Aspects of the δN formalism

David H. Lyth

Particle Theory and Cosmology Group

Physics Department

Lancaster University

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  - Cf. spectral tilt: $n - 1 = 2\eta - 6\epsilon$ (Liddle/DHL 1992)
- Trispectrum, even higher correlators, could be as important as the bispectrum
- Need to specify box size L (infrared cutoff)
  - But parameters run with L

The correlators

Spectrum $\mathcal{P}$, bispectrum[†] $f_{\rm NL}$, trispectrum[††] $\tau_{\rm NL}$:

$$
\begin{aligned}
\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}') K_1 \mathcal{P} \\
\frac{5}{3} \langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \rangle &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'') K_2 \mathcal{P}^2 f_{\rm NL} \\
\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \zeta_{\mathbf{k}'''} \rangle_{\rm c} &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}''+\mathbf{k}''') K_3 \mathcal{P}^3 \tau_{\rm NL}
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where the kinematic factors depend on the wave-vectors:

$$\begin{aligned} K_1 &\equiv 2\pi^2 / k^3 \\ K_2 &\equiv K_1(k) K_1(k') + \ 5 \mathrm{perms} \\ K_3 &\equiv K_2 K_1(|\mathbf{k} + \mathbf{k}''|) + 23 \mathrm{perms} \end{aligned}$$

† Komatsu/Spergel 2000; Maldacena 2003

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$$N = \int_{t_1}^{t} \frac{d \ln a(\mathbf{x},t)}{dt} dt$$

Salopek & Bond 1990; DHL, Malik & Sasaki 2005 (non-perturbative refs.)

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- Then

$$N(\mathbf{x}, t) = N(\phi_i(\mathbf{x}), \rho(t))$$

the expansion of a family of unperturbed universes

DHL, Malik & Sasaki 2005 (non-perturbative)

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- Slow-roll, GR $\Rightarrow \mathcal{P}_{\delta\phi} = (H/2\pi)^2$ and $\partial N/\partial\phi = V/V'$
  - First term of ζ dominates

$$\begin{aligned} \mathcal{P}(k) &= \frac{1}{2\epsilon_*}\left(\frac{H_*}{2\pi}\right)^2 \\ n - 1 &= 2\eta_* - 6\epsilon_* \end{aligned}$$

Non-gaussianity in the standard scenario

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  - non-linearity of ζ in terms of $\delta\phi$
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Comparison with Maldacena:

- He uses comoving slicing, computes $\mathcal{R} \to \zeta$ directly
- The δN approach instead uses two stages
  - Vacuum fluctuation converted to classical $\delta\phi$ (flat slicing)
  - Then δN gives ζ in terms of $\delta\phi$

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  - many curvaton candidates
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  (Dvali/Gruzinov/Zaldarriaga 2004, Kofman 2004, Bauer/Graesser/Salem 2005

Linear approximation; the spectrum

Linear in $\delta\phi_i$, NOT first-order cosmological perturbation theory

$$\zeta(\mathbf{x}, t) = \sum N_i(t)\, \delta\phi_i(\mathbf{x})$$
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$$\begin{aligned} \mathcal{P}_\zeta(k,t) &= \left(\frac{H_*}{2\pi}\right)^2 \left[\frac{1}{2\epsilon_*} + \sum N_{\sigma_i}^2(t)\right] \\ r &\equiv \mathcal{P}_{\text{tensor}}/\mathcal{P}_\zeta \leq 16\epsilon_* \qquad (\epsilon \equiv -\dot{H}/H^2) \end{aligned}$$

Starobinsky 1985, Sasaki & Stewart 1996

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- If one $\sigma_i \equiv \sigma$ dominates, $n - 1 = 2\eta_{\sigma\sigma} - 2\epsilon_*$.

Linear approximation: non-gaussianity

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- For slow-roll, correlator small (Seery & Lidsey 2005) leading to

$$\frac{3}{5} f_{\rm NL} = \frac{r}{32} f, \qquad 1 < f(k_1, k_2, k_3) < \frac{11}{6}$$

making $f_{\rm NL}$ unmeasurable (DHL & Zaballa 2005, Vernizzi & Wands 2006).

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- SL (06) same result for trispectrum
- For k- and ghost inflation, $f_{\rm NL}$ probably measurable.

Quadratic approximation

$$\zeta(\mathbf{x},t) = \sum N_i\, \delta\phi_i(\mathbf{x}) + \frac{1}{2}\sum N_{ij}\delta\phi_i\delta\phi_j$$

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$$\begin{aligned}
\mathcal{P}_\zeta &= \left(\frac{H_*}{2\pi}\right)^2 \sum N_i^2 + \ln(kL)\left(\frac{H_*}{2\pi}\right)^4 \mathrm{Tr}\,N^2 \\
\frac{3}{5}f_{\rm NL} &= \frac{\sum N_i N_j N_{ij}}{2(\sum N_i^2)^2} + \ln(kL)\mathcal{P}_\zeta \frac{\mathrm{Tr}\,N^3}{(\sum N_i^2)^3} \\
\tau_{\rm NL} &= 2\frac{N_i N_{ij} N_{jk} N_k}{(\sum N_i^2)^3} + \ln(kL)\mathcal{P}_\zeta \frac{\mathrm{Tr}\,N^4}{(\sum N_i^2)^4}
\end{aligned}$$

Infrared running

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If middle term negligible, non-gaussianity depends on the box size with $f_{\rm NL}$ and $\tau_{\rm NL} \propto \ln(kL)$.

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- Simplest case: $\frac{3}{5} f_{\rm NL} = -\frac{3}{4}$

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(ii) Two-component modular inflation Kadota & Stewart 03

$N \propto 1/\sigma$ gives negligible non-gaussianity DHL & Rodriguez 05

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4. Is Starobinsky's stochastic formalism an approximation to a non-perturbative version of Weinberg's analysis?