Tricritical points in field theory and statistical mechanics: from Potts models to finite density QCD

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Outline

Tricritical points: general results

3D three states Potts model in external field

(2+1)D three states Potts

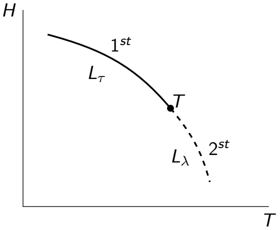
(3+1)D U(1) gauge theory

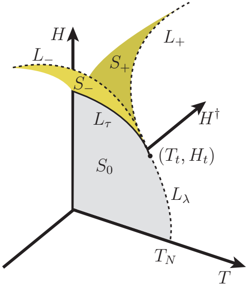
Imaginary chemical potential QCD

Conclusions

Tricritical point

- The point in which a line of first order transition becomes a line of second order transition.
  To have a line of 2^{nd} order transitions we must have two relevant variables and a marginal one. At the tricritical point the third variable becomes relevant.
- The point at which three-phase coexistence terminates in an extended parameter space.





Tricritical points in Landau theory

In the simplest case in which the order parameter is a scalar the free energy near a phase transition (small $|\eta|$) can be parametrized as

$$\mathcal{F} = \frac{1}{2}(\nabla\eta)^2 + \frac{\mu^2}{2}\eta^2 + \lambda\eta^4 + \kappa\eta^6$$

where $\kappa > 0$ to ensure stability.

- $\lambda > 0$, $\mu^2 = 0$ second order phase transition
- $\lambda < 0$, $\mu^2 = \frac{\lambda^2}{2\kappa}$ first order phase transition
- $\lambda = 0$, $\mu^2 = 0$ tricritical point

The upper critical dimension for tricritical points is 3 and the classical critical indices are (up to logarithmic corrections)

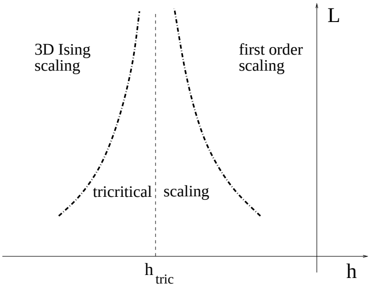
$$\alpha = \frac{1}{2} \quad \beta = \frac{1}{4} \quad \gamma = 1 \quad \delta = 4 \quad \nu = \frac{1}{2} \quad \eta = 0$$

see e.g. Landau & Lifshitz "Statistical Physics" §150

Scaling near a tricritical point

A tricritical point is an isolated point on a line of first/second order transitions. Where can we see tricritical scaling?

In a finite system of size L we have tricritical scaling in a neighborhood of h_{tric}, with the size of the neighborhood going to zero as $L \to \infty$. It can be shown that $L_c \propto |h - h_{tric}|^{-1}$ in the simplest case.



C. B., M. D'Elia Phys. Rev. D **82**, 114515 (2010).

The 3D three state Potts model in external field

The energy is

$$H = -\beta \sum_{\langle i,j \rangle} \delta(s_i, s_j) - h \sum_i \delta(s_i, s_h)$$

where $s_i \in \mathbb{N}$, $1 \leq s_i \leq 3$ and s_h is the external field direction. For $h = 0$ the transition is first order.

First order transitions are stable $\Rightarrow$ $h \approx 0$ is first order too.

For $h \to +\infty$ all spins are completely polarized along s_h, no residual symmetry. A critical endpoint is expected for $h > 0$.

F. Karsch, S. Stickan Phys. Lett. B **488**, 319 (2000).

For $h \to -\infty$ no spin is directed along s_h and the system becomes a 3D Ising model. A tricritical point is expected for $h < 0$.

C. B., M. D'Elia Phys. Rev. D **82**, 114515 (2010).

How to search for tricritical points

Possible strategies:

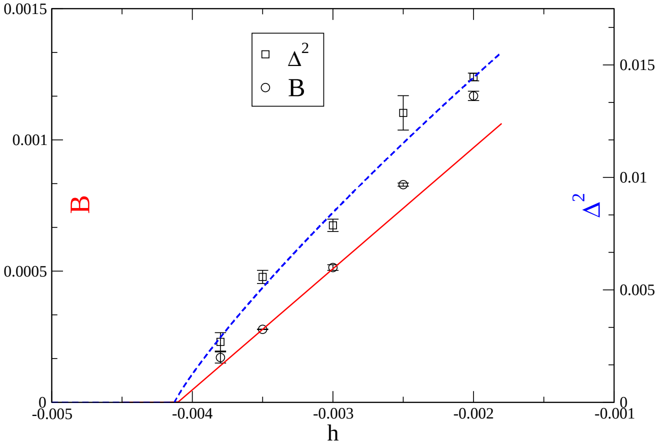
- estimate the discontinuities of the first order side and look for the point where the discontinuities vanish
- use RG invariant observables and look for crossing

Observables for the different approaches:

- susceptibilities (energy, order parameter), Binder-Challa-Landau cumulant of energy
- correlation length, Binder cumulant of the order parameter

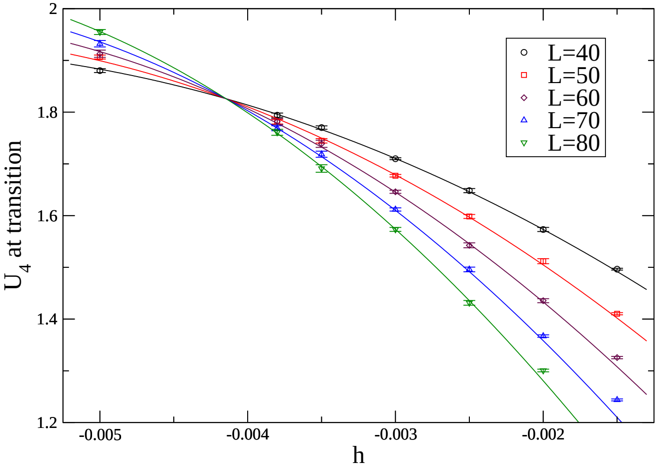
	ν	γ	α	γ/ν	α/ν
$3D$ Ising	0.6301(4)	1.2372(5)	0.110(1)	~ 1.963	~ 0.175
Tricritical	$1/2$	1	$1/2$	2	1
1^{st} Order	$1/3$	1	1	3	3

Vanishing of the gaps



$$\chi_{max} \sim \text{const} + \frac{\Delta^2}{4} L^3 \qquad B = \frac{2}{3} - B_4|_{min} = \frac{1}{3}\left(\frac{\Delta_E}{E}\right)^2$$

Crossing of the Binder cumulant



$$U_4 = \frac{\langle(\delta M)^4\rangle}{\langle(\delta M)^2\rangle^2}$$

(2+1)D three states Potts

No external field but one dimension is compactified and the lattice extent along this dimension is N_t.

$N_t = +\infty$ is the $3D$ model, first order transition. As far as the correlation length at the transition is $\lesssim N_t$ we expect first order.

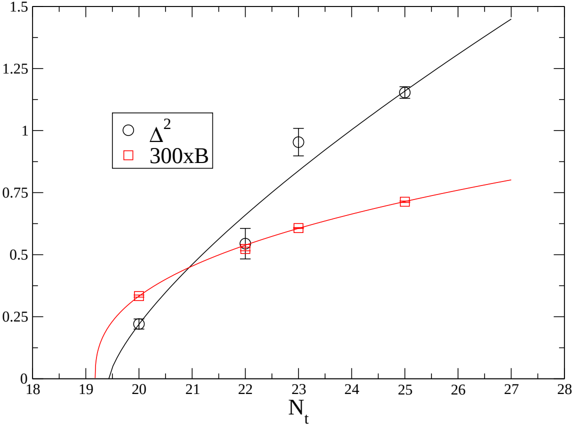
$N_t = 1$ is the $2D$ model, second order transition.

We expect a change in the order of the transition by varying the value of N_t.

P. de Forcrand, M. Fromm thesis

(2+1)D three states Potts

C. B., M. D'Elia in preparation



	ν	γ	α	γ/ν	α/ν
$2D\ Z_3$	5/6	13/9	1/3	26/15	6/15
Z_3 Tricritical	7/12	19/18	5/6	38/21	10/7
1^{st} Order	1/2	1	1	2	2

(3+1)D $U(1)$ gauge theory

$N_t = \infty$ is $4D$ $U(1)$ gauge theory, whose transition is first order and we expect first order also for large N_t. As far as the correlation length at the transition is $\lesssim N_t$ we expect first order.

$N_t = 1$ is $3D$ $U(1)$, whose transition is second order.

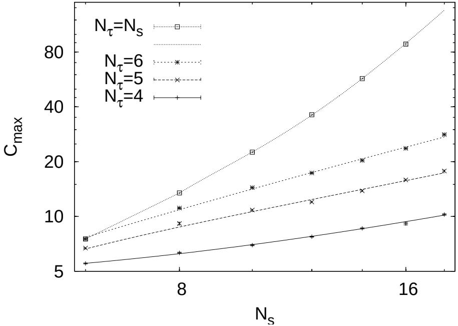
We expect a change in the order of the transition by varying the value of N_t.

	ν	γ	α	γ/ν	α/ν
$3D$ XY	0.67155(27)	1.3177(5)	-0.0146(8)	$\sim$ 1.962	$\sim$ -0.022
Tricritical	1/2	1	1/2	2	1
1^{st} Order	1/3	1	1	3	3

(3+1)D $U(1)$ gauge theory, previously confusing results

A high precision study of (3+1)D $U(1)$ gauge theory was performed in 2006 for temporal extent $N_t = 4, 5, 6$ and spatial $N_s \lesssim 18$ with the result that

"The exponents are consistent with 3D Gaussian values, but not with either first order transitions or the universality class of the 3D XY model."



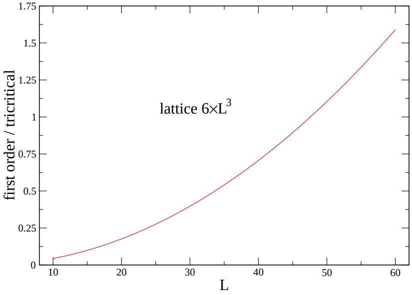
B. A. Berg, A. Bazavov Phys. Rev. D **74**, 094502 (2006).

First order behaviour was observed before on a lattice 6×48^3 but with very low statistics.

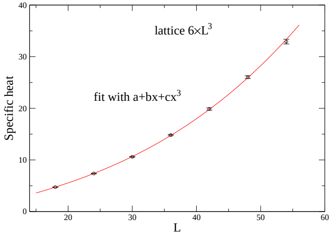
M. Vettorazzo, P. de Forcrand Phys. Lett. B **604**, 82 (2004).

Explanation of the previous results

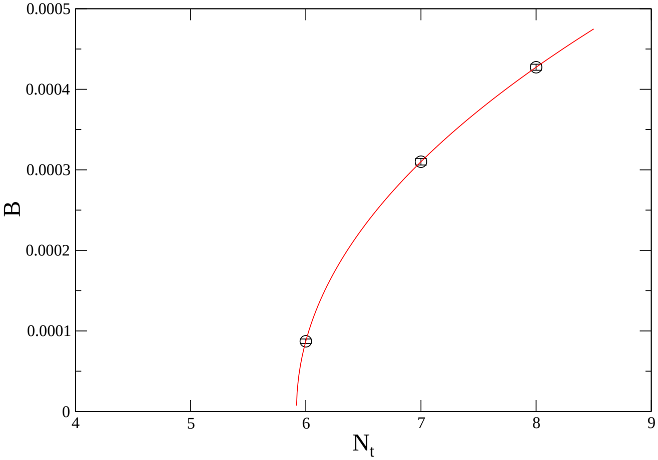
For $N_t = 6$ the transition is in fact first order, but very large lattices have to be used in order to see first order scaling since we are near the tricritical point.



$b = 0.2778(82)$
$c = 0.000123(3)$



The location of the tricritical point



C. B., M. D'Elia in preparation.

The Roberge-Weiss transition endpoint

We consider QCD at finite density with imaginary quark chemical potential (no sign problem)

$$\frac{1}{3}\mu_B = \mu_q \equiv i\mu_I \qquad \theta = \mu_I/T$$

It can be shown that $Z(T, \mu_I)$ is a periodic function of θ with period $2\pi/3$ ($3 = N_c$) and that at $\theta = (2k+1)\pi/N_c$ an exact Z_2 symmetry is present.

At low temperature this Z_2 symmetry is realized *à la* Wigner, while in the high temperature region it is spontaneously broken.

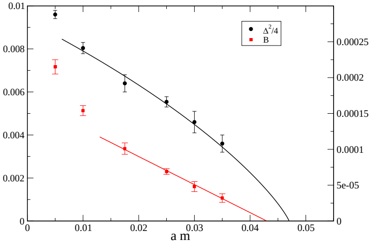
It was proposed that the structure of phase diagram of zero density QCD is determined by the RW endpoints.

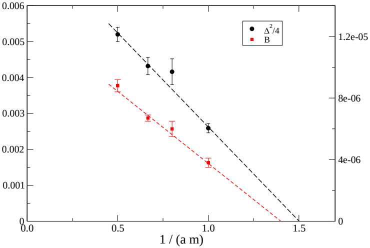
M. D'Elia, F. Sanfilippo Phys. Rev. D **80**, 111501(R) (2009).

P. de Forcrand, O. Philipsen Phys. Rev. Lett. **105**, 152001 (2010).

The order of the RW endpoint for $N_f = 2$

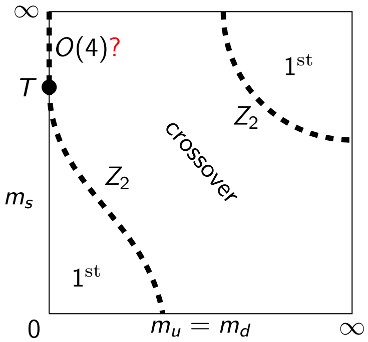
The transition is definitely first order for low and large quark masses but gets weaker at intermediate masses.
Since there is a change of symmetry there must be a transition for all mass values. Are first orders becoming weaker or are they turning second orders?





Two tricritical points, one at low and one at high mass.
C. B., G. Cossu, M. D'Elia, F. Sanfilippo Phys. Rev. D **83**, 054505 (2011).

The "accepted" QCD phase diagram

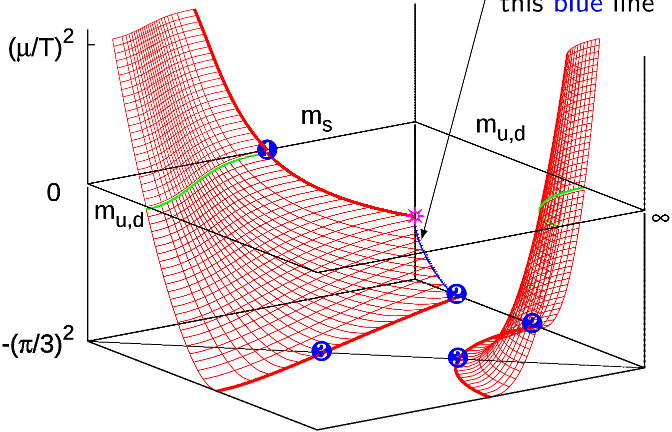


The first order regions shrink as a chemical potential is turned on.

P. de Forcrand, O. Philipsen Nucl. Phys. B **673**, 170 (2003).

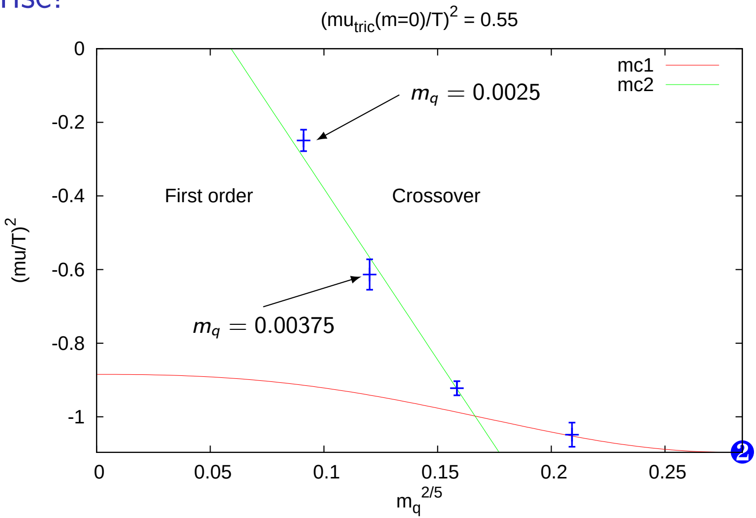
The following results are obtained by using simple staggered fermions with $N_t = 4$.

The extended “accepted” QCD phase diagram



We will look for this blue line

Surprise!



C. B, P. de Forcrand, M. D'Elia, O. Philipsen, F. Sanfilippo

work in progress

Not really a surprise . . .

previous studies supporting this picture

M. D'Elia, A. Di Giacomo, C. Pica Phys.Rev. D **72** 114510 (2005)
It is shown that $O(4)$ critical indices *are not* compatible with the chiral transition for $N_f = 2$

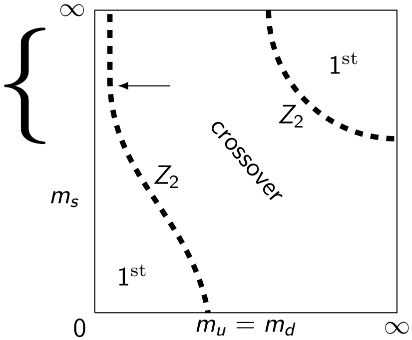
G. Cossu, M. D'Elia, A. Di Giacomo, C. Pica arXiv:0706.4470
It is shown that first order critical indices *are* compatible with the chiral transition for $N_f = 2$

"Personal" QCD phase diagram

In this region it can be misleading to look at $\bar{\psi}\psi$: we are near the tricritical line and

$$\left.\frac{\gamma}{\nu}\right|_{O(4)} \sim 1.977$$

$$\left.\frac{\gamma}{\nu}\right|_{tric} = 2$$



Conclusions

- tricritical points appear in very different physical systems
- the existence of tricritical points can explain some puzzling results
- the zero density QCD phase diagram can still have some surprise

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Thank you