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WALKING VS. CONFORMAL — RESULTS FROM THE SCHRÖDINGER FUNCTIONAL METHOD

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SU(2,3,4) gauge theories with $N_f = 2$ fermions in the $\mathbf{SYM}_2$ rep

1. Confining or conformal? And what lies in between
2. The running coupling at $m = 0$: Schrödinger Functional ($=$ background field method)
3. Phase diagrams on a finite lattice (m, "T" $\neq 0$)
4. Mass anomalous dimension $\gamma(g^2)$

POSSIBILITIES for IR PHYSICS

- Confinement & χSB $\Longrightarrow$ RUNNING [QCD]
  - or WALKING [ETC — extended technicolor]
- IRFP — conformal theory $\Longrightarrow$ STANDING STILL [unparticles?]

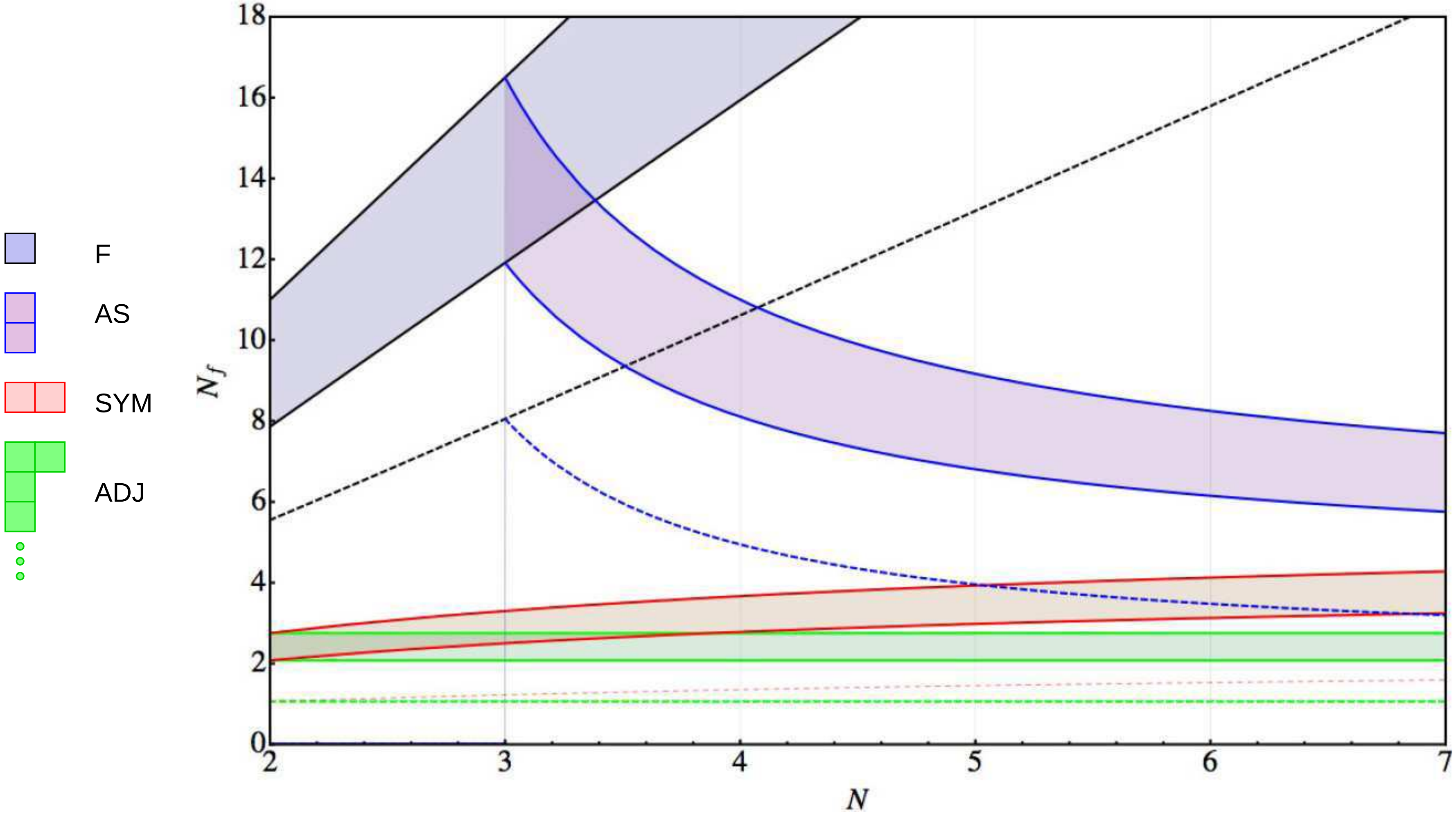
WALKING and IRFP [the *conformal window*] are HARD CASES:

- Running is slow — so strong coupling in IR is also strong coupling in UV (i.e., at lattice cutoff)
  i.e., we require $L >>>>>> a$ for a weak-coupling continuum limit
  OTHERWISE you are looking at a narrow range of scales!
- Scale invariance (approximate for WALKING) means all particle masses $\sim m_q^{1/y_m}$ with the same y_m. Hard to tell the two apart.
- Gauge coupling is irrelevant; m_q and $1/L$ are relevant couplings.
  $m_q \to 0$: really, really BAD finite-size effects.

Schrödinger functional turns finite volume from a *hindrance* to a *method*.

GAUGE GROUPS, REPs, and N_f

(Dietrich & Sannino, PRD 2007)



Our work: $N = 2, 3, 4$; REP=SYM=$\mathbf{3}, \mathbf{6}, \mathbf{10}$; $N_f = 2$ Is there an IRFP? Ladder approx says NO

THE β FUNCTION in the MASSLESS THEORY: the Schrödinger Functional

Continuum SF definition of $g(L)$: (Lüscher *et al.*, ALPHA collaboration)

- Hypercubical Euclidean box, volume L^4, massless limit
- Fix the gauge field on the two time boundaries
  $\Rightarrow$ background field — unique classical minimum of $S_{YM}^{cl} = \int d^4x\, F_{\mu\nu}^2$. Make sure L is the only scale.
- Calculate (if you can)

$$\begin{aligned}
\Gamma \equiv -\log Z &= \text{tree-level} + \text{one-loop} + \cdots \\
&= \left(\frac{1}{g^2(1/\mu)} + \frac{b_1}{32\pi^2} \log(\mu L) + \cdots \right) S_{YM}^{cl} \\
&\equiv \frac{1}{g^2(L)} S_{YM}^{cl} \qquad \text{nonperturbatively!}
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LATTICE THEORY:

- Wilson fermions
  $+$ clover term $+$ fat links (*nHYP* $=$ *normalized HYPercubic*)
- SF: fix spatial links U_i on time boundaries $t = 0, L$
  $+$ give fermions a spatial twist

A PROPOS CHIRAL SYMMETRY:

- Define m_q via AWI

$$\partial_\mu A^{a\mu} = 2m_q P^a \quad \Longrightarrow \quad m_q \equiv \frac{1}{2} \frac{\partial_4 \left\langle A_4^b(t)\ \mathcal{O}^b(t'=0,\vec{p}=0)\right\rangle}{\left\langle P^b(t)\ \mathcal{O}^b(t'=0,\vec{p}=0)\right\rangle}\Bigg|_{t=L/2}$$

- Find $\kappa_c(\beta)$ by setting $m_q = 0$. Work directly at κ_c: stabilized by SF BC's!

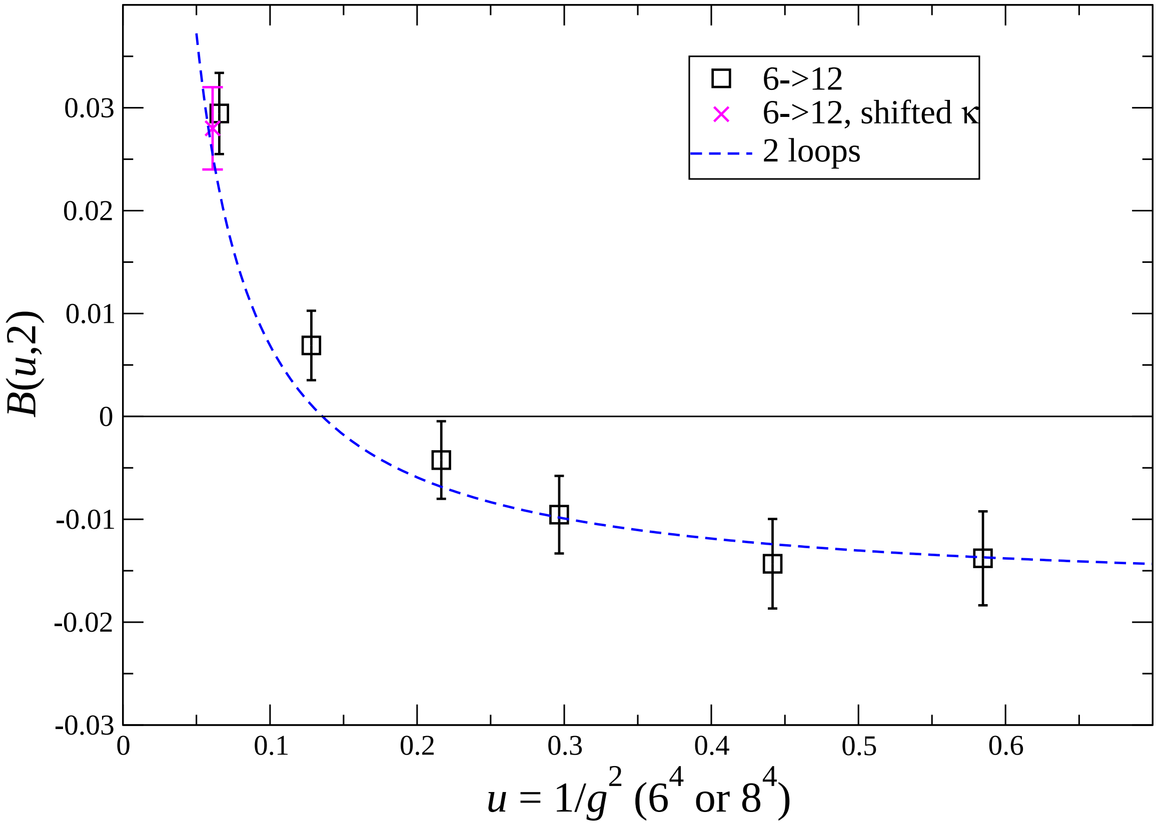
EXTRACTING PHYSICS

1. Fix lattice size L, bare couplings $\beta = 6/g_0^2$, $\kappa \equiv (8 + 2m_0 a)^{-1} = \kappa_c(\beta)$
2. Calculate $1/g^2(L)$ and $1/g^2(2L)$. Use common lattice spacing ($=$ UV cutoff) a.
3. Result: Discrete Beta Function

$$B(u,2) = \frac{1}{g^2(2L)} - \frac{1}{g^2(L)}\,,$$

a function of $u \equiv 1/g^2(L)$.

The DISCRETE BETA FUNCTION — SU(2)/triplet



$6^4 \longrightarrow 12^4$

S L O W running . . .

$B(u,2)$ crosses zero near the BZ coupling

$\Longrightarrow$ IRFP

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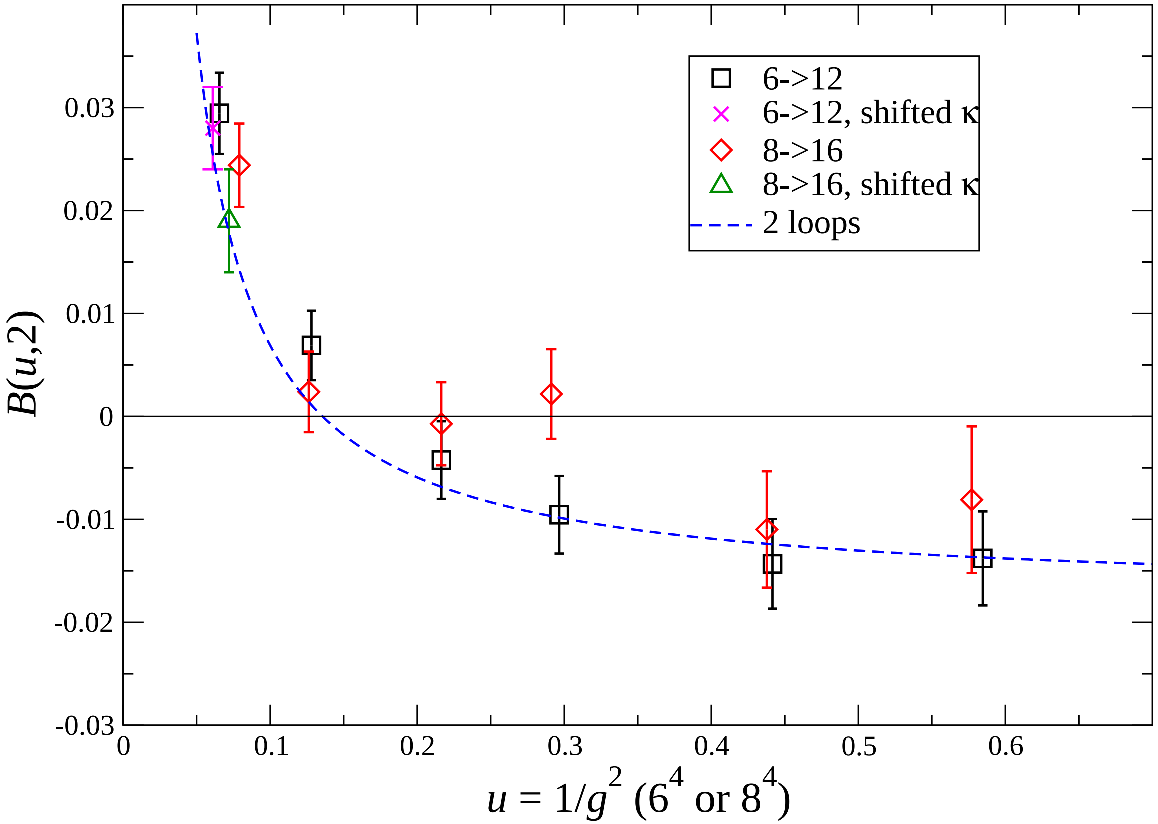

$6^4 \longrightarrow 12^4$

$8^4 \longrightarrow 16^4$

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SLOW RUNNING IS ALMOST *NO* RUNNING

Let $u(s) \equiv 1/g^2(s)$, and $\tilde{\beta}(u) \equiv du/d\log s = 2\beta(g^2)/g^4$. [We have been plotting $B(u,2) = u(2) - u(1)$.]
Slow running: $\tilde{\beta}(u(s)) \simeq \tilde{\beta}(u(1))$ — *quasi-conformal!*

Then

$$\frac{u(s) - u(1)}{\log s} \simeq \tilde{\beta}(u(1))$$

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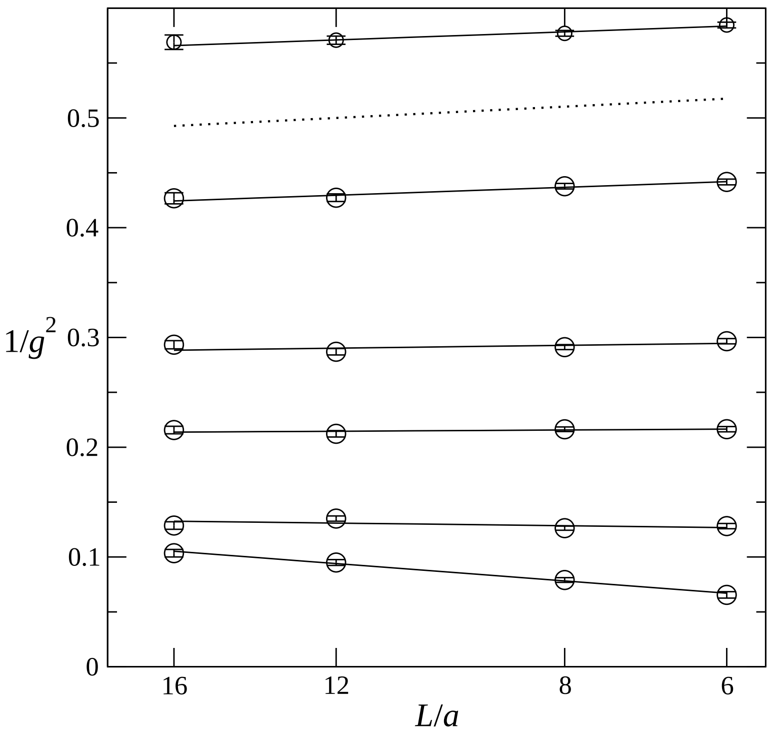
$\Longrightarrow$ linear fit to $1/g^2(\log L)$

(improved action is crucial)



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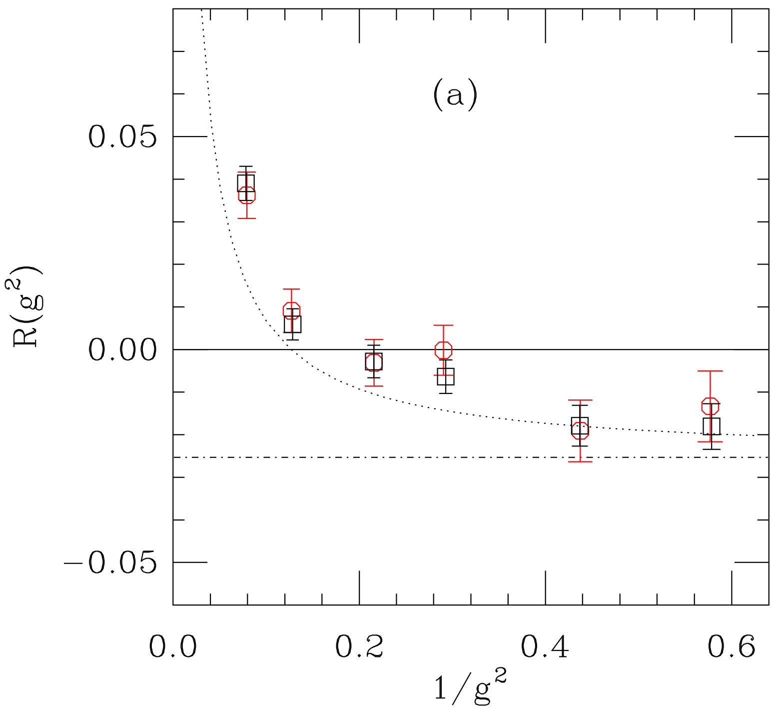
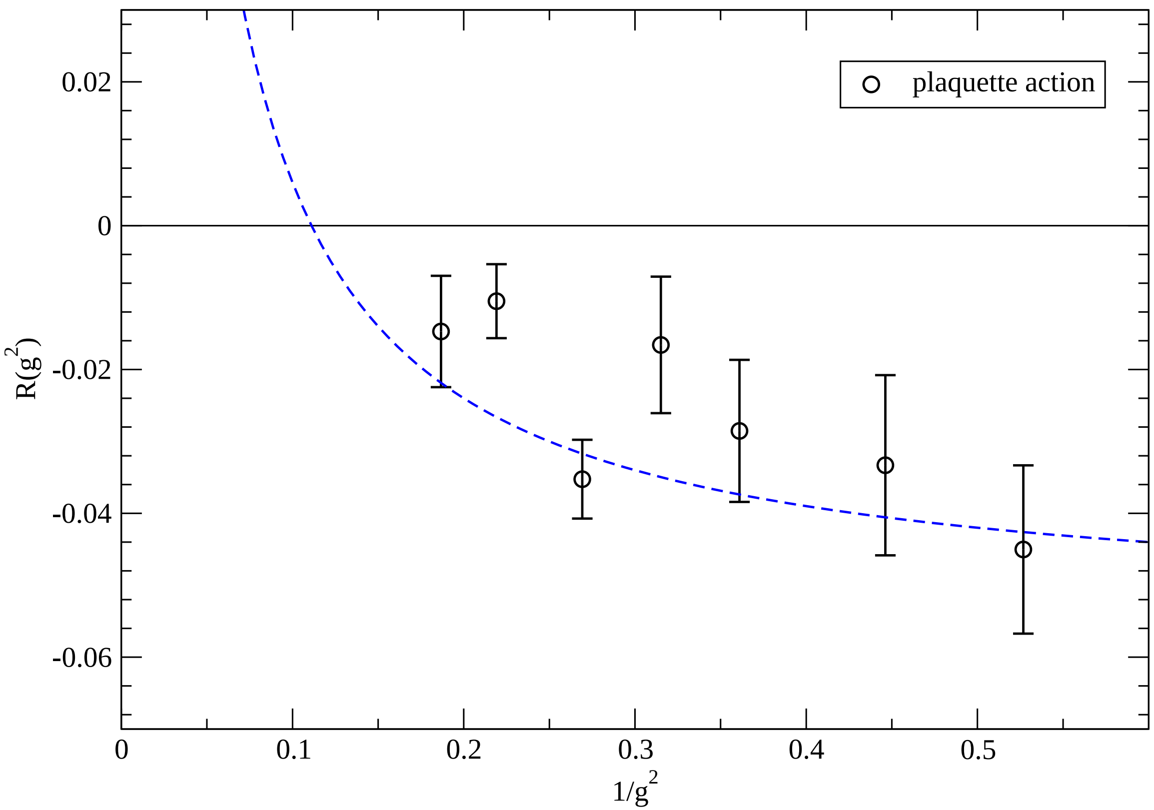
$$\frac{u(s) - u(1)}{\log s} \simeq \tilde\beta(u(1))$$

$\Longrightarrow$ collapse data for different s.

$\Rightarrow$ Reduced DBF $R(g^2) \simeq \tilde\beta(g^2)$



NOW FOR SU(3)/sextet



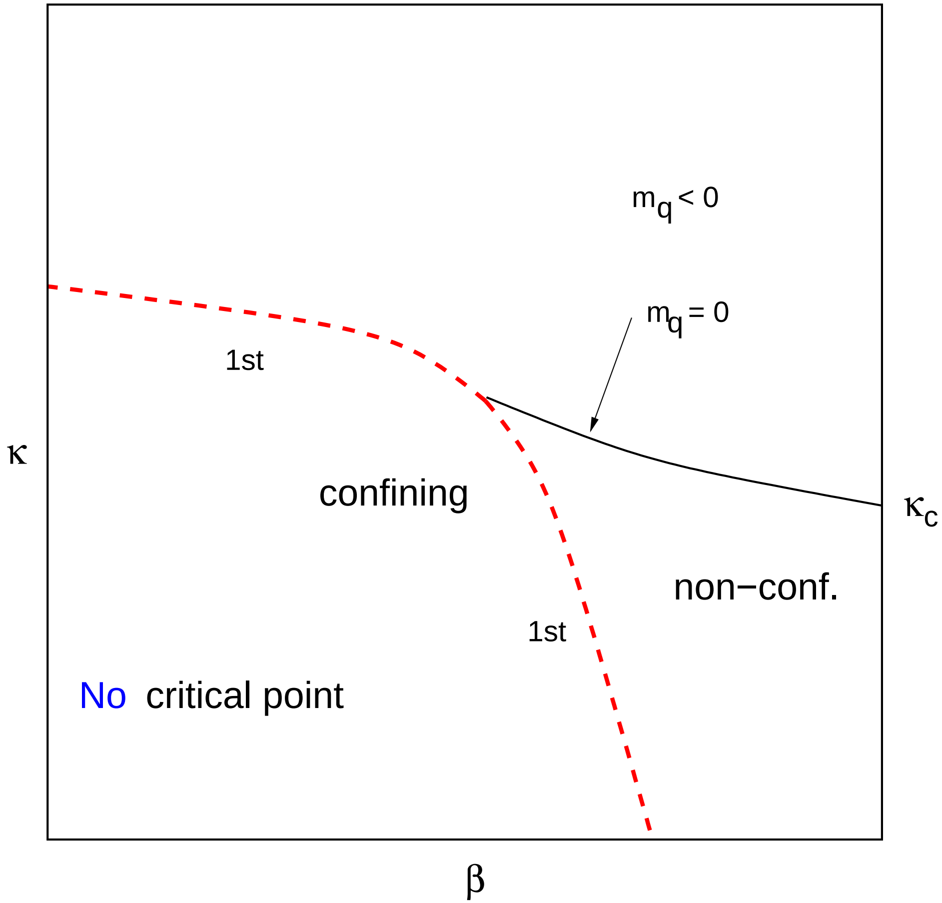
Fits from $L = 6, 8, 12, 16$

S L O W running ...

but does it cross zero?

Why did we stop?

PHASE DIAGRAM: (SU(3)/sextet)



THE WALL

in strong coupling:
m_q discontinuous in κ, never zero

cf. SU(3) with large N_f fund rep
(Iwasaki, Kanaya, Kaya, Sakai, and Yoshie 1992, 2003)

[cf. SU(2)/triplet: critical point at intersection]

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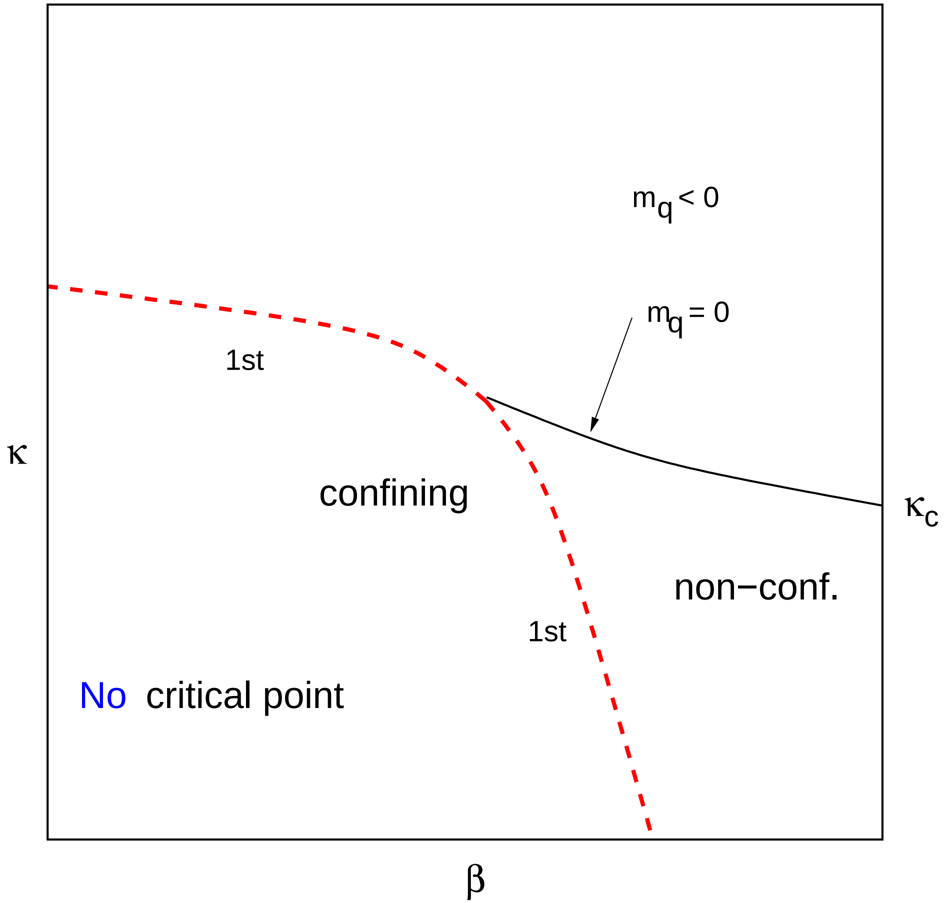
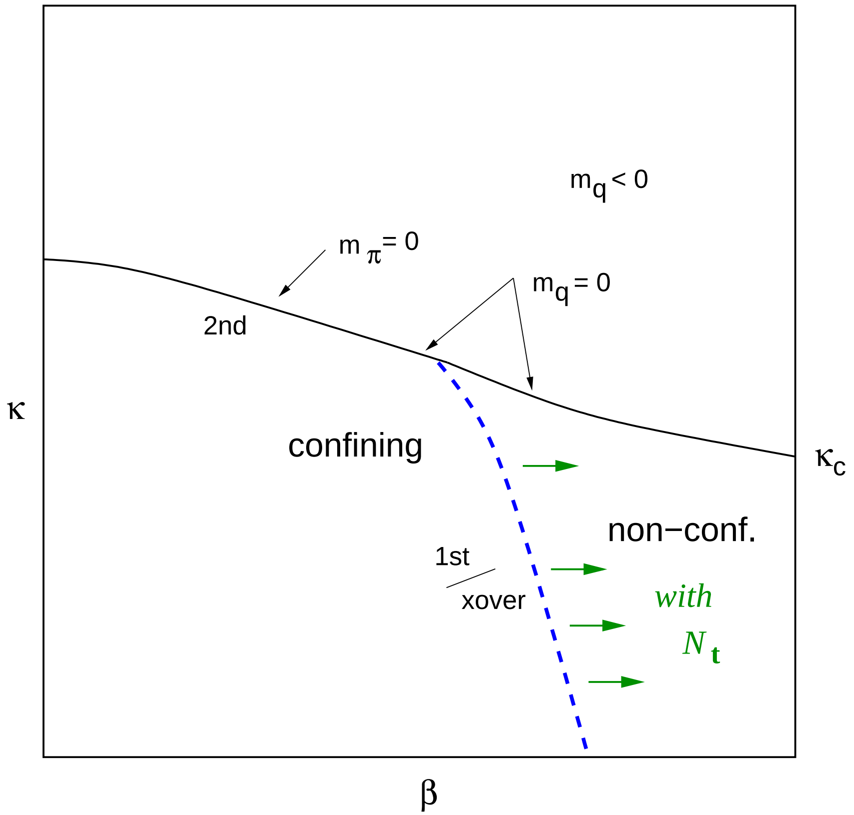


Cf. QCD



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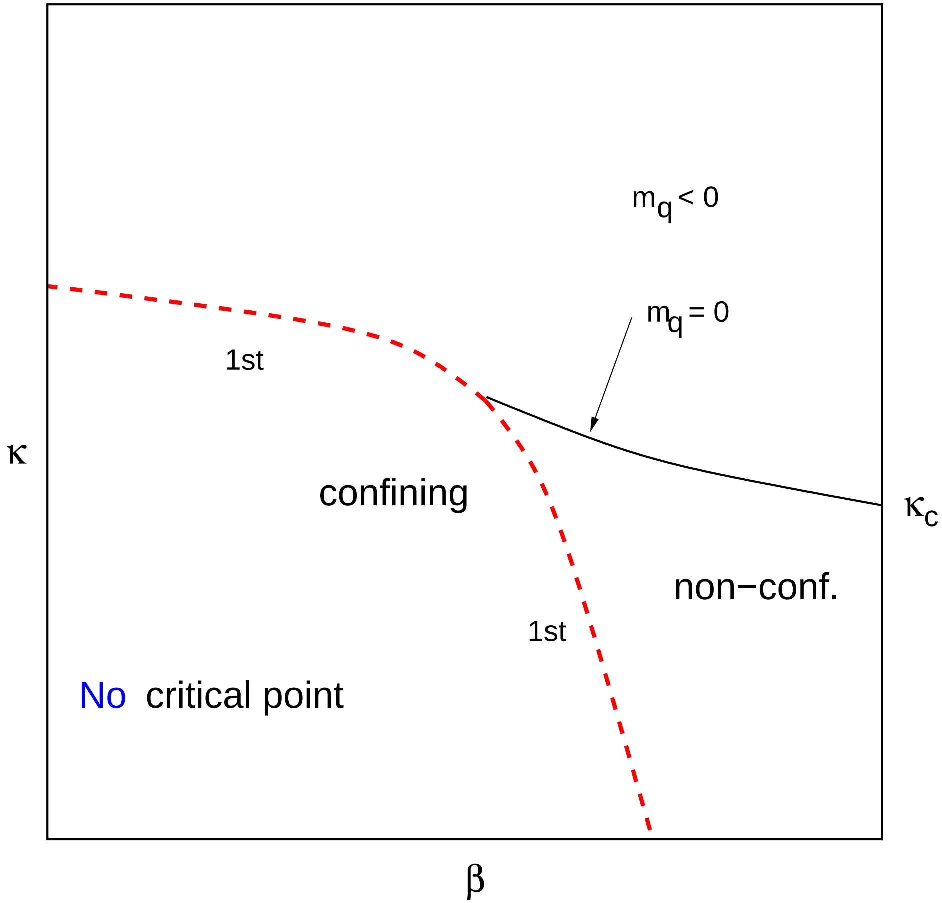


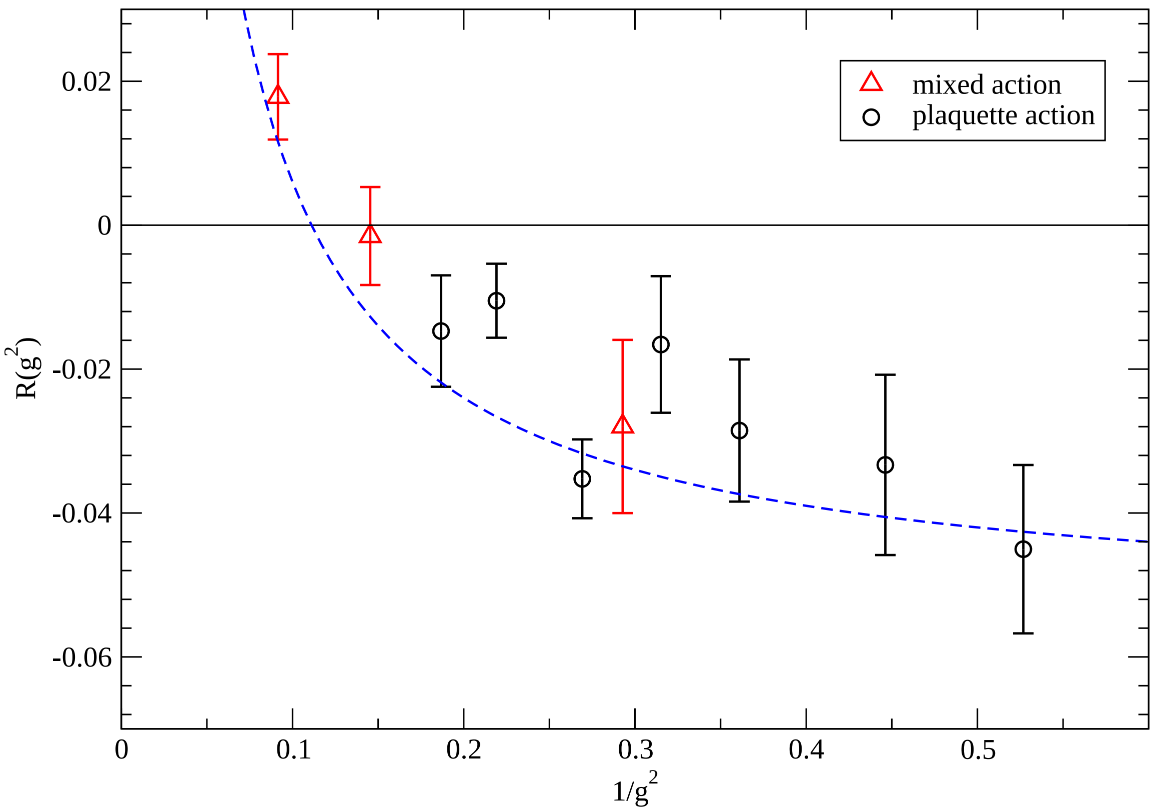
MOVING THE WALL:

Change the gauge action —

$$S_g = \frac{\beta}{2N_c}\sum \mathrm{Tr}\, U_p + \frac{\beta_f}{2d_f}\sum \mathrm{Tr}\, V_p$$

where V_p is made of fat links in the fermion rep (e.g. $\beta_f = +0.5$)

⟹ pushes the wall to stronger coupling:



An IRFP in the SU(3)/sextet theory*

*at low significance

MASS ANOMALOUS DIMENSION

Expected: $\gamma(g_*^2) \to 1$ at sill of conformal window (Cohen & Georgi 1988; Kaplan, Lee, Son, Stephanov 2010)

Work with correlation functions on lattice:

$$\left\langle P^b(t)\, \mathcal{O}^b(t'=0)\right\rangle\Big|_{t=L/2} = Z_P\, Z_{\mathcal{O}}\, e^{-m_\pi L/2}$$

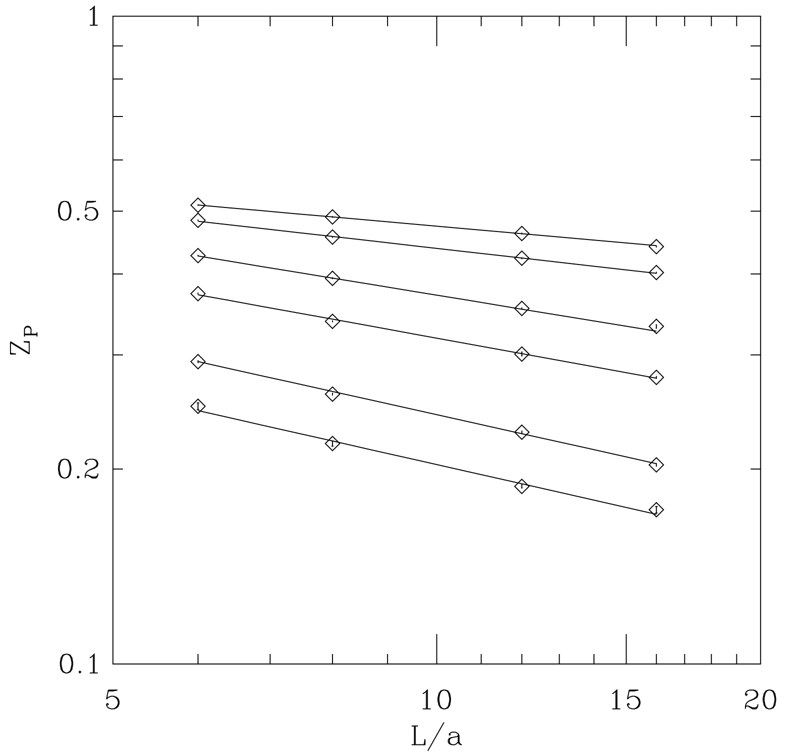
$$\left\langle \mathcal{O}^b(t=L)\, \mathcal{O}^b(t'=0)\right\rangle = Z_{\mathcal{O}}^2\, e^{-m_\pi L}$$

Take ratio, extract $Z_P(L)$, whence

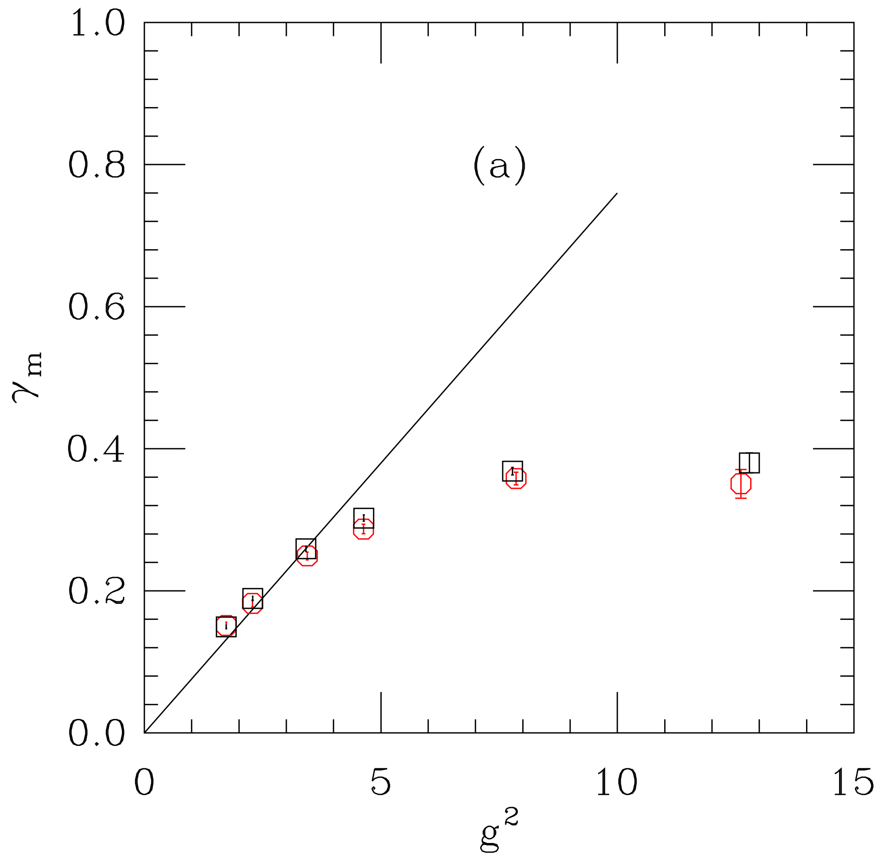
$$\frac{Z_P(L)}{Z_P(L_0)} = \left(\frac{L}{L_0}\right)^{-\gamma}$$

assuming $\gamma \simeq \text{const}$ as $L_0 \to L$,
since the running is *S L O W*

MASS ANOMALOUS DIMENSION — SU(2)/triplet



$\Longrightarrow$



slope $= -\gamma_m(g^2)$

Cf. one loop: $\gamma = \dfrac{6C_2(R)}{16\pi^2}\,g^2$

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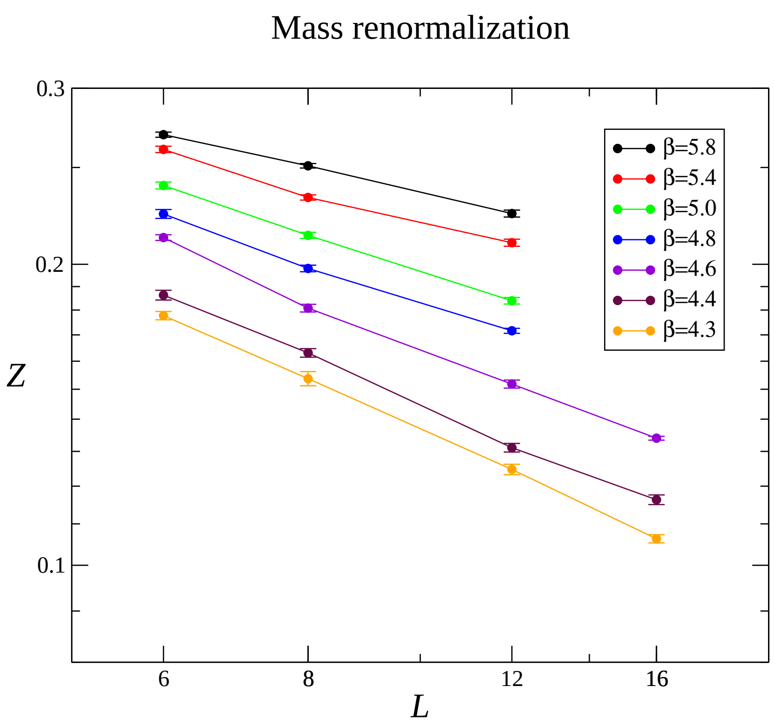

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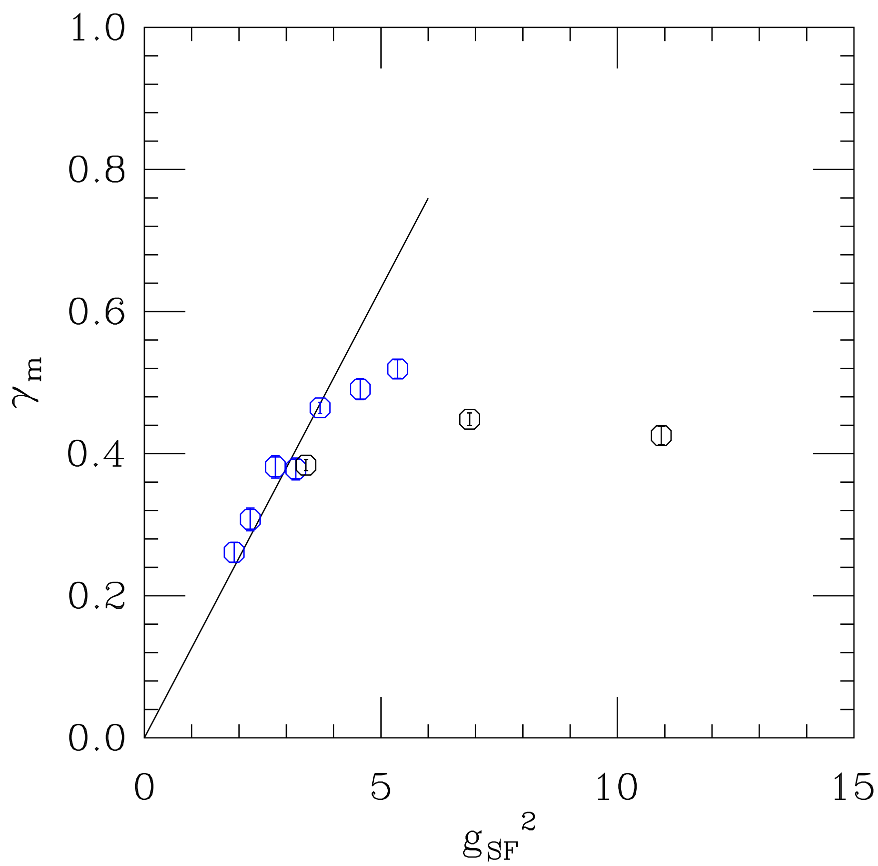

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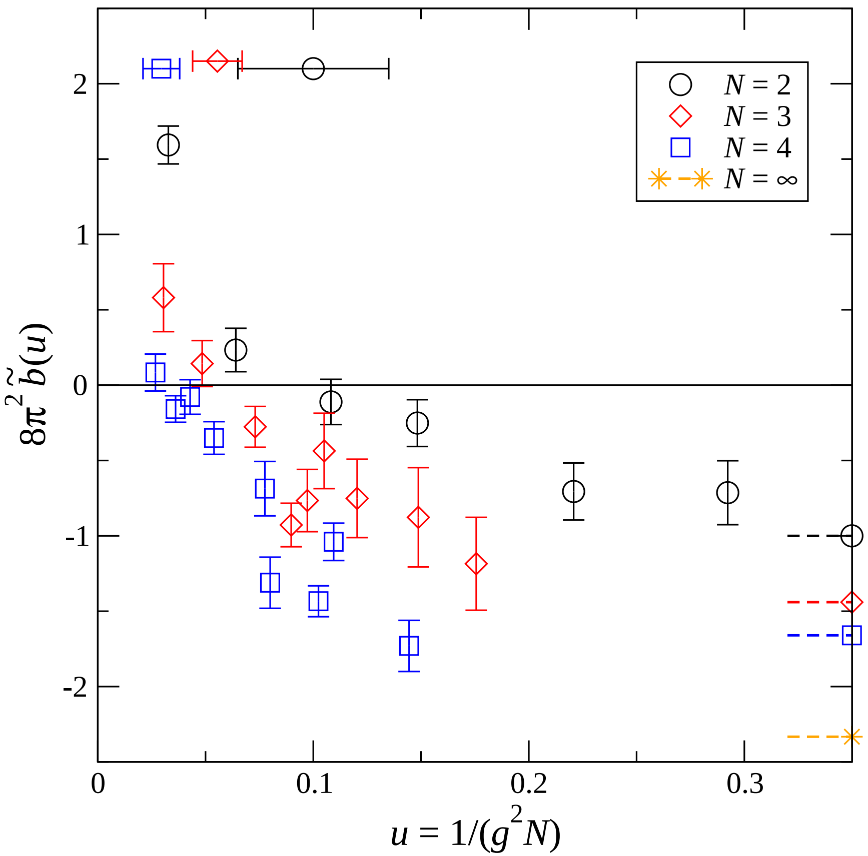
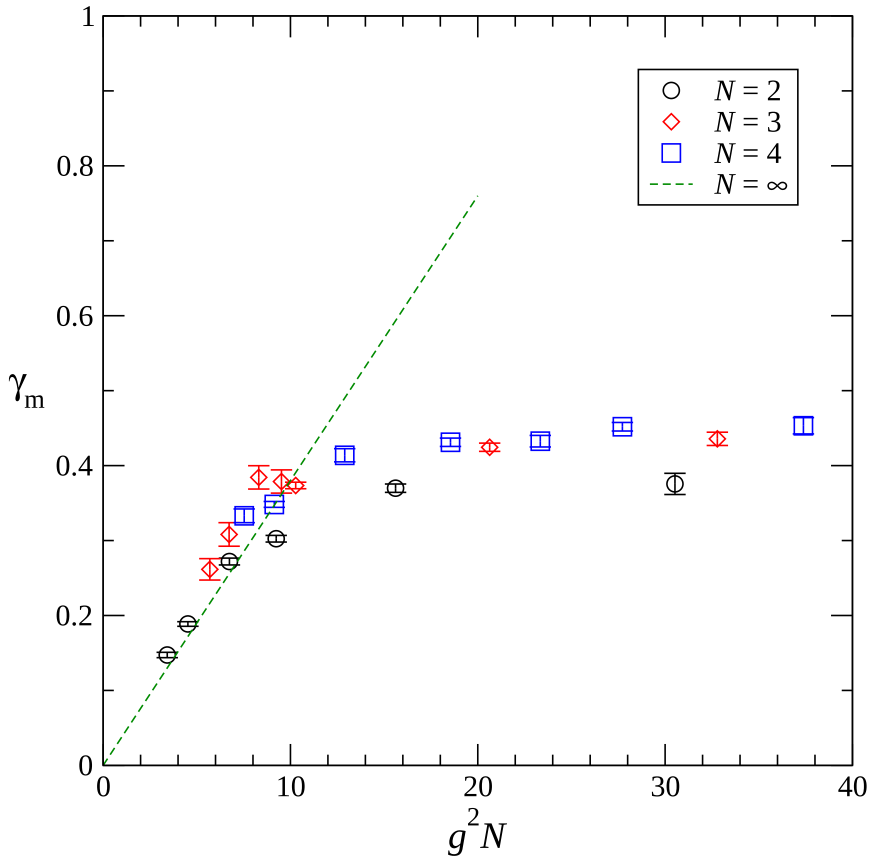
FINALLY, SU(4)/decuplet — compare all 3 theories





beta function $\tilde{b} \equiv \dfrac{d}{d \log s} \left(\dfrac{1}{g^2 N} \right)$

$\gamma \longrightarrow \sim 0.45$ — new universality?

SUMMARY

1. SU(2) gauge theory with $N_f = 2$ fermions in the $\mathbf{SYM}_2$ rep has an IRFP. SU(3), SU(4) might — at least, they run very slowly.
2. In each case, the mass anomalous dimension γ flattens out well short of 1.

THEORETICAL POINTS

Schwinger–Dyson eqns say these theories have no IRFP.

- Our fixed point(s) contradict the Schwinger–Dyson analysis.

SDEs also predict $\gamma \simeq 1$ near the sill of the conformal window (*walking technicolor*).

- For each $N = 2, 3, 4$ — $\gamma \lesssim 0.5$ means:
  1. We are deep in the conformal phase, or
  2. S–D eqns, model calculations are inapplicable here, too.

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FOR THE FUTURE

γ is much easier to calculate than β. More anomalous dimensions are waiting . . . ($\Longrightarrow$ "spectrum" of conformal theories)
. . . and also more gauge theories.