Physics of neutrino oscillations & flavor conversion

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Invisibles network INT Training lectures
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Matter effects:
Oscillations & flavor conversion

Original papers:

L. Wolfenstein, Phys. Rev. D17 (1978) 2369

L. Wolfenstein, in ``Neutrino-78'', Purdue Univ. C3, 1978.

adiabaticity

L. Wolfenstein, Phys. Rev. D20 (1979) 2634

V. D. Barger, K. Whisnant, S. Pakvasa, R.J.N. Phillips,
Phys.Rev. D22 (1980) 2718

enhancement of oscillations

S.P. Mikheev, A.Yu. Smirnov, Sov. J. Nucl.Phys. 42 (1985) 913-917,
Yad.Fiz. 42 (1985) 1441-1448

Resonance, Adiabaticity Solar nu

S.P. Mikheev, A.Yu. Smirnov, Nuovo Cim. C9 (1986) 17-26

S.P. Mikheev, A.Yu. Smirnov, Sov. Phys. JETP 64 (1986) 4-7,
Zh.Eksp.Teor.Fiz. 91 (1986) 7-13, arXiv:0706.0454 [hep-ph]

adiabatic formulas

S.P. Mikheev, A.Yu. Smirnov, 6th Moriond workshop, Tignes, Jan.
1986 p. 355

Earth matter effects, day night, atmospheric

... continued

H.A. Bethe, Phys.Rev.Lett. 56 (1986) 1305

A. Messiah, 6th Moriond workshop, Tignes Jan. 1986 p.373

Oscillations?

S. J. Parke, Phys.Rev.Lett. 57 (1986) 1275-1278

W.C. Haxton, Phys.Rev.Lett. 57 (1986) 1271-1274

S. P. Rosen, J. M. Gelb, Phys.Rev. D34 (1986) 969

P. Langacker, S.T. Petcov, G. Steigman, S. Toshev, Nucl.Phys. B282 (1987) 589

The MSW effect and matter effects in neutrino oscillations.
A.Yu. Smirnov, Phys. Scripta T121 (2005) 57-64, hep-ph/0412391

A. Y. Smirnov, hep-ph/0305106

P.C. de Holanda, A.Yu. Smirnov, Astropart.Phys. 21 (2004) 287, hep-ph/0309299

Quantum field theoretic approach to neutrino oscillations in matter.
E. Kh. Akhmedov, A. Wilhelm, arXiv:1205.6231 [hep-ph]

Observations of Matter effects

Solar neutrinos

Adiabatic conversion

Loss of coherence

Solar neutrinos do not oscillate

Adiabatic conversion $\neq$ oscillations

Atmospheric neutrinos

Suppression of ν_e - oscillations due to solar mass splitting in GeV range

Matter potential

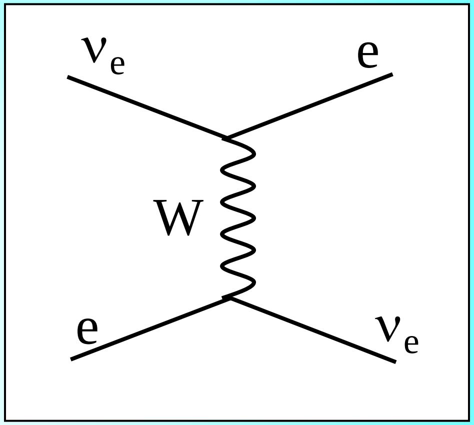
L. Wolfenstein, 1978

for ν_e ν_μ

at low energies Re A >> Im A
inelelastic interactions can be neglected

Elastic forward scattering ⇨ V_e, V_μ

potentials



Refraction index:

$$n - 1 = V / p$$

difference of potentials

$$V = V_e - V_\mu = \sqrt{2}\, G_F\, n_e$$

$V \sim 10^{-13}$ eV inside the Earth

for E = 10 MeV

$$n - 1 = \begin{cases} \sim 10^{-20} & \text{inside the Earth} \\ < 10^{-18} & \text{inside the Sun} \end{cases}$$

Refraction length:

$$l_0 = \frac{2\pi}{V}$$

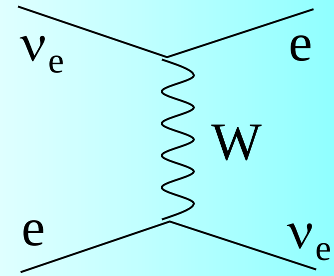
Matter potential

derivation

At low energies: neglect the inelastic scattering and absorption effect is reduced to the elastic forward scattering (refraction) described by the potential V:

$$H_{int}(\nu) = \langle \psi | H_{int} | \psi \rangle = V \bar{\nu} \nu$$

ψ is the wave function of the medium



CC interactions with electrons

$$H_{int} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^{\mu}(1 - \gamma_5) \nu \; \bar{e} \gamma_{\mu}(1 - \gamma_5) e$$

$\langle \bar{e} \gamma_0 (1 - \gamma_5) e \rangle = n_e$ - the electron number density

$\langle \bar{e} \vec{\gamma} e \rangle = n_e \vec{v}$

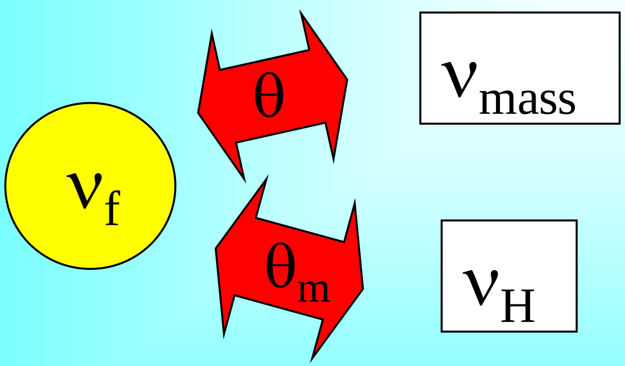
$\langle \bar{e} \vec{\gamma} \gamma_5 e \rangle = n_e \vec{\lambda}_e$ - averaged polarization vector of e

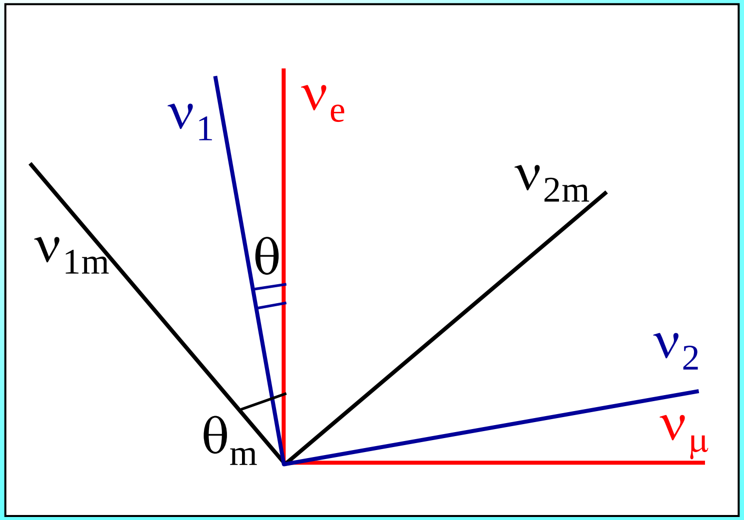
For unpolarized medium at rest:

$$V = \sqrt{2}\, G_F n_e$$

Mixing in matter

	in vacuum:		*in matter:*	
Effective Hamiltonian	H_0	⇨	$H = H_0 + V$	
Eigenstates	ν_1, ν_2	⇨	ν_{1m}, ν_{2m}	depend on n_e, E
Eigenvalues	$m_1^2/2E, m_2^2/2E$	⇨	H_{1m}, H_{2m}	instantaneous





Mixing angle determines flavors (flavor composition) of eigenstates of propagation

Evolution equation

$$i\,\frac{d\nu_f}{dt} = H_{tot}\,\nu_f \qquad \nu_f = \begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$H_{tot} = H_{vac} + V$ is the total Hamiltonian

$H_{vac} = \dfrac{M^2}{2E}$ is the vacuum (kinetic) part

$V = \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix}$ matter part $\qquad V_e = \sqrt{2}\,G_F n_e$

H_{tot}

$$i\,\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\dfrac{\Delta m^2}{2E}\cos 2\theta + V_e & \dfrac{\Delta m^2}{4E}\sin 2\theta \\ \dfrac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

Mixing in matter

Diagonalization of the Hamiltonian:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

$$V = \sqrt{2}\, G_F\, n_e$$

Mixing is maximal if

$$V = \frac{\Delta m^2}{2E} \cos 2\theta$$

Resonance condition

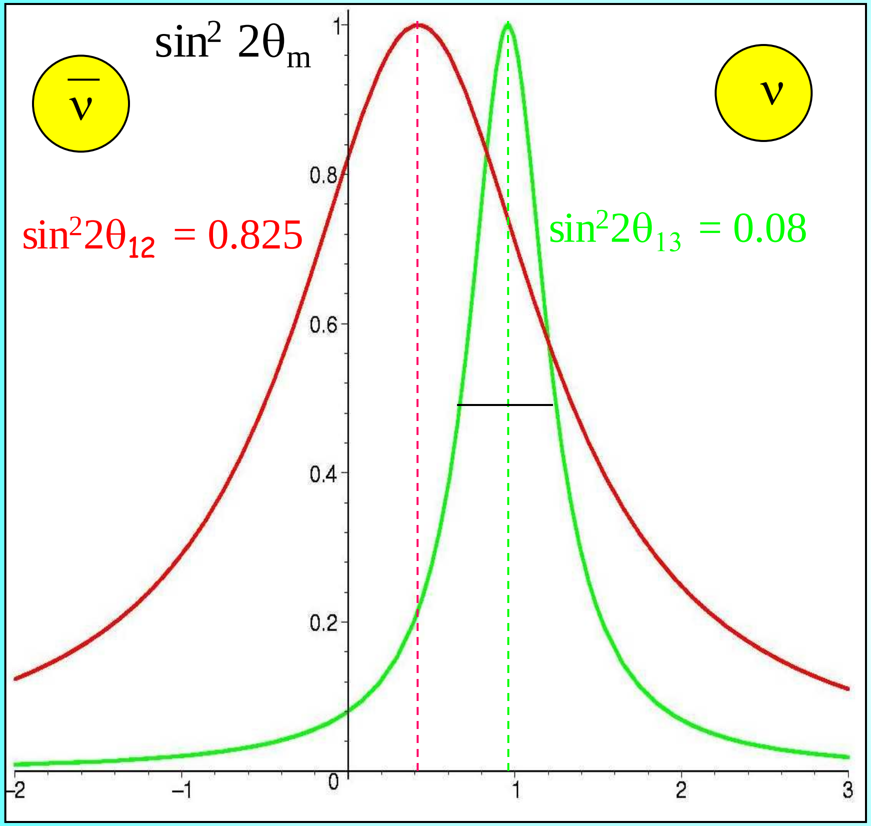
$$H_e = H_\mu$$

$$\sin^2 2\theta_m = 1$$

Difference of the eigenvalues

$$H_{2m} - H_{1m} = \frac{\Delta m^2}{2E} \sqrt{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

Resonance



$l_\nu / l_0 \sim n E$

In resonance:

$\sin^2 2\theta_m = 1$

Flavor mixing is maximal

$l_\nu = l_0 \cos 2\theta$

Vacuum oscillation length $\approx$ Refraction length

Resonance width: $\Delta n_R = 2 n_R \tan 2\theta$

Resonance layer: $n = n_R +/- \Delta n_R$

Level crossing

V. Rubakov, private comm.
N. Cabibbo, Savonlinna 1985
H. Bethe, PRL 57 (1986) 1271

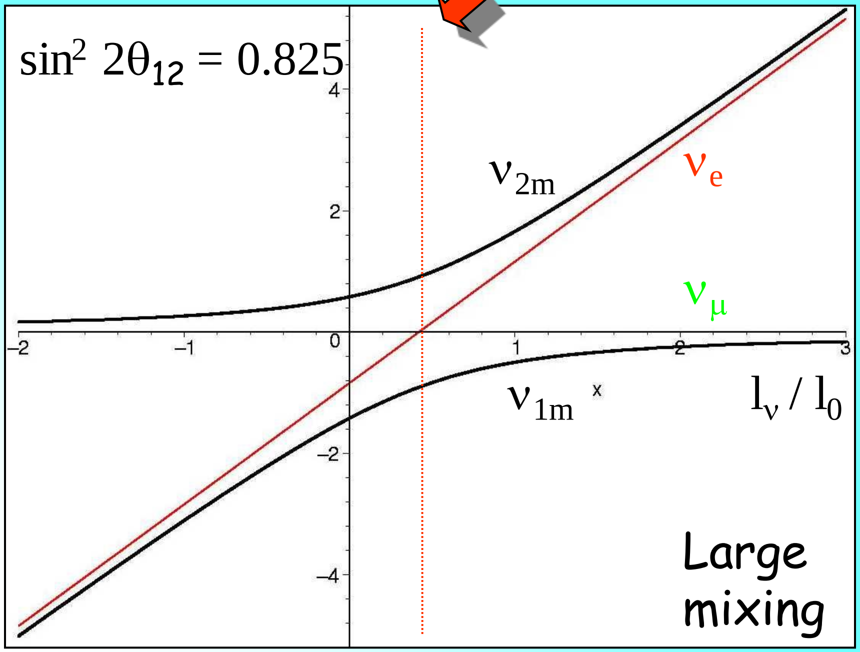
Dependence of the neutrino eigenvalues on the matter potential (density)

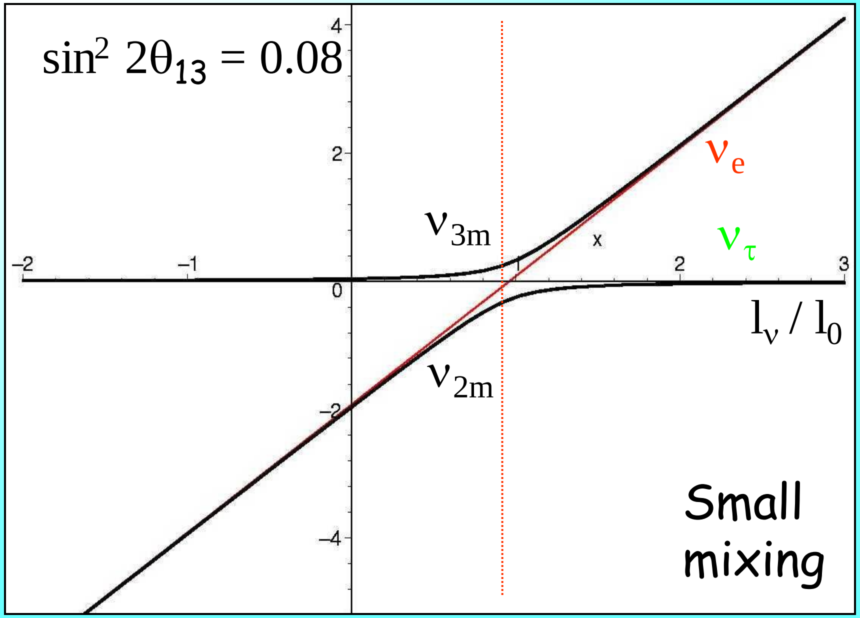
$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

$$\frac{l_\nu}{l_0} = \cos 2\theta$$

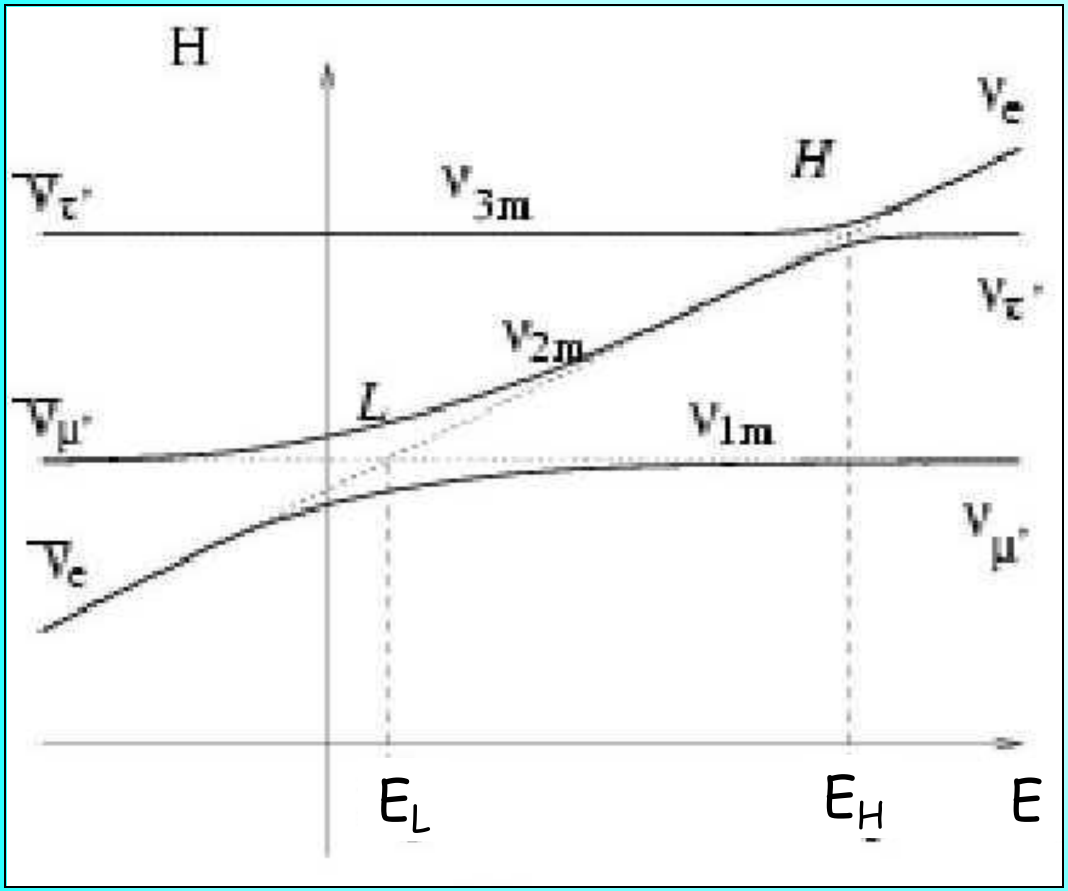
Crossing point - resonance
- the level split is minimal
- the oscillation length is maximal

H_{im} resonance





Level crossings



0.1 GeV

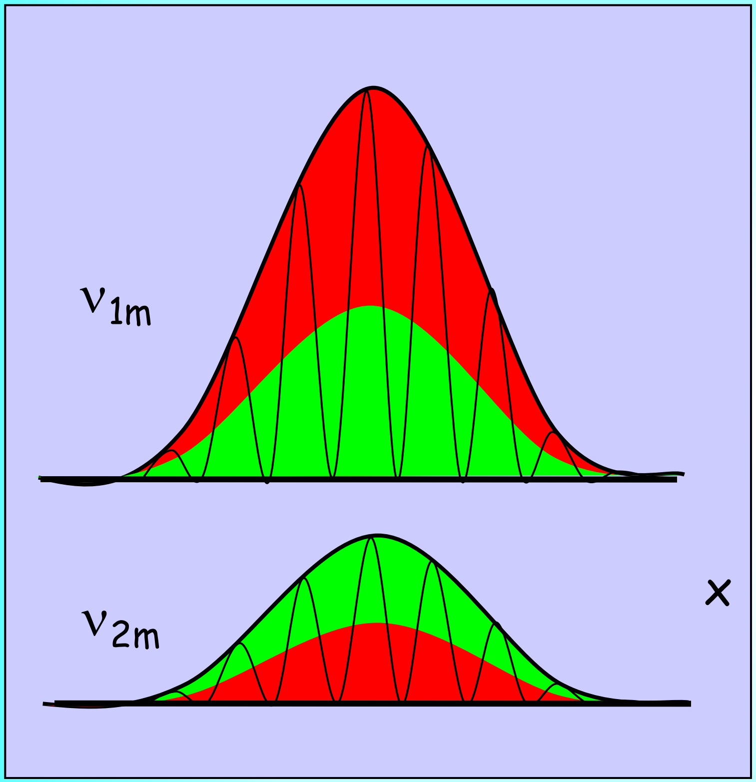
6 GeV

Normal mass hierarchy

Resonance region

High energy range

Oscillations in matter



$H_0 \rightarrow H = H_0 + V$

$\nu_k \rightarrow \nu_{mk}$

eigenstates of H_0

eigenstates of H

$\theta \rightarrow \theta_m$ (n)

Resonance - maximal mixing in matter – oscillations with maximal depth

$\theta_m = \pi/4$

Resonance condition:

$$V = \cos 2\theta \frac{\Delta m^2}{2E}$$

Constant density medium: the same dynamics

Mixing changed

phase difference changed

Oscillations in matter

Oscillation probability constant density

$$P(\nu_e \to \nu_a) = \sin^2 2\theta_m \sin^2\left(\frac{\pi L}{l_m}\right)$$

half-phase ϕ

Amplitude of oscillations

oscillatory factor

$\theta_m(E, n)$ - mixing angle in matter

$l_m(E, n)$ – oscillation length in matter

$$l_m = 2\pi/(H_{2m} - H_{1m})$$

In vacuum: $\theta_m \to \theta$, $l_m \to l_\nu$

Maximal effect: $\sin^2 2\theta_m = 1$ → MSW resonance condition

$\phi = \pi/2 + \pi k$

Oscillation length in matter

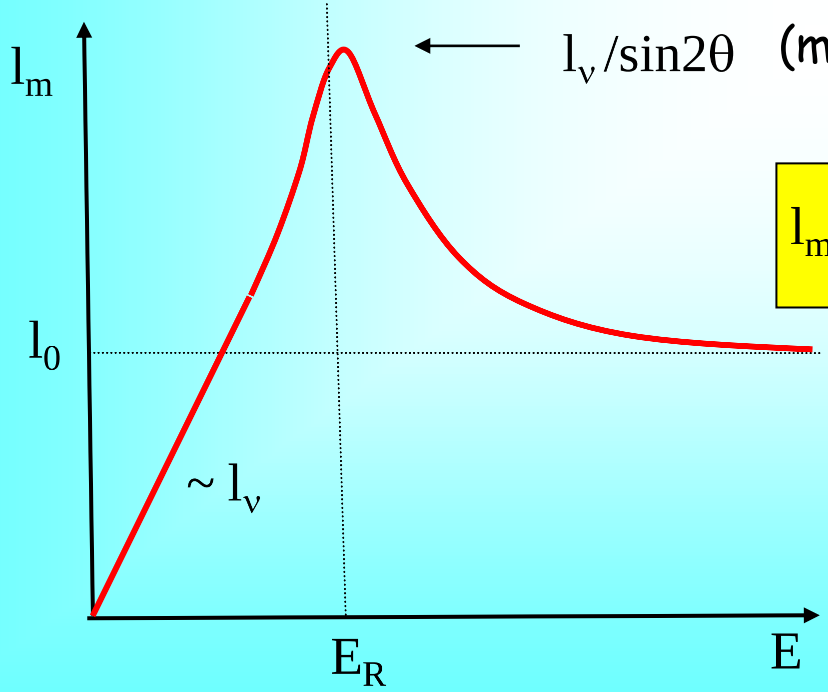
Oscillation length in vacuum

$$l_\nu = \frac{4\pi E}{\Delta m^2}$$

Refraction length

$$l_0 = \frac{2\pi}{\sqrt{2}\, G_F n_e}$$

- determines the phase produced by interaction with matter



$l_\nu / \sin 2\theta$ (maximum at $l_\nu = l_0 / \cos 2\theta$)

$$l_m = \frac{2\pi}{H_{2m} - H_{1m}}$$

shifts with respect resonance energy:

$$l_\nu (E_R) = l_0 \cos 2\theta$$

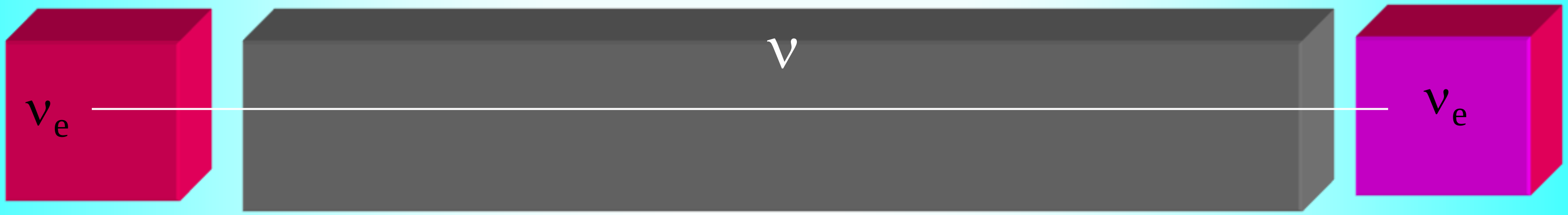
converges to the refraction length

Resonance enhancement of oscillations

Constant density

Resonance enhancement

Constant density



Source

$F_0(E)$

layer of length L

Detector

F(E)

Depth of oscillations determined by $\sin^2 2\theta_m$
as well as the oscillation length, l_m
depend on neutrino energy

For neutrinos propagating
in the mantle of the Earth

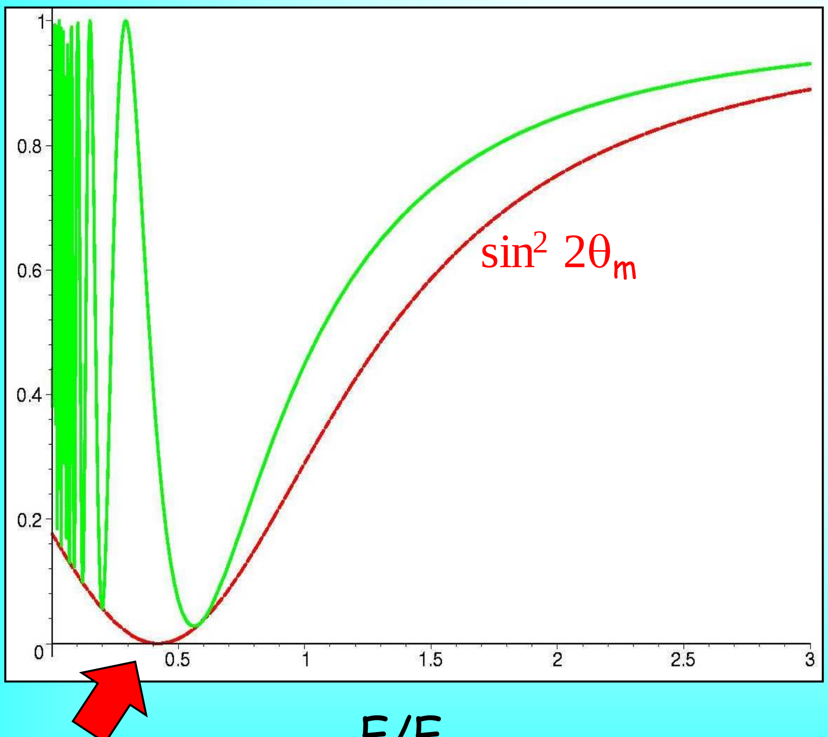
Large mixing $\sin^2 2\theta = 0.824$

Layer of length L $k = \pi L / l_0$

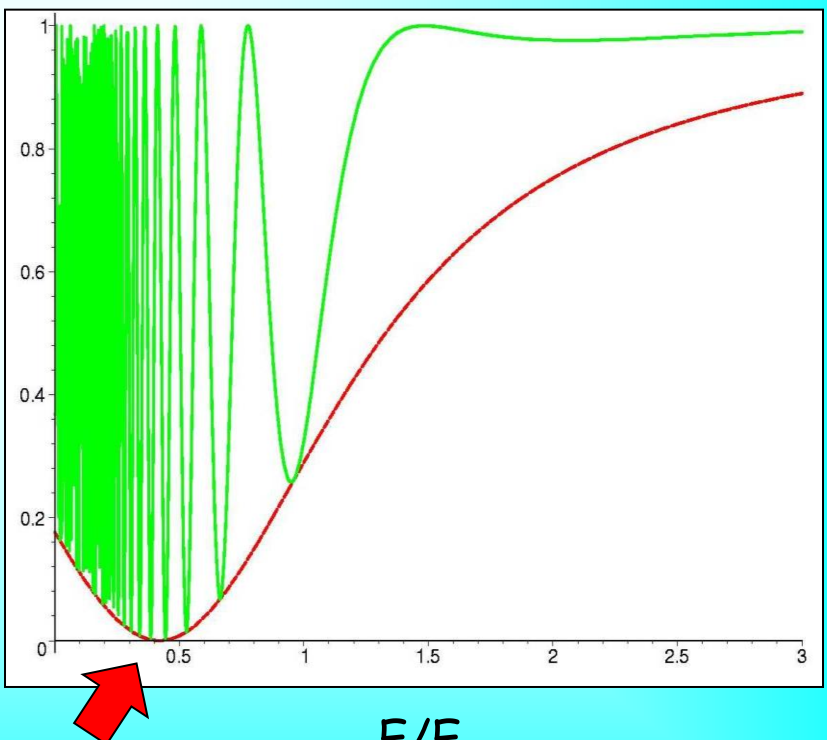
ν

thin layer $k = 1$ | thick layer $k = 10$

$\frac{F(E)}{F_0(E)}$

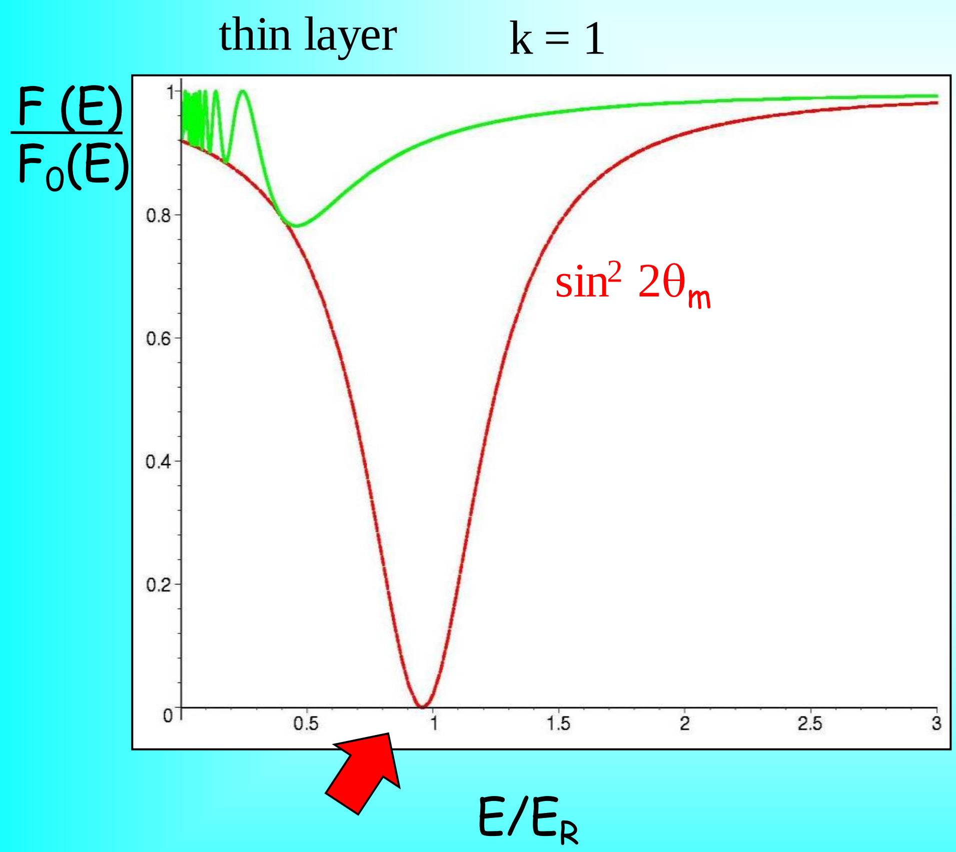


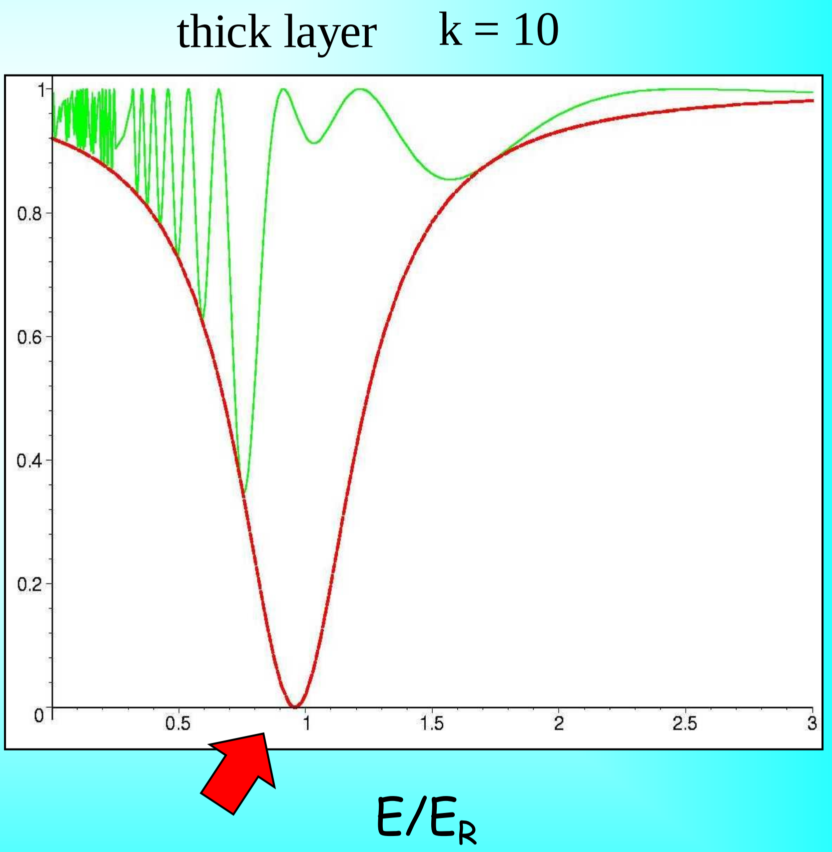
E/E_R



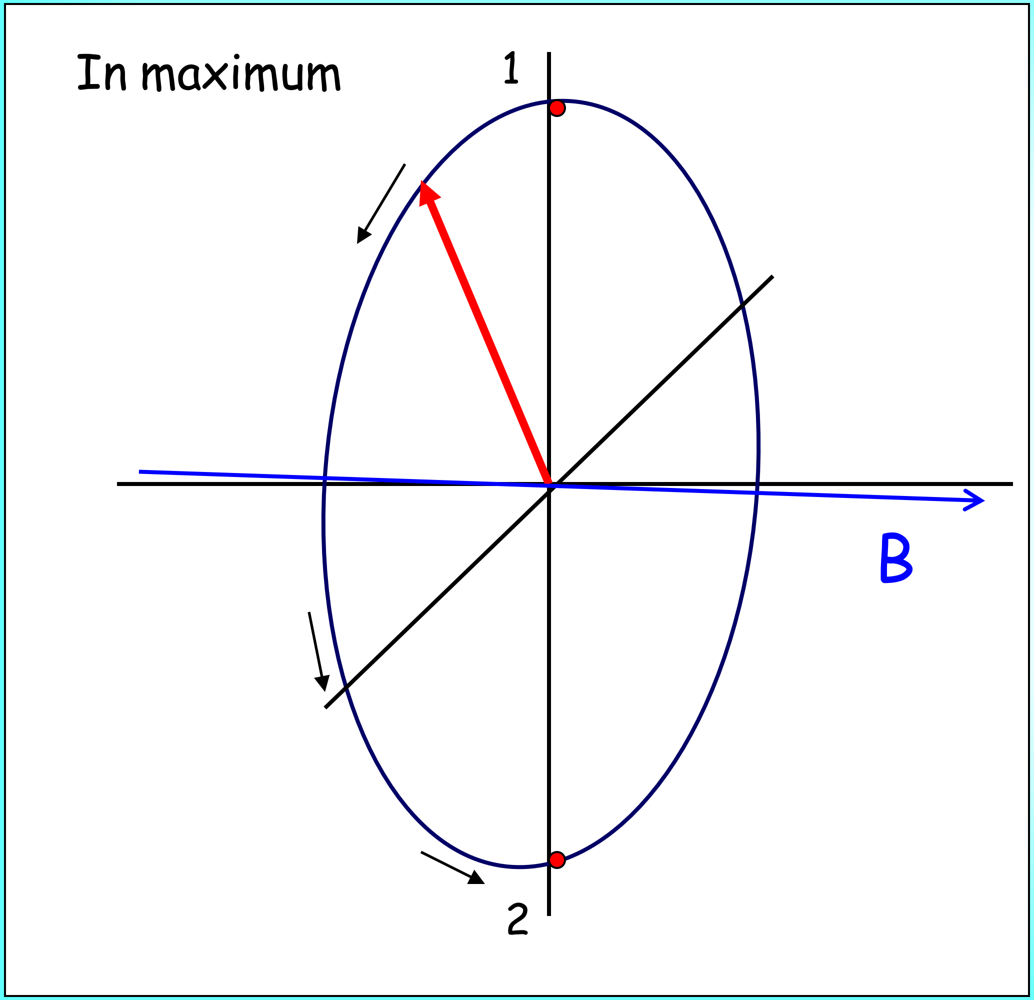
E/E_R

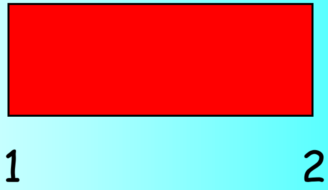
Small mixing $\sin^2 2\theta = 0.08$





Resonance enhancement





Adiabatic conversion

Varying density

Evolution equation for eigenstates

In non-uniform medium the Hamiltonian depends on time:

$H_{tot} = H_{tot}(n_e(t))$

$$i \frac{d\nu_f}{dt} = H_{tot}\, \nu_f$$

$$\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\nu_m = \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

Inserting $\nu_f = U(\theta_m)\, \nu_m$

$\theta_m = \theta_m(n_e(t))$

$$i \frac{d}{dt} \begin{pmatrix} \nu_{1m} \\ \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} 0 & i \frac{d\theta_m}{dt} \\ -i \frac{d\theta_m}{dt} & H_{2m} - H_{1m} \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \\ \nu_{2m} \end{pmatrix}$$

off=diagonal terms imply transitios

$\nu_{1m} \Leftrightarrow \nu_{2m}$

However

if $\left| \frac{d\theta_m}{dt} \right| \ll H_{2m} - H_{1m}$

off-diagonal elements can be neglected
no transitions between eigenstates
propagate independently

Adiabaticity

Adiabaticity condition $\left|\frac{d\theta_m}{dt}\right| \ll H_{2m} - H_{1m}$

External conditions (density) change slowly the system has time to adjust them

transitions between the neutrino eigenstates can be neglected

$\nu_{1m} \not\leftrightarrow \nu_{2m}$ → The eigenstates propagate independently

Shape factors of the eigenstates do not change

Crucial in the resonance layer:
- the mixing changes fast
- level splitting is minimal

$\Delta r_R > l_R$ if vacuum mixing is small

$l_R = l_\nu / \sin 2\theta$ oscillation length in resonance

$\Delta r_R = n_R / (dn/dx)_R \tan 2\theta$ width of the res. layer

If vacuum mixing is large, the point of maximal adiabaticity violation is shifted to larger densities

$n(a.v.) \rightarrow n_R^0 > n_R$

$n_R^0 = \Delta m^2 / 2\sqrt{2}\, G_F E$

Adiabatic parameter

$$\kappa = \frac{H_{2m} - H_{1m}}{\left|\frac{d\theta_m}{dt}\right|}$$

Adiabaticity condition:
$\kappa > 1$

most crucial in the resonance where
the mixing angle in matter changes fast

$$\kappa_R = \frac{\Delta r_R}{l_R}$$

$\Delta r_R = h_n \tan 2\theta$ is the width of the resonance layer

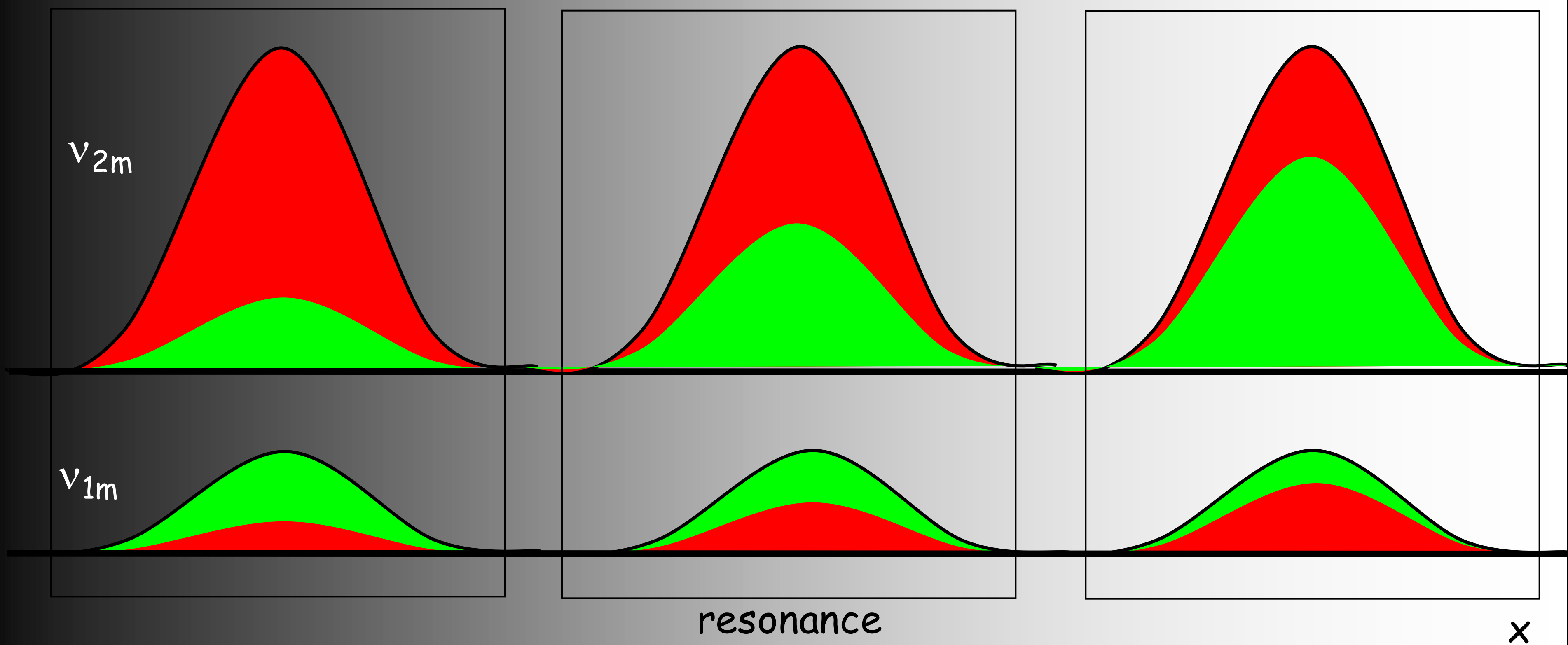
$h_n = \frac{n}{dn/dx}$ is the scale of density change

$l_R = l_\nu / \sin 2\theta$ is the oscillation length in resonance

Explicitly:

$$\kappa_R = \frac{\Delta m^2 \sin^2 2\theta \, h_n}{2E \cos 2\theta}$$

Adiabatic conversion



if density changes slowly

- the amplitudes of the wave packets do not change
- flavors of the eigenstates follow the density change

Adiabatic conversion probability

Sun, Supernova

From high to low densities

Initial state: $\nu(0) = \nu_e = \cos\theta_m^0\, \nu_{1m}(0) + \sin\theta_m^0\, \nu_{2m}(0)$

Mixing angle in matter in initial state

Adiabatic evolution to the surface of the Sun (zero density):

$\nu_{1m}(0) \rightarrow \nu_1$
$\nu_{2m}(0) \rightarrow \nu_2$

Final state: $\nu(f) = \cos\theta_m^0\, \nu_1 + \sin\theta_m^0\, \nu_2\, e^{-i\phi}$

Probability to find ν_e averaged over oscillations

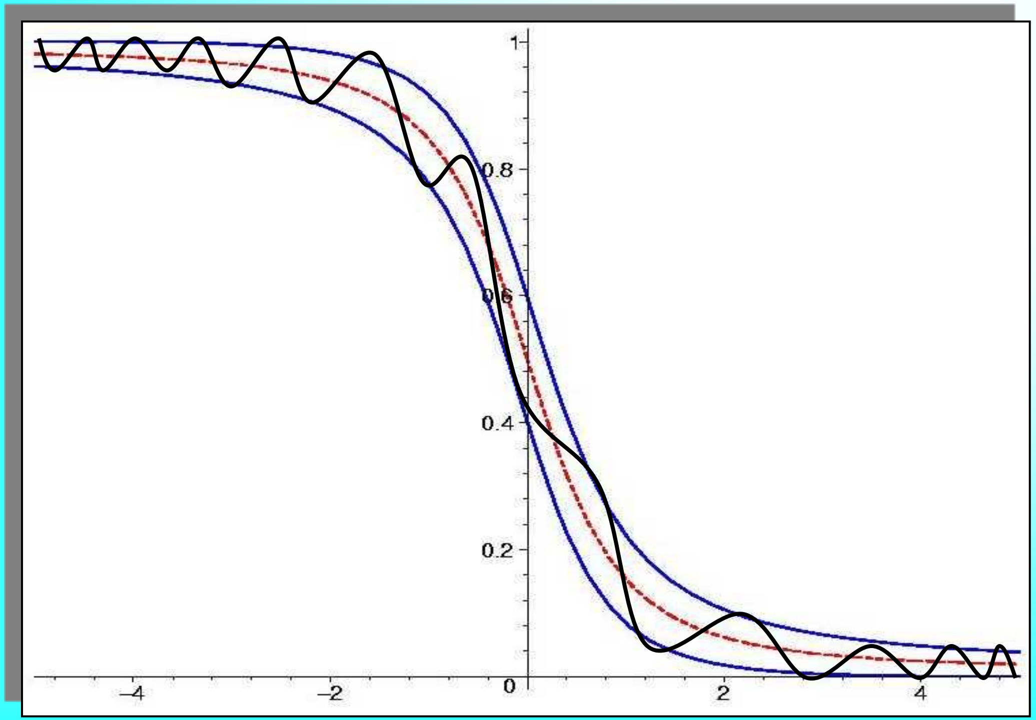
$P = |\langle \nu_e | \nu(f) \rangle|^2 = (\cos\theta \cos\theta_m^0)^2 + (\sin\theta \sin\theta_m^0)^2$

$= 0.5[\, 1 + \cos 2\theta_m^0 \cos 2\theta\,]$

$P = \sin^2\theta + \cos 2\theta \cos^2\theta_m^0$

Spatial picture

Adiabatic conversion

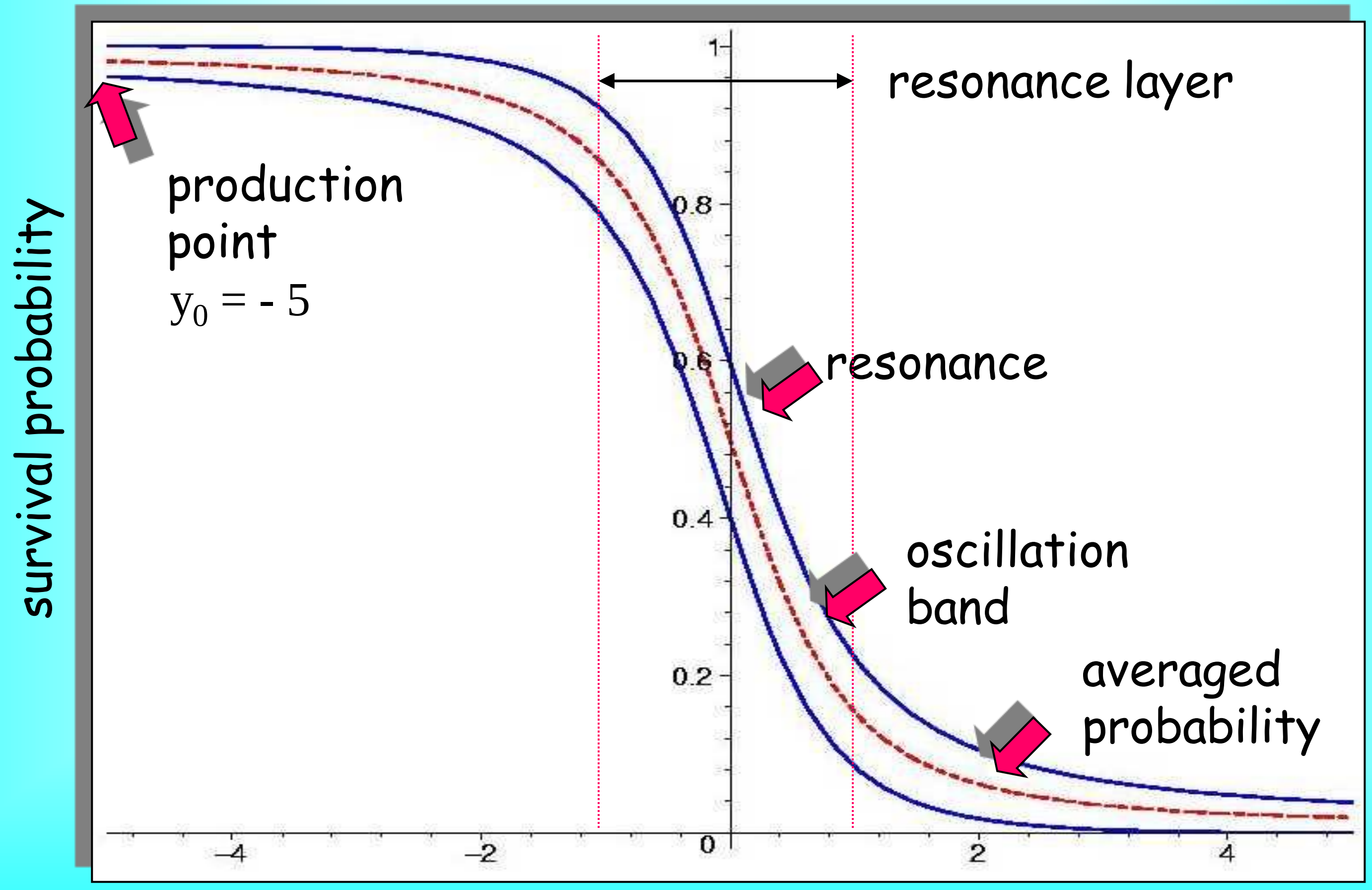


survival probability

distance

Spatial picture

The picture is universal in terms of variable $y = (n_R - n) / \Delta n_R$
no explicit dependence on oscillation parameters, density distribution, etc.
only initial value y_0 matters



$(n_R - n) / \Delta n_R$ (distance)

A Yu Smirnov

Adiabaticity violation

SN shock waves

If sterile neutrinos with small mixing exist

If density $n_e(t)$ changes fast

$$\left|\frac{d\theta_m}{dt}\right| \sim |H_{2m} - H_{1m}|$$

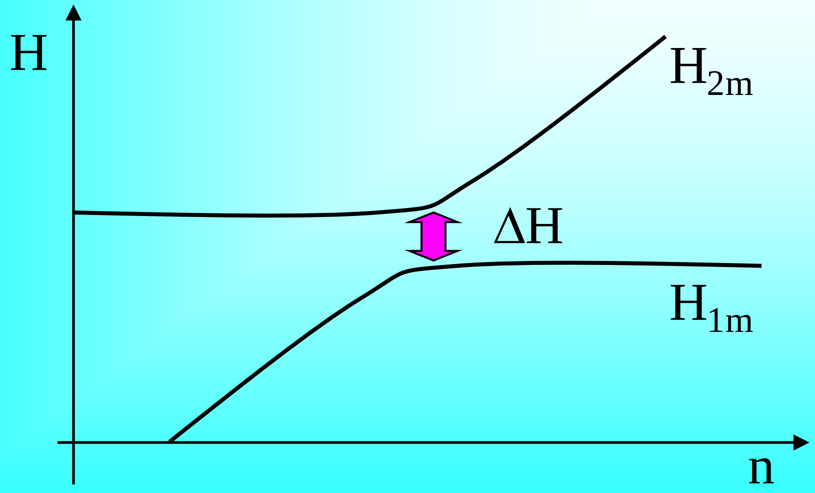
the off-diagonal terms in the Hamiltonian can not be neglected

transitions $\nu_{1m} \Leftrightarrow \nu_{2m}$

Admixtures of ν_{1m} ν_{2m} in a given neutrino state change

``Jump probability" penetration under barrier:

$$P_{12} = e^{-\frac{\Delta H}{E_n}}$$



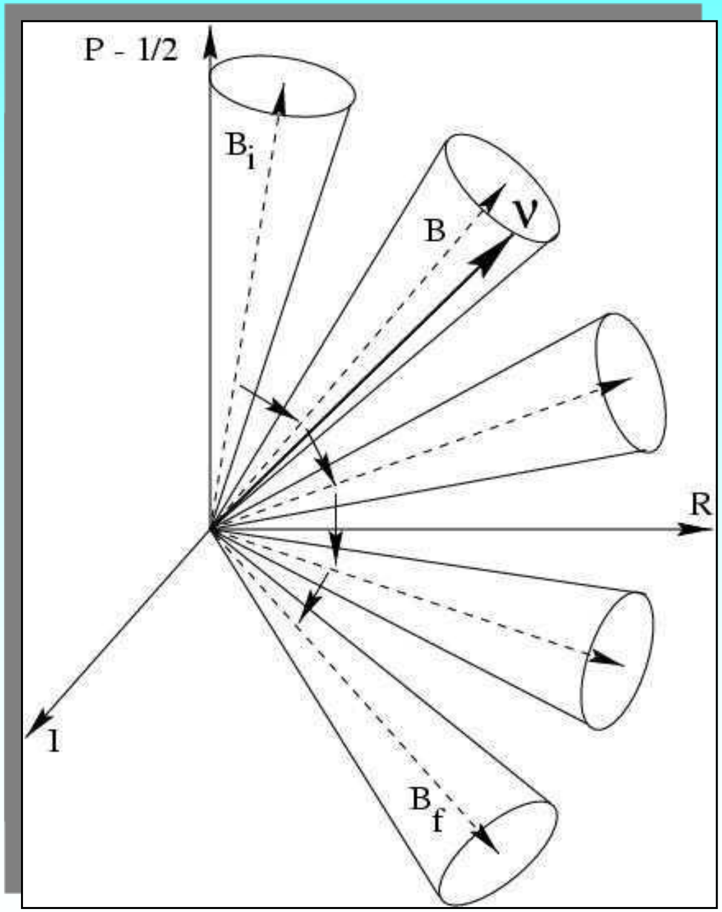
$E_n \sim 1/h_n$ is the energy associated to change of parameter (density)

$$P_{12} = e^{-\pi\kappa_R/2}$$

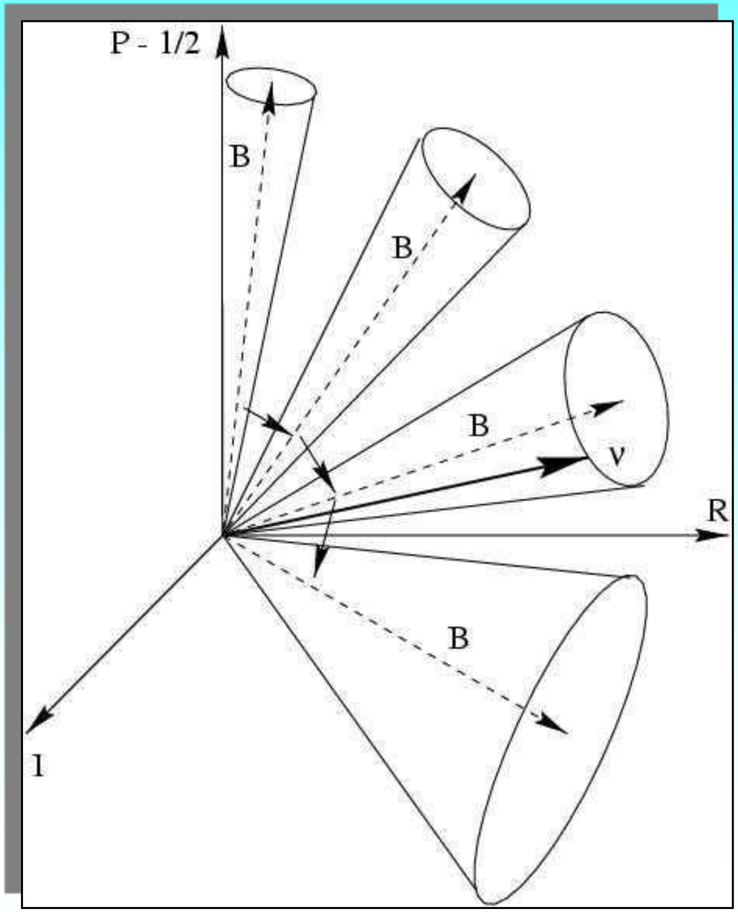
Landau-Zener

Adiabatic conversion

Pure adiabatic conversion



Partialy adiabatic conversion



Oscillations versus MSW

Different degrees of freedom

Oscillations

Vacuum or uniform medium with constant parameters

Phase difference increase between the eigenstates

ϕ

Mixing does not change

Adiabatic conversion

Non-uniform medium or/and medium with varying in time parameters

Change of mixing in medium = change of flavor of the eigenstates

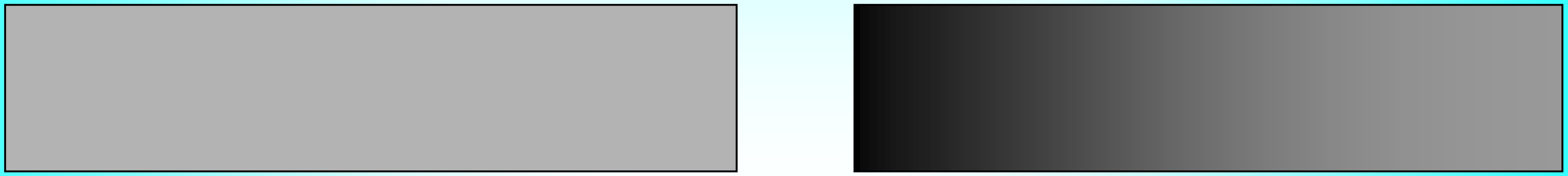
θ_m

Phase is irrelevant

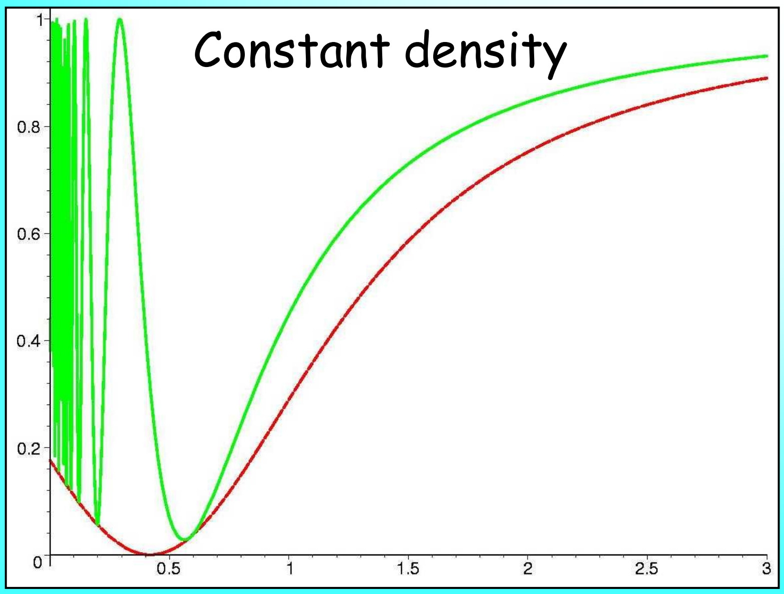
In non-uniform medium: interplay of both processes

Resonance oscillations vs. adiabatic conversion

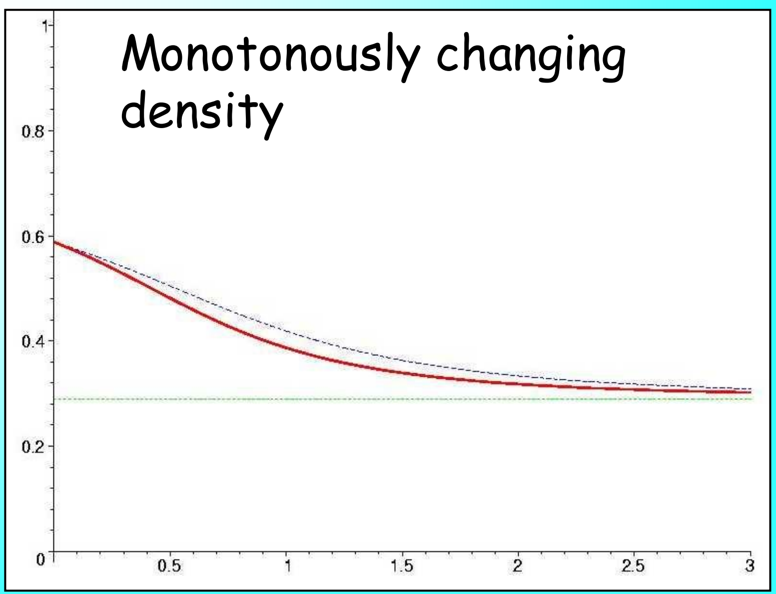
Passing through the matter filter ν



$\frac{F(E)}{F_0(E)}$



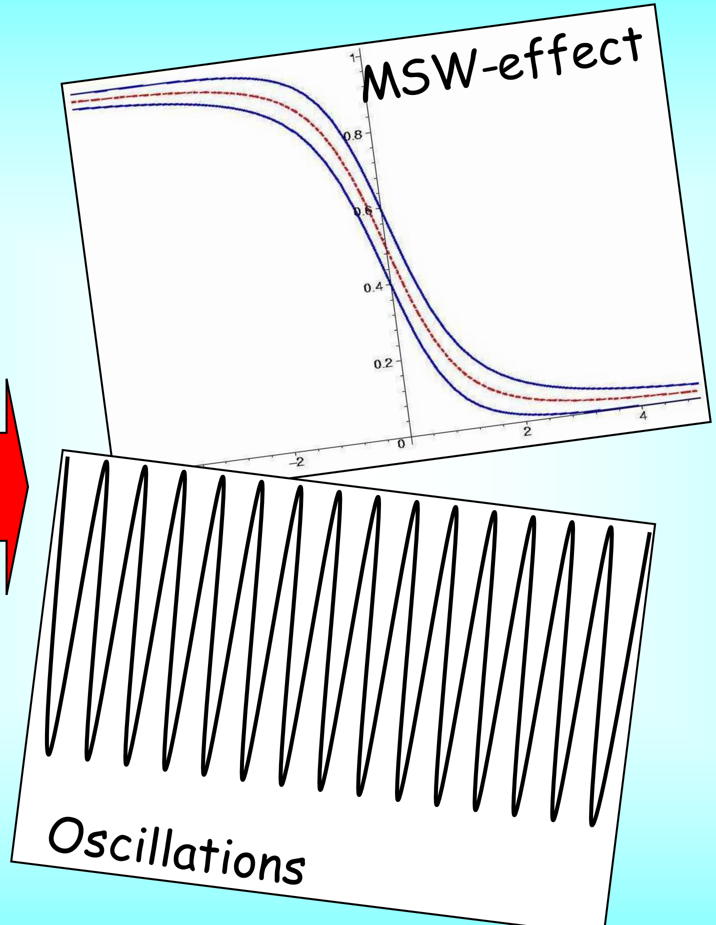
E/E_R



E/E_R

Two effects

Solar neutrinos
KamLAND
Atmospheric neutrinos
Double Chooz
Daya Bay
MINOS
K2K
RENO
Antares
T2K
DeepCore



Δm^2

θ

Can be resonantly enhanced in matter

Conclusion:

Adiabatic conversion is effect of change of mixing angle in matter in medium with slowly enough density change on the way of neutrino propagation

Conversion without oscillations

Resonance enhancement of oscillations occures in certain energy range in matter with constant density

nearly constant