

# Neutrinos, GUTs, and the Early Universe

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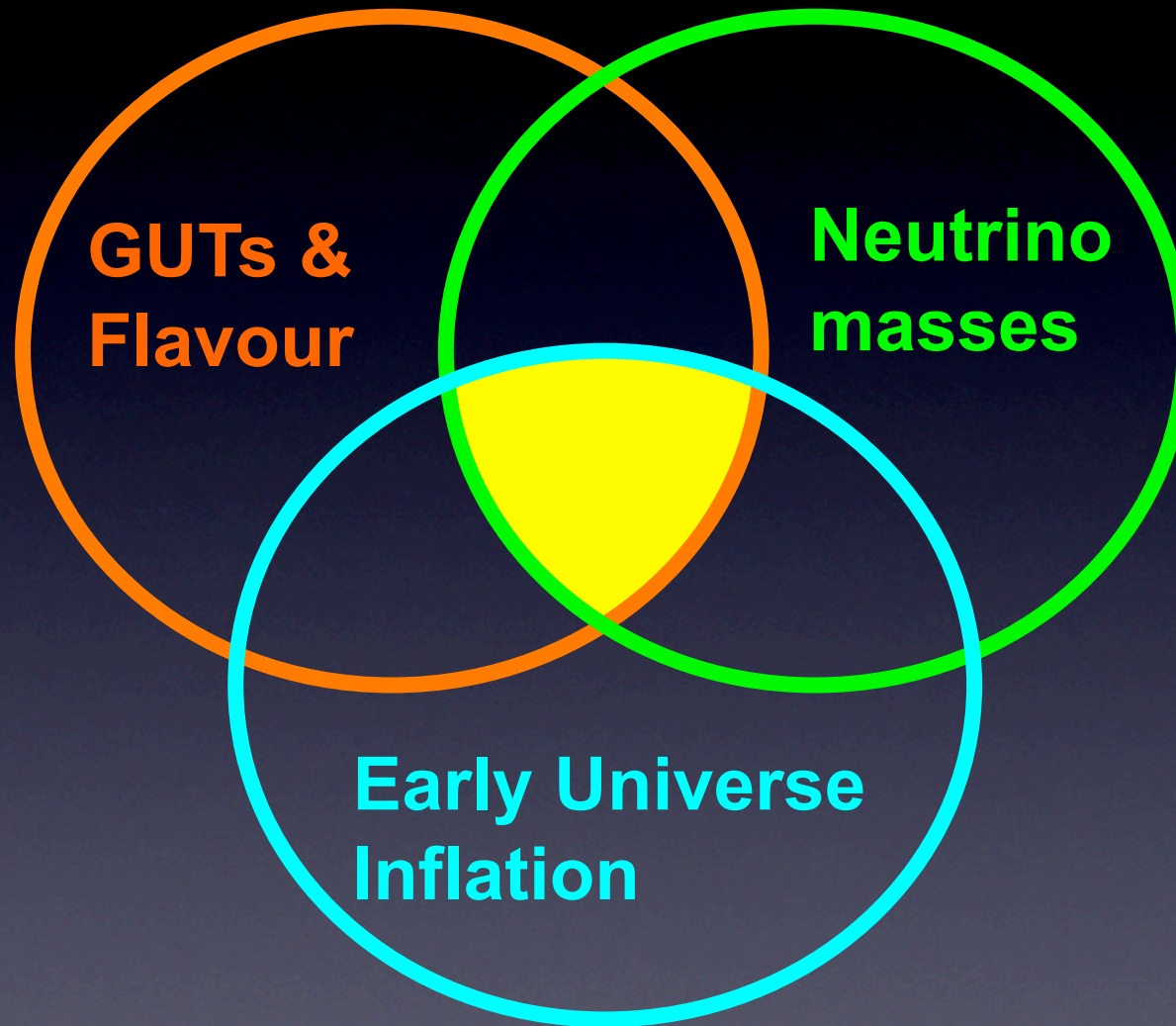
Max-Planck-Institut für Physik  
(Werner-Heisenberg-Institut)



“What is  $\nu$  ... ?”, Invisibles 12, Smirnov Fest  
GGI, Florence

June 26, 2012

# *Three challenges ...*





# *Overview: Two topics*

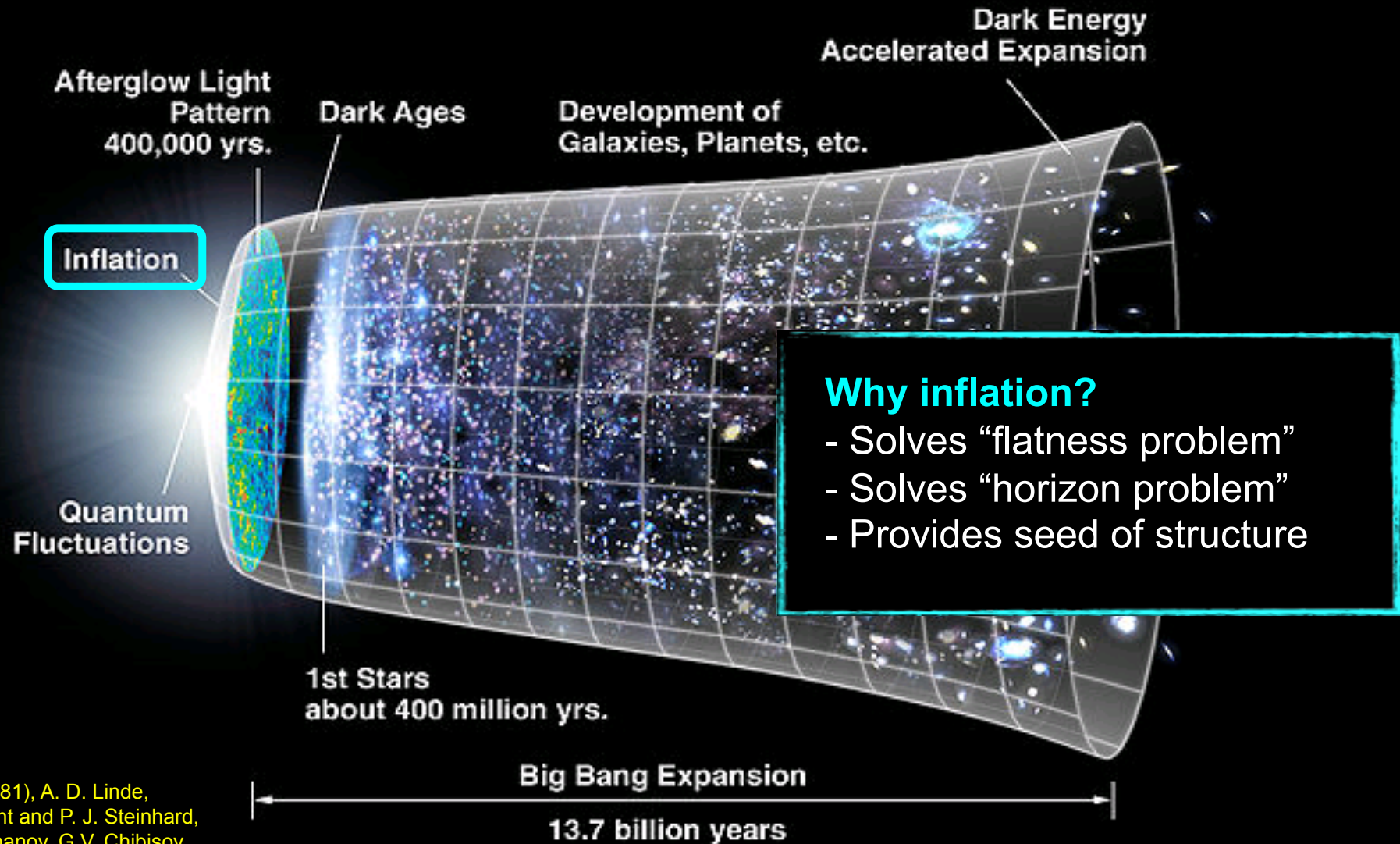
- Part 1: Neutrinos and inflation
- Part 2: Is there a footprint of GUTs in  $U_{\text{PMNS}}$ ?

First part of my talk:

# ***Neutrinos and inflation***



# Inflation = Era of accelerated expansion in the very early universe



A. Guth ('81), A. D. Linde,  
A. Albrecht and P. J. Steinhard,  
V.F. Mukhanov, G.V. Chibisov,  
A.H. Guth and S.Y. Pi,  
A.A. Starobinsky, S.W. Hawking

picture from WMAP website

# How can inflation be realised?

- Simple and attractive possibility: **Slowly rolling scalar field  $\phi$**  (minimally coupled to gravity)

$$T_{\mu\nu} = \partial_{\mu}\phi\partial_{\nu}\phi - g_{\mu\nu} \left( \frac{1}{2}\partial_{\rho}\phi\partial_{\rho}\phi + V(\phi) \right)$$

If the vacuum energy  $V(\phi)$  dominates:

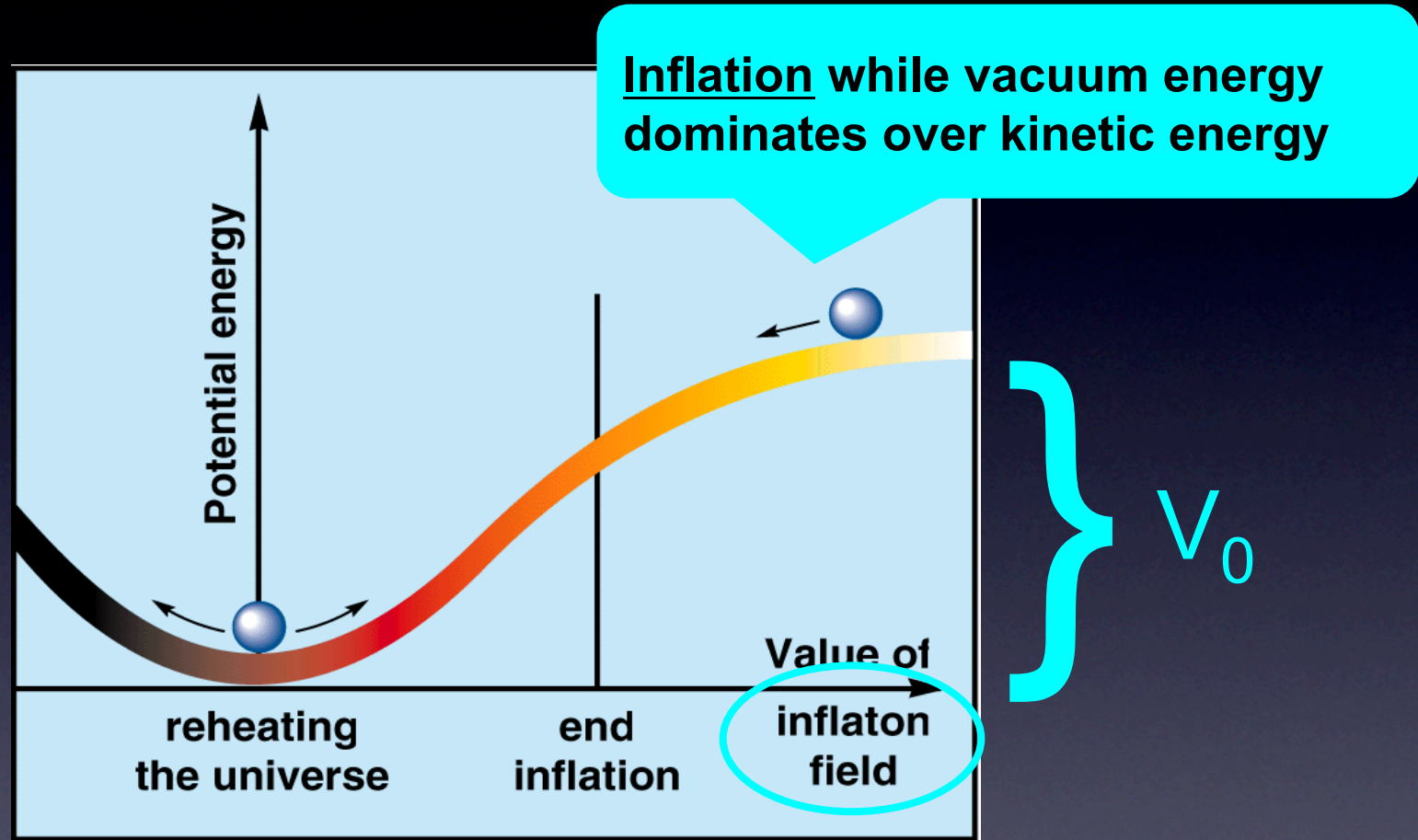
$$\Rightarrow a(t) = \exp \left( \sqrt{\frac{8\pi G_N V(\phi)}{3}} t \right)$$

and the universe “inflates”!

Important: **The field  $\phi$  is dynamical**  
 $\Rightarrow$  inflation can end!

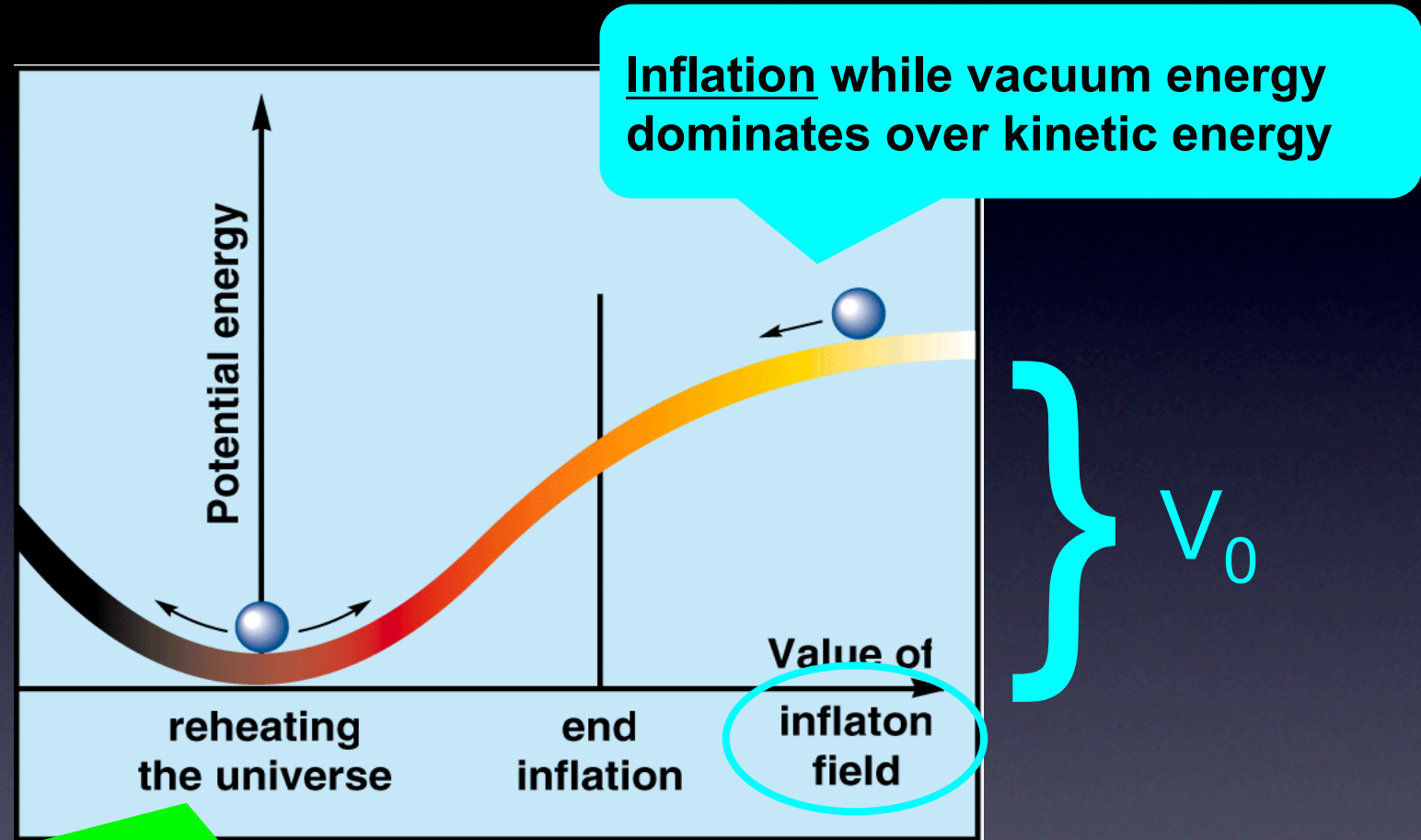


# Dynamics during and after inflation



Vacuum energy during inflation:  
 $(V_0)^{1/4} \sim 10^{16} \text{ GeV} \sim M_{\text{GUT}}$

# Dynamics during and after inflation



## Decays of the inflaton:

→ matter & antimatter, and possibly their asymmetry get produced!

Vacuum energy during inflation:  
 $(V_0)^{1/4} \sim 10^{16} \text{ GeV} \sim M_{\text{GUT}}$

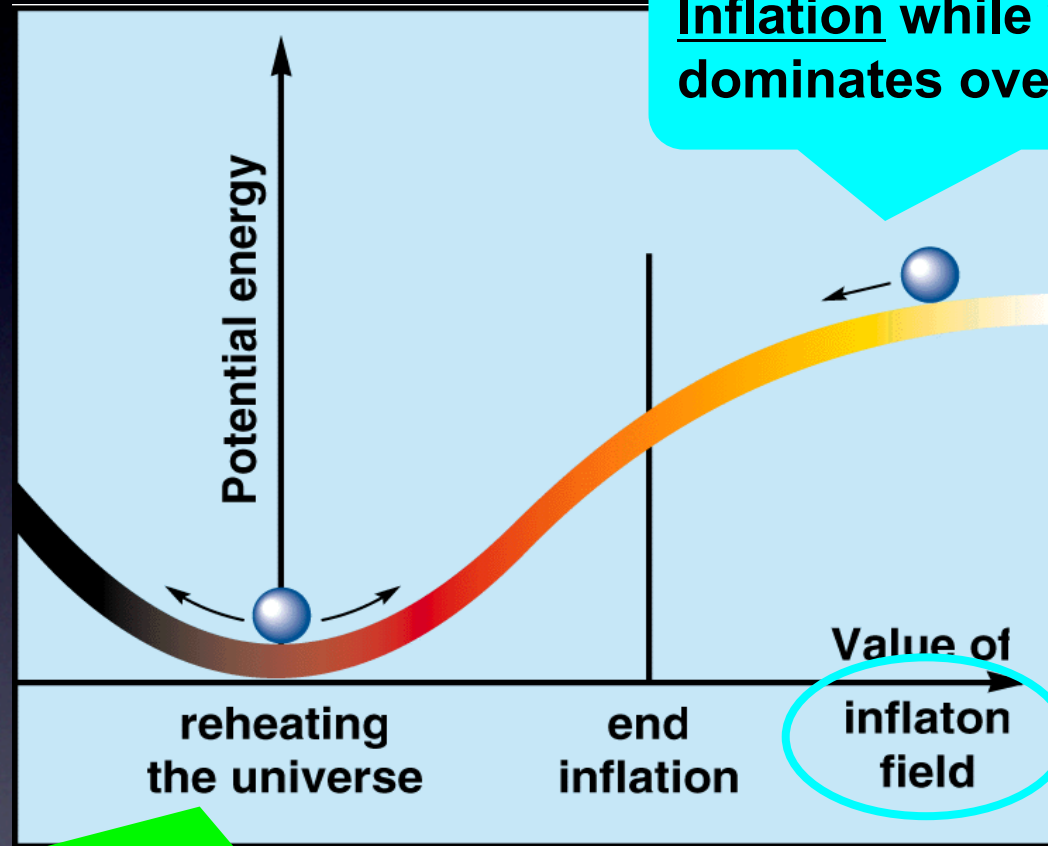


# Dynamics during and after inflation

## Requirements:

- more than  $\sim 60$  e-foldings of inflation
- consistency with CMB observables
- viable reheating

→ beyond the SM (or change gravity)



## Decays of the inflaton:

→ matter & antimatter, and possibly their asymmetry get produced!

Vacuum energy during inflation:  
 $(V_0)^{1/4} \sim 10^{16} \text{ GeV} \sim M_{\text{GUT}}$

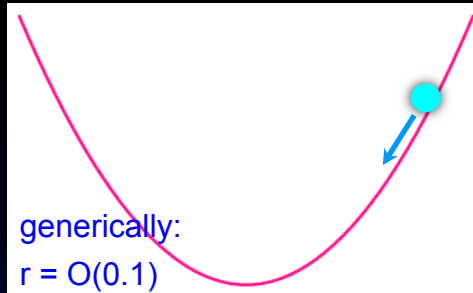
Two major questions:

***Which particle physics scenario can give rise to successful inflation?***

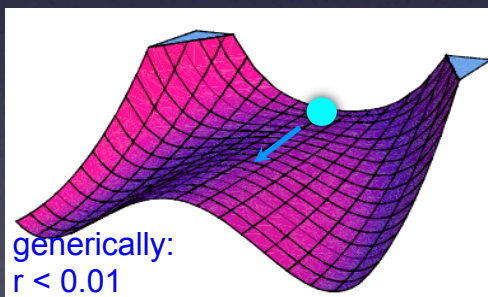
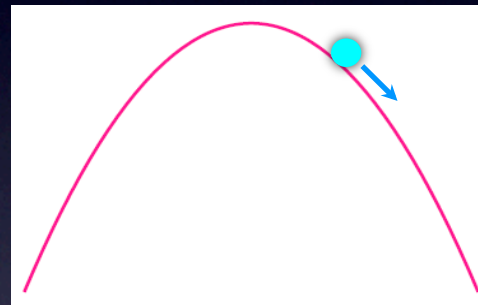
***And finally: Who is the inflaton particle?***



# Bottom-up considerations ...



'Large field' (chaotic) inflation



'Hybrid-type' inflation models

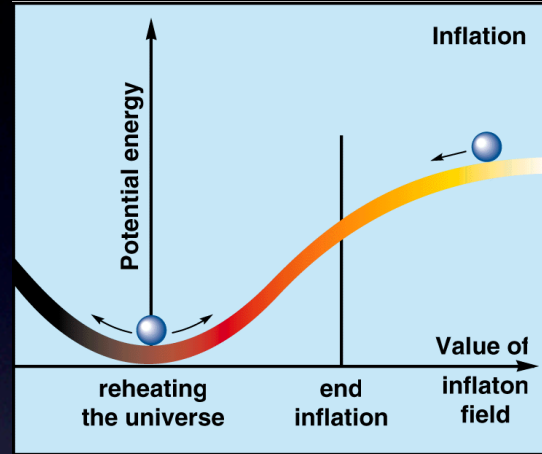
## Various considerations & viewpoints:

- Basic classes of inflation models
- Often: One just adds a singlet scalar field plus potential without connection to the rest of the theory ...
- Take SM particle (e.g. Higgs) and modify gravity (e.g. introduce non-minimal coupling to gravity ...)
- Look for candidates in extensions of the SM ...

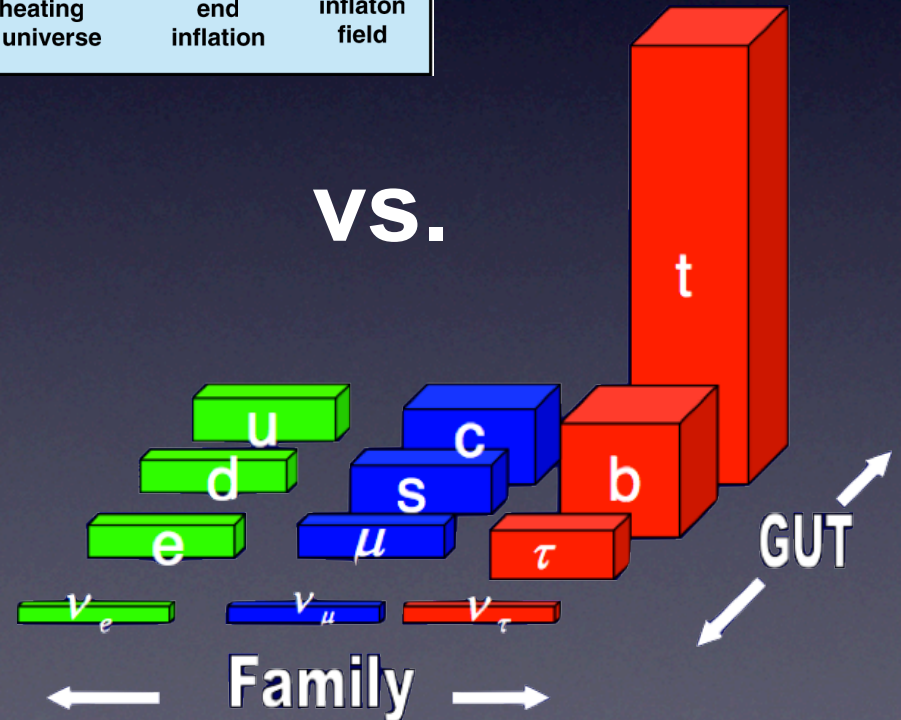
# Top-down viewpoints ...

## Various top-down possibilities:

- Grand Unified Theories (GUTs)?
- Family symmetries?
- Supersymmetry/Supergravity
- Seesaw mechanism for  $\nu$ -masses
- String theory, extra dimensions ...



VS.





# How does inflation fit into a more fundamental theory ... ?

## Various top-down possibilities:

- Grand Unified Theories (GUTs)?
- Family symmetries?
- Supersymmetry/Supergravity
- Seesaw mechanism for  $\nu$ -masses
- String theory, extra dimensions ...



## Various considerations & viewpoints:

- Basic classes of inflation models
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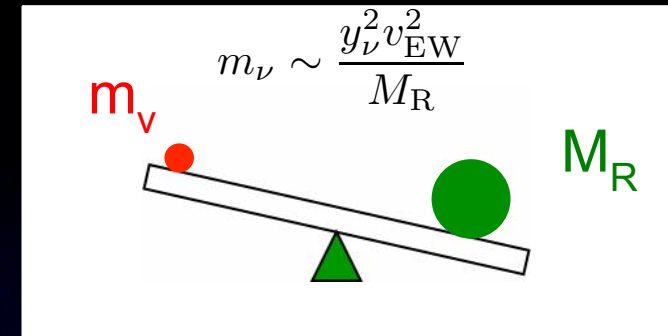
# Seesaw + SUSY $\rightarrow$ *The RH sneutrino as the inflaton*

The right-handed neutrino superfield:

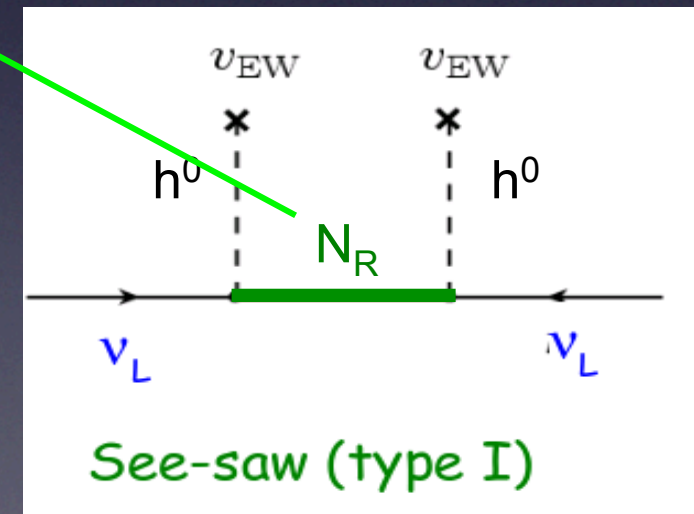
$$\nu_R = \tilde{N}_R + \sqrt{2}\theta N_R + \theta\theta F_{N_R}$$

The right-handed sneutrinos, i.e. the scalar superpartners of the RH neutrinos  $\rightarrow$  excellent candidates for acting as the inflaton field!

Framework: local supersymmetry = supergravity



P. Minkowski ('77), Mohapatra, Senjanovic, Yanagida, Gell-Mann, Ramond, Slansky, Schechter, Valle, ...





***Two possibilities for the origin of the large  
RH neutrino masses  $\leftrightarrow$  two options for  
realising inflation with RH sneutrinos***

# Origin of right-handed neutrino masses

I) Direct mass terms:

$$\mathcal{W}_{M_R} = M_R \nu_R \nu_R$$

II) Mass terms from spontaneous symmetry breaking

$$\mathcal{W}_{M_R} = \frac{\lambda}{M_{Pl}} \nu_R \nu_R H H$$



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For example:

In SO(10) GUTs:

$$\frac{1}{\Lambda} 16_i 16_j H_{\bar{16}} H_{\bar{16}}$$

In some  $A_4$  flavour models (with  $\theta^{(i)}$  flavons in 3 of  $A_4$ ):

$$\frac{1}{\Lambda} \nu_{Ri} \nu_{Rj} \theta^{(i)} \theta^{(j)}$$

# Chaotic Sneutrino Inflation

## I) Direct mass terms:

Murayama, Suzuki, Yanagida, Yokoyama ('93)

$$\mathcal{W}_{M_R} = M_R \nu_R \nu_R$$

Inflaton potential from:

$$|F_{\nu_R}|^2 = \left| \frac{\partial \mathcal{W}}{\partial \nu_R} \right|_{\theta=0} = |M_R \tilde{N}_R|^2$$

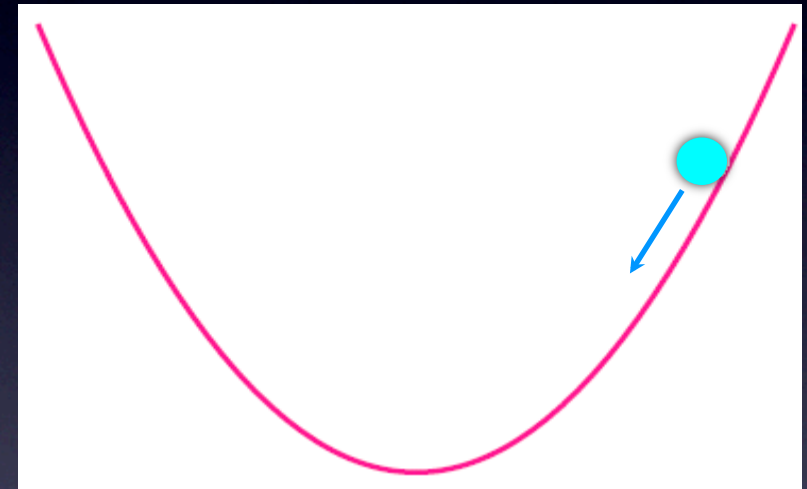
### Predictions for CMB observables:

$$n_s \approx 0.96, r \approx 0.16$$

### Predictions for neutrino physics:

$$M_R \sim 10^{13} \text{ GeV}$$

Note:  $\nu_R$  has to be a total singlet!



'Large field' (chaotic) sneutrino inflation

In supergravity:  
W+ suitable Kähler potential K



# Sneutrino Hybrid Inflation

## II) Mass term from spontaneous symmetry breaking (SSB)

$$\mathcal{W} = \kappa S(H^2 - M^2) + \frac{\lambda}{M_{\text{Pl}}} \nu_R \nu_R H H$$

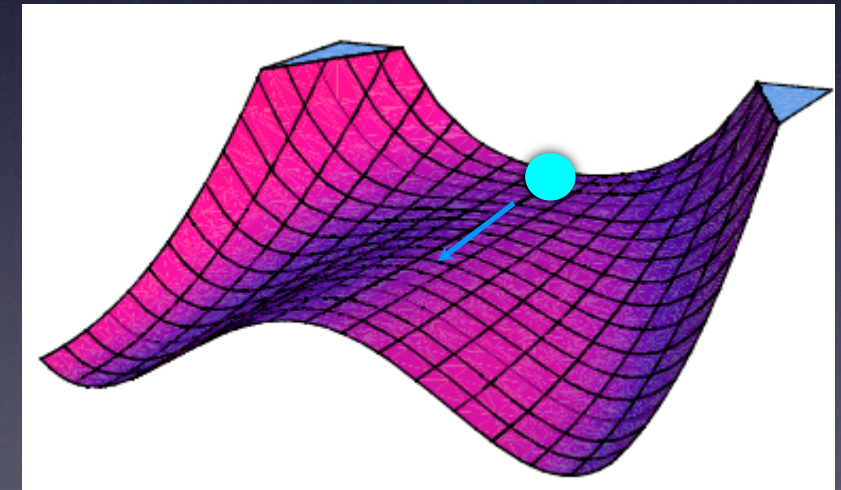
Additional term in  $\mathcal{W}$  is just a SUSY version of a SSB potential

$$|F_S|^2 \Rightarrow$$



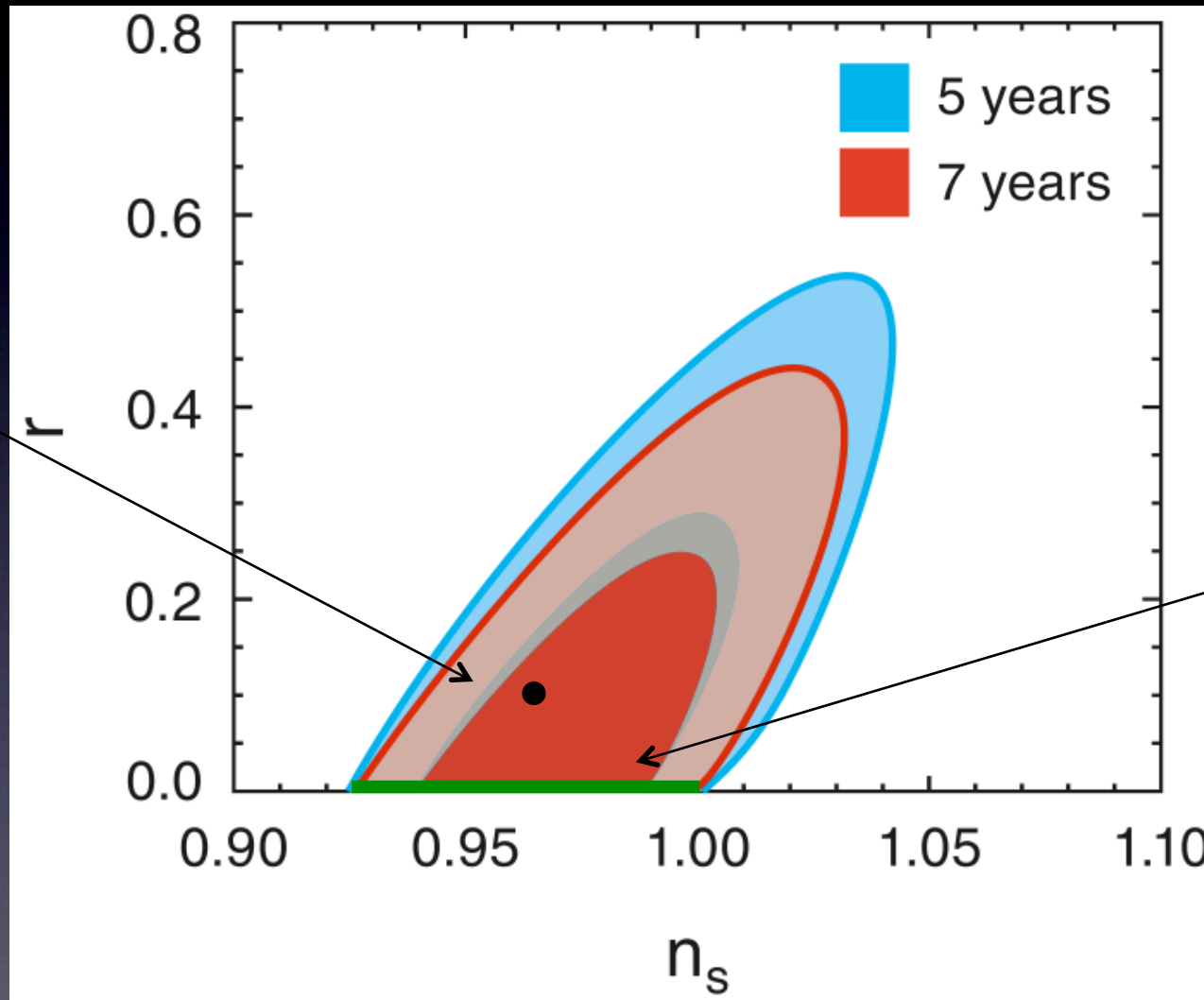
S.A., Bastero-Gil, King, Shafi ('04)

- i)  $\langle \tilde{N}_R \rangle \neq 0$  can stabilise  $H$  at  $\langle H \rangle = 0$  and leads to large vacuum energy  $V_0 \sim M^4$
- ii) Large masses for the RH (s)neutrinos when  $H$  gets a vev after inflation



'Hybrid-type' sneutrino inflation

# ***Chaotic ↔ Hybrid models can be distinguished by the results of the Planck satellite***



Prediction of  
**Chaotic  
Sneutrino  
Inflation**

Murayama,  
Suzuki,  
Yanagida,  
Yokoyama ('93)

Prediction of  
**Sneutrino  
Hybrid Inflation**

S.A., Bastero-Gil,  
King, Shafi ('04)

(WMAP '10, WMAP '08)



# Sneutrino Hybrid Inflation

$$\mathcal{W} = \kappa S (H^2 - M^2) + \frac{\lambda}{M_{\text{Pl}}} \nu_R \nu_R H H$$

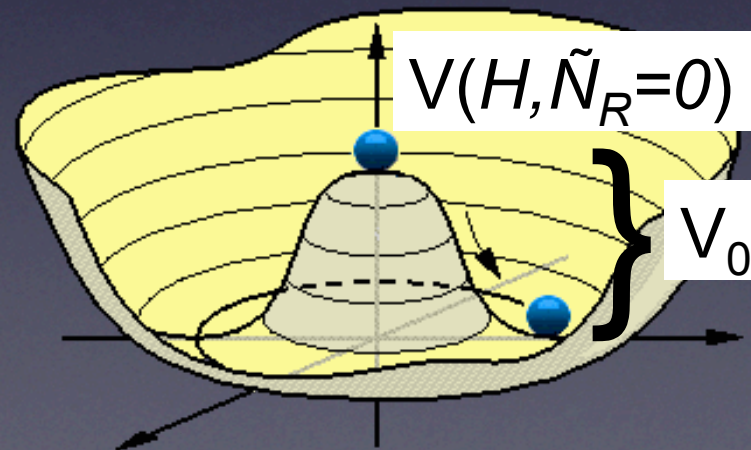
## Driving superfield

(its F-term generates the potential for H and provides the vacuum energy  $V_0$ ; **During and after inflation:**  $\langle S \rangle = 0$ .)

## Waterfall superfield

(contains the “waterfall field” (e.g. GUT- or Flavour-Higgs field) that **ends inflation by a 2<sup>nd</sup> order phase transition**)

$$|F_S|^2 \Rightarrow$$



In supergravity:  
 $\mathcal{W} + \text{suitable}$   
 Kähler potential  $K$

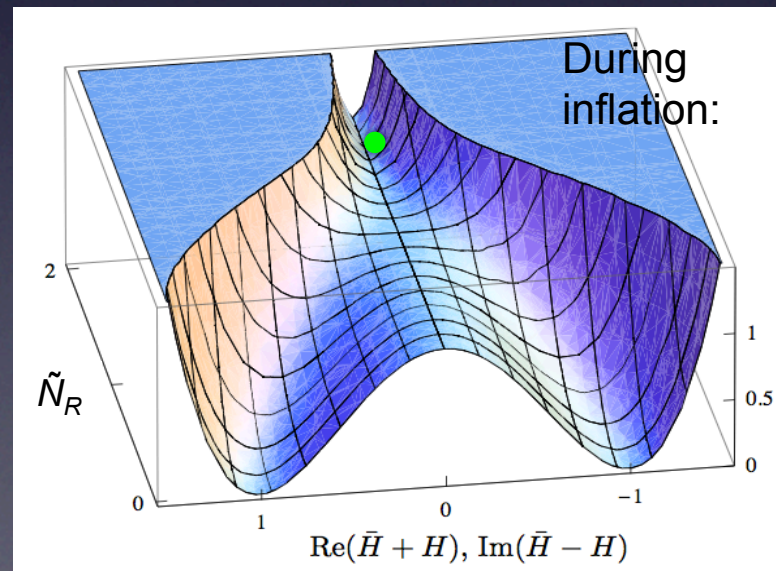
# Sneutrino Hybrid Inflation

$$|F_S|^2 \Rightarrow \mathcal{W} = \kappa S(H^2 - M^2) + \frac{\lambda}{M_{\text{Pl}}} \nu_R \nu_R H H$$



22

$\langle \tilde{N}_R \rangle > \tilde{N}_{R,\text{crit}}$

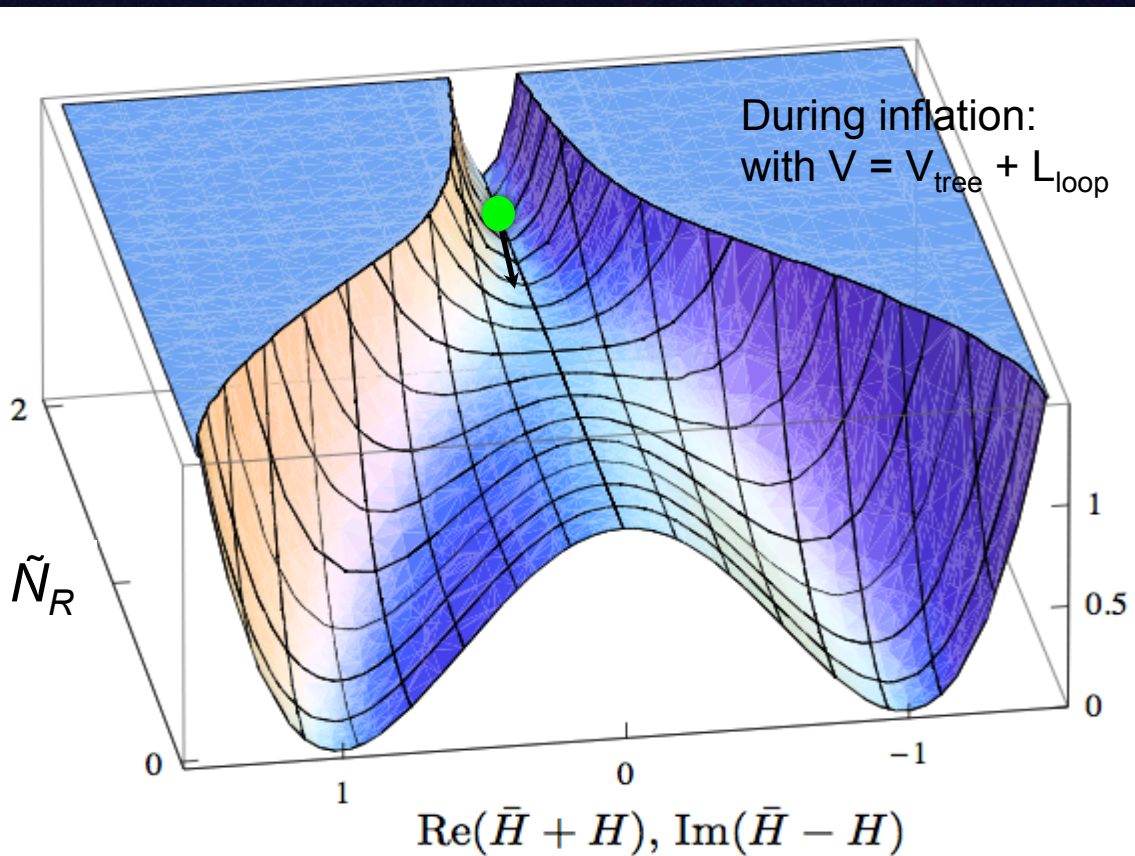


**Inflaton superfield**  
 ( $\nu_R$  contains the inflaton field  $\tilde{N}_R$  as scalar component;  
 For  $\langle \tilde{N}_R \rangle > \tilde{N}_{R,\text{crit}}$  it stabilises  $H$  at  $\langle H \rangle = 0$ )



# Sneutrino Hybrid Inflation

$$\mathcal{W} = \kappa S(H^2 - M^2) + \frac{\lambda}{M_{\text{Pl}}} \nu_R \nu_R H H$$

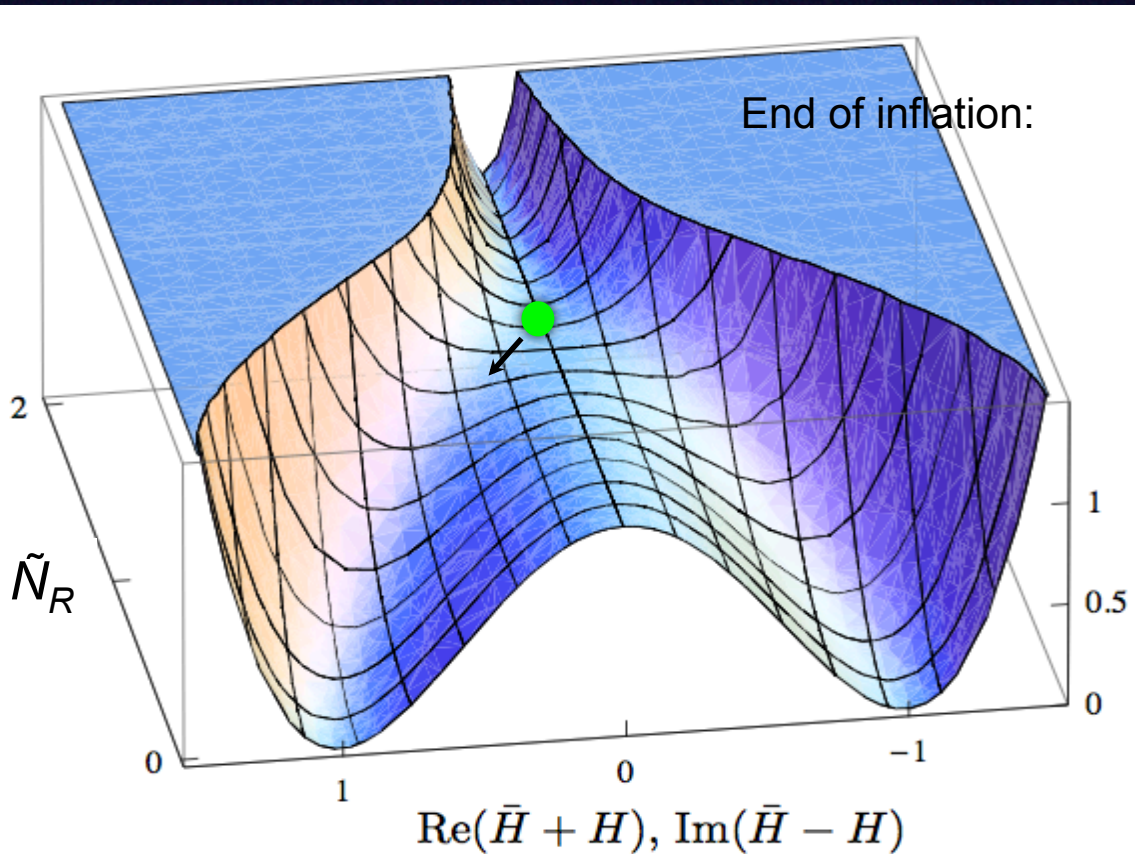


## Inflaton superfield

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# Sneutrino Hybrid Inflation

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## Inflaton superfield

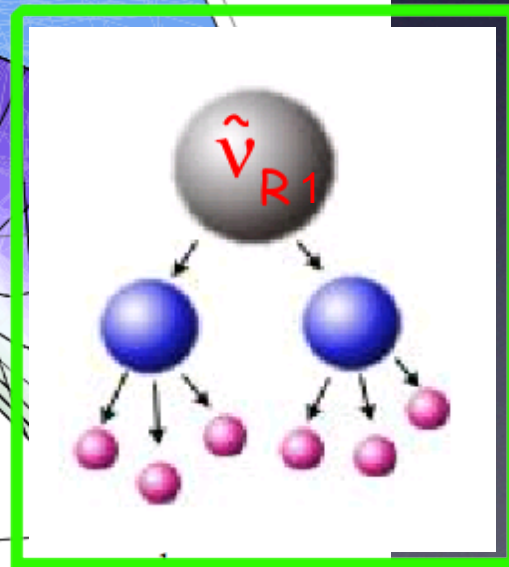
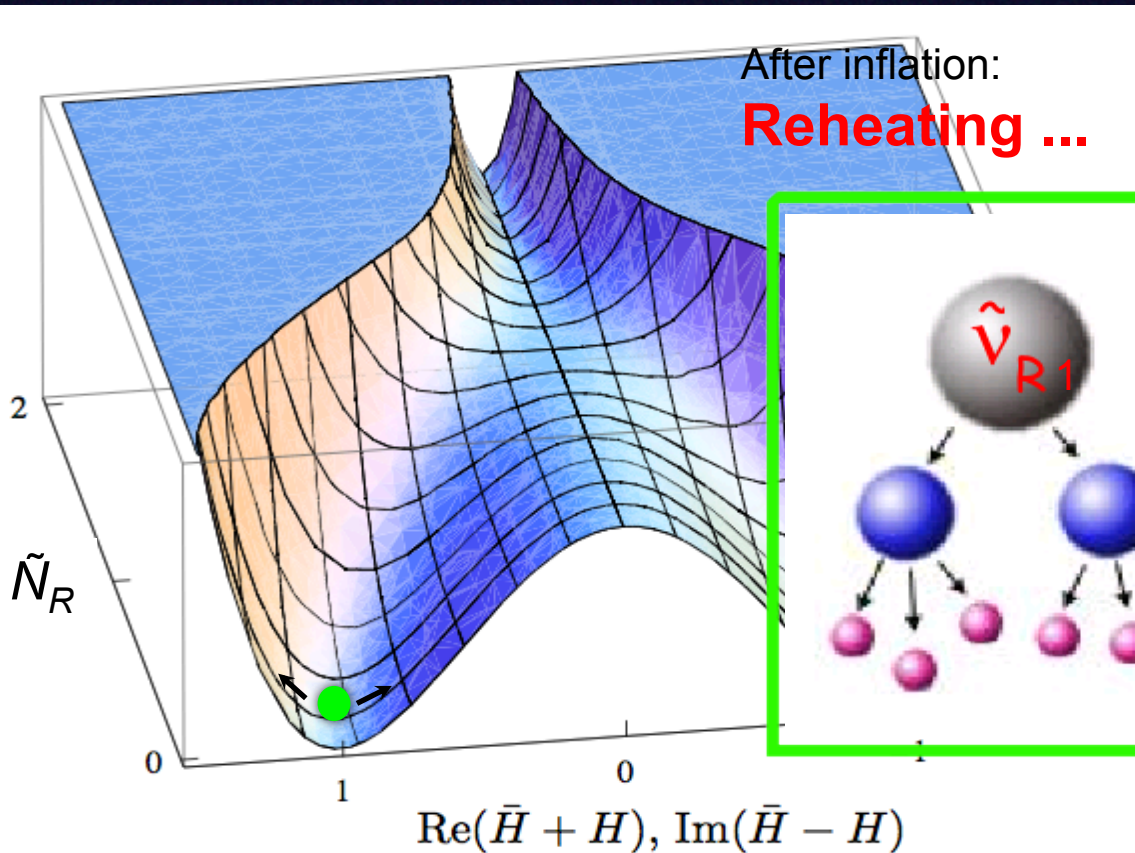
( $\nu_R$  contains the inflaton field  $\tilde{N}_R$  as scalar component; For  $\langle \tilde{N}_R \rangle > \tilde{N}_{R,\text{crit}}$  it stabilises  $H$  at  $\langle H \rangle = 0$ )



# Sneutrino Hybrid Inflation

$$\mathcal{W} = \kappa S (H^2 - M^2) + \frac{\lambda}{M_{\text{Pl}}} \nu_{Ri} \nu_{Ri} H H + (y_\nu)_{ij} \nu_{Ri} h L_j$$

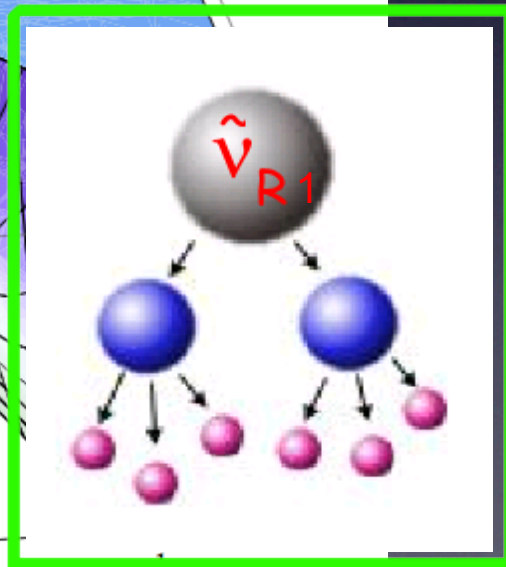
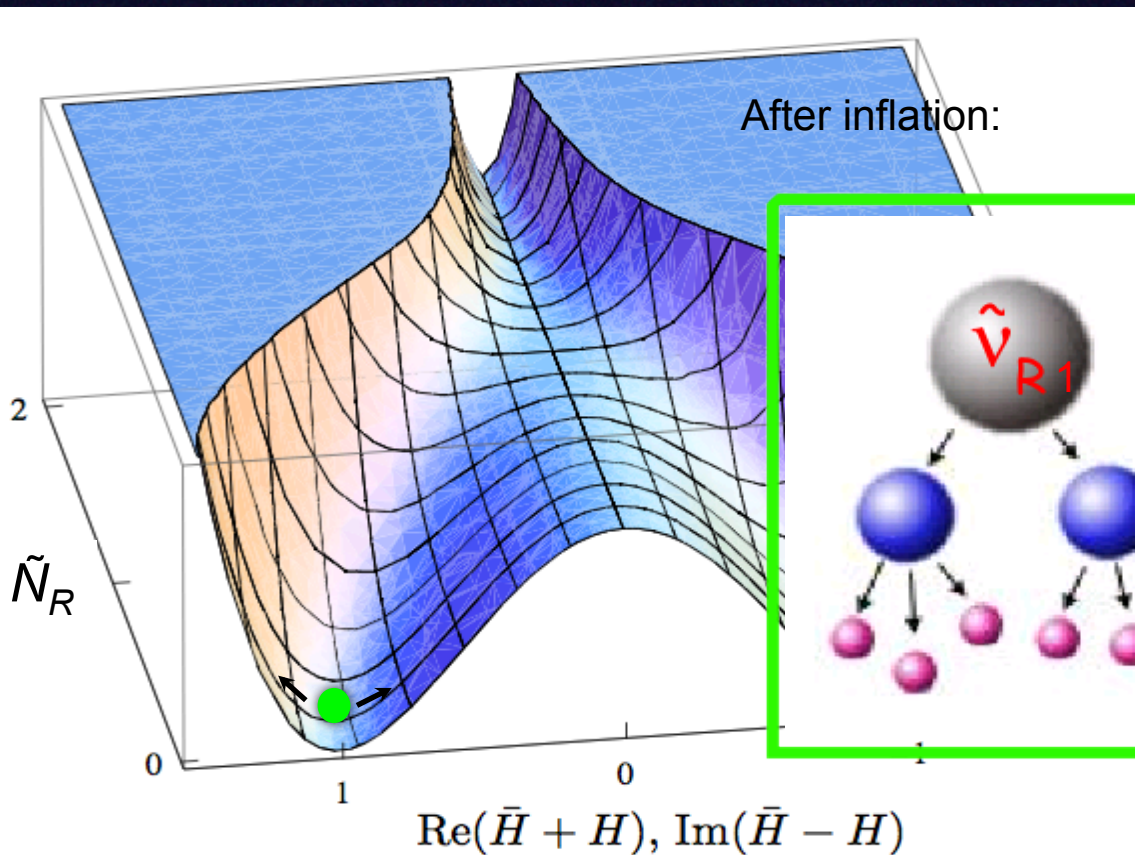
Neutrino Yukawa couplings



# Sneutrino Hybrid Inflation

$$\mathcal{W} = \kappa S (H^2 - M^2) + \frac{\lambda}{M_{\text{Pl}}} \nu_{Ri} \nu_{Ri} H H + (y_\nu)_{ij} \nu_{Ri} h L_j$$

Neutrino Yukawa couplings

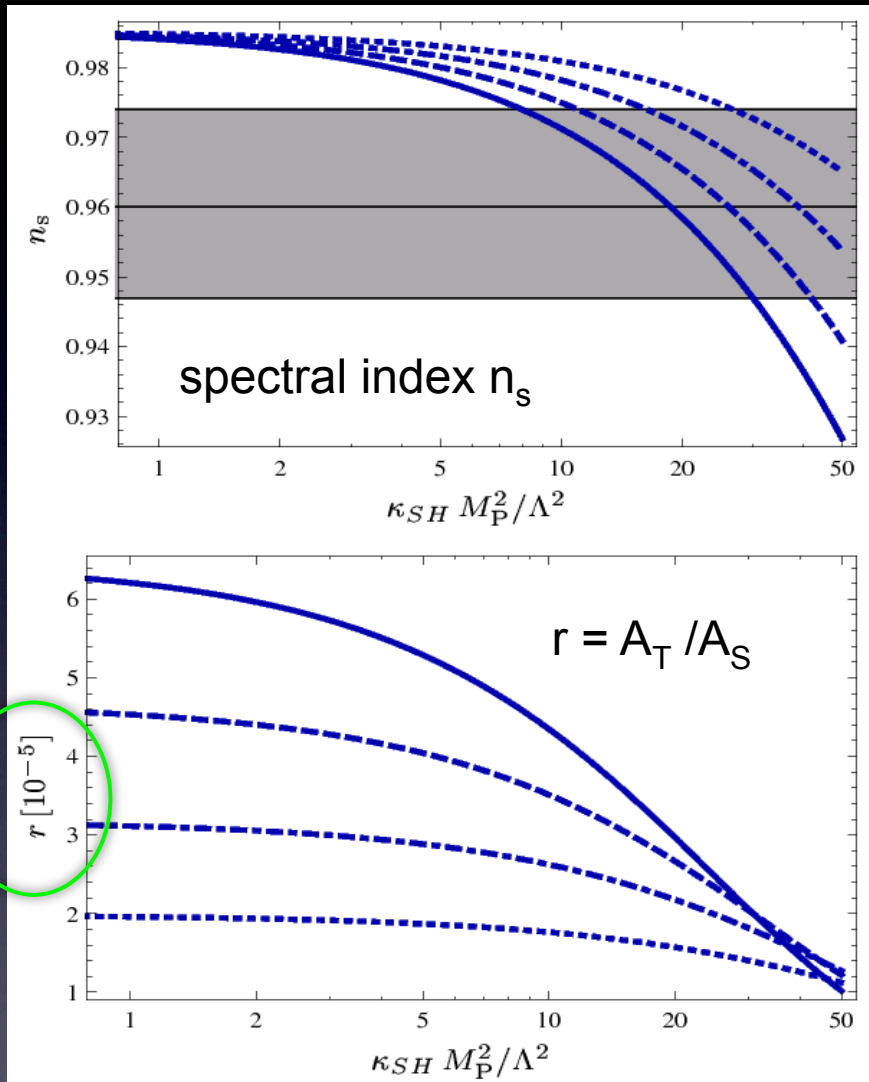


**Non-thermal leptogenesis after sneutrino inflation:**  
 very efficient way of generating the observed baryon asymmetry!

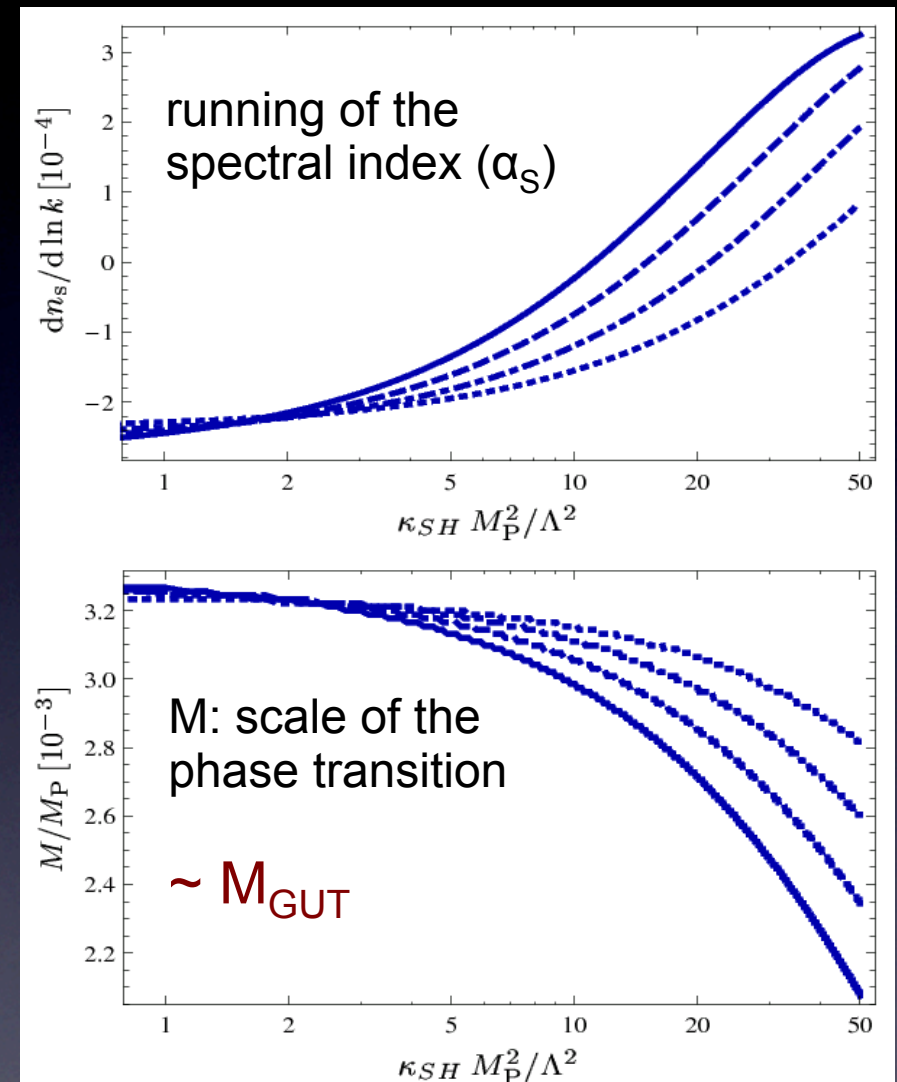
In Sneutrino Hybrid Inflation:  
 S.A., Baumann, Domcke, Kostka ('10)



# CMB observables



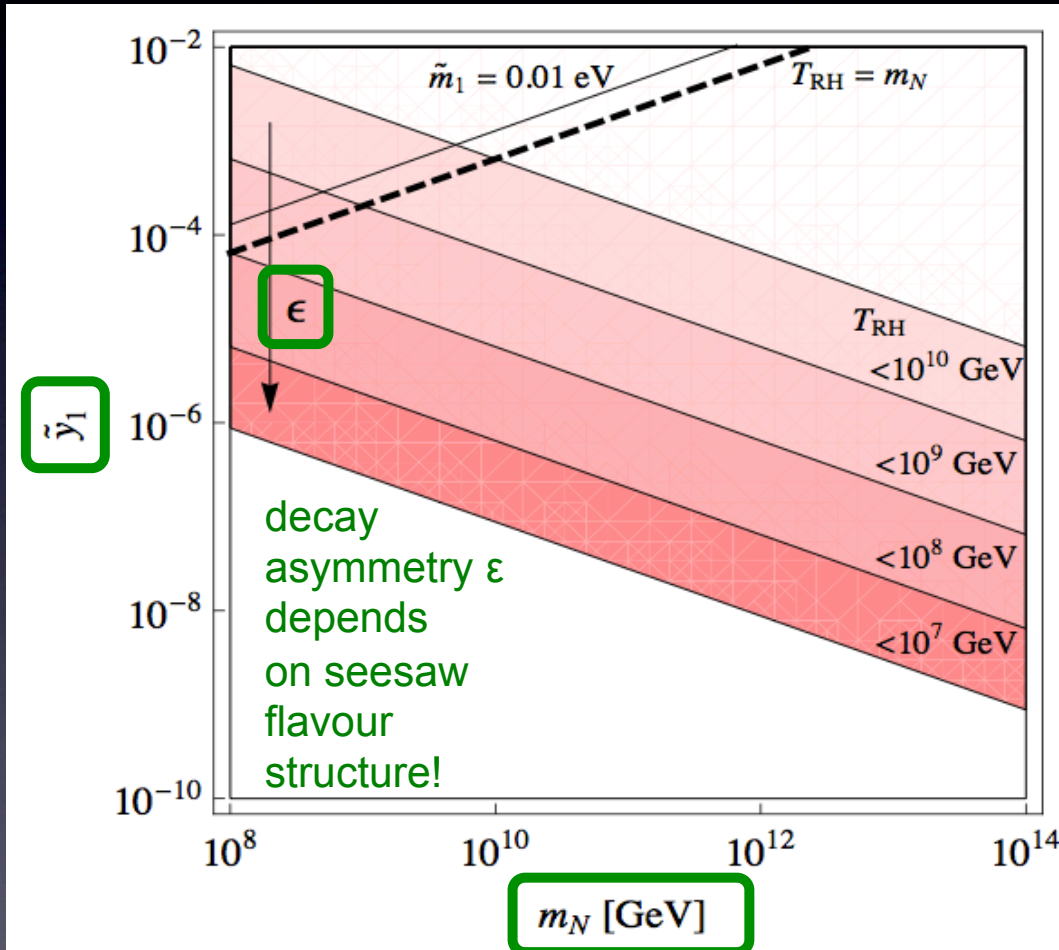
very small  
(as typical for Hybrid-type models)



Example: Predictions in a toy model ...

S.A., K. Dutta, P. M. Kostka ('09)

# Sneutrino hybrid inflation and non-thermal leptogenesis



- Reheating + Leptogenesis

$\tilde{N}_{R1} \leftrightarrow$  inflaton

$$\tilde{y}_1 \equiv \sqrt{(y_\nu y_\nu^\dagger)_{11}}$$

$m_N =$  (s)neutrino mass

$$\tilde{m}_1 = \tilde{y}_1^2 \langle \nu \rangle^2 / m_N$$

$$\epsilon < \frac{3}{8\pi} \frac{\sqrt{\Delta m_{\text{atm}}^2} m_N}{\langle \nu \rangle^2}$$

[S. Davidson, A. Ibarra '02]

S.A., Baumann, Domcke, Kostka ('10)



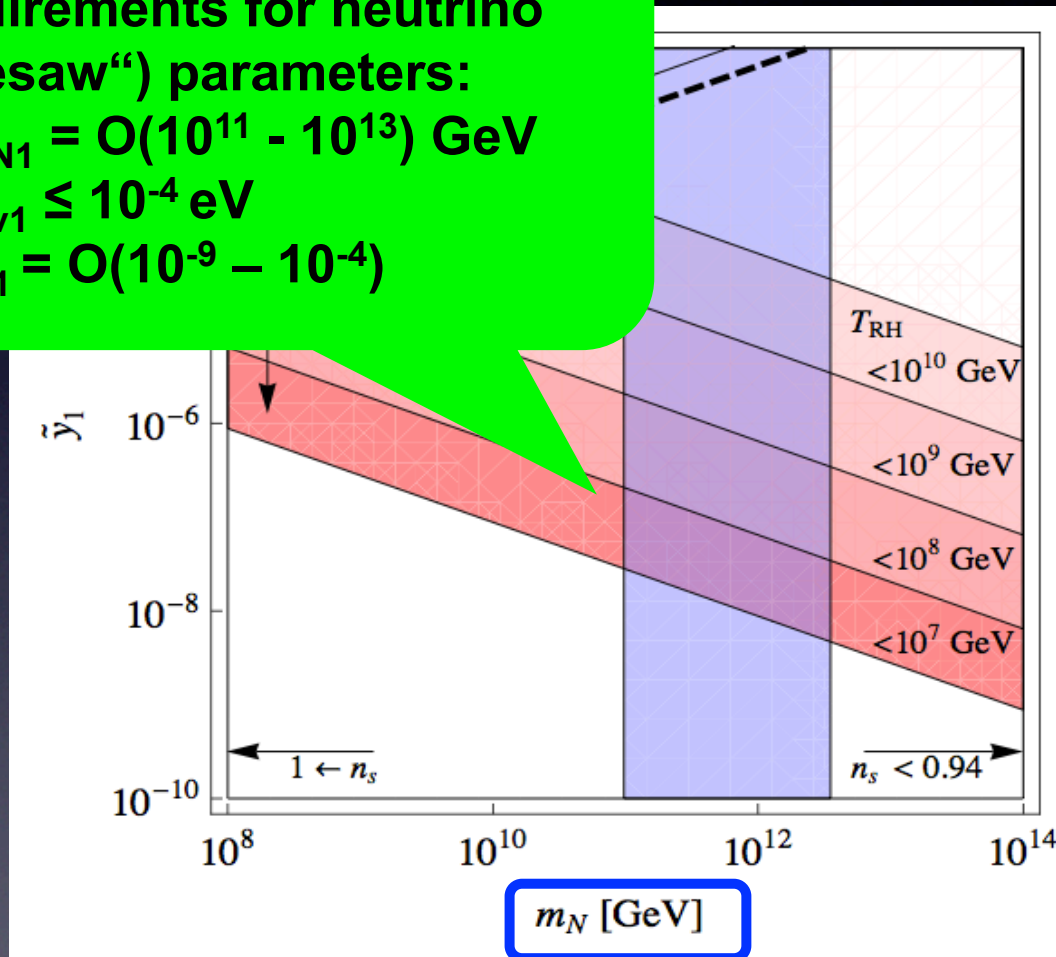
# Sneutrino hybrid inflation and non-thermal leptogenesis

Requirements for neutrino ("seesaw") parameters:

→  $m_{N1} = O(10^{11} - 10^{13}) \text{ GeV}$

→  $m_{\nu 1} \leq 10^{-4} \text{ eV}$

→  $y_{\nu 1} = O(10^{-9} - 10^{-4})$



- Reheating + Leptogenesis
- Inflation

$$\tilde{N}_{R1} \leftrightarrow \text{inflaton}$$

$$\tilde{y}_1 \equiv \sqrt{(y_\nu y_\nu^\dagger)_{11}}$$

$$m_N = \text{(s)neutrino mass}$$

$$\tilde{m}_1 = \tilde{y}_1^2 \langle \nu \rangle^2 / m_N$$

$$\epsilon < \frac{3}{8\pi} \frac{\sqrt{\Delta m_{\text{atm}}^2} m_N}{\langle \nu \rangle^2}$$

[S. Davidson, A. Ibarra '02]

in a simple example model of Sneutrino Hybrid Inflation ...

S.A., Baumann, Domcke, Kostka ('10)

# *Sneutrino Hybrid Inflation belongs to a more general class of models: **Matter Inflation***

$$\mathcal{W} = \kappa S (f(H) - M^2) + g(\phi, H)$$

Driving superfield

Waterfall superfield

Inflaton superfield  
(resides in the  
matter sector)

- Also referred to as “**Tribrid Inflation**”, because three fields play some role in the superpotential of the models ...

S.A., M. Bastero-Gil, K. Dutta, S. F. King, P. M. Kostka ('08)



# *Further developments & possibilities in Matter Inflation*

- Inflaton does not have to be a gauge singlet. It can also be a gauge non-singlet (e.g. a D-flat direction of GUT representations)

S.A., Bastero-Gil, Baumann, Dutta, King, Kostka ('10)

- The (2<sup>nd</sup> order) phase transition at the end of inflation can be ...

- ✓ ... a GUT phase transition

Dvali, Shafi, Schaefer ('94)

- ✓ ... the breaking of a family symmetry (e.g.  $A_4$ , ...)

S. A., King, Malinsky, Velasco-Sevilla, Zavala ('07)

- Possibilities for realising Matter Inflation in Heterotic String Theory ...

S. A., Halter, Erdmenger ('11)

# *Further developments & possibilities in Matter Inflation*

- No monopoles are generated at the end of inflation ...
  - ✓ ... if the inflaton is a gauge non-singlet (→ group broken during inflation)  
S.A., Bastero-Gil, Baumann, Dutta, King, Kostka ('10)
  - ✓ ... if a family symmetry is broken at the end of inflation (as in “flavon inflation”)  
S. A., King, Malinsky, Velasco-Sevilla, Zavala ('07)
  - ✓ ... in “pseudosmooth” versions of tribrid inflation  
S.A., Nolde, Ur Rehman ('12)
- ✓ The  $\eta$ -problem (→ “flatness problem” of the inflaton potential) can be solved in SUGRA by symmetry (e.g. by a Heisenberg or shift symmetry in the Kähler potential)  
S.A., M. Bastero-Gil, K. Dutta, S. F. King, P. M. Kostka ('08)  
S.A., K. Dutta, P. M. Kostka ('09)

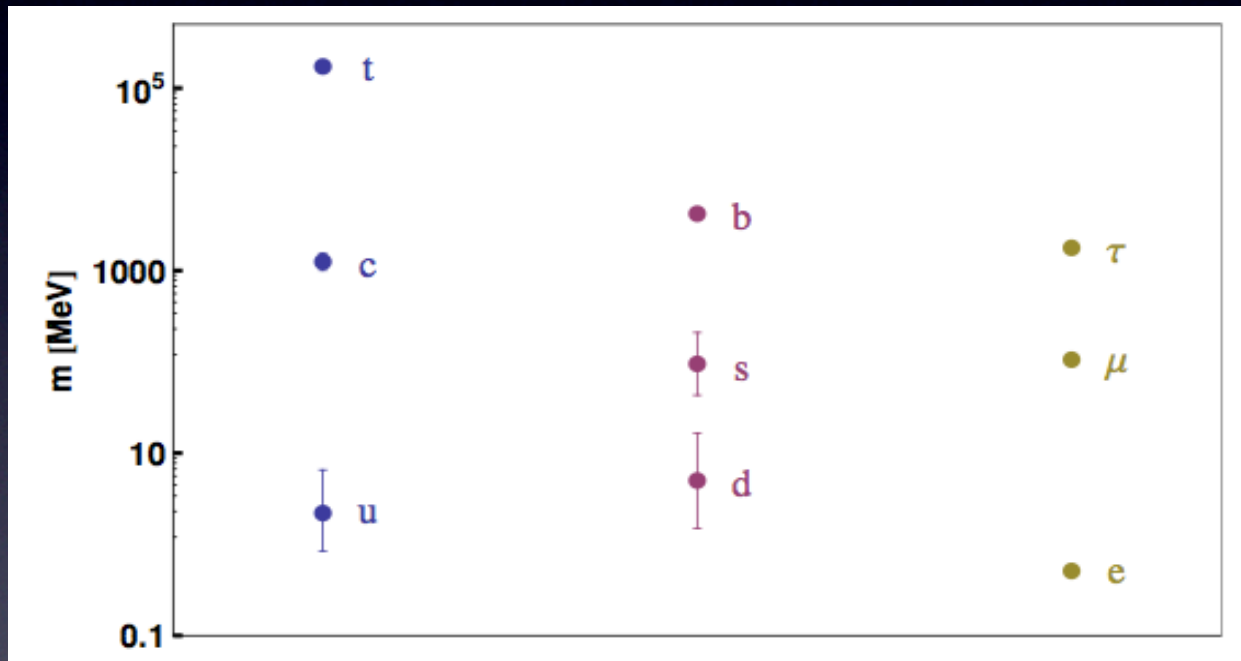


Second part of my talk:

***Are there  
GUT-footprints  
in fermion  
masses  
and mixings?***



- One motivation: Why are the observed masses of down-type quarks and charged leptons “similar” (but not equal)?



$$m_b \leftrightarrow m_\tau ?$$

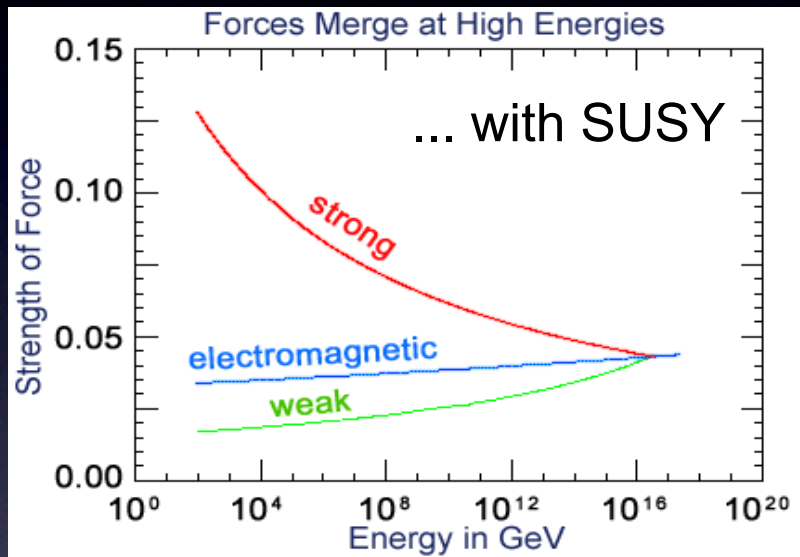
$$m_s \leftrightarrow m_\mu ?$$

(running masses at the top-mass scale; errors are 3 times the 1 $\sigma$  errors ...)



# GUTs: Unification of forces and of matter particles

## i) Unification of forces at $M_{\text{GUT}} \sim 10^{16}$ GeV



## ii) Unification of matter particles in joint representations of the GUT group

E.g. in SU(5) GUTs:

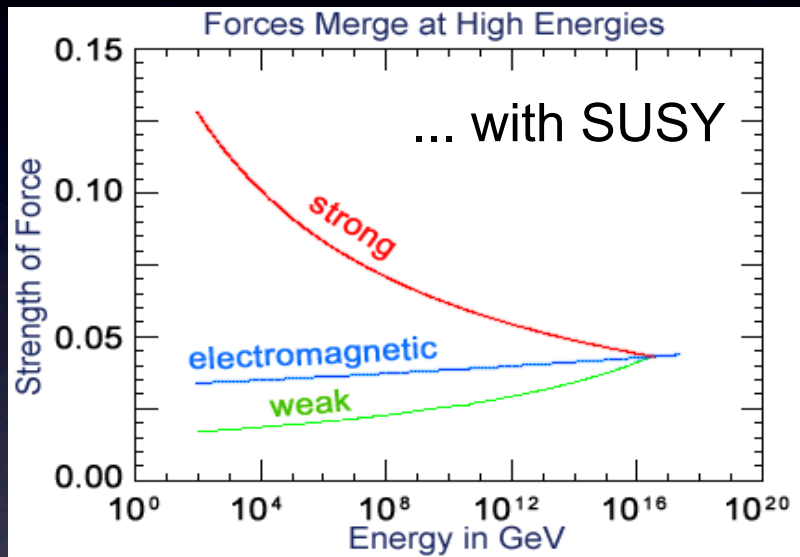
$$\bar{5}_i = (d_R^{cR} \quad d_R^{cB} \quad d_R^{cG} \quad e_L \quad -\nu_L)_i$$

$$10_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u_R^{cG} & u_R^{cB} & -u_L^R & -d_L^R \\ u_R^{cG} & 0 & -u_R^{cR} & -u_L^B & -d_L^B \\ -u_R^{cB} & u_R^{cR} & 0 & -u_L^G & -d_L^G \\ u_L^R & u_L^B & u_L^G & 0 & -e_R^c \\ d_L^R & d_L^B & d_L^G & e_R^c & 0 \end{pmatrix}_i$$

In SO(10) GUTs:  $\bar{5}_i + 10_i + \nu_{Ri}$  in  $16_i$

# GUTs: Unification of forces and of matter particles

## i) Unification of forces at $M_{\text{GUT}} \sim 10^{16}$ GeV



## ii) Unification of matter particles in joint representations of the GUT group

E.g. in SU(5) GUTs:

$$\bar{5}_i = (d_R^{cR} \quad d_R^{cB} \quad d_R^{cG} \quad e_L \quad -\nu_L)_i$$

One consequence:

Elements of the matrices  $M_d$  and  $M_e$  can be generated by one single operator

→ they differ only by group theoretical

Clebsch factors

$$10_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u_R^{cG} & u_R^{cB} & -u_L^R & -d_L^R \\ u_R^{cG} & 0 & -u_R^{cR} & -u_L^B & -d_L^B \\ -u_R^{cB} & u_R^{cR} & 0 & -u_L^G & -d_L^G \\ u_L^R & u_L^B & u_L^G & 0 & -e_R^c \\ d_L^R & d_L^B & d_L^G & e_R^c & 0 \end{pmatrix}_i$$

In SO(10) GUTs:  $\bar{5}_i + 10_i + \nu_{Ri}$  in  $16_i$



➤ This leads to GUT relations between the masses of down-type quarks and charged leptons (e.g. in SU(5) GUTs):

- Relations from fundamental operators:

$$\bar{\mathbf{5}}_3 \mathbf{10}_3 \langle \bar{H}_5 \rangle \Rightarrow \frac{m_\tau}{m_b} \Big|_{M_{\text{GUT}}} = 1 \quad \text{“bottom-tau unification”}$$

$$\bar{\mathbf{5}}_2 \mathbf{10}_2 \langle \bar{H}_{45} \rangle \Rightarrow \frac{m_\mu}{m_s} \Big|_{M_{\text{GUT}}} = 3 \quad \text{Georgi, Jarlskog ('79)}$$

SM Higgs particle in  
GUT representation  $H_{45}$

→ With effective operators new interesting GUT predictions for  $m_\tau/m_b$  and  $m_\mu/m_s$  can arise!

S. A., Spinrath ('09)

**Remark:**

In models aiming at explaining the fermion mass hierarchies, the masses  $\ll m_t$  are typically generated by effective operators!

Froggatt, Nielsen ('79)



➤ This leads to GUT relations between the masses of down-type quarks and charged leptons (e.g. in SU(5) GUTs):

- Relations from effective operators, e.g.:

S. A., Spinrath ('09)

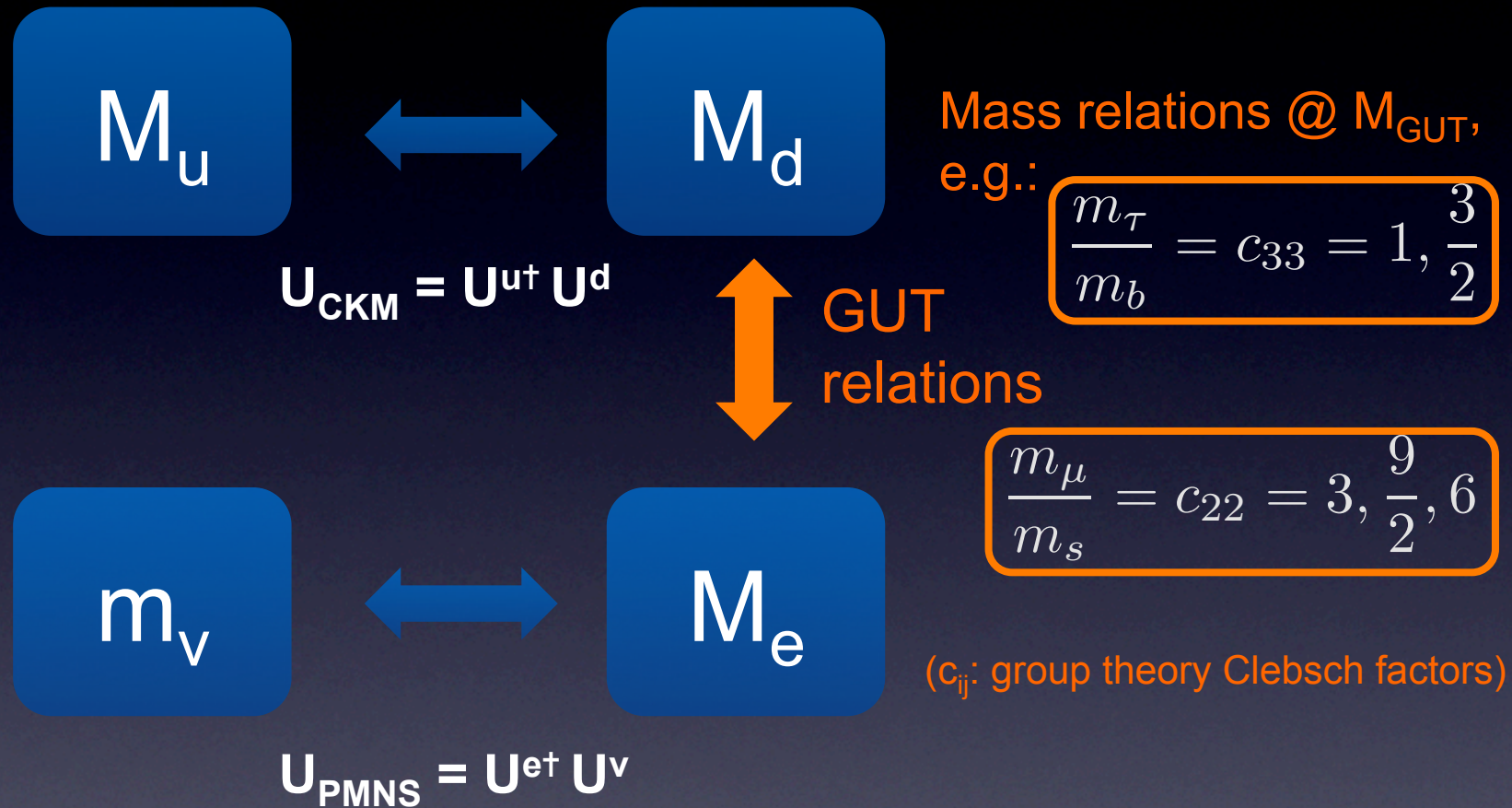
$$\bar{\mathbf{5}}_3 \frac{\langle H_{24} \rangle}{\Lambda} \mathbf{10}_3 \langle \bar{H}_5 \rangle \Rightarrow \frac{m_\tau}{m_b} \Big|_{M_{\text{GUT}}} = \frac{3}{2}$$

$$\bar{\mathbf{5}}_2 \frac{\langle H_{24} \rangle}{\Lambda} \mathbf{10}_2 \langle \bar{H}_{45} \rangle \Rightarrow \frac{m_\mu}{m_s} \Big|_{M_{\text{GUT}}} = \frac{9}{2}$$

$$\bar{\mathbf{5}}_2 \langle \bar{H}_5 \rangle \mathbf{10}_2 \frac{\langle H_{24} \rangle}{\Lambda} \Rightarrow \frac{m_\mu}{m_s} \Big|_{M_{\text{GUT}}} = 6$$

Their viability depends on RG running and threshold corrections (in SUSY)!

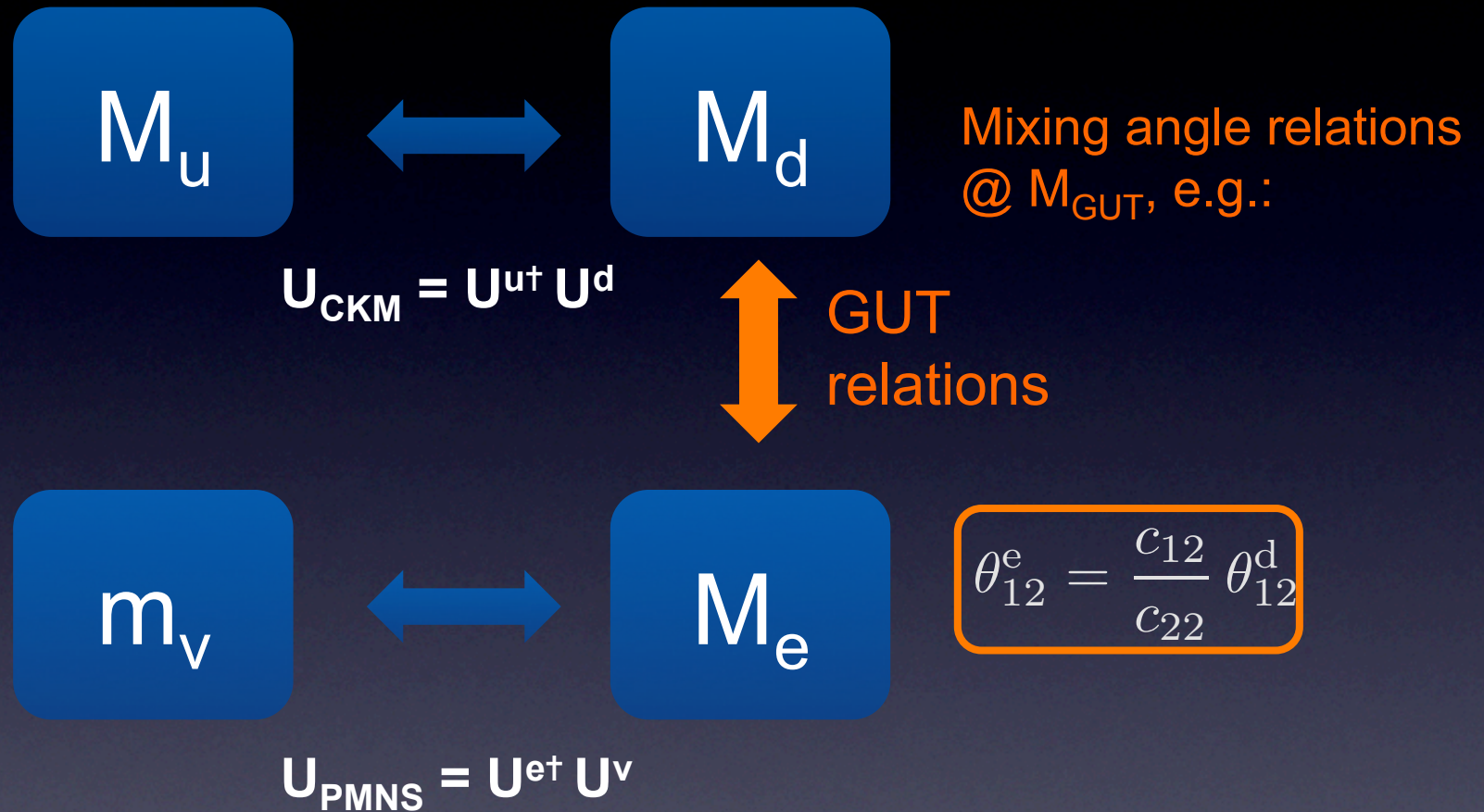
- The “GUT Higgs”  $H_{24}$  breaks the GUT symmetry to the SM
- The effective operators can explain the hierarchy of masses



Clebsch factors  $\in \{1, 3\}$  in GUTs: Georgi, Jarlskog ('79)  
 New possible Clebsch factors  $\in \{3/2, 3, 9/2, 6, \dots\}$ : S.A., Spinrath ('09)



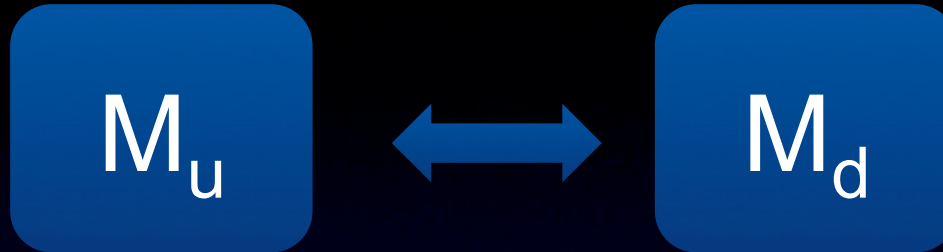
*In addition to GUT relations for the mass eigenvalues, there are also **relations** for the mixing angles ...*



Such GUT relations for the mixings can leave **footprints in the observable leptonic (PMNS) mixing parameters ...**

Possible new combinations of Clebsches leading to large  $\theta_{13}^{\text{PMNS}}$ : S.A., Maurer ('11) Mazocca, Petcov, Romanino, Spinrath ('11)

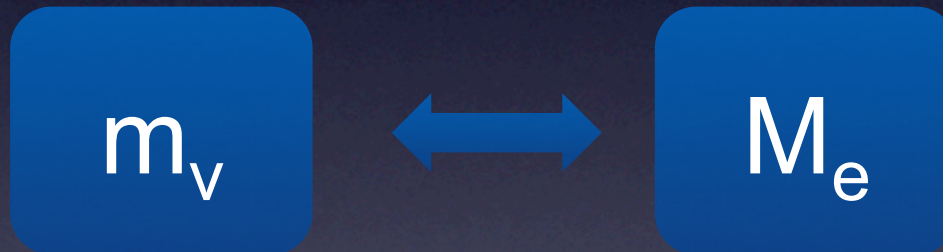




$$U_{CKM} = U^{u\dagger} U^d$$

Let us consider ...

Specific mixing pattern in  $m_\nu$ , e.g.:



$$U_{PMNS} = U^{e\dagger} U^\nu$$

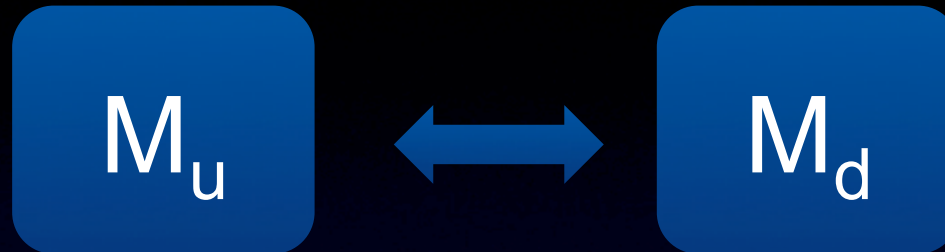
- Tri-bimaximal mixing:  $U^\nu = U_{TB}$
- Bimaximal mixing:  $U^\nu = U_{Bimax}$

with  $\theta_{13}^\nu \approx 0$

More hierarchical masses in  $M_u$  than in  $M_d \Rightarrow \theta_{ij}^u \ll \theta_{ij}^d \Rightarrow$  typically

$$\theta_{12}^d \cong \theta_C$$

and  $\theta_{13}^d, \theta_{23}^d \ll \theta_C$



$$U_{CKM} = U^{u\dagger} U^d$$

Let us consider  $U^v$  with:

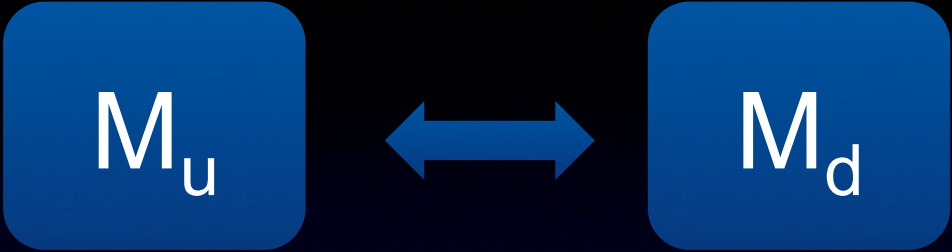
$$\theta_{13}^v \approx 0$$



$$U_{PMNS} = U^{e\dagger} U^v$$



$$\theta_{12}^d \cong \theta_C$$



$$U_{CKM} = U^{u\dagger} U^d$$

GUT relations

Let us consider  $U^{\nu}$  with:

$$\theta_{13}^{\nu} \approx 0$$

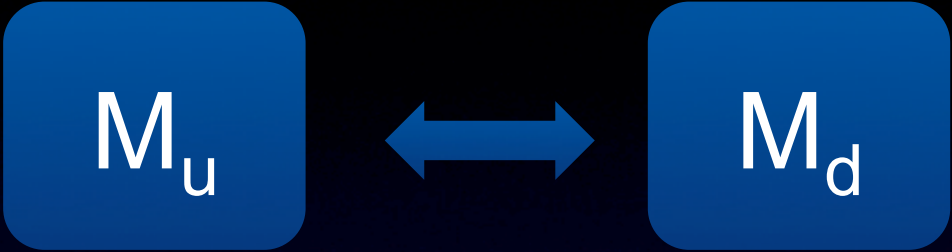


$$U_{PMNS} = U^{e\dagger} U^{\nu}$$

$$\theta_{12}^e = \frac{c_{12}}{c_{22}} \theta_{12}^d = \frac{c_{12}}{c_{22}} \theta_C$$

and  $\theta_{13}^e, \theta_{23}^e \ll \theta_C$

$$\theta_{12}^d \cong \theta_C$$



$$U_{CKM} = U^{u\dagger} U^d$$

GUT relations

Let us consider  $U^\nu$  with:

$$\theta_{13}^\nu \approx 0$$



$$U_{PMNS} = U^{e\dagger} U^\nu$$

$$\theta_{12}^e = \frac{c_{12}}{c_{22}} \theta_{12}^d = \frac{c_{12}}{c_{22}} \theta_C$$

and  $\theta_{13}^e, \theta_{23}^e \ll \theta_C$

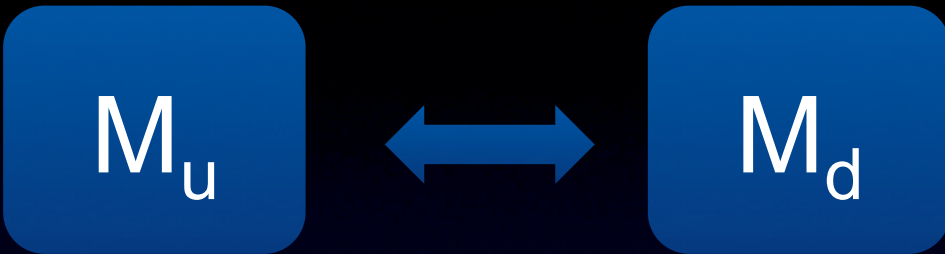
The PMNS mixing parameters result from  $U^\nu$  and charged lepton mixing effects ( $U^{e\dagger}$ )

⇒ Two main effects



Charged lepton mixing effects:  
works by many authors ...

$$\theta_{12}^d \cong \theta_C$$



$$U_{CKM} = U^{ut} U^d$$

GUT relations

Let us consider  $U^v$  with:

$$\theta_{13}^v \approx 0$$



$$\theta_{12}^e = \frac{c_{12}}{c_{22}} \theta_{12}^d = \frac{c_{12}}{c_{22}} \theta_C$$

and  $\theta_{13}^e, \theta_{23}^e \ll \theta_C$

$$U_{PMNS} = U^{e+} U^v$$

1) Prediction for  $\theta_{13}^{PMNS}$ :

under simple conditions in GUTs

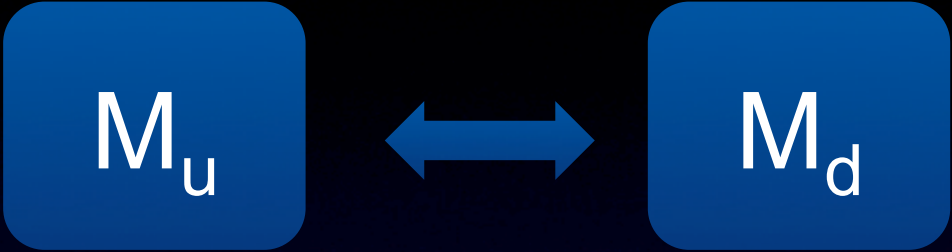
$$\theta_{13}^{PMNS} = \frac{\theta_{12}^e}{\sqrt{2}} = \frac{\theta_C}{\sqrt{2}} \frac{c_{12}}{c_{22}}$$

e.g.:  $\theta_{13}^{PMNS} = \frac{\theta_C}{\sqrt{2}}$

S.A., Maurer ('11); Mazocca, Petcov, Romanino, Spinrath ('11); King ('12); S.A., Gross, Maurer, Sluka ('12)

Remark:  $\theta_{13}^{PMNS} = \theta_C / \sqrt{2}$  also appears, e.g., in a variant of Quark-Lepton-Complementarity: Minakata, Smirnov ('04)

$$\theta_{12}^d \cong \theta_C$$



$$U_{CKM} = U^{u\dagger} U^d$$

GUT relations

Let us consider  $U^\nu$  with:

$$\theta_{13}^\nu \approx 0$$



$$U_{PMNS} = U^{e\dagger} U^\nu$$

$$\theta_{12}^e = \frac{c_{12}}{c_{22}} \theta_{12}^d = \frac{c_{12}}{c_{22}} \theta_C$$

and  $\theta_{13}^e, \theta_{23}^e \ll \theta_C$

2) "Lepton Mixing Sum Rule":

$$\theta_{12}^{PMNS} = \theta_{12}^\nu + \theta_{13}^{PMNS} \cos(\delta^{PMNS})$$

King ('05); Masina ('05); S.A., King ('05)



# Reconstructing $\theta_{12}^\nu$ using the lepton mixing sum rule

$$\theta_{12}^{\text{PMNS}} = \theta_{12}^\nu + \theta_{13}^{\text{PMNS}} \cos(\delta^{\text{PMNS}})$$

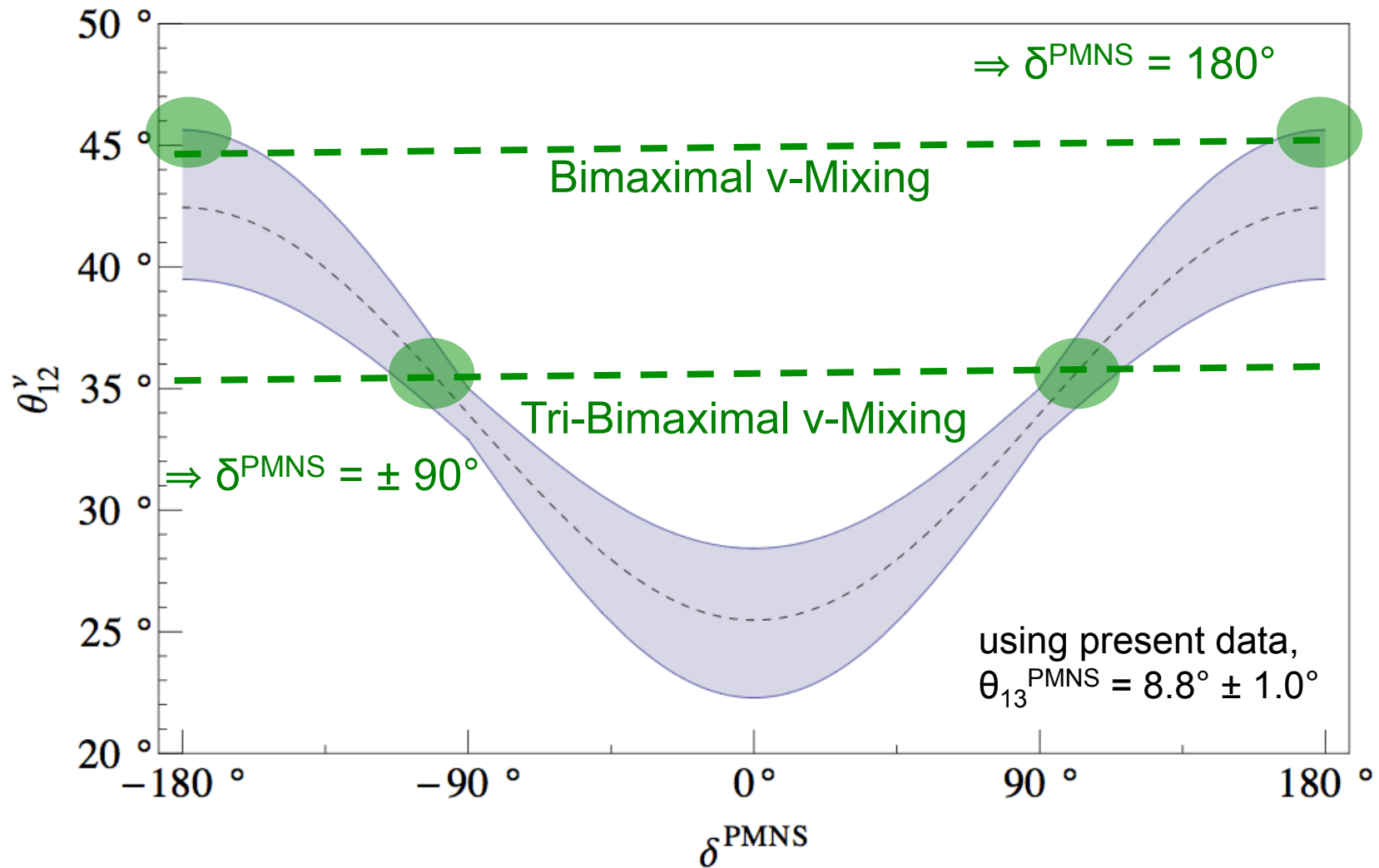


figure from S.A., Gross, Maurer, Sluka ('12)

# Note: Predictions emerge at high energies → RG running required

RGEs for the leptonic mixing angles (in LO in  $\theta_{13}$ ; in the MSSM):

$$\dot{\theta}_{12} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} s_{23}^2 \frac{|m_1 e^{i\varphi_1} + m_2 e^{i\varphi_2}|^2}{\Delta m_{\text{sol}}^2} + \mathcal{O}(\theta_{13})$$

$$\dot{\theta}_{13} \approx \frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m_{\text{atm}}^2} [m_1 \cos(\varphi_1 - \delta) - m_2 \cos(\varphi_2 - \delta)] + \mathcal{O}(\theta_{13})$$

$$\dot{\theta}_{23} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m_{\text{atm}}^2} [c_{12}^2 |m_2 e^{i\varphi_2} + m_3|^2 + s_{12}^2 |m_1 e^{i\varphi_1} + m_3|^2] + \mathcal{O}(\theta_{13})$$

S.A., Kersten, Lindner, Ratz ('03)

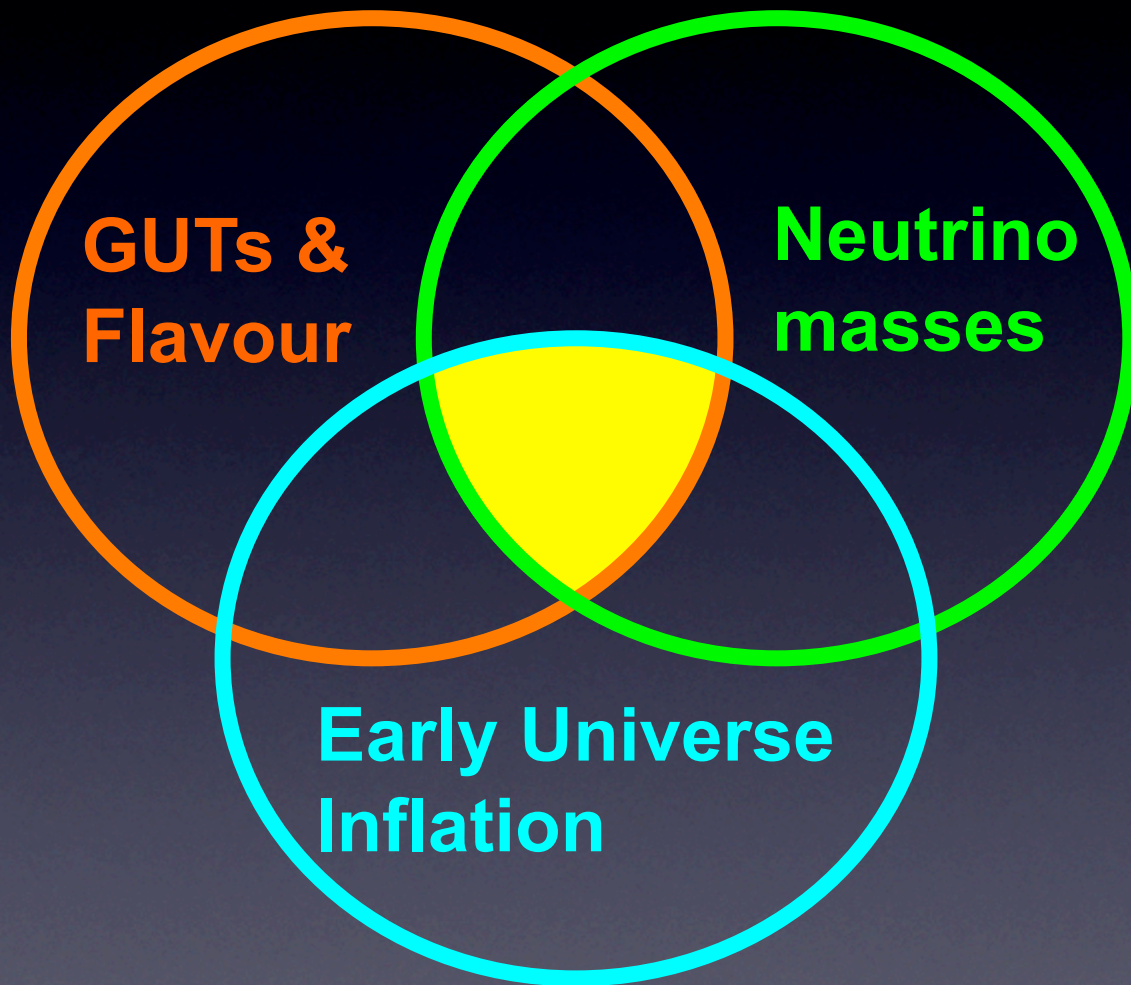
(RGEs in the SM: above equations times  $-3/2$ )

Including RG effects from  $Y_\nu$  (analytical formulae + software REAP):

S.A., Kersten, Lindner, Ratz, Schmidt ('05)



# Summary and conclusions



- The **RH sneutrino** is an attractive candidate for the **Inflation**:  
Chaotic vs Hybrid-like (= “Tribrid”)
- In **Sneutrino “Tribrid” Inflation**:  
The **end of inflation** can be associated with **GUT** or **family symmetry breaking**
- $\theta_{13}^{\text{PMNS}} \approx \theta_C / \sqrt{2}$  can emerge under simple conditions **from GUTs**
- **Predictions @ high energies** ( $M_{\text{GUT}}$ ): careful model analysis including **RG running required ... !**