Excited(ing) State Spectroscopy in Lattice QCD

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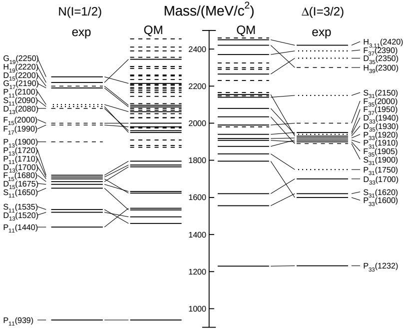
WHY STUDY EXCITED STATES?

Where are the 'missing' resonances? (N^*)

- Compare experiment (PDG '09) to quark model (Capstick, Roberts '00)
- Too many d.o.f in the QM? Reduced by diquarks (Jaffe)
- Experiment is mostly $\rightarrow N\pi$. States which couple weakly? N^* program at JLab.



Where are the 'QCD Exotica'?

- Hybrids?: Charmonia, GLuEX, BESIII
- Tetraquarks?

Spectroscopy in Lattice QCD

- Finite a, L, and T. Calculate Euclidean n-point functions
- Spectral rep. of two point functions:

$$C^{2pt}(t) = \langle 0|\mathcal{O}(t)\bar{\mathcal{O}}(0)|0\rangle = \sum_n A_n \mathrm{e}^{-E_n t} + \mathrm{O}(\mathrm{e}^{-ET}),$$

$$A_n = |\langle 0|\hat{\mathcal{O}}|n\rangle|^2$$

What are 'Resonances'?

- Def: Poles in the S-matrix on 2nd Riemannian sheet
- Often show up as 'bumps' in the cross section

Maiani-Testa No Go Theorem: S-matrix elements cannot be obtained from (I.V.) Euclidean correlators (except in principle at threshold).

Källén-Lehmann representation:

$$\Delta(p) = \int_0^\infty d\mu^2 \ \rho(\mu^2) \frac{1}{p^2 - \mu^2 + i\epsilon}$$

In I.V., $\rho(\mu^2)$ has δ-functions, and a continuum above threshold.

In F.V. $\rho(\mu^2)$ is discrete above threshold.

Behavior of F.V. energies below inelastic thresholds is well known (Lüscher '86). Generalizations:

- Moving frames: Gottlieb, Rummukainen '95
- Multiple 2-particle channels: Liu, Feng, He '05
- ...
- 2 Non-identical particles, moving frame: Leskovec, Prelovsek '12

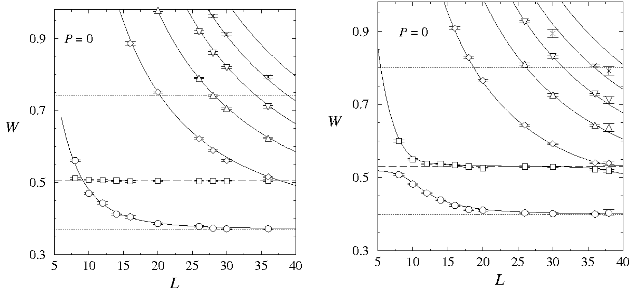
General prescription to extract I.V. resonance info above 3 (or more) particle thresholds lacking.

Below threshold, F.V energy corresponds to I.V. bound state up to $\mathcal{O}(e^{-m_\pi L})$.

Near threshold, F.V energies are distorted. Avoided level crossing occurs.

Example: taken from (Gottlieb, Rummukainen '95)

- Two scalars: $4m_\phi > m_\rho > 2m_\phi$, $\mathcal{L}_{int} = \frac{\lambda_1}{4!}\rho^4 + \frac{\lambda_2}{4!}\phi^4 + \frac{g}{2}\rho\phi^2$
- Spectrum from GEVP with single and multi-particle ops.
- Left: $g = 0$, Right: $g = 0.008$



The GEVP

Solve (Lüscher, Wolff ‘90)

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0),$$
$$C_{ij}(t) = \langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0)\rangle$$

Method 1:

- Avoids diagonalization at large t
- Solve GEVP for $t = t_*$. Discard $\lambda_n(t_*, t_0)$.
- Use $\{v_n(t_*, t_0)\}$ to rotate $C(t)$.
- Ensure that the off-diagonal elements remain small.

Method 2:

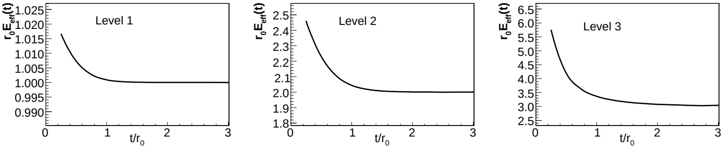
- Diagonalize on each t
- (Blossier, et al. ‘10): If $t_0 > t/2$

$$E_n^{eff}(t,t_0) = -\partial_t \log\lambda_n(t,t_0) = E_n + \mathcal{O}(\mathrm{e}^{-(E_{N+1}-E_n)t})$$

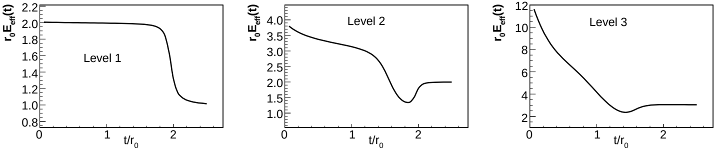
In practice, weakly coupled low-lying states are problematic

Example: $C_{ij}^{2pt}(t) = \sum_m \psi_{im}\psi_{jm}^* \mathrm{e}^{-E_m t}$

- Energies: $r_0 E_m = m, \; m = 1..20$
- A 3×3 GEVP, $\psi_{mi}, \; i = 1..3$ chosen empirically
- Solve for $E_n^{eff}(t, t_0 = t/2)$



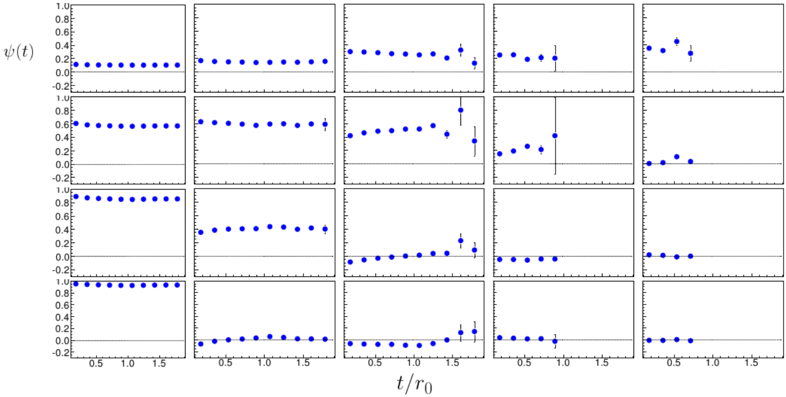
Reduce $\psi_{i1}, \; i = 1..3$ by 100:



Simple ops aren't enough

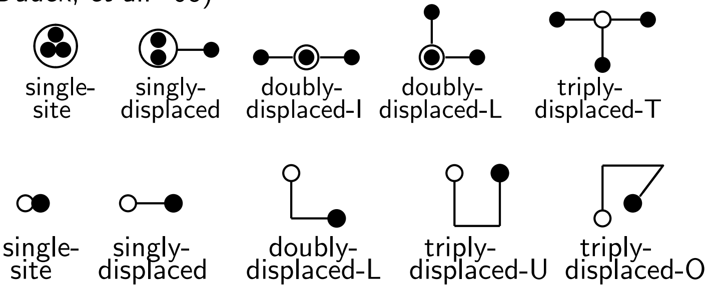
Using the GEVP, can access the $\psi_{im} = \lim_{t\to\infty} \psi_{eff}(t)$

Test case (JB, Donnellan, Sommer, '11): PS static-light mesons, $N_f = 0$, $a_s = a_t \sim 0.09\mathrm{fm}$, $L \sim 1.5\mathrm{fm}$, $m_q \sim m_s$



Ops. made from different smearings. Columns: $m = 1..5$, Rows: $r/r_0 = 0.0, 0.36, 0.62, 1.13$

Spatially Extended operators: (Basak, et al. '05), (Foley, et al. '07), (Dudek, et al. '08)

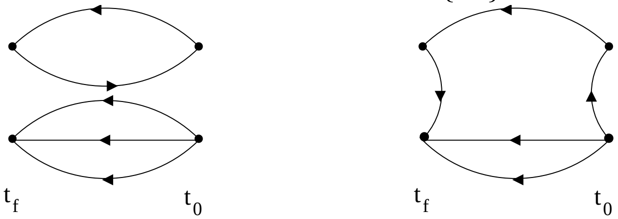


The Goal: For each symmetry channel pick a maximum energy.

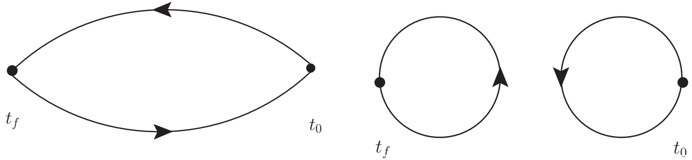
Include an op. for each (known) state below that energy. J=0, I=0 (Isoscalar-scalar channel):

- single σ-meson operators
- single glueball operators
- $I = 0$ two pion operators, moving and at rest
- $\bar{K} - K$ operators, moving and at rest
- ...

Needed to include multi-hadrons in $\{\mathcal{O}_i\}$



Needed even for isoscalar single hadrons



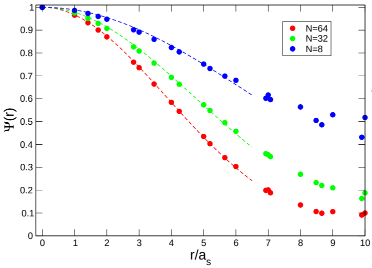
Distillation

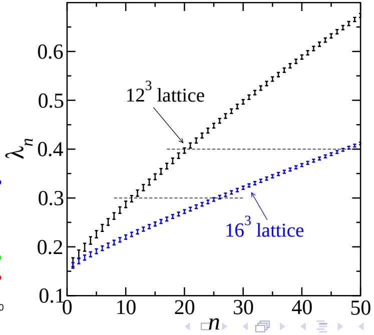
Exact smeared-smeared or point-smeared all-to-all propagators (Peardon, et al '09)

$$SQ^{-1}S = v_i K_{ij} v_j^\dagger$$

$$S = \sum_i^{N_{ev}} v_i^\dagger v_i, \quad \Delta v_i = \lambda_i v_i$$

Smearing controlled by λ_{max}. Requires $N_{inv} \sim N_{ev}$, but $N_{ev} \sim V$.





Calculating the eigenpairs

Solution of N_t 3d eigenproblems of a hermitian operator.

Use a Krylov-Spectral Restarted Lanczos (KRSL) algorithm (Wu, Simon '00)

Chebyshev acceleration is very helpful

$$B = 1 + \frac{2(\tilde{\Delta} + \lambda_C)}{\lambda_L - \lambda_C}, \quad A = T_n(B)$$

Cost is dominated by global re-orthog. of the Krlov space.

$$Cost \sim N_{ev}^2 * N_{itr} * V \sim V^4$$

Largest test: $L = 3.8\mathrm{fm}$, $N_{ev} = 384$, still a tiny fraction of the total cost.

Dominant cost is Dirac matrix inv.

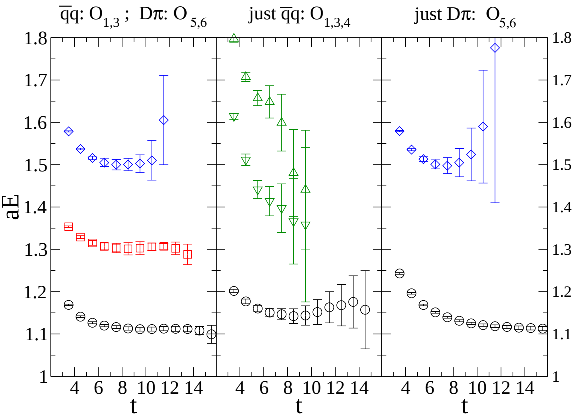
$$Inv.cost \sim N_{ev} * V \sim V^2$$

Distillation results

Results on small volumes ($< 2.4 - 2.9\mathrm{fm}$):

- N, Δ, and Ω baryons: (JB, et al. '10)
- $\pi\pi$-scattering: (Dudek, et al. '12)
- $D\pi$-scattering: (Mohler, et al. '12)
- $K\pi$-scattering: (Lang, et al. '12)
- ρ and a_1 meson decay: (Prelovsek, et al. '11)
- $I = 0$ mesons: (Dudek, et al. '11)
- Charmonium: (Liu, et al. '11)
- Hybrid Baryons: (Dudek, Edwards '12)

Improvement when adding multi-hadron ops. (Prelovsek, et al. '12): $a_s = a_t = 0.12\mathrm{fm}, m_\pi = 266\mathrm{MeV}, L_s = 2\mathrm{fm}$, D_0^* channel ($J^P = 0^+$)



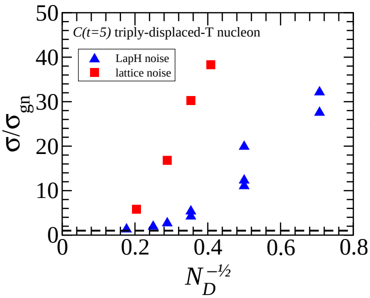
Left: single D_0^* ops and $D\pi$ ops
Middle: just single D_0^* ops
Right: just $D\pi$ ops

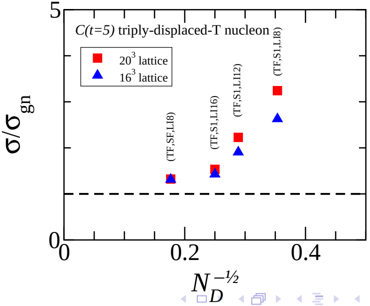
Introduce noise in the subspace (Morningstar, et al. '11)

$$\eta^{(r)}_{a\alpha}(\mathbf{x},t) = \rho^{(r)}_{\alpha i}(t) v^{i}_{at}(\mathbf{x})$$
$$SQ^{-1}S = E_r(\psi\eta^\dagger),\ \ \psi^{(r)} = SQ^{-1}\eta^{(r)}$$

Dilute in (α, i, t)-space:

$$\eta^{[d]} = P^{[d]}\eta,\ \ \psi^{[d]} = SQ^{-1}\eta^{[d]},\ SQ^{-1}S \quad = \sum_d E_r(\psi^{[d]}\eta^{[d]\dagger})$$





Correlator construction is simple and efficient!

- For connected lines: $N_{inv}/cfg. \sim 32 \times N_{t_0}$
- For disconnected lines: $N_{inv}/cfg. \sim 32 \times 16 \times (0.03\mathrm{fm}/a_t)$

Example: 4 connected lines, $N_{t_0} = 1$, $N_{inv}/cfg = 128$

$$\Omega_{ijk}(t_0) = a^{abc}_{\alpha\beta\gamma} \sum_{\mathbf{x}_0} e^{i\mathbf{p}\cdot\mathbf{x}_0} \eta^{[i]}_{a\alpha}(x_0) \eta^{[j]}_{b\beta}(x_0) \eta^{[k]}_{b\gamma}(x_0)$$

$$\Sigma_{ijk}(t, t_0) = a^{abc}_{\alpha\beta\gamma} \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \psi^{[i]}_{a\alpha t_0}(x) \psi^{[j]}_{b\beta t_0}(x) \psi^{[k]}_{b\gamma t_0}(x)$$

$$\mathcal{A}_{ij}(t, t_0) = \sum_{\mathbf{x}} \psi^{[i]\dagger}_{t_0}(x) A_0 \psi^{[j]}_{t_0}(x), \quad \omega_{ij}(t, t_0) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \psi^{[i]\dagger}_{t_0}(x) \Gamma \psi^{[j]}_{t_0}(x)$$

$$\rho_{ij}(t_0) = \sum_{\mathbf{x}_0} e^{i\mathbf{p}\cdot\mathbf{x}_0} \eta^{[i]\dagger}(x_0) \Gamma \eta^{[j]}(x_0)$$

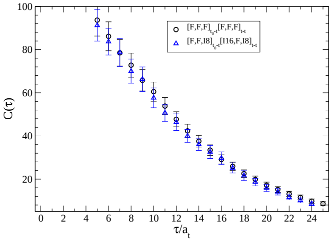
Correlation functions:

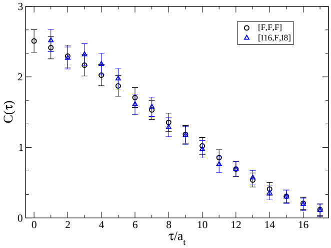
$$C_\pi(t - t_0) = \omega_{ij}\rho_{ij}, \; C_{f_\pi}(t - t_0) = \mathcal{A}_{ij}\rho_{ij}, \; C_N(t - t_0) = \Sigma_{ijk}\Omega^*_{ijk},$$
$$C^{I=2}_{\pi\pi}(t - t_0) = \omega_{ij}\, \rho_{jk}\, \omega_{k\ell}\, \rho_{\ell i} - C^2_\pi$$

Stochastic LapH and quark-disc. diagrams

Exact all-to-all is 'wasteful'(Wong, et al. '10):

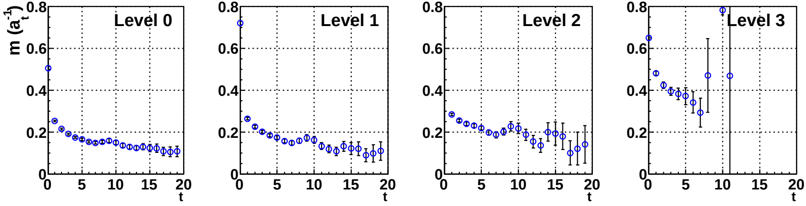
- HSC Lattice: $N_f = 2+1$, $a_s = .12\text{fm} = 3.5 a_t$, $m_\pi = 400\text{MeV}$, $L_s = 1.9\text{fm}$
- Left: 'Box' diagram for $\pi\pi$, Right: Disc. contribution to scalar
- For the scalar:
  - Distillation: $N_{inv}/cfg. = 16384$
  - Stochastic LapH: $N_{inv}/cfg. = 1024$





Scalar $I = 0$ channel (JB, D. Lenkner, et al. '11):

- HSC Lattice: $N_f = 2+1$, $a_s = .12\mathrm{fm} = 3.5a_t$, $m_\pi = 400\mathrm{MeV}$, $L_s = 1.9\mathrm{fm}$
- 5x5 GEVP: 2 single meson ops., 2 $\pi\pi$ ops., 1 glueball op.
- Results of a preliminary diagonalization



Conclusions

- All-to-all propagators can be stochastically estimated efficiently
  - Effort $\sim V$ with N_{inv} fixed, $N_{inv}/cfg \sim 1200$ is sufficient
  - Useful for 'ordinary' $C(t)$, bare current insertions.
- First finite box spectrum calculations are underway: a $0.9 - 0.12\mathrm{fm}$, $m_\pi > 260 - 300\mathrm{MeV}$, L $2.5 - 3\mathrm{fm}$
- In principle, $\delta(k)$ below > 3 hadron thresholds can be extracted. At $m_\pi \sim 300\mathrm{MeV}$, this covers interesting physics!
  - σ, $f_0(980)$
  - Roper resonance
  - $\Lambda(1405)$
- To Do:
  - Phase shifts above inelastic thresholds, similar issues to experiment
  - smaller a, m_π, larger L
  - Interaction with perturbative probes?