Lepton Mixing and Lepton Flavor Violation in Holographic Composite Higgs Models

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Outline

- Experimental status of lepton mixing
- Idea of non-trivially broken flavor group $G_f$
- Realization in composite Higgs models
- Explicit 5D model for Majorana neutrinos
- Comments about Dirac neutrinos
- Conclusions
Lepton Mixing

Parametrization of mixing matrix $U_{PMNS}$

$$U_{PMNS} = V(\theta_{ij}, \delta_{CP}) \cdot \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$V(\theta_{ij}, \delta_{CP}) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

with $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$
Special Lepton Mixing Patterns?

• $\mu\tau$ Symmetry

$$\sin^2 \theta_{12} \text{ free}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0$$

• Tri-Bimaximal mixing (TB mixing)

$$\sin^2 \theta_{12} = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0$$

• Golden Ratio \hspace{1em} (\phi = \frac{1}{2} (1 + \sqrt{5}))

$$\sin^2 \theta_{12} = \frac{1}{\sqrt{5\phi}} \approx 0.276, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0$$
Global Fits

• Fogli et al. (May ’12) at $3\sigma$ level

\[
\sin^2 \theta_{12} = 0.307^{+0.052}_{-0.048}, \quad \sin^2 \theta_{23} = 0.4(1)^{+0.24(5)}_{-0.07}, \quad \sin^2 \theta_{13} = 0.025^{+0.010}_{-0.010}
\]

• Forero et al. (May ’12) at $3\sigma$ level

normal hierarchy

\[
\sin^2 \theta_{12} = 0.32^{+0.05}_{-0.05}, \quad \sin^2 \theta_{23} = 0.49^{+0.15}_{-0.10}, \quad \sin^2 \theta_{13} = 0.026^{+0.010}_{-0.011}
\]

inverted hierarchy

\[
\sin^2 \theta_{12} = 0.32^{+0.05}_{-0.05}, \quad \sin^2 \theta_{23} = 0.53^{+0.11}_{-0.14}, \quad \sin^2 \theta_{13} = 0.027^{+0.010}_{-0.011}
\]
Status of Flavor Symmetries

... you could ask

• whether they are still interesting for lepton mixing
... you could ask

- whether they are still interesting for lepton mixing

My reply is

- there are "new" patterns with $\theta_{13} \neq 0$, e.g. from $\Delta(96)$ and $\Delta(384)$

\[
\begin{align*}
\sin^2 \theta_{12} &\approx 0.349, \\
\sin^2 \theta_{23} &\approx 0.349 (0.651), \\
\sin^2 \theta_{13} &\approx 0.045
\end{align*}
\]

\[
\begin{align*}
\sin^2 \theta_{12} &\approx 0.337, \\
\sin^2 \theta_{23} &\approx 0.424 (0.576), \\
\sin^2 \theta_{13} &\approx 0.011
\end{align*}
\]

- and of course all "old" patterns are usually just leading order results which are corrected in concrete models
Status of Flavor Symmetries

... you could ask

- whether they are still interesting for lepton mixing
- whether they are interesting at all
Status of Flavor Symmetries

... you could ask

• whether they are still interesting for lepton mixing

• whether they are interesting at all

My reply is

• flavor symmetries are not only useful for mixing, because in all extensions of the SM new particles and new interactions are present which induce processes strongly constrained by experiments, e.g.

\[ \mu \rightarrow e\gamma \text{ with } BR(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12} \]
Choice of Flavor Symmetry $G_f$

In the following I assume $G_f$ to be

- ... non-abelian,
- ... discrete,
- ... finite,
- ... global,
- ... explicitly or spontaneously broken to particular subgroups,
- ... of the form $X \times Z_N \ (N \geq 3)$ with $X$ non-abelian.
Lepton Mixing from Non-Trivial $G_f$ Breaking

Idea:
Derivation of the lepton mixing from how $G_f$ is broken
Interpretation as mismatch of embedding of different sub-groups $G_\nu$ and $G_e$ into $G_f$
**Lepton Mixing from Non-Trivial $G_f$ Breaking**

**Idea:**
Derivation of the lepton mixing from how $G_f$ is broken
Interpretation as mismatch of embedding of different sub-groups $G_\nu$ and $G_e$ into $G_f$

$G_f$

\[ \rightarrow \quad \rightarrow \]

- Neutrinos
  - Assume 3 generations
  - Of Majorana neutrinos
- Charged leptons
  - Distinguish 3 generations
Lepton Mixing from Non-Trivial $G_f$ Breaking

Idea:

Derivation of the lepton mixing from how $G_f$ is broken

Interpretation as mismatch of embedding of different sub-groups $G_\nu$ and $G_e$ into $G_f$

\[
G_f
\quad \downarrow \quad \downarrow
\]

neutrinos \hspace{1cm} charged leptons

\[
G_\nu = Z_2 \times Z_2 \hspace{1cm} G_e = Z_N \text{ with } N \geq 3
\]
Lepton Mixing from Non-Trivial $G_f$ Breaking

$G_f$

\[ G_\nu = Z_2 \times Z_2 \]

\[ G_e = Z_N \text{ with } N \geq 3 \]

Further requirements

- Two/three non-trivial angles $\Rightarrow$ irred 3-dim rep $\rho \sim 3$ of $G_f$
- Fix angles through $G_\nu, G_e \Rightarrow$ 3 generations transform differently under $G_\nu, G_e$
Lepton Mixing from Non-Trivial $G_f$ Breaking

• Neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix $m_\nu$ fulfills

  $\rho(g_{\nu,i})^T m_\nu \rho(g_{\nu,i}) = m_\nu$ with $i = 1, 2$

• Charged lepton sector: $Z_N$, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix $m_e$ fulfills

  $\rho(g_e)^\dagger m_e^\dagger m_e \rho(g_e) = m_e^\dagger m_e$
Lepton Mixing from Non-Trivial $G_f$ Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved and generated by
  \[
  \rho(g_{\nu,i}) = \Omega_\nu \rho(g_{\nu,i})^{diag} \Omega_\nu^\dagger \quad \text{with} \quad i = 1, 2
  \]
  \[
  \rho(g_{\nu,i})^{diag} = \text{diag}(\pm 1, \pm 1, \pm 1) \quad \text{and} \quad \Omega_\nu \quad \text{unitary}
  \]

- Charged lepton sector: $Z_N, N \geq 3$, preserved
  \[
  \rightarrow \text{charged lepton mass matrix } m_e \text{ fulfills}
  \]
  \[
  \rho(g_e)^\dagger m_e^\dagger m_e \rho(g_e) = m_e^\dagger m_e
  \]
Lepton Mixing from Non-Trivial $G_f$ Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix $m_\nu$ fulfills

  \[
  \rho(g_{\nu,i})^{\text{diag}} [\Omega_\nu^T m_\nu \Omega_\nu] \rho(g_{\nu,i})^{\text{diag}} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{with} \quad i = 1, 2
  \]

- Charged lepton sector: $Z_N$, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix $m_e$ fulfills

  \[
  \rho(g_e)^\dagger m_e^\dagger m_e \rho(g_e) = m_e^\dagger m_e
  \]
Lepton Mixing from Non-Trivial $G_f$ Breaking

• Neutrino sector: $Z_2 \times Z_2$ preserved
  \[ \rightarrow \text{neutrino mass matrix } m_\nu \text{ fulfills} \]
  \[ \Omega_T^T m_\nu \Omega_\nu \text{ is diagonal} \]

• Charged lepton sector: $Z_N$, $N \geq 3$, preserved
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Lepton Mixing from Non-Trivial $G_f$ Breaking

• Neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix $m_\nu$ fulfills

  $\Omega^T_\nu m_\nu \Omega_\nu$ is diagonal

• Charged lepton sector: $Z_N, N \geq 3$, preserved and generated by

  $\rho(g_e) = \Omega_e \rho(g_e)^{diag} \Omega_e^\dagger$ with $\Omega_e$ unitary

  $\rho(g_e)^{diag} = \text{diag} (\omega_N^{n_e}, \omega_N^{n_\mu}, \omega_N^{n_\tau})$

  and $n_e \neq n_\mu \neq n_\tau$ and $\omega_N = e^{2\pi i/N}$
Lepton Mixing from Non-Trivial $G_f$ Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved
  \[ \rightarrow \text{neutrino mass matrix } m_\nu \text{ fulfills } \]
  \[ \Omega^T_\nu m_\nu \Omega_\nu \text{ is diagonal} \]

- Charged lepton sector: $Z_N$, $N \geq 3$, preserved
  \[ \rightarrow \text{charged lepton mass matrix } m_e \text{ fulfills } \]
  \[ (\rho(g_e)^{diag})^* \left[ \Omega^\dagger_e m^\dagger_e m_e \Omega_e \right] \rho(g_e)^{diag} = \left[ \Omega^\dagger_e m^\dagger_e m_e \Omega_e \right] \]
Lepton Mixing from Non-Trivial $G_f$ Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved
  \[ \rightarrow \text{neutrino mass matrix } m_\nu \text{ fulfills} \]
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- Charged lepton sector: $Z_N$, $N \geq 3$, preserved
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  \[ \Omega^\dagger_e m^\dagger_e m_e \Omega_e \text{ is diagonal} \]
Lepton Mixing from Non-Trivial $G_f$ Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved
  \[ \Omega_{\nu}^T m_{\nu} \Omega_{\nu} \text{ is diagonal} \]

- Charged lepton sector: $Z_N$, $N \geq 3$, preserved
  \[ \Omega_{e}^\dagger m_{e}^\dagger m_{e} \Omega_{e} \text{ is diagonal} \]

- Conclusion: $\Omega_{\nu,e}$ diagonalize $m_{\nu}$ and $m_{e}^\dagger m_{e}$
  \[ U_{PMNS} = \Omega_{e}^\dagger \Omega_{\nu} \]
Lepton Mixing from Non-Trivial $G_f$ Breaking

$$U_{PMNS} = \Omega^\dagger_e \Omega_\nu$$

- 3 unphysical phases are removed by $\Omega_e \rightarrow \Omega_e K_e$
- Neutrino masses are made real and positive through $\Omega_\nu \rightarrow \Omega_\nu K_\nu$
- Permutations of columns of $\Omega_e, \Omega_\nu$ are possible: $\Omega_{e,\nu} \rightarrow \Omega_{e,\nu} P_{e,\nu}$

\[\downarrow\]

Predictions:
- Mixing angles up to exchange of rows/columns
- Dirac CP phase $\delta_{CP}$ up to $\pi$
- Majorana phases undetermined
Lepton Mixing from Non-Trivial $G_f$ Breaking

Basis used in the following:

- $\rho(g_e)$ is diagonal and thus $\Omega_e$ a permutation
- $\rho(g_{\nu,i})$ are non-diagonal and $\Omega_{\nu}$ is called $V$
- Choose $\rho(g_{\nu,i})^{diag}$ to be

$$
\rho(g_{\nu,1})^{diag} = \begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{pmatrix}, \quad
\rho(g_{\nu,2})^{diag} = \begin{pmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{pmatrix}
$$
Idea of Composite Higgs Models

- Assume strong sector with composite Higgs in order to resolve hierarchy problem
- Higgs VEV breaks electroweak gauge group
- Higgs might be a pseudo Goldstone boson, then it is naturally light (realization in 5D: gauge-Higgs unification)
- SM fermions are part of the elementary sector
- SM fermions get mass upon mixing with fermionic resonances $\Psi$ of composite sector (idea of partial compositeness)

Generic form of Lagrangian:

$$\mathcal{L}_{tot} = \mathcal{L}_{el} + \mathcal{L}_{comp} + \mathcal{L}_{mix}$$
Composite Higgs Models

- Lagrangian:
  \[ \mathcal{L}_{tot} = \mathcal{L}_{el} + \mathcal{L}_{comp} + \mathcal{L}_{mix} \]

- Assume
  - \( \mathcal{L}_{mix} \) invariant under \( G_f \)
  - \( \mathcal{L}_{el} \) invariant under \( G_\nu \)
  - \( \mathcal{L}_{comp} \) invariant under \( G_e \)

- Degrees of freedom in elementary sector
  - 3 LH lepton doublets \( l^\alpha_L \)
  - 3 RH charged leptons \( l^\alpha_R \)
  - 3 RH neutrinos \( \nu^\alpha_R \)
Composite Higgs Models

\( l^\alpha_L \sim 3 \) and \( \nu^\alpha_R \sim 3 \) under non-abelian group \( X \subset G_f \)

\[ \rightarrow \mathcal{L}_{el} \text{ contains Majorana mass term fixed by } G_\nu = Z_2 \times Z_2 \subset X \subset G_f \]

\[ -\frac{1}{2} \left( \bar{\nu}^c_\alpha M_{\alpha\beta} \nu^\beta_R + h.c. \right) \quad \text{with } M = V^* M_D V^\dagger, \quad M_D \text{ diagonal} \]

Charged lepton mass hierarchy through partial compositeness

\[ \rightarrow l^\alpha_R \sim 1 \text{ under } X \]

Distinct RH charged leptons through additional \( Z_N \) symmetry

\[ \rightarrow G_f = X \times Z_N \text{ and } l^\alpha_R \sim (1, \omega^m_N) \]
Composite Higgs Models

Thus take

- 3 LH lepton doublets $l^\alpha_L \sim (3, 1)$ under $(X, Z_N)$
- 3 RH charged leptons $l^\alpha_R \sim (1, \omega^\alpha N)$ under $(X, Z_N)$
- 3 RH neutrinos $\nu^\alpha_R \sim (3, 1)$ under $(X, Z_N)$

Assume linear coupling of elementary fields to fermionic resonances $\Psi$ of composite sector

$$
\mathcal{L}_{mix} = \frac{\lambda_{l_L}}{\Lambda \gamma_{l_L}} \bar{l}_L \gamma l_L, R \Psi^\alpha_{l,L,R} + \frac{\lambda_{l_R}}{\Lambda \gamma_{l_R}} \bar{l}_R \gamma l_R, L \Psi^\alpha_{l,L,R} + \frac{\lambda_{\nu_R}}{\Lambda \gamma_{\nu_R}} \bar{\nu}_R \gamma \nu_R, L \Psi^\alpha_{\nu,R,L} + h.c.
$$

implies that $\Psi^\alpha_{l_L} \sim l^\alpha_L$, $\Psi^\alpha_{l_R} \sim l^\alpha_R$, $\Psi^\alpha_{\nu_R} \sim \nu^\alpha_R$ under SM and $G_f$
Composite Higgs Models

• We expect upon integrating out the resonances a charged lepton mass matrix of the form

\[ M_{l,\alpha\beta} \sim \frac{\lambda_{l_L}}{\Lambda \gamma_{lL}} \frac{\lambda_{l_R}^\beta}{\Lambda \gamma_{lR}} \langle \bar{\Psi}_{l_R}^\beta \Psi_{l_L}^\alpha \rangle \]

• For no \( G_f \) breaking the condensate has to vanish

• For \( G_f \) broken to \( G_e = Z_N^{(D)} \) instead we find

\[ \langle \bar{\Psi}_{l_R}^\beta \Psi_{l_L}^\alpha \rangle \sim b_\alpha \delta_{\alpha\beta} v_H \mu^{\gamma_{l_R}^\alpha + \gamma_{lL}^\alpha} , \]

since \( l_L^\alpha, \Psi_{l_L}^\alpha, l_R^\alpha, \Psi_{l_R}^\alpha, \nu_R^\alpha, \Psi_{\nu_R}^\alpha \sim \omega_N^{n_\alpha} \) under \( Z_N^{(D)} \)

which is the diagonal subgroup of \( Z_N \subset X \) and the external \( Z_N \)
Composite Higgs Models: Mass Matrices

- Thus we generate a charged lepton mass matrix of the form

$$M_{l, \alpha\beta} \sim b_\alpha v_H \lambda_L \lambda_R^\alpha \delta_{\alpha\beta} \left( \frac{\mu}{\Lambda} \right)^{\gamma_{iR} + \gamma_{iL}}$$

- At the same time the Dirac neutrino mass matrix takes the form

$$M^{D}_\nu, \alpha\beta \sim \frac{\hat{b}_\alpha v_H \lambda_L}{\tilde{b}_\alpha} \delta_{\alpha\beta} \left( \frac{\mu}{\Lambda} \right)^{\gamma_{iL}}$$

- Then the light neutrino mass matrix is

$$M_{\nu, \alpha\beta} \sim \hat{b}_\alpha \hat{b}_\beta v^2_H \lambda^2_L \lambda^2_{\nu_R} \left( \frac{\mu}{\Lambda} \right)^{2(\gamma_{\nu_R} + \gamma_{iL})} \left( VM_D^{-1} V^T \right)_{\alpha\beta}$$

- Result for lepton mixing, if $\hat{b}$ are universal: $U_{PMNS} = V$
Explicit Warped GHU Models: Setup

<table>
<thead>
<tr>
<th>SU(2) x U(1)Y</th>
<th>SO(5) x U(1)X</th>
<th>SO(4) x U(1)X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z2 x Z2 x ZN</td>
<td>X x ZN</td>
<td>ZN(D)</td>
</tr>
<tr>
<td>GaUV</td>
<td>Ga</td>
<td>GalR</td>
</tr>
</tbody>
</table>

UV | Bulk | IR
Explicit Warped GHU Models: Symmetries

• Additional symmetries $G_a, G_{a,UV}$ and $G_{a,IR}$

$$G_a = Z'_3 \times Z''_3, \quad G_{a,UV} = Z''_3, \quad G_{a,IR} = Z'_3$$

for minimizing terms in bulk, on UV and IR brane

• Examples of symmetries $X \times Z_N$

$$(X, Z_N) = (S_4, Z_3) \quad \rightarrow \quad \text{TB mixing}$$

$$(X, Z_N) = (A_5, Z_5) \quad \rightarrow \quad \text{Golden Ratio}$$

$$(X, Z_N) = (\Delta(96), Z_3) \quad \rightarrow \quad \text{Mixing with } \sin^2 \theta_{13} \approx 0.045$$

$$(X, Z_N) = (\Delta(384), Z_3) \quad \rightarrow \quad \text{Mixing with } \sin^2 \theta_{13} \approx 0.011$$
Explicit Warped GHU Models: Particles

Particle content \((X = 0)\)

\[
\xi_{l,\alpha} \sim (5, 3, 1, \omega_3, \omega_3), \quad \xi_{e,\alpha} \sim (10, 1, \omega_N^{\alpha}, \omega_3, \omega_3), \quad \xi_{\nu,\alpha} \sim (1, 3, 1, \omega_3, 1)
\]

with

\[
\xi_{l,\alpha} = \left( \begin{array}{c}
\tilde{L}_{1,\alpha L} (-+), L_{\alpha L} (++)
\end{array} \right), \quad \xi_{e,\alpha} = \left( \begin{array}{c}
x_{\alpha L} (+-)
\tilde{\nu}_{\alpha L} (+-) \quad Z_{\alpha L} (+-)
n_{\alpha L} (--)
\end{array} \right)
\]

\[
\xi_{\nu,\alpha} = \nu_{\alpha L} (--)
\]

\[
\left[ \tilde{L}_{2,\alpha L} (+-), \hat{L}_{\alpha L} (+-) \right]
\]
Explicit Warped GHU Models: Particles

<table>
<thead>
<tr>
<th></th>
<th>Bulk</th>
<th>UV</th>
<th>IR</th>
</tr>
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<tbody>
<tr>
<td>$G_f \times G_a$</td>
<td>$G_{f,UV} \times G_{a,UV}$</td>
<td>$G_{f,IR} \times G_{a,IR}$</td>
<td></td>
</tr>
<tr>
<td>$\xi_{l,\alpha}$</td>
<td>$(3, 1, \omega_3, \omega_3)$</td>
<td>$(1, -1, 1, \omega_3)$</td>
<td>$(\omega_{N}^{n_{\alpha}}, \omega_3)$</td>
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</table>
Explicit Warped GHU Models: Parameters

• Bulk: Mass parameters $c_l$, $c_\alpha$ ($\alpha = 1, 2, 3$) and $c_\nu$

• Lagrangian on IR brane

\[
\left(\frac{R}{R'}\right)^4 \sum_{\alpha=e,\mu,\tau} \left( m_{l,IR,\alpha} \left( \overline{L}_{1,\alpha} L_{2,\alpha R} + \overline{L}_{\alpha L} \hat{L}_{\alpha R} \right) + m_{\nu,IR,\alpha} \overline{\nu}_L \nu_{\alpha R} + h.c. \right)
\]

which contains as independent complex parameters $m_{l,IR,\alpha}$ and $m_{\nu,IR,\alpha} = m_{\nu,IR,0}(1 + \delta_\alpha)$

• Lagrangian on UV brane

\[
\frac{1}{2} \overline{\nu}_{\alpha R}^c M_{UV,\alpha \beta} \nu_{\beta R} + h.c.
\]

with $M_{UV} = V^* m_{UV} V^\dagger$ and $m_{UV,\alpha}$ are free complex parameters
Explicit Warped GHU Models: Lepton Mixing

\[ X = S_4 \quad V = U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \]

\[ X = A_5 \quad V = U_{GR} = \begin{pmatrix} \cos \theta_{12}^{GR} & -\sin \theta_{12}^{GR} & 0 \\ \frac{\sin \theta_{12}^{GR}}{\sqrt{2}} & \frac{\cos \theta_{12}^{GR}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin \theta_{12}^{GR}}{\sqrt{2}} & \frac{\cos \theta_{12}^{GR}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \]

with \[ \tan \theta_{12}^{GR} = 1/\phi \]
Explicit Warped GHU Models: Lepton Mixing

\[ X = \Delta(96) : \quad V = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{1}{2}(\sqrt{3} + 1) & 1 & \frac{1}{2}(\sqrt{3} - 1) \\ \frac{1}{2}(\sqrt{3} - 1) & 1 & -\frac{1}{2}(\sqrt{3} + 1) \\ 1 & 1 & 1 \end{pmatrix} \]

\[ X = \Delta(384) : \quad V = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{1}{2}\sqrt{4 + \sqrt{2} + \sqrt{6}} & 1 & -\frac{1}{2}\sqrt{4 - \sqrt{2} - \sqrt{6}} \\ \frac{1}{2}\sqrt{4 + \sqrt{2} - \sqrt{6}} & 1 & -\frac{1}{2}\sqrt{4 - \sqrt{2} + \sqrt{6}} \\ \sqrt{1 - \frac{1}{\sqrt{2}}} & 1 & \sqrt{1 + \frac{1}{\sqrt{2}}} \end{pmatrix} \]
Explicit Warped GHU Models: Mixing Angles

To plot the effect of $\delta_\alpha$ choose the parametrization

$$S_4 , A_5 : \delta_e = 0, \delta_\mu = \delta, \delta_\tau = 0$$

$$\Delta(96) , \Delta(384) : \delta_e = \delta, \delta_\mu = \delta, \delta_\tau = 0$$

$$\Delta(96) , \Delta(384) : \delta_e = \delta, \delta_\mu = 0, \delta_\tau = \delta$$

Fix parameters $m_{UV,\alpha} \geq 0$ through

$$m_0 = 0.01 \text{ eV} , \quad \Delta m_{\text{sol}}^2 = 7.59 \times 10^{-5} \text{ eV}^2 , \quad \Delta m_{\text{atm}}^2 = 2.40 \times 10^{-3} \text{ eV}^2$$

And all BKT zero, $m_{IR,\alpha}^l = 1 , \quad 0.3 \leq m_{IR,0}^\nu \leq 1 , \quad h = v_H / f_H = 1/3$

$$c_l = 0.52 , \quad c_\nu = -0.365 , \quad c_\alpha \text{ from charged lepton masses}$$
Explicit Warped GHU Models: Mixing Angles

\[ \sin^2 \theta_{12} \]

\[ \Delta(96) \]
\[ \Delta(384) \]

\( S_4 \)
\( A_5 \)

\(-0.10\) \(-0.05\) \(0.00\) \(0.05\) \(0.10\) \(\delta\)
Explicit Warped GHU Models: Mixing Angles
Explicit Warped GHU Models: Mixing Angles

... and the results for $\theta_{13}$

\[ S_4, A_5 : \quad \sin^2 \theta_{13} \ll 10^{-3} \]
\[ \Delta(96) : \quad 0.04 \lesssim \sin^2 \theta_{13} \lesssim 0.053 [0.047] \]
\[ \Delta(384) : \quad 0.01 \lesssim \sin^2 \theta_{13} \lesssim 0.012 \]

Easy way out of the tuning problem of $\delta_\alpha$:

\[ Z_2 \] exchange symmetry on IR brane

\[ \hat{\nu}_\alpha(x, R') \leftrightarrow \nu_\alpha(x, R'), \tilde{L}_{1,\alpha}(x, R') \leftrightarrow \tilde{L}_{2,\alpha}(x, R'), L_\alpha(x, R') \leftrightarrow \hat{L}_\alpha(x, R') \]

leads to $|m^l_{IR,\alpha}| = 1$ and $|m^\nu_{IR,\alpha}| = 1$
Explicit Warped GHU Models: LFV

• With no BKTs the only source of flavor violation is

\[ \frac{1}{2} \nu^c_{\alpha R} \mathcal{M}_{UV, \alpha \beta} \nu_{\beta R} \]

• Use KK decomposition

\[
\frac{1}{2} \left( \sum_{m=0}^{\infty} N^{(m)c}_{\alpha R}(x) f^{(m)}_{\nu, \alpha R}(R) \right) \mathcal{M}_{UV, \alpha \beta} \left( \sum_{n=0}^{\infty} f^{(n)}_{\nu, \beta R}(R) N^{(n)}_{\beta R}(x) \right)
\]

to show that only heavy RH Majorana neutrinos are sensitive to flavor violation

\[ \rightarrow \text{LFV processes suppressed by the large mass of RH neutrinos} \]
Explicit Warped GHU Models: BKTs

In general kinetic terms are present on branes

- at the IR brane they are flavor diagonal due to $Z_N^{(D)}$
- at the UV brane those of fields in 3 of $X$ are flavor violating most important is

$$\mathcal{L}_{BKT} = \bar{L}_L(x, R)(R\hat{Z}_l)i\nabla L_L(x, R) \quad \text{with} \quad \hat{Z}_l = V \text{diag}(\hat{z}_{el}, \hat{z}_{\mu l}, \hat{z}_{\tau l})V^\dagger$$

- Note $\hat{Z}_l$ scale with value of wave function of $L_L$ at UV brane

$$\left( \log^{-1} \frac{R'}{R} + \delta_c \right) \hat{Z}_l \simeq \left( \frac{1}{35} + \delta_c \right) \hat{Z}_l \quad \text{for} \quad c_l = 1/2 + \delta_c$$

→ relevant suppression of LFV processes
Explicit Warped GHU Models: $\mu \rightarrow e\gamma$

$Z_2$ invariant model

$$\delta \hat{z} = 3(\hat{Z}_l)_{e\mu}$$

BR$(\mu \rightarrow e\gamma)$
Explicit Warped GHU Models: EDMs

• Consider first the case without BKTs
• Bulk mass parameters are real
• Apply the following phase transformations

\[ \xi_{\nu,\alpha} \rightarrow e^{-i\theta^\nu_\alpha} \xi_{\nu,\alpha}, \quad \xi_{e,\alpha} \rightarrow e^{-i\theta^l_\alpha} \xi_{e,\alpha} \]

with \( m_{\text{IR},\alpha} = |m_{\text{IR},\alpha}| e^{i\theta^l_\alpha} \)

• Phases are then only present in UV localized Majorana mass
• Use same argument as in the case of LFV processes

\[ \rightarrow \text{EDMs are highly suppressed} \]
Explicit Warped GHU Models: EDMs

- Considering the dominant BKT we see

\[ \bar{L}_L(x, R)(R\hat{Z}_l)i\bar{\partial}L_L(x, R) \]

remains untouched, i.e. does not acquire a phase

- But subleading BKTs exist

\[ \bar{\nu}_R(x, R)(R\hat{Z}_\nu)i\bar{\partial}\nu_R(x, R) \]

in which the phases \( \theta^\nu_\alpha \) appear

- Still their contribution to EDMs is negligible
Model with Dirac Neutrinos

Just change the boundary conditions of $\hat{\nu}_\alpha L$ contained in $\xi_{l,\alpha}$

$$\xi_{l,\alpha} = \begin{pmatrix} [\tilde{L}_{1,\alpha L} (-+), L_{\alpha L} (++)] \\ \hat{\nu}_{\alpha L} (+-) \end{pmatrix}$$

Then we can write a Dirac mass term at the UV brane

$$\overline{\hat{\nu}_{\alpha L}} M_{UV, \alpha \beta} \nu_{\beta R} + h.c.$$ with $M_{UV} = V^* m_{UV} V^\dagger$ and $m_{UV,\alpha}$ free parameters

NB: Setup requires slight extension of auxiliary symmetry

$$G_a = Z_5 \times Z_3', \ G_{a,UV} = Z_3', \ G_{a,IR} = Z_5$$
Naively Dirac is in better shape than Majorana model, but presence of lightish KK modes of $\hat{\nu}_\alpha L(\pm)$

- ... leads to a too large deviation of the gauge coupling $Z\nu_L \bar{\nu}_L$ measured through invisible $Z$ decay width

$$\frac{\delta g^\alpha_{\nu_L}}{g^\alpha_{\nu_L}} = -\frac{h^2}{2} \quad \text{with} \quad h = \frac{v_H}{f_H}$$

$\rightarrow h \lesssim 1/10$ would be required

- ... can induce deviations of $U_{PMNS}$ from unitarity comparable to experimental bounds ($h = 1/3$)

- ... leads to LFV processes like $\mu \rightarrow e\gamma$
Model with Dirac Neutrinos: $\mu \rightarrow e\gamma$

with BKT, parameters similar to Majorana case, $h = 1/3$, $c_\nu = 1.33$
Conclusions

• Idea of predicting lepton mixing through non-trivial breaking of flavor symmetry

• Context of composite Higgs models very suitable for implementation of idea

• Construction of explicit warped GHU models for Majorana and Dirac neutrinos

• Model with Majorana neutrinos successful; only assume $|\delta_{\alpha}| \lesssim 0.1$

• Model with Dirac neutrinos: lightish state leads to too large $\frac{\delta g^\alpha_{\nu L}}{g^\alpha_{\nu L}}$ and deviation of $U_{PMNS}$ from unitarity