$\mathbf{S_3}$ and the angle $\theta_{\mathbf{13}}$

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Outline

1. *Different strategies to go Beyond SM*
2. S_3 *as a family symmetry*
3. *A particular realization of* S_3
4. *Quark mixing*
5. *Lepton mixing: the angle* θ_{13}
6. *Outlook*

Beyond SM

- *Fermion masses and CP violation*
- *Neutrino physics*
- *Hierarchy problem (EW and Planck scale)*
- a_μ: *magnetic anomalous moment of muon*
- *Dark matter*

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Early LHC: It may be that flavour is decoupled from the hierarchy problem . . .

Bottom up approach

1. *Propose an effective model for fermion mixing at the EW scale*
2. *Later, address the rest of the problems*

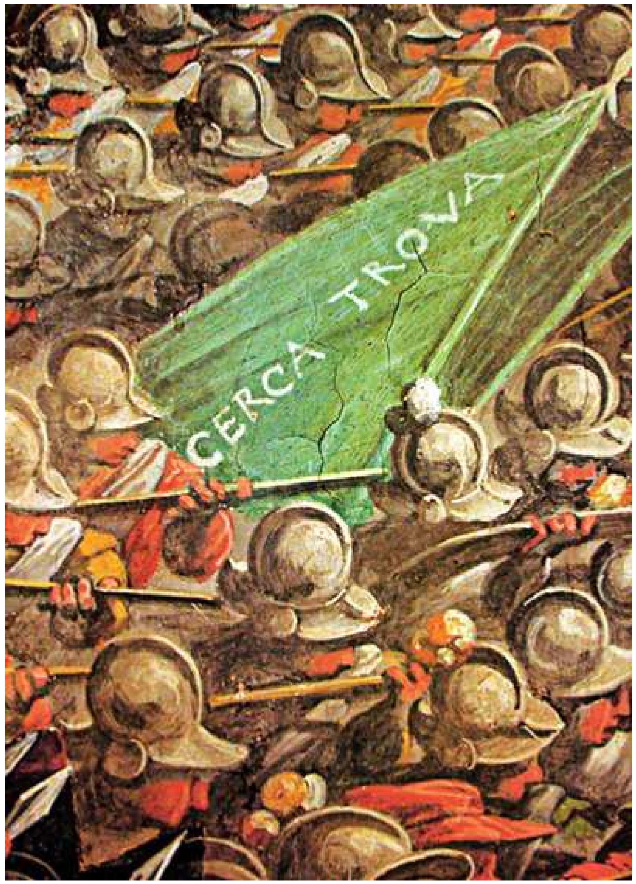
S_3 as a family symmetry

- *Rescued from old ideas:*
  Pakvasa 77, Wyler, Derman, Frere 78
  A. and M. Mondragon, et. al 1998, Kubo, Branco, Paes, etc
- *Two main classes*
  - S_3 *extension (that is, must be broken)*
    $S_3 \supset S_{3L} \times S_{3R} \supset S_3^{\mathrm{diag.}} \supset S_2$
  - S_3*-SM:* S_3 *realized as an exact symmetry, possible due to the addition of Higgs doublets.*
- *Fertile patch: keep exploring (Review in 1205.4755)*

Time for advertisement: it is worth to keep exploring: "Look-for" then "find" (cerca-trova)

Cerca trova

Maurizio Seracini: lost painting beneath one of the walls of a room at Palazzo Vecchio



Detail of Giorgio Vasari's wall painting (Battle of Marciano) at Palazzo Vecchio

Cerca trovi: Davinci's Battaglia di Anghiari

After 30 years of research, Seracini may have found the, for long time considered lost, painting of Davinci's La battaglia di Anghiari (left: a copy of Rubens, right: an sketch of Davinci for the painting)

A particular realization of S_3: S_3-SM

To appear soon (A. Mondragón, F. González-Canales, U. J. Saldaña and L. V-S)

1. *S_3 is considered to be a global symmetry of the theory, even after the Higgs mechanism $\to$ the invariance under permutations of the 3 families can be seen as a hidden symmetry that makes its appearance only through the observed mass spectrum*
2. *The scalar sector: four Higgs $SU(2)_L$ doublets belonging to the S_3 reducible representation, $4 = \mathbf{2} \oplus \mathbf{1}_S \oplus \mathbf{1}_A$*
   - *Four effective $SU(2)_L$ doublets*
   - *Three effective $SU(2)_L$ doublets*
3. *What is so far this construction achieved?*
   - *Through the solution of 9 equations with 9 variables (in each sector), one can get exact formulas of mixing angles in terms of the physical mass ratios of fermions*
   - *Excelent fits of quark and lepton mixing*
4. *Next steps: reduce the number of parameters to be predictive and explore EWSB*

- *The assignment of fermion families in S_3 is suggested by the observed mass hierarchy*

$$\begin{pmatrix} f_{I(L,R)} \\ f_{II(L,R)} \end{pmatrix} \sim \mathbf{2}; \quad f_{III(L,R)} \equiv g_{L,R} \sim 1_{\mathbf{S}},$$

$$f_{IIIL} = (b_L, t_L) \quad f_{IIIR} = t_R,$$

$$\begin{pmatrix} f_{IL} \\ f_{IIL} \end{pmatrix} = \begin{pmatrix} (u_L, d_L) \\ (c_L, s_L) \end{pmatrix}, \quad \begin{pmatrix} f_{IR} \\ f_{IIR} \end{pmatrix}_{f=u} = \begin{pmatrix} u_R \\ c_R \end{pmatrix}, \quad \begin{pmatrix} f_{IR} \\ f_{IIR} \end{pmatrix}_{f=d} = \begin{pmatrix} d_R \\ s_R \end{pmatrix},$$

- *Higgs sector*

$$\begin{pmatrix} H_{1W} \\ H_{2W} \end{pmatrix} \sim \mathbf{2}; \quad H_{SW} \sim 1_{\mathbf{S}}; \quad H_{AW} \sim 1_{\mathbf{A}}.$$

- *Yukawa interactions*

$$\mathcal{L}_{Y_f} = Y_1^f \,(\overline{g}_{SW} g_{SR} H_{SW}) + \frac{1}{\sqrt{2}} Y_2^f (\overline{f}_{1W} f_{1R} + \overline{f}_{2W} f_{2R}) H_{SW} +$$
$$\frac{1}{2} Y_3^f \left[(\overline{f}_{1W} H_{2W} + \overline{f}_{2W} H_{1W}) f_{1R} + (\overline{f}_{1W} H_{1W} - \overline{f}_{2W} H_{2W}) f_{2R} \right] +$$
$$\frac{1}{\sqrt{2}} Y_4^f (\overline{f}_{1W} f_{2R} - \overline{f}_{2W} f_{1R}) H_{AW} + \frac{1}{\sqrt{2}} Y_5^f (\overline{f}_{1W} H_{1W} + \overline{f}_{2W} H_{2W}) g_{SR} +$$
$$\frac{1}{\sqrt{2}} Y_6^f \,[\overline{g}_{SW} (H_{1W} f_{1R} + H_{2W} f_{2R})] + h.c., \quad l, f = d,$$
$$f = u, \nu, \; H_{\bullet W} \to i\sigma_2 H^*_{\bullet W}$$

- *Just as in the SM, Higgs acquire VEVs and CP violation comes from complex Yukawa couplings*

$$\langle 0|H_{1W}|0\rangle \equiv w_1, \quad \langle 0|H_{2W}|0\rangle \equiv w_2, \quad \langle 0|H_{SW}|0\rangle \equiv v_S, \quad \langle 0|H_{AW}|0\rangle \equiv v_A.$$

- *Effective mass Lagrangian after EWSB (same for leptons)*

$$\mathcal{L}_q = -(\bar{u}_L, \bar{c}_L, \bar{t}_L)\mathcal{M}^u_{S_3}\begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} - (\bar{d}_L, \bar{s}_L, \bar{b}_L)\mathcal{M}^d_{S_3}\begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + h.c.,$$

$$\mathcal{M}^f_{S_3} = \begin{pmatrix} \sqrt{2}Y^f_2 v_S + Y^f_3 w_2 & Y^f_3 w_1 + \sqrt{2}Y^f_4 v_A & \sqrt{2}Y^f_5 w_1 \\ Y^f_3 w_1 - \sqrt{2}Y^f_4 v_A & \sqrt{2}Y^f_2 v_S - Y^f_3 w_2 & \sqrt{2}Y^f_5 w_2 \\ \sqrt{2}Y^f_6 w_1 & \sqrt{2}Y^f_6 w_2 & 2Y^f_1 v_S \end{pmatrix}.$$

- *We know that if the matrix looks like*

$$\mathcal{M}^f = \begin{pmatrix} 0 & A^f & 0 \\ A^{f*} & |B^f| & C^f \\ 0 & C^{f*} & |D^f| \end{pmatrix} \quad \textit{(Think Fritzsch)}$$

it produces a good phenomenology (easy to obtain in these scenarios)

- *So, let us just make a rotation*

$$\mathcal{M}^f \equiv \mathcal{R}(\theta)_{12} \mathcal{M}^f_{S_3} \mathcal{R}(\theta)^T_{12}$$

and identify when we can then achieve a matrix with two texture zeroes

$$Y_5^f = Y_6^{f*}, \quad \textit{arg}(Y_4^f) = \pm\frac{\pi}{2}, \quad \textit{arg}(Y_1^f) = \textit{arg}(Y_2^f) = \textit{arg}(Y_3^f) = 0$$

$$\sqrt{2} Y_2^f v_s = -Y_3^f w_2 \cos^2\theta (1 - 3\tan^2\theta)$$

- *We do not alter the mass matrices as given by the S_3 symmetry if we perform the same rotation in both, up and down sectors and another different rotation in up and neutrino sector*
- *Once we identify this, we can easily and exactly diagonalize the mass matrices, writing*

$$
\begin{aligned}
\mathcal{M}^f &= \mathcal{P}_f^\dagger \bar{\mathcal{M}}^f \mathcal{P}_f, \\
\bar{\mathcal{M}}^f &= m_3^f \begin{pmatrix} 0 & \sqrt{\dfrac{\widetilde{m}_1^f \widetilde{m}_2^f}{1-\delta_f}} & 0 \\ \sqrt{\dfrac{\widetilde{m}_1^f \widetilde{m}_2^f}{1-\delta_f}} & \widetilde{m}_1^f - \widetilde{m}_2^f + \delta_f & \sqrt{\dfrac{\delta_f}{1-\delta_f}\xi_1^f \xi_2^f} \\ 0 & \sqrt{\dfrac{\delta_f}{1-\delta_f}\xi_1^f \xi_2^f} & 1-\delta_f \end{pmatrix}; \\
\widetilde{m}_i^f &\equiv \frac{m_i^f}{m_3^f}, \quad \xi_1^f \equiv 1 - \widetilde{m}_1^f - \delta_f, \quad \xi_2^f \equiv 1 + \widetilde{m}_2^f - \delta_f,
\end{aligned}
$$

Quark mixing

- *In this parameterization, we can obtain exact formulas for CKM elements*

$$V_{ud}^{th} = \sqrt{\frac{\widetilde{m}_c \widetilde{m}_s \xi_1^u \xi_1^d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{m}_u \widetilde{m}_d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_1^u \xi_1^d} + \sqrt{\delta_u \delta_d \xi_2^u \xi_2^d} e^{i\phi_2} \right) e^{i\phi_1},$$

$$V_{us}^{th} = -\sqrt{\frac{\widetilde{m}_c \widetilde{m}_d \xi_1^u \xi_2^d}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{m}_u \widetilde{m}_s}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_1^u \xi_2^d} + \sqrt{\delta_u \delta_d \xi_2^u \xi_1^d} e^{i\phi_2} \right) e^{i\phi_1},$$

$$V_{ub}^{th} = \sqrt{\frac{\widetilde{m}_c \widetilde{m}_d \widetilde{m}_s \delta_d \xi_1^u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} + \sqrt{\frac{\widetilde{m}_u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\delta_d \xi_1^u} - \sqrt{\delta_u \xi_2^u \xi_1^d \xi_2^d} e^{i\phi_2} \right) e^{i\phi_1},$$

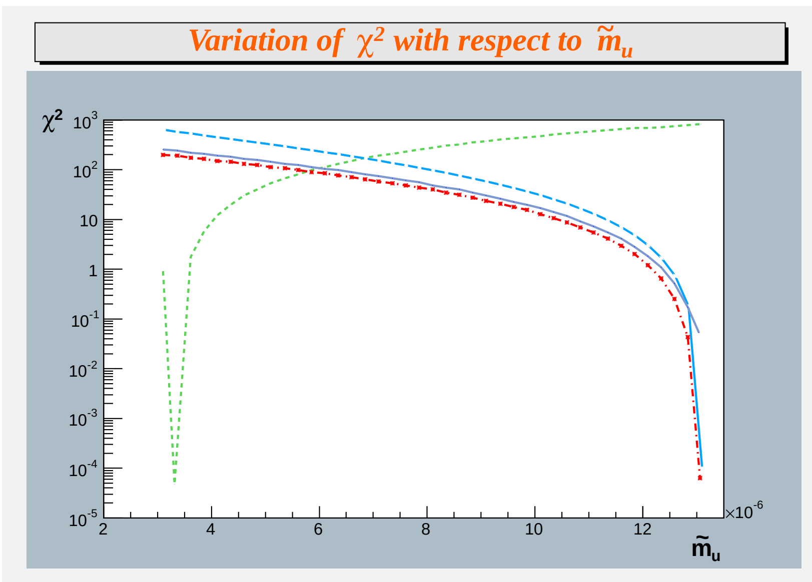
- *Which allow us to make a precision* χ^2 *test*

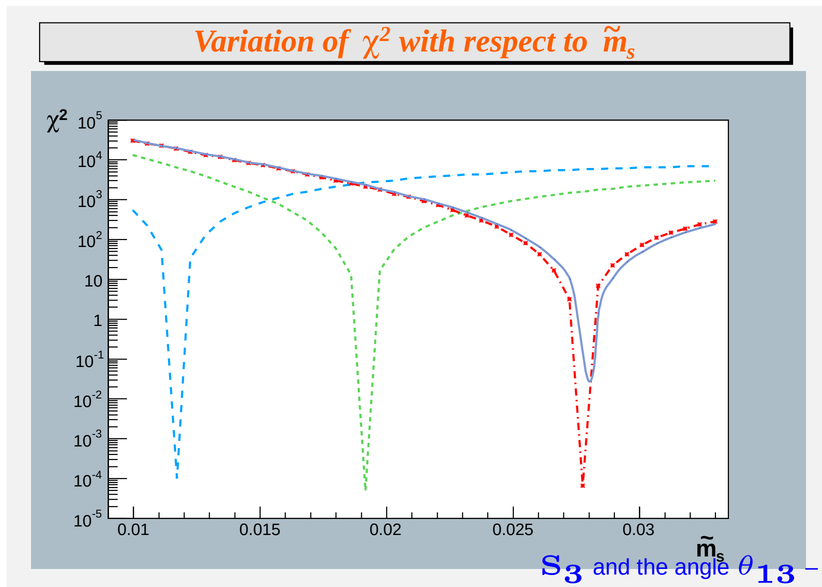
$$\chi^2 = \frac{\left(V_{ud}^{th} - V_{ud}\right)^2}{\sigma_{V_{ud}}^2} + \frac{\left(V_{us}^{th} - V_{us}\right)^2}{\sigma_{V_{us}}^2} + \frac{\left(V_{ub}^{th} - V_{ub}\right)^2}{\sigma_{V_{ub}}^2} + \frac{\left(\mathcal{J}_q^{th} - \mathcal{J}_q\right)^2}{\sigma_{\mathcal{J}_q}^2},$$

$\tilde{m}_u$ and $\tilde{m}_s$ have the biggest uncertainties, so have a big impact on the χ^2 value

	Mass ratios at M_Z
$\widetilde{m_u}\,(M_Z)$	0.0000081 ± 0.0000025
$\widetilde{m_c}\,(M_Z)$	0.0036 ± 0.0004
$\widetilde{m_d}\,(M_Z)$	0.000998 ± 0.000188
$\widetilde{m_s}\,(M_Z)$	0.021 ± 0.006

Fits with 3 or 4 Higgs doublets





Lepton mixing

- *See-saw* $M_{\nu_L} = M_{\nu D} M_{\nu_R}^{-1} M_{\nu D}^T$
- *The mixing matrix has exactly the same form of the quark sector with the obvious replacements*

$$V_{e1}^{th} = \sqrt{\frac{\widetilde{m}_\mu \widetilde{m}_{\nu_2} f_{l1} f_{\nu 1}}{\mathcal{D}_{l1}\mathcal{D}_{\nu 1}}} + \sqrt{\frac{\widetilde{m}_e \widetilde{m}_{\nu_1}}{\mathcal{D}_{l1}\mathcal{D}_{\nu 1}}} \left(\sqrt{(1-\delta_l)(1-\delta_\nu) f_{l1} f_{\nu 1}} e^{i\Phi_1} + \sqrt{\delta_l \delta_\nu f_{l2} f_{\nu 2}} e^{i\Phi_2} \right),$$
$$V_{e2}^{th} = -\sqrt{\frac{\widetilde{m}_\mu \widetilde{m}_{\nu_1} f_{l1} f_{\nu 2}}{\mathcal{D}_{l1}\mathcal{D}_{\nu 2}}} + \sqrt{\frac{\widetilde{m}_e \widetilde{m}_{\nu_2}}{\mathcal{D}_{l1}\mathcal{D}_{\nu 2}}} \left(\sqrt{(1-\delta_l)(1-\delta_\nu) f_{l1} f_{\nu 2}} e^{i\Phi_1} + \sqrt{\delta_l \delta_\nu f_{l2} f_{\nu 1}} e^{i\Phi_2} \right),$$
$$V_{e3}^{th} = \sqrt{\frac{\widetilde{m}_\mu \widetilde{m}_{\nu_1} \widetilde{m}_{\nu_2} \delta_\nu f_{l1}}{\mathcal{D}_{l1}\mathcal{D}_{\nu 3}}} + \sqrt{\frac{\widetilde{m}_e}{\mathcal{D}_{l1}\mathcal{D}_{\nu 3}}} \left(\sqrt{\delta_\nu (1-\delta_l)(1-\delta_\nu) f_{l1}} e^{i\Phi_1} - \sqrt{\delta_e f_{l2} f_{\nu 1} f_{\nu 2}} e^{i\Phi_2} \right),$$

- *Neutrino masses (example of values with 3 Higgs doublets)*

$$\widetilde{m}_{\nu_1} = \sqrt{1 - \frac{(\Delta m_{32}^2 + \Delta m_{21}^2)}{m_{\nu_3}^2}},\ \widetilde{m}_{\nu_2} = \sqrt{1 - \frac{\Delta m_{32}^2}{m_{\nu_3}^2}}.$$

$$m_{\nu_1} = \left(3.22^{+0.67}_{-0.39}\right)\times 10^{-3}\ \text{eV},\ m_{\nu_2} = \left(9.10^{+0.25}_{-0.13}\right)\times 10^{-3}\ \text{eV},\ m_{\nu_3} = \left(4.92^{+0.21}_{-0.22}\right)\times 10^{-2}\ \text{eV}.$$

Mixing angles

- *3 Higgs doublets:* M_{1R} *and* M_{2R} *degenerate (our case has the same features as the one in 1205.4755)*

$$\sin^2\theta_{12}^{l^{th}} \approx \frac{f_{\nu 2}\left\{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)+2\sqrt{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\cos\Phi_1\right\}}{(1+\widetilde{m}_{\nu_2})(1-\delta_\nu)\left(1+\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}\right)\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)},$$

$$\sin^2\theta_{23}^{l^{th}} \approx \frac{\delta_\nu+\delta_e f_{\nu 2}-\sqrt{\delta_\nu\delta_e f_{\nu 2}}\cos(\Phi_1-\Phi_2)}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})},$$

$$\sin^2\theta_{13}^{l^{th}} \approx \frac{\delta_\nu\left\{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}+\frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)}-2\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)}}\cos\Phi_1\right\}}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})}.$$

-

$$\theta_{12}^{l^{th}} = \left(34.43^{+0.85}_{-0.98}\right)^\circ, \quad \theta_{23}^{l^{th}} = \left(43.60^{+1.97}_{-2.22}\right)^\circ, \quad \theta_{13}^{l^{th}} = \left(6.80^{+0.95}_{-0.66}\right)^\circ,$$

The angle θ_{13}

- *4 Higgs doublets:* M_{1R} *and* M_{2R} *not degenerate*

$$m_{\nu_2} = 0.056 \text{ eV}, \quad m_{\nu_1} = 0.053 \text{ eV}, m_{\nu_3} = 0.048 \text{ eV},$$
$$\delta_l = \pi/2, \quad \mu_0 = 0.049 \text{ eV}, \quad d = 8 \times 10^{-5} \ \text{ eV},$$

$$\sin^2 \theta^l_{13} \approx \frac{(\mu_0 + 2d - m_{\nu_3})(\mu_0 - m_{\nu_3})}{(m_{\nu_1} - m_{\nu_3})(m_{\nu_2} - m_{\nu_3})}.$$

$$\theta^l_{13} \approx 9.8^\circ$$

Daya Bay $\rightarrow \ \theta^l_{13} \approx 9.1^\circ \qquad d = \frac{2|\lambda||\mu_2^\nu|^2}{|\overline{M}|}$, μ_2^ν *free parameter*

$$\lambda = \frac{1}{2}\left(\frac{M_2 - M_1}{M_1 + M_2}\right), \ \overline{M} = 2\frac{M_1 M_2}{M_2 + M_1}.$$

Counting of free parameters

- *Quark sector: there are 10 observables*
- *Lepton sector: there are $10+2$ observables*
- *Quark sector: (both models o.k. but different qualities of the fit)*
  - *3 Higgs-doublets: 9 free parameters*
  - *4 Higgs-doublets: 10 parameters*
- *Lepton sector:*
  - *3 Higgs-doublets: M_{1R} and M_{2R} almost degenerate, θ^l_{13} small: 9 free parameters*
  - *4 Higgs-doublets: M_{1R} not M_{2R} degenerate : 10 free parameters*

$$\theta^l_{13} \approx 9.8^\circ$$

- *Status: We can accommodate observables to a great degree*

Outlook

- *Try to reduce parameters*
- *FCNC under control, but precise values still to be computed*
- *Interesting contributions to the* a_μ
- *Understand the values of the VEVs of the Higgs* $\leftrightarrow$ *EWSB*