neutrino oscillation and neutrino velocity

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I have only 3 papers with Alexei..

1. H. Minakata and A. Yu Smirnov:
Neutrino Velocity and Neutrino Oscillations, Physical Review D in press. [arXiv:1202.0953 [hep-ph]].

23. H. Minakata and A. Yu Smirnov:
Neutrino Mixing and Quark-Lepton Complementarity, Physical Review D **70** (2004) 073009-1-12 [hep-ph/0405088].

54. H. Minakata and A. Yu Smirnov:
High Energy Cosmic Neutrinos and the Equivalence Principle, Physical Review D **54** (1996) 3698-3705 [arXiv:hep-ph/9601311].

I am a relatively minor speaker in the Smirnov Fest ➔ Thank you for the invitation

I learned a lot from him during these collaborating works

How good is MSW?

A challenge to Alexei

Constraining ε_{ee} with solar ν

Update of the analysis by Fogli-Lisi, NJP 04

$G_F \rightarrow A_{MSW}\, G_F$

$A_{MSW} = 1+\varepsilon_{ee}$

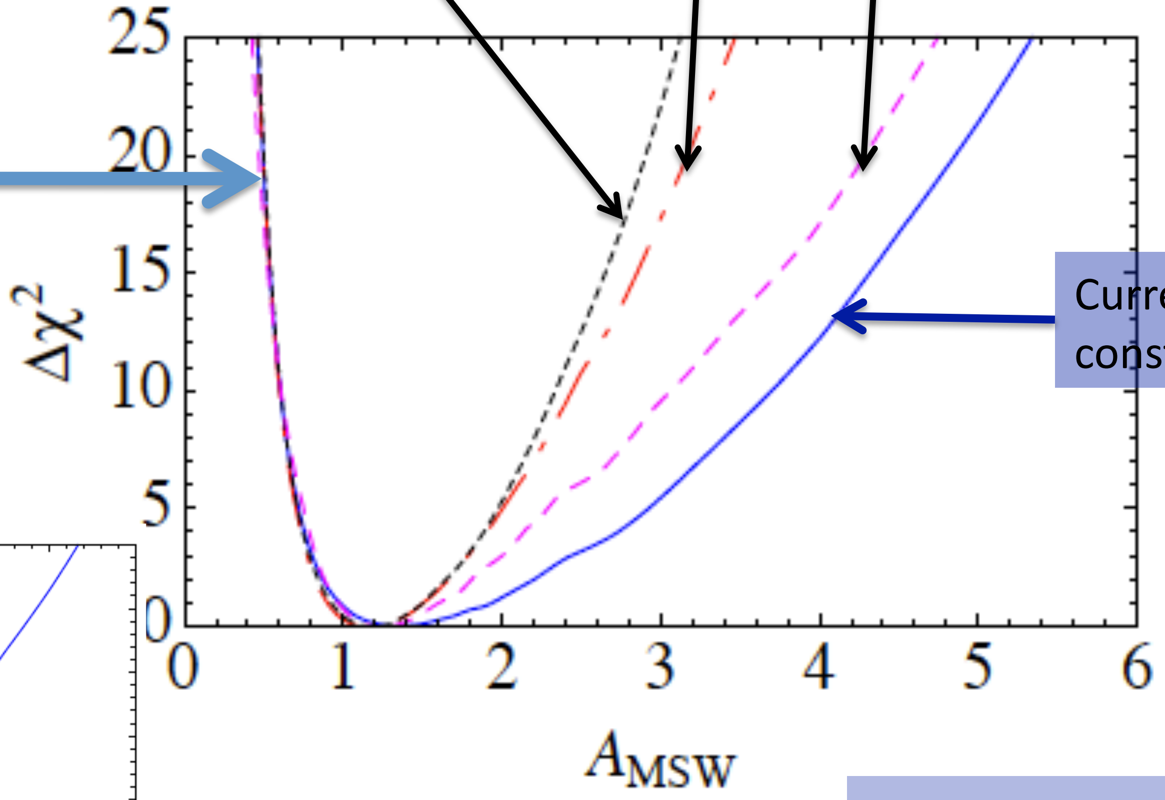
+Be5%-pep3%

+SK-upturn-3σ

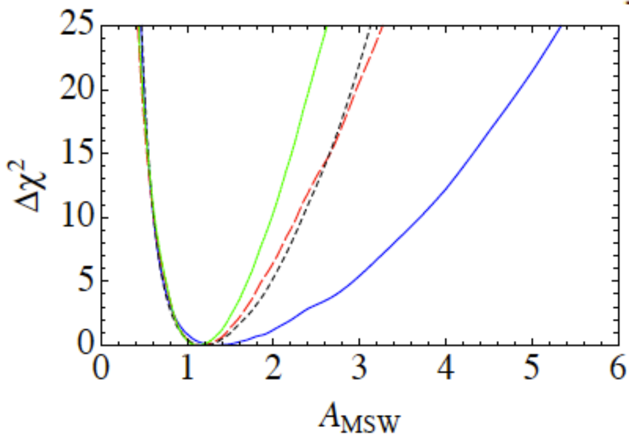
+SK-d/n-2σ

Alexei wall

Current constraint



+low E-spectrum



HM-Pena-Garay **arXiv: 1009.4869 [hep-ph]**

Quark lepton complementarity

Large θ_{13} in QLC context

QLC based on observation: $\theta_{12} + \theta_C = \pi/4$

"bimaximal minus CKM mixing."

Bi-maximal mixing from neutrinos

$$U_\nu = R_{23}^m R_{12}^m, \qquad U_l = V^{\mathrm{CKM}}.$$

$$U_{\mathrm{MNS}} = V^{\mathrm{CKM}\dagger}\Gamma_\delta R_{23}^m R_{12}^m$$
$$= R_{12}^{\mathrm{CKM}\dagger} R_{13}^{\mathrm{CKM}\dagger} R_{23}^{\mathrm{CKM}\dagger}\Gamma_\delta R_{23}^m R_{12}^m,$$

$$\sin\theta_{13} \simeq \frac{1}{\sqrt{2}}\sin\theta_C$$

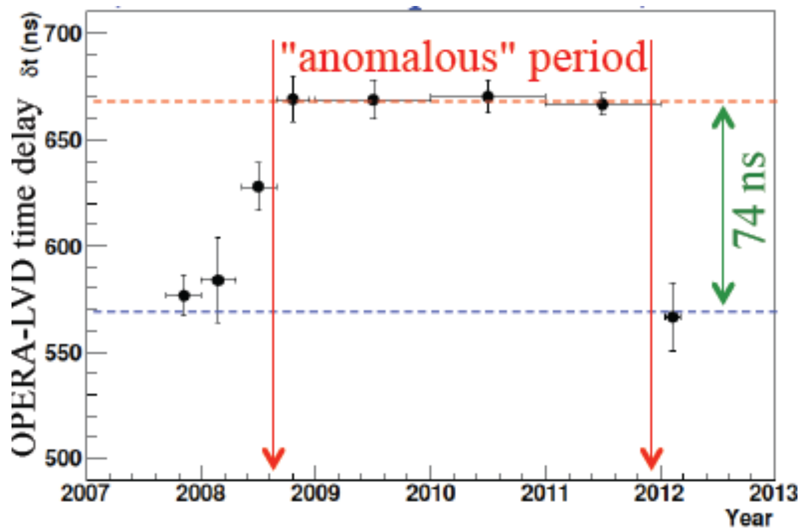
HM-A.Smirnov 04

$$\sin^2\theta_{13} = 0.026 \pm 0.008$$

Predicted value of θ_{13} in this model agrees well with experiments !

Superluminal neutrino?

Apparently no except for..



Neutrino velocity after OPERA?

- Still interesting subject
- Topics of our newest publication
- It is a new subject to me
- Can we see neutrino wave packet?
- Future measurement of neutrino and light velocities

Nothing anomalous because we take special relativity

v=dE/dp in matter

$$E_i^m = p + \frac{\lambda_i}{2p} \; (i = 1, 2, 3)$$

a=$\sqrt{2}G_FN_ep$

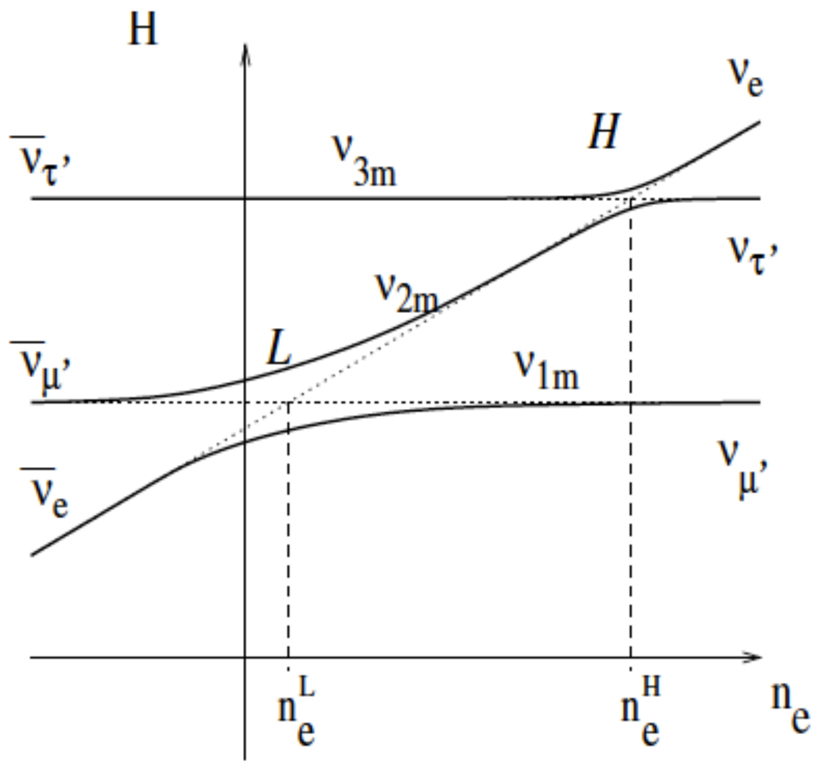
$$v_i - 1 = -\frac{\lambda_i}{2p^2} + \frac{1}{2p}\frac{d\lambda_i}{da}\frac{da}{dp} = -\frac{\lambda_i}{2p^2} + \frac{1}{2p^2}\frac{d\lambda_i}{da}a = -\frac{\lambda_i}{2p^2}\left[1 - \frac{d(\log \lambda_i)}{d(\log a)}\right]$$

$$\frac{d(\log \lambda_i)}{d(\log a)} < 1.$$

because

$$\lambda_1 = \frac{1}{3}s - \frac{1}{3}\sqrt{s^2 - 3t}\left[u + \sqrt{3(1-u^2)}\right],$$
$$\lambda_2 = \frac{1}{3}s - \frac{1}{3}\sqrt{s^2 - 3t}\left[u - \sqrt{3(1-u^2)}\right],$$
$$\lambda_3 = \frac{1}{3}s + \frac{2}{3}u\sqrt{s^2 - 3t},$$

$$s = \Delta_{21} + \Delta_{31} + a,$$
$$t = \Delta_{21}\Delta_{31} + a[\Delta_{21}(1 - s_{12}^2c_{13}^2) + \Delta_{31}(1 - s_{13}^2)],$$
$$u = \cos\left[\frac{1}{3}\cos^{-1}\left(\frac{2s^3 - 9st + 27a\Delta_{21}\Delta_{31}c_{12}^2c_{13}^2}{2(s^2-3t)^{3/2}}\right)\right]$$



Neutrino wave packet is large

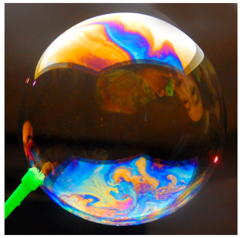
Quantum states in a decay tunnel

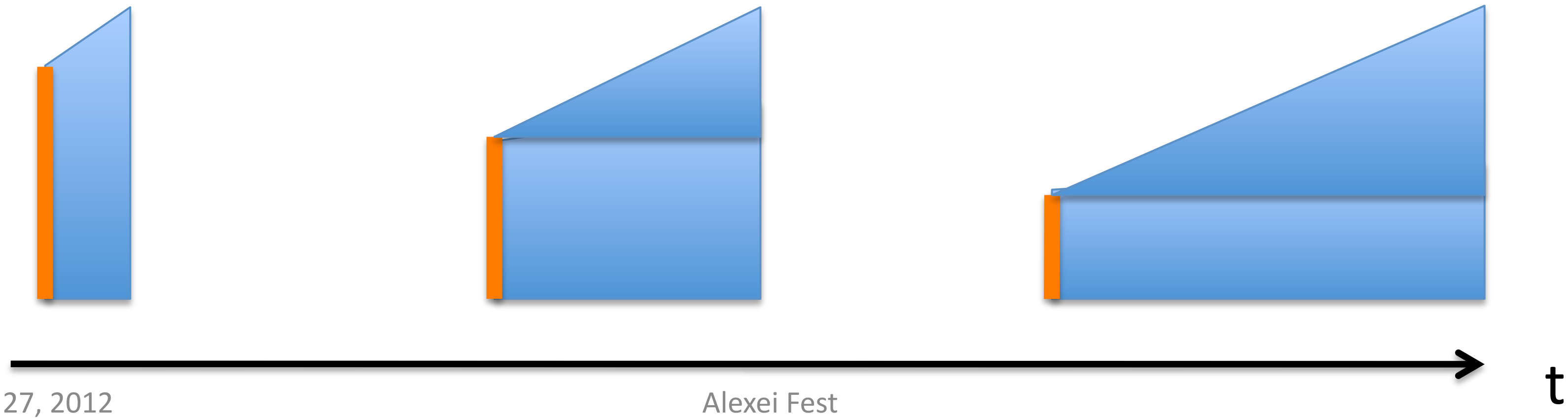
- Suppose pions are produced at the target and will decay in flight in vacuum
- Quantum states in a decay tunnel is a superposition of un-decayed pion and $\mu + \nu_\mu$
- Unless pion (or muon) is observed we do not know which states, π or $(\mu + \nu_\mu)$, is realized in the tunnel
- Then, ν_μ state is a single quantum state at anywhere in the decay tunnel

Energy loss = 0.2% in JPARC decay tunnel (110m)

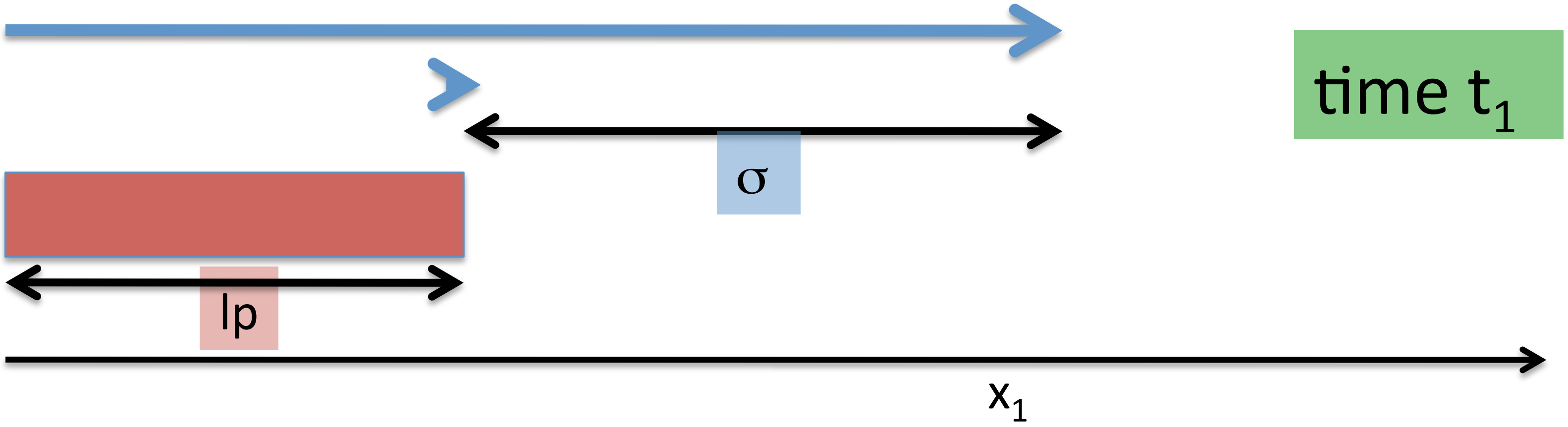
Neutrino wave packet is large

- How big is the ν_μ wave packet?
- Once "coherence over decay tunnel" is agreed the size is determined by motion of ν_μ wave
- "soap bubble" model of ν wave packet





How big is the ν_μ wave packet?



- At t=0 pion is produced
- At time t_1 when ν_μ emitted at the end of decay pipe, the first emitted ν_μ is at $x_1 = lp + \sigma = v_\nu t_1$
- $t_1 = lp / v_\pi$ or $lp = v_\pi t_1$
- By eliminating t_1 we obtain $v_\nu/v_\pi = (lp + \sigma)/lp$, or $\sigma/lp = v_\nu/v_\pi - 1 \sim m_\pi^2/2p^2$
- packet size $\sigma \sim lp/\gamma_\pi^2$ (dimension is from lp)

Numerically ..

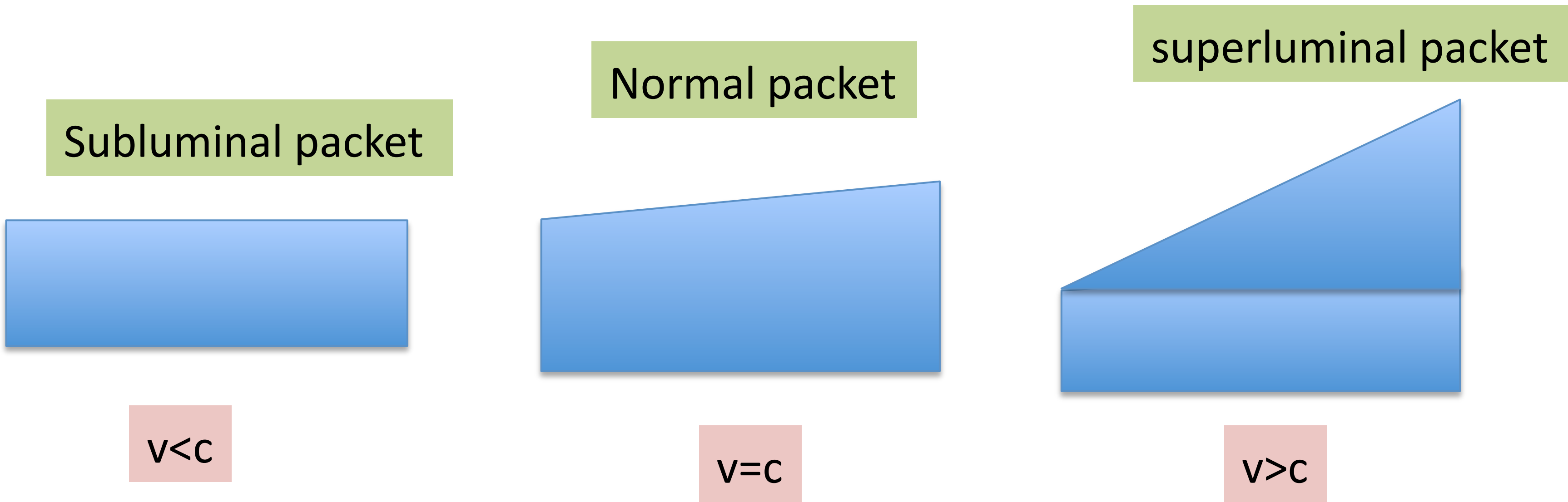
- packet size σ ~ lp $/\gamma_{\pi}^{2}$
- σ ~ 50 cm $(\gamma_{\pi}/10)^{-2}$ (lp/100 m) or
- σ ~ 18 cm $(E/1GeV)^{-2}$ (lp/100 m)
- Note: γ_{π}=16.8 (E/1GeV) for pion decay, so γ_{π}=10 correspond to E=~ 600MeV
- Neutrino wave packet is large!
- Time difference 1m/3x10^{8}m= 3 ns, so seeing the profile of ν wave packet may not be too difficult
- interesting experimental challenge

Superluminal neutrinos ?

Yes !

Is there possibility of superluminal neutrino propagation?

- Yes
- But how?
- By modulation of wave packet shape in flight; center of mass moves faster/slower



If ν oscillate it is easy ..

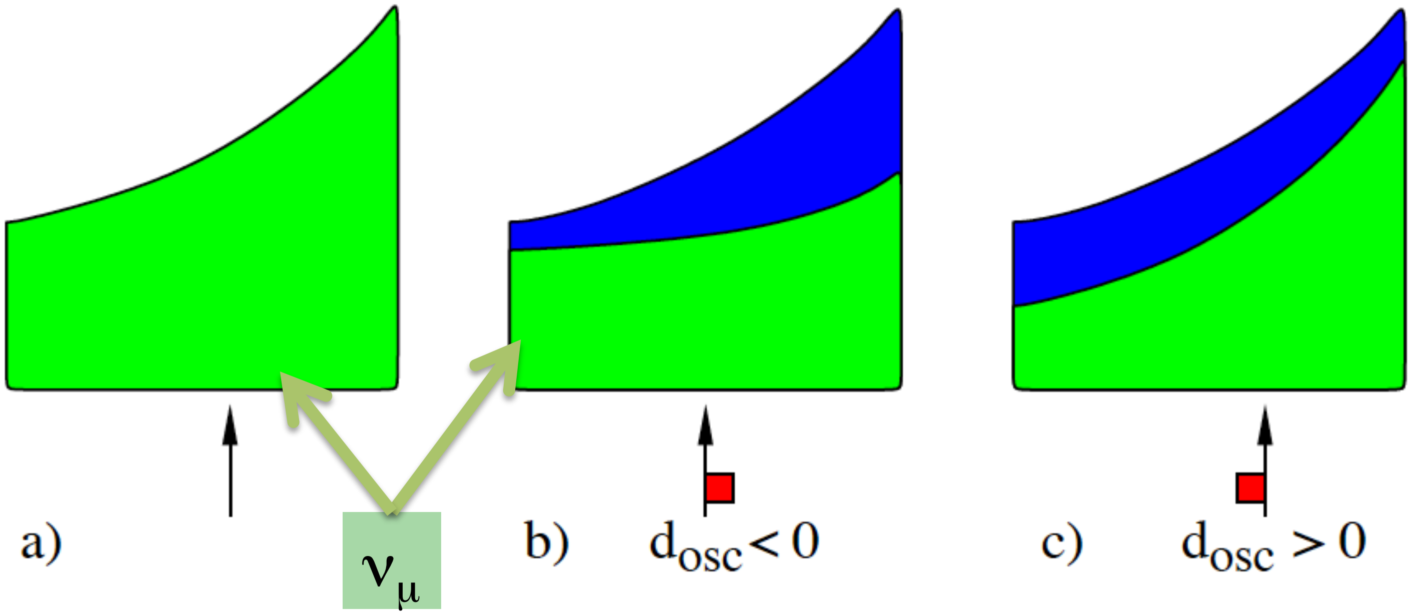


FIG. 1: The wave packets of the muon neutrino from pion decay without oscillations (a) and with oscillations in two different moments of time (b),(c). The lower (green) parts of the shape factors show the ν_μ fraction, whereas the upper (blue) parts correspond to the ν_τ fraction which appears due to oscillations. The arrows indicate positions of "centers of mass" of the ν_μ parts. The red boxes show shifts of the centers due to oscillations with respect to the center in the no-oscillation case. The pannel (b) corresponds to the baselines $0 < L < l_{osc}/2$, when the front edge of the wave packet is suppressed. The pannel (c) is for $l_{osc}/2 < L < l_{osc}$, when the trailing edge is suppressed.

Shape factor and phase factor

$E_k(p) = E_k(p_k) + v_k(p - p_k)$

$\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$

Inserting into $\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$

Alexei course I

$$\Psi_k \sim e^{ip_k x - iE_k(p_k)t} g_k(x - v_k t)$$

Phase factor

$$e^{i\phi_k}$$

$$\phi_k = p_k x - E_k t$$

Depends on mean characteristics p_k and corresponding energy:

$$E_k(p_k) = \sqrt{p_k^2 + m_k^2}$$

Shape factor

$$g_k(x - v_k t) = \int dp\, f_k(p)\, e^{ip(x - v_k t)}$$

Depends on x and t only in combination $(x - v_k t)$ and therefore Describes propagation of the wave packet with group velocity v_k without change of the shape

Wave packet picture

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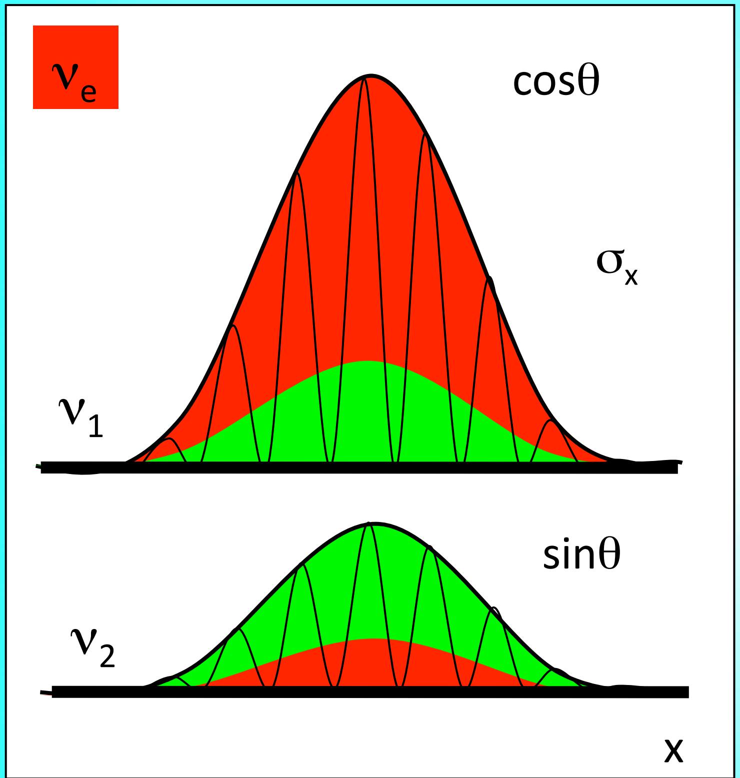


$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$
$$\nu_\mu = - \sin\theta\, \nu_1 + \cos\theta\, \nu_2$$

opposite phase

$$\nu_1 = \cos\theta\, \nu_e - \sin\theta\, \nu_\mu$$
$$\nu_2 = \cos\theta\, \nu_\mu + \sin\theta\, \nu_\varepsilon$$

Interference of the same flavor parts

$\phi = 0$

Main, effective frequency

$$|\nu(x,t)\rangle = \cos\theta\, g_1(x - v_1 t)\, e^{i\phi_1} |\nu_1\rangle + \sin\theta\, g_2(x - v_2 t)\, e^{i\phi_2} |\nu_2\rangle$$

$\phi = \phi_2 - \phi_1$

Oscillation phase

Exponential-shaped packet

$$\langle x\rangle_{osc} = -\frac{\sin^2 2\theta}{2P_{\mu\mu}}\frac{\sigma}{y^2+\phi_p^2}\left(\kappa_s\sin\phi - \kappa_c\cos\phi\right)$$

$$\langle x(t)\rangle \equiv \frac{\int dx\; x\; |\psi_{\nu_\mu}(x,t)|^2}{\int dx\; |\psi_{\nu_\mu}(x,t)|^2}$$

$$\kappa_s = \frac{y}{1-e^{-y}}\left[\frac{-2y\phi_p e^{-y} + q\sin\phi_p - n\cos\phi_p}{y^2+\phi_p^2} - \left(\frac{1}{1-e^{-y}} - \frac{1}{y}\right)\left(\phi_p e^{-y} + y\sin\phi_p - \phi_p\cos\phi_p\right)\right]$$

$$\kappa_c = \frac{y}{1-e^{-y}}\left[\frac{(y^2-\phi_p^2)e^{-y} + q\cos\phi_p + n\sin\phi_p}{y^2+\phi_p^2} - \left(\frac{1}{1-e^{-y}} - \frac{1}{y}\right)\left(-ye^{-y} + y\cos\phi_p + \phi_p\sin\phi_p\right)\right]$$

$$y = \frac{l_p}{l_{decay}} \approx \frac{l_p\Gamma_0}{\gamma_\pi} \simeq 1.28\times\left(\frac{\gamma_\pi}{10}\right)^{-1}\left(\frac{l_p}{100\text{m}}\right)$$

$$q \equiv y^2(y-1) + \phi_p^2(y+1), \qquad n \equiv \phi_p(y^2 - 2y + \phi_p^2)$$

$$\sigma\phi_p = 1.25\times 10^{-2}\text{cm}\left(\frac{\Delta m^2}{10^{-3}\text{eV}^2}\right)\left(\frac{E}{1\text{GeV}}\right)^{-1}\left(\frac{\gamma_\pi}{10}\right)^{-2}\left(\frac{l_p}{100\text{m}}\right)^2$$

$$\phi_p = \frac{\Delta m^2}{2E}l_p \approx 2.5\times 10^{-4}\left(\frac{\Delta m^2}{10^{-3}\text{eV}^2}\right)\left(\frac{E}{1\text{GeV}}\right)^{-1}\left(\frac{l_p}{100\text{m}}\right)$$

$\phi_p = 2p\ (lp/l_\nu)$
=phase variation
inside WP

Box-shaped packet

$$\langle x(t)\rangle_{osc} \approx -\frac{\sin^2 2\theta}{24P_{\mu\mu}}\sigma\phi_p \sin\left(\phi + \frac{\phi_p}{2}\right)$$

$\phi = \Delta m^2 t/2E$

$$\sigma\phi_p = 1.25 \times 10^{-2}\mathrm{cm}\ \left(\frac{\Delta m^2}{10^{-3}\mathrm{eV}^2}\right)\left(\frac{E}{1\mathrm{GeV}}\right)^{-1}\left(\frac{\gamma_\pi}{10}\right)^{-2}\left(\frac{l_p}{100\mathrm{m}}\right)^2$$

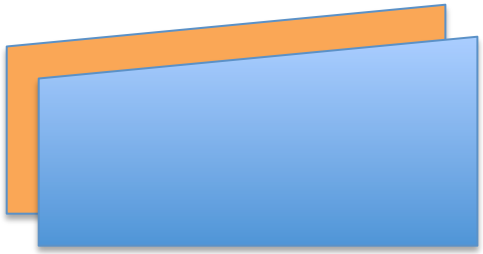
ϕ_p = 2p (lp/l_ν)= ν phase variation inside wave packet

$$\phi_p = \frac{\Delta m^2}{2E} l_p \approx 2.5 \times 10^{-4}\left(\frac{\Delta m^2}{10^{-3}\mathrm{eV}^2}\right)\left(\frac{E}{1\mathrm{GeV}}\right)^{-1}\left(\frac{l_p}{100\mathrm{m}}\right)$$

Effect not very large ...

$$\langle x(t)\rangle \equiv \frac{\int dx\ x\ |\psi_{\nu_\mu}(x,t)|^2}{\int dx\ |\psi_{\nu_\mu}(x,t)|^2};$$

The effect people computed before is even smaller



2 mass eigenstate separate as time goes ➔ modulation of packet shape

$$\langle x(t)\rangle_{shift-a} = \cos\phi \, \frac{\sin^2 2\theta}{2P_{\mu\mu}} \left(\frac{\Delta vt}{2}\right) F(y,\epsilon)$$

F~O(1)

$$\Delta vt \approx \frac{\Delta m^2}{2E^2} L = 0.5 \times 10^{-13} \left(\frac{\Delta m^2}{10^{-3}\mathrm{eV}^2}\right) \left(\frac{E}{1\mathrm{GeV}}\right)^{-2} \left(\frac{L}{10^3\mathrm{km}}\right) \ \mathrm{cm}$$

Extremely small ➔ utterly negligible

Super(sub) luminal shift for various experimental settings

TABLE I: The values of $y = l_{form}/l_{decay}$, sine of the oscillation phase, ϕ, the wave packet length, σ, as well as the contributions to the distance of ν_μ propagation from the mass terms, d_{mass}, from the relative shift of the wave packets, $d_{shift-s}$, and from oscillations, d_{osc}. We use $\Delta m^2 = 2.5\times10^{-3}\text{eV}^2$, and $\sin^2 2\theta = 0.97$.

Experiment	E (GeV)	y	$\sin\phi$	σ (cm)	$-d_{mass}$ (cm)	$d_{shift-s}$ (cm)	d_{osc} (cm)
OPERA	17	0.491	0.270	0.671	1.58×10^{-16}	2.80×10^{-17}	-1.0×10^{-5}
OPERA	1	1	-0.99	23.3	4.56×10^{-14}	1.65×10^{-14}	$+1.6 \times 10^{-3}$
MINOS	3	1	1.00	7.7	5.10×10^{-15}	1.70×10^{-15}	-4.6×10^{-4}
MINOS	1	1	-0.995	23.3	4.59×10^{-14}	1.62×10^{-14}	$+1.54 \times 10^{-3}$
T2K	0.6	1	0.052	38.8	5.12×10^{-14}	2.94×10^{-13}	-2.2×10^{-3}
T2K	0.4	1	-0.997	58	1.14×10^{-13}	4.25×10^{-14}	$+3.9 \times 10^{-3}$

Ways to enhance the effect?

- Low energy beams (stopped mu decay ..)
- If there is sterile ν of $\Delta m^2 \sim 1 eV^2$ then $\sigma \sim 10$ cm
- Tune the energy / baseline so that $1/P_{\mu\mu}$ large etc.
- But it appears that $\langle x \rangle \sim O(\sigma)$

What we have learned?

What we have learned

- Our computation provides the basic framework for future research on ν velocity
- Since ν wave packet is large, one would expect rich quantum physics (but may not easy to measure..)
- We have shown that ν can travel with superluminal velocity (sublumi-superlumi periodically)
- Having ~1m (3 ns) difference seems difficult
- Not easy to design experiments to detect this effect

Further prospects?

Any further prospects ?

- Suppose that neutrinos and gamma are emitted from a distant source
- If we can measure arrival times of both ν and γ it would tell us new ν interactions in the intergalactic space, or space-time probed by ν and γ; "new gravity" ..
- here accuracy of time measurement is not the issue
- The issue is how we can guarantee that ν and γ are emitted at the "same time"
- Variable Blazer/GRB like object?