Lepton Mixing and Lepton Flavor Violation in Holographic Composite Higgs Models

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Outline

- Experimental status of lepton mixing
- Idea of non-trivially broken flavor group G_f
- Realization in composite Higgs models
- Explicit 5D model for Majorana neutrinos
- Comments about Dirac neutrinos
- Conclusions

Lepton Mixing

Parametrization of mixing matrix U_{PMNS}

$$U_{PMNS} = V(\theta_{ij}, \delta_{CP}) \cdot \begin{pmatrix} e^{i\,\chi_1} & 0 & 0 \\ 0 & e^{i\,\chi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$V(\theta_{ij}, \delta_{CP}) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

with $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Special Lepton Mixing Patterns ?

- $\mu\tau$ Symmetry

$$\sin^2\theta_{12} \quad \text{free}\;, \quad \sin^2\theta_{23} = \frac{1}{2}\;, \quad \sin^2\theta_{13} = 0$$

- Tri-Bimaximal mixing (TB mixing)

$$\sin^2\theta_{12} = \frac{1}{3}\;, \quad \sin^2\theta_{23} = \frac{1}{2}\;, \quad \sin^2\theta_{13} = 0$$

- Golden Ratio $\quad (\phi = \frac{1}{2}\,(1+\sqrt{5}))$

$$\sin^2\theta_{12} = \frac{1}{\sqrt{5}\phi} \approx 0.276\;, \quad \sin^2\theta_{23} = \frac{1}{2}\;, \quad \sin^2\theta_{13} = 0$$

Global Fits

- Fogli et al. (May '12) at 3σ level

$$\sin^2\theta_{12} = 0.307^{+0.052}_{-0.048}\ , \quad \sin^2\theta_{23} = 0.4(1)^{+0.24(5)}_{-0.07}\ , \quad \sin^2\theta_{13} = 0.025^{+0.010}_{-0.010}$$

- Forero et al. (May '12) at 3σ level

normal hierarchy

$$\sin^2\theta_{12} = 0.32^{+0.05}_{-0.05}\ , \quad \sin^2\theta_{23} = 0.49^{+0.15}_{-0.10}\ , \quad \sin^2\theta_{13} = 0.026^{+0.010}_{-0.011}$$

inverted hierarchy

$$\sin^2\theta_{12} = 0.32^{+0.05}_{-0.05}\ , \quad \sin^2\theta_{23} = 0.53^{+0.11}_{-0.14}\ , \quad \sin^2\theta_{13} = 0.027^{+0.010}_{-0.011}$$

Status of Flavor Symmetries

... you could ask

- whether they are still interesting for lepton mixing

Status of Flavor Symmetries

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- whether they are still interesting for lepton mixing

My reply is

- there are "new" patterns with $\theta_{13} \neq 0$, e.g. from $\Delta(96)$ and $\Delta(384)$

$$\sin^2\theta_{12} \approx 0.349\,, \quad \sin^2\theta_{23} \approx 0.349\,(0.651)\,, \quad \sin^2\theta_{13} \approx 0.045$$
$$\sin^2\theta_{12} \approx 0.337\,, \quad \sin^2\theta_{23} \approx 0.424\,(0.576)\,, \quad \sin^2\theta_{13} \approx 0.011$$

- and of course all "old" patterns are usually just leading order results which are corrected in concrete models

Status of Flavor Symmetries

... you could ask

- whether they are still interesting for lepton mixing
- whether they are interesting at all

Status of Flavor Symmetries

... you could ask

- whether they are still interesting for lepton mixing
- whether they are interesting at all

My reply is

- flavor symmetries are not only useful for mixing, because in all extensions of the SM new particles and new interactions are present which induce processes strongly constrained by experiments, e.g.

$$\mu \to e\gamma \text{ with } BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$

Choice of Flavor Symmetry G_f

In the following I assume G_f to be

- ... non-abelian,
- ... discrete,
- ... finite,
- ... global,
- ... explicitly or spontaneously broken to particular subgroups,
- ... of the form $X \times Z_N$ ($N \geq 3$) with X non-abelian.

Lepton Mixing from Non-Trivial G_f Breaking

Idea:

Derivation of the lepton mixing from how G_f is broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos	charged leptons
G_ν	G_e

Lepton Mixing from Non-Trivial G_f Breaking

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neutrinos	charged leptons
assume 3 generations of Majorana neutrinos	distinguish 3 generations

Lepton Mixing from Non-Trivial G_f Breaking

<u>Idea</u>:

Derivation of the lepton mixing from how G_f is broken

Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos	charged leptons
$G_\nu = Z_2 \times Z_2$	$G_e = Z_N$ with $N \geq 3$

Lepton Mixing from Non-Trivial G_f Breaking

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos — $G_\nu = Z_2 \times Z_2$

charged leptons — $G_e = Z_N$ with $N \geq 3$

Further requirements

- Two/three non-trivial angles $\Rightarrow$ irred 3-dim rep $\rho \sim \mathbf{3}$ of G_f
- Fix angles through G_ν, G_e $\Rightarrow$ 3 generations transform differently under G_ν, G_e

Lepton Mixing from Non-Trivial G_f Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved

$\rightarrow$ neutrino mass matrix m_ν fulfills

$$\rho(g_{\nu,i})^T m_\nu \rho(g_{\nu,i}) = m_\nu \quad \text{with} \quad i = 1, 2$$

- Charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$\rho(g_e)^\dagger m_e^\dagger m_e \rho(g_e) = m_e^\dagger m_e$$

Lepton Mixing from Non-Trivial G_f Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved and generated by

$$\rho(g_{\nu,i}) = \Omega_\nu \rho(g_{\nu,i})^{diag} \Omega_\nu^\dagger \quad \text{with} \quad i = 1, 2$$

$$\rho(g_{\nu,i})^{diag} = \text{diag}\,(\pm 1, \pm 1, \pm 1) \quad \text{and} \quad \Omega_\nu \quad \text{unitary}$$

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$$\rho(g_e)^\dagger m_e^\dagger m_e \rho(g_e) = m_e^\dagger m_e$$

Lepton Mixing from Non-Trivial G_f Breaking

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- Charged lepton sector: Z_N, $N \geq 3$, preserved and generated by

  $$\rho(g_e) = \Omega_e \rho(g_e)^{diag} \Omega_e^\dagger \quad \text{with} \quad \Omega_e \quad \text{unitary}$$

  $$\rho(g_e)^{diag} = \text{diag}\left(\omega_N^{n_e}, \omega_N^{n_\mu}, \omega_N^{n_\tau}\right)$$

  $$\text{and} \quad n_e \neq n_\mu \neq n_\tau \quad \text{and} \quad \omega_N = e^{2\pi i/N}$$

Lepton Mixing from Non-Trivial G_f Breaking

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$$\Omega_\nu^T m_\nu \Omega_\nu \quad \text{is diagonal}$$

- Charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$\left(\rho(g_e)^{diag}\right)^\star \left[\Omega_e^\dagger m_e^\dagger m_e \Omega_e\right] \rho(g_e)^{diag} = \left[\Omega_e^\dagger m_e^\dagger m_e \Omega_e\right]$$

Lepton Mixing from Non-Trivial G_f Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

  $\Omega_\nu^T m_\nu \Omega_\nu$ is diagonal

- Charged lepton sector: Z_N, $N \geq 3$, preserved

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  $\Omega_e^\dagger m_e^\dagger m_e \Omega_e$ is diagonal

Lepton Mixing from Non-Trivial G_f Breaking

- Neutrino sector: $Z_2 \times Z_2$ preserved

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- Charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$\Omega_e^\dagger m_e^\dagger m_e \Omega_e \quad \text{is diagonal}$$

- Conclusion: $\Omega_{\nu,e}$ diagonalize m_ν and $m_e^\dagger m_e$

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu$$

Lepton Mixing from Non-Trivial G_f Breaking

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- Neutrino masses are made real and positive through $\Omega_\nu \to \Omega_\nu K_\nu$
- Permutations of columns of Ω_e, Ω_ν are possible: $\Omega_{e,\nu} \to \Omega_{e,\nu} P_{e,\nu}$

$$\Downarrow$$

Predictions:
Mixing angles up to exchange of rows/columns
Dirac CP phase δ_{CP} up to π
Majorana phases undetermined

Lepton Mixing from Non-Trivial G_f Breaking

Basis used in the following:

- $\rho(g_e)$ is diagonal and thus Ω_e a permutation
- $\rho(g_{\nu,i})$ are non-diagonal and Ω_ν is called V
- Choose $\rho(g_{\nu,i})^{diag}$ to be

$$\rho(g_{\nu,1})^{diag} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} , \quad \rho(g_{\nu,2})^{diag} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Idea of Composite Higgs Models

- Assume strong sector with composite Higgs in order to resolve hierarchy problem
- Higgs VEV breaks electroweak gauge group
- Higgs might be a pseudo Goldstone boson, then it is naturally light (realization in 5D: gauge-Higgs unification)
- SM fermions are part of the elementary sector
- SM fermions get mass upon mixing with fermionic resonances Ψ of composite sector (idea of partial compositeness)

Generic form of Lagrangian:

$$\mathcal{L}_{tot} = \mathcal{L}_{el} + \mathcal{L}_{comp} + \mathcal{L}_{mix}$$

Composite Higgs Models

- Lagrangian:

$$\mathcal{L}_{tot} = \mathcal{L}_{el} + \mathcal{L}_{comp} + \mathcal{L}_{mix}$$

- Assume
  - $\mathcal{L}_{mix}$ invariant under G_f
  - $\mathcal{L}_{el}$ invariant under G_ν
  - $\mathcal{L}_{comp}$ invariant under G_e
- Degrees of freedom in elementary sector
  - 3 LH lepton doublets l_L^α
  - 3 RH charged leptons l_R^α
  - 3 RH neutrinos ν_R^α

Composite Higgs Models

$l_L^\alpha \sim \mathbf{3}$ and $\nu_R^\alpha \sim \mathbf{3}$ under non-abelian group $X \subset G_f$

$\rightarrow \mathcal{L}_{el}$ contains Majorana mass term fixed by $G_\nu = Z_2 \times Z_2 \subset X \subset G_f$

$$-\frac{1}{2}\left(\overline{\nu_R^c}^\alpha M_{\alpha\beta} \nu_R^\beta + h.c.\right) \quad \text{with} \quad M = V^\star M_D V^\dagger\,, \quad M_D \ \text{diagonal}$$

Charged lepton mass hierarchy through partial compositeness

$\rightarrow l_R^\alpha \sim \mathbf{1}$ under X

Distinct RH charged leptons through additional Z_N symmetry

$\rightarrow G_f = X \times Z_N$ and $l_R^\alpha \sim (\mathbf{1}, \omega_N^{n_\alpha})$

Composite Higgs Models

Thus take

- 3 LH lepton doublets $l_L^\alpha \sim (\mathbf{3}, 1)$ under (X, Z_N)
- 3 RH charged leptons $l_R^\alpha \sim (\mathbf{1}, \omega_N^{n_\alpha})$ under (X, Z_N)
- 3 RH neutrinos $\nu_R^\alpha \sim (\mathbf{3}, 1)$ under (X, Z_N)

Assume linear coupling of elementary fields to fermionic resonances Ψ of composite sector

$$\mathcal{L}_{mix} = \frac{\lambda_{l_L}}{\Lambda^{\gamma_{lL}}} \bar{l}_L^\alpha \Psi_{l_L,R}^\alpha + \frac{\lambda_{l_R}^\alpha}{\Lambda^{\gamma_{lR}^\alpha}} \bar{l}_R^\alpha \Psi_{l_R,L}^\alpha + \frac{\lambda_{\nu_R}}{\Lambda^{\gamma_{\nu R}}} \bar{\nu}_R^\alpha \Psi_{\nu_R,L}^\alpha + h.c.$$

implies that $\Psi_{l_L}^\alpha \sim l_L^\alpha$, $\Psi_{l_R}^\alpha \sim l_R^\alpha$, $\Psi_{\nu_R}^\alpha \sim \nu_R^\alpha$ under SM and G_f

Composite Higgs Models

- We expect upon integrating out the resonances a charged lepton mass matrix of the form

$$M_{l,\alpha\beta} \simeq \frac{\lambda_{l_L}}{\Lambda^{\gamma_{lL}}} \frac{\lambda^{\beta}_{l_R}}{\Lambda^{\gamma^{\beta}_{lR}}} \langle \bar{\Psi}^{\beta}_{l_R} \Psi^{\alpha}_{l_L} \rangle$$

- For no G_f breaking the condensate has to vanish
- For G_f broken to $G_e = Z_N^{(D)}$ instead we find

$$\langle \bar{\Psi}^{\beta}_{l_R} \Psi^{\alpha}_{l_L} \rangle \sim b_\alpha \delta_{\alpha\beta} v_H \, \mu^{\gamma^{\alpha}_{lR} + \gamma_{lL}} \ ,$$

since $l^\alpha_L \, , \, \Psi^\alpha_{l_L} \, , \, l^\alpha_R \, , \, \Psi^\alpha_{l_R} \, , \, \nu^\alpha_R \, , \, \Psi^\alpha_{\nu_R} \sim \omega_N^{n_\alpha}$ under $Z_N^{(D)}$
which is the diagonal subgroup of $Z_N \subset X$ and the external Z_N

Composite Higgs Models: Mass Matrices

- Thus we generate a charged lepton mass matrix of the form

$$M_{l,\alpha\beta} \sim b_\alpha v_H \lambda_{l_L} \lambda^\alpha_{l_R} \delta_{\alpha\beta} \left(\frac{\mu}{\Lambda}\right)^{\gamma^\alpha_{lR}+\gamma_{lL}}$$

- At the same time the Dirac neutrino mass matrix takes the form

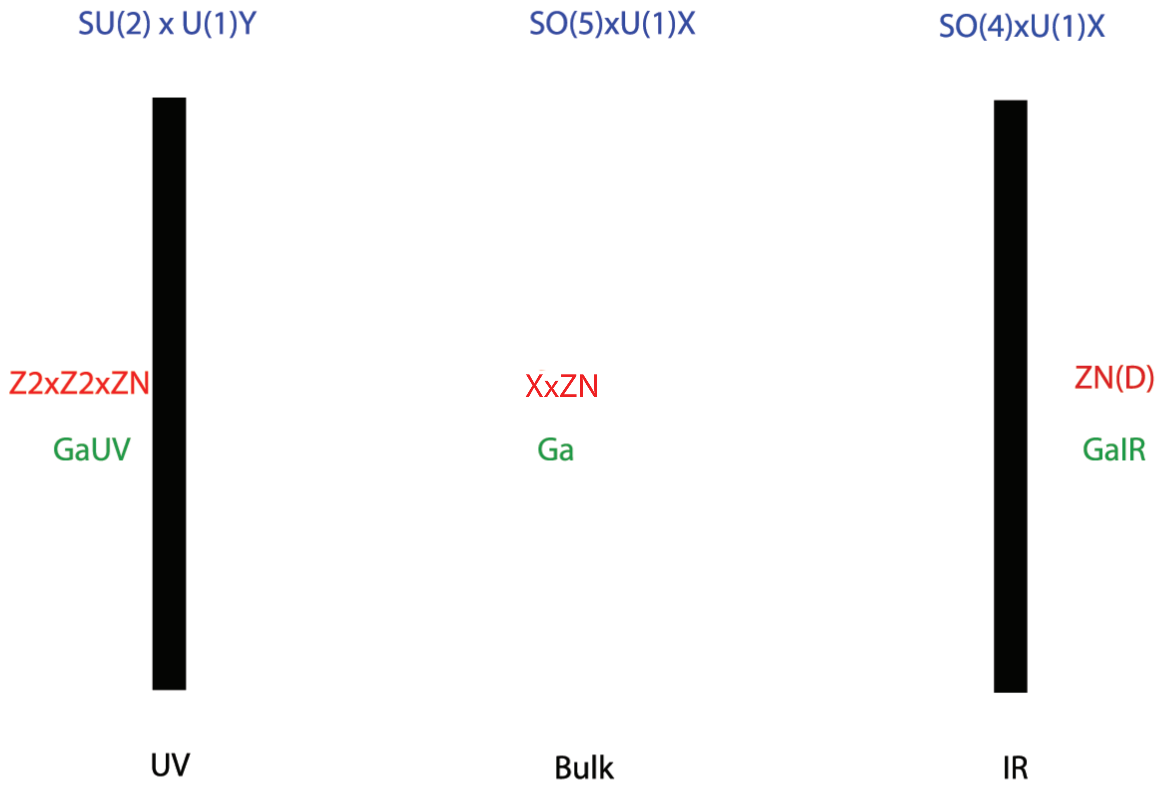
$$M^D_{\nu,\alpha\beta} \sim \frac{\hat{b}_\alpha v_H \lambda_{l_L}}{\tilde{b}_\alpha} \delta_{\alpha\beta} \left(\frac{\mu}{\Lambda}\right)^{\gamma_{lL}}$$

- Then the light neutrino mass matrix is

$$M_{\nu,\alpha\beta} \simeq \hat{b}_\alpha \hat{b}_\beta v_H^2 \lambda^2_{l_L} \lambda^2_{\nu_R} \left(\frac{\mu}{\Lambda}\right)^{2(\gamma_{\nu R}+\gamma_{lL})} \left(V M_D^{-1} V^T\right)_{\alpha\beta}$$

- Result for lepton mixing, if $\hat{b}$ are universal: $U_{PMNS} = V$

Explicit Warped GHU Models: Setup



Explicit Warped GHU Models: Symmetries

- Additional symmetries G_a, $G_{a,\mathrm{UV}}$ and $G_{a,\mathrm{IR}}$

$$G_a = Z_3' \times Z_3'' \; , \quad G_{a,\mathrm{UV}} = Z_3'' \; , \quad G_{a,\mathrm{IR}} = Z_3'$$

  for minimizing terms in bulk, on UV and IR brane

- Examples of symmetries $X \times Z_N$

$$\begin{array}{lclcl} (X, Z_N) & = & (S_4, Z_3) & \rightarrow & \text{TB mixing} \\ (X, Z_N) & = & (A_5, Z_5) & \rightarrow & \text{Golden Ratio} \\ (X, Z_N) & = & (\Delta(96), Z_3) & \rightarrow & \text{Mixing with } \sin^2\theta_{13} \approx 0.045 \\ (X, Z_N) & = & (\Delta(384), Z_3) & \rightarrow & \text{Mixing with } \sin^2\theta_{13} \approx 0.011 \end{array}$$

Explicit Warped GHU Models: Particles

Particle content ($X = 0$)

$$\xi_{l,\alpha} \sim (\mathbf{5}, \mathbf{3}, 1, \omega_3, \omega_3)\ , \quad \xi_{e,\alpha} \sim (\mathbf{10}, \mathbf{1}, \omega_N^{n_\alpha}, \omega_3, \omega_3)\ , \quad \xi_{\nu,\alpha} \sim (\mathbf{1}, \mathbf{3}, 1, \omega_3, 1)$$

with

$$\xi_{l,\alpha} = \begin{pmatrix} \left[\widetilde{L}_{1,\alpha L}\,(-+)\,,\ L_{\alpha L}\,(++) \right] \\ \\ \hat{\nu}_{\alpha L}\,(-+) \end{pmatrix}, \quad \xi_{e,\alpha} = \begin{pmatrix} x_{\alpha L}\,(+-) & \\ \widetilde{\nu}_{\alpha L}\,(+-) & Z_{\alpha L}\,(+-) \\ e_{\alpha L}\,(--) & \\ \left[\widetilde{L}_{2,\alpha L}\,(+-)\,,\ \hat{L}_{\alpha L}\,(+-) \right] & \end{pmatrix}$$

$$\xi_{\nu,\alpha} = \nu_{\alpha L}\,(--)$$

Explicit Warped GHU Models: Particles

	Bulk	UV	IR
	$G_f \times G_a$	$G_{f,\mathrm{UV}} \times G_{a,\mathrm{UV}}$	$G_{f,\mathrm{IR}} \times G_{a,\mathrm{IR}}$
$\xi_{l,\alpha}$	$(\mathbf{3}, 1, \omega_3, \omega_3)$	$(1, -1, 1, \omega_3)$ $(-1, 1, 1, \omega_3)$ $(-1, -1, 1, \omega_3)$	$(\omega_N^{n_\alpha}, \omega_3)$
$\xi_{e,\alpha}$	$(\mathbf{1}, \omega_N^{n_\alpha}, \omega_3, \omega_3)$	$(1, 1, \omega_N^{n_\alpha}, \omega_3)$	$(\omega_N^{n_\alpha}, \omega_3)$
$\xi_{\nu,\alpha}$	$(\mathbf{3}, 1, \omega_3, 1)$	$(1, -1, 1, 1)$ $(-1, 1, 1, 1)$ $(-1, -1, 1, 1)$	$(\omega_N^{n_\alpha}, \omega_3)$

Explicit Warped GHU Models: Parameters

- Bulk: Mass parameters c_l, c_α ($\alpha = 1, 2, 3$) and c_ν
- Lagrangian on IR brane

$$\left(\frac{R}{R'}\right)^4 \sum_{\alpha=e,\mu,\tau} \left(m^l_{\mathrm{IR},\alpha} \left(\overline{\widetilde{L}}_{1,\alpha L} \widetilde{L}_{2,\alpha R} + \overline{L}_{\alpha L} \hat{L}_{\alpha R} \right) + m^\nu_{\mathrm{IR},\alpha} \overline{\hat{\nu}}_{\alpha L} \nu_{\alpha R} + h.c. \right)$$

which contains as independent complex parameters

$m^l_{\mathrm{IR},\alpha}$ and $m^\nu_{\mathrm{IR},\alpha} = m^\nu_{\mathrm{IR},0}(1 + \delta_\alpha)$

- Lagrangian on UV brane

$$\frac{1}{2} \overline{\nu^c_{\alpha R}} \mathcal{M}_{\mathrm{UV},\alpha\beta} \nu_{\beta R} + h.c.$$

with $\mathcal{M}_{\mathrm{UV}} = V^\star m_{\mathrm{UV}} V^\dagger$ and $m_{\mathrm{UV},\alpha}$ are free complex parameters

Explicit Warped GHU Models: Lepton Mixing

$$X = S_4 \quad : \quad V = U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$

$$X = A_5 \quad : \quad V = U_{GR} = \begin{pmatrix} \cos\theta_{12}^{\rm GR} & -\sin\theta_{12}^{\rm GR} & 0 \\ \frac{\sin\theta_{12}^{\rm GR}}{\sqrt{2}} & \frac{\cos\theta_{12}^{\rm GR}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}^{\rm GR}}{\sqrt{2}} & \frac{\cos\theta_{12}^{\rm GR}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

with $\tan\theta_{12}^{\rm GR} = 1/\phi$

Explicit Warped GHU Models: Lepton Mixing

$$X = \Delta(96) \quad : \quad V = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{1}{2}(\sqrt{3}+1) & 1 & \frac{1}{2}(\sqrt{3}-1) \\ \frac{1}{2}(\sqrt{3}-1) & 1 & -\frac{1}{2}(\sqrt{3}+1) \\ 1 & 1 & 1 \end{pmatrix}$$

$$X = \Delta(384) \quad : \quad V = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{1}{2}\sqrt{4+\sqrt{2}+\sqrt{6}} & 1 & -\frac{1}{2}\sqrt{4-\sqrt{2}-\sqrt{6}} \\ \frac{1}{2}\sqrt{4+\sqrt{2}-\sqrt{6}} & 1 & -\frac{1}{2}\sqrt{4-\sqrt{2}+\sqrt{6}} \\ \sqrt{1-\frac{1}{\sqrt{2}}} & 1 & \sqrt{1+\frac{1}{\sqrt{2}}} \end{pmatrix}$$

Explicit Warped GHU Models: Mixing Angles

To plot the effect of δ_α choose the parametrization

$$
\begin{aligned}
S_4\,,\; A_5 \quad &: \quad \delta_e = 0, \delta_\mu = \delta, \delta_\tau = 0 \\
\Delta(96)\,,\; \Delta(384) \quad &: \quad \delta_e = \delta, \delta_\mu = \delta, \delta_\tau = 0 \\
\Delta(96)\,,\; \Delta(384) \quad &: \quad \delta_e = \delta, \delta_\mu = 0, \delta_\tau = \delta
\end{aligned}
$$

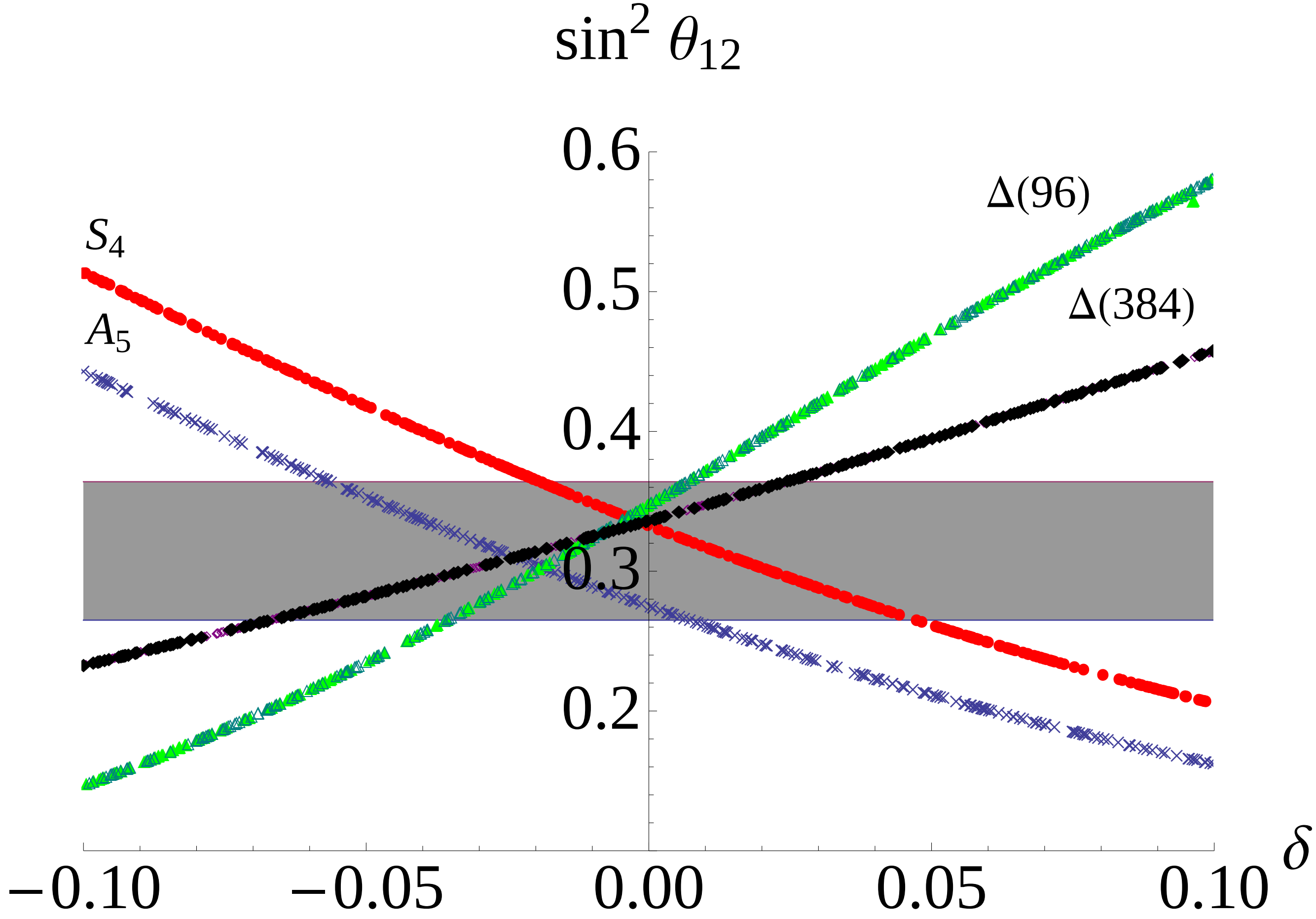
Fix parameters $m_{\mathrm{UV},\alpha} \geq 0$ through

$m_0 = 0.01\,\text{eV}\;,\quad \Delta m^2_{\mathrm{sol}} = 7.59\times 10^{-5}\,\text{eV}^2\;,\quad \Delta m^2_{\mathrm{atm}} = 2.40\times 10^{-3}\,\text{eV}^2$

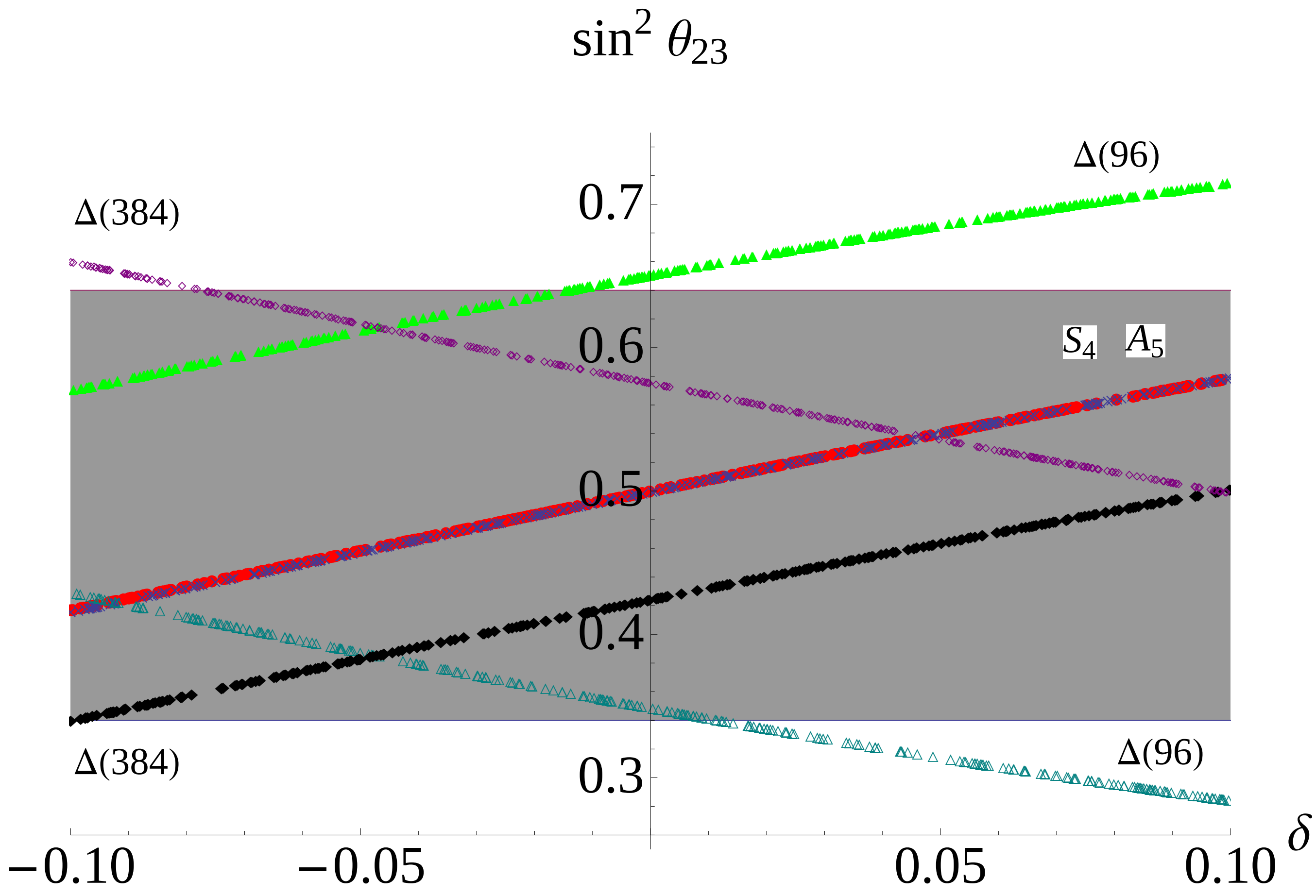
And all BKT zero, $m^l_{\mathrm{IR},\alpha} = 1\;,\quad 0.3 \leq m^\nu_{\mathrm{IR},0} \leq 1$, $h = v_H/f_H = 1/3$

$c_l = 0.52$, $c_\nu = -0.365$, c_α from charged lepton masses

Explicit Warped GHU Models: Mixing Angles



Explicit Warped GHU Models: Mixing Angles



Explicit Warped GHU Models: Mixing Angles

... and the results for θ_{13}

$$\begin{aligned} S_4\,,\,A_5 \quad &: \quad \sin^2\theta_{13} \ll 10^{-3} \\ \Delta(96) \quad &: \quad 0.04 \lesssim \sin^2\theta_{13} \lesssim 0.053\,[0.047] \\ \Delta(384) \quad &: \quad 0.01 \lesssim \sin^2\theta_{13} \lesssim 0.012 \end{aligned}$$

Easy way out of the tuning problem of δ_α:

Z_2 exchange symmetry on IR brane

$$\hat{\nu}_\alpha(x,R') \leftrightarrow \nu_\alpha(x,R'), \tilde{L}_{1,\alpha}(x,R') \leftrightarrow \tilde{L}_{2,\alpha}(x,R'), L_\alpha(x,R') \leftrightarrow \hat{L}_\alpha(x,R')$$

leads to $|m^l_{\mathrm{IR},\alpha}| = 1$ and $|m^\nu_{\mathrm{IR},\alpha}| = 1$

Explicit Warped GHU Models: LFV

- With no BKTs the only source of flavor violation is

$$\frac{1}{2}\overline{\nu^c_{\alpha R}}\mathcal{M}_{\mathrm{UV},\alpha\beta}\nu_{\beta R}$$

- Use KK decomposition

$$\frac{1}{2}\Big(\sum_{m=0}^{\infty}\overline{N^{(m)c}_{\alpha R}}(x)f^{(m)}_{\nu,\alpha R}(R)\Big)\mathcal{M}_{\mathrm{UV},\alpha\beta}\Big(\sum_{n=0}^{\infty}f^{(n)}_{\nu,\beta R}(R)N^{(n)}_{\beta R}(x)\Big)$$

to show that only heavy RH Majorana neutrinos are sensitive to flavor violation

$\rightarrow$ LFV processes suppressed by the large mass of RH neutrinos

Explicit Warped GHU Models: BKTs

In general kinetic terms are present on branes

- at the IR brane they are flavor diagonal due to $Z_N^{(D)}$
- at the UV brane those of fields in $\mathbf{3}$ of X are flavor violating most important is

$$\mathcal{L}_{BKT} = \bar{L}_L(x,R)(R\hat{Z}_l)i\not{D}L_L(x,R) \quad \text{with} \quad \hat{Z}_l = V\,\mathrm{diag}\,(\hat{z}_{el}, \hat{z}_{\mu l}, \hat{z}_{\tau l})V^\dagger$$

- Note $\hat{Z}_l$ scale with value of wave function of L_L at UV brane

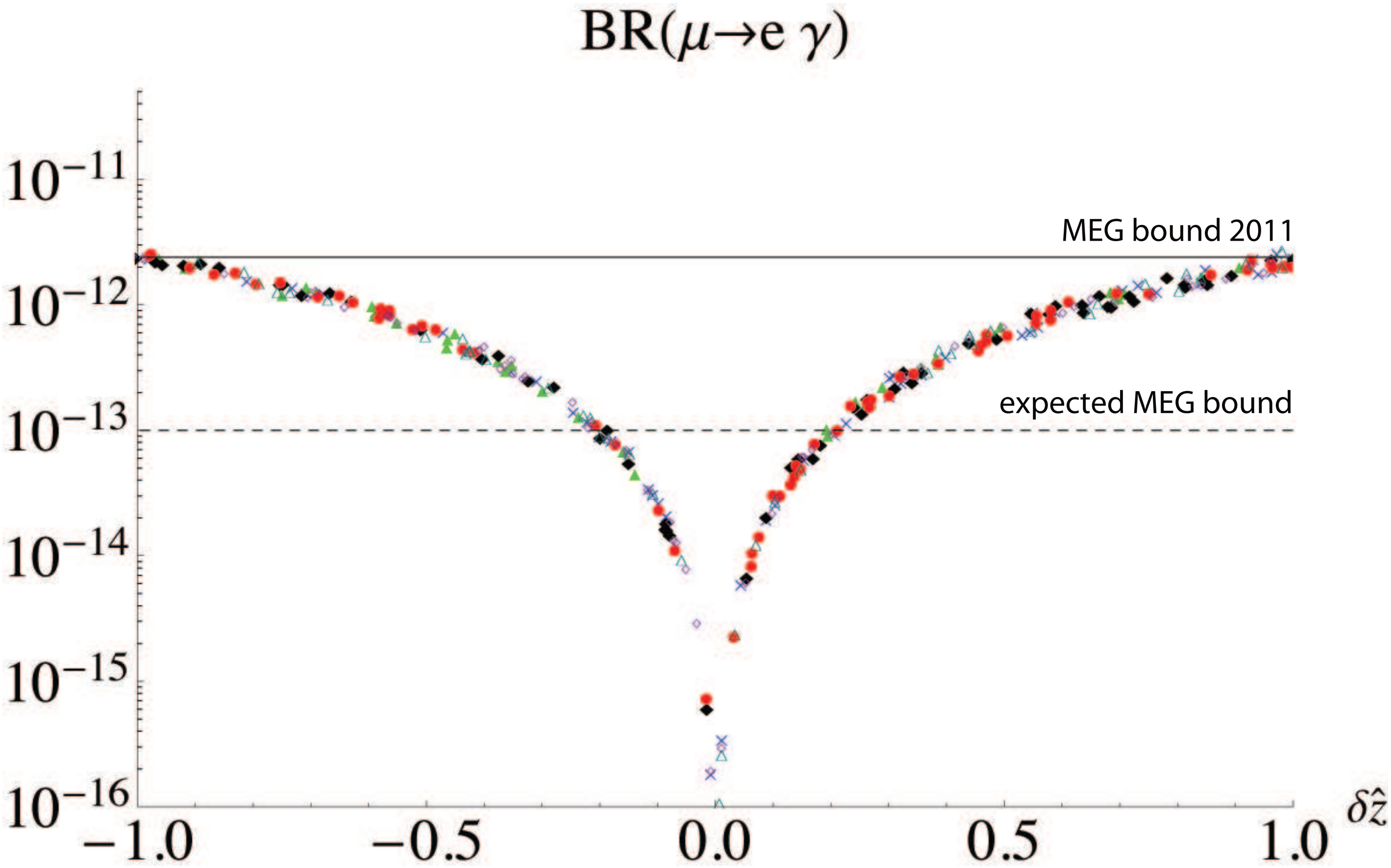
$$\left(\log^{-1}\frac{R'}{R} + \delta_c\right)\hat{Z}_l \simeq \left(\frac{1}{35} + \delta_c\right)\hat{Z}_l \quad \text{for} \quad c_l = 1/2 + \delta_c$$

$\rightarrow$ relevant suppression of LFV processes

Explicit Warped GHU Models: $\mu \to e\gamma$

Z_2 invariant model

$$\delta\hat{z} = 3(\hat{Z}_l)_{e\mu}$$



Explicit Warped GHU Models: EDMs

- Consider first the case without BKTs
- Bulk mass parameters are real
- Apply the following phase transformations

$$\xi_{\nu,\alpha} \to e^{-i\theta^\nu_\alpha} \xi_{\nu,\alpha} \,, \quad \xi_{e,\alpha} \to e^{-i\theta^l_\alpha} \xi_{e,\alpha}$$

$$\text{with} \quad m^{l,\nu}_{\mathrm{IR},\alpha} = |m^{l,\nu}_{\mathrm{IR},\alpha}| e^{i\theta^{l,\nu}_\alpha}$$

- Phases are then only present in UV localized Majorana mass
- Use same argument as in the case of LFV processes

$\to$ EDMs are highly suppressed

Explicit Warped GHU Models: EDMs

- Considering the dominant BKT we see

$$\bar{L}_L(x,R)(R\hat{Z}_l)i\not{D}L_L(x,R)$$

remains untouched, i.e. does not acquire a phase

- But subleading BKTs exist

$$\bar{\nu}_R(x,R)(R\hat{Z}_\nu)i\not{\partial}\nu_R(x,R)$$

in which the phases θ_α^ν appear

- Still their contribution to EDMs is negligible

Model with Dirac Neutrinos

Just change the boundary conditions of $\hat{\nu}_{\alpha L}$ contained in $\xi_{l,\alpha}$

$$\xi_{l,\alpha} = \begin{pmatrix} \left[\widetilde{L}_{1,\alpha L}\,(-+)\,,\; L_{\alpha L}\,(++) \right] \\ \\ \hat{\nu}_{\alpha L}\,(+-) \end{pmatrix}$$

Then we can write a Dirac mass term at the UV brane

$$\overline{\hat{\nu}_{\alpha L}} \mathcal{M}_{\mathrm{UV},\alpha\beta} \nu_{\beta R} + h.c.$$

with $\mathcal{M}_{\mathrm{UV}} = V^{\star} m_{\mathrm{UV}} V^{\dagger}$ and $m_{\mathrm{UV},\alpha}$ free parameters

NB: Setup requires slight extension of auxiliary symmetry

$$G_a = Z_5 \times Z_3' \;, \;\; G_{a,\mathrm{UV}} = Z_3' \;, \;\; G_{a,\mathrm{IR}} = Z_5$$

Model with Dirac Neutrinos

Naively Dirac is in better shape than Majorana model,
but presence of lightish KK modes of $\hat{\nu}_{\alpha L}\ (+-)$

- ... leads to a too large deviation of the gauge coupling $Z\nu_L\bar{\nu}_L$ measured through invisible Z decay width

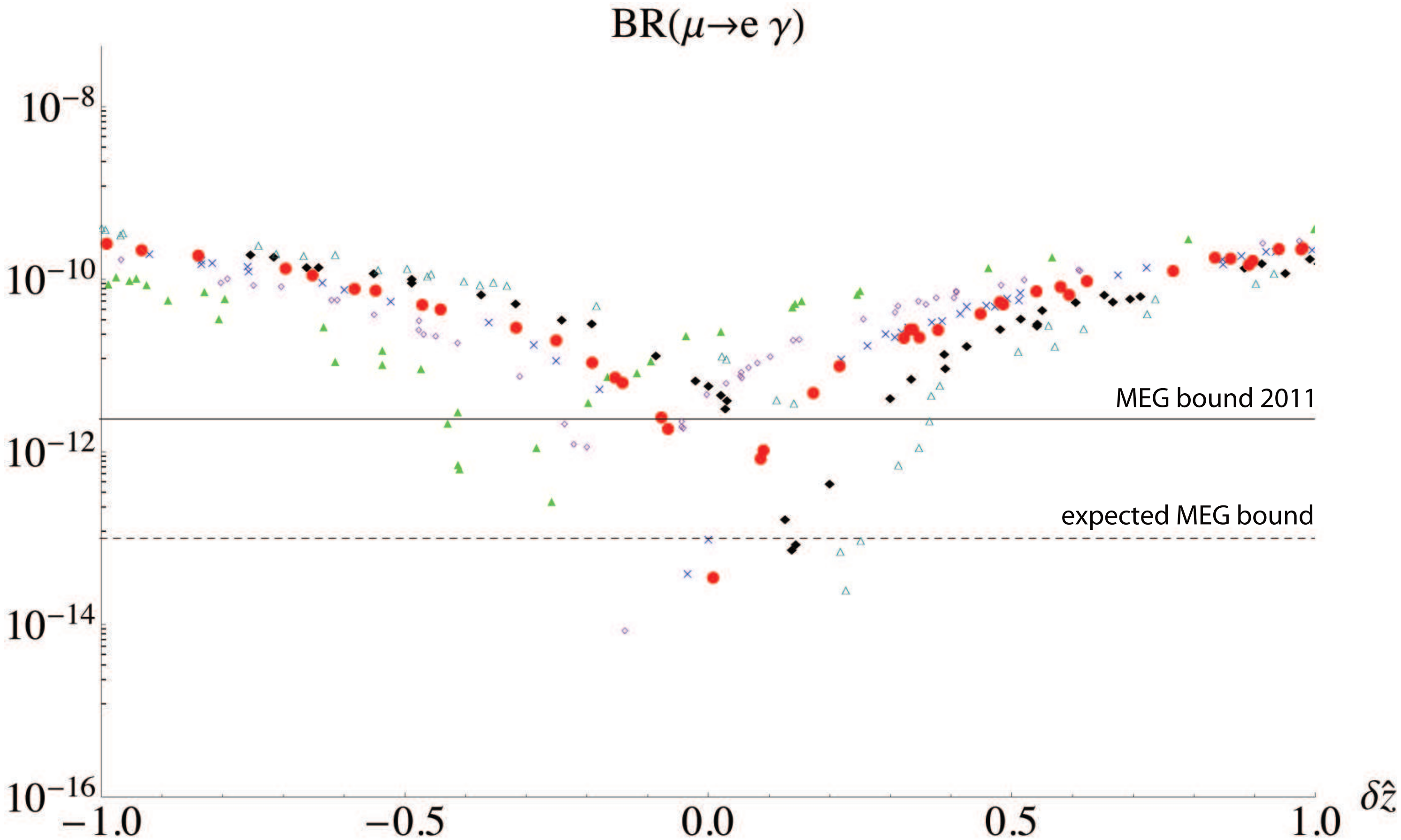
$$\frac{\delta g^{\alpha}_{\nu_L}}{g^{\alpha}_{\nu_L}} = -\frac{h^2}{2} \quad \text{with} \quad h = \frac{v_H}{f_H}$$

$\rightarrow h \lesssim 1/10$ would be required

- ... can induce deviations of U_{PMNS} from unitarity comparable to experimental bounds ($h = 1/3$)
- ... leads to LFV processes like $\mu \rightarrow e\gamma$

Model with Dirac Neutrinos: $\mu \to e\gamma$

with BKT, parameters similar to Majorana case, $h = 1/3$, $c_\nu = 1.33$



Conclusions

- Idea of predicting lepton mixing through non-trivial breaking of flavor symmetry
- Context of composite Higgs models very suitable for implementation of idea
- Construction of explicit warped GHU models for Majorana and Dirac neutrinos
- Model with Majorana neutrinos successful; only assume $|\delta_\alpha| \lesssim 0.1$
- Model with Dirac neutrinos: lightish state leads to too large $\frac{\delta g^\alpha_{\nu_L}}{g^\alpha_{\nu_L}}$ and deviation of U_{PMNS} from unitarity