A Minimal Model of Neutrino Flavor

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Work in progress w/Christoph Luhn (IPPP, Durham) &
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Neutrino Mixing Matrix

- Neutrinos have mass and the different flavors can mix

  Super-Kamiokande Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of atmospheric neutrinos," *Phys. Rev. Lett.* **81** (1998) 1562–1567, hep-ex/9807003

  SNO Collaboration, Q. R. Ahmad *et al.*, "Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory," *Phys. Rev. Lett.* **89** (2002) 011301, nucl-ex/0204008

- Charged lepton and neutrino mass matrices cannot be simultaneously diagonalized

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

- Pontecorvo-Maki-Nakagawa-Sakata matrix

  B. Pontecorvo, "Mesonium and antimesonium," *Sov. Phys. JETP* **6** (1957) 429

  Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog. Theor. Phys.* **28** (1962) 870–880

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{D_L U_L^\dagger}_{U_{\text{PMNS}}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrino Mixing Matrix

What we know about the mixing angles ...

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{13} s_{23} c_{12} e^{i\delta} & c_{23} c_{12} - s_{13} s_{23} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - s_{13} c_{23} c_{12} e^{i\delta} & -s_{23} c_{12} - s_{13} c_{23} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

D. Forero, M. Tortola, and J. Valle, "Global status of neutrino oscillation parameters after recent reactor measurements," 1205.4018

Parameter	Best Fit	1σ range	3σ range
θ_{12}	$34.45°$	$33.40° - 35.37°$	$31.31° - 37.46°$
θ_{23}	$44.43°$	$41.55° - 49.02°$	$38.65° - 53.13°$
θ_{13}	$9.28°$	$8.53° - 9.80°$	$7.03° - 10.94°$

Tribimaximal Mixing

- Until recently, our best guess was tribimaximal mixing (TBM)

P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Phys. Lett.* **B530** (2002) 167, hep-ph/0202074

$$U_{\mathrm{PMNS}} \overset{?}{=} U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$\hookrightarrow \quad \theta_{12} = 35.26^\circ, \ \ \theta_{23} = 45^\circ, \ \ \theta_{13} = 0^\circ$

- Agreement still quite good for θ_{12}, θ_{23}, but $\theta_{13} = 0^\circ$ excluded @5σ

Forero et al, 1205.4018, **DAYA-BAY** Collaboration, 1203.1669, **RENO** Collaboration, 1204.0626

Parameter	Tribimaximal	Global fit 1σ	Daya Bay	Reno	
θ_{12}	35.26°	$33.40^\circ - 35.37^\circ$	-	-	✓
θ_{23}	45.00°	$41.55^\circ - 49.02^\circ$	-	-	✓
θ_{13}	0.00°	$8.53^\circ - 9.80^\circ$	8.8°	9.8°	✗

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Post-Daya Bay Confusion

- Regular patter U_{PMNS} is suggestive of a family symmetry

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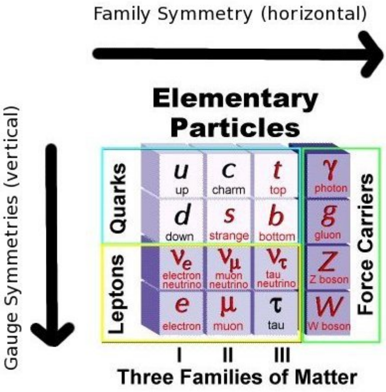
- Daya bay and Reno rule out tribimaximal Mixing
- What are our options?
  - Give up family symmetries? Some remarks towards the end ...
  - Look for groups that give $\theta_{13} \neq 0^\circ$
    R. d. A. Toorop, F. Feruglio, and C. Hagedorn, "Discrete Flavour Symmetries in Light of T2K," *Phys.Lett.* **B703** (2011) 447–451, 1107.3486
  - Keep TBM and calculate higher corrections $\rightarrow$ This talk!

Discrete Flavor Symmetries

- Introduce relations between families of quarks and leptons
- But which discrete group do we take for the family symmetry?



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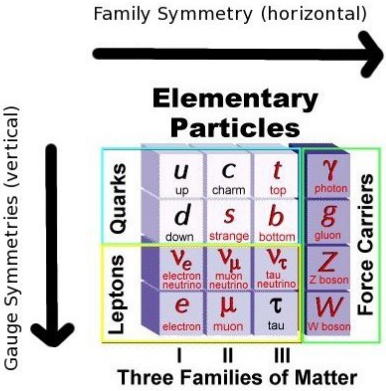
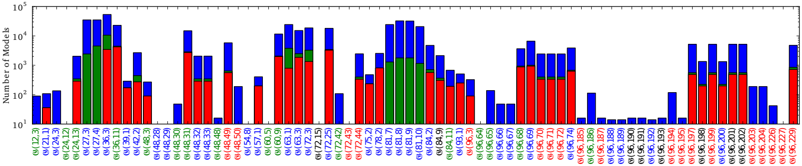


Discrete Flavor Symmetries

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

- We scanned 76 discrete groups
- Many papers on $A_4 \times \mathbb{Z}_n$, $n \geq 3$. Connection between A_4 and TBM?
- $\Delta(96)$ gives $\theta_{13} \neq 0^\circ$ but is large
- We identified the smallest group that gives TBM!



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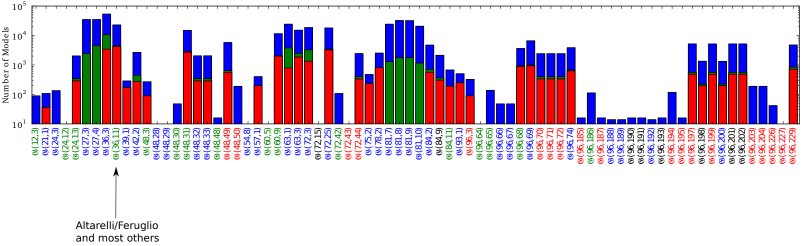

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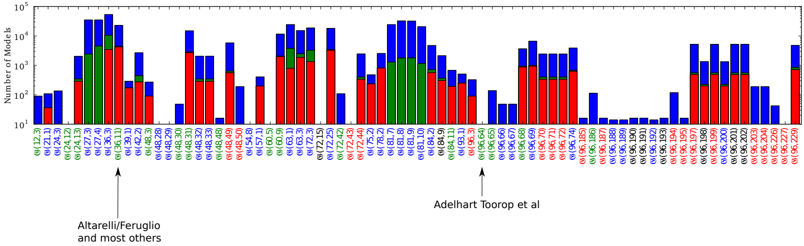

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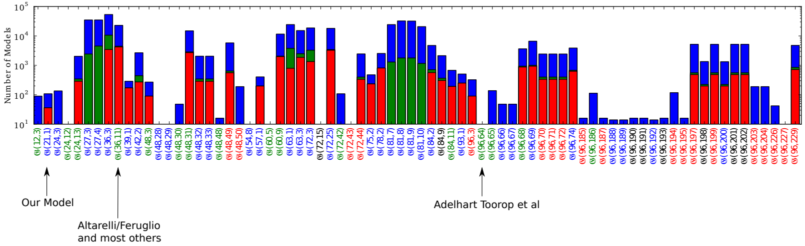

Discrete Flavor Symmetries

C. Luhn, K. M. Parattu, A. Wingerter, *work in progress*

❶ Symmetries of the model

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times T_7 \times \mathrm{U}(1)_R$$

❷ Particle content and charges

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	T_7	$\mathrm{U}(1)_R$
L	$(\mathbf{2}, \text{-}1)$	$\mathbf{3}$	1
e	$(\mathbf{1}, 2)$	$\mathbf{1}$	1
μ	$(\mathbf{1}, 2)$	$\mathbf{1'}$	1
τ	$(\mathbf{1}, 2)$	$\mathbf{1''}$	1
h_u	$(\mathbf{2}, 1)$	$\mathbf{1}$	0
h_d	$(\mathbf{2}, \text{-}1)$	$\mathbf{1}$	0
φ	$(\mathbf{1}, 0)$	$\mathbf{3}$	0
$\widetilde{\varphi}$	$(\mathbf{1}, 0)$	$\mathbf{3'}$	0

❸ Breaking the family symmetry

$$\langle \varphi \rangle = (v_\varphi, v_\varphi, v_\varphi), \quad \langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$$

All you need to know about T_7

Tensor Products

$\mathbf{1} \times \mathbf{1} = \mathbf{1}$
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Contractions

$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3},$

$$z = \begin{pmatrix} \frac{1}{3}\sqrt{3}x_1y_1 + \frac{1}{3}\sqrt{3}x_2y_3 + \frac{1}{3}\sqrt{3}x_3y_2 \\ x_1y_2\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) + x_2y_1\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) + x_3y_3\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) \\ x_1y_3\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) + x_2y_2\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) + x_3y_1\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) \end{pmatrix}$$

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Our T_7 Model at Leading Order

➢ Terms that are invariant, have 2 leptons and mass dimension $\leq$ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3}' \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}') \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}'$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3 \\ & \quad + y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2 \\ & \quad + y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1 \end{aligned}$$

➢ Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Our T_7 Model at Leading Order

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3'} \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}) \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3 \\ & \quad + y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2 \\ & \quad + y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1 \end{aligned}$$

➢ Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Our T_7 Model at Leading Order

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$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3'} \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}) \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3 \\ & \quad + y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2 \\ & \quad + y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1 \end{aligned}$$

➢ Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Our T_7 Model at Leading Order

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3'} \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}) \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3 \\ & \quad + y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2 \\ & \quad + y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1 \end{aligned}$$

➢ Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Our T_7 Model at Leading Order

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\tilde\varphi}}{\sqrt{6}\Lambda} \times \begin{array}{c c} & \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{array}{c} L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} & \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \end{array}, \quad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c c} & \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{array}{c} L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} & \begin{pmatrix} \sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde\varphi} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi \\ -\frac{1}{2}\sqrt{2}y_2 v_\varphi & \sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde\varphi} \\ -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde\varphi} & \sqrt{2}y_2 v_\varphi \end{pmatrix} \end{array}$$

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form-diagonalizable! Mixing does not depend on A, B, masses do!

➢ Mixing angles: $\theta_{12} = 35.26°$, $\theta_{23} = 45°$, $\theta_{13} = 0°$ Tribimaximal ✔

Our T_7 Model at Leading Order

- Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\widetilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{array}{c c} & \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{array}{c} L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} & \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \end{array}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c c} & \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{array}{c} L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} & \begin{pmatrix} \sqrt{2} y_2 v_\varphi + 2 y_1 v_{\widetilde{\varphi}} & -\frac{1}{2}\sqrt{2} y_2 v_\varphi & -\frac{1}{2}\sqrt{2} y_2 v_\varphi \\ -\frac{1}{2}\sqrt{2} y_2 v_\varphi & \sqrt{2} y_2 v_\varphi & -\frac{1}{2}\sqrt{2} y_2 v_\varphi + 2 y_1 v_{\widetilde{\varphi}} \\ -\frac{1}{2}\sqrt{2} y_2 v_\varphi & -\frac{1}{2}\sqrt{2} y_2 v_\varphi + 2 y_1 v_{\widetilde{\varphi}} & \sqrt{2} y_2 v_\varphi \end{pmatrix} \end{array}$$

- Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

- Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

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Our T_7 Model at Leading Order

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\tilde\varphi}}{\sqrt{6}\Lambda} \times \begin{array}{c|ccc} & e & \mu & \tau \\ \hline L_1^{(2)} & y_e & 0 & 0 \\ L_2^{(2)} & 0 & y_\mu & 0 \\ L_3^{(2)} & 0 & 0 & y_\tau \end{array}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c|ccc} & L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \\ \hline L_1^{(1)} & \sqrt{2}A + 2B & -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A \\ L_2^{(1)} & -\frac{1}{2}\sqrt{2}A & \sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B \\ L_3^{(1)} & -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B & \sqrt{2}A \end{array}$$

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$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

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Our T_7 Model at Leading Order

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\tilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{array}{c c} & \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{array}{c} L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} & \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \end{array}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c c} & \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{array}{c} L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} & \begin{pmatrix} \sqrt{2}A + 2B & -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A \\ -\frac{1}{2}\sqrt{2}A & \sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B \\ -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B & \sqrt{2}A \end{pmatrix} \end{array}$$

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

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➢ Mixing angles: $\theta_{12} = 35.26^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$ Tribimaximal ✔

Next-to-Leading-Order Corrections

- Remember leading-order superpotential:

> - Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:
>
> $$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$
>
> $$\mathbf{3'} \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}) \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}$$

- Superpotential (now mass dimension $\leq$6 or 7)

$$\begin{aligned}
&C_9\; L\, e\, h_d\, \widetilde{\varphi} \;+\; C_{14}\, L\, \mu\, h_d\, \widetilde{\varphi} \;+\; C_{19}\, L\, \tau\, h_d\, \widetilde{\varphi} \;+\\
&C_{10}\, L\, e\, h_d\, \varphi\, \varphi \;+\; C_{11}\, L\, e\, h_d\, \varphi\, \widetilde{\varphi} \;+\; C_{12}\, L\, e\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} \;+\; C_{13}\, L\, e\, h_u\, h_d\, h_d\, \widetilde{\varphi} \;+\\
&C_{15}\, L\, \mu\, h_d\, \varphi\, \varphi \;+\; C_{16}\, L\, \mu\, h_d\, \varphi\, \widetilde{\varphi} \;+\; C_{17}\, L\, \mu\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} \;+\; C_{18}\, L\, \mu\, h_u\, h_d\, h_d\, \widetilde{\varphi} \;+\\
&C_{20}\, L\, \tau\, h_d\, \varphi\, \varphi \;+\; C_{21}\, L\, \tau\, h_d\, \varphi\, \widetilde{\varphi} \;+\; C_{22}\, L\, \tau\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} \;+\; C_{23}\, L\, \tau\, h_u\, h_d\, h_d\, \widetilde{\varphi} \;+\\
&C_1\; L\, L\, h_u\, h_u\, \varphi \;+\; C_2\; L\, L\, h_u\, h_u\, \widetilde{\varphi} \;+\\
&C_3\; (L\, L)_{\mathbf{3}}\, h_u\, h_u\, \varphi\, \varphi \;+\; C_4\; (L\, L)_{\mathbf{3'}}\, h_u\, h_u\, \varphi\, \varphi \;+\\
&C_7\; (L\, L)_{\mathbf{3}}\, h_u\, h_u\, \widetilde{\varphi}\, \widetilde{\varphi} \;+\; C_8\; (L\, L)_{\mathbf{3'}}\, h_u\, h_u\, \widetilde{\varphi}\, \widetilde{\varphi} \;+\\
&C_5\; (L\, L)_{\mathbf{3}}\, h_u\, h_u\, \varphi\, \widetilde{\varphi} \;+\; C_6\; (L\, L)_{\mathbf{3'}}\, h_u\, h_u\, \varphi\, \widetilde{\varphi}
\end{aligned}$$

- Leading-order terms, neglected terms, NLO terms

Next-to-Leading-Order Corrections

- The Charged Lepton Sector

$$\Delta M_\ell = -\frac{1}{3\sqrt{2}} v_d \Big[C_{10} \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} + C_{11} \begin{pmatrix} 1&0&0\\1&0&0\\1&0&0 \end{pmatrix} + C_{12} \begin{pmatrix} 1&0&0\\0&0&0\\0&0&0 \end{pmatrix}$$

$$+ C_{15} \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} + C_{16} \begin{pmatrix} 0&1&0\\0&1&0\\0&1&0 \end{pmatrix} + C_{17} \begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix}$$

$$+ C_{20} \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} + C_{21} \begin{pmatrix} 0&0&1\\0&0&1\\0&0&1 \end{pmatrix} + C_{22} \begin{pmatrix} 0&0&0\\0&0&0\\0&0&1 \end{pmatrix} \Big]$$

- The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{24\sqrt{3}} \Big[C_3 \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} + 3\sqrt{2} C_4 \begin{pmatrix} 2 & -\omega^2 & -\omega \\ -\omega^2 & 2\omega & -1 \\ -\omega & -1 & 2\omega^2 \end{pmatrix} + 4 C_7 \begin{pmatrix} 1&0&0\\0&0&1\\0&1&0 \end{pmatrix}$$

$$+ C_8 \begin{pmatrix} 2&0&0\\0&0&-1\\0&-1&0 \end{pmatrix} + 4 C_5 \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + \sqrt{2} C_6 \begin{pmatrix} 2 & -\omega & -\omega^2 \\ -\omega & 2\omega^2 & -1 \\ -\omega^2 & -1 & 2\omega \end{pmatrix} \Big]$$

Switching the constants on and off

	$\exp(2\pi i)$			$\exp(2\pi i/3)$			$\exp(2\pi i/5)$		
	θ_{12}	θ_{23}	θ_{13}	θ_{12}	θ_{23}	θ_{13}	θ_{12}	θ_{23}	θ_{13}
C_3	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_4	28.52	45.00	3.47	37.73	56.24	8.90	32.28	57.91	8.50
C_5	35.34	45.00	3.48	35.42	41.51	5.04	35.41	41.80	4.83
C_6	31.22	45.00	2.18	36.92	37.73	5.57	33.64	36.76	5.60
C_7	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_8	33.57	45.00	0.00	36.13	45.00	0.00	34.72	45.00	0.00
C_{10}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{11}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{12}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{15}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{16}	31.68	44.95	3.58	37.08	44.83	4.10	33.85	44.90	3.79
C_{17}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{20}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{21}	31.03	51.15	4.22	37.40	42.38	4.97	33.52	47.78	4.52
C_{22}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00

Switching the constants on and off

➢ The Charged Lepton Sector

$$\Delta M_\ell = -\frac{1}{3\sqrt{2}} v_d \Big[C_{10} \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} + C_{11} \begin{pmatrix} 1&0&0\\1&0&0\\1&0&0 \end{pmatrix} + C_{12} \begin{pmatrix} 1&0&0\\0&0&0\\0&0&0 \end{pmatrix}$$

$$+ C_{15} \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} + C_{16} \begin{pmatrix} 0&1&0\\0&1&0\\0&1&0 \end{pmatrix} + C_{17} \begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix}$$

$$+ C_{20} \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} + C_{21} \begin{pmatrix} 0&0&1\\0&0&1\\0&0&1 \end{pmatrix} + C_{22} \begin{pmatrix} 0&0&0\\0&0&0\\0&0&1 \end{pmatrix} \Big]$$

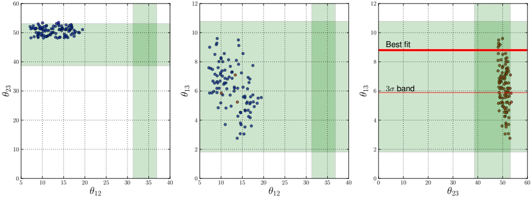
➢ The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{24\sqrt{3}} \Big[C_3 \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} + 3\sqrt{2} C_4 \begin{pmatrix} 2 & -\omega^2 & -\omega \\ -\omega^2 & 2\omega & -1 \\ -\omega & -1 & 2\omega^2 \end{pmatrix} + 4 C_7 \begin{pmatrix} 1&0&0\\0&0&1\\0&1&0 \end{pmatrix}$$

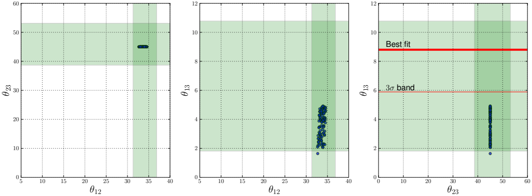
$$+ C_8 \begin{pmatrix} 2&0&0\\0&0&-1\\0&-1&0 \end{pmatrix} + 4 C_5 \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + \sqrt{2} C_6 \begin{pmatrix} 2 & -\omega & -\omega^2 \\ -\omega & 2\omega^2 & -1 \\ -\omega^2 & -1 & 2\omega \end{pmatrix} \Big]$$

Varying the Constants

- No constraints on the C_i

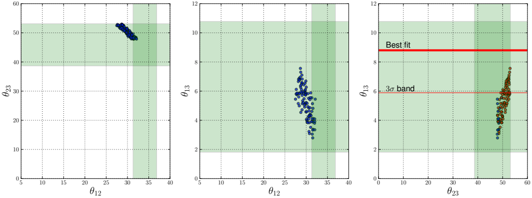


- $C_4 = C_6 = C_{16} = C_{21} = 0$

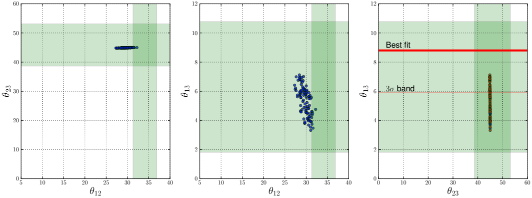


Varying the Constants

- $C_4 = C_6 = C_{16} = 0, \; C_{21} \neq 0$



- $C_4 = C_6 = C_{21} = 0, \; C_{16} \neq 0$



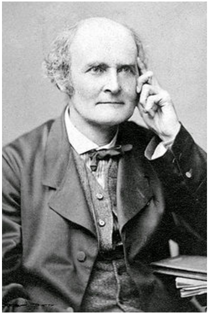
Which Flavor Symmetry?

When would we claim that a given flavor symmetry is specifically well-suited to describe neutrino mixing?

- First of all, it should reproduce the data!
  TBM is excluded at 5σ. What does this mean for A_4 (or T_7)?
- It should convincingly reproduce the data!
  - One can (probably) always tweak TBM models to give $\theta_{13} \neq 0$.
  - θ_{12}, θ_{23} in agreement w/TBM. Is that enough?
  - Angles should be insensitive to C_i $\longrightarrow$ Form-diagonalizable?
  - Or: Correlation between neutrino masses and angles
- Ideally, flavor symmetry arises from some more fundamental theory
- The choice of group should be guided by above principles
- "Intuition", geometric imagination and "easiness" may be misleading $\longrightarrow$ Consider the case of A_4

Which Flavor Symmetry?

Arthur Cayley (1821-1895) is the first to systematically construct groups; in 1854, he determined all groups of order 4 and 6 . . .



The Small Groups library

All groups (423,164,062) of order $\leq$ 2000 except 1024

Hans Ulrich Besche, Bettina Eick and Eamonn O'Brien

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[1, 1]	1	✗	---	---	---	✗
[2, 1]	$\mathbb{Z}_2$	✗	---	---	---	✗
[3, 1]	$\mathbb{Z}_3$	✗	---	---	---	✗
[4, 1]	$\mathbb{Z}_4$	✗	---	---	---	✗
[4, 2]	$\mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[5, 1]	$\mathbb{Z}_5$	✗	---	---	---	✗
[6, 1]	S_3	✗	✔	✔	✔	✗
[6, 2]	$\mathbb{Z}_6$	✗	---	---	---	✗
[7, 1]	$\mathbb{Z}_7$	✗	---	---	---	✗
[8, 1]	$\mathbb{Z}_8$	✗	---	---	---	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[8, 2]	$\mathbb{Z}_4 \times \mathbb{Z}_2$	✗	---	---	---	✗
[8, 3]	D_4	✗	✔	✔	✔	✗
[8, 4]	Q_8	✗	✔	✔	✔	✗
[8, 5]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[9, 1]	$\mathbb{Z}_9$	✗	---	---	---	✗
[9, 2]	$\mathbb{Z}_3 \times \mathbb{Z}_3$	✗	---	---	---	✗
[10, 1]	D_5	✗	✔	✔	✔	✗
[10, 2]	$\mathbb{Z}_{10}$	✗	---	---	---	✗
[11, 1]	$\mathbb{Z}_{11}$	✗	---	---	---	✗
[12, 1]	$\mathbb{Z}_3 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[12, 2]	$\mathbb{Z}_{12}$	✗	---	---	---	✗
[12, 3]	A_4	✔	✔	✗	✗	✔
[12, 4]	D_6	✗	✔	✔	✔	✗
[12, 5]	$\mathbb{Z}_6 \times \mathbb{Z}_2$	✗	---	---	---	✗
[13, 1]	$\mathbb{Z}_{13}$	✗	---	---	---	✗
[14, 1]	D_7	✗	✔	✔	✔	✗
[14, 2]	$\mathbb{Z}_{14}$	✗	---	---	---	✗
[15, 1]	$\mathbb{Z}_{15}$	✗	---	---	---	✗
[16, 1]	$\mathbb{Z}_{16}$	✗	---	---	---	✗
[16, 2]	$\mathbb{Z}_4 \times \mathbb{Z}_4$	✗	---	---	---	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[16, 3]	$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✗	✔	✗
[16, 4]	$\mathbb{Z}_4 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✗	✔	✗
[16, 5]	$\mathbb{Z}_8 \times \mathbb{Z}_2$	✗	---	---	---	✗
[16, 6]	$\mathbb{Z}_8 \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✔	✔	✗
[16, 7]	D_8	✗	✔	✔	✔	✗
[16, 8]	QD_8	✗	✔	✔	✔	✗
[16, 9]	Q_{16}	✗	✔	✔	✔	✗
[16, 10]	$\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[16, 11]	$\mathbb{Z}_2 \times D_4$	✗	✔	✗	✔	✗
[16, 12]	$\mathbb{Z}_2 \times Q_8$	✗	✔	✗	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[16, 13]	$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✔	✔	✗
[16, 14]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[17, 1]	$\mathbb{Z}_{17}$	✗	---	---	---	✗
[18, 1]	D_9	✗	✔	✔	✔	✗
[18, 2]	$\mathbb{Z}_{18}$	✗	---	---	---	✗
[18, 3]	$\mathbb{Z}_3 \times S_3$	✗	✔	✔	✔	✗
[18, 4]	$(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes_\varphi \mathbb{Z}_2$	✗	✗	✗	✗	✗
[18, 5]	$\mathbb{Z}_6 \times \mathbb{Z}_3$	✗	---	---	---	✗
[19, 1]	$\mathbb{Z}_{19}$	✗	---	---	---	✗
[20, 1]	$\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[20, 2]	$\mathbb{Z}_{20}$	✗	---	---	---	✗
[20, 3]	$\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$	✗	✗	✗	✗	✗
[20, 4]	D_{10}	✗	✔	✔	✔	✗
[20, 5]	$\mathbb{Z}_{10} \times \mathbb{Z}_2$	✗	---	---	---	✗
[21, 1]	$\mathbb{Z}_7 \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
[21, 2]	$\mathbb{Z}_{21}$	✗	---	---	---	✗
[22, 1]	D_{11}	✗	✔	✔	✔	✗
[22, 2]	$\mathbb{Z}_{22}$	✗	---	---	---	✗
[23, 1]	$\mathbb{Z}_{23}$	✗	---	---	---	✗
[24, 1]	$\mathbb{Z}_3 \rtimes_\varphi \mathbb{Z}_8$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[24, 2]	$\mathbb{Z}_{24}$	✗	---	---	---	✗
[24, 3]	SL(2, 3)	✔	✔	✔	✔	✗
[24, 4]	$\mathbb{Z}_3 \rtimes_\varphi Q_8$	✗	✔	✔	✔	✗
[24, 5]	$\mathbb{Z}_4 \times S_3$	✗	✔	✔	✔	✗
[24, 6]	D_{12}	✗	✔	✔	✔	✗
[24, 7]	$\mathbb{Z}_2 \times (\mathbb{Z}_3 \rtimes_\varphi \mathbb{Z}_4)$	✗	✔	✗	✔	✗
[24, 8]	$(\mathbb{Z}_6 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✔	✔	✗
[24, 9]	$\mathbb{Z}_{12} \times \mathbb{Z}_2$	✗	---	---	---	✗
[24, 10]	$\mathbb{Z}_3 \times D_4$	✗	✔	✔	✔	✗
[24, 11]	$\mathbb{Z}_3 \times Q_8$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[24, 12]	S_4	✔	✔	✗	✗	✔
[24, 13]	$\mathbb{Z}_2 \times A_4$	✔	✔	✗	✗	✔
[24, 14]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times S_3$	✗	✔	✗	✔	✗
[24, 15]	$\mathbb{Z}_6 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[25, 1]	$\mathbb{Z}_{25}$	✗	---	---	---	✗
[25, 2]	$\mathbb{Z}_5 \times \mathbb{Z}_5$	✗	---	---	---	✗
[26, 1]	D_{13}	✗	✔	✔	✔	✗
[26, 2]	$\mathbb{Z}_{26}$	✗	---	---	---	✗
[27, 1]	$\mathbb{Z}_{27}$	✗	---	---	---	✗
[27, 2]	$\mathbb{Z}_9 \times \mathbb{Z}_3$	✗	---	---	---	✗

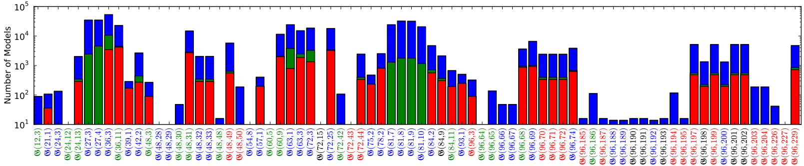
Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[27, 3]	$(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
[27, 4]	$\mathbb{Z}_9 \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
[27, 5]	$\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3$	✗	---	---	---	✗
[28, 1]	$\mathbb{Z}_7 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✔	✔	✗
[28, 2]	$\mathbb{Z}_{28}$	✗	---	---	---	✗
[28, 3]	D_{14}	✗	✔	✔	✔	✗
[28, 4]	$\mathbb{Z}_{14} \times \mathbb{Z}_2$	✗	---	---	---	✗
[29, 1]	$\mathbb{Z}_{29}$	✗	---	---	---	✗
[30, 1]	$\mathbb{Z}_5 \times S_3$	✗	✔	✔	✔	✗
[30, 2]	$\mathbb{Z}_3 \times D_5$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842



- 50% of groups have tribimaximal models
- Smallest group that can produce TBM: $\mathfrak{G}(21,1) = T_7$
- Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$
- A_4 has nice geometric interpretation, but what does that mean? That humans like to think in terms of geometry?
- There is probably no special connection between A_4 and TBM!

Conclusions

- We started from TBM and NLO corrections give $\theta_{13} \neq 0^\circ$
- Model is very economical:
  - Family symmetry T_7 is second-smallest group w/3-dim irreps
  - No extra (shaping) symmetries, i.e. no extra $\mathrm{U}(1)$ or $\mathbb{Z}_N$
  - Only 2 flavon fields
- Vacuum stabilization kind of a headache
  $\longrightarrow$ Either more symmetry or more flavons
- Which flavor symmetry to use?
  - Daya Bay & Reno have shattered the TBM paradigm
  - Probably there is no special connection between A_4 and TBM
  - Need to look beyond smallest groups and also critically review motivation for family symmetries

Backup

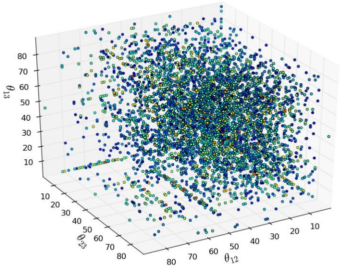
Backup

Slides

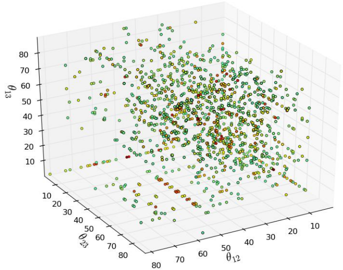
How did we generate this set of models?

Group is $A_4 \times \mathbb{Z}_3$

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842



(a) The 5528 bins that are ≥ 1.



(b) The 1287 bins that are ≥ 1000.

How did we generate this set of models?

❶ Fix the family symmetry (we considered 76 groups)

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times A_4 \times \mathbb{Z}_3 \times \mathrm{U}(1)_R$$

❷ Constant particle content; scan over all representations!

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	$A_4 \times \mathbb{Z}_3$	$\mathrm{U}(1)_R$
L	$(\mathbf{2},\text{-}1)$	$\mathbf{3'}$	1
e	$(\mathbf{1},\ 2)$	$\mathbf{1'}$	1
μ	$(\mathbf{1},\ 2)$	$\mathbf{1^{(8)}}$	1
τ	$(\mathbf{1},\ 2)$	$\mathbf{1^{(5)}}$	1
h_u	$(\mathbf{2},\ 1)$	$\mathbf{1}$	0
h_d	$(\mathbf{2},\text{-}1)$	$\mathbf{1}$	0
φ_T	$(\mathbf{1},\ 0)$	$\mathbf{3}$	0
φ_S	$(\mathbf{1},\ 0)$	$\mathbf{3'}$	0
ξ	$(\mathbf{1},\ 0)$	$\mathbf{3'}$	0

❸ Partial scan over vevs

$$\langle\varphi_T\rangle = (0/1, 0/1, 0/1), \quad \langle\varphi_S\rangle = (0/1, 0/1, 0/1), \quad \langle\xi\rangle = (0/1, 0/1, 0/1)$$

How did we generate this set of models?

➢ We consider 2 models equivalent, if their Lagrangians are the same after contracting the family indices, but before the vevs are substituted

➢ In this sense, we have 39,900 inequivalent models/Lagrangians

➢ 22,932 models have non-singular charged lepton and neutrino mass matrices:

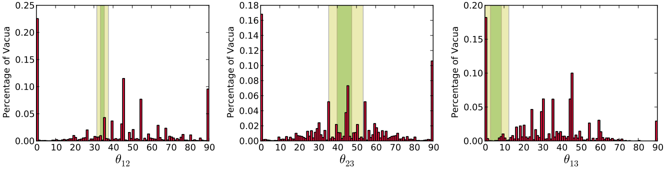
$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \qquad \hat{M}_\nu = U_L M_\nu U_R^\dagger, \qquad U_{\mathrm{PMNS}} \equiv D_{\mathrm{L}} U_{\mathrm{L}}^\dagger$$

➢ 4,481 consistent w/experiment at 3σ level (19.5%) (obsolete!)

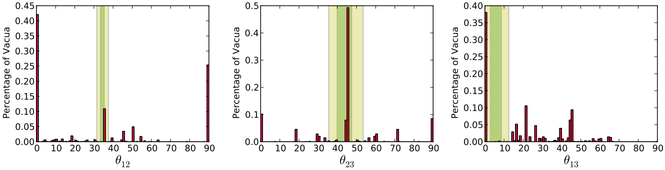
➢ 4,233 are tribimaximal (18.5%)

➢ Probably largest set of viable neutrino models ever constructed!

How did we generate this set of models?

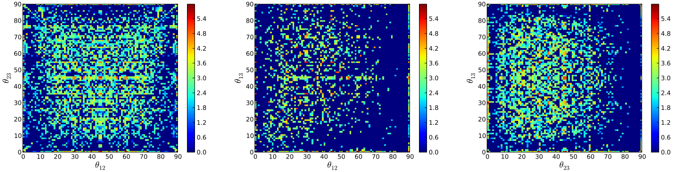


(c) Number of models that give θ_{ij} with no constraints on the other 2 angles. Each histogram has 15992118 entries.

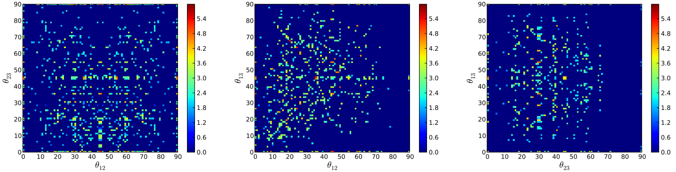


(d) Number of models that give θ_{ij} with the other 2 angles restricted to their 3σ interval. The histograms have 838289, 148886 and 225844 entries, respectively.

How did we generate this set of models?

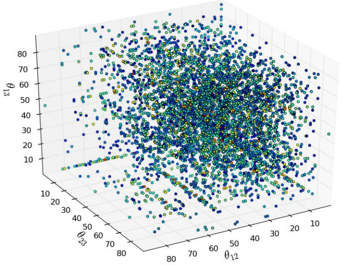


(e) Number of models that give θ_{ij} and θ_{mn} with no constraint on the remaining angle. Each histogram has 15992118 entries.

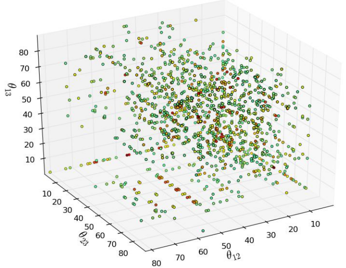


(f) Number of models that give θ_{ij} and θ_{mn} with the remaining angle restricted to its 3σ interval. The histograms have 2941000, 3675600 and 1057170 entries, respectively.

How did we generate this set of models?

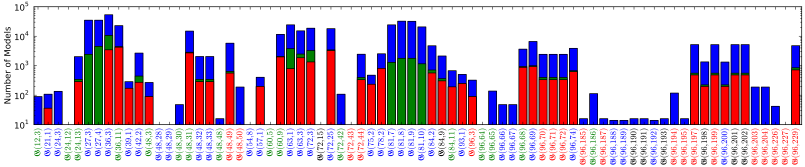


(g) The 5528 bins that are ≥ 1.



(h) The 1287 bins that are ≥ 1000.

How did we generate this set of models?



- Histogram bars: All models, 3σ, TBM
- 38 groups (50%) have tribimaximal models
- Smallest group that can produce TBM: $\mathfrak{G}(21,1) = T_7$
- Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$. Special?
- Group names: $\mathfrak{g} \subset \mathrm{U}(3)$, $\mathfrak{g} \supset A_4$, $A_4 \subset \mathfrak{g} \subset \mathrm{U}(3)$