# The Potential of Minimal Flavour Violation

Rodrigo Alonso

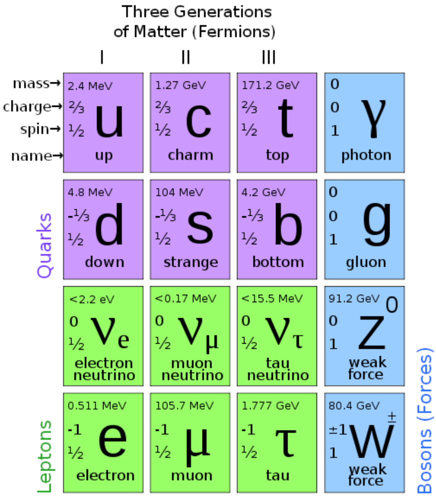
What is $\nu$?, GGI, Florence, Italy, 19/06/2012

Departamento de Física Teórica
Universidad Autónoma de Madrid
& IFT-UAM/CSIC

*based on the work with M. B. Gavela, L. Merlo , S. Rigolin & D. Hernández arXiv:1103.2915, arXiv:1206.3167*

# Outline

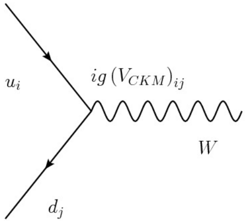
# The Flavour Puzzle




- Why 3 generations? CP violation?
- Visible part of the universe $\to$ $1^{st}$ generation

# Mixing

Generations connect with each other through mixing matrices




**Quarks**

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix}$$

**Leptons**

$$U_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ -0.4 & 0.6 & 0.7 \\ 0.4 & -0.6 & 0.7 \end{pmatrix}$$

Why is the mixing patern so different for leptons and quarks?

One can ask, optimistically:
will **B**eyond the **S**tandard **M**odel shed light on the flavour puzzle?

One can ask, optimistically:
will **B**eyond the **S**tandard **M**odel shed light on the flavour puzzle?

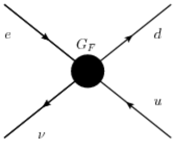
whereas in practice :
can **B**eyond the **S**tandard **M**odel accommodate flavour data and still be within reach?

A model independent way to treat new physics:
*Effective Field Theory*

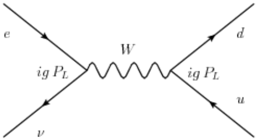
# Effective Field Theory

Fermi's Theory of beta decay

$$\mathcal{L}_{em} + G_F \bar{e}\gamma_\mu \nu \bar{u}\gamma^\mu d$$




$$\Downarrow\Downarrow\Downarrow$$

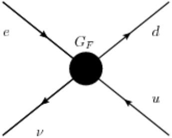



$$\mathcal{L}_{em} + \frac{g^2}{M_W^2} \bar{e}\gamma_\mu \nu_L \bar{u}\gamma^\mu d_L$$

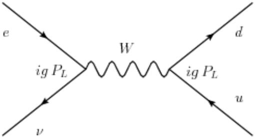
# Effective Field Theory

Fermi's Theory of beta decay

$$\mathcal{L}_{em} + G_F \bar{e}\gamma_\mu \nu \bar{u}\gamma^\mu d$$



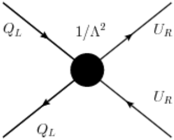

$$\Downarrow\Downarrow\Downarrow$$




$$\mathcal{L}_{em} + \frac{g^2}{M_W^2} \bar{e}\gamma_\mu \nu_L \bar{u}\gamma^\mu d_L$$

BSM physics can be parametrized in the same (Gauge Invariant ) way

$$\mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{O}^{d=5} + \frac{1}{\Lambda^2}\mathcal{O}^{d=6} + \dots$$




which is a valid description until we reach the scale $\Lambda$

$$\Downarrow$$

???

As an example the operator:

$$\mathcal{L} = \cdots + \frac{1}{\Lambda^2} \bar{Q}_L \mathbf{c} \gamma_\nu Q_L \bar{\ell}_L \gamma^\nu \ell_L + \cdots$$

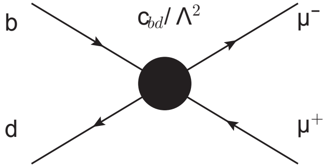
$$= \cdots + \frac{1}{\Lambda^2} \begin{pmatrix} \bar{d}_L \\ \bar{s}_L \\ \bar{b}_L \end{pmatrix}^T \begin{pmatrix} c_{dd} & c_{ds} & c_{db} \\ c_{ds}^* & c_{ss} & c_{sb} \\ c_{db}^* & c_{sb}^* & c_{bb} \end{pmatrix} \gamma^\nu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \bar{\mu} \gamma_\nu \mu + \cdots$$

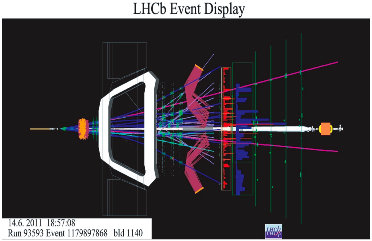
As an example the operator:

$$\mathcal{L} = \cdots + \frac{1}{\Lambda^2} \bar{Q}_L \mathbf{c} \gamma_\nu Q_L \bar{\ell}_L \gamma^\nu \ell_L + \cdots$$

$$= \cdots + \frac{1}{\Lambda^2} \begin{pmatrix} \bar{d}_L \\ \bar{s}_L \\ \bar{b}_L \end{pmatrix}^T \begin{pmatrix} c_{dd} & c_{ds} & c_{db} \\ c^*_{ds} & c_{ss} & c_{sb} \\ c^*_{db} & c^*_{sb} & c_{bb} \end{pmatrix} \gamma^\nu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \bar{\mu}\gamma_\nu \mu + \cdots$$

contributes to $B^0_s$ , $B^0 \to \mu^- \mu^+$;




Marconi; Planck 2012

LHCb : $BR(B^0 \to \mu^- \mu^+) < 10^{-9} \quad \Rightarrow$

$$\boxed{c_{\alpha\beta} \sim 1 \Rightarrow \Lambda \gtrsim 10^3 \text{TeV}}$$

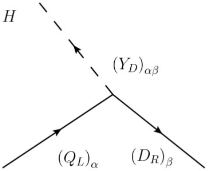
# Minimal Flavour Violation (MFV)

...new physics may have a definite flavour pattern, an option is:

Minimal Flavour Violation

The MFV hypothesis: *The Yukawa couplings are the **only** sources of flavour violation in and **beyond** the Standard Model*[1].

$$-\mathcal{L}_{Yukawa} = \overline{Q}_L Y_U U_R \tilde{H} + \overline{Q}_L Y_D D_R H + \overline{\ell}_L Y_E E_R H + h.c. + (\nu\,\text{mass})$$




$$Y_U = \text{Diag}(y_u, y_c, y_t),\ Y_D = V_{CKM}\text{Diag}(y_d, y_s, y_b),\ Y_E = \text{Diag}(y_e, y_\mu, y_\tau)$$

[1] Georgi & Chivukula 1987; D'Ambrosio, Giudice, Isidori, & Strumia, 2002; Cirigliano, Grinstein, Isidori & Wise 2005.

# Minimal Flavour Violation; Realization

- Generations are distinguished by masses; in the limit of zero mass ($\not{\mathcal{L}}_{Yuk}$) the SM presents an extended symmetry group :

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark} \times \overbrace{SU(3)_{\ell_L} \times SU(3)_{E_R}}^{Lepton} \times \cdots$$

$$D_R = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \qquad D_R \sim (1, 3, 1 \cdots)$$

# Minimal Flavour Violation; Realization

- Generations are distinguished by masses; in the limit of zero mass ($\not{\mathcal{L}}_{Yuk}$ ) the SM presents an extended symmetry group :

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark} \times \overbrace{SU(3)_{\ell_L} \times SU(3)_{E_R}}^{Lepton} \times \cdots$$

$$D_R = \left( \begin{array}{c} d_R \\ s_R \\ b_R \end{array} \right) \qquad\qquad D_R \sim (1, 3, 1 \cdots)$$

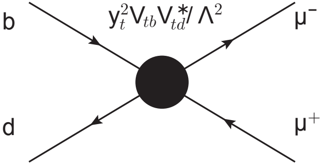
- The Yukawa couplings break the symmetry, unless

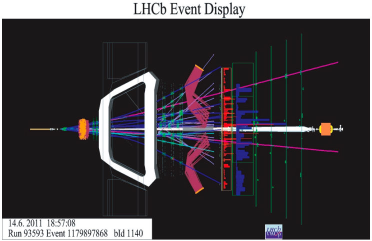
$$\overline{Q}_L Y_D D_R H \qquad\qquad Y_D \sim (3, \bar{3}, 1) \quad \text{'SPURIONS'}$$

## Formal Invariance under $\mathcal{G}_f$ :

$$\mathcal{L} = \cdots + \frac{1}{\Lambda^2}\,\bar{Q}_L\, Y_U Y_U^\dagger \gamma_\nu Q_L \bar{\ell}_L \gamma^\nu \ell_L \cdots$$

$$= \cdots + \frac{y_t^2}{\Lambda^2} \begin{pmatrix} \bar{d}_L \\ \bar{s}_L \\ \bar{b}_L \end{pmatrix} \begin{pmatrix} |V_{td}|^2 & V_{td}V_{ts}^* & V_{td}V_{tb}^* \\ V_{ts}V_{td}^* & |V_{ts}|^2 & V_{ts}V_{tb}^* \\ V_{tb}V_{td}^* & V_{tb}V_{ts}^* & |V_{tb}|^2 \end{pmatrix} \gamma^\nu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \bar{\mu}\gamma_\nu\mu_L$$

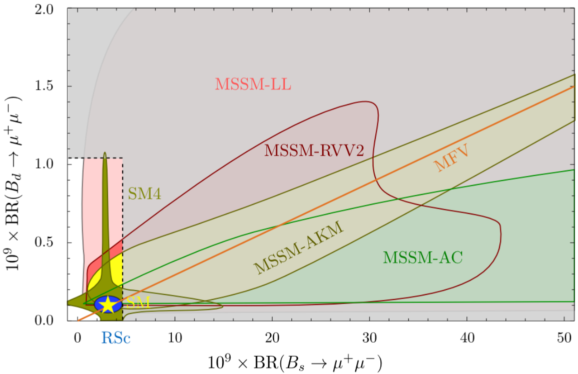



Marconi; Planck 2012

$$B_s^0\,,\; B_d^0 \to \mu^+\mu^- \qquad \Rightarrow \qquad \boxed{\Lambda \sim \text{TeV}}$$

# predictivity



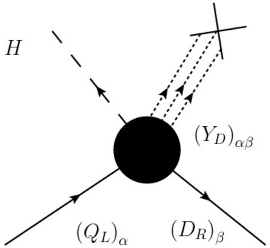

Straub, Moriond 2012

$$\frac{\Gamma(B^0 \to \mu^-\mu^+)}{\Gamma(B_s^0 \to \mu^-\mu^+)} \simeq \frac{V_{td}V_{td}^*}{V_{ts}V_{ts}^*}$$

can we go one step further in the direction of Minimal Flavour Violation?

# The Dynamics Behind MFV

Transformation properties suggest that the Yukawa couplings have a *dynamical origin* .




The Yukawa couplings arise with the v.e.v.s[2] of fields that transform for real under the flavour symmetry:

$$\boxed{Y = \langle\Sigma\rangle/\Lambda_f\,,\ \Sigma \sim (..3, \bar{3}, ..)}\,,\ \text{or } Y \sim \left\langle\Sigma^2\right\rangle/\Lambda_f^2,\ \text{or } Y \sim \left\langle\Sigma^{-1}\right\rangle/\Lambda_f^{-1}$$

[2] 1997 Anselm & Berezhiani, 2009 Feldmann, Jung & Mannel

# Quarks

# The Dynamics Behind MFV: Quarks

the flavour symmetry: $SU(3)_L \times SU(3)_{D_R} \times SU(3)_{U_R}$

Straightforward case : The Yukawas are the vev of 1 field $Y \sim \langle \Sigma \rangle$

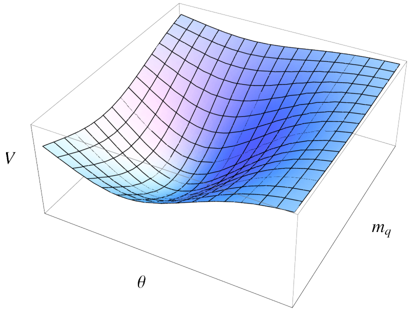
$$\Sigma_d \sim (3, \bar{3}, 1) \qquad \Sigma_u \sim (3, 1, \bar{3})$$

$$\boxed{\frac{\langle \Sigma_d \rangle}{\Lambda_f} = Y_D = V_{CKM} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}}, \quad \boxed{\frac{\langle \Sigma_u \rangle}{\Lambda_f} = Y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}}.$$

... but these vevs are acquired somehow...

# The Potential

the Potential $V(\Sigma_u, \Sigma_d)$
is the culprit of fixing $\langle \Sigma_{u,d} \rangle \Leftrightarrow V_{CKM}$ & $m_q$




unfortunately no mixing can come out of this potential
more on this later...

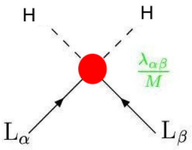
Gavela, Merlo, Rigolin, R.A. 2011

# Leptons

# What is $\nu$?

$\nu$'s are special:
**The** leading operator in our EFT approach

$$\mathcal{L}^{d=5} = \frac{\lambda}{\Lambda}\bar{\ell}_L \tilde{H}\, \tilde{H}^T \ell_L^c$$

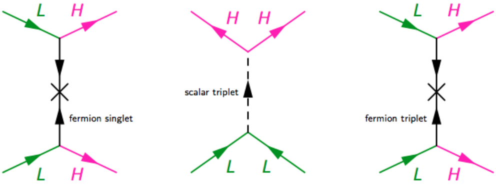



$$\mathcal{O}^{d=5} \quad \text{EWSB} \quad \rightarrow \quad \bar{\nu} m_\nu \nu^c$$

MAJORANA PARTICLES

# Looking inside the Weinberg Operator

- Three types of models yield the Weinberg operator at tree level




Type I Type II Type III

## Type I

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L \, , \, \bar{N}^c) \begin{pmatrix} 0 & \tilde{H} Y_\nu \\ (\tilde{H} Y_\nu)^T & \Lambda \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \end{pmatrix}$$

# The Dynamics Behind MFV: Leptons

We'll pick a predictive Inverse Seesaw Model: s

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & \tilde{H}Y & \tilde{H}Y' \\ (\tilde{H}Y)^T & 0 & \Lambda \\ (\tilde{H}Y')^T & \Lambda & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

$$|Y'| \ll |Y| \;\Rightarrow \text{approx LN} \qquad \Lambda_{LN} = \Lambda/\,|Y'|\,, \quad \Lambda_{fl} = \Lambda\,.$$

# The Dynamics Behind MFV: Leptons

We'll pick a predictive Inverse Seesaw Model: s

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & \tilde{H}Y & \tilde{H}Y' \\ (\tilde{H}Y)^T & 0 & \Lambda \\ (\tilde{H}Y')^T & \Lambda & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

$$|Y'| \ll |Y| \;\Rightarrow \text{approx LN} \qquad \Lambda_{LN} = \Lambda/\,|Y'|\,, \quad \Lambda_{fl} = \Lambda\,.$$

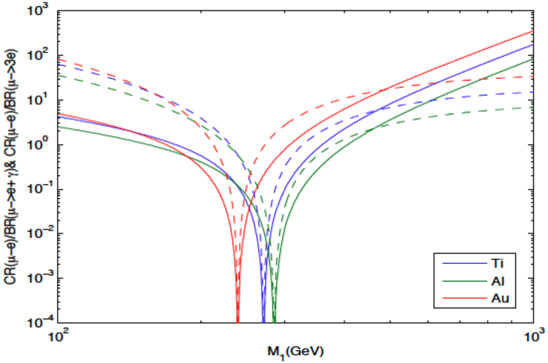
the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}}\, U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

Gavela, Hambye, P. Hernández, D. Hernández
Raidal, Strumia, Turzynski

# LMFV

"predictivity"




Dihn, Ibarra, Molinaro, Petcov, 2012

# LMFV

$$\text{when } Y, Y' \to 0 \;\Rightarrow \mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Lambda \\ 0 & \Lambda & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the flavour symmetry is:

$$SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N \ .$$

# LMFV

$$\text{when } Y, Y' \to 0 \;\Rightarrow \mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Lambda \\ 0 & \Lambda & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the flavour symmetry is:

$$SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N \;.$$

Fully restored at high energies with the introduction of the scalar fields:

$$Y_E = \frac{\langle \Sigma_E \rangle}{\Lambda_f} \sim (3\,,\,\bar{3}\,,1)\,; \quad (Y\,,\,Y') = \frac{\langle \chi \rangle}{\Lambda} \sim (3\,,\,1\,,2)\,. \tag{1}$$

whose vev's are

$$\langle \Sigma_E \rangle \propto \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}\,, \quad \langle \chi \rangle \propto U_{PMNS} \begin{pmatrix} 0 & 0 \\ \sqrt{m_{\nu_2}} & 0 \\ 0 & \sqrt{m_{\nu_3}} \end{pmatrix} \begin{pmatrix} -iy & iy' \\ y & y' \end{pmatrix}$$

what is the potential for the mixing parameters now?

# Invariant terms of the Potential: Mixing

Let's focus on the mixing parameters, the only invariant that depends on them is, at renormalizable level $(|Y'| << |Y|)$ :

$$\mathrm{Tr}\left(\Sigma_E \Sigma_E^\dagger \chi \chi^\dagger\right) \propto \Big\{ \sum_{l,i} |U_{PMNS}^{li}|^2 m_l^2 m_{\nu_i} + \Big[ i\, e^{2i\alpha} \sum_{l,i<j} U_{PMNS}^{li} (U_{PMNS}^{lj})^* m_l^2 \sqrt{m_{\nu_i} m_{\nu_j}} + c.c. \Big] \Big\} .$$

whereas for quarks:

$$\mathrm{Tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger\right) \propto \sum_{i,j} |U_{CKM}^{ij}|^2 m_{u_i}^2 m_{d_j}^2 ,$$

# Minimum of the Potential: 2 Generations

## Leptons

The invariant containing the angle (2 family case & $(|Y'| << |Y|)$):

$$\mathrm{Tr}\left(\Sigma_E \Sigma_E^\dagger \chi \chi^\dagger\right) \propto (m_\mu^2 - m_e^2)\left((m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\alpha \sin 2\theta\right) ,$$

Renormalizable level: $\partial_\theta V = 0$ yields:

$$\boxed{\mathrm{tg} 2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}}, \qquad \sin 2\theta \cos 2\alpha = 0 , \qquad \boxed{\alpha = \pm\pi/4}$$

# Minimum of the Potential: 2 Generations

## Leptons

The invariant containing the angle (2 family case & $(|Y'| << |Y|)$):

$$\mathrm{Tr}\left(\Sigma_E \Sigma_E^\dagger \chi \chi^\dagger\right) \propto (m_\mu^2 - m_e^2)\left((m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\alpha \sin 2\theta\right) ,$$

Renormalizable level: $\partial_\theta V = 0$ yields:

$$\boxed{\mathrm{tg} 2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}}, \qquad \sin 2\theta \cos 2\alpha = 0 , \quad \boxed{\alpha = \pm\pi/4}$$

## Quarks

Let's bring back the quark invariant (Bifundamental):

$$\mathrm{Tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger\right) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

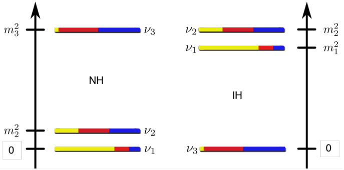
which yields

$$\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2) \sin 2\theta = 0}.$$

# 3 Family Case

For the three family case:

- Only one angle can be set different from 0 as $m_{\nu_1} = 0$ imposes strong hierarchies with $m_{\nu_2}, m_{\nu_3}$. This is a peculiarity of the model.




- Such angle lies in the experimentally allowed region for an inverse Yukawa relation: $Y^{-1} \sim \Sigma$.
  - ! Suggesting: this happens in gauged flavour symmetry models [3] which also solve the problem of goldstones...

[3] Berezhiani, Khlopov 1990, Grinstein, Redi, Villadoro 2011, Feldmann 2011

# Summary

1. The distinctive Majorana character of neutrinos within a Seesaw Model makes the potential of MFV different from that of quarks.
2. This difference allows for *maximal angles in the limit of degenerate neutrino masses* (but Majorana phase $\neq 0$ ! ).
3. The realistic 3-family scenario points towards an inverse relation of Yukawas and scalar fields and degenerate neutrino mass spectrum.

# Summary

1. The distinctive Majorana character of neutrinos within a Seesaw Model makes the potential of MFV different from that of quarks.
2. This difference allows for *maximal angles in the limit of degenerate neutrino masses* (but Majorana phase $\neq 0$ ! ).
3. The realistic 3-family scenario points towards an inverse relation of Yukawas and scalar fields and degenerate neutrino mass spectrum.

Grazie