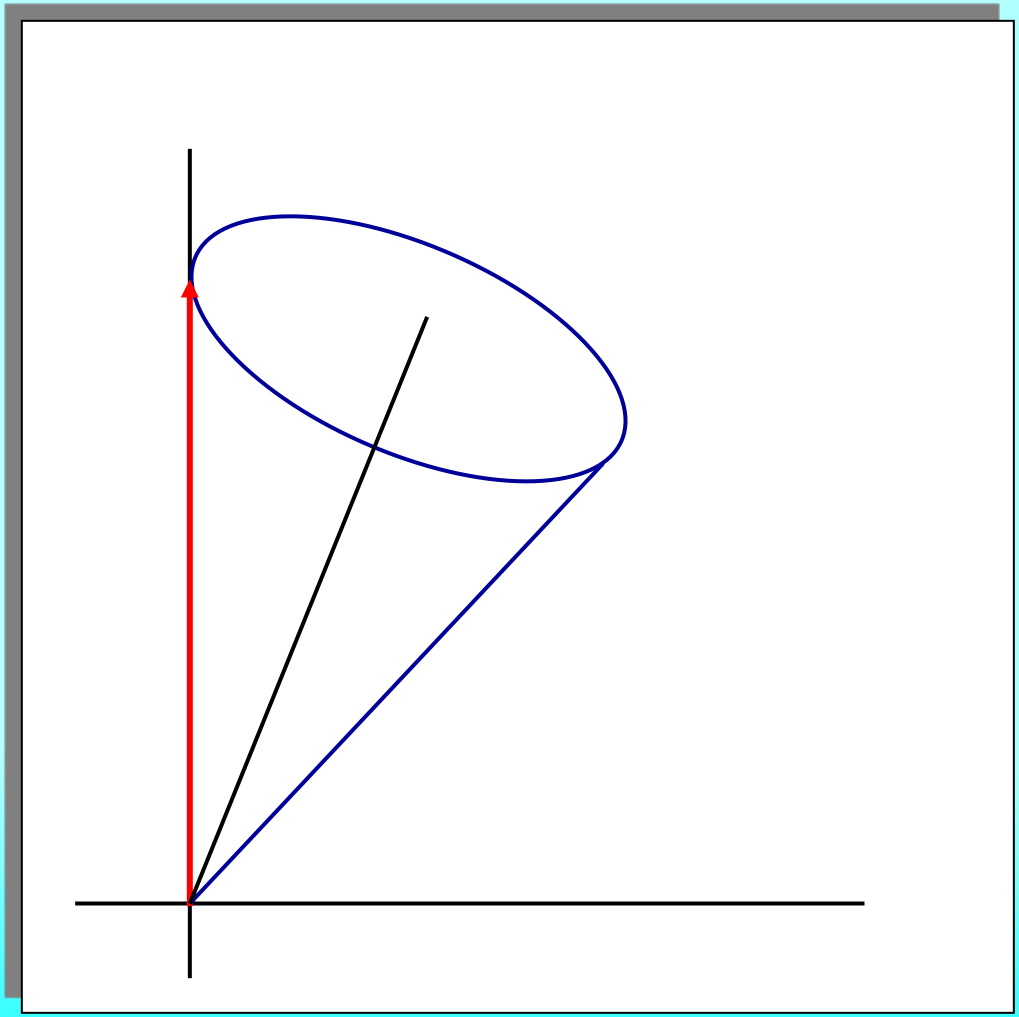
Evolution equation

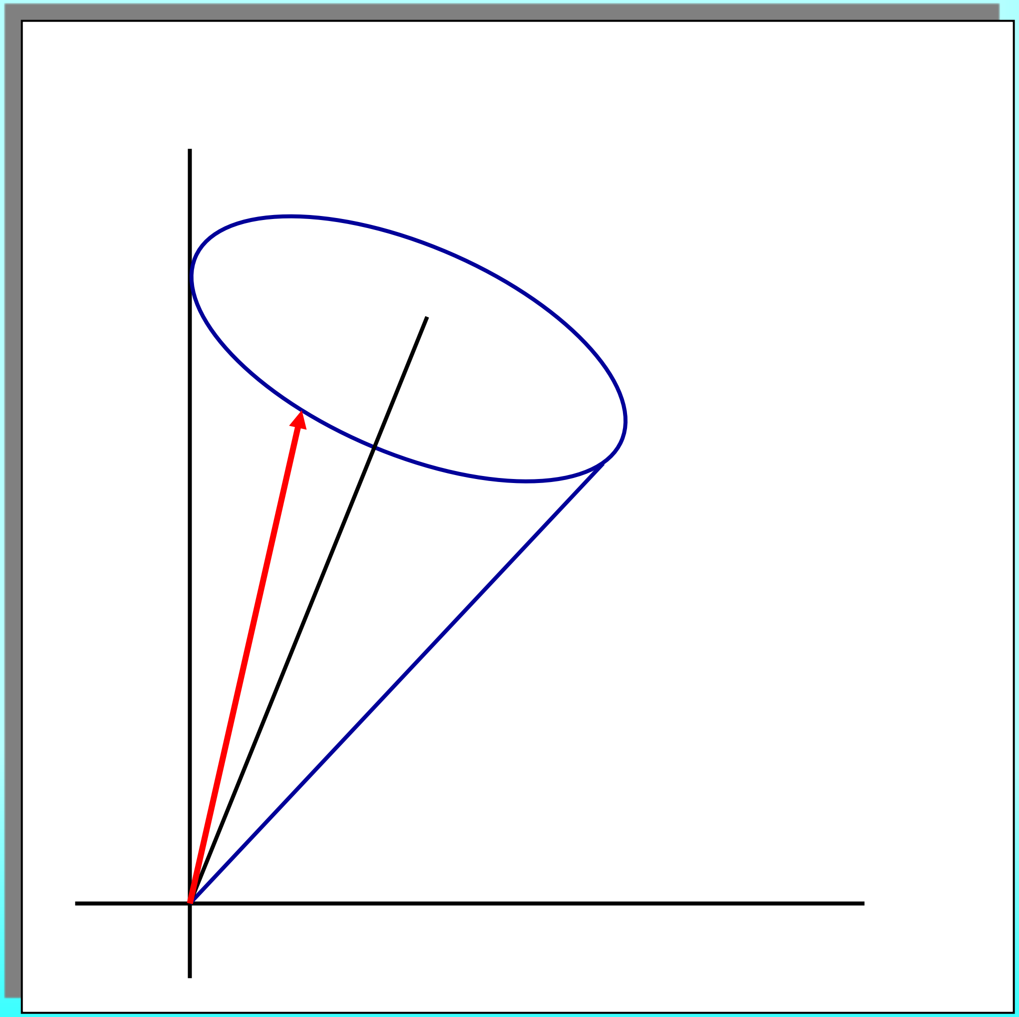
$$\frac{d\vec{\nu}}{dt} = (\vec{B} \times \vec{\nu})$$

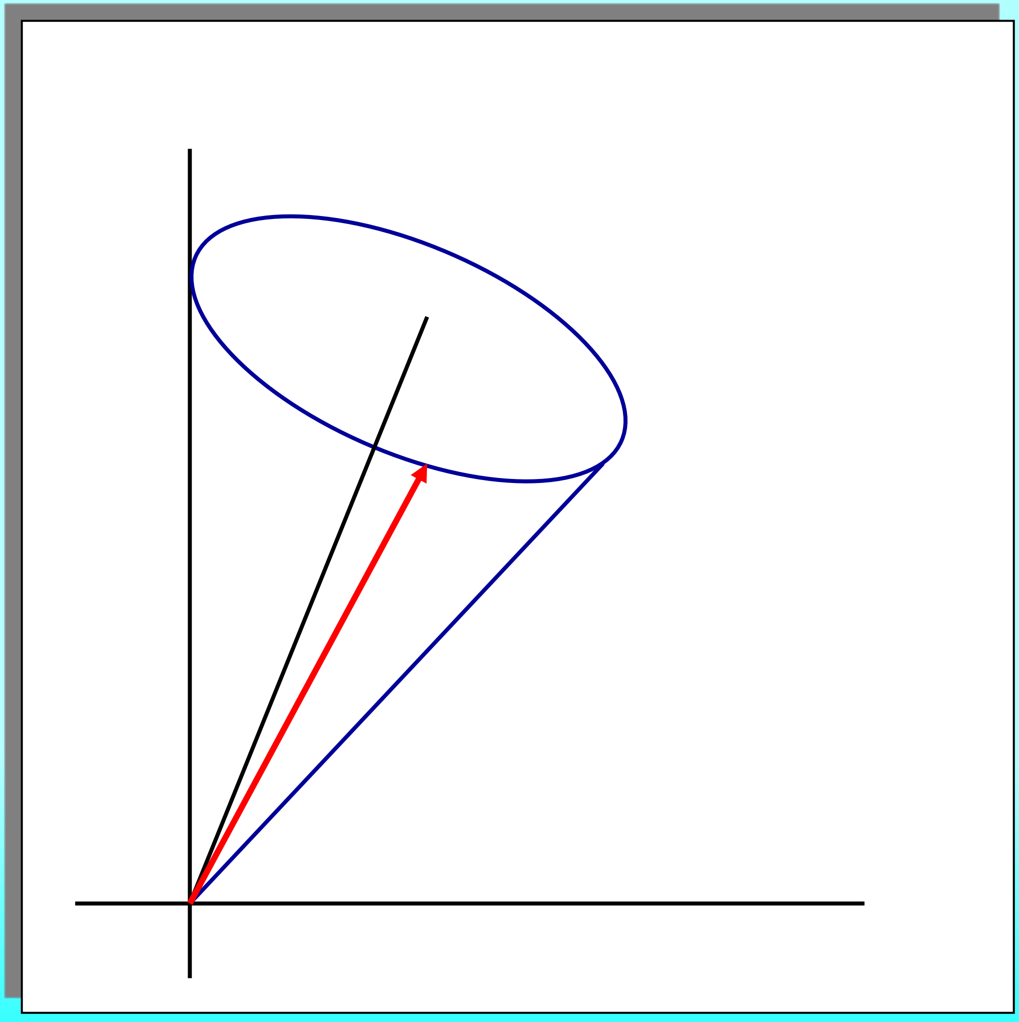
$\phi = 2\pi t / l_m$ - phase of oscillations

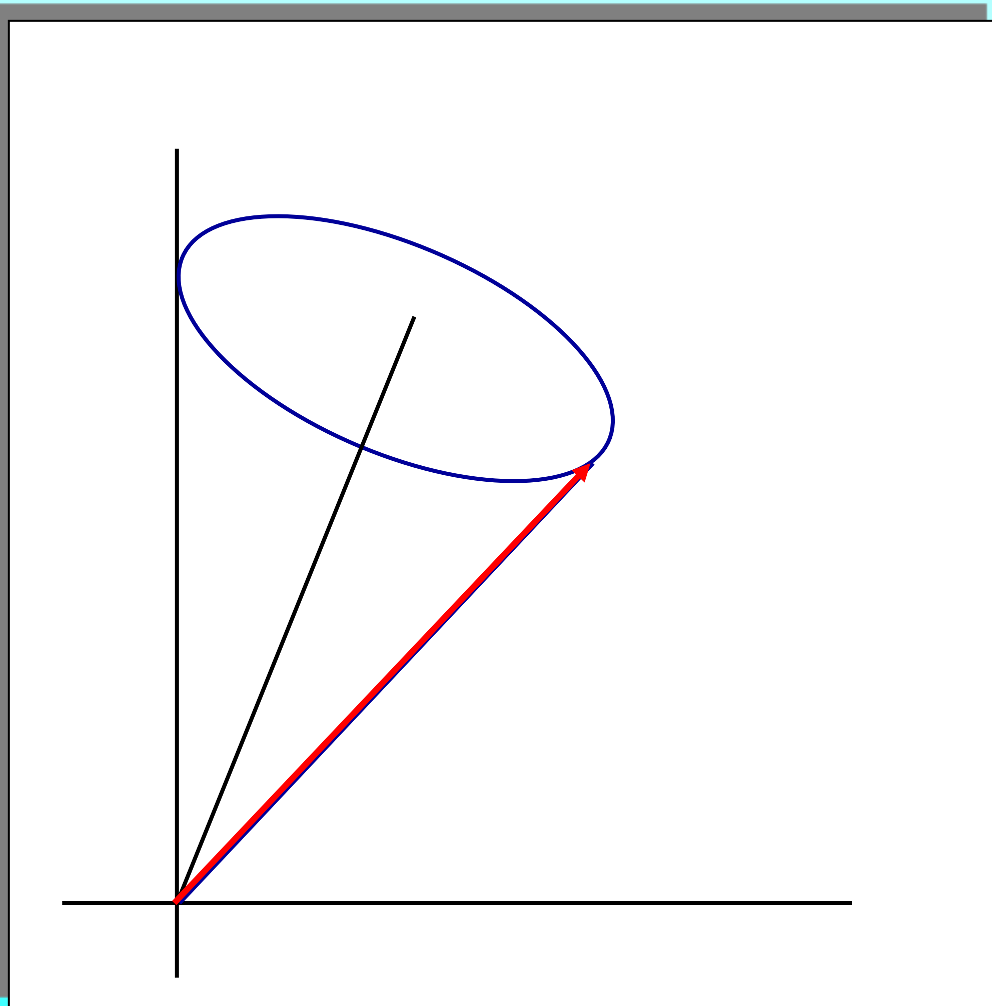
$P = \nu_e^+ \nu_e = \nu_Z + 1/2 = \cos^2\theta_Z/2$ probability to find ν_e

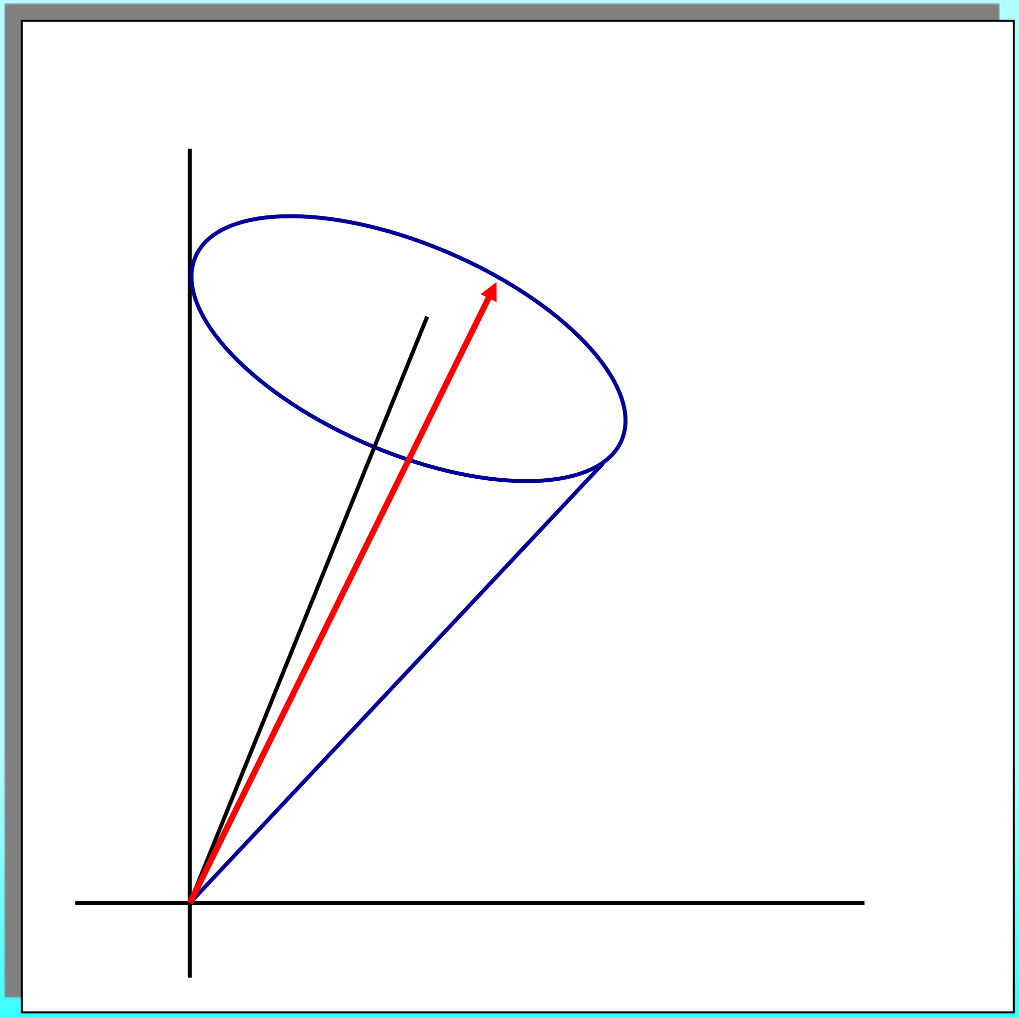
Oscillations

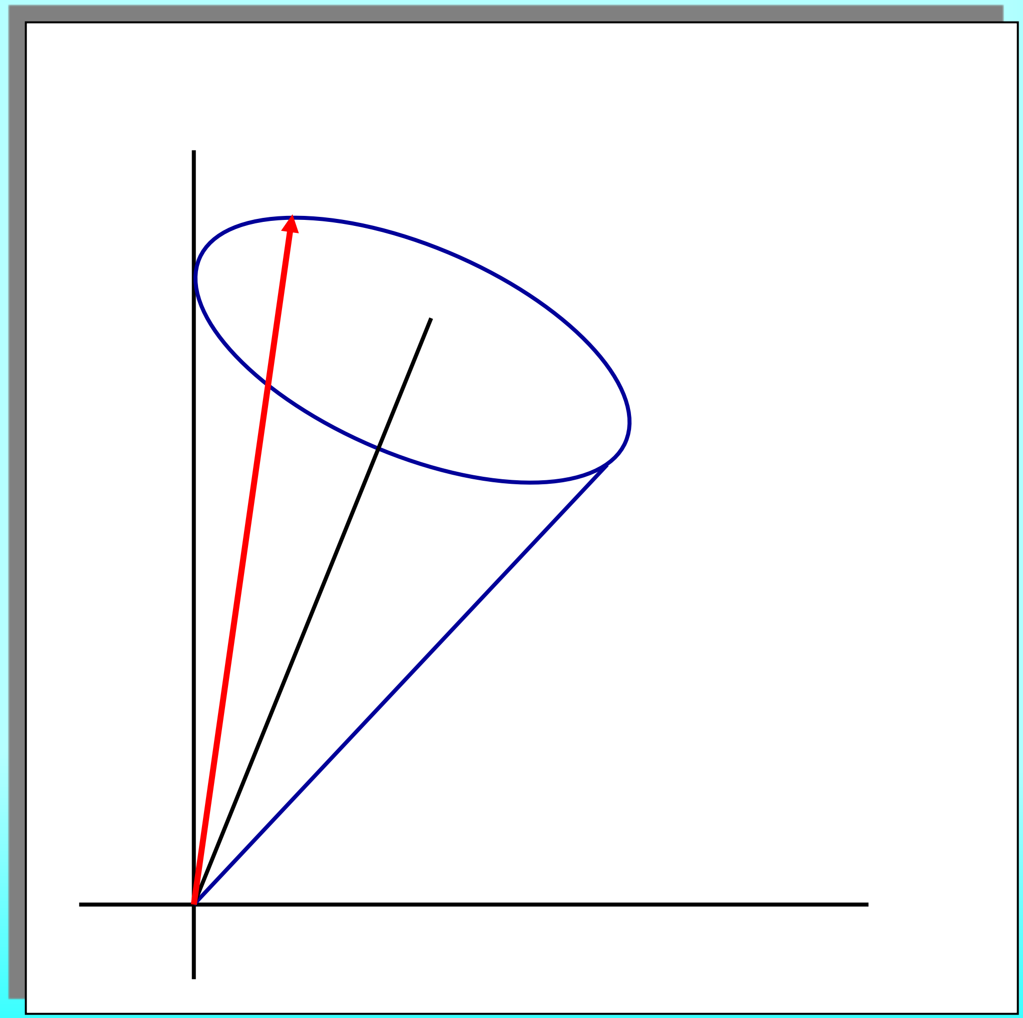


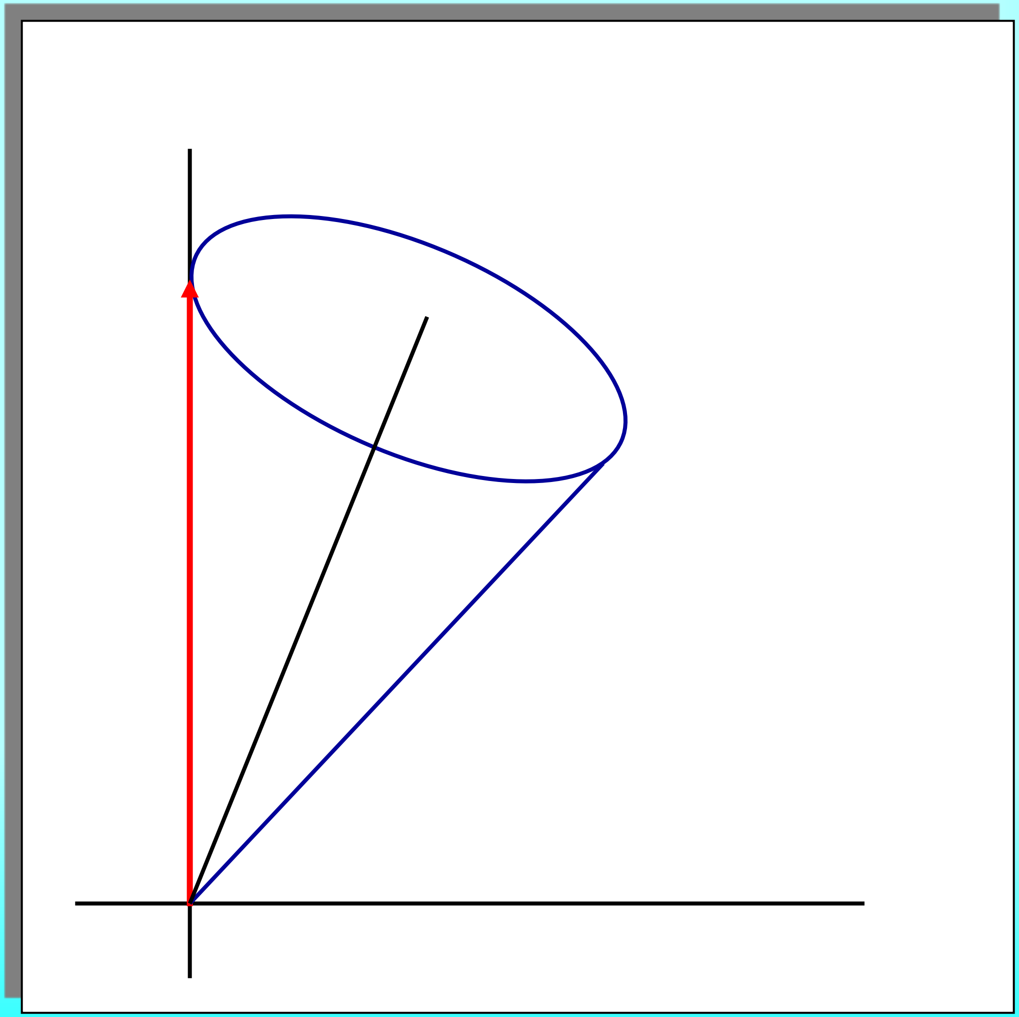








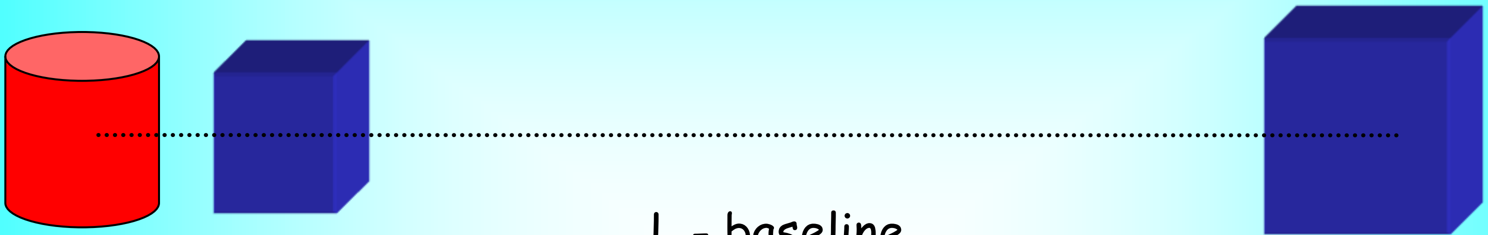




Conclusion:

Oscillations is effect of
monotonous increase of phase difference
between eigenstates of propagation (mass eigenstates)
In course of propagation in space-time

Experimental set-up



ν_α

ν_α disappearance

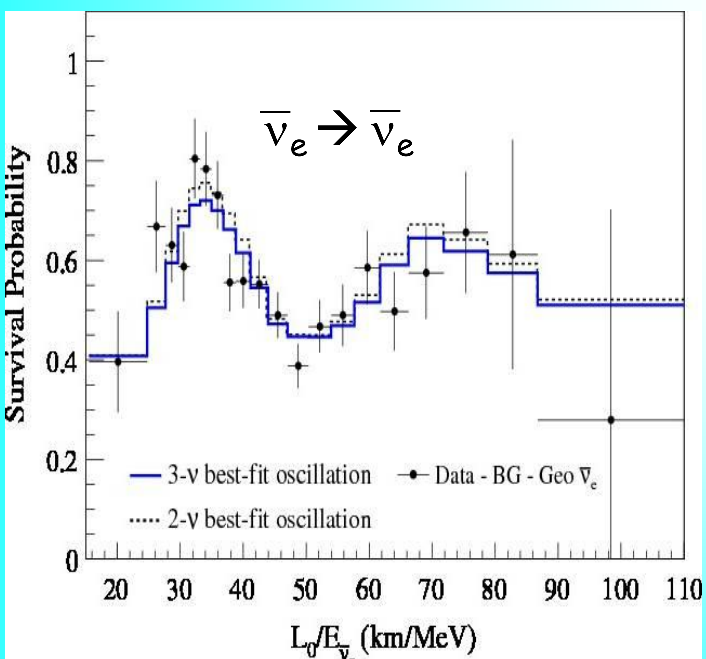
ν_β appearance

Oscillation probability – periodic function of

- Distance L and
- Inverse energy 1/E

Observation of oscillations

KamLAND



T2K