New Developments in

Collective Neutrino Oscillations

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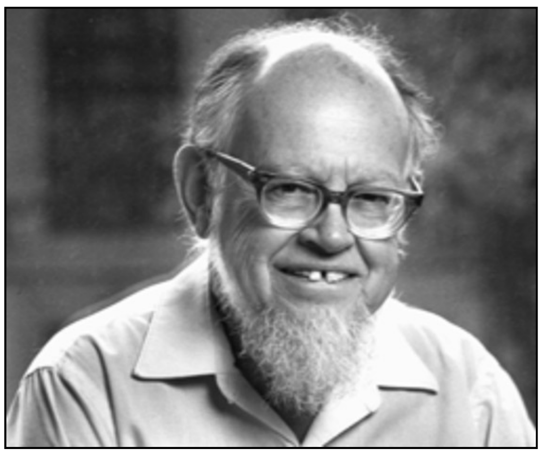
3400 citations

Neutrino oscillations in matter

L. Wolfenstein
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

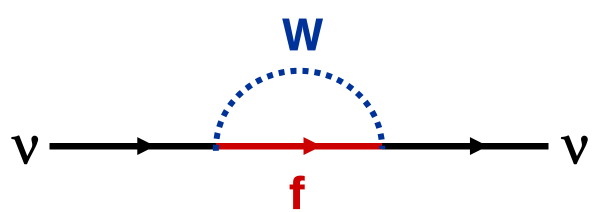
(Received 6 October 1977; revised manuscript received 5 December 1977)

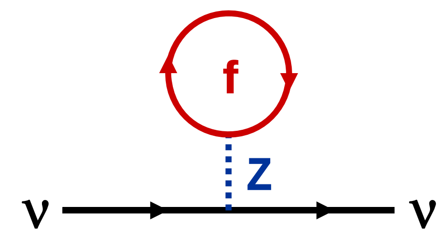
The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein

Neutrinos in a medium suffer flavor-dependent refraction



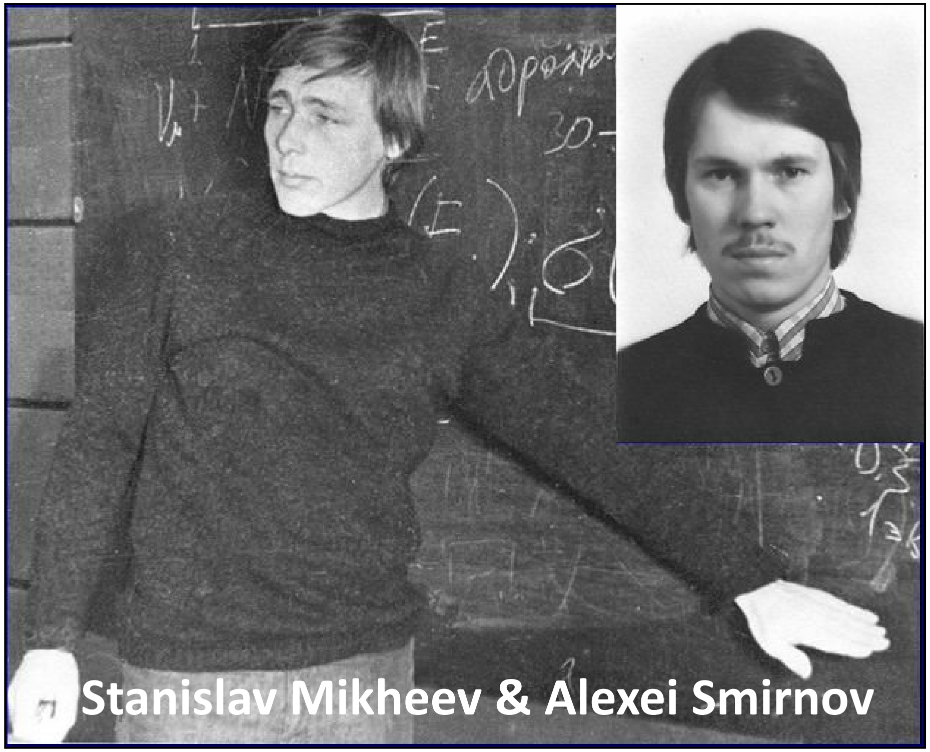


$$V_{\rm weak} = \sqrt{2} G_{\rm F} \times \begin{cases} N_{\rm e} - N_{\rm n}/2 & \text{for } \nu_{\rm e} \\ -N_{\rm n}/2 & \text{for } \nu_\mu \end{cases}$$

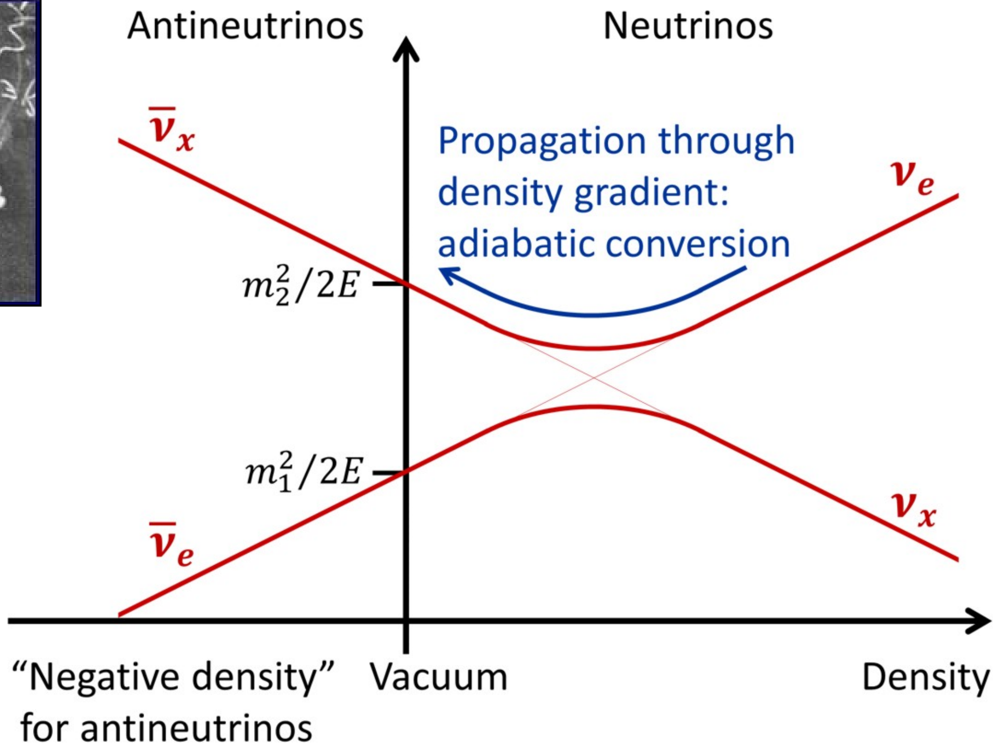
Typical density of Earth: 5 g/cm^3

$$\Delta V_{\rm weak} \approx 2 \times 10^{-13}\ {\rm eV} = 0.2\ {\rm peV}$$

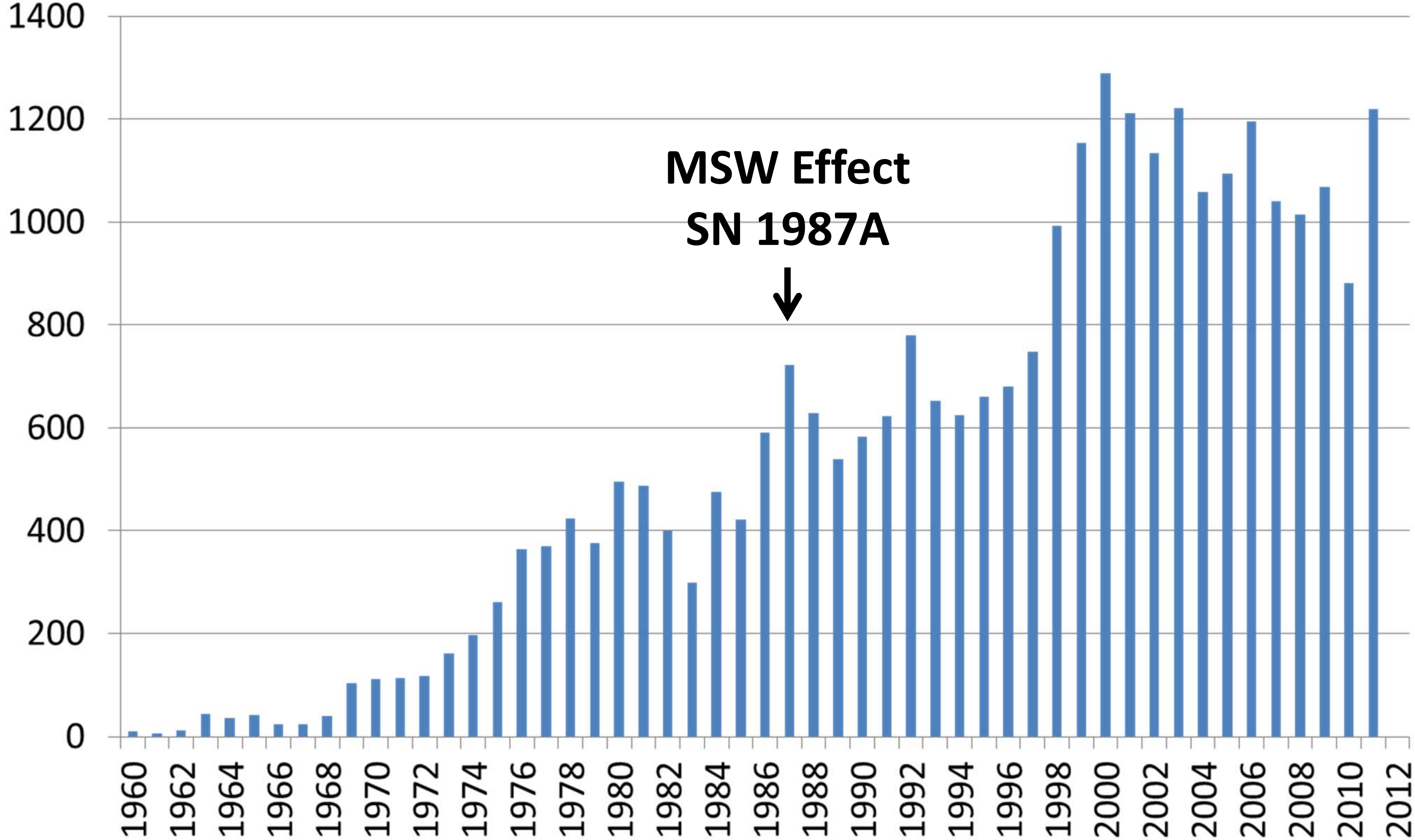
Mikheev-Smirnov-Wolfenstein (MSW) effect



Eigenvalues of Hamiltonian for 2-flavor oscillations



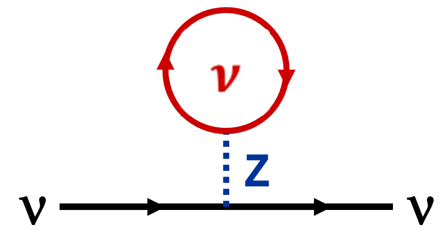
MSW Resonance in Neutrino Physics



inSPIRE entries with "Neutrino" in title

2-flavor neutrino evolution as an effective 2-level problem

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$



Effective mixing Hamiltonian

$$H = \frac{M^2}{2E} + \sqrt{2}G_\mathrm{F}\begin{pmatrix} N_e - \frac{N_n}{2} & 0 \\ 0 & -\frac{N_n}{2}\end{pmatrix} + \sqrt{2}G_\mathrm{F}\begin{pmatrix} N_{\nu_e} & N_{\langle\nu_e|\nu_\mu\rangle} \\ N_{\langle\nu_\mu|\nu_e\rangle} & N_{\nu_\mu}\end{pmatrix}$$

Mass term in flavor basis: causes vacuum oscillations

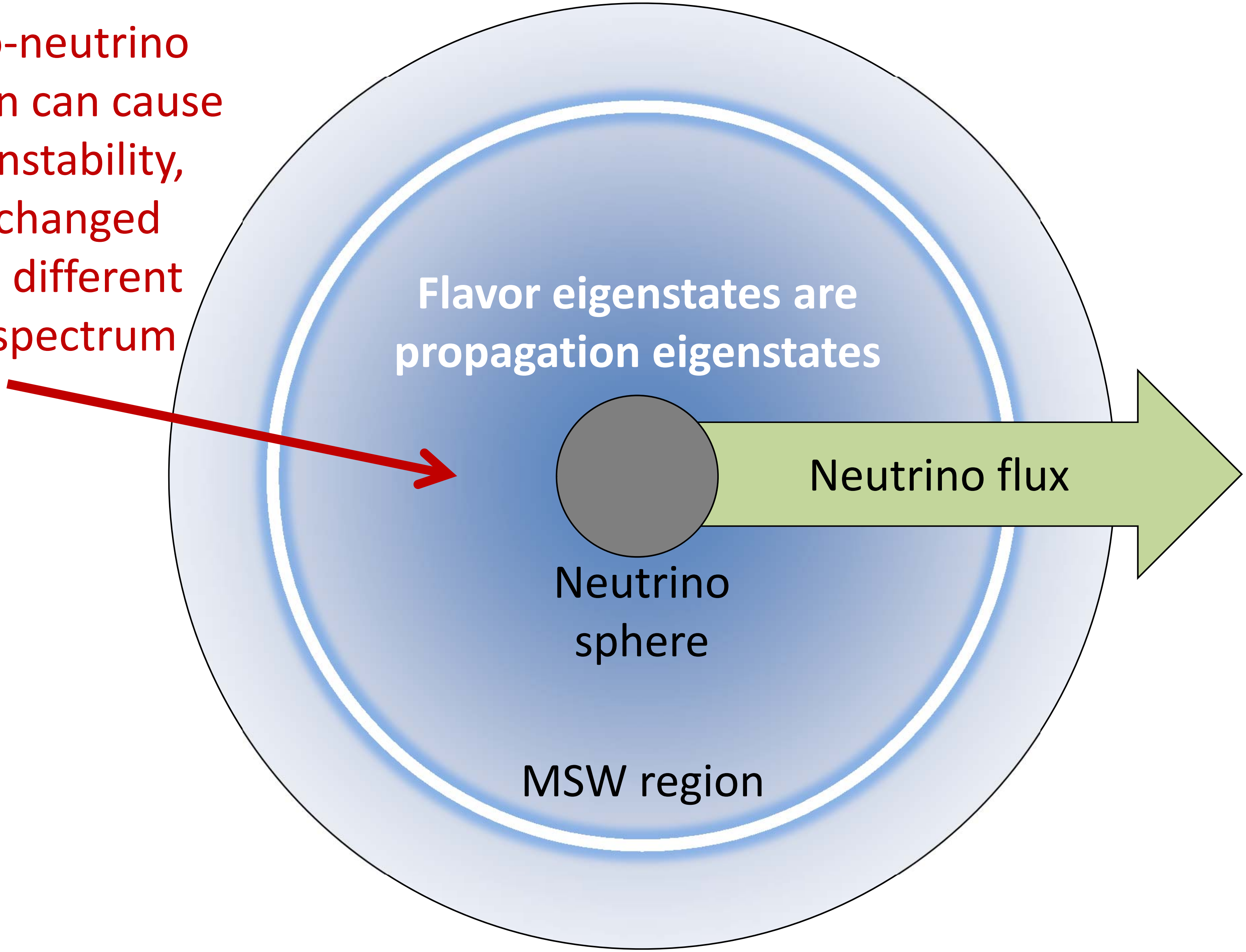
Wolfenstein's weak potential, causes MSW conversion together with vacuum term

Flavor-off-diagonal potential, caused by flavor oscillations. (J.Pantaleone, PLB 287:128,1992)

Flavor oscillations feed back on the Hamiltonian: Nonlinear effects!

Flavor Oscillations in Core-Collapse Supernovae

Neutrino-neutrino refraction can cause a flavor instability, flavor exchanged between different parts of spectrum



Collective Supernova Nu Oscillations since 2006

Two seminal papers in 2006 triggered a torrent of activities

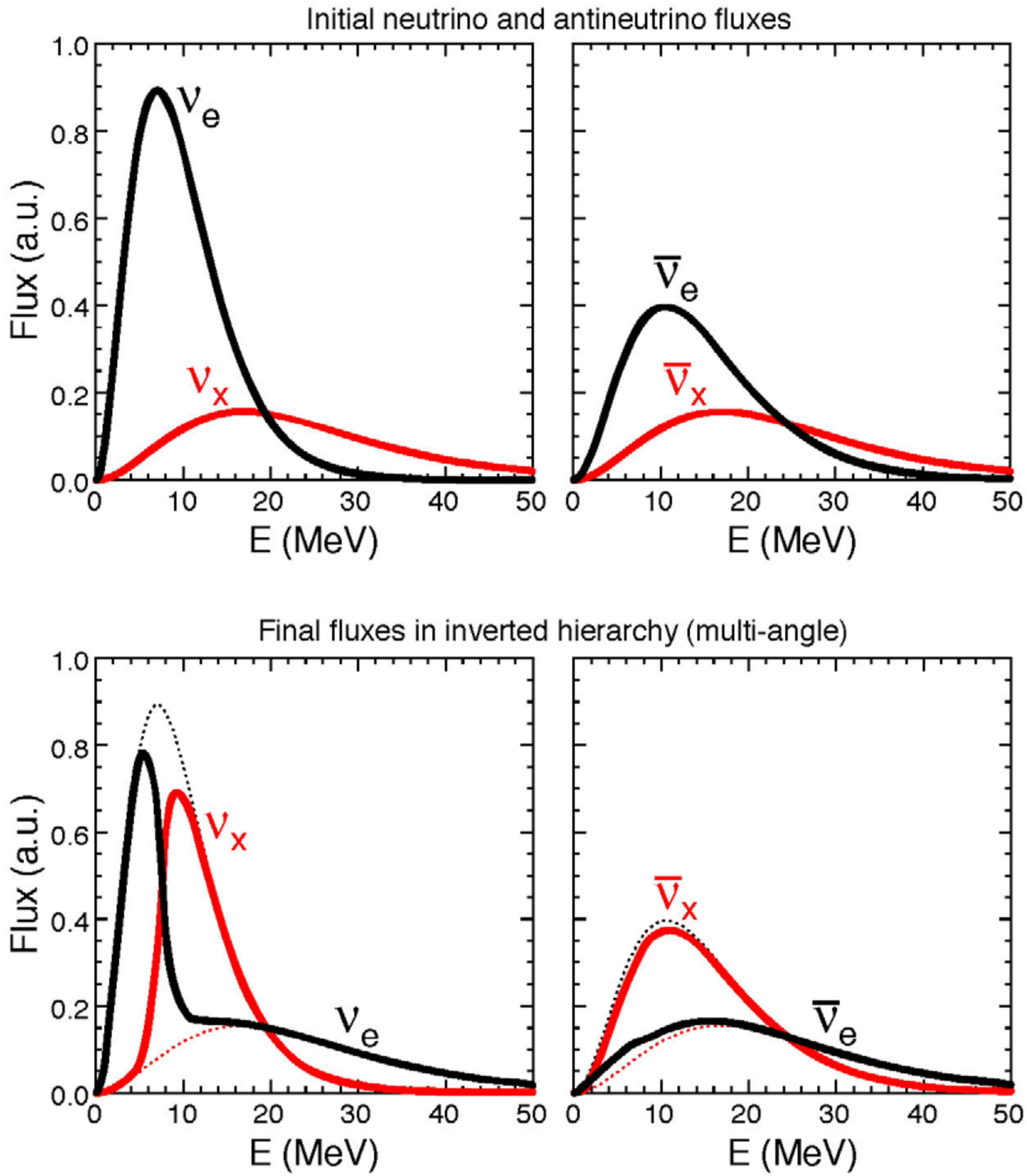
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

Balantekin, Gava & Volpe [0710.3112]. Balantekin & Pehlivan [astro-ph/0607527]. Blennow, Mirizzi & Serpico [0810.2297]. Cherry, Fuller, Carlson, Duan & Qian [1006.2175, 1108.4064]. Cherry, Wu, Fuller, Carlson, Duan & Qian [1109.5195]. Cherry, Carlson, Friedland, Fuller & Vlasenko [1203.1607]. Chakraborty, Choubey, Dasgupta & Kar [0805.3131]. Chakraborty, Fischer, Mirizzi, Saviano, Tomàs [1104.4031, 1105.1130]. Choubey, Dasgupta, Dighe & Mirizzi [1008.0308]. Dasgupta & Dighe [0712.3798]. Dasgupta, Dighe & Mirizzi [0802.1481]. Dasgupta, Dighe, Raffelt & **Smirnov** [0904.3542]. Dasgupta, Dighe, Mirizzi & Raffelt [0801.1660, 0805.3300]. Dasgupta, Mirizzi, Tamborra & Tomàs [1002.2943]. Dasgupta, Raffelt & Tamborra [1001.5396]. Dasgupta, O'Connor & Ott [1106.1167]. Duan, Fuller, Carlson & Qian [astro-ph/0608050, 0703776, 0707.0290, 0710.1271]. Duan, Fuller & Qian [0706.4293, 0801.1363, 0808.2046, 1001.2799]. Duan, Fuller & Carlson [0803.3650]. Duan & Kneller [0904.0974]. Duan & Friedland [1006.2359]. Duan, Friedland, McLaughlin & Surman [1012.0532]. Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl [0807.0659]. Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl [0706.2498, 0712.1137]. Fogli, Lisi, Marrone & Mirizzi [0707.1998]. Fogli, Lisi, Marrone & Tamborra [0812.3031]. Friedland [1001.0996]. Gava & Jean-Louis [0907.3947]. Gava & Volpe [0807.3418]. Galais, Kneller & Volpe [1102.1471]. Galais & Volpe [1103.5302]. Gava, Kneller, Volpe & McLaughlin [0902.0317]. Hannestad, Raffelt, Sigl & Wong [astro-ph/0608695]. Wei Liao [0904.0075, 0904.2855]. Lunardini, Müller & Janka [0712.3000]. Mirizzi, Pozzorini, Raffelt & Serpico [0907.3674]. Mirizzi & Serpico [1111.4483]. Mirizzi & Tomàs [1012.1339]. Pehlivan, Balantekin, Kajino & Yoshida [1105.1182]. Pejcha, Dasgupta & Thompson [1106.5718]. Raffelt [0810.1407, 1103.2891]. Raffelt & Sigl [hep-ph/0701182]. Raffelt & **Smirnov** [0705.1830, 0709.4641]. Raffelt & Tamborra [1006.0002]. Sawyer [hep-ph/0408265, 0503013, 0803.4319, 1011.4585]. Sarikas, Raffelt, Hüdepohl & Janka [1109.3601]. Sarikas, Tamborra, Raffelt, Hüdepohl & Janka [1204.0971]. Saviano, Chakraborty, Fischer, Mirizzi [1203.1484]. Wu & Qian [1105.2068].

Spectral Split

Initial fluxes at neutrino sphere

After collective trans-formation



Figures from Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Explanations in

Raffelt & Smirnov arXiv:0705.1830 and 0709.4641

Duan, Fuller, Carlson & Qian arXiv:0706.4293 and 0707.0290

Three Ways to Describe Flavor Oscillations

Schrödinger equation in terms of "flavor spinor"

$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2}{2E} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Neutrino flavor density matrix

$$\rho = \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{pmatrix}$$

Equivalent commutator form of Schrödinger equation

$$i\partial_t \rho = [H, \rho]$$

Expand 2×2 Hermitean matrices in terms of Pauli matrices

$\rho = \frac{1}{2}[\mathrm{Tr}(\rho) + \mathbf{P} \cdot \boldsymbol{\sigma}]$ and $H = \frac{\Delta m^2}{2E} \mathbf{B} \cdot \boldsymbol{\sigma}$ with $\mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)$

Equivalent spin-precession form of equation of motion

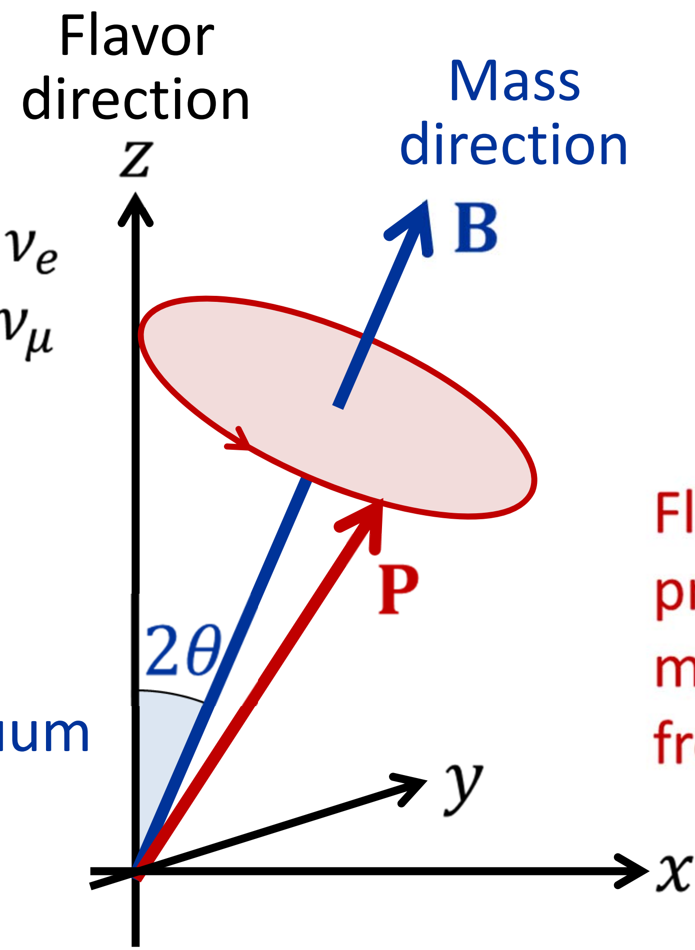
$\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P}$ with $\omega = \frac{\Delta m^2}{2E}$

$\mathbf{P}$ is "polarization vector" or "Bloch vector"

Flavor Oscillation as Spin Precession

↑ Spin up ν_e

↓ Spin down ν_μ



Twice the vacuum mixing angle

Flavor polarization vector precesses around the mass direction with frequency $\omega = \Delta m^2/2E$

Collective Nu Oscillations as a Many-Body Problem

Hamiltonian for interacting "flavor spins" (*classical* in mean-field approach)

$$H = \sum_{i=1}^{N} \omega_i \mathbf{B} \cdot \mathbf{P}_i + \lambda \mathbf{L} \cdot \sum_{i=1}^{N} \mathbf{P}_i + \mu \sum_{i,j=1}^{N} (1 - \cos\theta_{ij})\, \mathbf{P}_i \cdot \mathbf{P}_j$$

- $\mathbf{B}$: Unit vector in mass direction
- $\mathbf{L}$: Unit vector in flavor direction
- $(1-\cos\theta_{ij})$: Multi-angle effects from current-current structure

"Spin-pairing H" for isotropic system (or single angle), ignoring matter effect

$$H = \sum_{i=1}^{N} \omega_i \mathbf{B} \cdot \mathbf{P}_i + \mu \mathbf{P}_{\rm tot}^2$$

BCS theory (using Anderson's pseudo-spin), nuclear physics, …

Integrable system (as many "Gaudin invariants" as spins)

→ Pehlivan, Balantekin, Kajino & Yoshida [arxiv:1105.1182] for introduction

N-mode coherent solutions ("Normal and anomalous solitons")

- Emil Yuzbashian, Phys. Rev. **B** 78, 184507 (2008) — Super-conductivity (BCS)
- Georg Raffelt, Phys. Rev. **D** 83, 105022 (2011) — Collective Nus

Inverse-Energy Spectrum

Fermi-Dirac energy spectrum

$$\frac{dN}{dE} \propto \frac{E^2}{e^{E/T-\eta}+1}$$

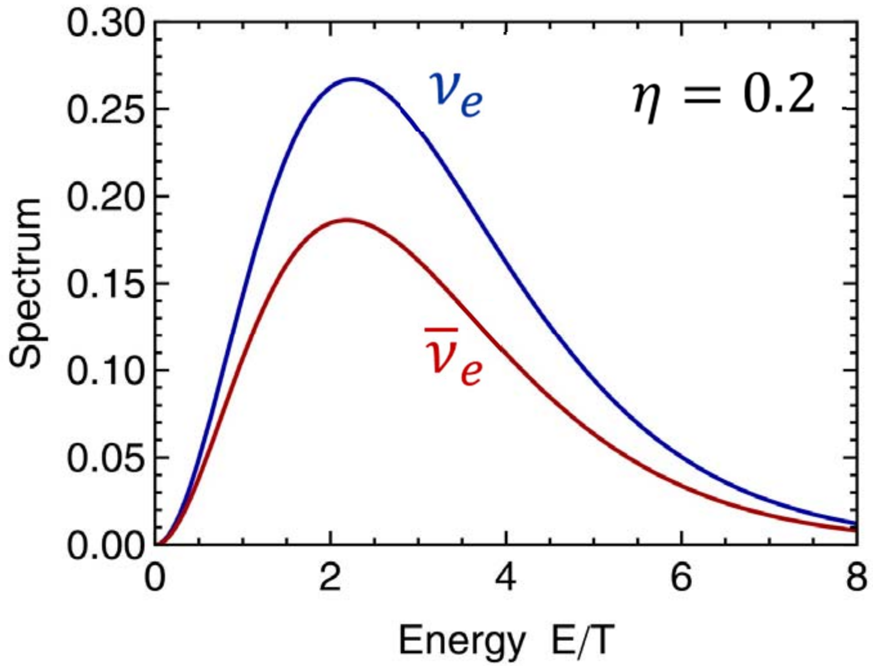
η degeneracy parameter, $-\eta$ for $\overline{\nu}$

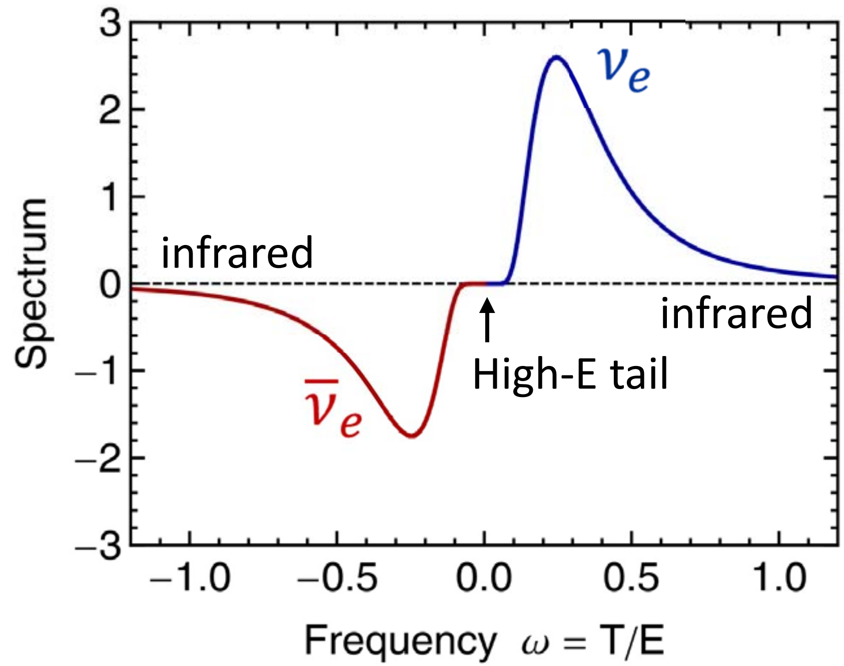
Same spectrum in terms of $\omega = T/E$

- Antineutrinos $E \to -E$
- and dN/dE negative (flavor isospin convention)

  $\omega > 0$: $\nu_e = \uparrow$ and $\nu_\mu = \downarrow$

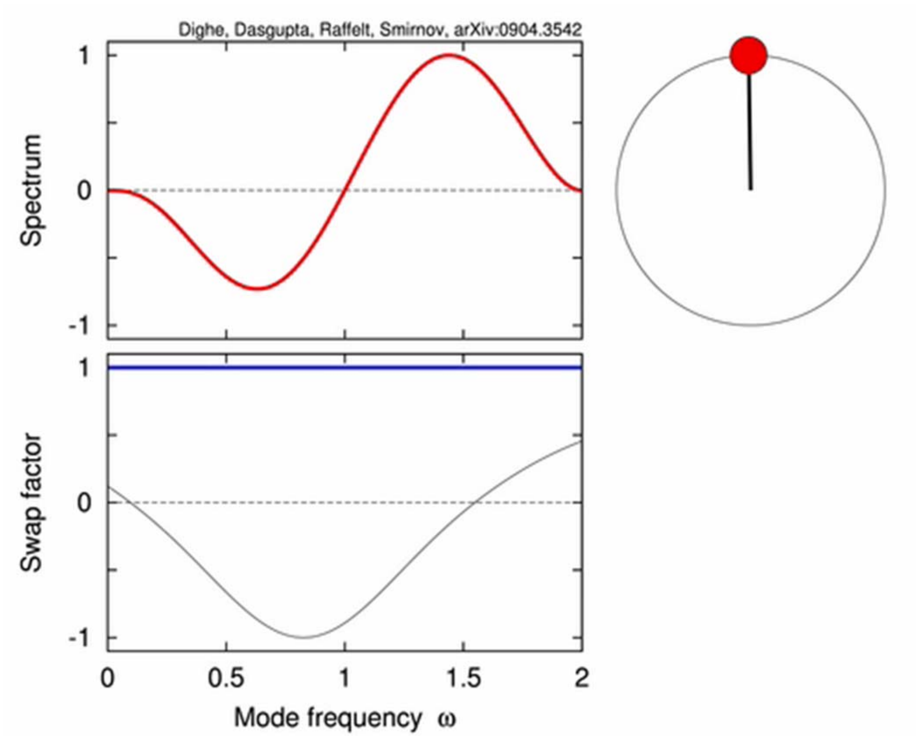
  $\omega < 0$: $\overline{\nu}_e = \downarrow$ and $\overline{\nu}_\mu = \uparrow$



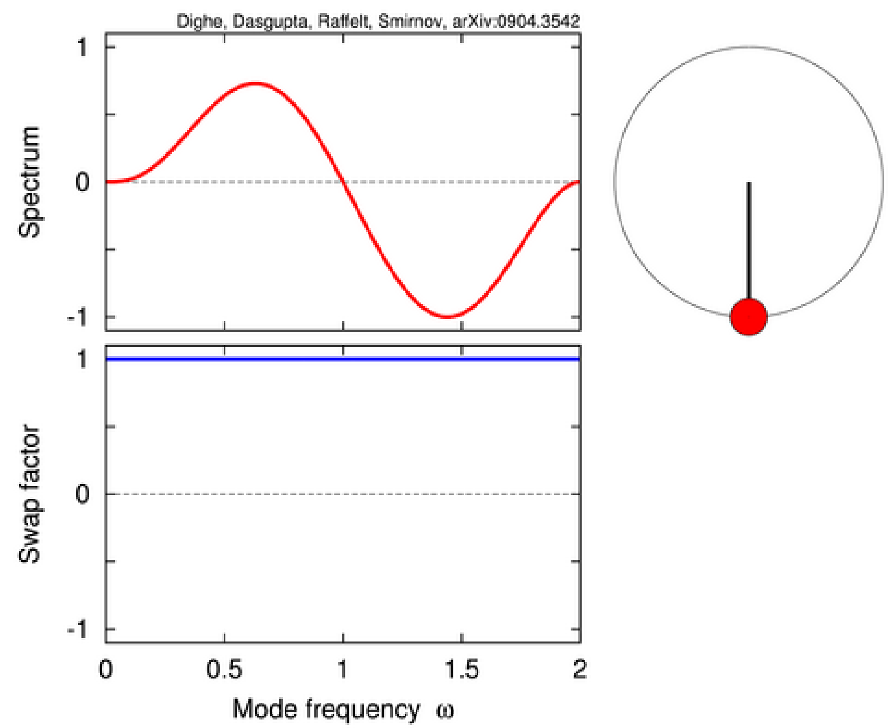


Flavor Pendulum

Single “positive” crossing
(potential energy at a maximum)



Single “negative” crossing
(potential energy at a minimum)



Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542
For movies see http://www.mppmu.mpg.de/supernova/multisplits

Self-Induced Flavor Conversion

- Flavor content exchanged between different momentum modes (or nus and anti-nus changing together)
- No net flavor conversion of ensemble
- Instability required to get started: Exponential growth of the off-diagonal density matrix parts

→ Linearized Stability Analysis

Sawyer, arXiv:0803.4319 – Banerjee, Dighe & Raffelt, arXiv:1107.2308

Linearized Stability Analysis

Schrödinger equation for flavor matrices of neutrino fluxes $\Phi_{\omega,u}$

$\omega = \pm\Delta m^2/2E$ $\quad u = \sin^2(\text{emission angle})$ $\quad v_u = \text{radial velocity at } r$

$$\mathrm{i}\partial_r \Phi_{\omega,u} = \left[\frac{\omega + \sqrt{2}G_\mathrm{F}N_\ell}{v_u} + \frac{\sqrt{2}G_\mathrm{F}}{4\pi r^2}\int d\omega' du'\, \Phi_{\omega',u'}\frac{1 - v_u v_{u'}}{v_u v_{u'}}, \Phi_{\omega,u}\right]$$

Linearize in small off-diagonal flux terms and Fourier transform

$$\Phi_{\omega,u} = \frac{g_{\omega,u}}{2}\begin{pmatrix} 1 & Q_{\omega,u}\, e^{-\mathrm{i}\Omega r} \\ Q^*_{\omega,u}\, e^{\mathrm{i}\Omega r} & -1 \end{pmatrix}$$

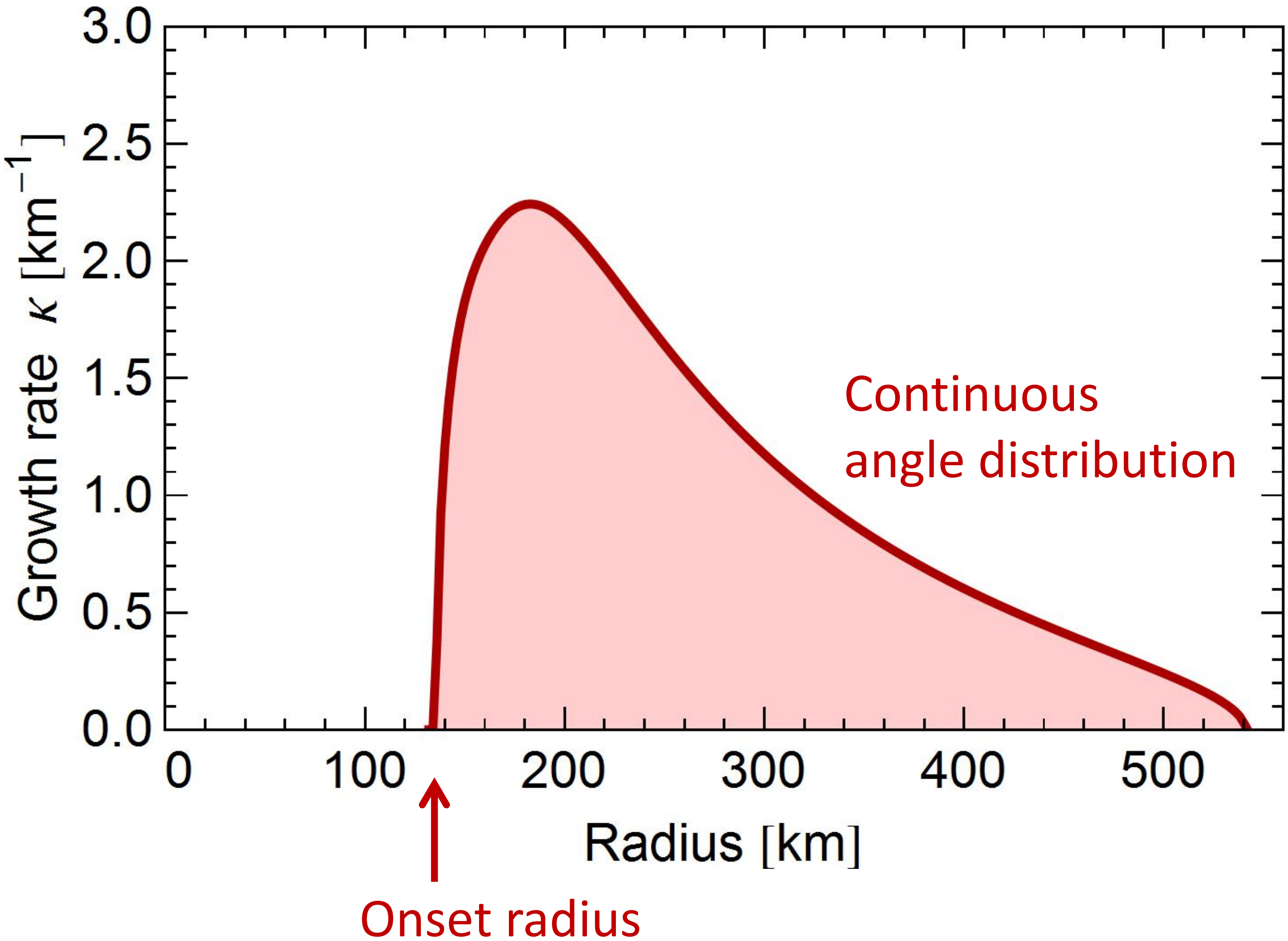
Eigenvalue equation for $Q_{\omega,u}$ in terms of eigenfrequency $\Omega = \gamma + \mathrm{i}\,\kappa$

$$\left[\omega + u\left(\lambda + \int d\omega' du'\, g_{\omega',u'}\right) - \Omega\right] Q_{\omega,u} = \mu \int d\omega' du'\, (u + u') g_{\omega',u'} Q_{\omega',u'}$$

Straightforward to solve for exponential growth rate κ

Banerjee, Dighe & Raffelt, arXiv:1107.2308

Stability Analysis for Schematic SN Example



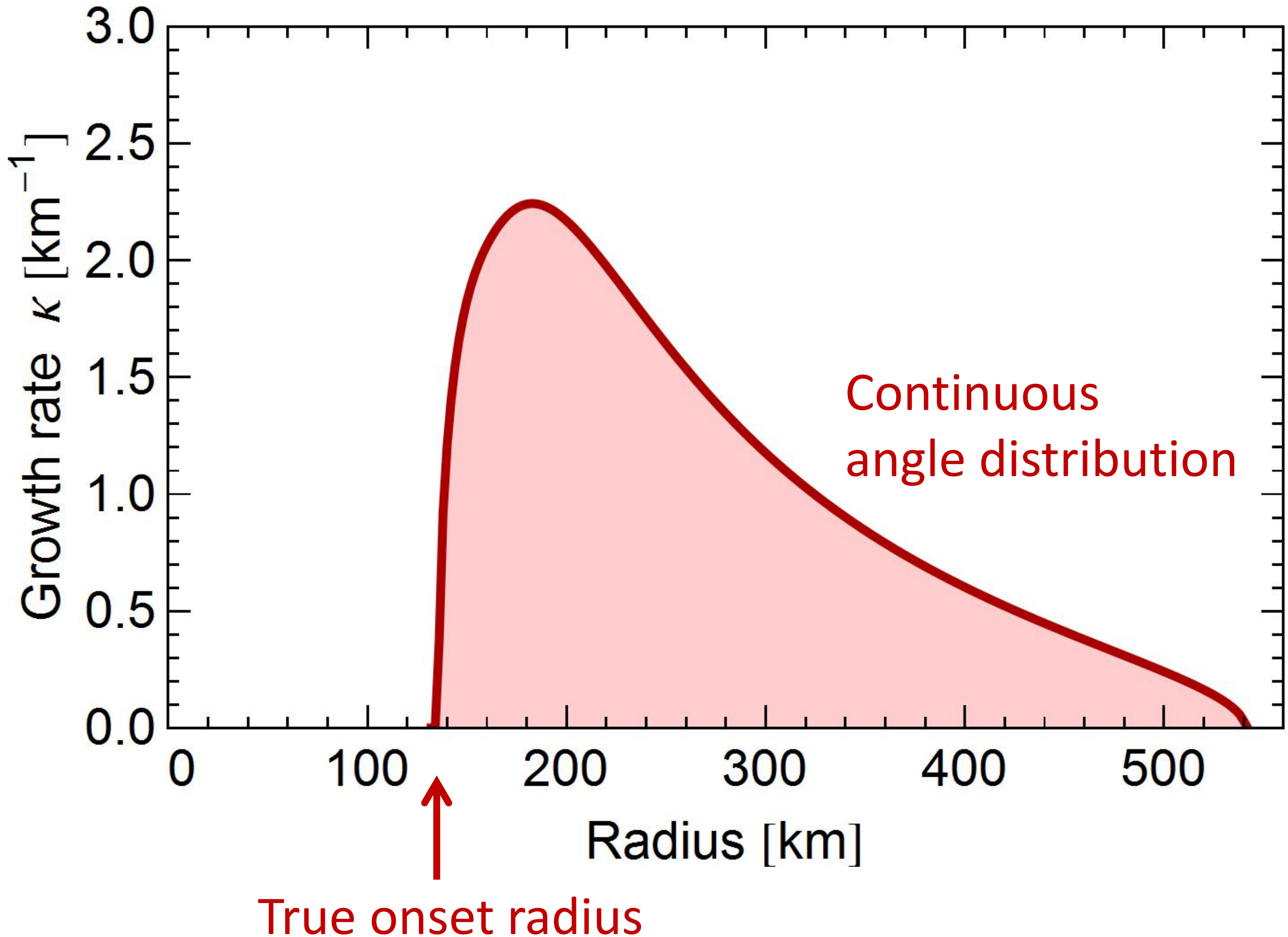
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Paradox of Numerical Solutions

Represent neutrino field by discrete energy and angle modes

- Number of energy modes chosen to fit desired precision
- $N_a \gg 1$ of angle modes required
  N_a too small: Unphysical solutions

New Instabilities for Discrete Angle Modes



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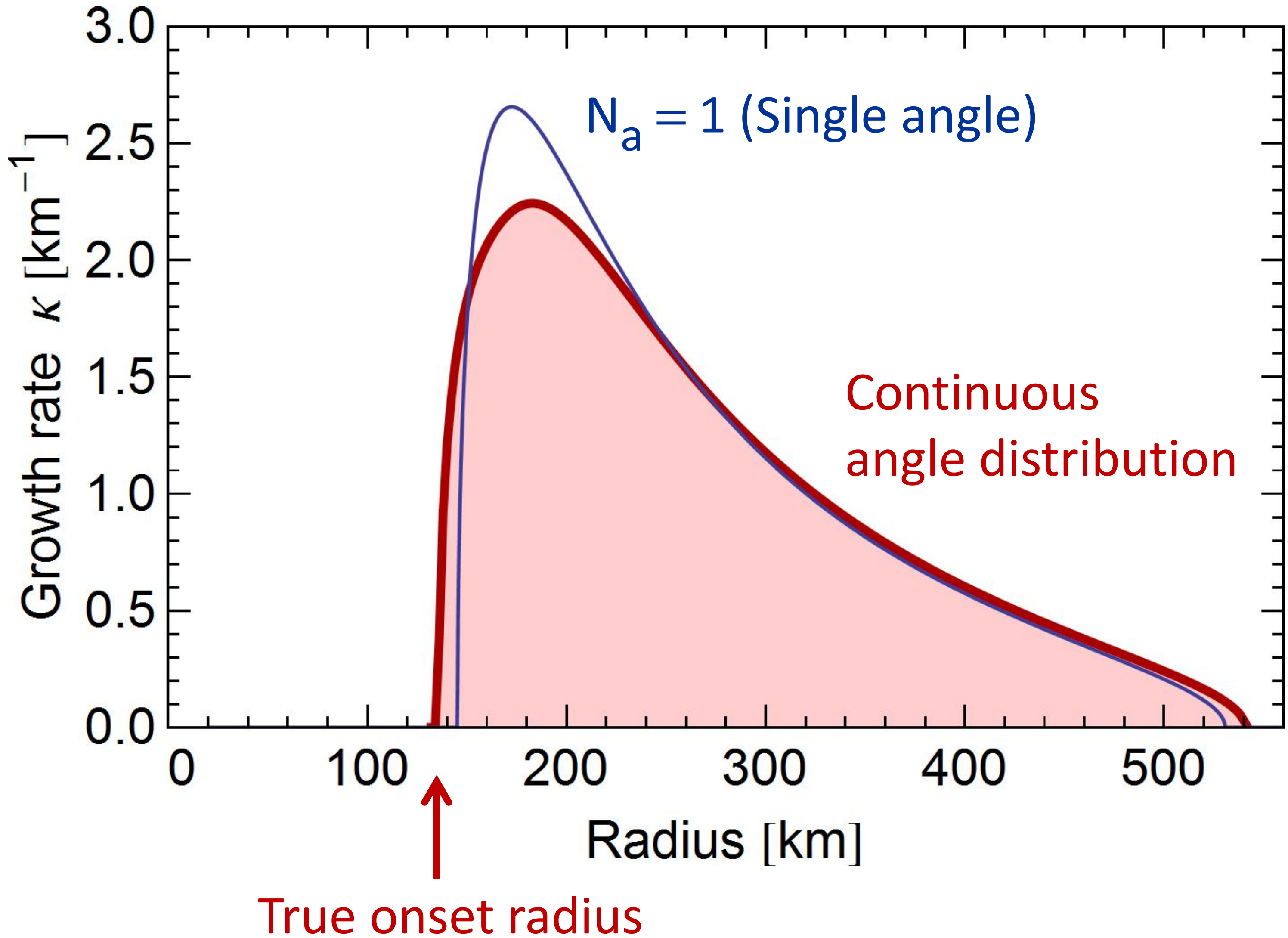

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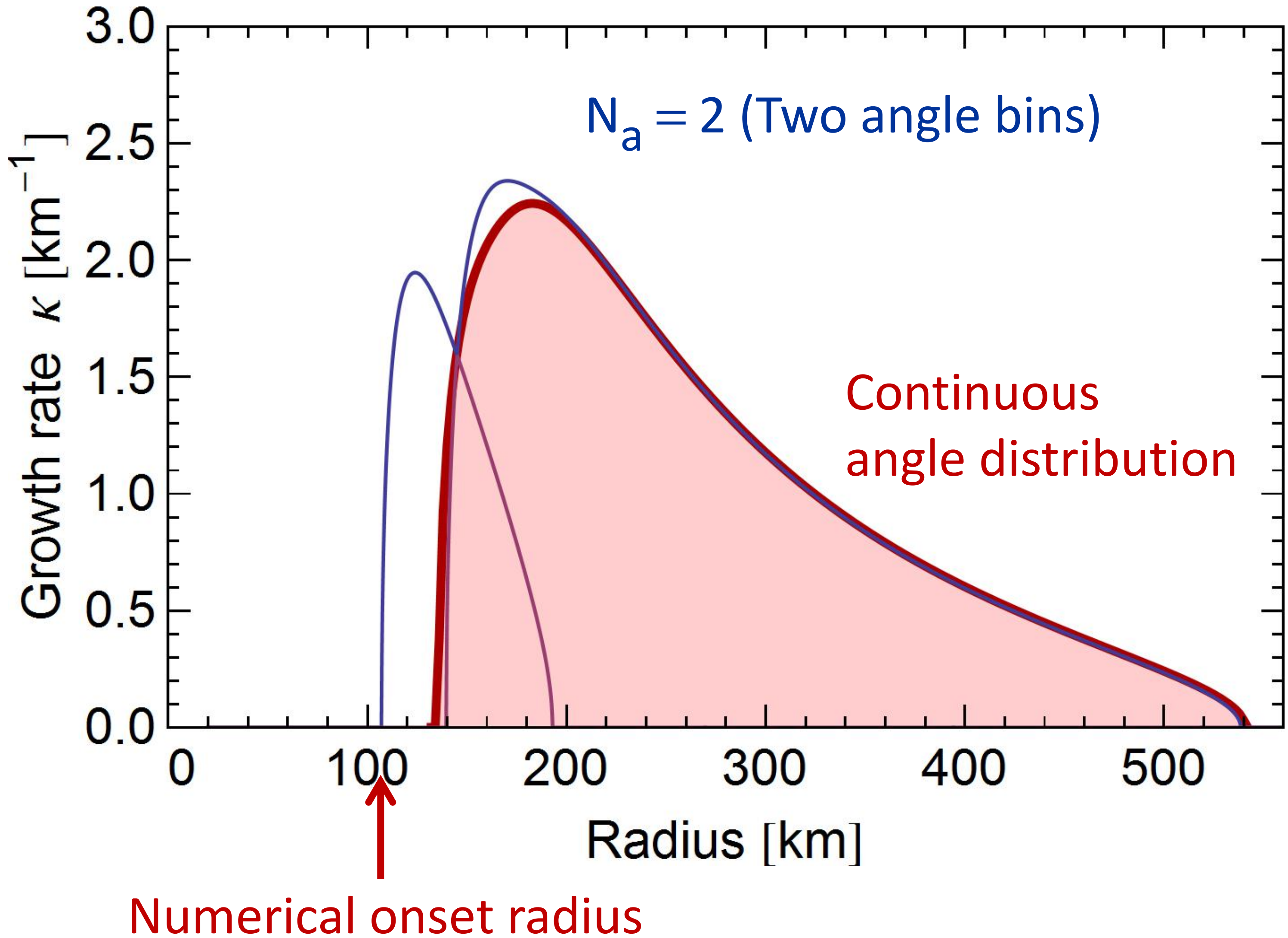

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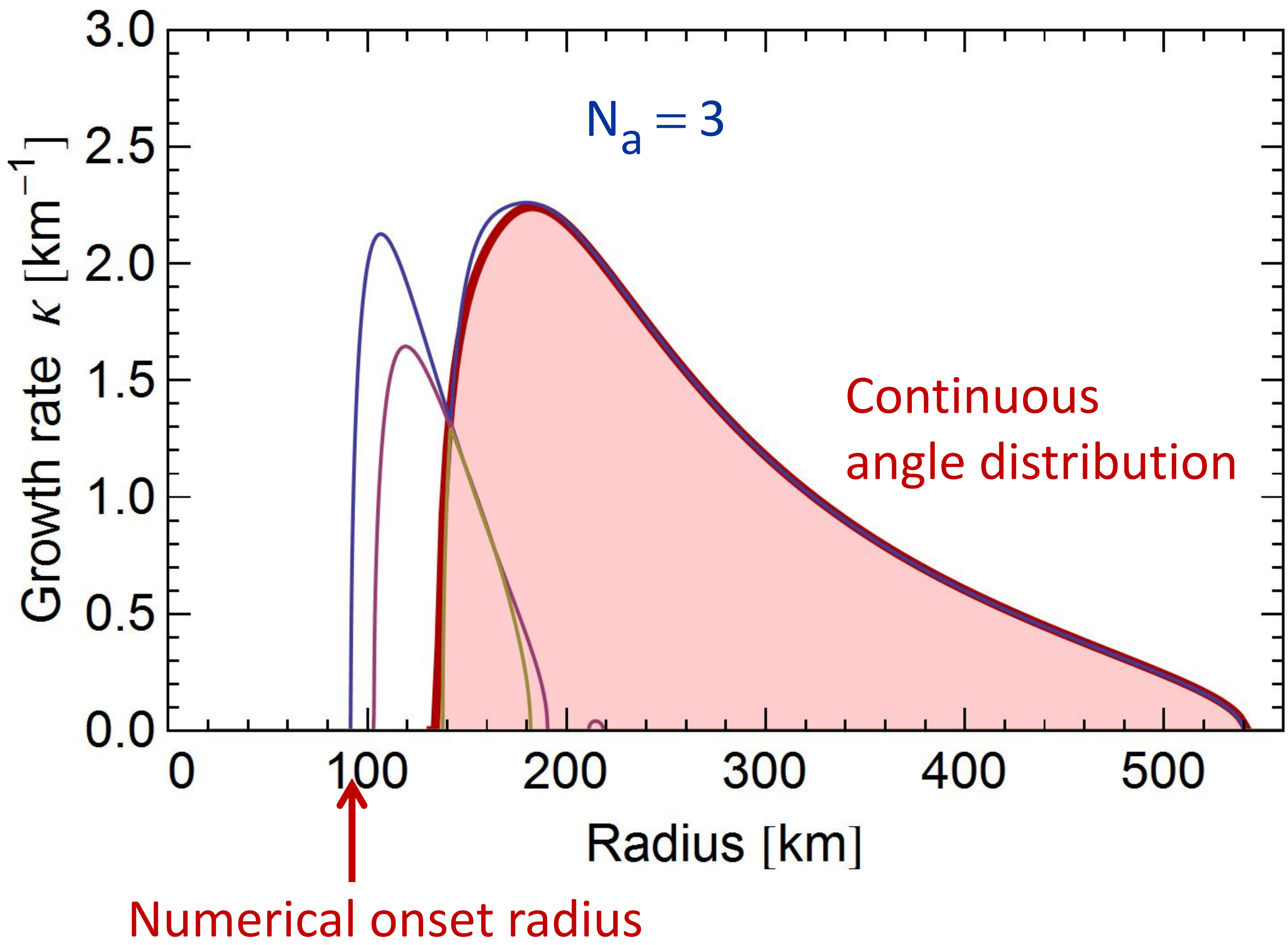

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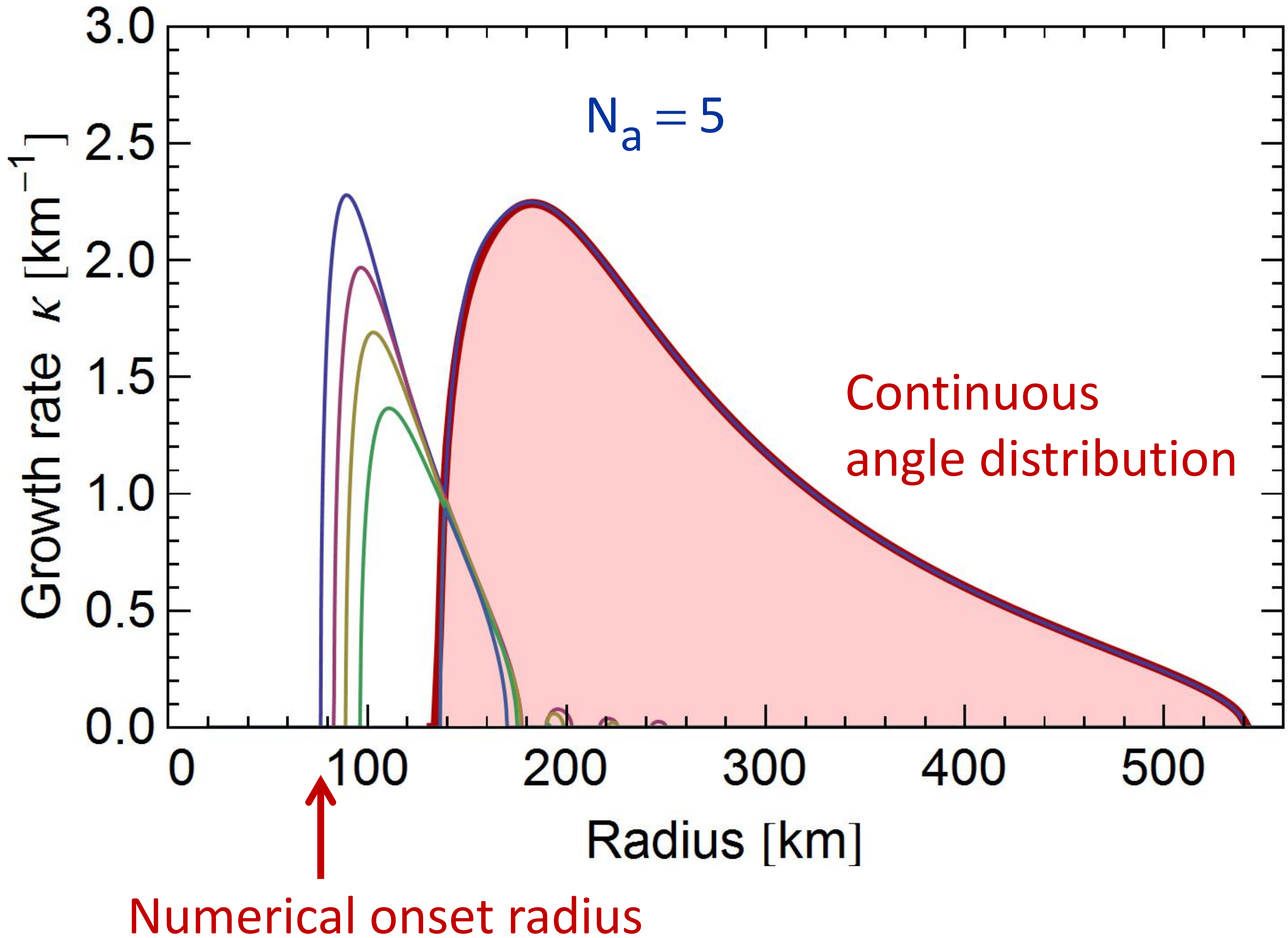
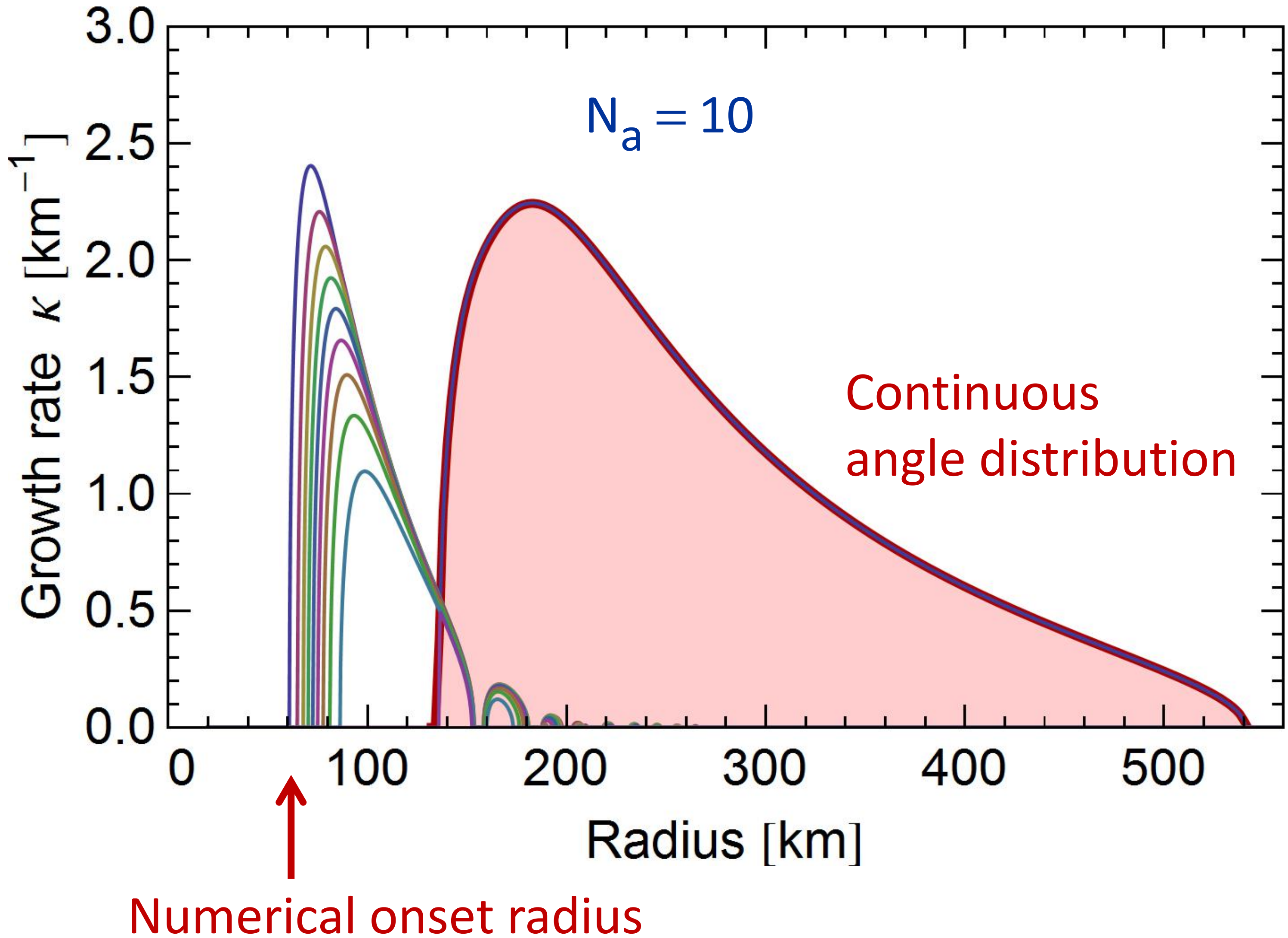


Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

New Instabilities for Discrete Angle Modes



Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

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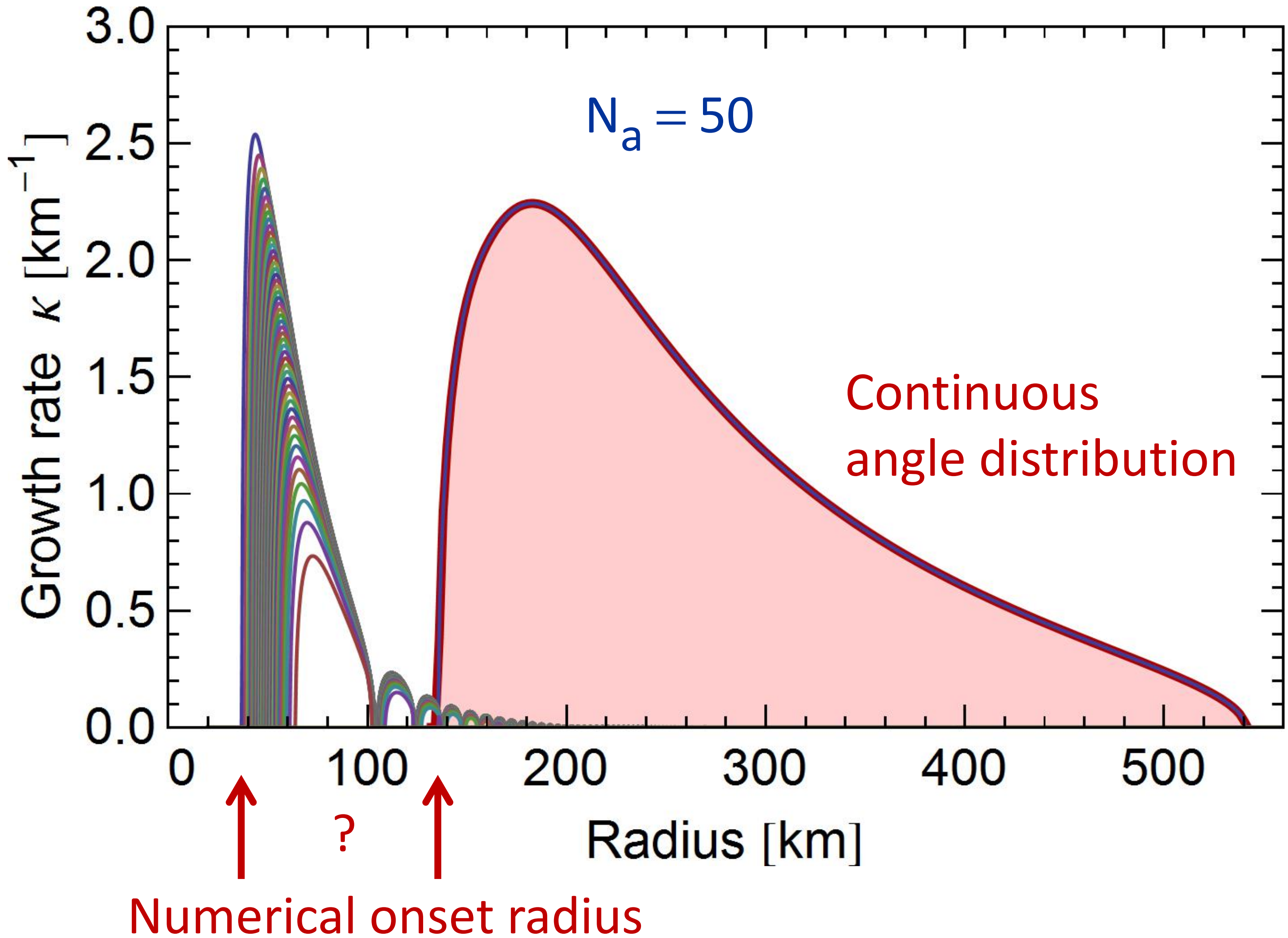

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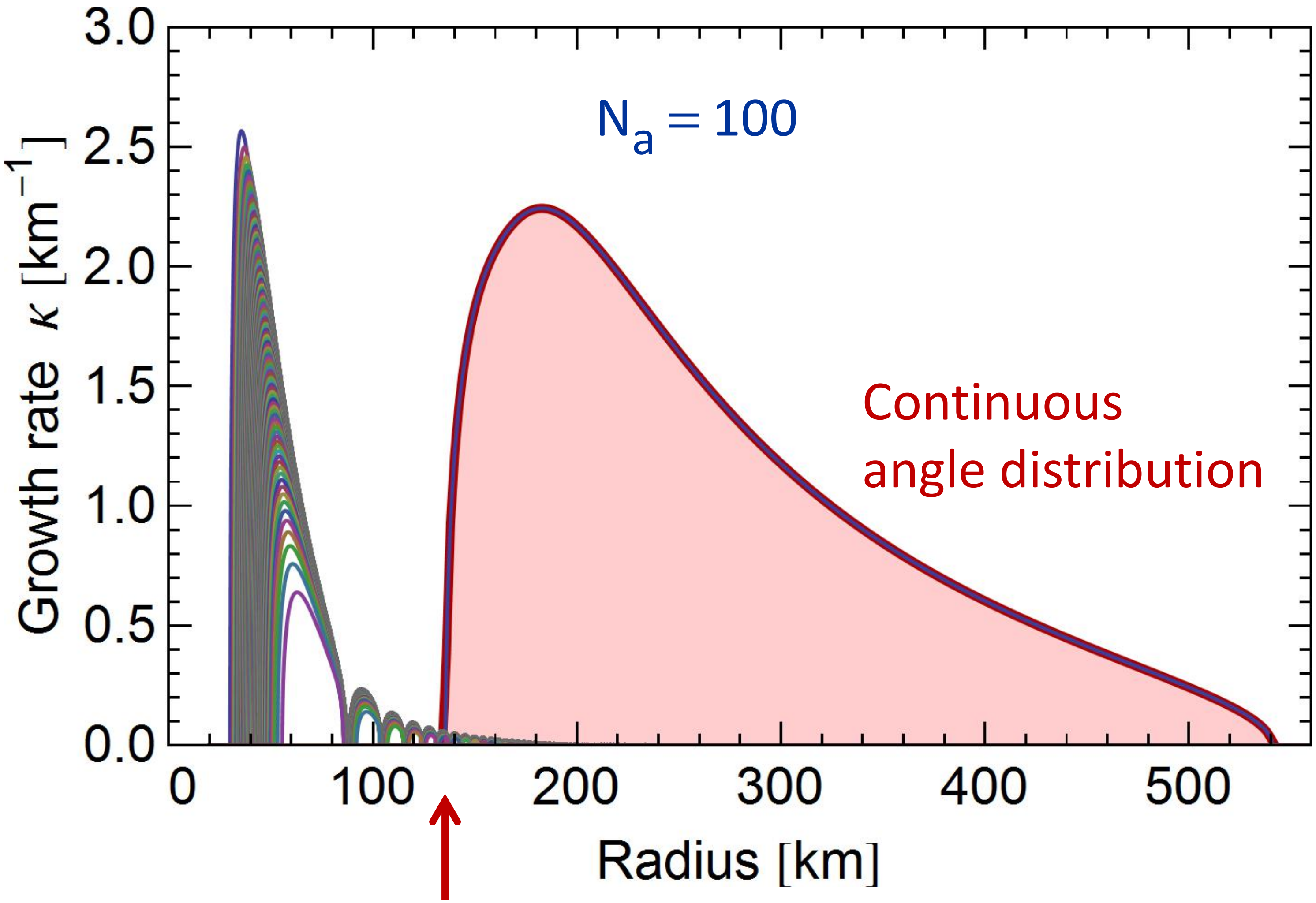
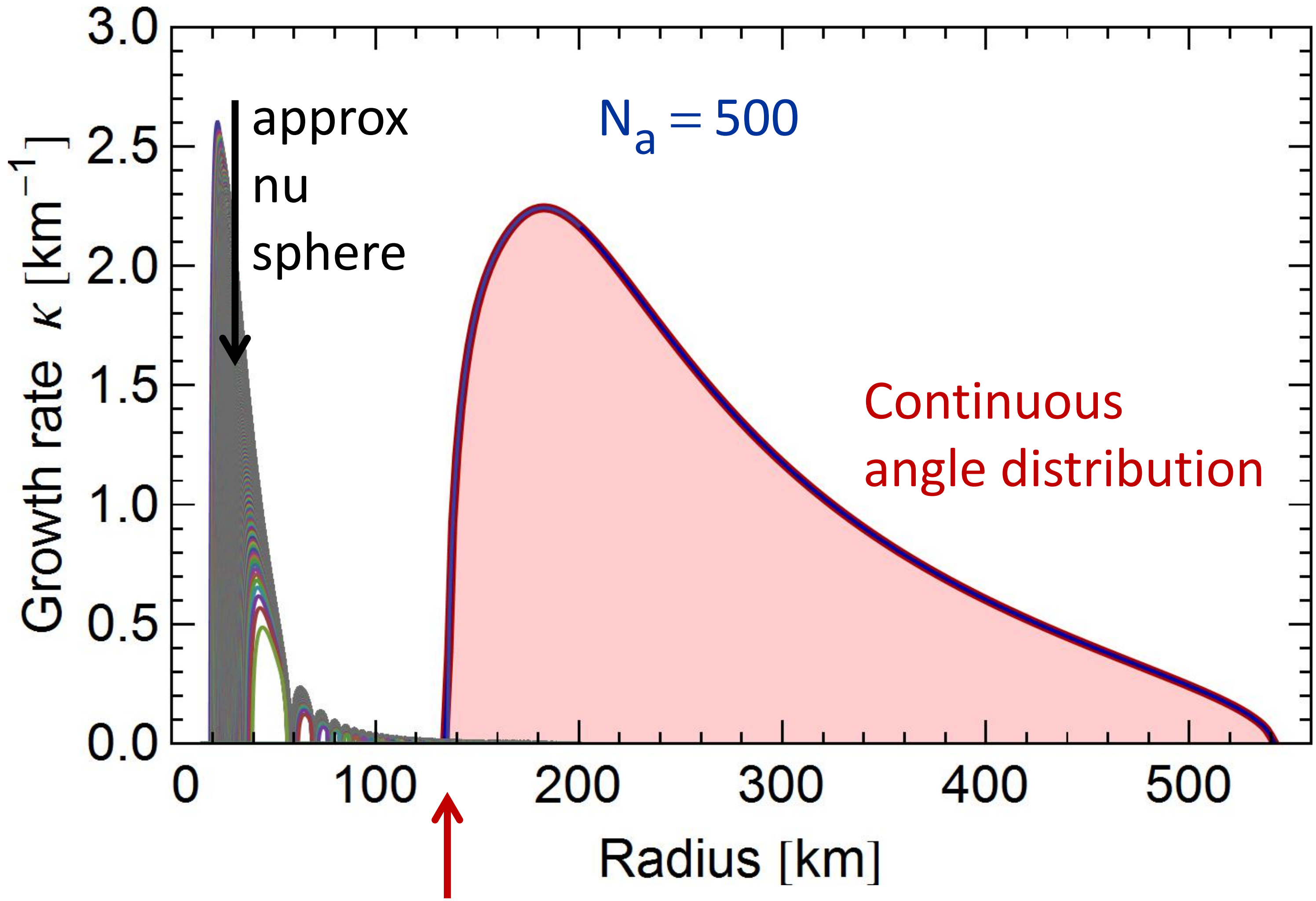
New Instabilities for Discrete Angle Modes



Numerical onset radius (start integration not too deep)

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

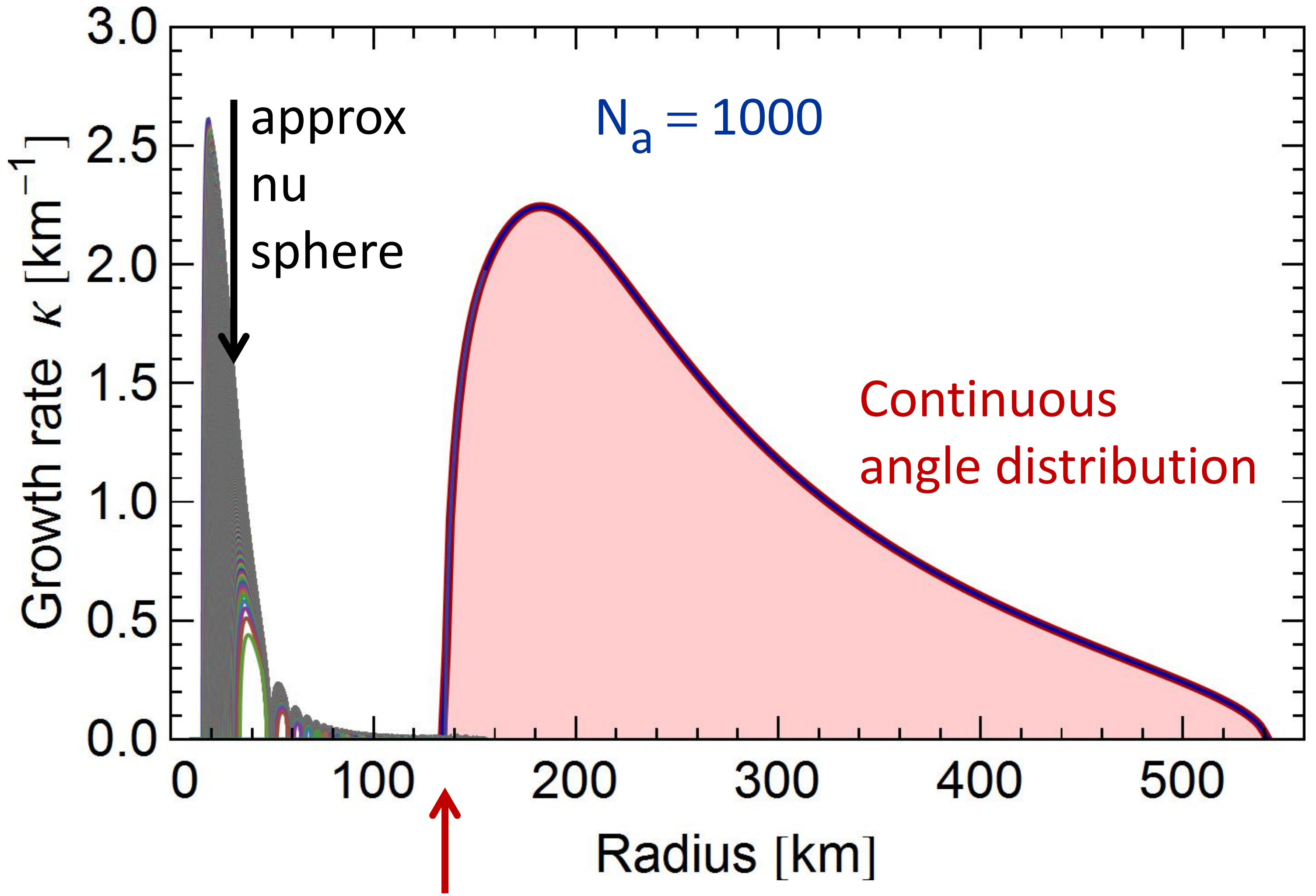
New Instabilities for Discrete Angle Modes



Numerical onset radius (start integration at nu sphere?)

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

New Instabilities for Discrete Angle Modes



Numerical onset radius (start integration at nu sphere?)

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

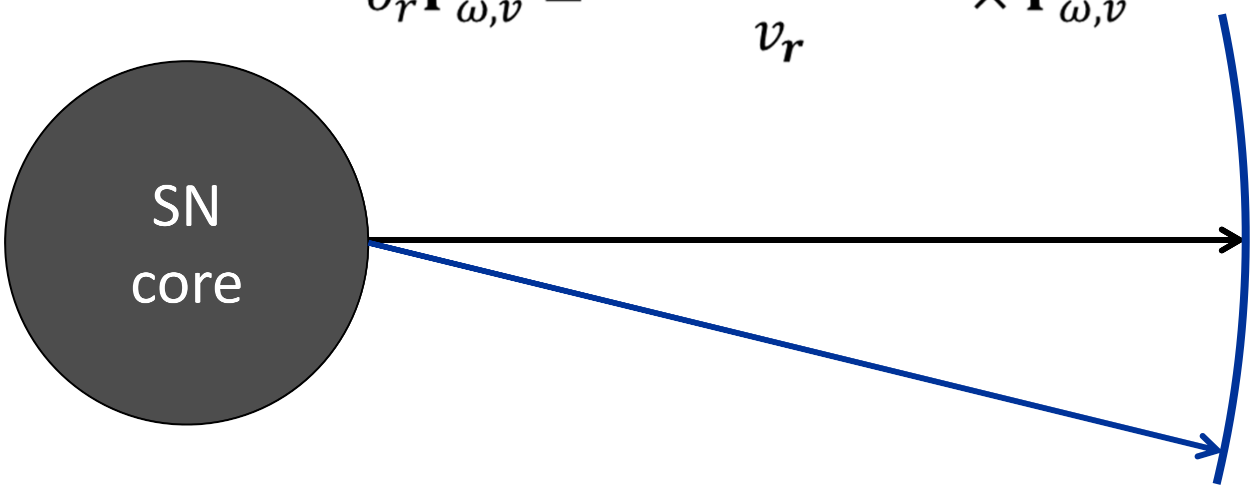
Multi-Angle Matter Effect

Liouville form of oscillation equation

$$\dot{\mathbf{P}}_{\omega,\mathbf{v}} + (\mathbf{v}\cdot\nabla_r)\,\mathbf{P}_{\omega,\mathbf{v}} = (\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}) \times \mathbf{P}_{\omega,\mathbf{v}}$$

Drops out for stationary solutions — $\lambda = \sqrt{2}G_\mathrm{F}N_e$ — $\mu = \sqrt{2}G_\mathrm{F}N_\nu$

$$\partial_r \mathbf{P}_{\omega,v} = \frac{\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}}{v_r} \times \mathbf{P}_{\omega,v}$$

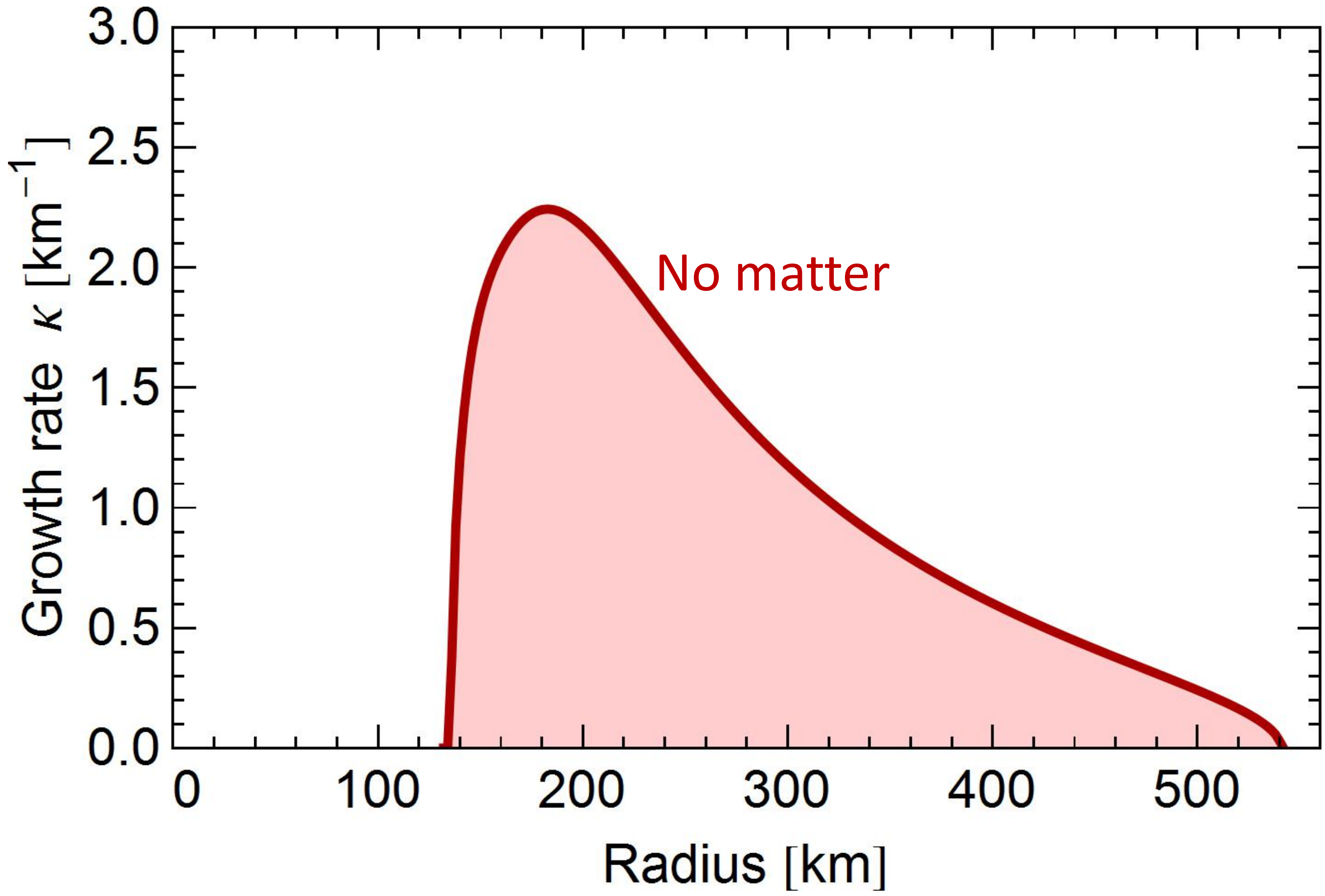


Longer path to
same radial distance:
Larger matter effect

Self-induced conversion suppressed for $N_e \gtrsim N_\nu$

Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659

Multi-Angle Matter Suppression



Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

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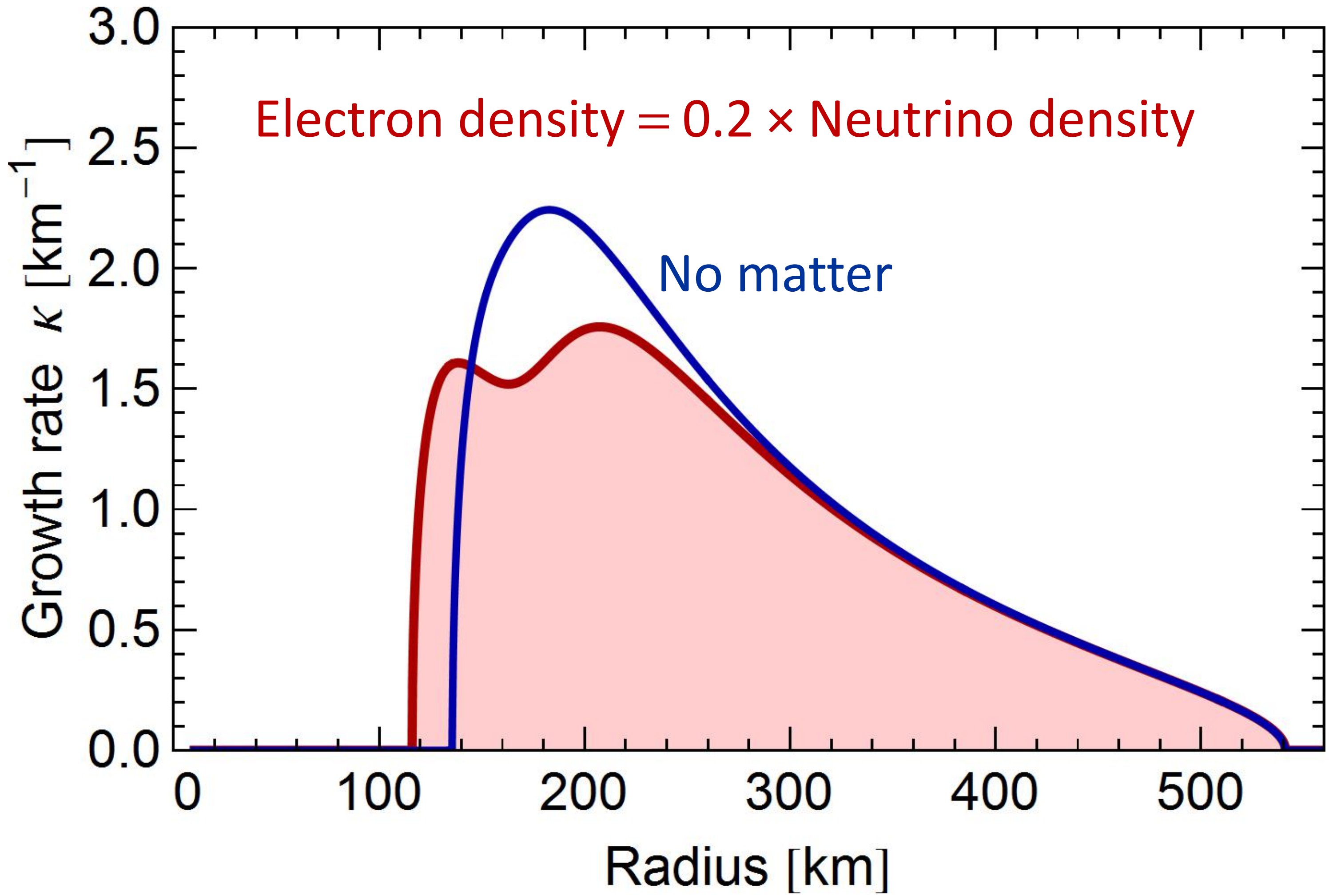

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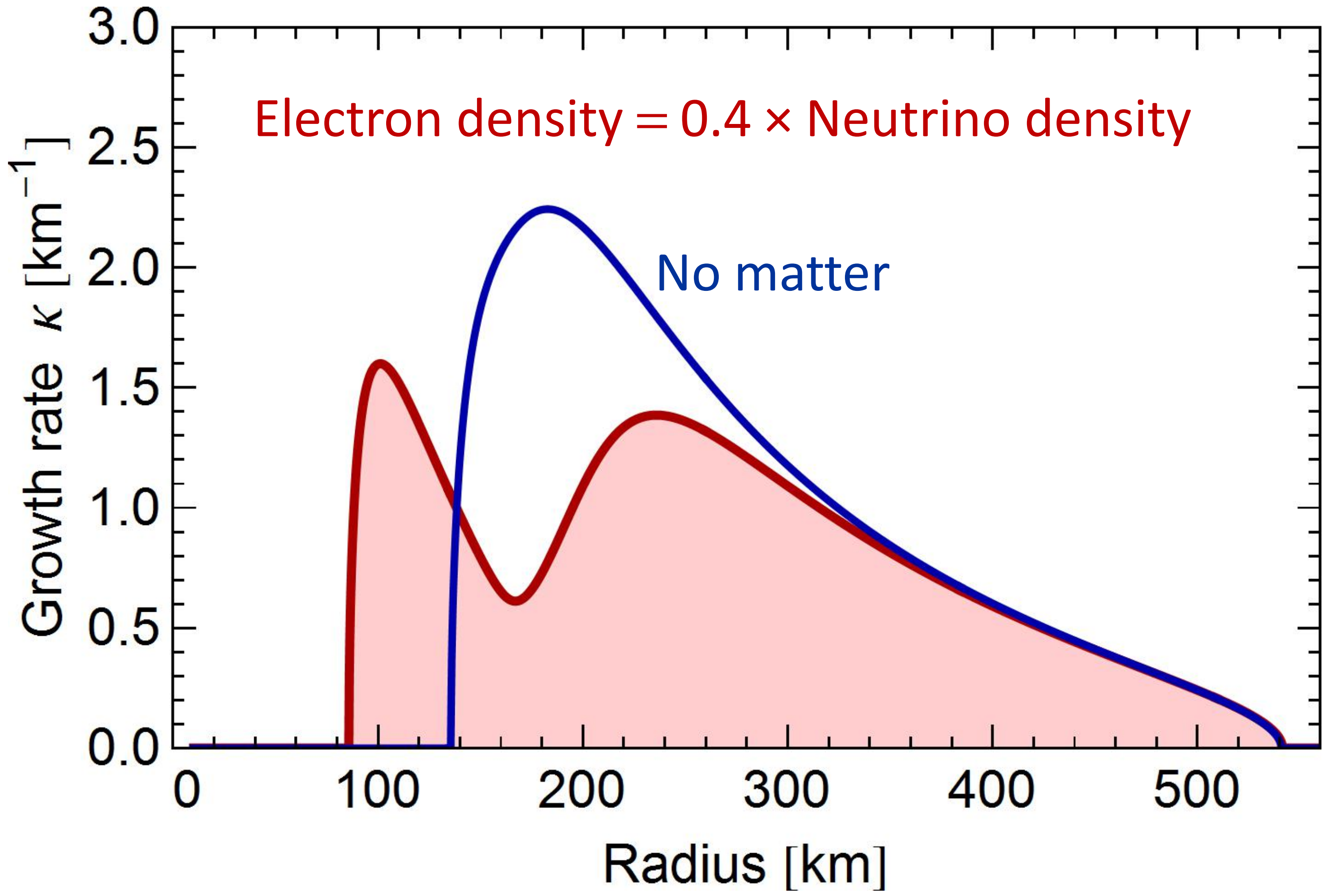

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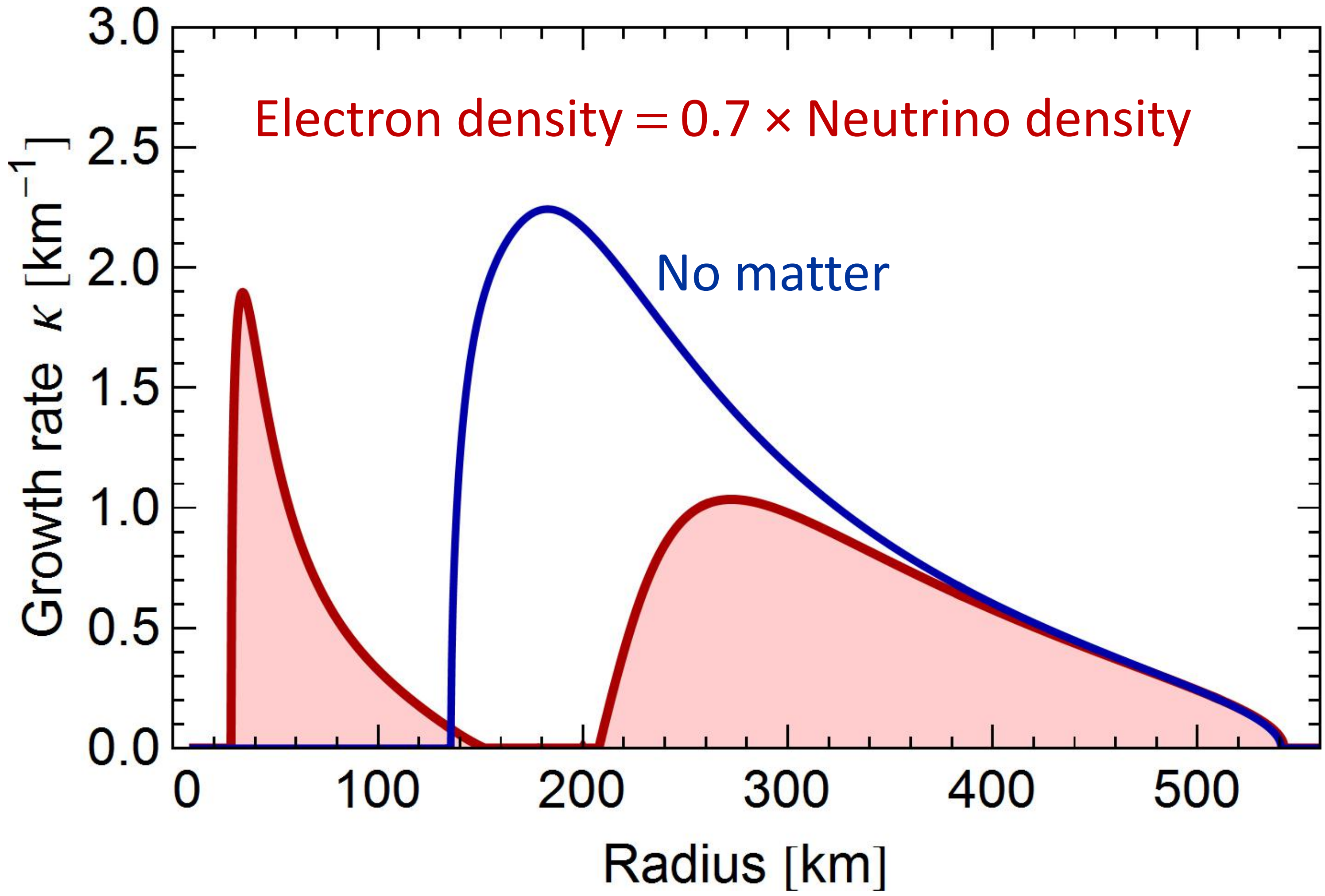

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

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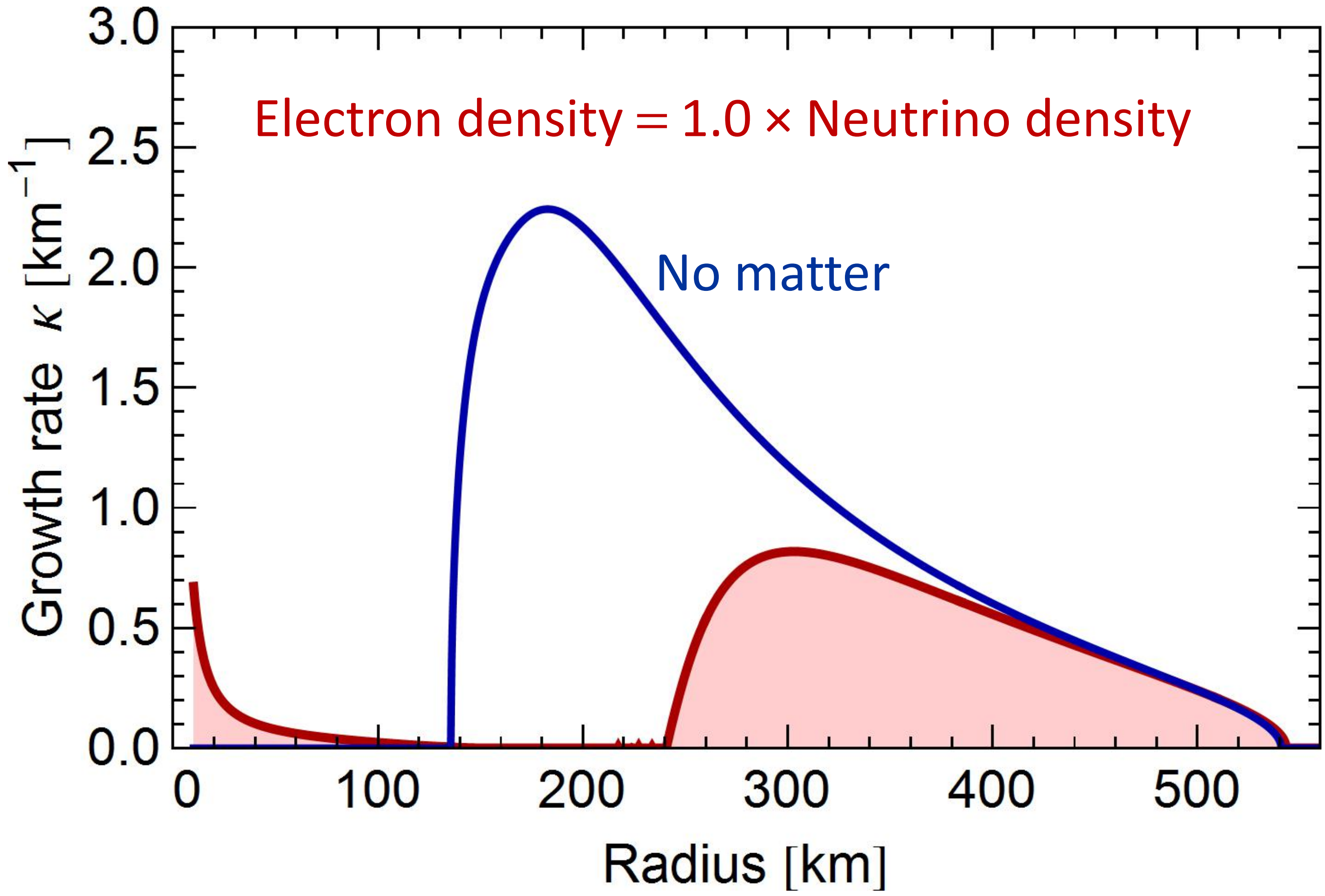

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

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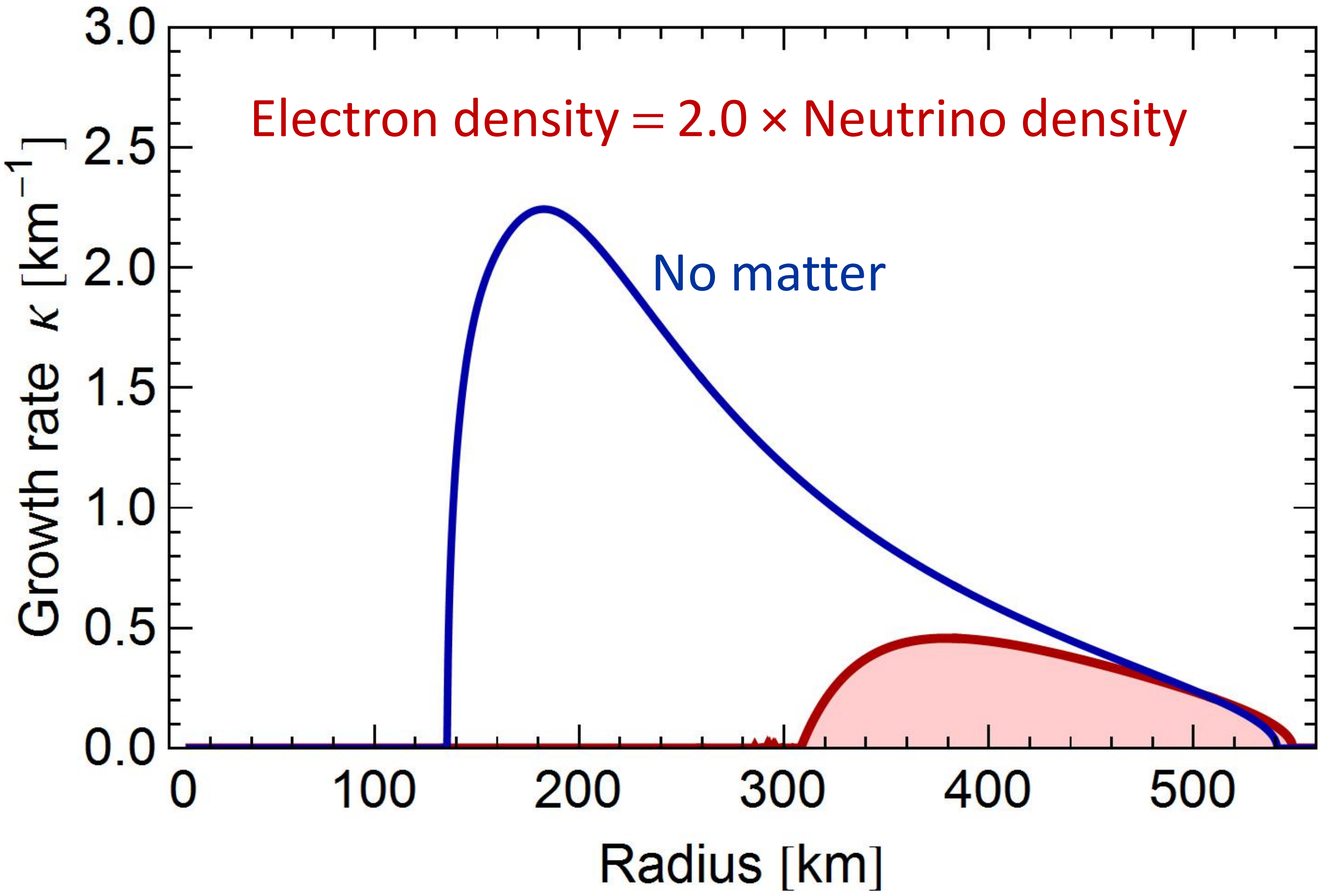

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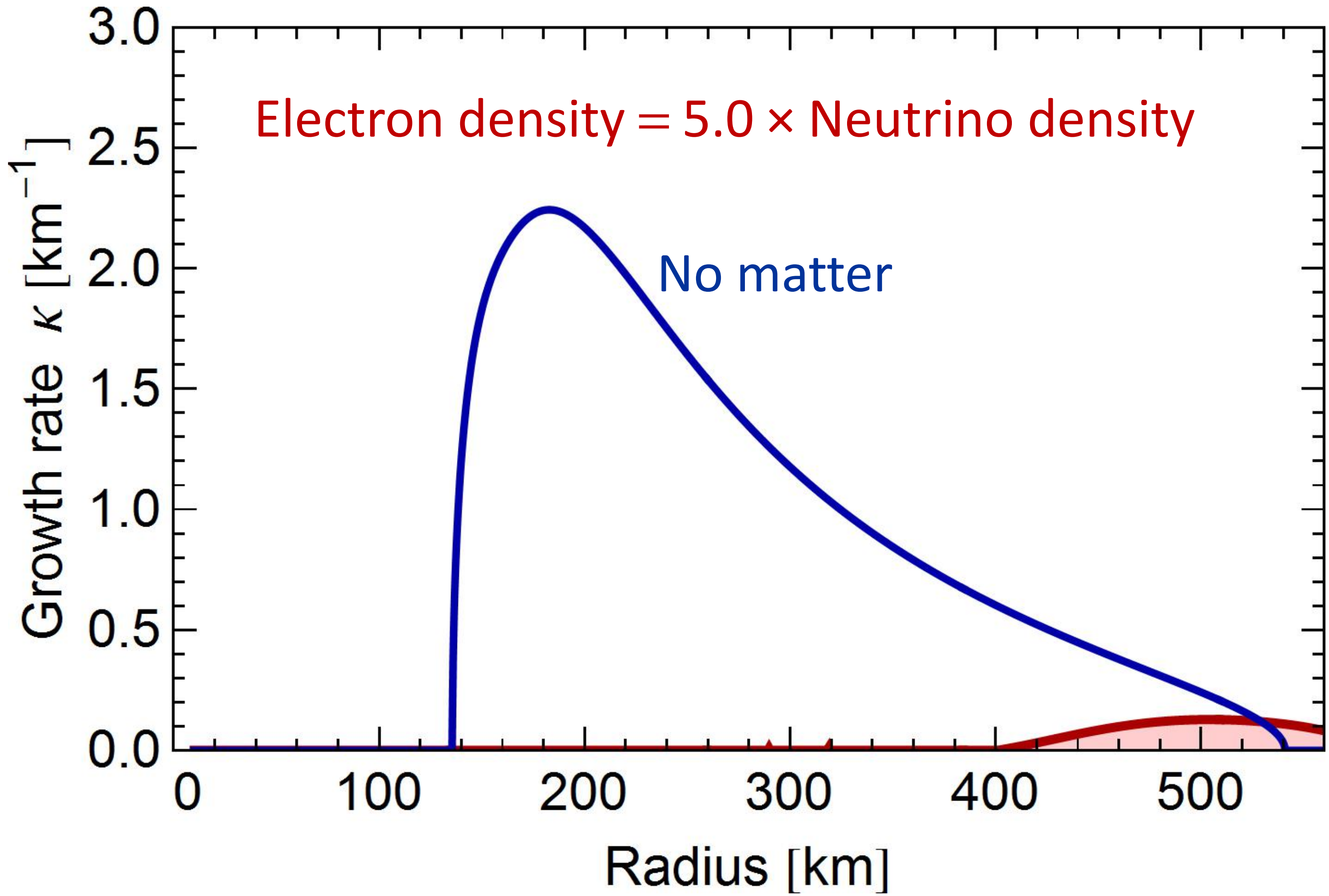
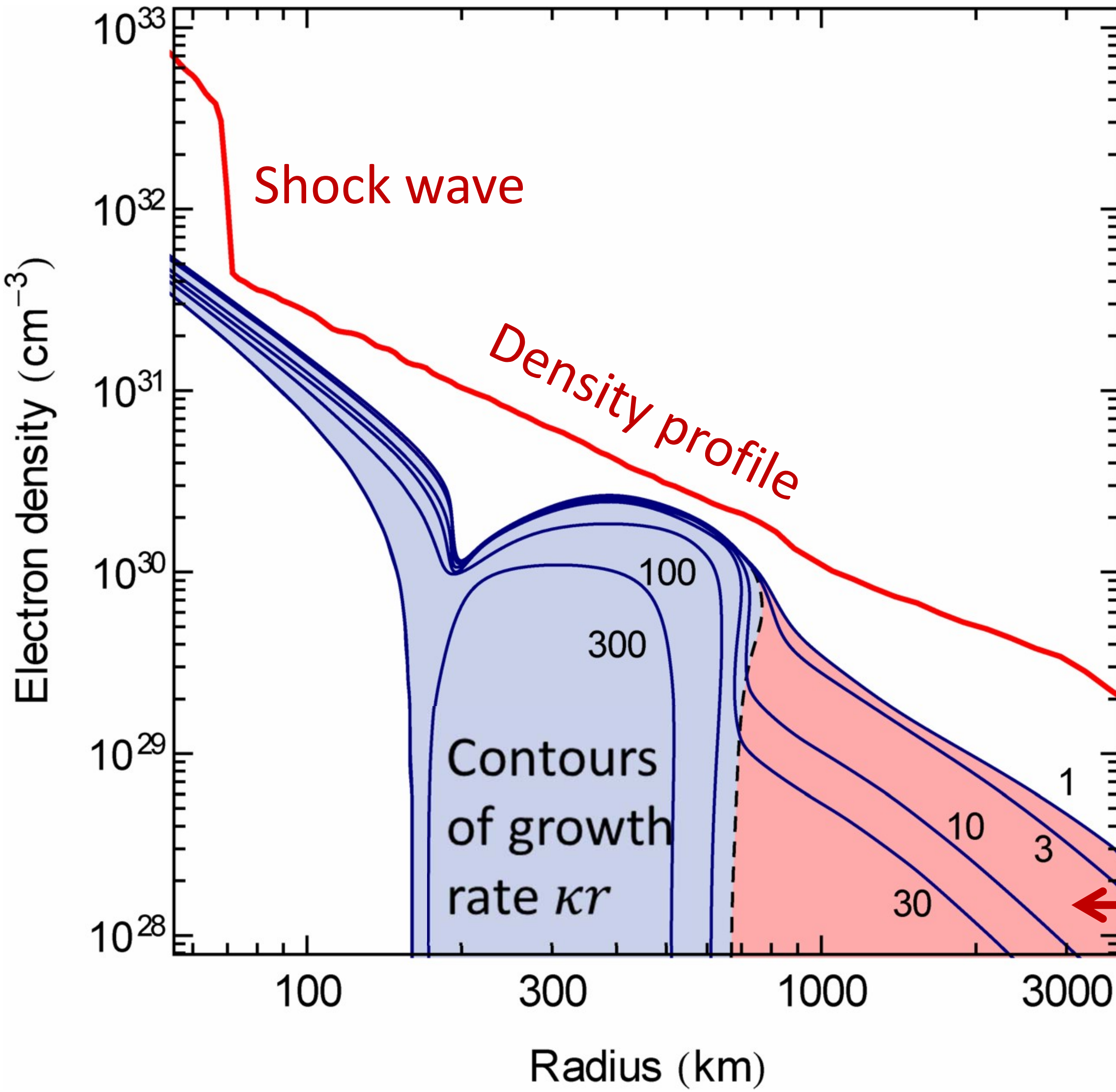


Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression in Realistic Model



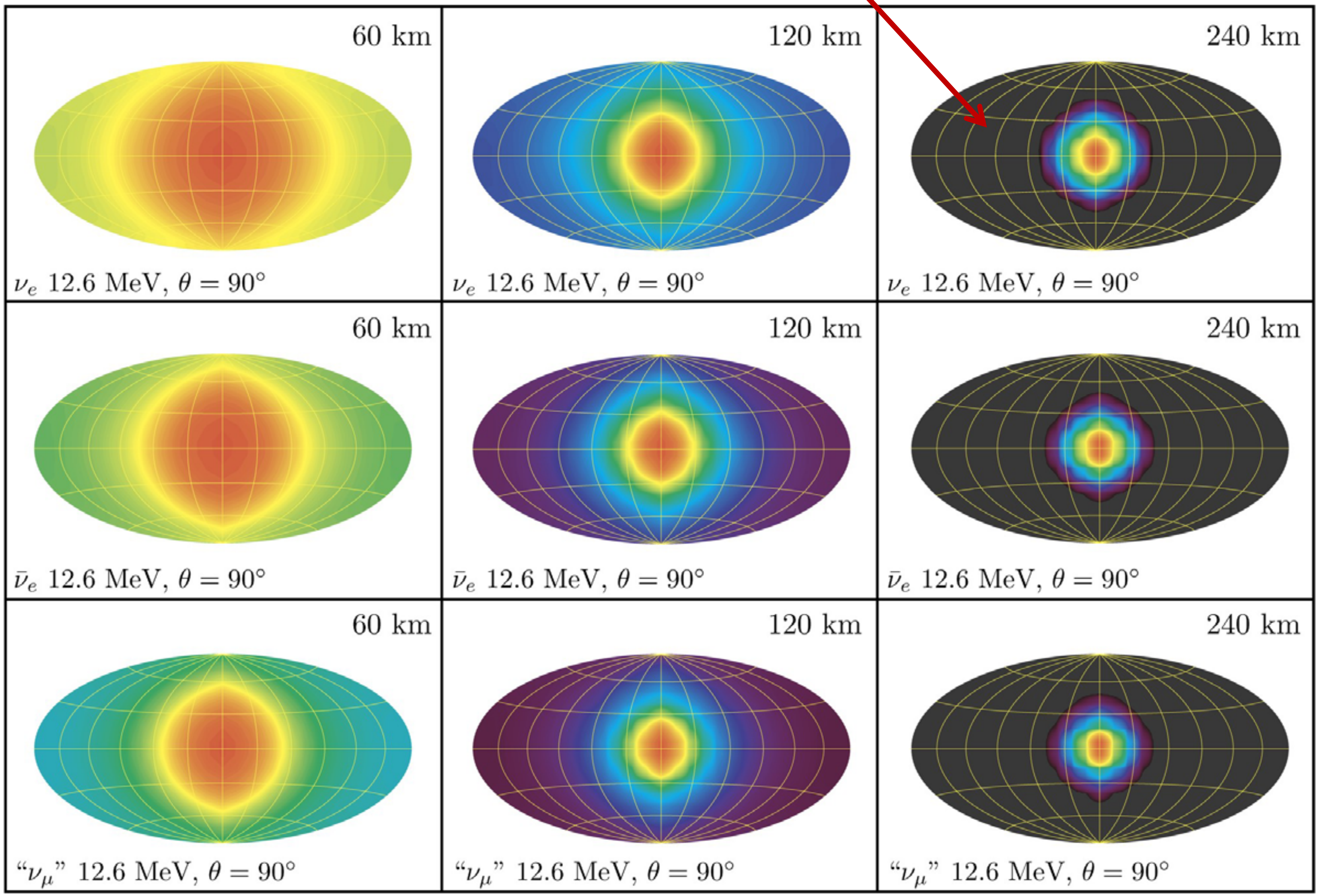
The tested Garching $15\ M_\odot$ accretion-phase models are stable against collective flavor conversion

Halo contribution

Sarikas, Tamborra, Raffelt, Hüdepohl & Janka, arXiv:1109.3601, 1204.0971

Neutrino Radiation Field

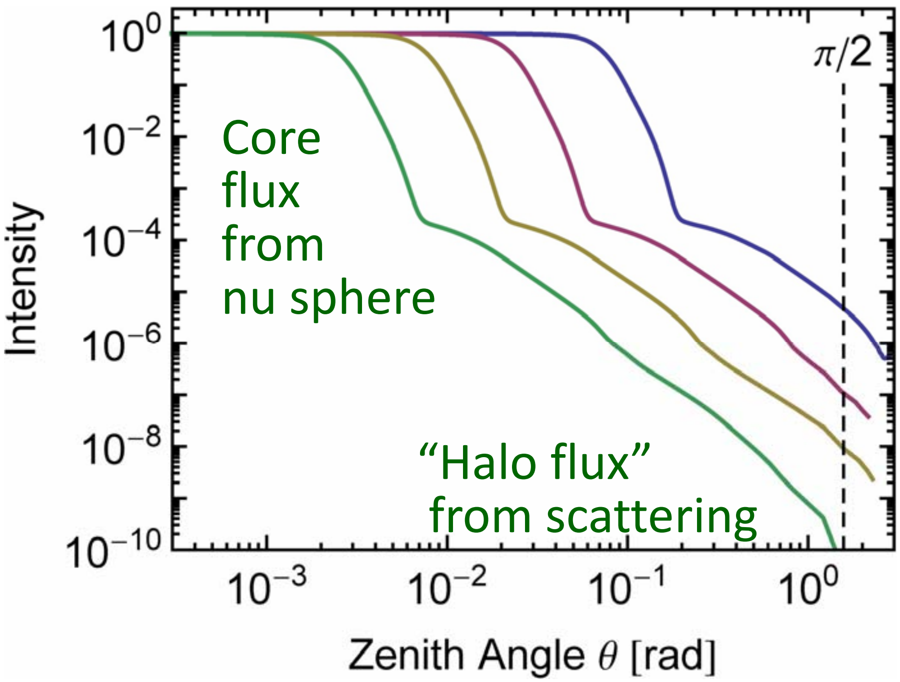
Small “scattering halo” important for nu-nu refraction? (Cherry et al., arXiv:1203.1607)



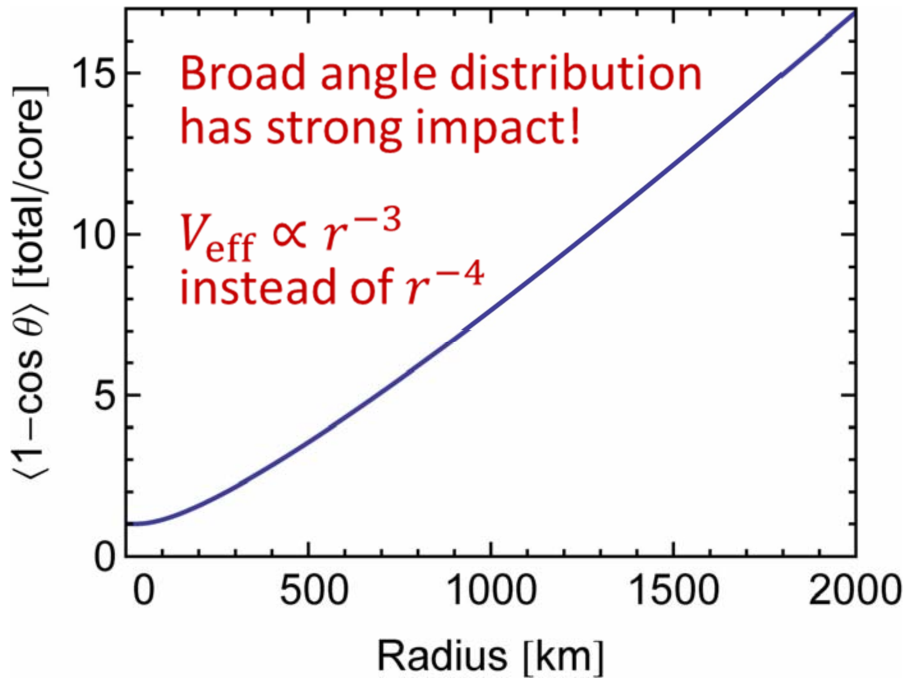
Picture from Ott, Burrows, Dessart & Livne, arXiv:0804.0239

Scattered SN Neutrinos as a Source of Refraction

SN neutrino angle distribution seen at 10^4, 3000, 1000 and 300 km

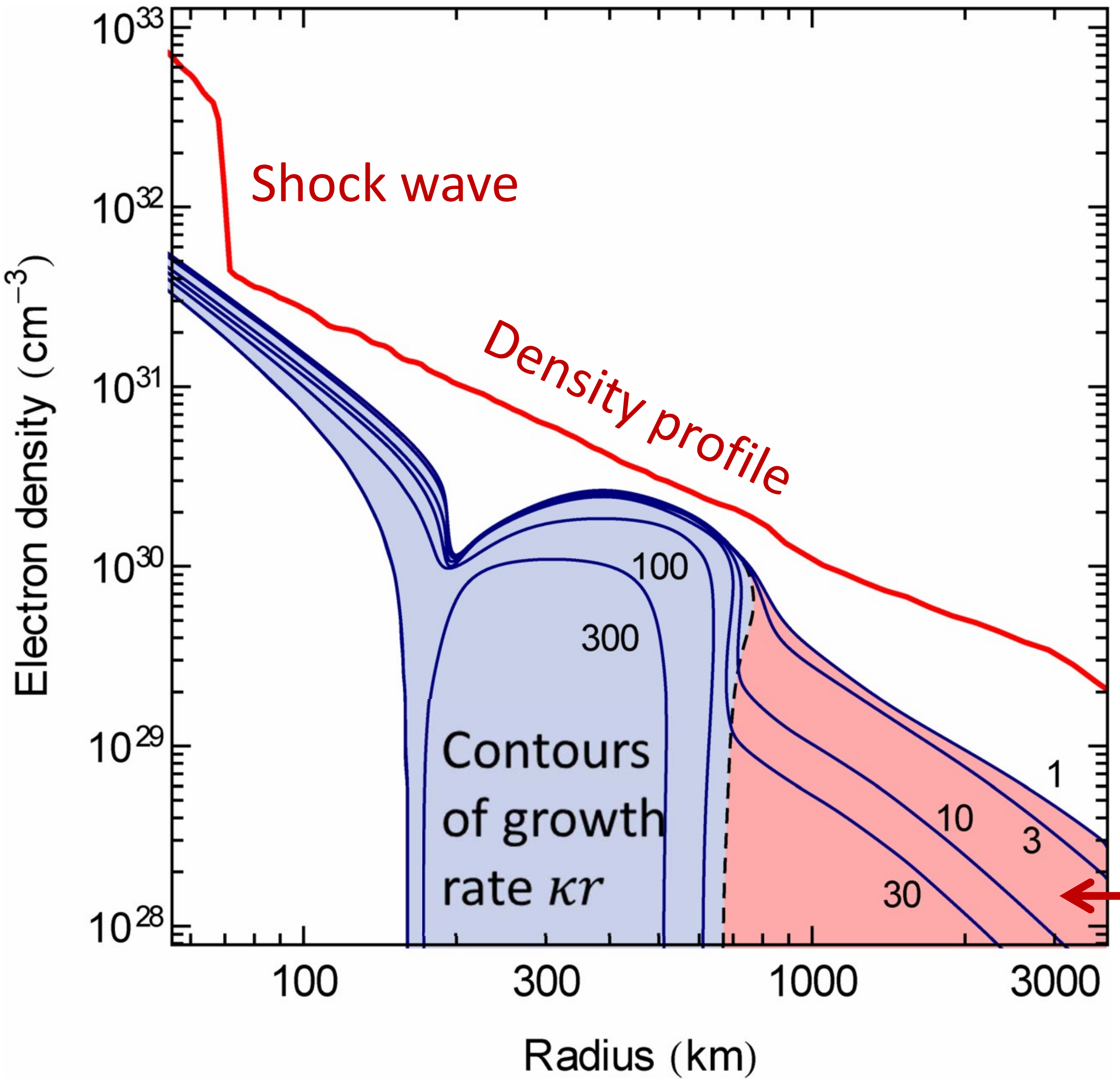


Relative importance of halo for nu-nu refractive potential



Cherry, Carlson, Friedland, Fuller & Vlasenko, arXiv:1203.1607
Sarikas, Tamborra, Raffelt, Hüdepohl & Janka, arXiv:1204.0971

Multi-Angle Matter Suppression in Realistic Model



The tested Garching $15\ M_\odot$ accretion-phase model is stable against collective flavor conversion

Halo contribution

Sarikas, Tamborra, Raffelt, Hüdepohl & Janka, arXiv:1109.3601, 1204.0971

Summary

Supernova neutrino flavor evolution remains a complicated subject

- Axial symmetry was always assumed – too symmetric?
- Numerical treatments challenging
- Novel role for neutrino “scattering halo”?
- Simultaneous space and time dependence important?

Theoretical developments

- Analogy to BCS theory
- Linearized stability analysis provides many conceptual insights
- And practical results

Working hypothesis for SN neutrinos

- Multi-angle matter effect can prevent instability (accretion phase)
- No collective conversion during early accretion phase
- Can test for nu mass hierarchy (because θ_{13} is large) in the spirit of Dighe & Smirnov


More theory progress is needed to reliably interpret neutrino signal of next galactic supernova!