Neutrino Flavour Models and Impact on LFV

Luca Merlo

26.06.2012, WHAT IS υ? INVISIBLES12 and Alexei Smirnov Fest

Outline

- News on neutrino mixings
- Impact on neutrino flavour models (discrete symmetries)
- Implications for LFV transitions in supersymmetric models and correlation with the muon g-2 discrepancy

based on: **Altarelli, Feruglio, LM & Stamou, arXiv:1205.4670**
Altarelli, Feruglio & LM, arXiv:1205.5133
Bazzocchi & LM, arXiv:1205.5135

- Digression: a couple of alternative attempts

based on: **Alonso, Gavela, D.Hernandez & LM, arXiv:1206.3167**
Altarelli, Feruglio, Masina & LM, to appear

Recent Results of Global Fits

Very recent global fit: Fogli *et al.* 1205.5254 (Only 3 active neutrinos...)

(see also [Tortola *et al.* 1205.4018])

$$\Delta m^2_{\rm sol} = (7.54^{+0.26}_{-0.22}) \times 10^{-5}\,{\rm eV}^2$$

$$\Delta m^2_{\rm atm} = (2.43^{+0.07}_{-0.09})[2.42^{+0.07}_{-0.10}] \times 10^{-3}\,{\rm eV}^2$$

$$\sin^2\theta_{12} = 0.307^{+0.018}_{-0.016}$$

$$\sin^2\theta_{23} = 0.398^{+0.030}_{-0.026}[0.408^{+0.035}_{-0.030}]$$

$$\sin^2\theta_{13} = 0.0245^{+0.0034}_{-0.0031}[0.0246^{+0.0034}_{-0.0031}]$$

$$\delta = \pi(0.89^{+0.29}_{-0.44})[0.90^{+0.32}_{-0.43}]$$

[Talks by Walter & Wang & Schwetz]

Neutrino Mass Patterns

In the past:

- large atmospheric angle
- only upper bound on the reactor angle

→ $\sin^2\theta_{23} = \frac{1}{2}$, $\sin^2\theta_{13} = 0$ **mu-tau symmetry**

This suggests a fundamental structure of nature!

Neutrino Mass Patterns

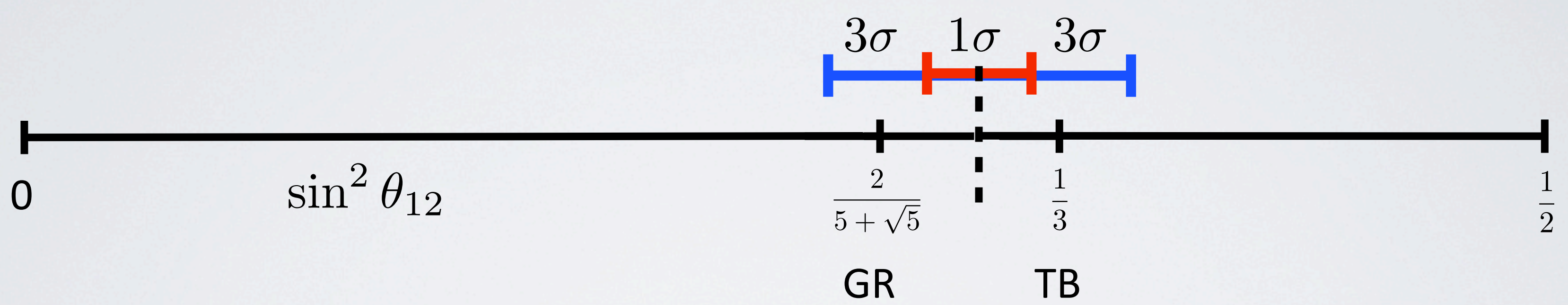
In the past:

- large atmospheric angle
- only upper bound on the reactor angle

$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0$$

mu-tau symmetry

This suggests a fundamental structure of nature!



TRI-BIMAXIMAL (TB) [Harrison, Perkins & Scott 2002; Zhi-Zhong Xing 2002]

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \sin^2\theta_{12} = \frac{1}{3} \quad \longrightarrow \quad \theta_{12} = 35.26^\circ$$

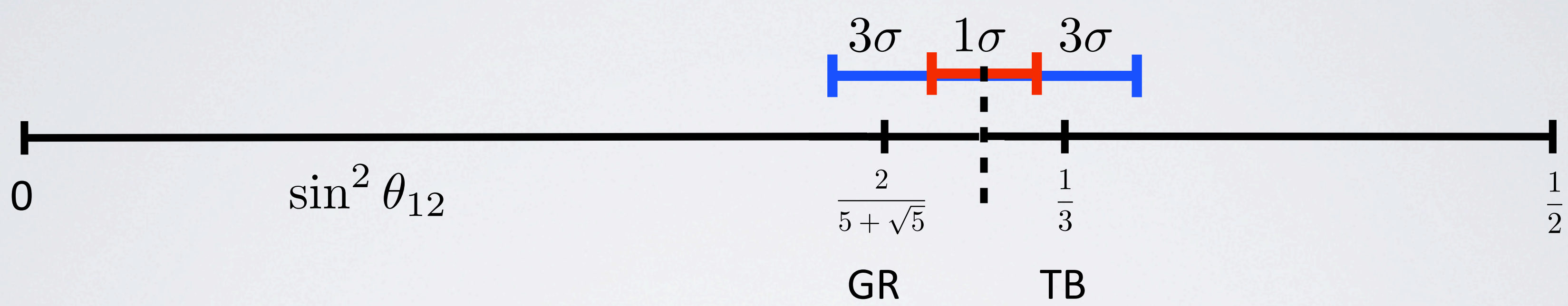
Neutrino Mass Patterns

In the past:

- large atmospheric angle
- only upper bound on the reactor angle

→ $\sin^2\theta_{23} = \frac{1}{2}$ $\sin^2\theta_{13} = 0$ **mu-tau symmetry**

This suggests a fundamental structure of nature!



TRI-BIMAXIMAL (TB) [Harrison, Perkins & Scott 2002; Zhi-Zhong Xing 2002]

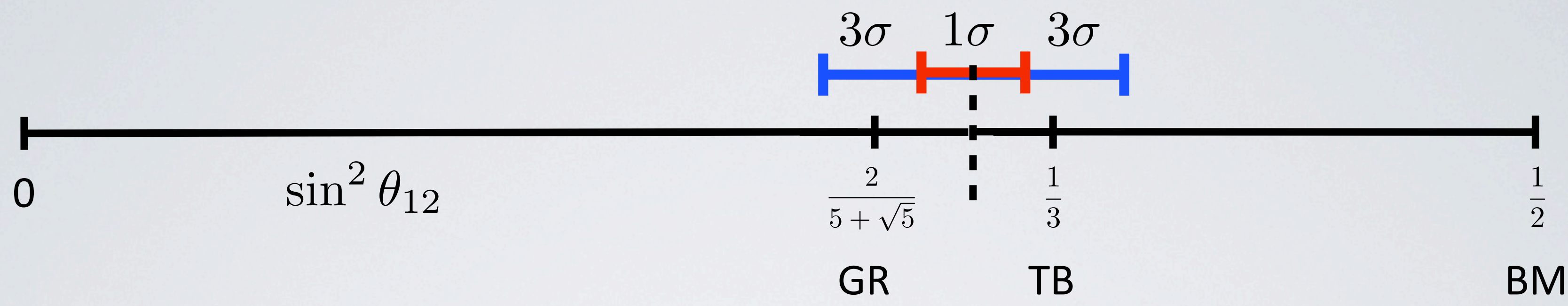
$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \sin^2\theta_{12} = \frac{1}{3} \quad \longrightarrow \quad \theta_{12} = 35.26^\circ$$

GOLDEN RATIO (GR) [Kajiyama, Raidal & Strumia 2007]

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \tan\theta_{12} = \frac{1}{\phi} \quad \longrightarrow \quad \theta_{12} = 31.72^\circ$$

$$\phi \equiv \frac{1+\sqrt{5}}{2}$$

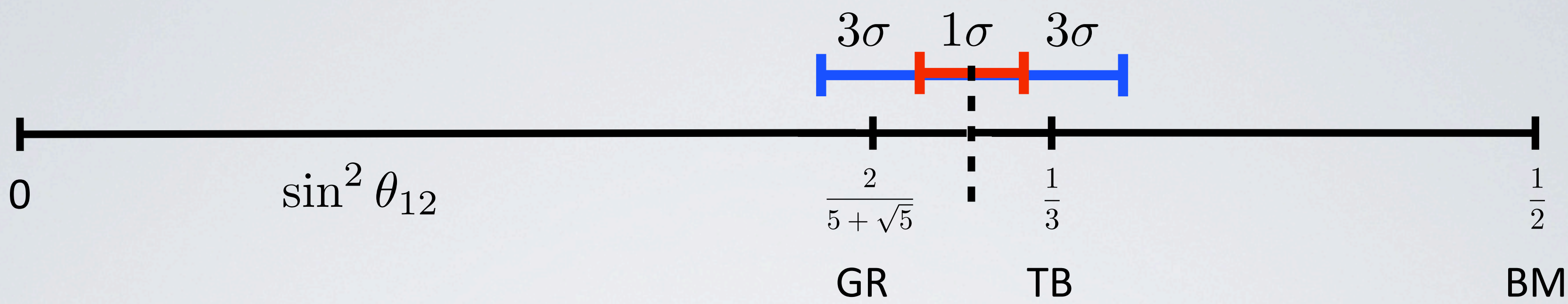
Neutrino Mass Patterns



BIMAXIMAL (BM) [Vissani 1997; Barger et al. 1998]

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \sin^2\theta_{12} = \frac{1}{2} \longrightarrow \theta_{12} = 45^\circ$$

Neutrino Mass Patterns



BIMAXIMAL (BM) [Vissani 1997; Barger et al. 1998]

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \sin^2\theta_{12} = \frac{1}{2} \quad \longrightarrow \quad \theta_{12} = 45^\circ$$

Maybe related to the **Quark-Lepton Complementarity**:
[Smirnov; Raidal; Minakata & Smirnov 2004]

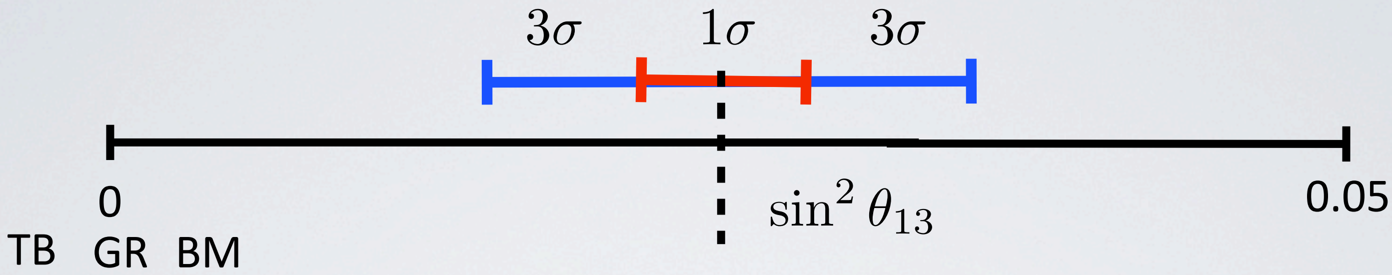
$$\pi/4 \approx \theta_{12} + \lambda$$

$$\longrightarrow \quad \theta_{12}^{Exp} \approx \theta_{12}^{BM} - \lambda$$

[Altarelli, Feruglio and LM 2009, Adelhart, Bazzocchi and LM 2010, Meloni 2011]

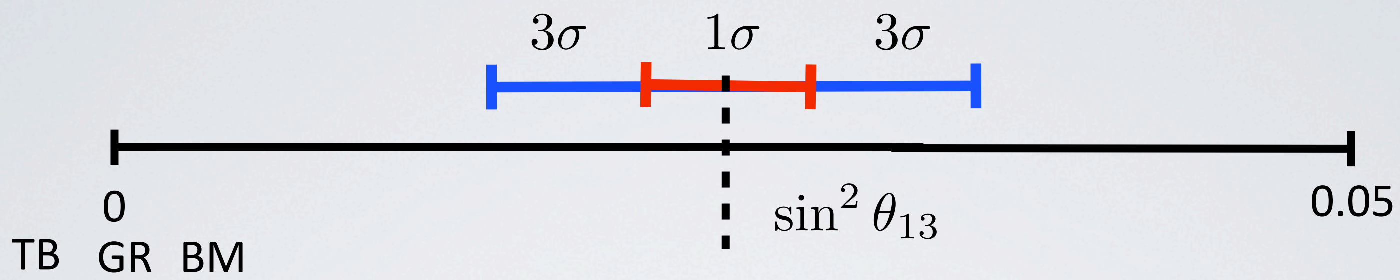
Need of Corrections!!

$$\sin^2\theta_{13} = 0.0245^{+0.0034}_{-0.0031}\,[0.0246^{+0.0034}_{-0.0031}]$$



Need of Corrections!!

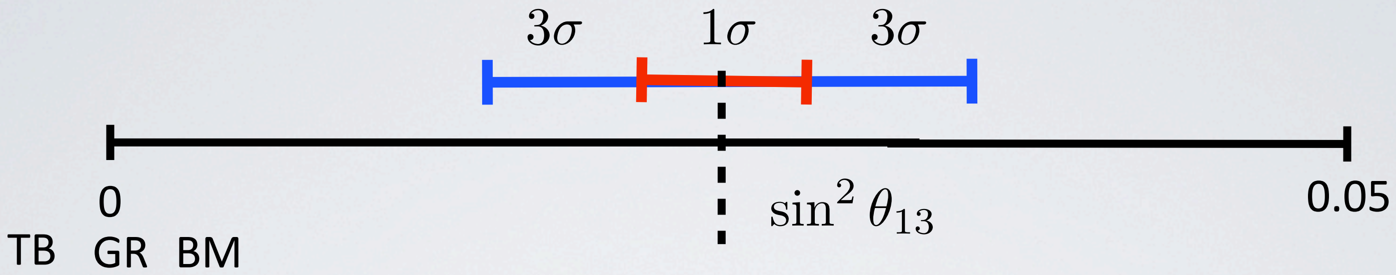
$$\sin^2\theta_{13} = 0.0245^{+0.0034}_{-0.0031}[0.0246^{+0.0034}_{-0.0031}]$$



Are these predictive patterns completely ruled out?

Need of Corrections!!

$$\sin^2\theta_{13} = 0.0245^{+0.0034}_{-0.0031} [0.0246^{+0.0034}_{-0.0031}]$$



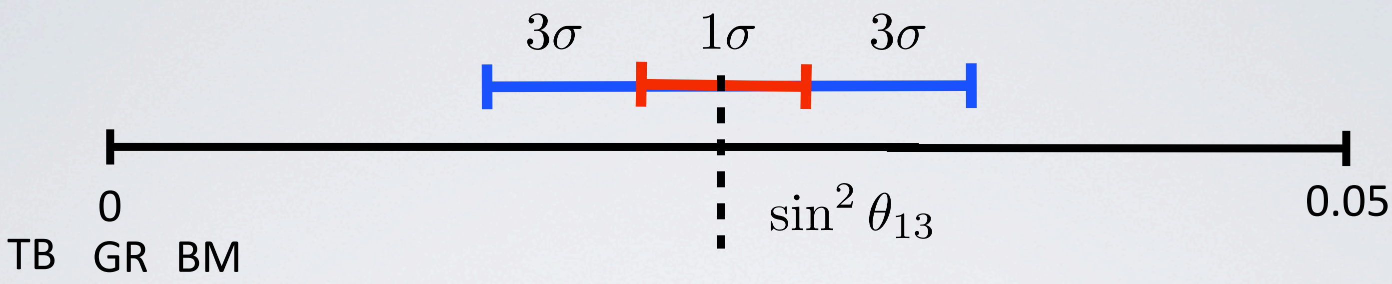
Are these predictive patterns completely ruled out?

Such corrections can arise from the charged lepton and/or from the neutrino sectors:

$$m_e = m_e^{(0)} + \delta m_e \qquad\qquad m_\nu = m_\nu^{(0)} + \delta m_\nu$$

Need of Corrections!!

$$\sin^2\theta_{13} = 0.0245^{+0.0034}_{-0.0031}[0.0246^{+0.0034}_{-0.0031}]$$



Are these predictive patterns completely ruled out?

Such corrections can arise from the charged lepton and/or from the neutrino sectors:

$$m_e = m_e^{(0)} + \delta m_e \qquad\qquad m_\nu = m_\nu^{(0)} + \delta m_\nu$$

in the basis in which the LO masses satisfy to

$$m_e^{diag} = m_e^{(0)} \qquad m_\nu^{diag} = U_\nu^{0T}\, m_\nu^{(0)}\, U_\nu^0 \qquad U_\nu^0 = \{U_{TB}, U_{GR}, U_{BM}\}$$

then the NLO corrections are encoded in

$$(m_e^{diag})^2 = \delta U_e^\dagger\, m_e^\dagger\, m_e\, \delta U_e$$

$$m_\nu^{diag} = \delta U_\nu^T\, U_\nu^{0T}\, m_\nu\, U_\nu^0\, \delta U_\nu$$

$$\delta U = \begin{pmatrix} 1 & c_{12}\,\xi & c_{13}\,\xi \\ -c_{12}^*\,\xi & 1 & c_{23}\,\xi \\ -c_{13}^*\,\xi & -c_{23}^*\,\xi & 1 \end{pmatrix}$$

Typical Tri-Bimaximal

In typical TB (GR) models, the corrections are democratic in all the angles:

$$c^e_{12} \approx c^e_{23} \approx c^e_{13} \qquad\qquad c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13}$$

$$\xi^e \approx \xi^\nu \equiv \xi$$

A_4: Altarelli & Feruglio 2005
T': Feruglio, Hagedorn, LM & Lin 2007
S_4: Bazzocchi, LM & Morisi 2009

Typical Tri-Bimaximal

In typical TB (GR) models, the corrections are democratic in all the angles:

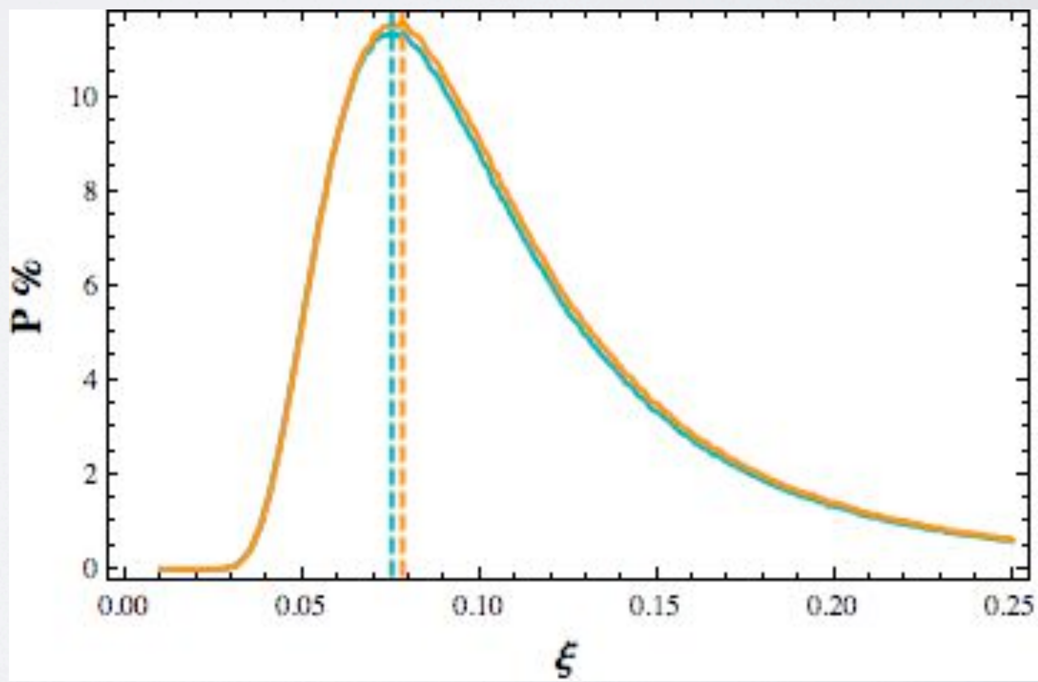
$$c^e_{12} \approx c^e_{23} \approx c^e_{13} \qquad\qquad c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13}$$

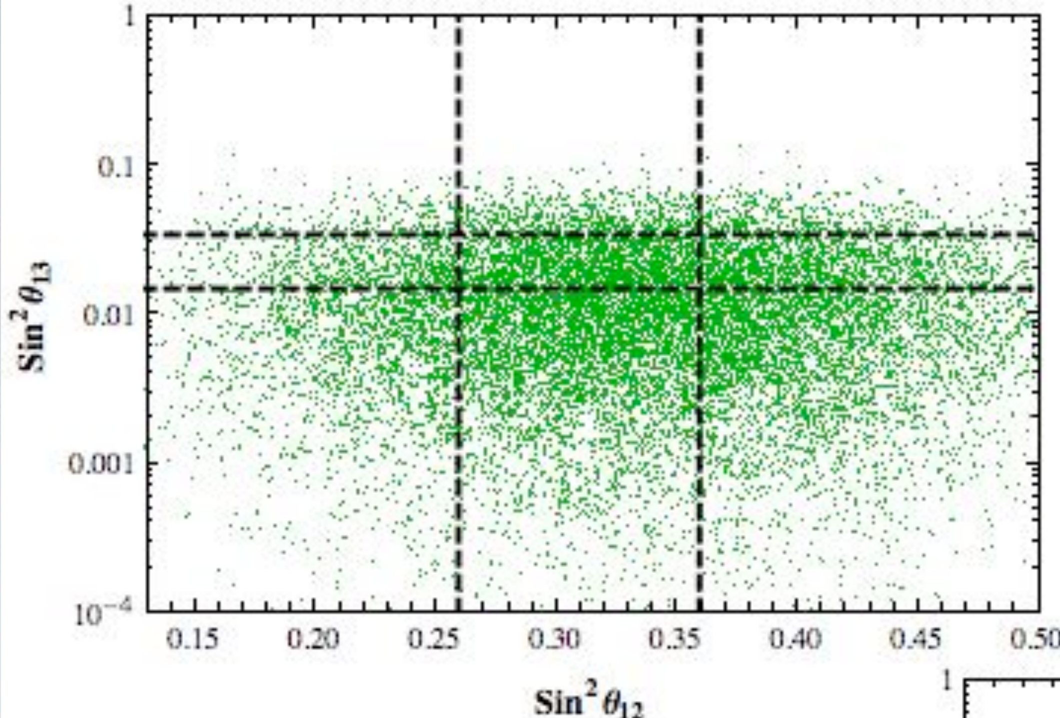
$$\xi^e \approx \xi^\nu \equiv \xi$$

A_4: Altarelli & Feruglio 2005
T': Feruglio, Hagedorn, LM & Lin 2007
S_4: Bazzocchi, LM & Morisi 2009

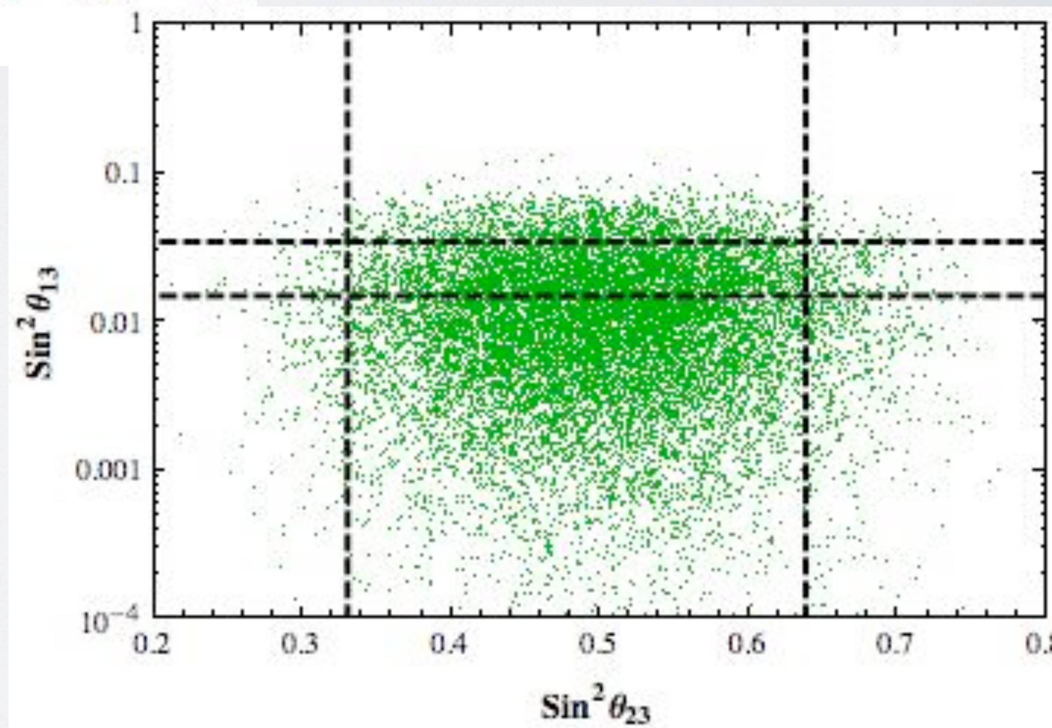
To maximize the success rate for all the three mixing angles inside the 3σ:

$$\xi \simeq 0.075$$





1
0.1
0.01
0.001
10^{-4}
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
$\mathrm{Sin}^2\theta_{13}$
$\mathrm{Sin}^2\theta_{12}$



1
0.1
0.01
0.001
10^{-4}
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\mathrm{Sin}^2\theta_{13}$
$\mathrm{Sin}^2\theta_{23}$

Special Tri-Bimaximal

In special TB models, the corrections are specific in certain flavour directions:

A_4: Lin 2009

$$\xi^\nu \gg \xi^e$$

$$c^\nu_{12} = c^\nu_{23} = 0 \qquad c^\nu_{13} \neq 0$$

$$c^e_{12} \approx c^e_{23} \approx c^e_{13}$$

Special Tri-Bimaximal

In special TB models, the corrections are specific in certain flavour directions:

A_4: Lin 2009

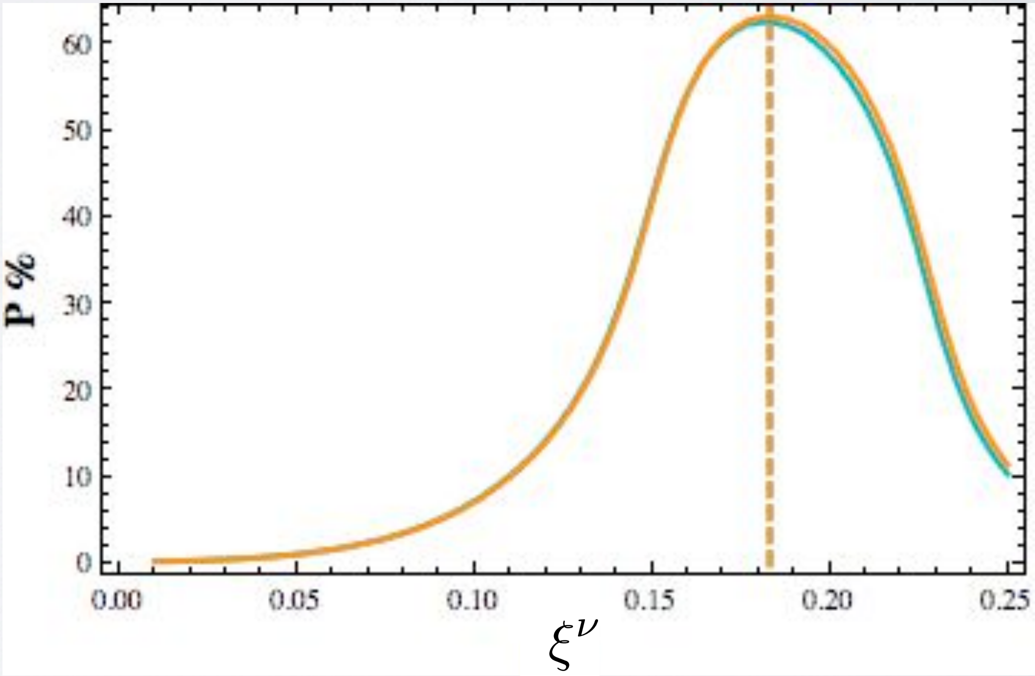
$$\xi^\nu \gg \xi^e$$

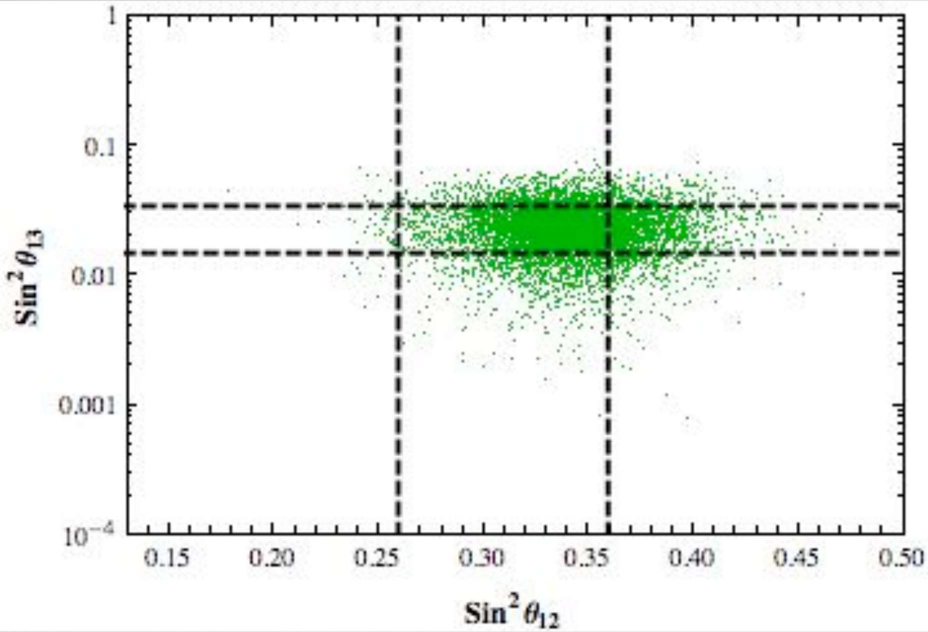
$$c^\nu_{12} = c^\nu_{23} = 0 \qquad c^\nu_{13} \neq 0$$

$$c^e_{12} \approx c^e_{23} \approx c^e_{13}$$

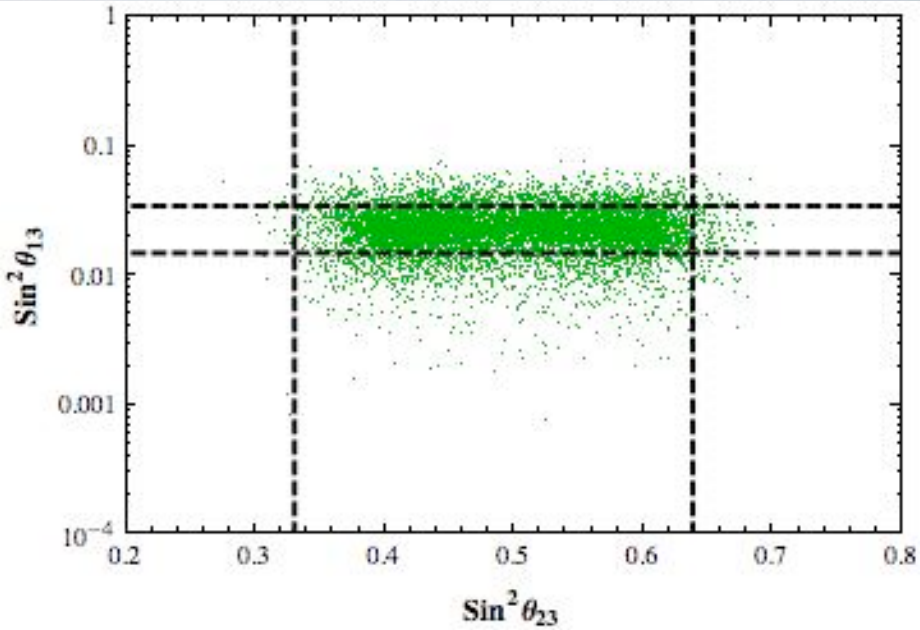
To maximize the success rate for all the three mixing angles inside the 3 σ:

$$\xi^\nu \simeq 0.18$$

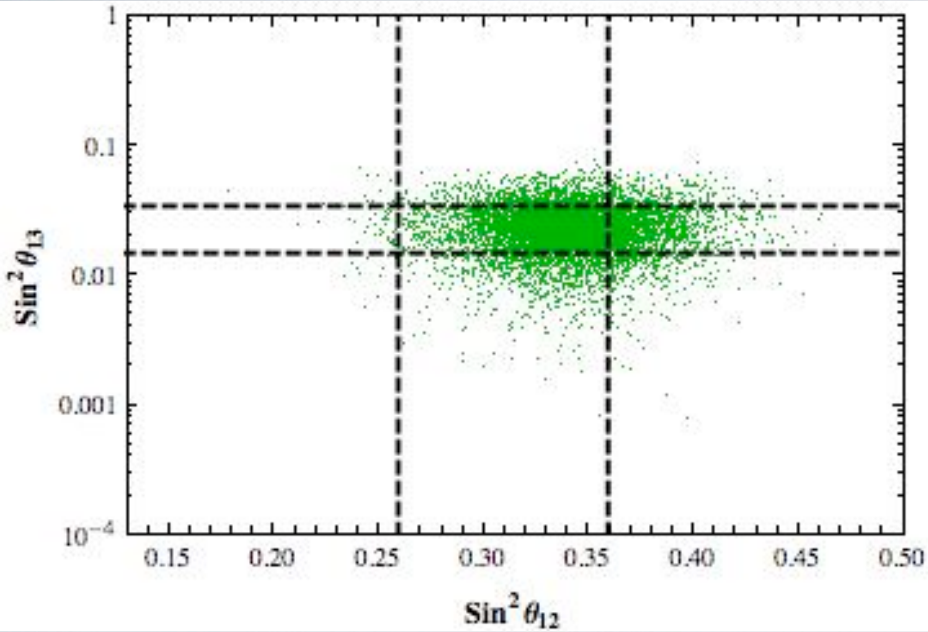


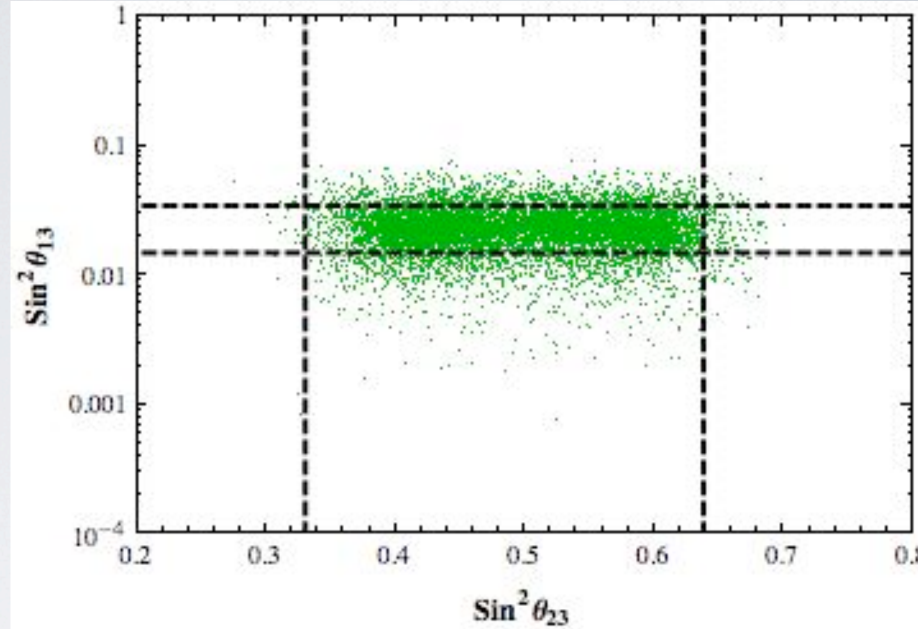


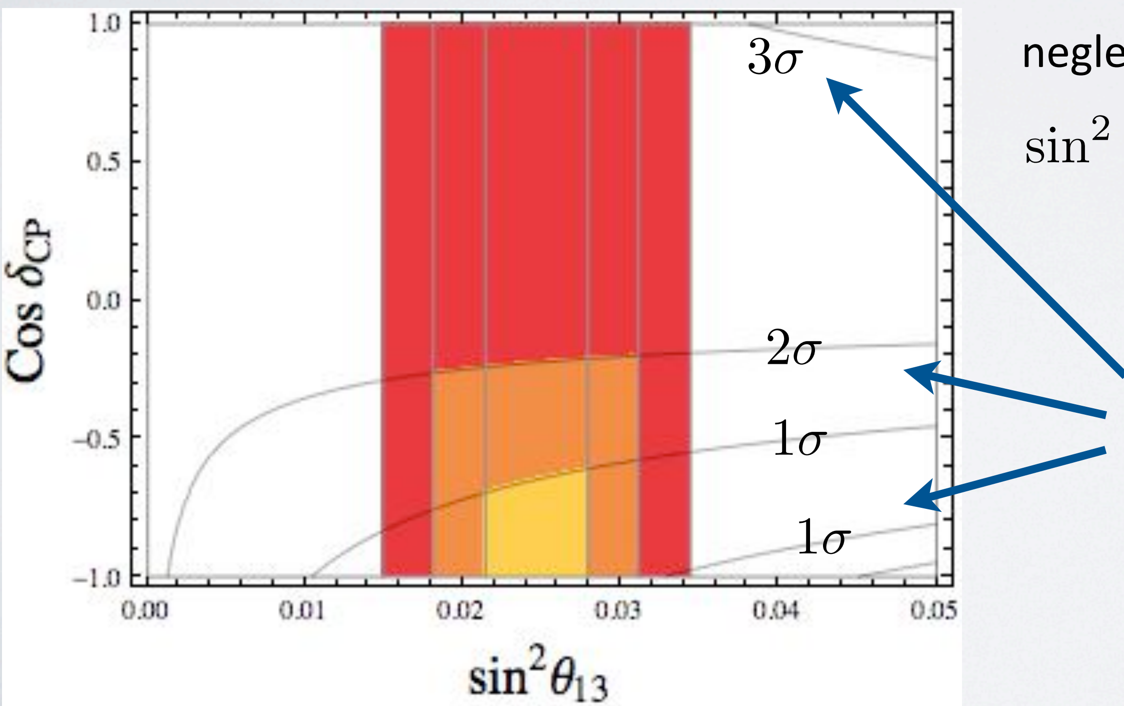
1
0.1
0.01
0.001
10^{-4}
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
$\mathrm{Sin}^2\theta_{13}$
$\mathrm{Sin}^2\theta_{12}$



1
0.1
0.01
0.001
10^{-4}
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\mathrm{Sin}^2\theta_{13}$
$\mathrm{Sin}^2\theta_{23}$







neglecting the subleading corrections:

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\theta_{13}\cos\delta_{CP}$$

[General context:
D. Hernandez & Smirnov 2012]

contours of constant $\sin^2\theta_{23}$

Bimaximal

Also in BM models, the corrections are specific in certain flavour directions:

S4: Altarelli, Feruglio and LM 2009
Adelhart, Bazzocchi and LM 2010

$$\xi^\nu \ll \xi^e$$

$$c^e_{12},\, c^e_{13} \neq 0 \qquad c^e_{13} = 0$$

$$c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13}$$

Bimaximal

Also in BM models, the corrections are specific in certain flavour directions:

S4: Altarelli, Feruglio and LM 2009
Adelhart, Bazzocchi and LM 2010

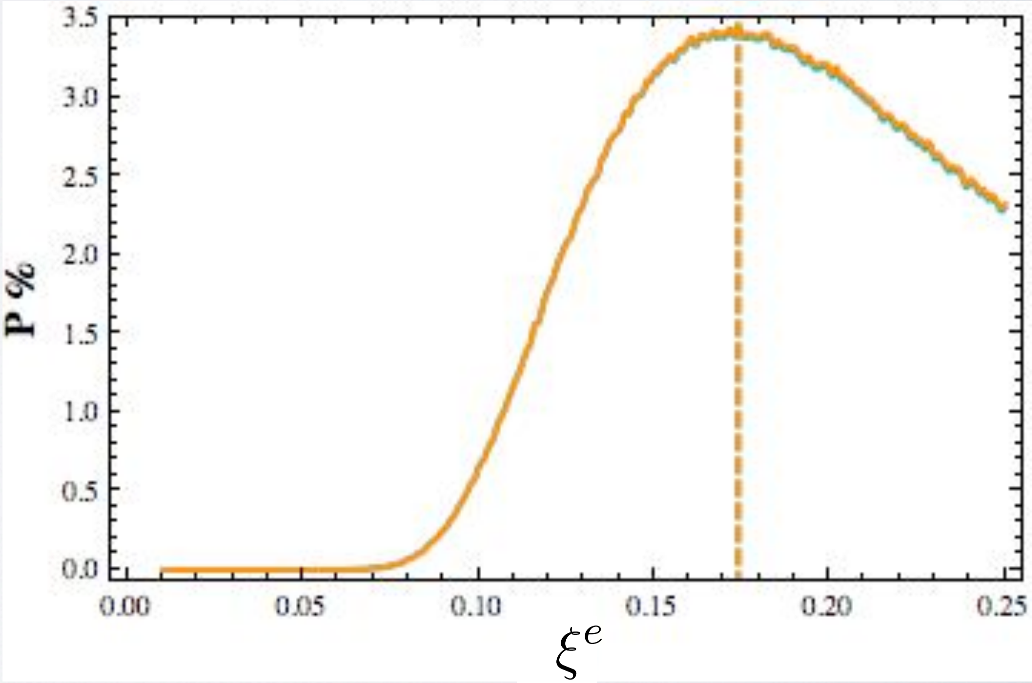
$$\xi^\nu \ll \xi^e$$

$$c^e_{12},\, c^e_{13} \neq 0 \qquad c^e_{13} = 0$$

$$c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13}$$

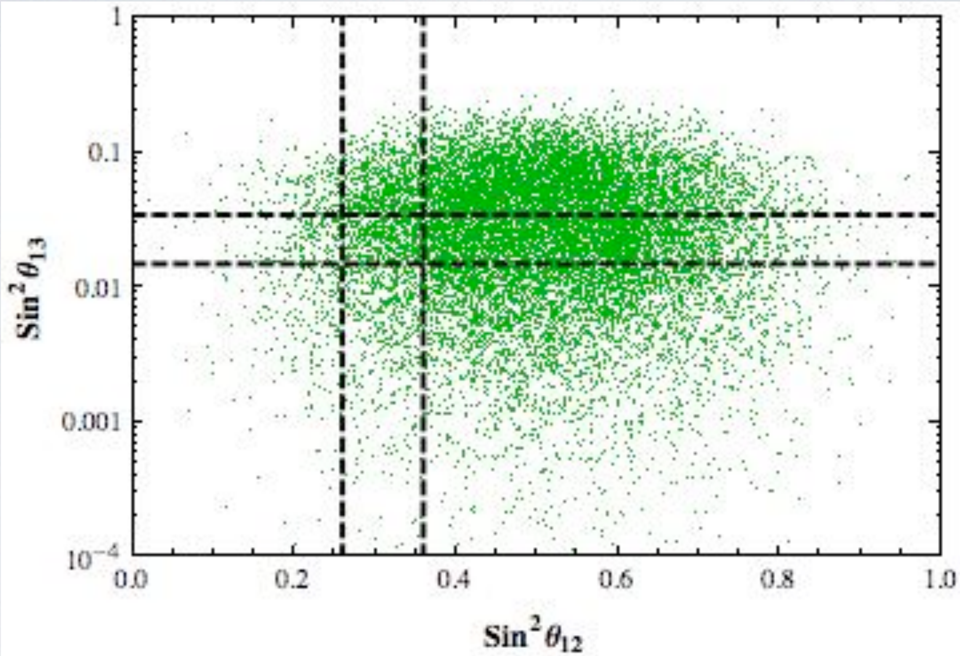
To maximize the success rate for all the three mixing angles inside the 3 σ:

$$\xi^e \simeq 0.17$$

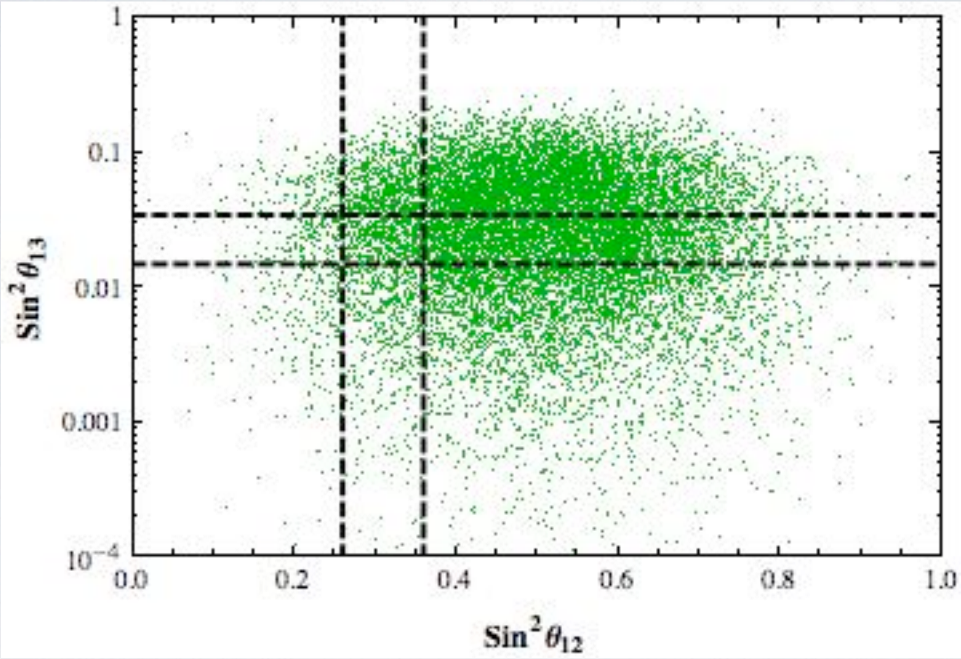


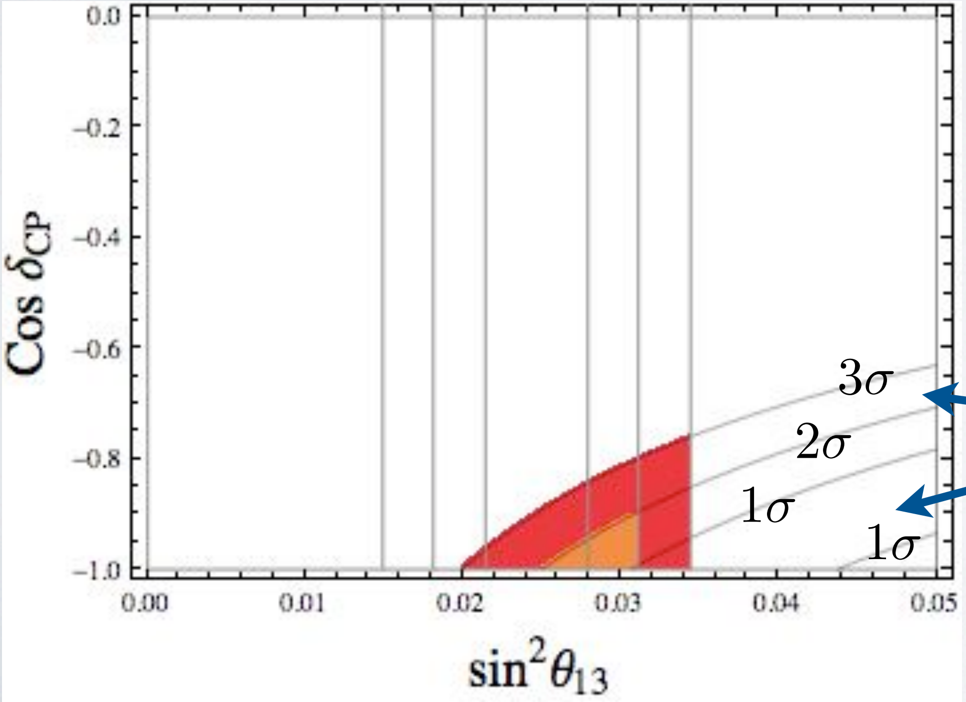
(Similar results for the **self-complementarity** when the corrections come from the neutrino sector instead of the charged lepton sector.)

[Bazzocchi & LM, arXiv:1205.5135]



1
0.1
0.01
0.001
10^{-4}
0.0
0.2
0.4
0.6
0.8
1.0
$\mathrm{Sin}^2\theta_{13}$
$\mathrm{Sin}^2\theta_{12}$





neglecting the subleading corrections:

$$\sin^2\theta_{12} = \frac{1}{2} + \sin\theta_{13}\,\cos\delta_{CP}$$

contours of constant $\sin^2\theta_{23}$

- Which is the meaning of ξ ?
- How can we achieve these flavour structures?

Basic Points on Model Building

[Talk by King;
Alternative way:
talk by Ma]

Basic Points on Model Building

[Talk by King; Alternative way: talk by Ma]

- **Flavour Symmetries** to introduce these flavour structures

Basic Points on Model Building

[Talk by King; Alternative way: talk by Ma]

- **Flavour Symmetries** to introduce these flavour structures
- **Flavour Symmetries cannot be exact**: the Yukawas do not show any symmetry

Basic Points on Model Building

[Talk by King; Alternative way: talk by Ma]

- **Flavour Symmetries** to introduce these flavour structures
- **Flavour Symmetries cannot be exact**: the Yukawas do not show any symmetry
- Starting from a Yukawa Lagrangian invariant under a Flavour Symmetry, masses and mixings arise only through a **symmetry breaking mechanism**:

$$\mathcal{L}_Y = \frac{(Y_e[\varphi^n])_{ij}}{\Lambda_f^n}\, e_i^c\, H^\dagger\, \ell_j + \frac{(Y_\nu[\varphi^m])_{ij}}{\Lambda_f^m}\, \frac{(\ell_i\, \tilde{H}^*)(\tilde{H}^\dagger\, \ell_j)}{2\Lambda_L}$$

where φ are new heavy scalar fields, singlets under SM, called **flavons**

Basic Points on Model Building

[Talk by King; Alternative way: talk by Ma]

- **Flavour Symmetries** to introduce these flavour structures
- **Flavour Symmetries cannot be exact**: the Yukawas do not show any symmetry
- Starting from a Yukawa Lagrangian invariant under a Flavour Symmetry, masses and mixings arise only through a **symmetry breaking mechanism**:

$$\mathcal{L}_Y = \frac{(Y_e[\varphi^n])_{ij}}{\Lambda_f^n}\, e_i^c\, H^\dagger\, \ell_j + \frac{(Y_\nu[\varphi^m])_{ij}}{\Lambda_f^m}\, \frac{(\ell_i\, \tilde{H}^*)(\tilde{H}^\dagger\, \ell_j)}{2\Lambda_L}$$

where φ are new heavy scalar fields, singlets under SM, called **flavons**

- **Suitable** Spontaneous Symmetry Breaking $\varphi \to \langle\varphi\rangle$

$$\frac{\langle\varphi^{e,\nu}\rangle}{\Lambda_f} \approx \xi^{e,\nu}$$

Basic Points on Model Building

[Talk by King; Alternative way: talk by Ma]

- **Flavour Symmetries** to introduce these flavour structures
- **Flavour Symmetries cannot be exact**: the Yukawas do not show any symmetry
- Starting from a Yukawa Lagrangian invariant under a Flavour Symmetry, masses and mixings arise only through a **symmetry breaking mechanism**:

$$\mathcal{L}_Y = \frac{(Y_e[\varphi^n])_{ij}}{\Lambda_f^n}\, e_i^c\, H^\dagger\, \ell_j + \frac{(Y_\nu[\varphi^m])_{ij}}{\Lambda_f^m}\, \frac{(\ell_i\, \tilde{H}^*)(\tilde{H}^\dagger\, \ell_j)}{2\Lambda_L}$$

where φ are new heavy scalar fields, singlets under SM, called **flavons**

- **Suitable** Spontaneous Symmetry Breaking $\varphi \to \langle\varphi\rangle$

$$\frac{\langle\varphi^{e,\nu}\rangle}{\Lambda_f} \approx \xi^{e,\nu}$$

- At LO the PMNS can take one of the previous predictive patterns

Basic Points on Model Building

[Talk by King; Alternative way: talk by Ma]

- **Flavour Symmetries** to introduce these flavour structures
- **Flavour Symmetries cannot be exact**: the Yukawas do not show any symmetry
- Starting from a Yukawa Lagrangian invariant under a Flavour Symmetry, masses and mixings arise only through a **symmetry breaking mechanism**:

$$\mathcal{L}_Y = \frac{(Y_e[\varphi^n])_{ij}}{\Lambda_f^n}\, e_i^c\, H^\dagger\, \ell_j + \frac{(Y_\nu[\varphi^m])_{ij}}{\Lambda_f^m}\, \frac{(\ell_i\, \tilde{H}^*)(\tilde{H}^\dagger\, \ell_j)}{2\Lambda_L}$$

where φ are new heavy scalar fields, singlets under SM, called **flavons**

- **Suitable** Spontaneous Symmetry Breaking $\varphi \to \langle\varphi\rangle$

$$\frac{\langle\varphi^{e,\nu}\rangle}{\Lambda_f} \approx \xi^{e,\nu}$$

- At LO the PMNS can take one of the previous predictive patterns
- At NLO, some corrections arise and they are proportional to the VEV of the flavons: larger is the VEV and larger are the corrections

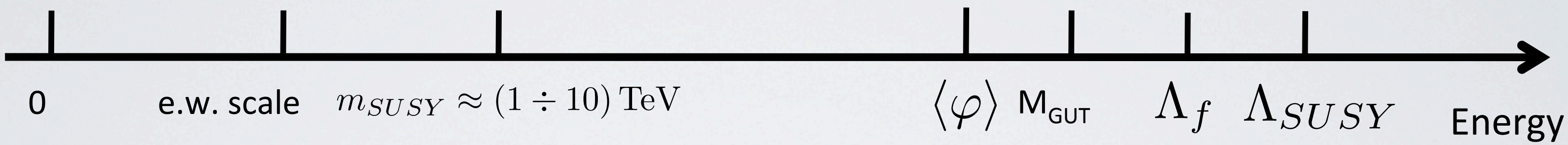
Are there consequences of so large ξ ?

Impact on LFV

Low Energy Observables:
- ν masses
- ν oscillations

- $(g\text{-}2)_\mu$ discrepancy
- dark matter
- gauge coupling unification
- hierarchy problem

- GUTs
- flavour symmetries
- ν^c
- superheavy gauge bosons



The Flavour symmetry at the high-scale affects the low-energy observables **indirectly**:

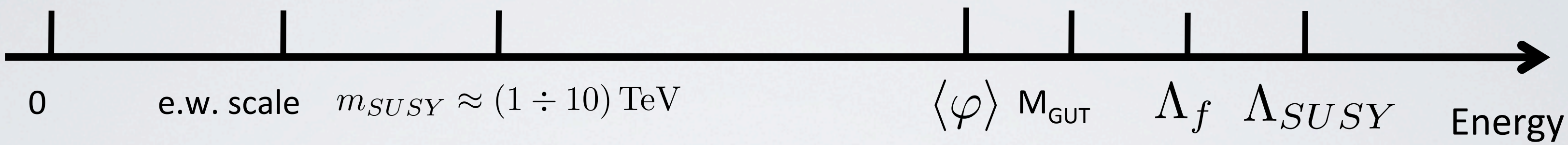
- the flavons φ do not lead to direct contributions (suppressed by the heavy mass)
- the soft-SUSY breaking parameters are governed by the flavour symmetry and its breaking mechanism

Impact on LFV

Low Energy Observables:
- ν masses
- ν oscillations

- $(g\text{-}2)_\mu$ discrepancy
- dark matter
- gauge coupling unification
- hierarchy problem

- GUTs
- flavour symmetries
- ν^c
- superheavy gauge bosons



The Flavour symmetry at the high-scale affects the low-energy observables **indirectly**:

- the flavons φ do not lead to direct contributions (suppressed by the heavy mass)
- the soft-SUSY breaking parameters are governed by the flavour symmetry and its breaking mechanism

⟶ non-universal boundary conditions for the soft terms

⟶ different results w.r.t. CMSSM scenario

$BR(\mu \to e\gamma)$

We focus on the radiative decay $\mu \to e\gamma$:

$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12} \quad @\, 95\%\, \text{C.L.}$$

$BR(\mu \to e\gamma)$

We focus on the radiative decay $\mu \to e\gamma$:

$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12} \quad @\,95\%\,\text{C.L.}$$

The normalized BR is defined by:

$$R_{ij} = \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{SUSY}^4} \left[\left| A_L^{ij} \right|^2 + \left| A_R^{ij} \right|^2 \right]$$

MI

$$A_L^{ij} = a_{LL} \left(\delta_{ij}\right)_{LL} + a_{RL} \frac{m_{SUSY}}{m_i} \left(\delta_{ij}\right)_{RL}$$

$$A_R^{ij} = a_{RR} \left(\delta_{ij}\right)_{RR} + a_{LR} \frac{m_{SUSY}}{m_i} \left(\delta_{ij}\right)_{LR}$$

$BR(\mu \to e\gamma)$

We focus on the radiative decay $\mu \to e\gamma$:

$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12} \quad @\, 95\%\, \text{C.L.}$$

The normalized BR is defined by:

$$R_{ij} = \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{SUSY}^4} \left[\left| A_L^{ij} \right|^2 + \left| A_R^{ij} \right|^2 \right]$$

MI

$$A_L^{ij} = a_{LL} \left(\delta_{ij}\right)_{LL} + a_{RL} \frac{m_{SUSY}}{m_i} \left(\delta_{ij}\right)_{RL}$$

$$A_R^{ij} = a_{RR} \left(\delta_{ij}\right)_{RR} + a_{LR} \frac{m_{SUSY}}{m_i} \left(\delta_{ij}\right)_{LR}$$

The $a_{CC'}$ are loop factors of the SUSY parameters:

$$\tan\beta = \{2, 25\} \qquad \begin{cases} a_{LL} = \{2, 27\} \\ a_{RR} = \{-1.9, -0.6\} \\ a_{RL} = a_{LR} = 0.3 \end{cases}$$

The $(\delta_{ij})_{CC'}$ depend on the soft parameters:

$$(\delta_{ij})_{CC'} = \frac{\left(m^2_{CC'}\right)_{ij}}{m^2_{SUSY}}$$

The $(\delta_{ij})_{CC'}$ depend on the soft parameters:

$$(\delta_{ij})_{CC'} = \frac{(m^2_{CC'})_{ij}}{m^2_{SUSY}}$$

where the soft masses are defined by

$$-\mathcal{L}_m \supset \quad (\bar{\tilde{e}} \quad \tilde{e}^c) \begin{pmatrix} m^2_{eLL} & m^2_{eLR} \\ m^2_{eRL} & m^2_{eRR} \end{pmatrix} \begin{pmatrix} \tilde{e} \\ \bar{\tilde{e}}^c \end{pmatrix} + \bar{\tilde{\nu}}\, m^2_{\nu LL}\, \tilde{\nu}$$

The $(\delta_{ij})_{CC'}$ depend on the soft parameters:

$$(\delta_{ij})_{CC'} = \frac{(m^2_{CC'})_{ij}}{m^2_{SUSY}}$$

where the soft masses are defined by

$$-\mathcal{L}_m \supset \quad (\bar{\tilde{e}} \quad \tilde{e}^c) \begin{pmatrix} m^2_{eLL} & m^2_{eLR} \\ m^2_{eRL} & m^2_{eRR} \end{pmatrix} \begin{pmatrix} \tilde{e} \\ \bar{\tilde{e}}^c \end{pmatrix} + \bar{\tilde{\nu}}\, m^2_{\nu LL}\, \tilde{\nu}$$

- $m^2_{(e,\nu)LL}$ and m^2_{eRR} are hermitian matrices from the Kähler potential
- $m^2_{eLR} = (m^2_{eRL})^\dagger$ from the superpotential

generated from the SUSY Lagrangian analytically continuing all the coupling constants into superspace.

The $(\delta_{ij})_{CC'}$ depend on the soft parameters:

$$(\delta_{ij})_{CC'} = \frac{(m^2_{CC'})_{ij}}{m^2_{SUSY}}$$

where the soft masses are defined by

$$-\mathcal{L}_m \supset \quad (\bar{\tilde{e}} \quad \tilde{e}^c) \begin{pmatrix} m^2_{eLL} & m^2_{eLR} \\ m^2_{eRL} & m^2_{eRR} \end{pmatrix} \begin{pmatrix} \tilde{e} \\ \bar{\tilde{e}}^c \end{pmatrix} + \bar{\tilde{\nu}}\, m^2_{\nu LL}\, \tilde{\nu}$$

- $m^2_{(e,\nu)LL}$ and m^2_{eRR} are hermitian matrices from the Kähler potential
- $m^2_{eLR} = (m^2_{eRL})^\dagger$ from the superpotential

generated from the SUSY Lagrangian analytically continuing all the coupling constants into superspace.

→ **The MI of these mass matrices are governed by** ξ

- Typical TB (GR) models

$$\xi \simeq 0.075$$
$$SR \sim 12\%$$

$\longrightarrow$

$$R_{ij} = \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{SUSY}^4} \left|a_{LL} + a_{RL}\right|^2 \mathcal{O}\left(\xi^4\right)$$

$$R_{\mu e} \approx R_{\tau e} \approx R_{\tau\mu}$$

- Special TB models

$$\xi^\nu \simeq 0.18$$
$$SR \sim 64\%$$

$\longrightarrow$

$$R_{ij} = \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{SUSY}^4} \left|a_{LL} + a_{RL}\right|^2 \mathcal{O}\left(\xi^{\nu 4}\right)$$

$$R_{\mu e} \approx R_{\tau e} \approx R_{\tau\mu}$$

- BM models

$$\xi^e \simeq 0.17$$
$$SR \sim 3.4\%$$

$\longrightarrow$

$$R_{ij} = \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{SUSY}^4} \left|a_{LL} + a_{RL}\right|^2 \times \begin{cases} \mathcal{O}\left(\xi^{e2}\right) & ij = 21, 31 \\ \mathcal{O}\left(\xi^{e4}\right) & ij = 32 \end{cases}$$

$$R_{\mu e} \approx R_{\tau e} \gg R_{\tau\mu}$$

$m_0 = 200\ \text{GeV}$ & $\tan\beta = 15$

$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$

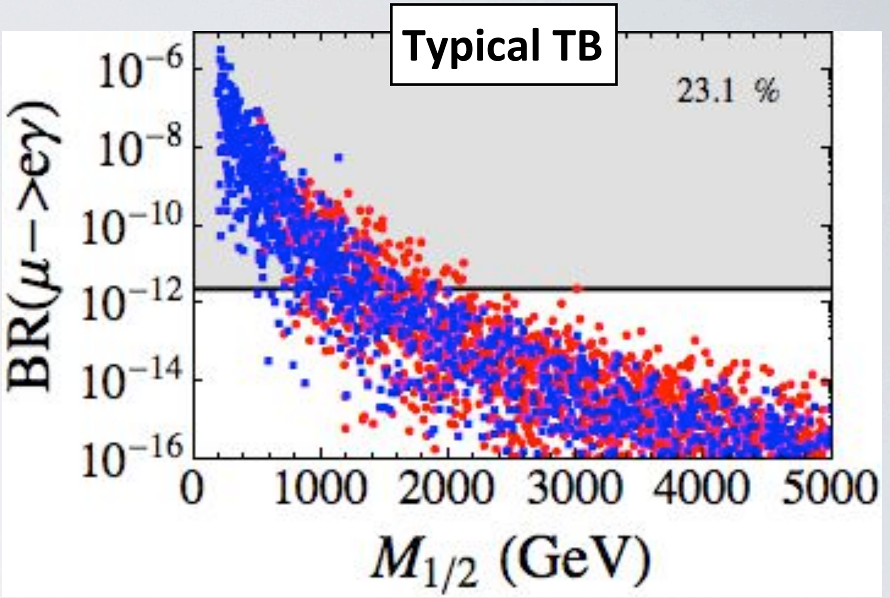
→ $M_{1/2} \lesssim 400\ \text{GeV}$

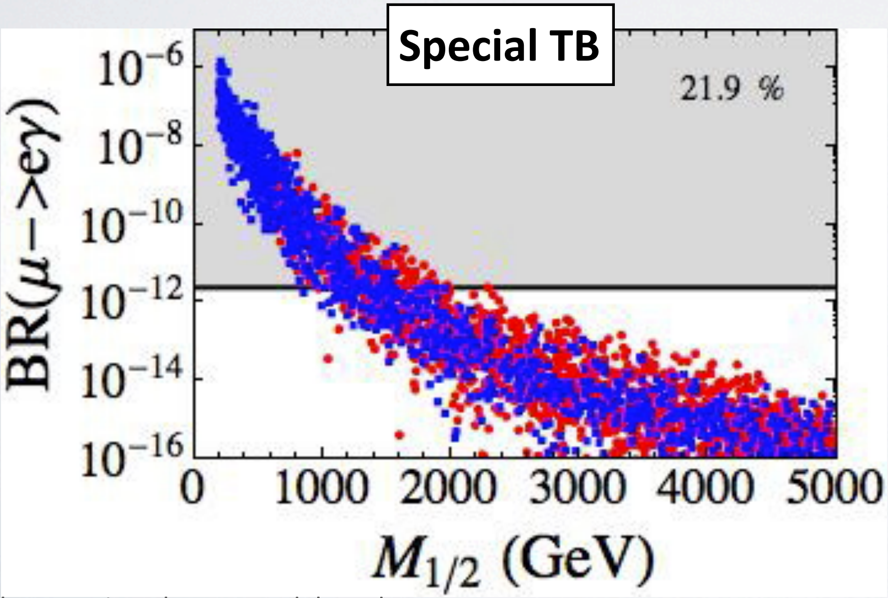
$\chi^0 \approx 156\ \text{GeV}$

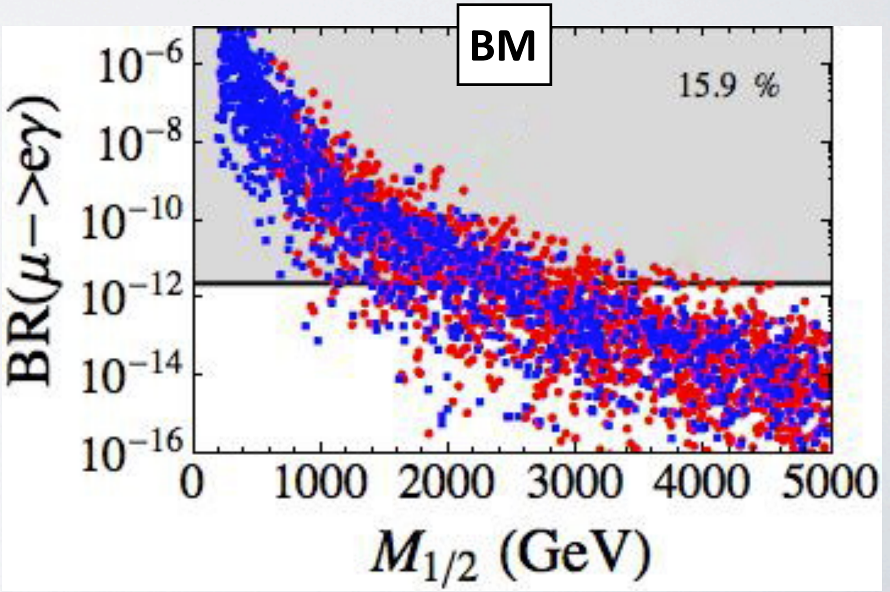
$\chi^\pm \approx 306\ \text{GeV}$

$\tilde{\ell}_R \approx [160, 350]\ \text{GeV}$

$\tilde{\ell}_L \approx [230, 500]\ \text{GeV}$







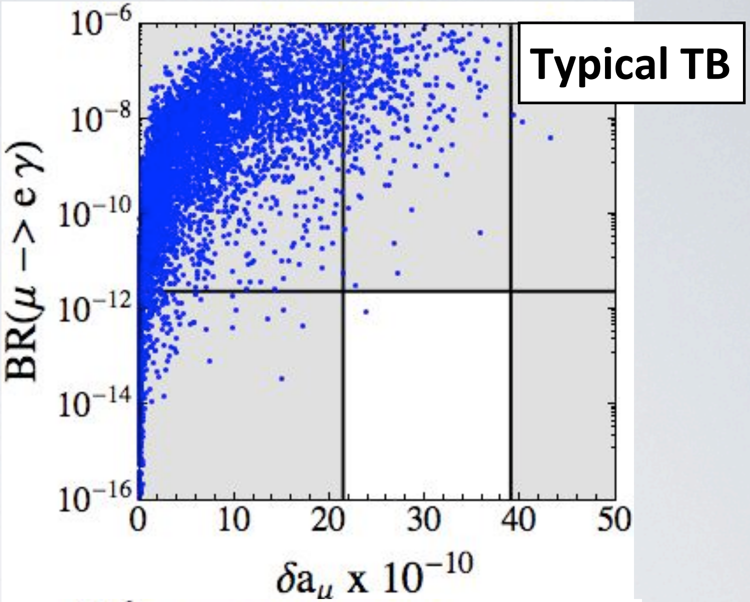
$BR(\mu \to e\gamma)$ & $(g-2)_\mu$

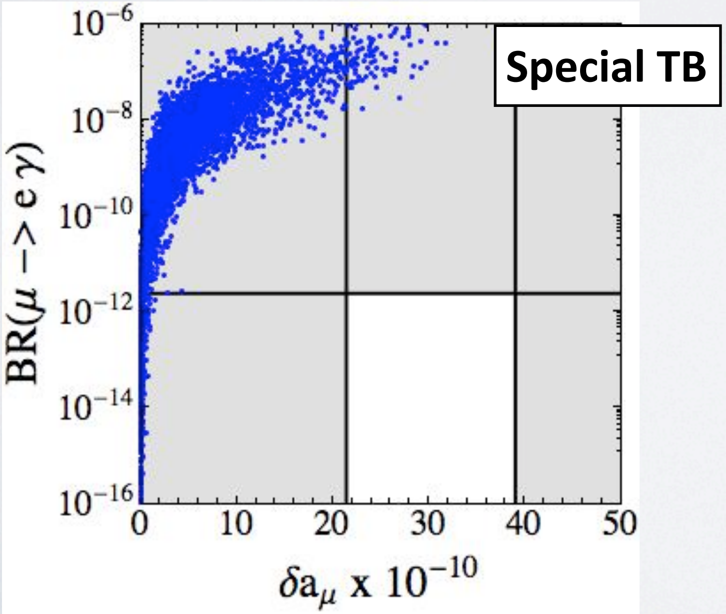
$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$

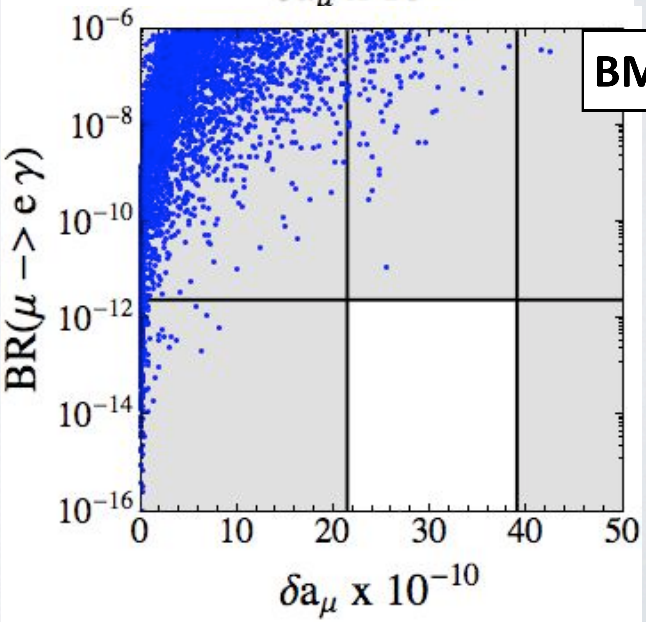
$$\delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = 302(88) \times 10^{-11}$$

$$\tan\beta \in [2, 15]$$

$$m_0,\ M_{1/2} \in [200, 5000]\text{GeV}$$







Intermediate Conclusions

Intermediate Conclusions

- Discrete symmetries can accommodate neutrino mixing patterns and TB, GR & BM can still be taken as starting point

Intermediate Conclusions

- Discrete symmetries can accommodate neutrino mixing patterns and TB, GR & BM can still be taken as starting point
- LFV analysis gives strong constraints
- No possibility to satisfy $BR(\mu \to e\gamma)$ and δa_μ

Intermediate Conclusions

- Discrete symmetries can accommodate neutrino mixing patterns and TB, GR & BM can still be taken as starting point
- LFV analysis gives strong constraints
- No possibility to satisfy $BR(\mu \to e\gamma)$ and δa_μ
- **Are TB, GR & BM the flavour structure of nature? or only accidents?**

Intermediate Conclusions

- Discrete symmetries can accommodate neutrino mixing patterns and TB, GR & BM can still be taken as starting point
- LFV analysis gives strong constraints
- No possibility to satisfy $BR(\mu \to e\gamma)$ and δa_μ

- **Are TB, GR & BM the flavour structure of nature? or only accidents?**
- The new data on θ_{13} have put severe doubts on their **naturalness**:

Intermediate Conclusions

- Discrete symmetries can accommodate neutrino mixing patterns and TB, GR & BM can still be taken as starting point
- LFV analysis gives strong constraints
- No possibility to satisfy $BR(\mu \to e\gamma)$ and δa_μ

- **Are TB, GR & BM the flavour structure of nature? or only accidents?**
- The new data on θ_{13} have put severe doubts on their **naturalness**:
  - All need large corrections in their simplest versions
  - The special TB (the most successful) needs dynamical tricks

Intermediate Conclusions

- Discrete symmetries can accommodate neutrino mixing patterns and TB, GR & BM can still be taken as starting point
- LFV analysis gives strong constraints
- No possibility to satisfy $BR(\mu \to e\gamma)$ and δa_μ

- **Are TB, GR & BM the flavour structure of nature? or only accidents?**
- The new data on θ_{13} have put severe doubts on their **naturalness**:
  - All need large corrections in their simplest versions
  - The special TB (the most successful) needs dynamical tricks
  - Alternatives: Other patterns? [Toorop, Feruglio, Hagedorn 2011 (1°&2°); ecc...]

Intermediate Conclusions

- Discrete symmetries can accommodate neutrino mixing patterns and TB, GR & BM can still be taken as starting point
- LFV analysis gives strong constraints
- No possibility to satisfy $BR(\mu \to e\gamma)$ and δa_μ

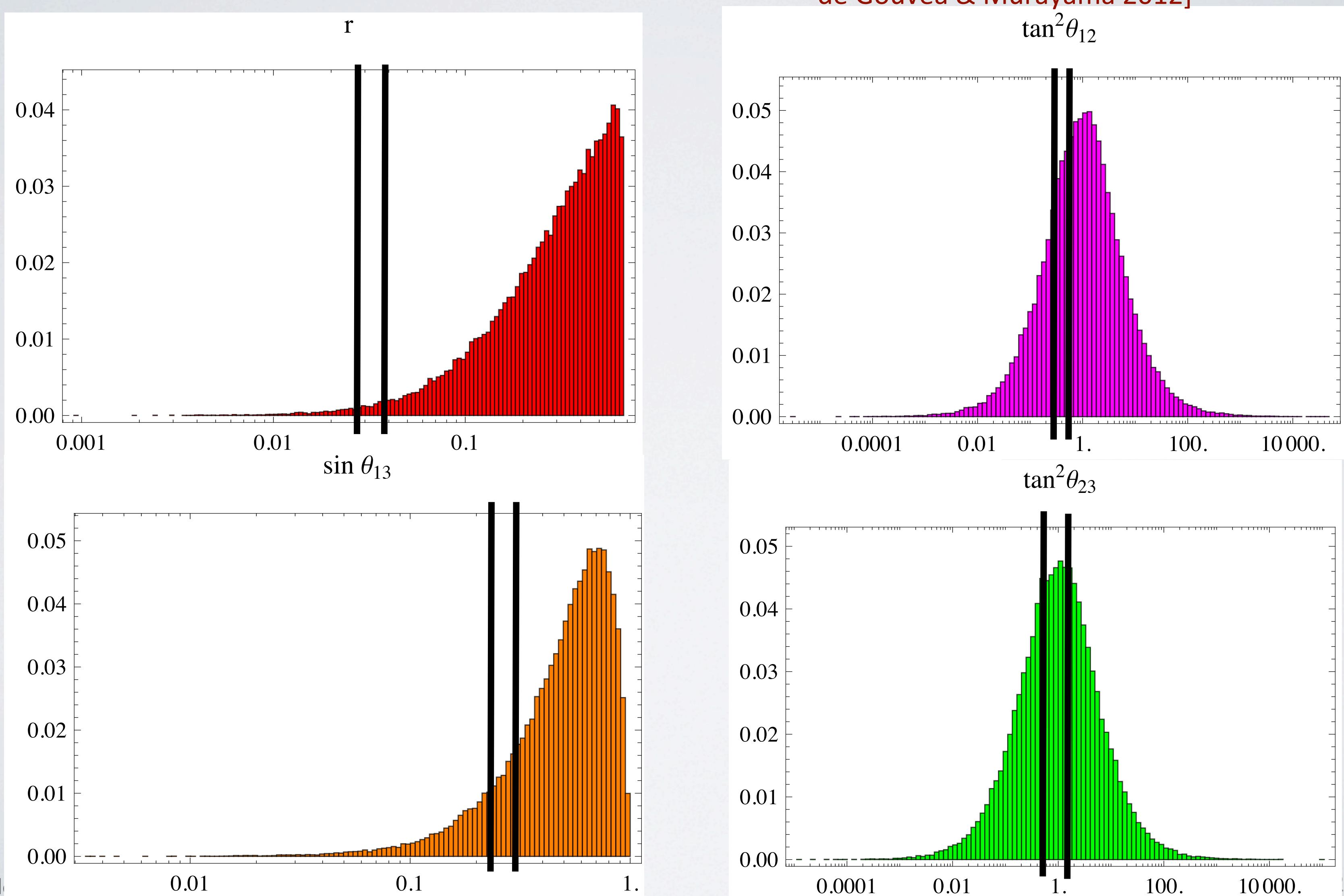
- **Are TB, GR & BM the flavour structure of nature? or only accidents?**
- The new data on θ_{13} have put severe doubts on their **naturalness**:
  - All need large corrections in their simplest versions
  - The special TB (the most successful) needs dynamical tricks
  - Alternatives: Other patterns? [Toorop, Feruglio, Hagedorn 2011 (1°&2°); ecc...]

    Anarchy? Something in between?

Anarchy?

Are these patterns only numerical accidents? If **Yes** what?

→ **Anarchy** [Hall, Murayama & Weiner 1999; de Gouvea & Murayama 2012]

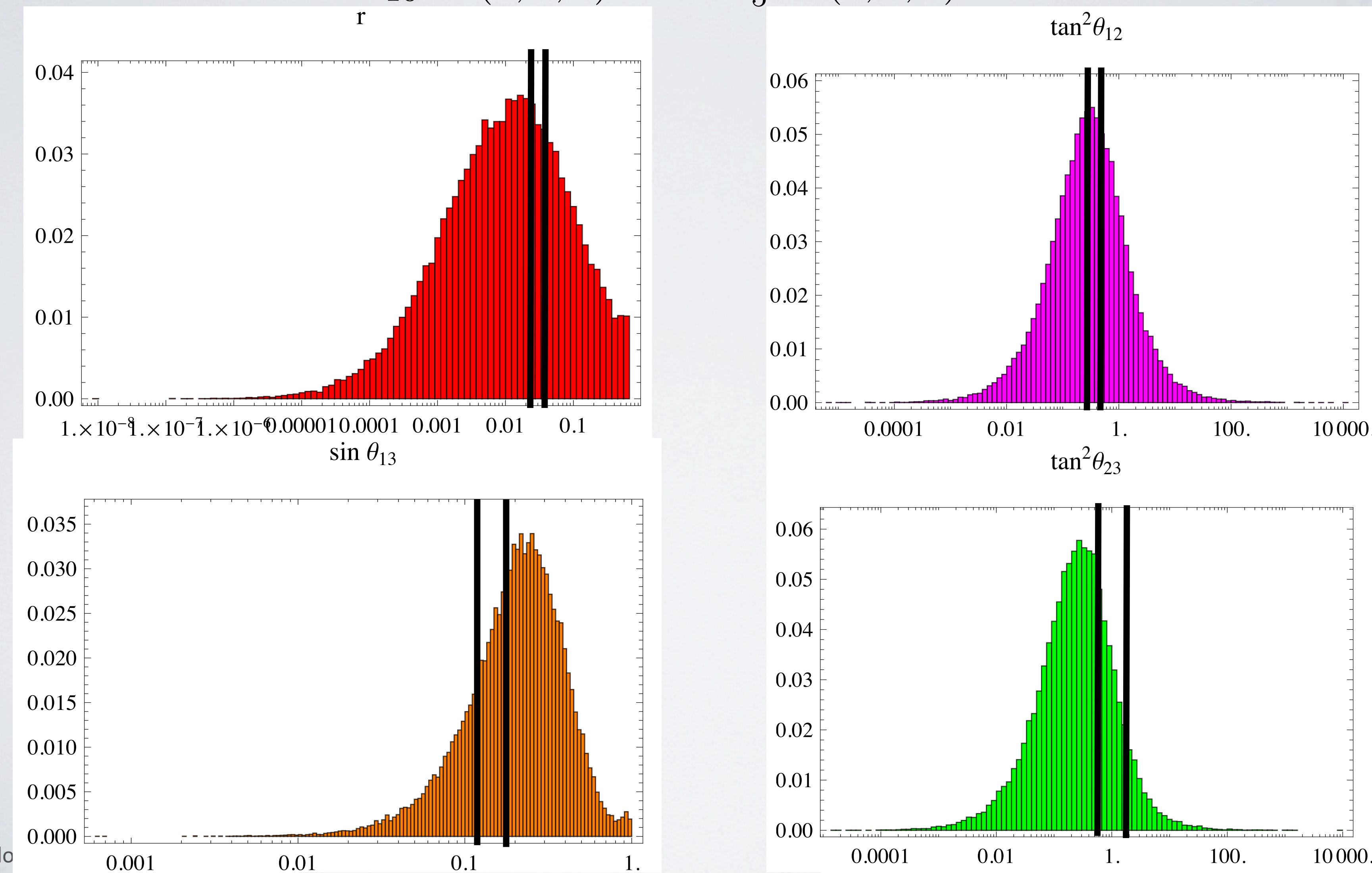


Anarchy?: Better Hierarchy!

[Altarelli, Feruglio, Masina & LM to appear]

Consider a simple U(1) as flavour symmetry, in a SU(5) inspired context: $SU(5) \times U(1)$

$$\Psi_{10} = (5,3,0) \qquad \Psi_{\bar{5}} = (2,1,0)$$

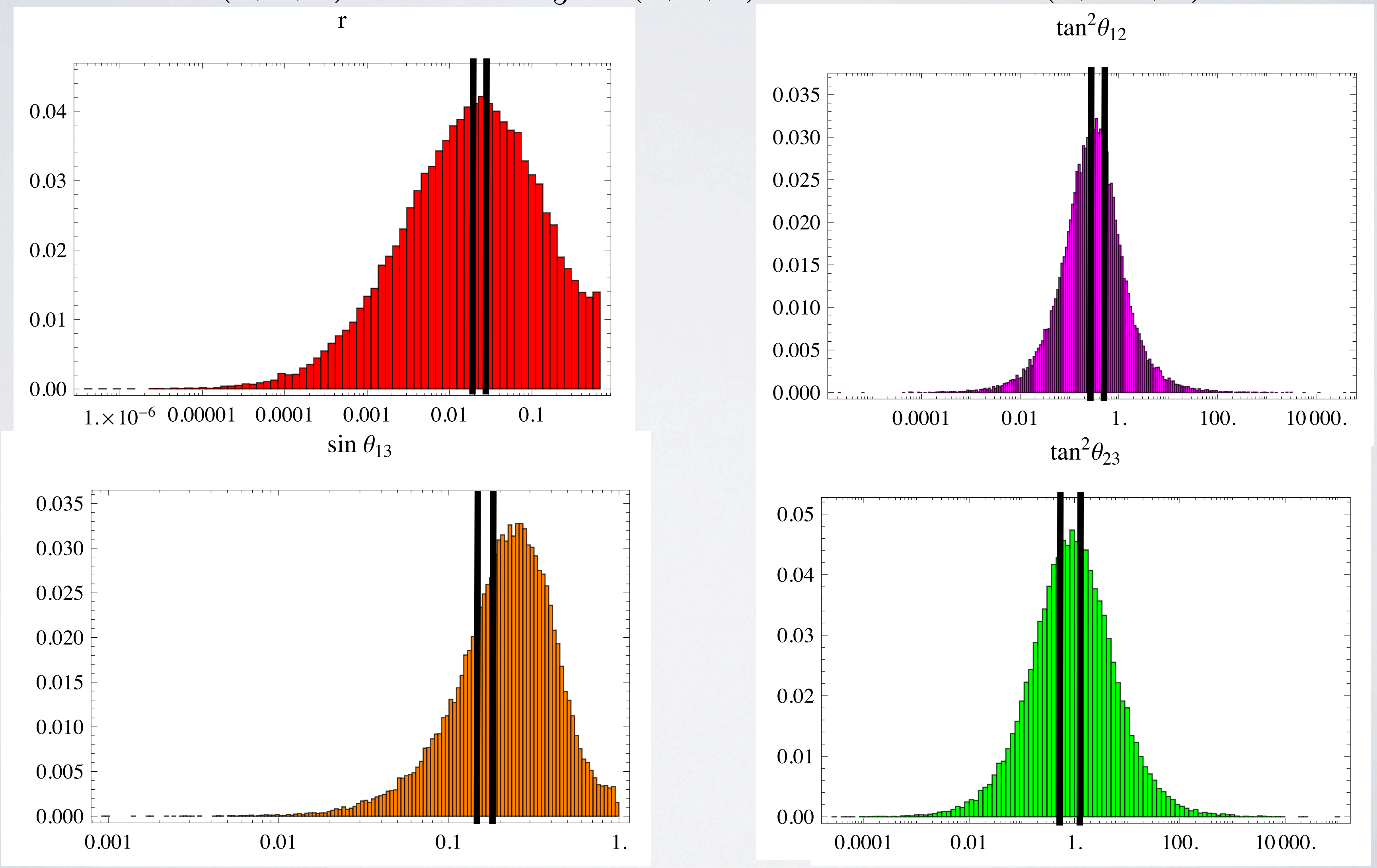


Anarchy?: Better Hierarchy!

[Altarelli, Feruglio & Masina 2002; Altarelli, Feruglio, Masina & LM to appear]

Consider a simple U(1) as flavour symmetry, in a SU(5) inspired context: $SU(5) \times U(1)$

$\Psi_{10} = (5, 3, 0)$ $\Psi_{\overline{5}} = (2, 0, 0)$ $\Psi_1 = (1, -1, 0)$



Intermediate Conclusions

- Discrete symmetries can accommodate neutrino mixing patterns and TB, GR & BM can still be taken as starting point
- LFV analysis gives strong constraints
- No possibility to satisfy $BR(\mu \to e\gamma)$ and δa_μ

- **Are TB, GR & BM the flavour structure of nature? or only accidents?**
- The new data on θ_{13} have put severe doubts on their **naturalness**:
  - All need large corrections in their simplest versions
  - The special TB (most successful) needs dynamical tricks
  - Alternatives: Other patterns? [Toorop, Feruglio, Hagedorn 2011 (1°&2°); ecc...]

    Anarchy? Something in between?

    New correlations?

New Correlations?

[Alonso, Gavela, D.Hernandez & LM 1206.3167]

The **minimisation of the scalar potential**, that explains the VEV of the flavons in a particularly predictive **MLFV scenario** (2 RH neutrinos), links the **neutrino spectrum, the mixing angles and the Majorana phase**: main responsible **Majorana Nature**

[Gavela, Hambye, D.Hernandez & P.Hernandez 2009]

New Correlations?

[Alonso, Gavela, D.Hernandez & LM 1206.3167]

The **minimisation of the scalar potential**, that explains the VEV of the flavons in a particularly predictive **MLFV scenario** (2 RH neutrinos), links the **neutrino spectrum, the mixing angles and the Majorana phase**: main responsible **Majorana Nature**

[Gavela, Hambye, D.Hernandez & P.Hernandez 2009]

$$\mathcal{G}_{fl} \sim SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N$$

promoting $\quad Y_E \sim (3\,,\,\bar{3}\,,\,1) \qquad \tilde{Y}_\nu \sim (3\,,\,1\,,\,2)$

$$-\mathcal{L}_{mass} = \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T + \Lambda(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c.$$

New Correlations?

[Alonso, Gavela, D.Hernandez & LM 1206.3167]

The **minimisation of the scalar potential**, that explains the VEV of the flavons in a particularly predictive **MLFV scenario** (2 RH neutrinos), links the **neutrino spectrum, the mixing angles and the Majorana phase**: main responsible **Majorana Nature**

[Gavela, Hambye, D.Hernandez & P.Hernandez 2009]

$$\mathcal{G}_{fl} \sim SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N$$

promoting $\quad Y_E \sim (3, \bar{3}, 1) \qquad \tilde{Y}_\nu \sim (3, 1, 2)$

$$-\mathcal{L}_{mass} = \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T + \Lambda(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c.$$

The scalar potential is extremized when:

$$(y^2 - y'^2)\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0$$

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{y^2 - y'^2}{y^2 + y'^2} \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}$$

New Correlations?

[Alonso, Gavela, D.Hernandez & LM 1206.3167]

The **minimisation of the scalar potential**, that explains the VEV of the flavons in a particularly predictive **MLFV scenario** (2 RH neutrinos), links the **neutrino spectrum, the mixing angles and the Majorana phase**: main responsible **Majorana Nature**

[Gavela, Hambye, D.Hernandez & P.Hernandez 2009]

$$\mathcal{G}_{fl} \sim SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N$$

promoting $\quad Y_E \sim (3\,,\,\bar{3}\,,\,1) \qquad \tilde{Y}_\nu \sim (3\,,\,1\,,\,2)$

$$-\mathcal{L}_{mass} = \overline{\ell}_L \phi Y_E E_R + \overline{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T + \Lambda(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c.$$

The scalar potential is extremized when:

$$(y^2 - y'^2)\sqrt{m_{\nu_2} m_{\nu_1}}\,\sin 2\theta \cos 2\alpha = 0$$

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{y^2 - y'^2}{y^2 + y'^2} \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}$$

2 family case:

$$\alpha = \pm\pi/4$$

$m_{\nu_2} \approx m_{\nu_1} \longrightarrow \theta_{12}$ large

New Correlations?

[Alonso, Gavela, D.Hernandez & LM 1206.3167]

The **minimisation of the scalar potential**, that explains the VEV of the flavons in a particularly predictive **MLFV scenario** (2 RH neutrinos), links the **neutrino spectrum, the mixing angles and the Majorana phase**: main responsible **Majorana Nature**

[Gavela, Hambye, D.Hernandez & P.Hernandez 2009]

$$\mathcal{G}_{fl} \sim SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N$$

promoting $\quad Y_E \sim (3, \bar{3}, 1) \qquad \tilde{Y}_\nu \sim (3, 1, 2)$

$$-\mathcal{L}_{mass} = \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T + \Lambda(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c.$$

The scalar potential is extremized when:

$$(y^2 - y'^2)\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0$$

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{y^2 - y'^2}{y^2 + y'^2} \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}$$

2 family case:

$$\alpha = \pm\pi/4$$

$m_{\nu_2} \approx m_{\nu_1} \longrightarrow \theta_{12}$ large

3 family case:

$$\alpha = \pm\pi/4$$

IH: $m_{\nu_2} \approx m_{\nu_1} \longrightarrow \theta_{12}$ large, but negative

New Correlations?

[Alonso, Gavela, D.Hernandez & LM 1206.3167]

The **minimisation of the scalar potential**, that explains the VEV of the flavons in a particularly predictive **MLFV scenario** (2 RH neutrinos), links the **neutrino spectrum, the mixing angles and the Majorana phase**: main responsible **Majorana Nature**

[Gavela, Hambye, D.Hernandez & P.Hernandez 2009]

$$\mathcal{G}_{fl} \sim SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N$$

promoting $\quad Y_E \sim (3, \bar{3}, 1) \qquad \tilde{Y}_\nu \sim (3, 1, 2)$

$$-\mathcal{L}_{mass} = \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T + \Lambda(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c.$$

The scalar potential is extremized when:

$$(y^2 - y'^2)\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0$$

$$\mathrm{tg} 2\theta = \sin 2\alpha \frac{y^2 - y'^2}{y^2 + y'^2} \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}$$

2 family case:

$$\alpha = \pm\pi/4$$

$m_{\nu_2} \approx m_{\nu_1} \longrightarrow \theta_{12}$ large

3 family case:

$$\alpha = \pm\pi/4$$

work in progress

IH: $m_{\nu_2} \approx m_{\nu_1} \longrightarrow \theta_{12}$ large, but negative

Thanks for your attention

Backup Slides

Typical Tri-Bimaximal

$$\xi^e \approx \xi^\nu \equiv \xi$$

$$c^e_{12} \approx c^e_{23} \approx c^e_{13}$$

$$c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13}$$

$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{R}e(c^e_{23})\,\xi + \frac{1}{\sqrt{3}}\left(\mathcal{R}e(c^\nu_{13}) - \sqrt{2}\,\mathcal{R}e(c^\nu_{23})\right)\,\xi$$

$$\sin^2\theta_{12} = \frac{1}{3} - \frac{2}{3}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi + \frac{2\sqrt{2}}{3}\,\mathcal{R}e(c^\nu_{12})\,\xi$$

$$\sin\theta_{13} = \frac{1}{6}\left|3\sqrt{2}\,(c^e_{12} - c^e_{13}) + 2\sqrt{3}\left(\sqrt{2}\,c^\nu_{13} + c^\nu_{23}\right)\right|\,\xi$$

Special Tri-Bimaximal

$$\xi^\nu \gg \xi^e$$

$$c^\nu_{12} = c^\nu_{23} = 0 \qquad c^\nu_{13} \neq 0$$

$$c^e_{12} \approx c^e_{23} \approx c^e_{13}$$

$$\delta_{CP} \approx \arg c^\nu_{13}$$

$$\sin\theta_{13} = \left|\sqrt{\frac{2}{3}}\, c^\nu_{13}\, \xi^\nu + \frac{c^e_{12} - c^e_{13}}{\sqrt{2}}\, \xi^e\right|$$

$$\sin^2\theta_{12} = \frac{1}{3} + \frac{2}{9}\,|c^\nu_{13}\,\xi^\nu|^2 - \frac{2}{3}\,\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi^{e2}$$

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{3}}\,|c^\nu_{13}\,\xi^\nu|\,\cos\delta_{CP} + \mathcal{R}e(c^e_{23})\,\xi^{e2}$$

Bimaximal

$$\xi^\nu \ll \xi^e$$

$$c^e_{12},\, c^e_{13} \neq 0 \qquad\qquad c^e_{13} = 0$$

$$c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13}$$

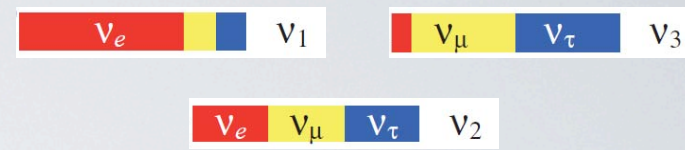
$$\delta_{CP} = \pi + \arg\left(c^e_{12} - c^e_{13}\right)$$

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\,|c^e_{12} - c^e_{13}|\,\xi^e$$

$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}\,\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi^e$$

$$\sin^2\theta_{23} = \frac{1}{2}$$

Typical Tri-Bimaximal



$$\sin^2\theta_{12}^{TB} = 1/3$$
$$\sin^2\theta_{23}^{TB} = 1/2$$
$$\sin\theta_{13}^{TB} = 0$$

$$U^{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

In the basis of diagonal charged leptons:

$$M_\nu^{TB} = \begin{pmatrix} x & y & y \\ y & z & x+y-z \\ y & x+y-z & z \end{pmatrix}$$

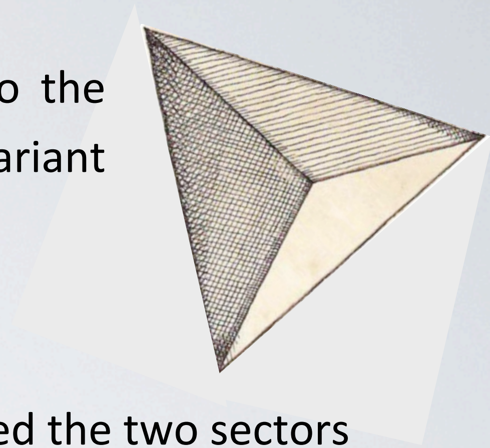
mu-tau sym
magic sym

Discrete Symmetries:

[A4: Adhikary; Altarelli; Aristizabal Sierra; Babu; Bazzocchi; Bertuzzo; Di Bari; Branco; Brahmachari; Chen; Choubey; Ciafaloni; Csaki; Delaunay; Felipe; Feruglio; Frampton; Frigerio; Ghosal; Grimus; Grojean; Grossmann; Hagedorn; He; Hirsch; Honda; Joshipura; Kaneko; Keum; King; Koide; Kuhbock; Lavoura; Lin; Ma; Machado; Malinsky; Matsuzaki; de Medeiros Varzielas; Meloni; LM; Mitra; Molinaro; Morisi; Nardi; Parida; Paris; Petcov; Pleitez; Picariello; Rajasekaran; Riazzudin, Romao; Serodio; Skadhauge; Tanimoto; Torrente-Lujan; Urbano; Valle; Villanova del Moral; Volkas; Yin; Zee; ...;
S_4, T', $\Delta(3n^2)$: de Adelhart Toorop; Altarelli, Bazzocchi; Chen; Ding; Hagedorn; Feruglio; Frampton; Kephart; King; Lam; Lin; Luhn; Ma; Mahanthappa; Matsuzaki; de Medeiros Varzielas; LM; Morisi; Nasri; Ramond; Ross;
...]

The Altarelli-Feruglio Model

[Altarelli & Feruglio 2005]



A_4 is the group of even permutations of 4 objects isomorphic to the group of the rotations which leave a regular tetrahedron invariant (Subgroup of SO(3)).

It has 12 elements and 4 representations: 3, 1, 1’, 1’’

$$G_f = A_4 \times G_{aux}$$

- keeps separated the two sectors
- explains the hierarchy

$$m_e \ll m_\mu \ll m_\tau$$

$\langle \varphi_\ell \rangle$ → $G_\ell = Z_3$ → M_e^{diag}

$\langle \varphi_\nu \rangle$ → $G_\nu = Z_2$ → M_ν^{TB}

$$\Rightarrow \quad U = U_e^\dagger U_\nu = U^{TB}$$

The Altarelli-Feruglio Model

[Altarelli & Feruglio 2005]

	Matter fields				Higgs	Flavons			
	ℓ	e^c	μ^c	τ^c	$h_{u,d}$	θ	φ_T	φ_S	ξ
A_4	3	1	$1''$	$1'$	1	1	3	3	1

$$w_e = y_e \frac{\theta^2}{\Lambda^3} e^c \left(\varphi_T \ell\right) h_d + y_\mu \frac{\theta}{\Lambda^2} \mu^c \left(\varphi_T \ell\right)' h_d + y_\tau \frac{1}{\Lambda} \tau^c \left(\varphi_T \ell\right)'' h_d$$

$$w_\nu = x_a \frac{\xi}{\Lambda} \frac{h_u \ell h_u \ell}{\Lambda_L} + x_b \left(\frac{\varphi_S}{\Lambda} \frac{h_u \ell h_u \ell}{\Lambda_L} \right)$$

Expansion in ϕ/Λ

vacuum alignment:

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (u, 0, 0)$$

$$\frac{\langle \varphi_S \rangle}{\Lambda} = c_b (u, u, u)$$

$$\frac{\langle \xi \rangle}{\Lambda} = c_a u$$

$$\frac{\langle \theta \rangle}{\Lambda} = t$$

$$M_e = \mathrm{diag}(y_e\, t^2,\, y_\mu\, t,\, y_\tau)\, v_d\, u$$

$$\frac{m_e}{m_\mu} = \frac{m_\mu}{m_\tau} = t \approx 0.05$$

$$M_\nu = \begin{pmatrix} a + \frac{2}{3} b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3} b & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & \frac{2}{3} b \end{pmatrix} v_u^2$$

$$M_\nu^{diag} = v_u^2 \mathrm{diag}(a + b,\, a,\, -a + b)$$

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{35}$$

With RH Neutrino

When RH neutrinos are present in the spectrum, their RGE are important:

$$\left(m^2_{eLL}\right)_{ij} \simeq -\frac{1}{8\pi^2}\left(3\,m_0^2 + A_0^2\right)\sum_k (\hat{Y}_\nu^\dagger)_{ik}\log\left(\frac{\Lambda}{M_k}\right)(\hat{Y}_\nu)_{kj}$$

If the RH neutrinos transform as 3dim irreducible representations then

$$\rho(g)\ Y_\nu^\dagger Y_\nu\ \rho(g)^\dagger = Y_\nu^\dagger Y_\nu \to \left[\rho(g),\, Y_\nu^\dagger Y_\nu\right] = 0 \to Y_\nu^\dagger Y_\nu \propto 1 \to Y_\nu \text{ is unitary}$$

Writing the usual type I See-Saw relation:

$$m_\nu = \frac{v^2}{2}\hat{Y}_\nu^T M^{-1}\hat{Y}_\nu$$

$$\longrightarrow \quad \hat{Y}_\nu = k\ U^\dagger + \ldots \qquad M^{-1} = \frac{2}{|k|^2 v^2} m_\nu^{diag}$$

$$\longrightarrow \quad \left(m^2_{eLL}\right)_{ij} \simeq -\frac{|k|^2}{8\pi^2}\left(3\,m_0^2 + A_0^2\right)\left[U_{i2}\log\frac{m_2}{m_1}U^*_{j2} + U_{i3}\log\frac{m_3}{m_1}U^*_{j3}\right] + \ldots$$

Very predictive relation: it only depends on the LO mixing pattern and neutrino spectrum

- **TB pattern**

$$(m^2_{eLL})_{\mu e} \propto \frac{1}{3}\log\left(\frac{m_2}{m_1}\right)$$

$$(m^2_{eLL})_{\tau e} \propto \frac{1}{3}\log\left(\frac{m_2}{m_1}\right)$$

$$(m^2_{eLL})_{\tau \mu} \propto \frac{1}{3}\log\left(\frac{m_2}{m_1}\right) - \frac{1}{2}\log\left(\frac{m_3}{m_1}\right)$$

- **GR pattern**

$$(m^2_{eLL})_{\mu e} \propto -\frac{1}{\sqrt{10}}\log\left(\frac{m_2}{m_1}\right)$$

$$(m^2_{eLL})_{\tau e} \propto -\frac{1}{\sqrt{10}}\log\left(\frac{m_2}{m_1}\right)$$

$$(m^2_{eLL})_{\tau \mu} \propto \frac{5+\sqrt{5}}{20}\log\left(\frac{m_2}{m_1}\right) - \frac{1}{2}\log\left(\frac{m_3}{m_1}\right)$$

- **BM pattern**

$$(m^2_{eLL})_{\mu e} \propto \frac{1}{4}\sqrt{\frac{3}{2}}\log\left(\frac{m_2}{m_1}\right)$$

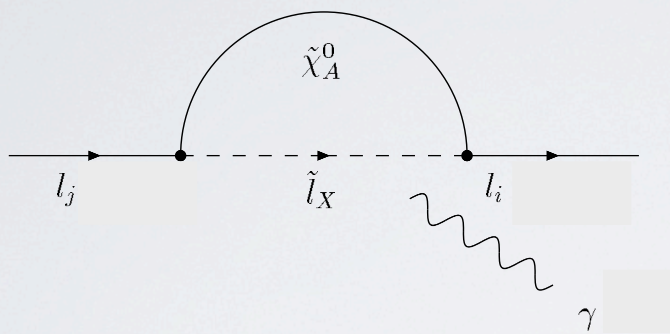
$$(m^2_{eLL})_{\tau e} \propto \frac{1}{4}\sqrt{\frac{3}{2}}\log\left(\frac{m_2}{m_1}\right)$$

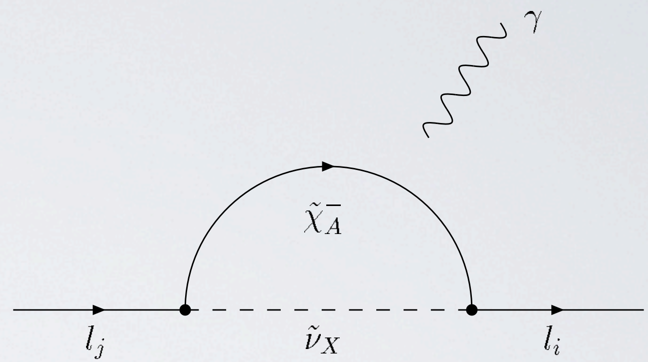
$$(m^2_{eLL})_{\tau \mu} \propto \frac{3}{8}\log\left(\frac{m_2}{m_1}\right) - \frac{1}{2}\log\left(\frac{m_3}{m_1}\right)$$

→ Expressing all the neutrino masses in terms of the lightest one, these quantities depend on only **1 parameter**

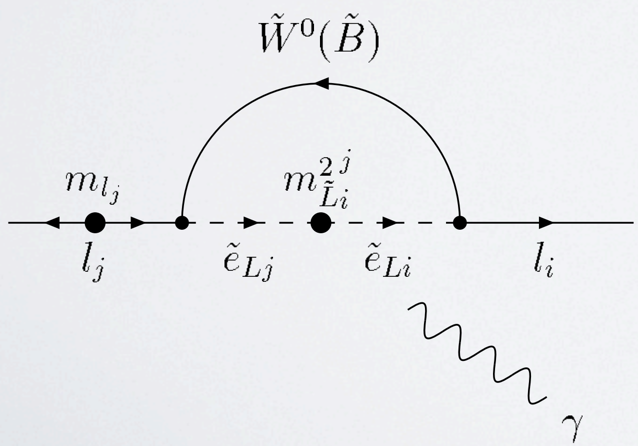
Mass Insertion Approximation

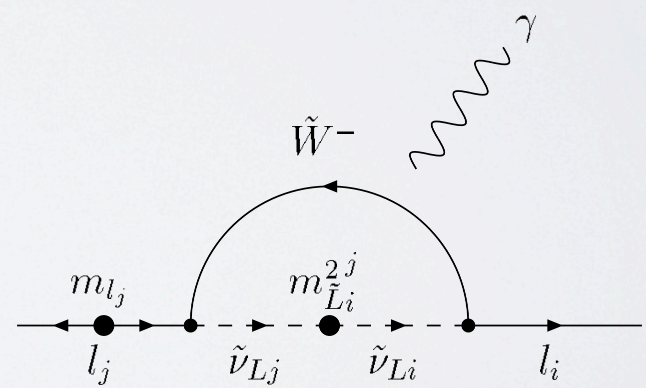
To get EDM, MDM and the LFV transitions we should calculate diagrams as:





A Good analytical approach is the Mass Insertion approximation:





The $(\delta_{ij})_{CC'}$ depend on the soft parameters:

$$(\delta_{ij})_{CC'} = \frac{\left(m^2_{CC'}\right)_{ij}}{m^2_{SUSY}}$$

where the soft masses are defined by

$$-\mathcal{L}_m \supset \quad \left(\bar{\tilde{e}} \quad \tilde{e}^c\right) \begin{pmatrix} m^2_{eLL} & m^2_{eLR} \\ m^2_{eRL} & m^2_{eRR} \end{pmatrix} \begin{pmatrix} \tilde{e} \\ \bar{\tilde{e}}^c \end{pmatrix} + \bar{\tilde{\nu}}\, m^2_{\nu LL}\, \tilde{\nu}$$

- $m^2_{(e,\nu)LL}$ and m^2_{eRR} are hermitian matrices from the Kähler potential
- $m^2_{eLR} = (m^2_{eRL})^\dagger$ from the superpotential

generated from the SUSY Lagrangian analytically continuing all the couplings constants into superspace:

$$\mathcal{L} \supset \int d^2\theta\, d^2\bar{\theta}\, \bar{\ell}\ell \to \int d^2\theta\, d^2\bar{\theta}\, \left(1 + k\, m_0^2\, \theta^2 \bar{\theta}^2\right) \bar{\ell}\ell$$

$$\mathcal{L} \supset \int d^2\theta\, y_e\, e^c\, \ell\, h_d \to \int d^2\theta\, \left(y_e + x_e\, m_0\, \theta^2\right)\, e^c\, \ell\, h_d$$

The flavour is encoded into the soft masses:

$$\mathcal{L} \supset \int d^2\theta\, d^2\bar{\theta}\, \left(1 + k\, m_0^2\, \theta^2\bar{\theta}^2\right) \left(\bar{\ell}\ell + \bar{\ell}\ell \frac{\varphi^n}{\Lambda_f^n}\right)$$

→ Non-canonical kinetic terms

$$\rightarrow \quad (m^2_{eLL})_K = \begin{pmatrix} 1 & \mathcal{O}(\xi^n) & \mathcal{O}(\xi^n) \\ \mathcal{O}(\xi^n) & 1 & \mathcal{O}(\xi^n) \\ \mathcal{O}(\xi^n) & \mathcal{O}(\xi^n) & 1 \end{pmatrix} m_0^2$$

$$\mathcal{L} \supset \int d^2\theta\, \left(Y_e + A_e\, m_0 \theta^2\right)_{ij}\, e_i^c\, \ell_j\, h_d$$

$$\rightarrow \quad Y_e = \begin{pmatrix} y_e & y_e\, \mathcal{O}(\xi^n) & y_e\, \mathcal{O}(\xi^n) \\ y_\mu\, \mathcal{O}(\xi^n) & y_\mu & y_\mu\, \mathcal{O}(\xi^n) \\ y_\tau\, \mathcal{O}(\xi^n) & y_\tau\, \mathcal{O}(\xi^n) & y_\tau \end{pmatrix}$$

$$\rightarrow \quad m^2_{eRL} = \begin{pmatrix} y_e & y_e\, \mathcal{O}(\xi^n) & y_e\, \mathcal{O}(\xi^n) \\ y_\mu\, \mathcal{O}(\xi^n) & y_\mu & y_\mu\, \mathcal{O}(\xi^n) \\ y_\tau\, \mathcal{O}(\xi^n) & y_\tau\, \mathcal{O}(\xi^n) & y_\tau \end{pmatrix} m_0\, v_d$$

same flavour structure but different coefficients

SUSY Parameters

Many parameters: $M_1, M_2, \mu, \tan\beta, m_L^2, m_R^2, A_0$

All of them are not independent: $m_L^2(\Lambda_f) = m_R^2(\Lambda_f) = A_0 \equiv m_0$

$$\tan\beta \approx 100\,\eta\,y_\tau$$

SUGRA context: $m_L^2(m_W) \simeq m_L^2(\Lambda_f) + 0.5M_2^2(\Lambda_f) + 0.04M_1^2(\Lambda_f)$

$$m_R^2(m_W) \simeq m_R^2(\Lambda_f) + 1.5M_1^2(\Lambda_f)$$

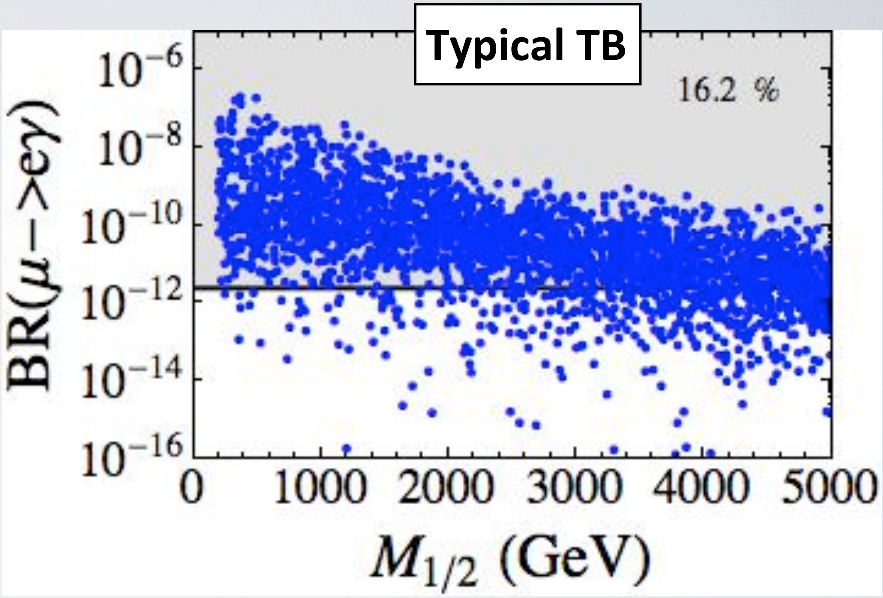
$$M_i(m_W) \simeq \frac{\alpha_i(m_W)}{\alpha_i(\Lambda_f)} M_i(\Lambda_f)$$

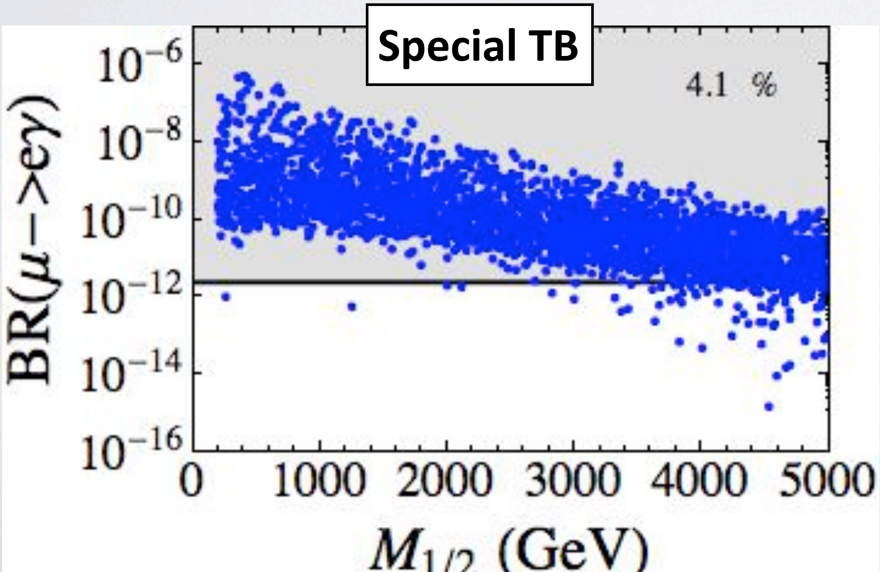
$$\longrightarrow \quad M_i(\Lambda_f) \equiv M_{1/2} \qquad \alpha_i(\Lambda_f) = \frac{1}{25}$$

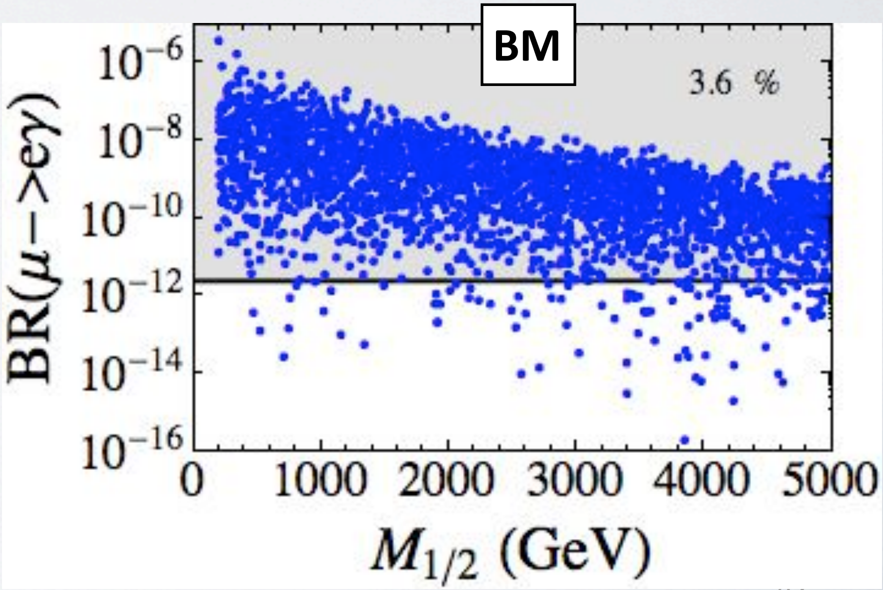
$$|\mu|^2 \simeq \frac{1 + 0.5\tan^2\beta}{\tan^2\beta - 1} m_0^2 + \frac{0.5 + 3.5\tan^2\beta}{\tan^2\beta - 1} M_{1/2}^2 - \frac{1}{2} m_Z^2$$

$m_0 = 5000\ \text{GeV}$ & $\tan\beta = 15$

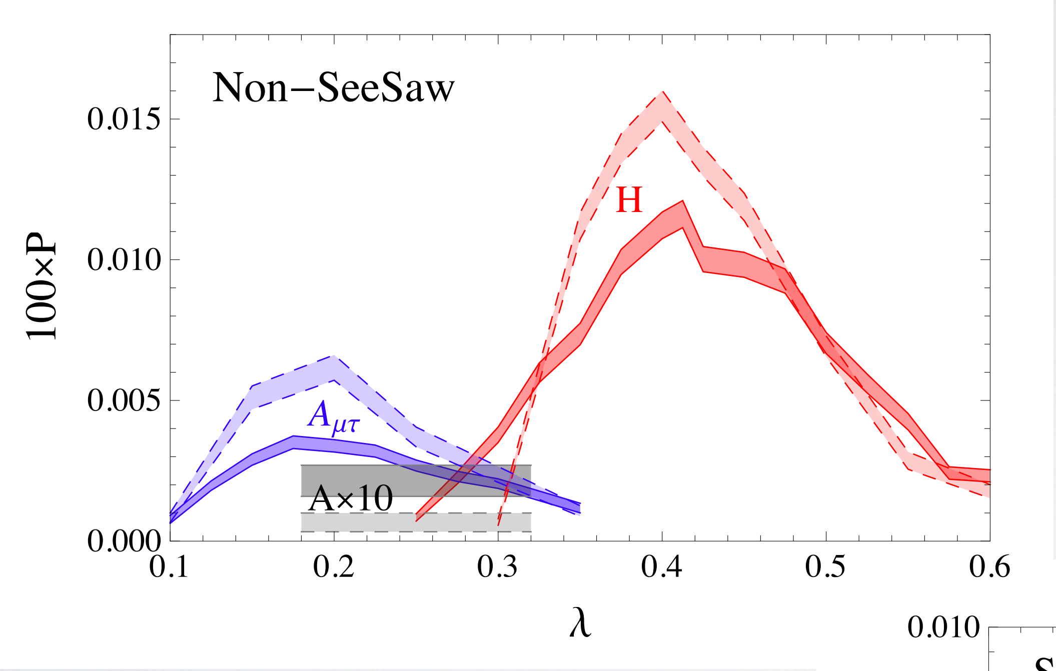
$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$

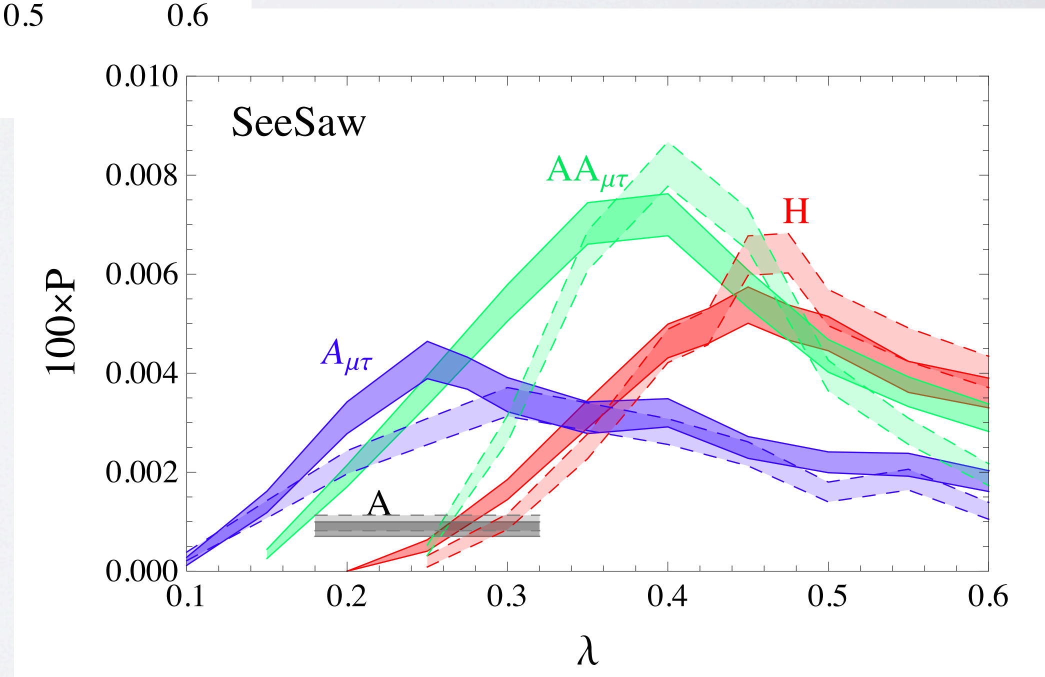






Anarchy vs. Hierarchy?





Scalar Potential

operators:

$$\mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right) \qquad \mathrm{Tr}\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right) \qquad \det\left(\mathcal{Y}_E\right)$$

$$\mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2 \qquad \mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)$$

$$\mathrm{Tr}\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^2 \qquad \mathrm{Tr}\left(\mathcal{Y}_\nu\sigma_2\mathcal{Y}_\nu^\dagger\right)^2$$

scalar potential:

$$\begin{aligned} V = & -\mu^2\cdot\mathbf{X}^2 + \left(\mathbf{X}^2\right)^\dagger\,\lambda\,\mathbf{X}^2 + \left(\mu_D \det\left(\mathcal{Y}_E\right) + h.c.\right) + \\ & + \lambda_E\,\mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2 + g\,\mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right) + \\ & + h\,\mathrm{Tr}\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^2 + h'\,\mathrm{Tr}\left(\mathcal{Y}_\nu\sigma_2\mathcal{Y}_\nu^\dagger\right)^2 \end{aligned}$$

the mixing term for 2 family case:

$$\begin{aligned} g\mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right) \propto g\Big\{ & (m_e^2+m_\mu^2)(y^2+y'^2)(m_{\nu_2}+m_{\nu_1})+ \\ & + (m_\mu^2-m_e^2)\Big[(m_{\nu_2}-m_{\nu_1})(y^2+y'^2)\cos 2\theta+ \\ & + (y^2-y'^2)2\sqrt{m_{\nu_2}m_{\nu_1}}\sin 2\alpha\sin 2\theta\Big]\Big\} \end{aligned}$$