θ-dependence of the deconfinement temperature in Yang-Mills theories

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August 28 2012

New Frontiers in
Lattice Gauge Theories

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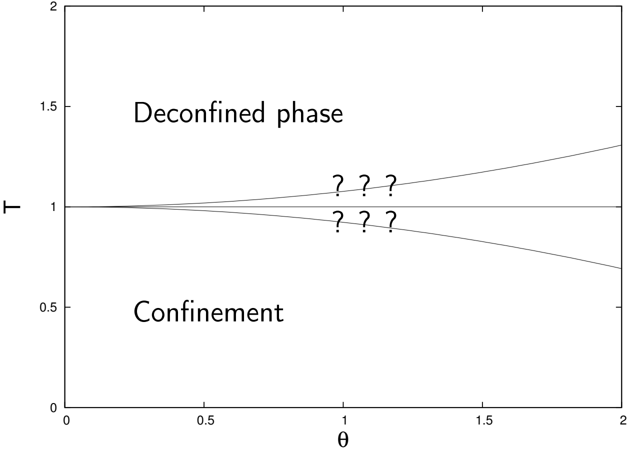
Based on: hep-lat/1205.0538v1

Outline

- 1) Introduction to the problem.
- 2) Topological θ-term and sign problem.
- 3) The lattice discretization.
- 4) Numerical results from LGT.
- 5) Large N_c estimate.
- 6) Conclusions.

1) Introduction.

SU(3) gauge theory phase diagram in the $T - \theta$ plane.



Does T_c depend on θ? Is it growing or decreasing?

1) Introduction.

Our aim:

1) Study if and how the deconfinement transition temperature depends on the topological θ-term.

$$\frac{T_c(\theta)}{T_c(0)} = 1 - R_\theta \theta^2 + O(\theta^4)$$

2) Perform a large-N_c estimation of this dependence.

3) Compare these calculations.

2) Topological θ-term and sign problem.

We consider the following continuum action in euclidean metric:

$$S = S_{YM} + S_\theta$$

The pure gauge term:

$$S_{YM} = -\frac{1}{4}\int d^4x\ F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)$$

and the topological θ-term:

$$S_\theta = -i\theta\frac{g_0^2}{64\pi^2}\int d^4x\ \epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x) \equiv -i\theta\int d^4x\ q(x) \equiv -i\theta Q[A]$$

2) Topological θ-term and sign problem.

LGT techniques are based on the possibility to interpret the partition function integrand

$$Z(T,\theta) = \int D[A] \; e^{-S_{YM}+i\theta Q[A]}$$

as a probability distribution for the fields A_μ^a.

But it is complex! Bad news... sign problem!

Anyhow LGT are preferred ways to probe the non-perturbative properties of YM theories.
Can we somehow re-arrange things so that we can apply LGT techniques to such a model?

2) Topological θ-term and sign problem.

Via an imaginary $\theta = i\theta_I$ term we can "solve" the sign problem.
[Azcoiti et al., PRL 2002; Alles and Papa, PRD 2008; Horsley et al., arxiv:0808.1428 [hep-lat]; Panagopoulos and Vicari, JHEP 2011]

Analyticity around $\theta = 0$ is supported by the current knowledge of the vacuum free energy derivatives with respect to θ evaluated at $\theta = 0$.
[Alles, D'Elia and Di Giacomo, PRD 2005; Vicari and Panagopoulos, Physics Reports 2008]
Studying the dependence on θ_I we will have access to a (small) range of real θ via analytic continuation.

The continuum partition function to be put on the lattice is:

$$Z(T,\theta) = \int D[A] \; e^{-S_{YM} - \theta_I Q[A]}$$

3) The lattice discretization.

The topological charge operator can be discretized as:

$$Q_L[U] = \frac{-1}{2^9\pi^2} \sum_{n}^{\text{Lattice}} \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \tilde{\epsilon}_{\mu\nu\rho\sigma} \mathrm{Tr}\left(\Pi_{\mu\nu}(n)\Pi_{\rho\sigma}(n)\right)$$

Using the Wilson action for S_{YM} the lattice partition function is:

$$\boxed{Z(T,\theta) = \int D[U] e^{-S^L_{YM}[U] - \theta_L Q_L[U]}}$$

Due to a finite multiplicative renormalization Q_L is related to the integer valued Q by :

$$Q_L = Z(\beta) Q + O(a^2)$$

[Campostrini, Di Giacomo and Panagopoulos, Phys Lett B 1988]

So the θ-term is also

$$S_\theta \equiv -\theta_L Q_L = -\theta_L Z(\beta) Q = -\theta_I Q$$

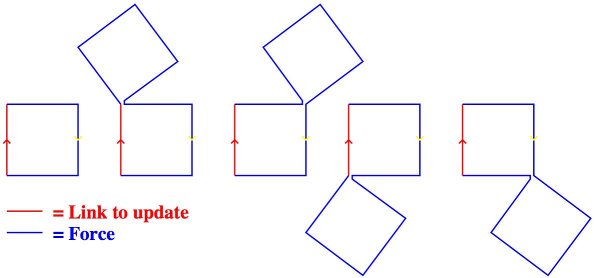
3) The lattice discretization.

In this simple action each link appears linearly.

$\Downarrow$

We can exploit standard Heatbath and Overrelaxation algorithms.

It is necessary to modify the staples definition. Pictorically:



With more complicated topological charge definitions on the lattice such standard algorithms wouldn't have been applicable.

4) Numerical results from LGT.

$\mathbb{Z}_3$ center symmetry holds also when we introduce the topological term in the action.

Deconfinement $\rightarrow$ spontaneous breaking of $\mathbb{Z}_3$ center symmetry.

Order parameter: Polyakov loop

$$L(\beta, \theta_L) = \langle L \rangle_{\beta,\theta_L} = \left\langle \frac{1}{V_s} \sum_{n_x, n_y, n_z} \mathrm{Tr} \left(\prod_{i=0}^{N_t - 1} U_t(n_x, n_y, n_z, i) \right) \right\rangle_{\beta,\theta_L}$$

At a fixed θ_L we find the transition in correspondence of the susceptibility peak:

$$\chi_L(\beta, \theta_L) = V_s \left(\langle L^2 \rangle_{\beta,\theta_L} - \langle L \rangle^2_{\beta,\theta_L} \right)$$

1) $Z(\beta)$ in order to determine $\theta_I = Z(\beta)\theta_L$.

Compute Q_L via the operator previously defined.
Compute Q via *cooling* algorithm.
Evaluate:

$$Z(\beta) = \frac{\langle Q_L Q \rangle_\beta}{\langle Q^2 \rangle_\beta}$$

as proposed in [Panagopoulos and Vicari, JHEP 2011]

Simulations were performed on a symmetric 16^4 lattice for 8 values of β spanning in $5.7 - 6.3$.
The results were checked for some β on a symmetric 24^4 lattice.

4) Numerical results from LGT: ingredients for R_θ.

2) $\beta_c(\theta_I)$ in order to measure $T_c(\theta_I)/T_c(0)$.

For various θ_L we search β_c via a Lorentzian fit.
Using the non-perturbative determination of $a(\beta)$ in [Boyd et al., Nucl Phys B 1996] we have:

$$\frac{T_c(\theta_I)}{T_c(0)} = \frac{a(\beta_c(\theta = 0))}{a(\beta_c(\theta_I))}$$

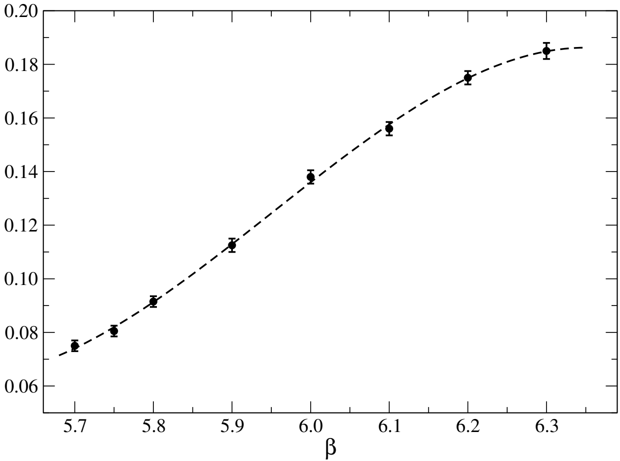
Where $\theta_I = Z(\beta_c)\theta_L$.
Simulations have been performed for various lattice spacings in order to approach the continuum limit.

We choose $a \simeq 1/(4T_c(0))$, $a \simeq 1/(6T_c(0))$ and $a \simeq 1/(8T_c(0))$.

The lattices we have used are $16^3 \times 4$, $24^3 \times 6$ and $32^3 \times 8$.

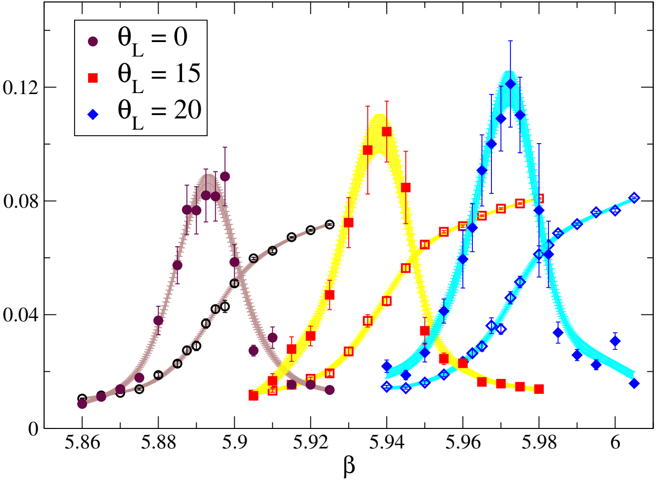
4) Numerical results from LGT: $Z(\beta)$.

Simulation on 16^4 lattice and polinomial cubic interpolation.



4) Numerical results from LGT: $\beta_c(\theta_l)$.

Determination of β_c e.g. on the $24^3 \times 6$ lattice.
L and χ_L data and β-reweighting analysis.



Weak increase in χ_L peak.

$\downarrow$

Stronger transition?

4) Numerical results from LGT: $\beta_c(\theta_I)$.

lattice	θ_L	β_c	θ_I	$T_c(\theta_I)/T_c(0)$
$16^3 \times 4$	0	5.6911(4)	0	1
$16^3 \times 4$	5	5.6934(6)	0.370(10)	1.0049(11)
$16^3 \times 4$	10	5.6990(7)	0.747(15)	1.0171(12)
$16^3 \times 4$	15	5.7092(7)	1.141(20)	1.0395(11)
$16^3 \times 4$	20	5.7248(6)	1.566(30)	1.0746(10)
$16^3 \times 4$	25	5.7447(7)	2.035(30)	1.1209(10)
$24^3 \times 6$	0	5.8929(8)	0	1
$24^3 \times 6$	5	5.8985(10)	0.5705(60)	1.0105(24)
$24^3 \times 6$	10	5.9105(5)	1.168(12)	1.0335(18)
$24^3 \times 6$	15	5.9364(8)	1.836(18)	1.0834(23)
$24^3 \times 6$	20	5.9717(8)	2.600(24)	1.1534(24)
$32^3 \times 8$	0	6.0622(6)	0	1
$32^3 \times 8$	5	6.0684(3)	0.753(8)	1.0100(11)
$32^3 \times 8$	8	6.0813(6)	1.224(15)	1.0312(14)
$32^3 \times 8$	10	6.0935(11)	1.551(20)	1.0515(21)
$32^3 \times 8$	12	6.1059(21)	1.890(24)	1.0719(34)
$32^3 \times 8$	15	6.1332(7)	2.437(30)	1.1201(17)

θ_I values spanning in $[0; 2.5]$

Typical statistics for each size and for each θ_L:

$\sim 10^5 - 10^6$

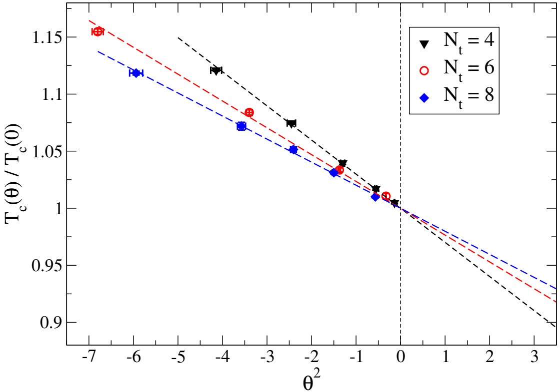
4) Numerical results from LGT: R_θ.

We find:

$R_\theta^{N_t=4} = 0.0299(7)$
$\chi^2/d.o.f. \sim 0.3$

$R_\theta^{N_t=6} = 0.0235(5)$
$\chi^2/d.o.f. \sim 1.6$

$R_\theta^{N_t=8} = 0.0204(5)$
$\chi^2/d.o.f. \sim 0.7$



T_c increases for imaginary coupling then, by analytic continuation, it decreases for real θ.

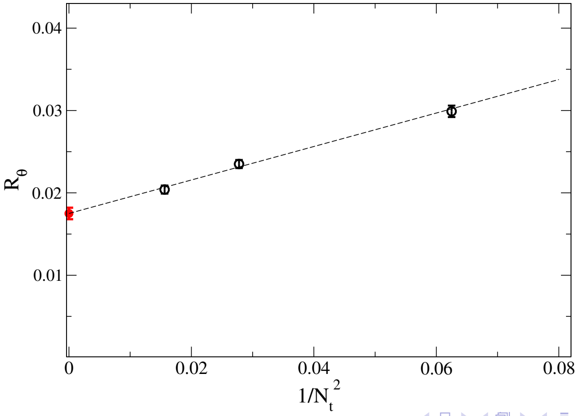
4) Numerical results from LGT: continuum extrapolation.

Assuming quadratic finite lattice spacing corrections to R_θ:

$$R_\theta^{N_t} = R_\theta^{\text{cont}} + c/N_t^2$$

we can extrapolate to the continuum limit to get

$$R_\theta^{\text{cont}} = 0.0175(7) \text{ with } \chi^2/d.o.f. \sim 1$$



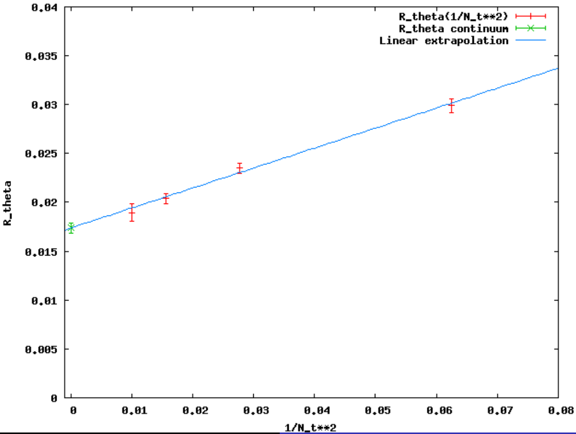
4) Numerical results from LGT: continuum extrapolation.

Preliminary finer lattice spacing: for $N_t = 10$ we found

$$R_\theta^{N_t=10} = 0.0190(9)$$

Extrapolation towards the continuum limit leads to:

$$R_\theta^{\text{cont}} = 0.0174(5) \text{ with } \chi^2/d.o.f. \sim 0.7$$



5) Large N_c estimate.

$\boxed{1^{st}\text{-order transition}} \longrightarrow$

2 phases with different free energy densities crossing at T_c.
$f_c(T_c) = f_d(T_c)$
$f'_c(T_c) \neq f'_d(T_c)$

Close to T_c and using $t = (T - T_c)/T_c$ the free energies are:

$$\frac{f_c(t)}{T} = A_c t + O(t^2) \qquad \frac{f_d(t)}{T} = A_d t + O(t^2)$$

From the usual relations:

$$Z = e^{-\frac{V_s f(T)}{T}} \qquad \epsilon(T) = \frac{T^2}{V_s}\partial_T \log Z$$

we easily find that the slope difference is related to the latent heat

$$\Delta\epsilon = \epsilon_d(T_c) - \epsilon_c(T_c) = T_c(A_c - A_d)$$

5) Large N_c estimate.

When we have $\theta \neq 0$ the free energy density is modified by

$$f(T,\theta) = f(T,\theta=0) + \frac{\chi(T)\theta^2}{2} + O(\theta^4)$$

In the large N_c limit $\chi(T)$ is a step function:

$$\chi(T < T_c) = \chi(T=0) \equiv \chi \neq 0 \qquad \chi(T > T_c) = 0$$

[Alles, D'Elia and Di Giacomo, Phys Lett B '96-'97-'00; Del Debbio, Vicari and Panagopoulos, JHEP 2004; Lucini, Teper and Wenger, Nucl Phys B 2005]

This modifies the free energies in:

$$\frac{f_c(t)}{T} = A_c t + \frac{\chi\theta^2}{2T} \qquad \frac{f_d(t)}{T} = A_d t$$

T_c is found when $f_c = f_d \longrightarrow \frac{T_c(\theta)}{T_c(0)} = 1 - \frac{\chi}{2\Delta\epsilon}\theta^2$

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T_c is found when $f_c = f_d \longrightarrow \frac{T_c(\theta)}{T_c(0)} = 1 - R_\theta^{large\ N_c}\theta^2$

5) Large N_c estimate.

From the large N_c estimates in [Lucini, Teper and Wenger, JHEP 2005]:

$$\frac{\chi}{\sigma^2} = 0.0221(14) \qquad \frac{\Delta\epsilon}{N_c^2 T_c^4} = 0.344(72) \qquad \frac{T_c}{\sqrt{\sigma}} = 0.5978(38)$$

we can evaluate $R_\theta^{large\ N_c}$:

$$R_\theta^{large\ N_c} = \frac{\chi}{2\Delta\epsilon} = \frac{0.253(56)}{N_c^2} + O(\frac{1}{N_c^4})$$

The argument in [Witten, PRL 1998] supports this dependence on N_c.
Large-N_c limit $\rightarrow$ expansion variable $\frac{\theta}{N_c} \rightarrow R_\theta \theta^2 \rightarrow R_\theta \propto \frac{1}{N_c^2}$
Let's recall both our results and compare them in the case $N_c = 3$.

$$\boxed{R_\theta^{\text{cont}} = 0.0175(7) \qquad R_\theta^{large\ N_c}(N_c = 3) = 0.0281(62)}$$

A possible different and complementary approach to the problem is to perform reweighting analysis for real θ starting from $\theta = 0$:

$$\langle O \rangle_\theta = \frac{\int DU\, e^{-S_W + i\theta Q} O}{\int DU\, e^{-S_W + i\theta Q}} = \frac{\int DU\, e^{-S_W + i\theta Q} O}{\int DU\, e^{-S_W + i\theta Q}} \cdot \frac{\int DU\, e^{-S_W}}{\int DU\, e^{-S_W}} =$$

$$= \frac{\int DU\, e^{-S_W + i\theta Q} O}{\int DU\, e^{-S_W}} \cdot \frac{\int DU\, e^{-S_W}}{\int DU\, e^{-S_W + i\theta Q}} = \frac{\langle O e^{i\theta Q} \rangle_{\theta=0}}{\langle e^{i\theta Q} \rangle_{\theta=0}}$$

This approach allows us to explore a small range of real θ like the analytic continuation method.

Preliminary tests have shown a good agreement between the two methods for the coarsest lattice spacing ($N_t = 4$).

6^-) Correlation between Q and L.

If we imagine to reweight in θ for the Polyakov loop L

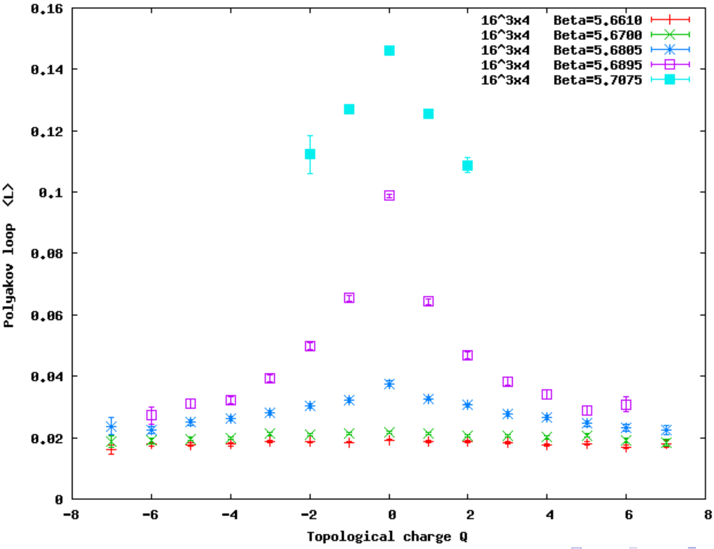
$$\langle L \rangle_\theta = \frac{\langle L e^{i\theta Q} \rangle_{\theta=0}}{\langle e^{i\theta Q} \rangle_{\theta=0}} = \frac{\sum_Q L(Q) p(Q) e^{i\theta Q}}{\sum_Q p(Q) e^{i\theta Q}}$$

we realize that $\langle L \rangle_\theta$ can depend on θ only if L depends on Q: actually θ and Q are conjugated variables.

We expect L to depend on Q: true in the coarsest lattice ($N_t = 4$) if away from the thermodynamic limit (e.g. $N_s = 16$).

Study the transition in different sectors to look for possible T_c dependence on Q.

6^-) Correlation between Q and L.



6) Conclusions

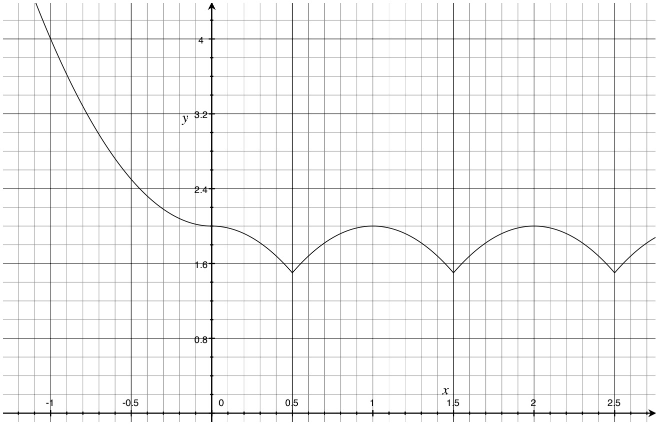
- Use of imaginary θ_I parameter to cure sign problem for LGT.
- Deconfinement transition temperature dependence on θ_I.
- Determination of the quadratic coefficient R_θ^{cont}.
- Large N_c estimate and comparison.

Perspectives:

- Finer lattice spacings to improve continuum limit approach. ✓
- Weaker transition? Finite size scaling study.
- Extend the analysis to $SU(2)$ and $SU(4)$.
- Reweighting for real θ starting from $\theta = 0$. ✓

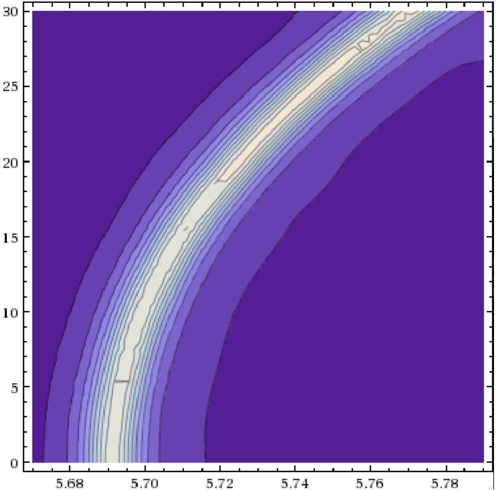
7) Backup: conjectured phase diagram.

At least in the large N_c limit when only $O((\theta/N_c)^2)$ terms are relevant we can suppose the phase diagram to show 2π-periodicity and cusps in $\theta = (2k+1)\pi$.



7) Backup: move along $\theta_I = \text{const}$

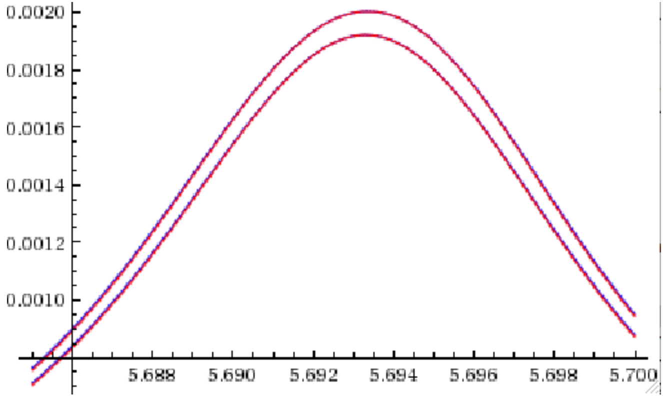
Reweighting analysis on all $16^3 \times 4$ data.



We obtain a $3D$ plot for the polyakov loop suceptibility:

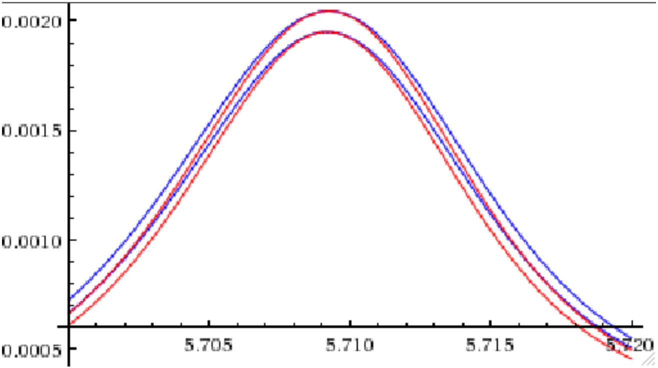
7) Backup: move along $\theta_I = \text{const}$

Moving along constant θ_I instead of constant θ_L.
For $\theta_I \simeq 0.37$ and $\theta_L = 5.0$



7) Backup: move along $\theta_I = \text{const}$

Moving along constant θ_I instead of constant θ_L.
For $\theta_I \simeq 1.14$ and $\theta_L = 15.0$



Moving along constant θ_I instead of constant θ_L.
For $\theta_I \simeq 2.04$ and $\theta_L = 25.0$