

# QCD Critical Point : Inching Towards Continuum

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Introduction

Lattice QCD Results

Searching Experimentally

Summary

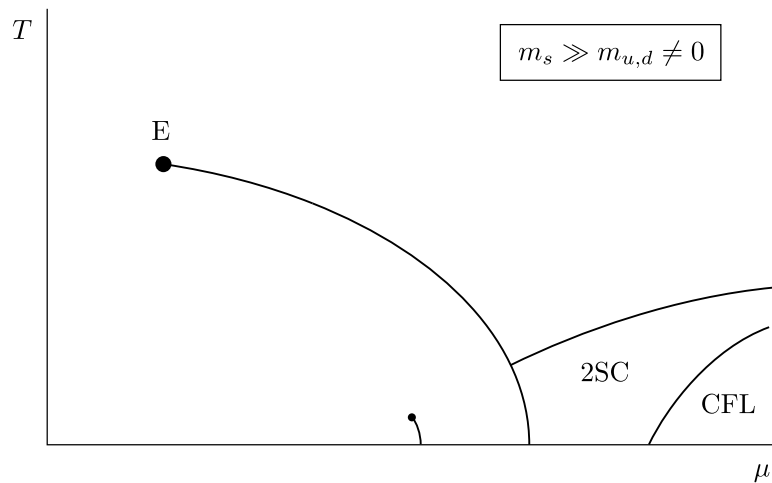
*\* Work done with Saumen Datta & Sourendu Gupta*

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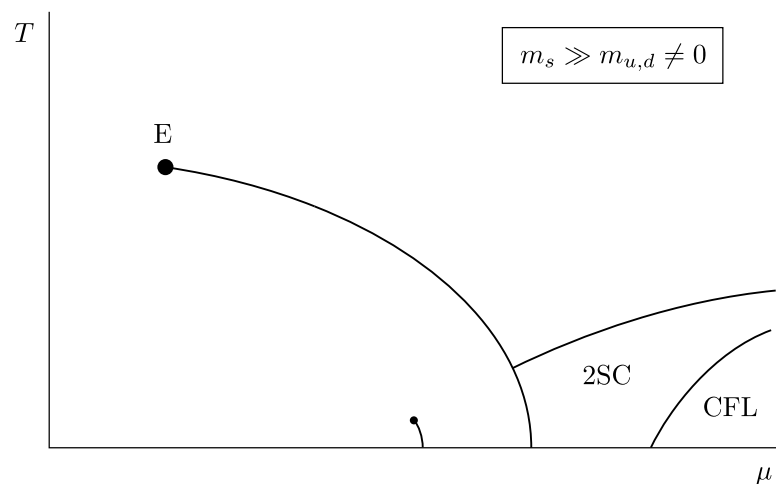


From Rajagopal-Wilczek Review

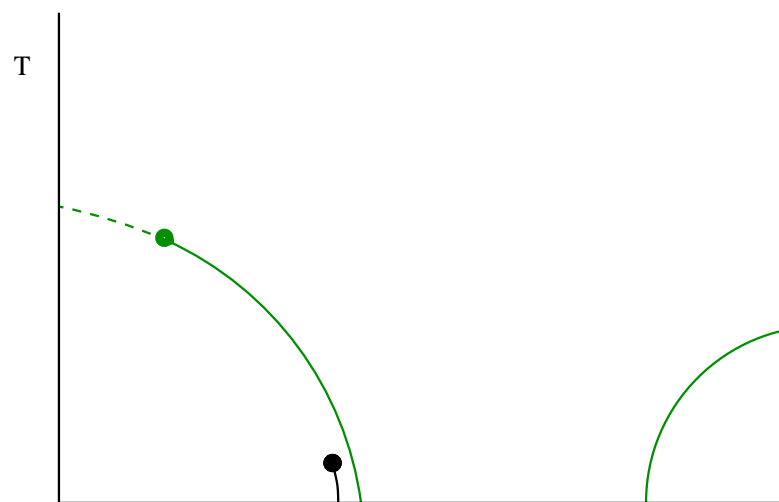
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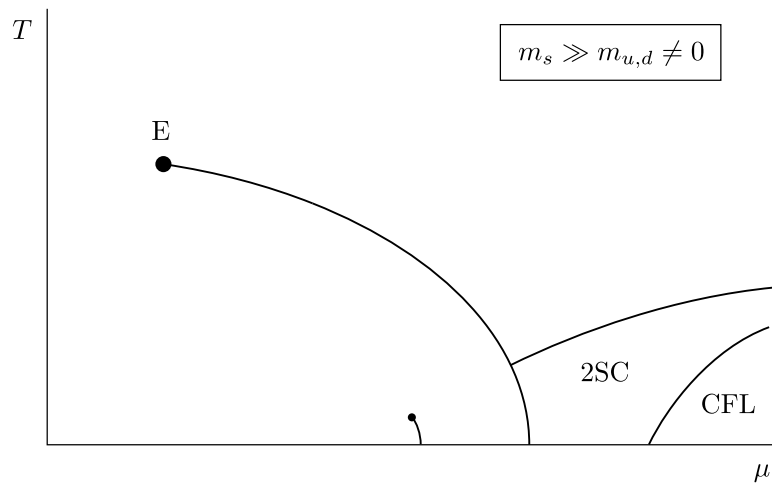
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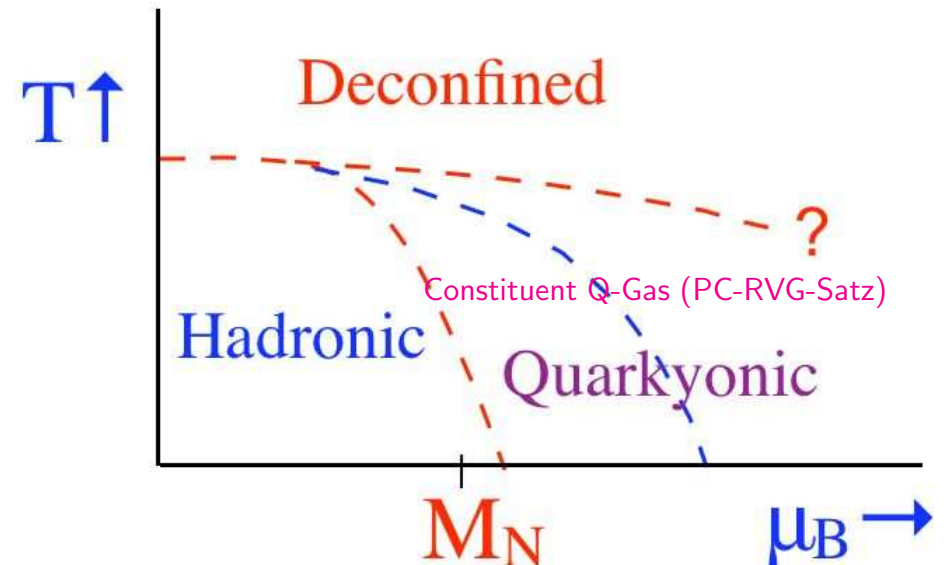
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... but could, however, be ... (McLerran-Pisarski 2007; Castorina-RVG-Satz 2010)



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- Between the theorists, to claim a patch on the QCD phase diagram,
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♡ Maybe it is the synergy between one or more of them that will eventually lead us to the holy grail.

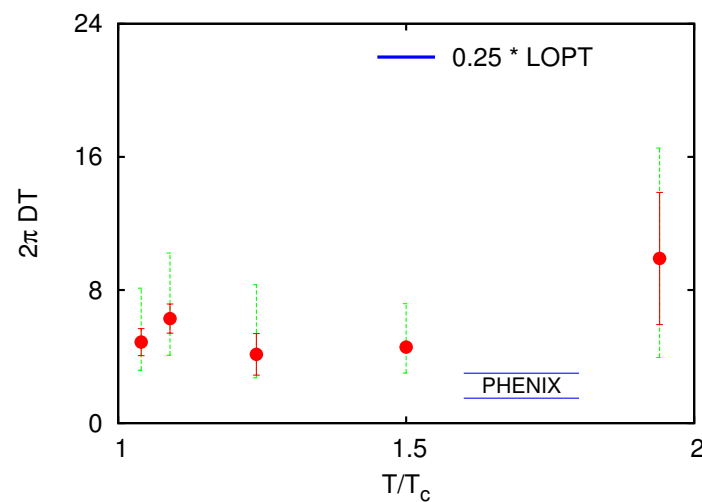
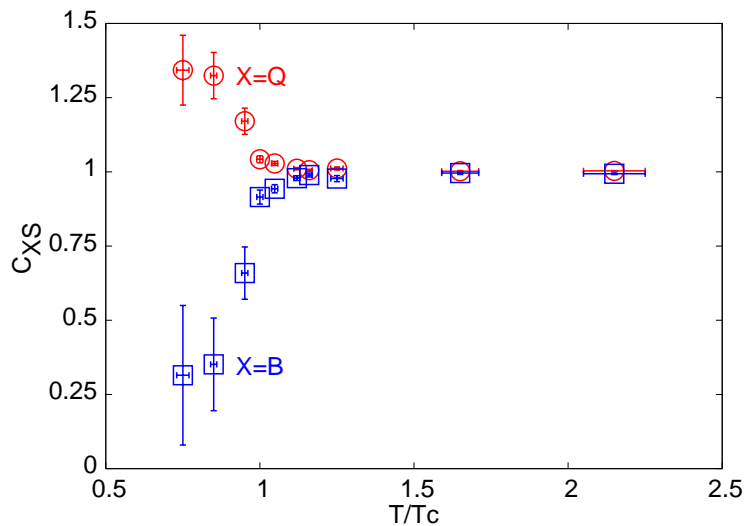


# Lattice QCD Results

- Lattice QCD – Most Reliable and Completely parameter-free way to extract non-perturbative physics relevant to Heavy Ion Colliders.
- The Transition Temperature  $T_c$ , the Equation of State (used now in ‘elliptic flow’ analysis), and the Wróblewski Parameter  $\lambda_s$  etc. (Wuppertal-Budapest, HotQCD, GG '02)

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- Flavour Correlations ( $C_{BS}$ ) and Charm Diffusion Coefficient  $D$  are some more such examples for RHIC Physics. (Gavai-Gupta, PRD 2006 & Banerjee et al. PRD 2012)



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- Introduction of  $\mu$  a la Bloch & Wettig (PRL 2006 & PRD2007)
- Unfortunately breaks chiral symmetry ! (Banerjee, Gavai & Sharma PRD 2008; PoS (Lattice 2008); PRD 2009 )
- Good News : Problem Solved !  
Overlap Lattice Action with exact chiral invariance at nonzero  $\mu$  and any  $a$  now exists (Gavai & Sharma , arXiv : 1111.5944; PLB in press, Narayanan-Sharma JHEP '11).

- Using chiral projectors for overlap fermions as,  $\psi_L = [1 - \gamma_5(1 - aD_{ov})]\psi/2$  &  $\psi_R = [1 + \gamma_5(1 - aD_{ov})]\psi/2$ , leaving the antiquark field decomposition as in the continuum, the overlap action for nonzero  $\mu$  is

$$\begin{aligned}
 S^F &= \sum_n [\bar{\psi}_{n,L}(aD_{ov} + a\mu\gamma^4)\psi_{n,L} + \bar{\psi}_{n,R}(aD_{ov} + a\mu\gamma^4)\psi_{n,R}] \\
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- Easy to check that under the chiral transformations,  $\delta\psi = i\alpha\gamma_5(1 - aD_{ov})\psi$  and  $\delta\bar{\psi} = i\alpha\bar{\psi}\gamma_5$ , it is invariant for all values of  $a\mu$  and  $a$ .
- Order parameter exists for all  $\mu$  and  $T$ . It is

$$\langle \bar{\psi}\psi \rangle = \lim_{am \rightarrow 0} \lim_{V \rightarrow \infty} \left\langle \text{Tr} \frac{(1 - aD_{ov}/2)}{[aD_{ov} + (am + a\mu\gamma^4)(1 - aD_{ov}/2)]} \right\rangle .$$

# The $\mu \neq 0$ problem : The Measure

Simulations can be done IF  $\text{Det } M > 0$ . However,  $\det M$  is a complex number for any  $\mu \neq 0$  : The Phase/sign problem



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Several Approaches proposed in the past two decades :

- Two parameter Re-weighting (Z. Fodor & S. Katz, JHEP 0203 (2002) 014 ).
- Imaginary Chemical Potential (Ph. de Forcrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505 ).
- Taylor Expansion (C. Allton et al., PR D66 (2002) 074507 & D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506 ).
- Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, Pos LAT2005 (2006) 167. )
- Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work ).

# Why Taylor series expansion?

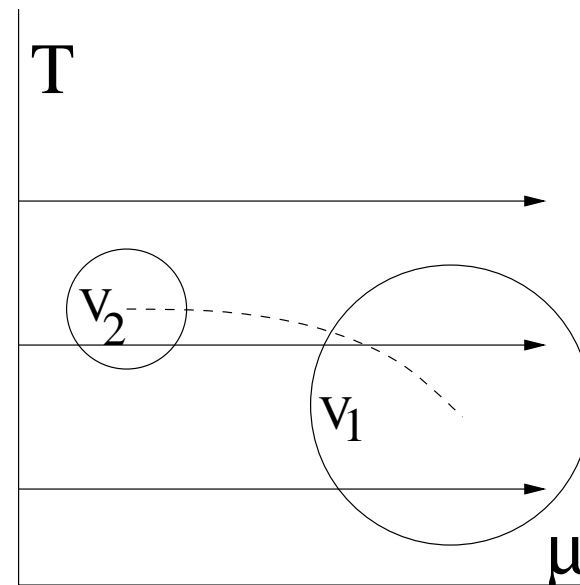
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We studied volume dependence at several  $T$  to i) bracket the critical region and then ii) tracked its change as a function of volume.

# Details of Expansion

Standard definitions yield various number densities and susceptibilities :

$$n_i = \frac{T}{V} \frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \quad \text{and} \quad \chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations,  $\lambda_s \dots$ )

Denoting higher order susceptibilities by  $\chi_{n_u, n_d}$ , the pressure  $P$  has the expansion in  $\mu$ :

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu, T)}{T^4} - \frac{P(0, T)}{T^4} = \sum_{n_u, n_d} \chi_{n_u, n_d} \frac{1}{n_u!} \left( \frac{\mu_u}{T} \right)^{n_u} \frac{1}{n_d!} \left( \frac{\mu_d}{T} \right)^{n_d}$$

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- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives estimate of the location of nearest critical point.
- Successive estimates for the radius of convergence obtained from these using  $\sqrt[n]{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}T^2}}$  or  $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}T^2}\right)^{1/n}$ . We use both and terms up to 8th order in  $\mu$ .
- All coefficients of the series must be POSITIVE for the critical point to be at real  $\mu$ , and thus physical.
- We (Gavai-Gupta '05, '09) use up to 8<sup>th</sup> order. B-RBC so far has up to 6<sup>th</sup> order.
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# The Susceptibilities

All susceptibilities can be written as traces of products of  $M^{-1}$  and various derivatives of  $M$ .

At leading order,

$$\chi_{20} = \left(\frac{T}{V}\right) [\langle \mathcal{O}_2 + \mathcal{O}_{11} \rangle], \quad \chi_{11} = \left(\frac{T}{V}\right) [\langle \mathcal{O}_{11} \rangle]$$

Here  $\mathcal{O}_2 = \text{Tr } M^{-1}M'' - \text{Tr } M^{-1}M'M^{-1}M'$ , and  $\mathcal{O}_{11} = (\text{Tr } M^{-1}M')^2$ , and the traces are estimated by a stochastic method (Gottlieb et al., PRL '87):

$\text{Tr } A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$ , and  $(\text{Tr } A)^2 = 2 \sum_{i>j=1}^L (\text{Tr } A)_i (\text{Tr } A)_j / L(L-1)$ , where  $R_i$  is a complex vector from a set of  $N_v$  subdivided in  $L$  independent sets.

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Higher order NLS are more involved. E.g., at the 8th order, terms involve operators up to  $\mathcal{O}_8$  which in turn have terms up to 8 quark propagators and combinations of  $M'$  and  $M''$ .

In fact, the entire evaluation of the  $\chi_{80}$  needs 20 inversions of Dirac matrix.

This can be reduced to 8 inversions using an action linear in  $\mu$  (Gavai-Sharma PRD 2012 & PRD 2010), leading still to results in agreement with that exponential in  $\mu$ .

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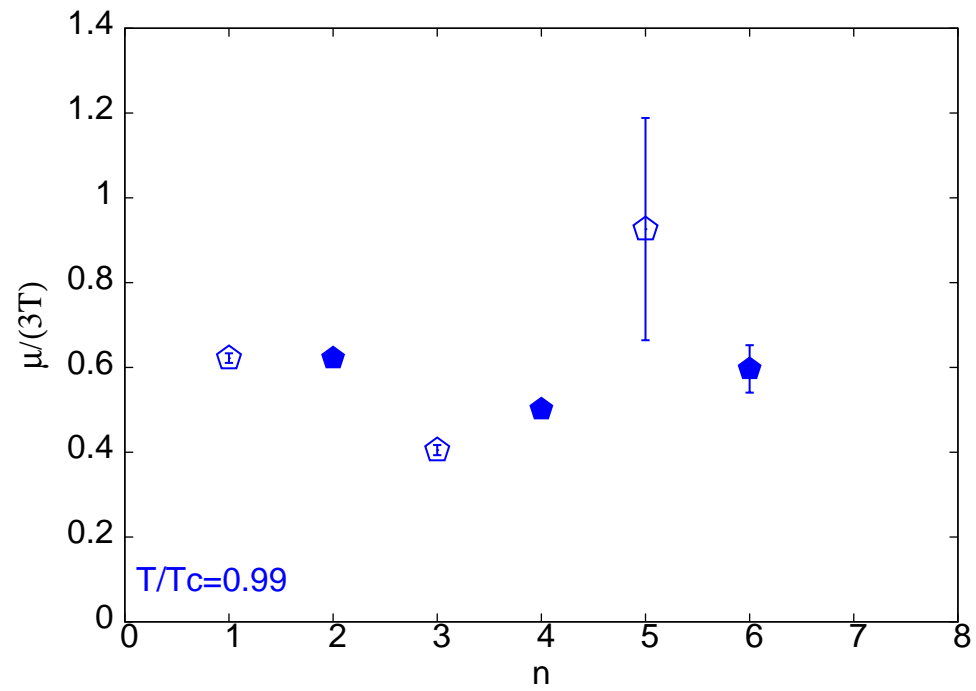
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# Our Simulations & Results

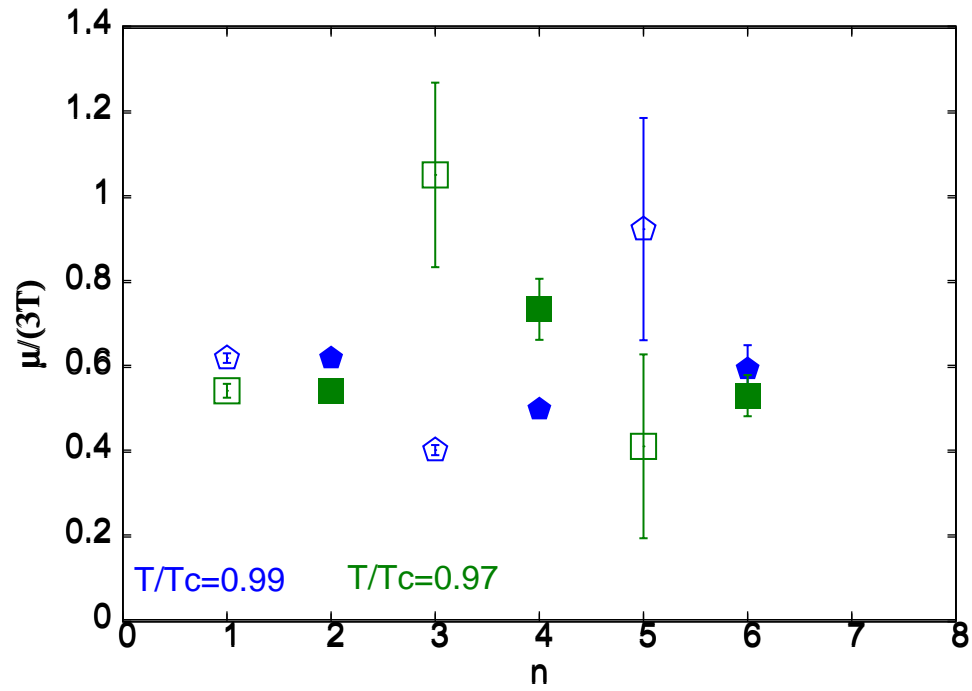
- Staggered fermions with  $N_f = 2$  of  $m/T_c = 0.1$ ; R-algorithm used.
- $m_\pi/m_\rho = 0.31 \pm 0.01$  (MILC); Kept the same as  $a \rightarrow 0$  (on all  $N_t$ ).
- Earlier Lattice :  $4 \times N_s^3$ ,  $N_s = 8, 10, 12, 16, 24$  (Gavai-Gupta, PRD 2005)  
Finer Lattice :  $6 \times N_s^3$ ,  $N_s = 12, 18, 24$  (Gavai-Gupta, PRD 2009).

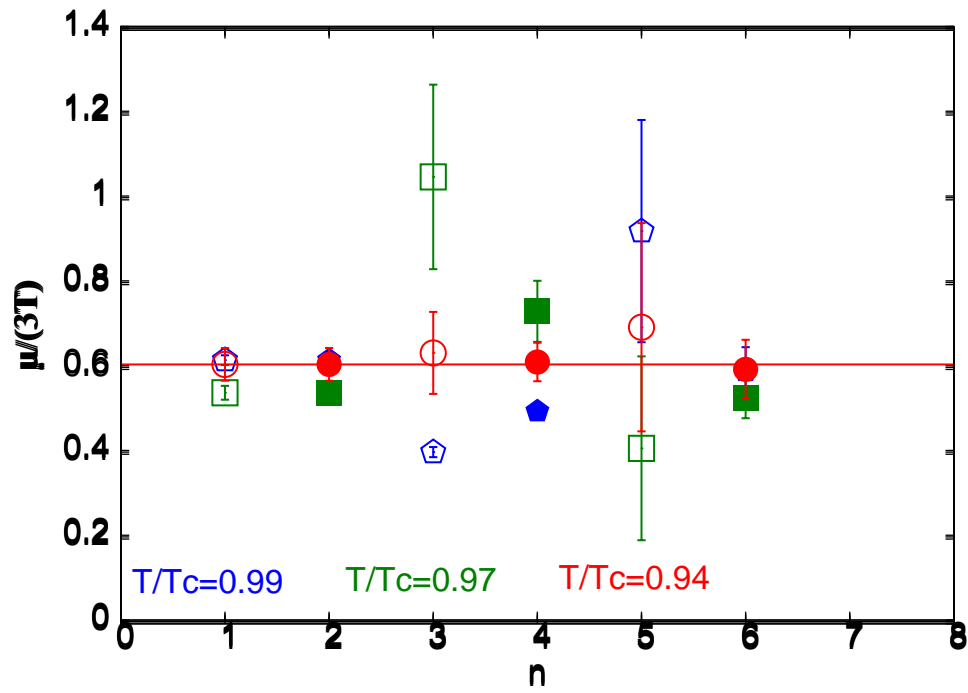
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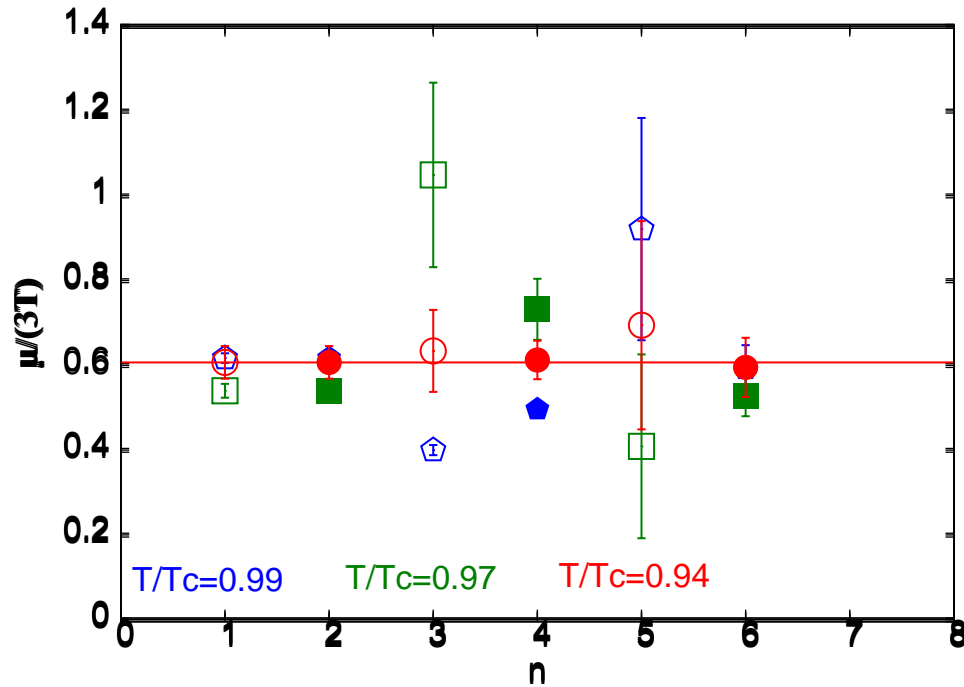
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Finer Lattice :  $6 \times N_s^3$ ,  $N_s = 12, 18, 24$  (Gavai-Gupta, PRD 2009).
- Even finer Lattice :  $8 \times 32^3$  — This Talk (Datta-RVG-Gupta, '12)  
Aspect ratio,  $N_s/N_t$ , maintained four to reduce finite volume effects.
- Simulations made at  $T/T_c = 0.90, 0.92, 0.94, 0.96, 0.98, 1.00, 1.02, 1.12, 1.5$  and 2.01. Typical stat. 100-200 in max autocorrelation units.
- $T_c$  — defined by the peak of Polyakov loop susceptibility.











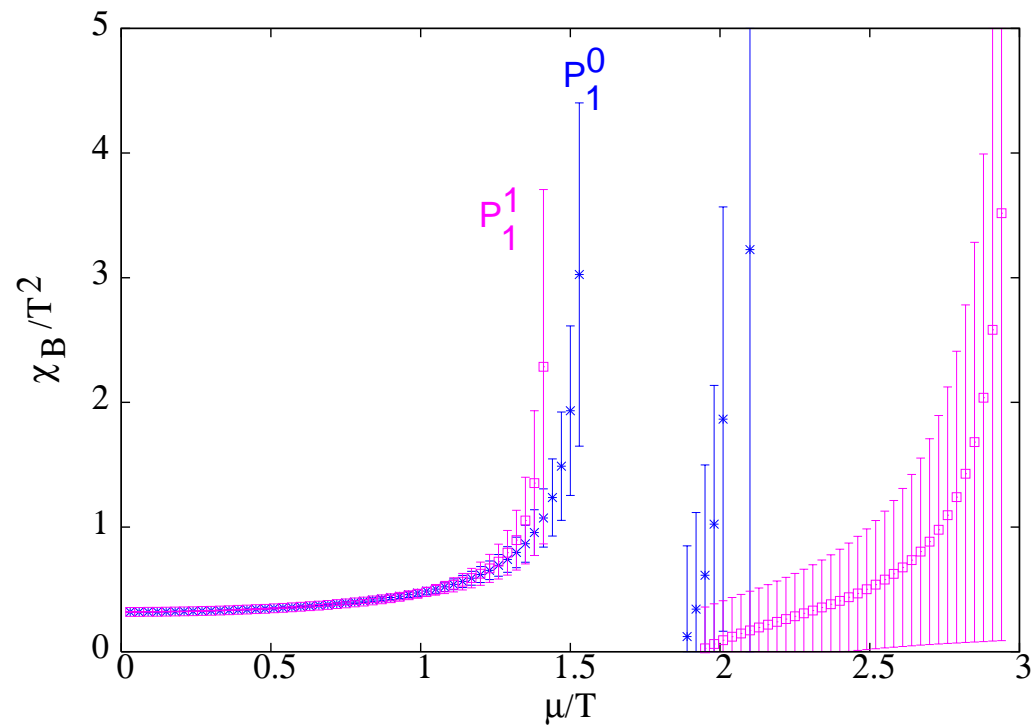
- $\frac{T^E}{T_c} = 0.94 \pm 0.01$ , and  $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$  for finer lattice: Our earlier coarser lattice result was  $\mu_B^E/T^E = 1.3 \pm 0.3$ . Infinite volume result:  $\downarrow$  to 1.1(1)
- Critical point at  $\mu_B/T \sim 1 - 2$ .

## Cross Check on $\mu^E/T^E$

♠ Use Padé approximants for the series to estimate the radius of convergence.

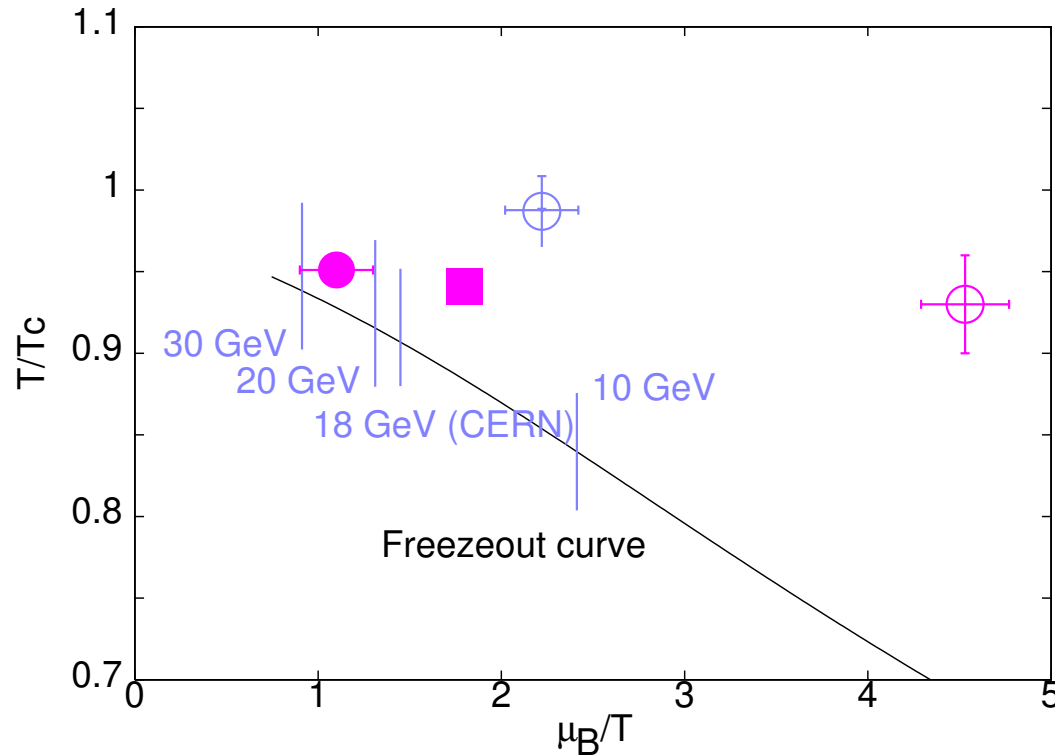
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♡ Consistent Window with our other estimates.

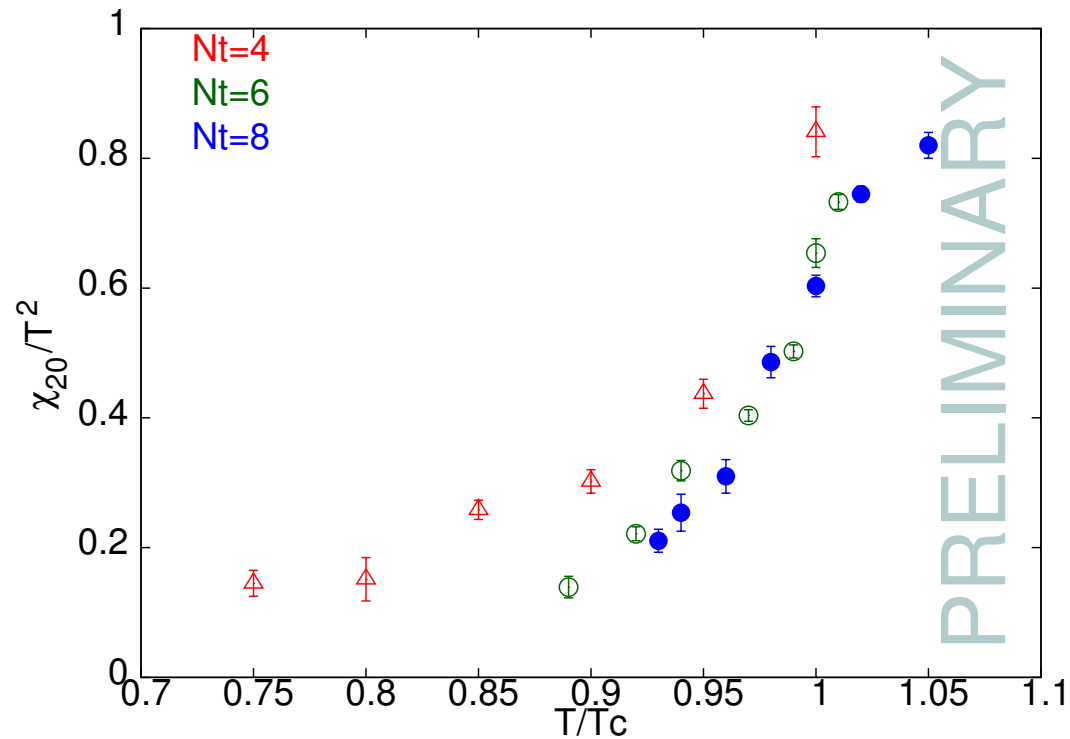
# Critical Point : Story thus far



♠  $N_f = 2$  (magenta) and  $2+1$  (blue) (Fodor-Katz, JHEP '04).

♡  $N_t = 4$  Circles (GG '05 & Fodor-Katz JHEP '02),  $N_t = 6$  Box (GG '09).

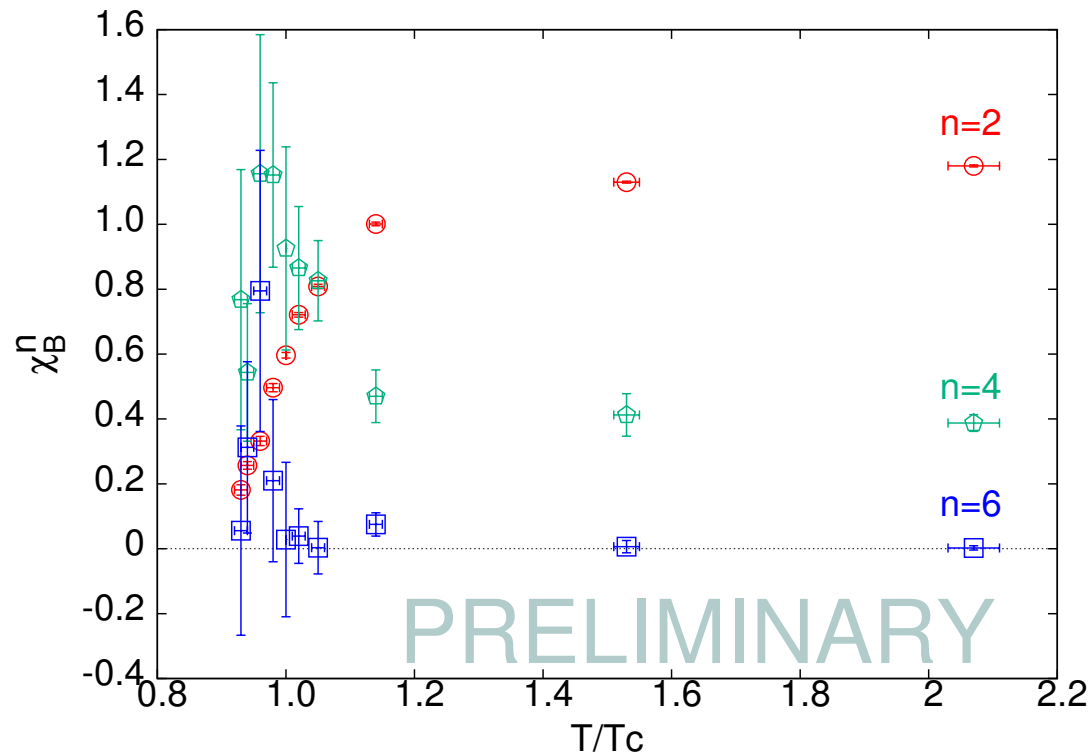
## $\chi_2$ for $N_t = 8, 6,$ and $4$ lattices



♠  $N_t = 8$  (Datta-Gavai-Gupta, QM12) and 6 (GG, PRD '09) results agree.

♡  $\beta_c(N_t = 8)$  agrees with Gottlieb et al. PR D47,1993.

## $\chi_B^n$ for $N_t = 8$ lattice

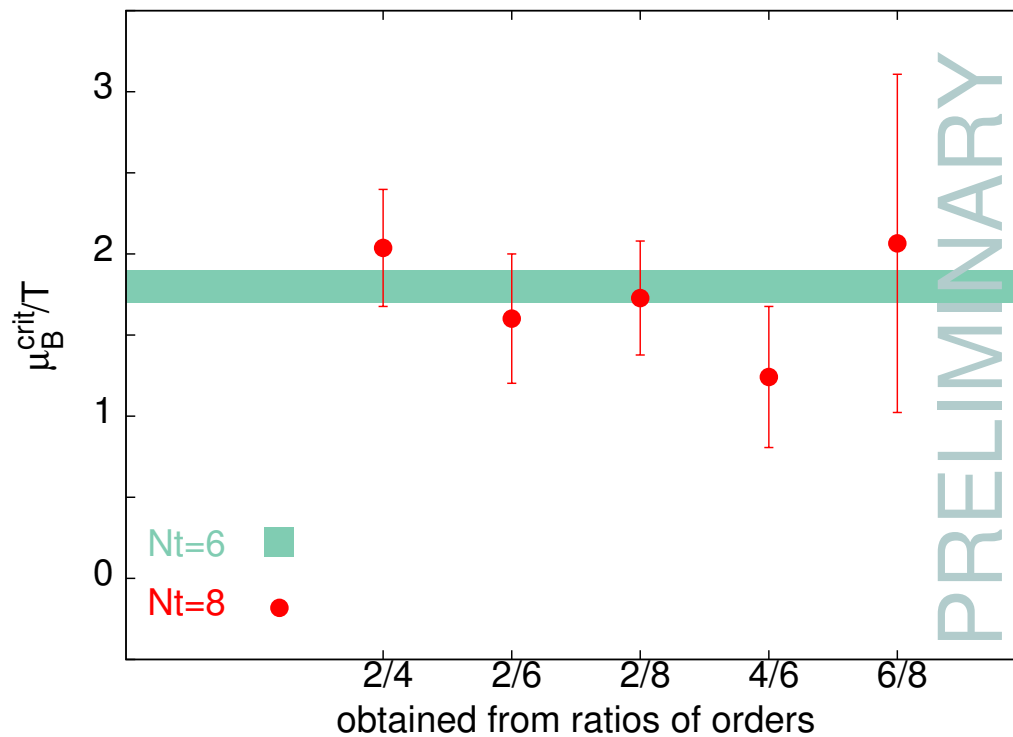


♠ 100 configurations & 1000 vectors at each point employed.

♡ More statistics coming in critical region. Window of positivity in anticipated region.

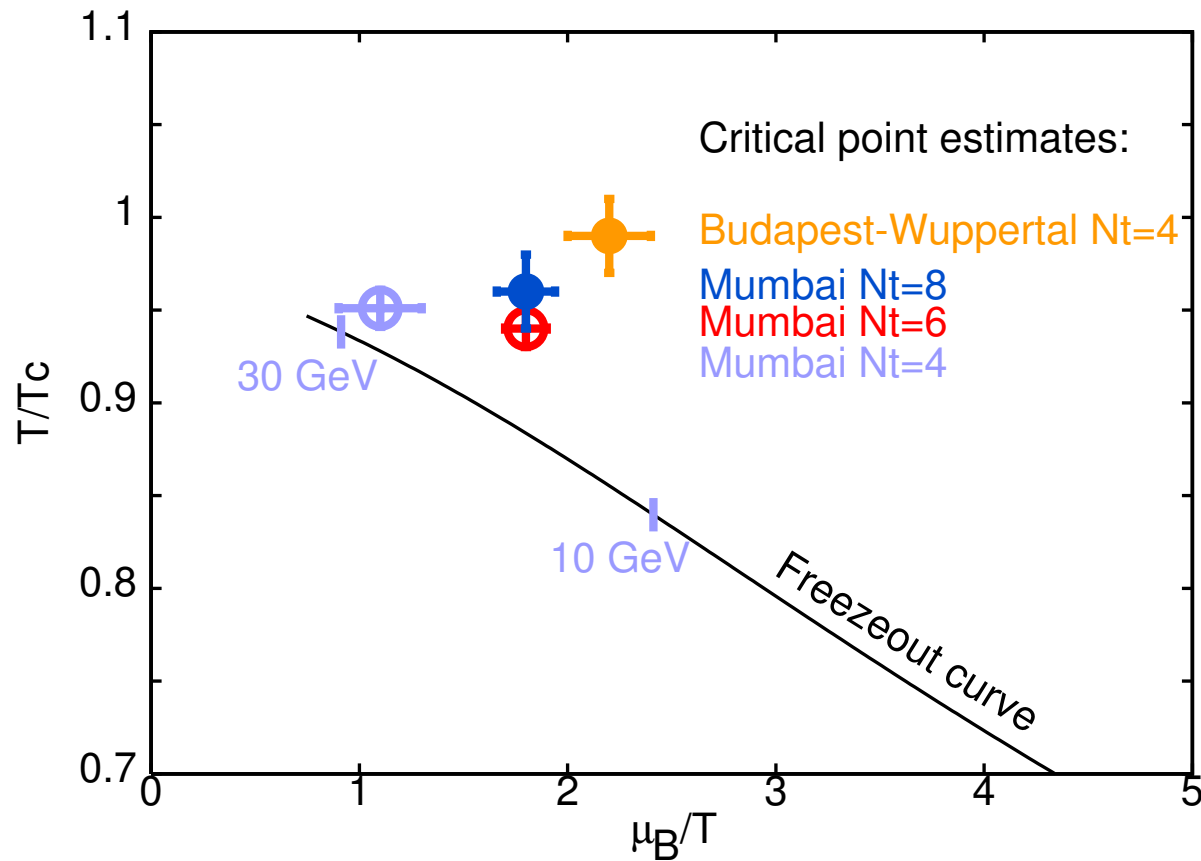


# Radius of Convergence result



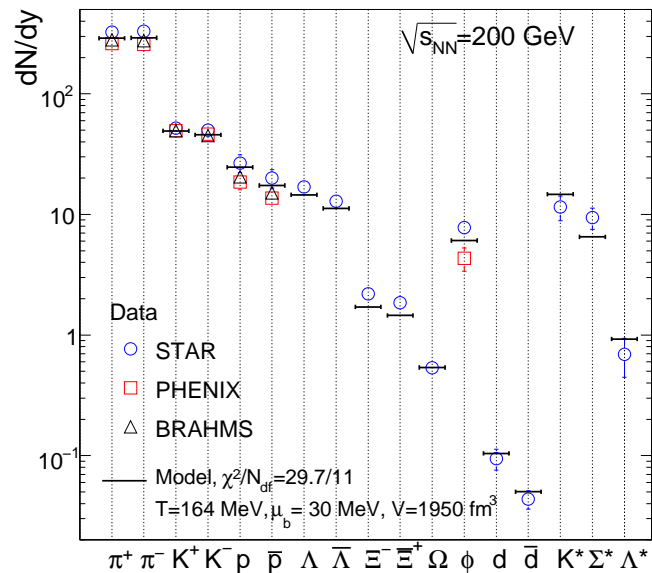
- ♠ At our  $(T_E, \mu_E)$  for  $N_t = 6$ , the ratios display constancy for  $N_t = 8$  as well.
- ♡ Currently : Similar results at neighbouring  $T/T_c \implies$  a larger  $\Delta T$  at same  $\mu_B^E$ .

# Critical Point : Inching Towards Continuum



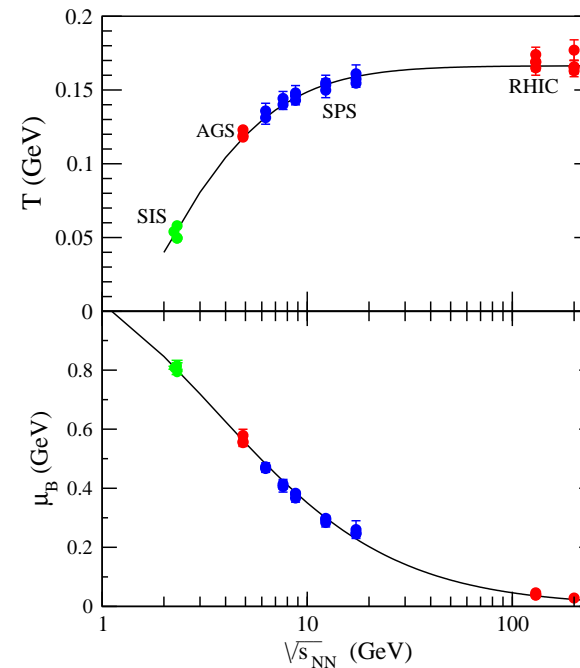
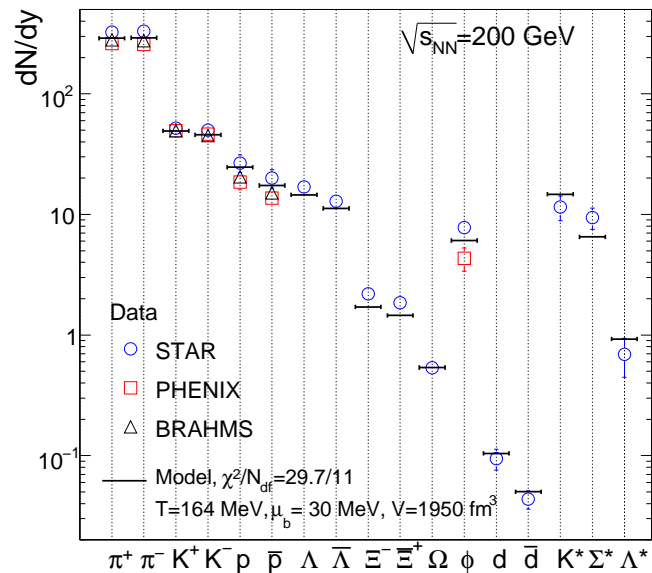
# Lattice predictions along the freezeout curve

- Hadron yields well described using Statistical Models, leading to a freezeout curve in the  $T$ - $\mu_B$  plane. (Andronic, Braun-Munzinger & Stachel, PLB 2009 ; Oeschler, Cleymans, Redlich & Wheaton, 2009)



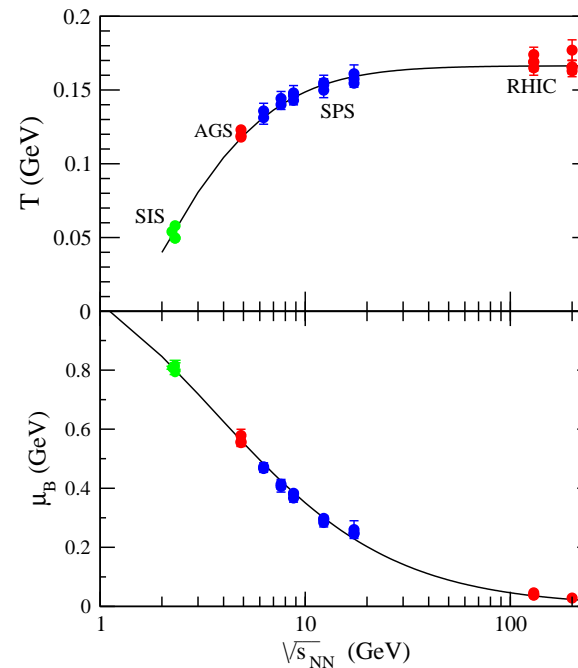
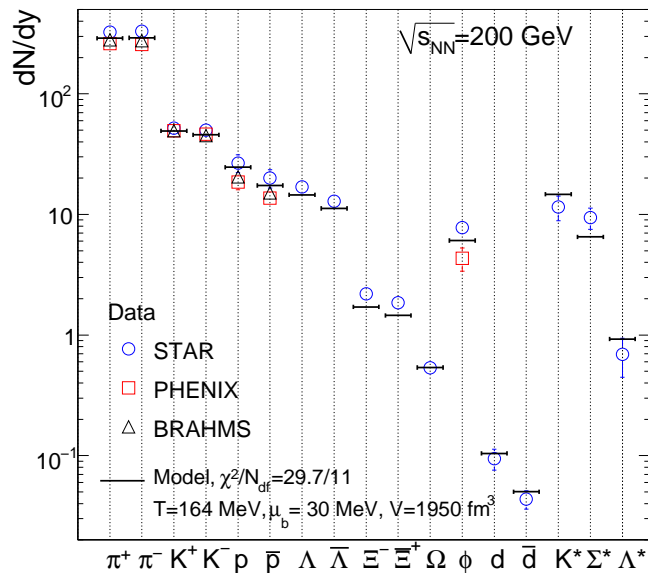
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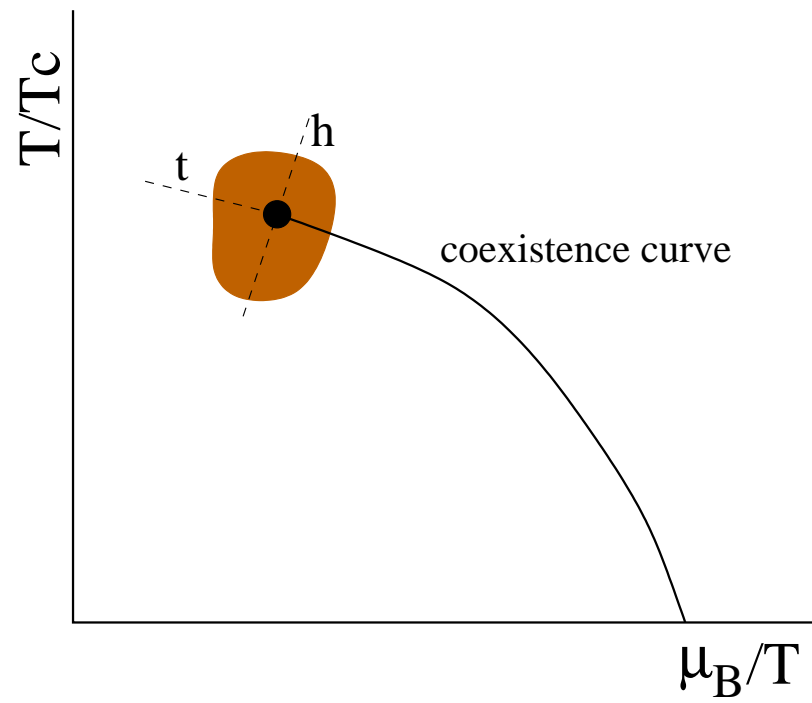


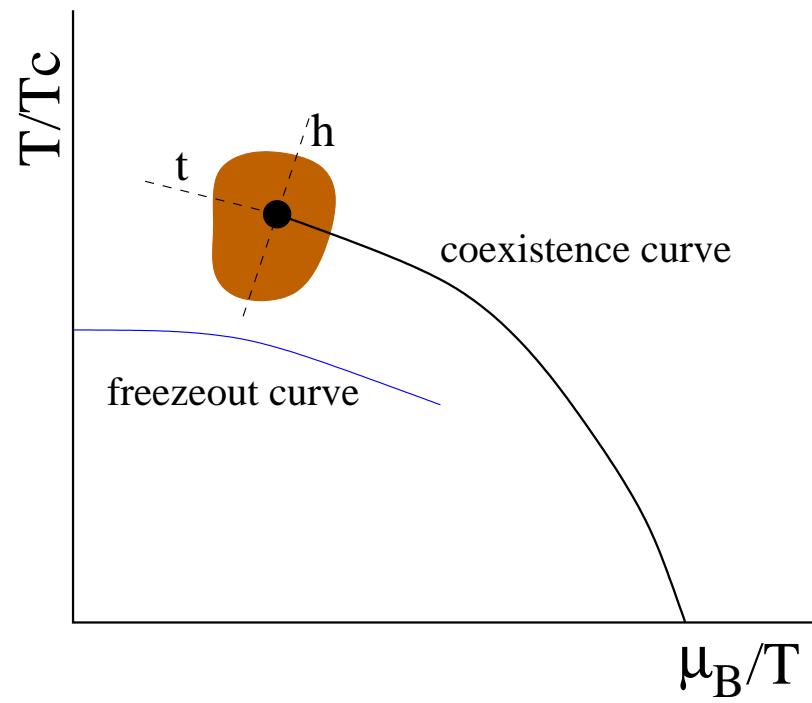
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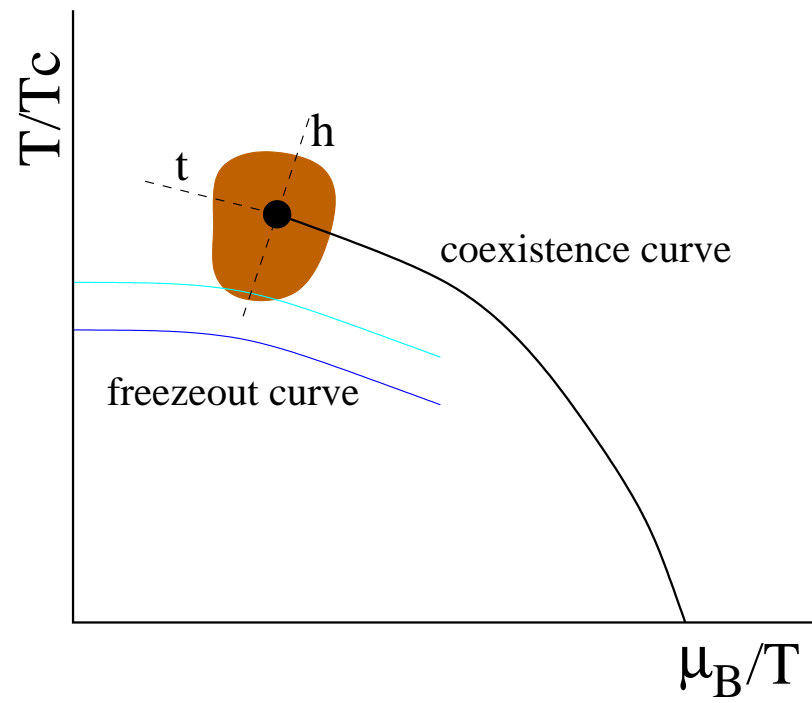
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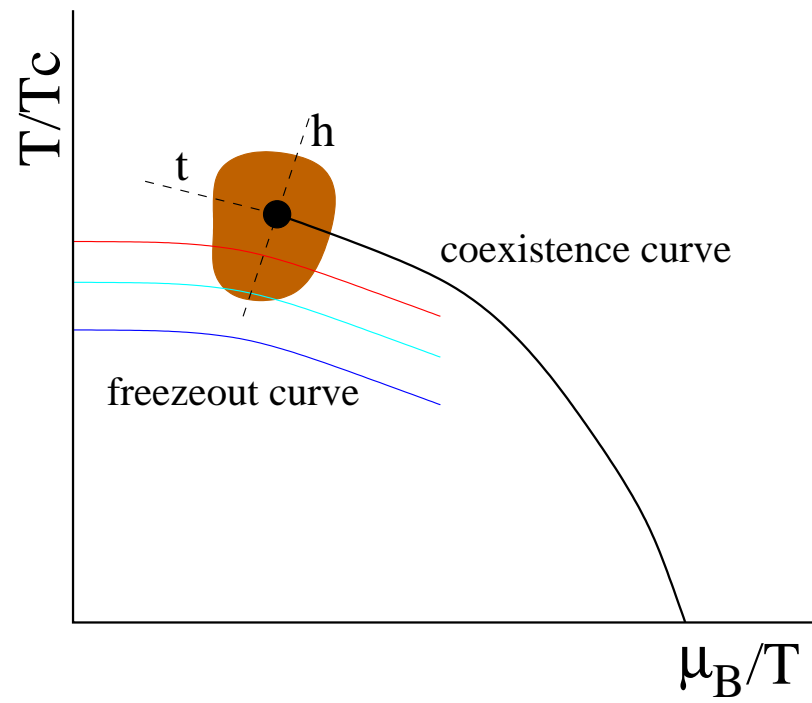
- Our Key Proposal : Use the freezeout curve from hadron abundances to *predict* fluctuations using lattice QCD along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)

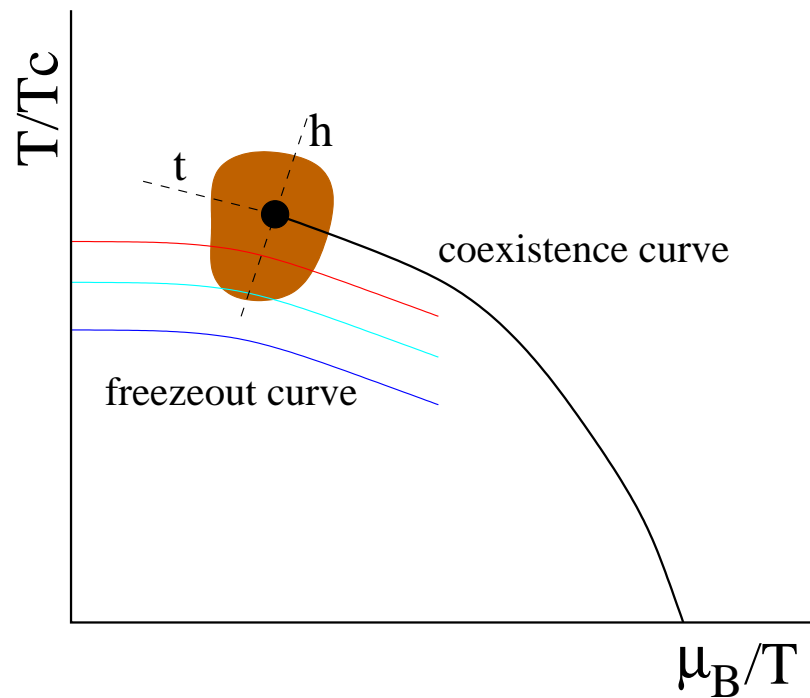




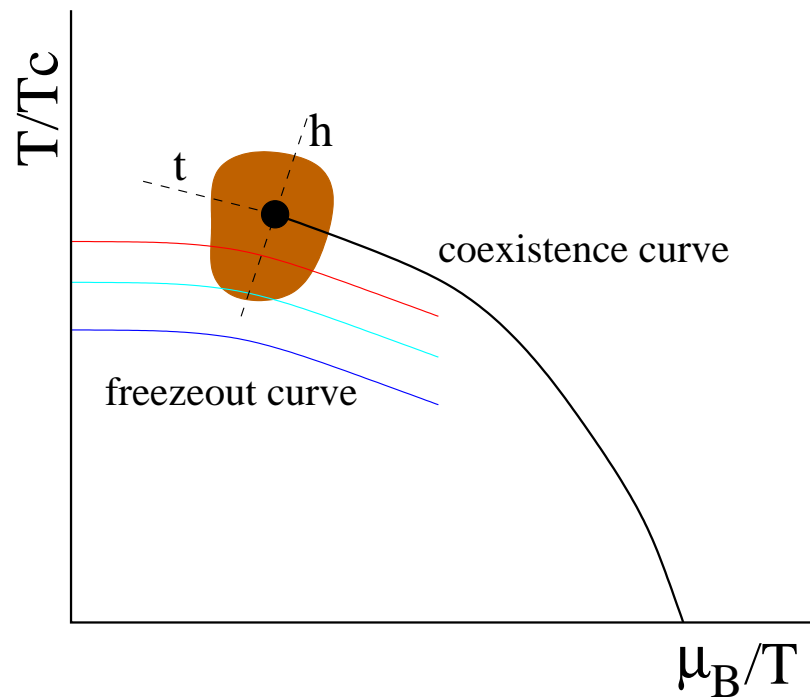






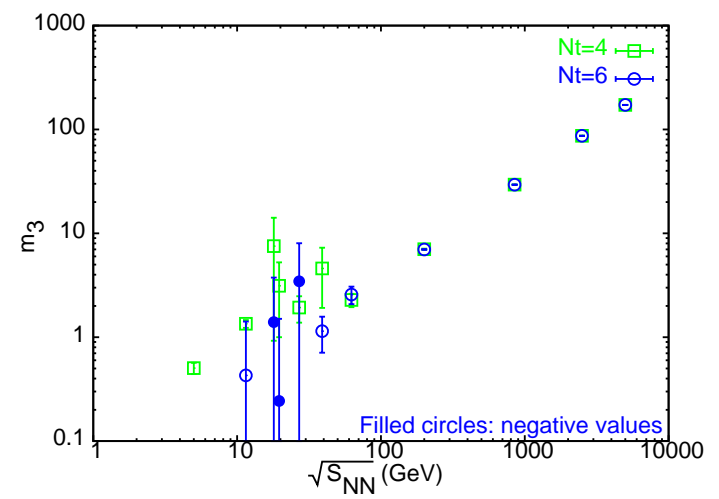
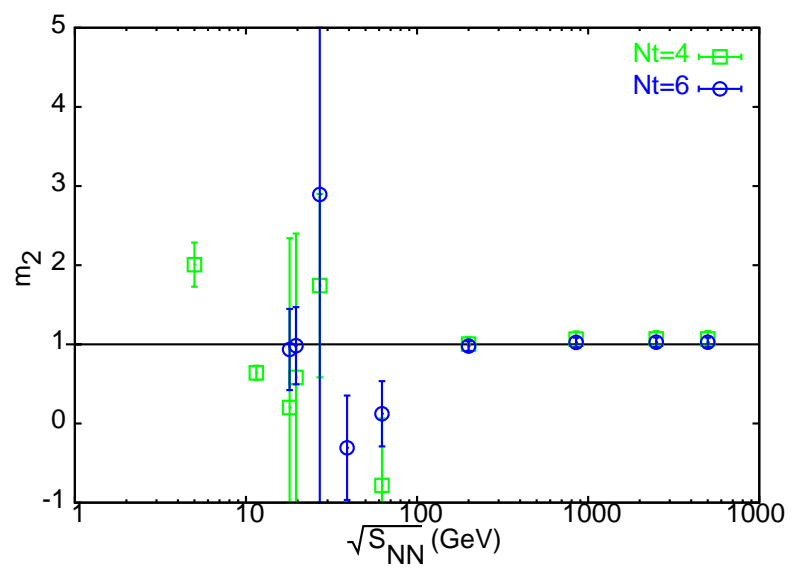
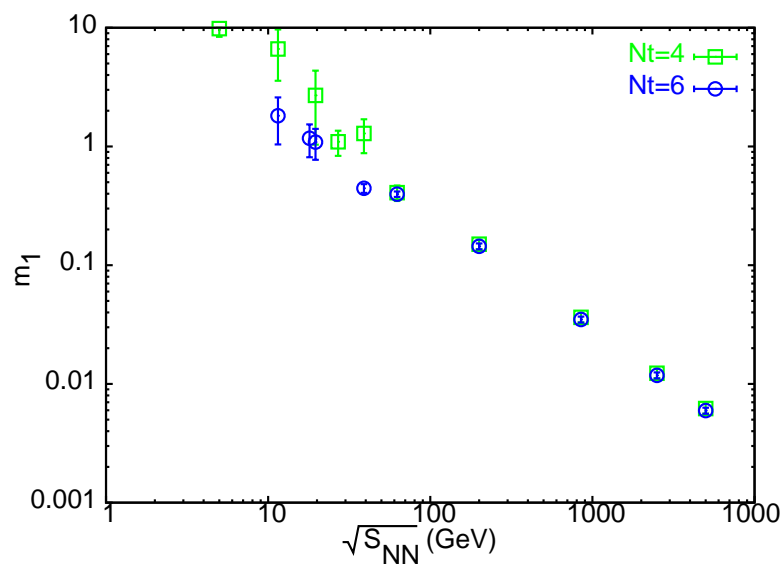


- Use the freezeout curve to relate  $(T, \mu_B)$  to  $\sqrt{s}$  and employ lattice QCD predictions along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)

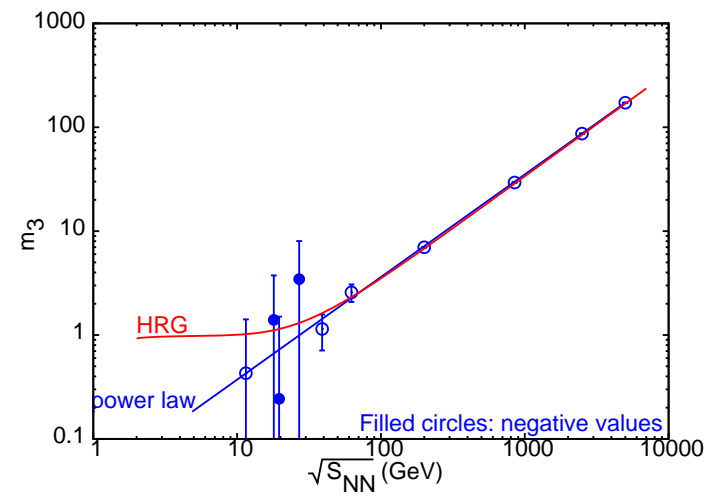
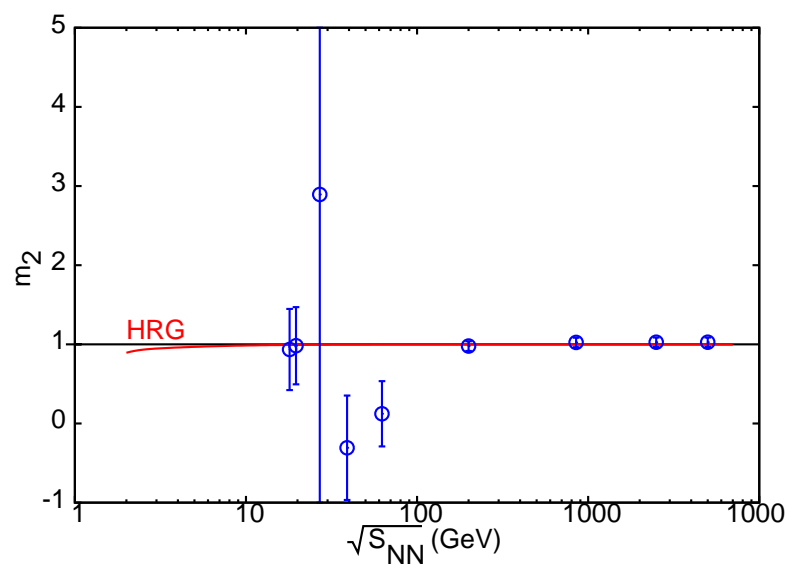
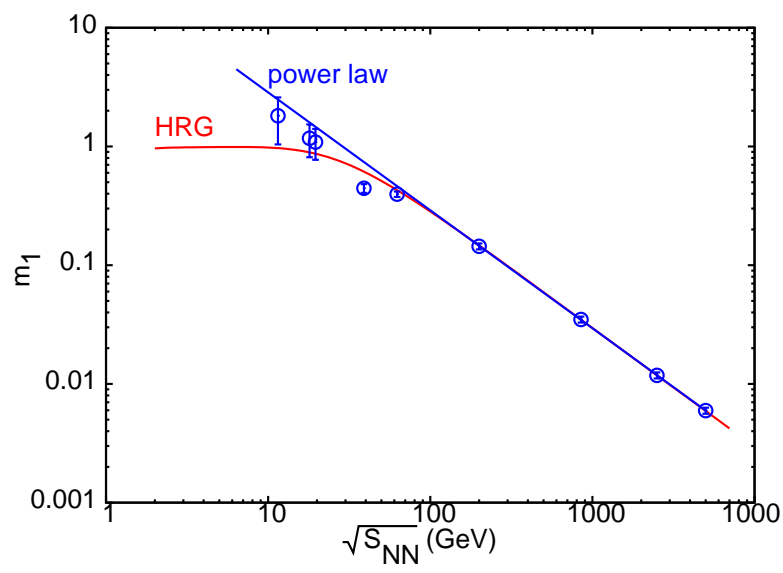


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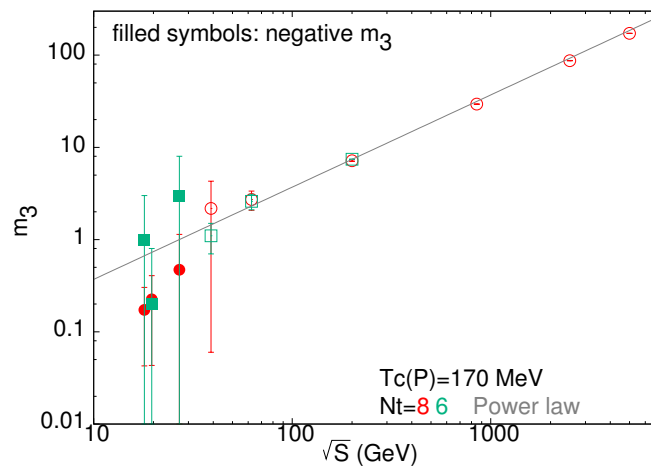
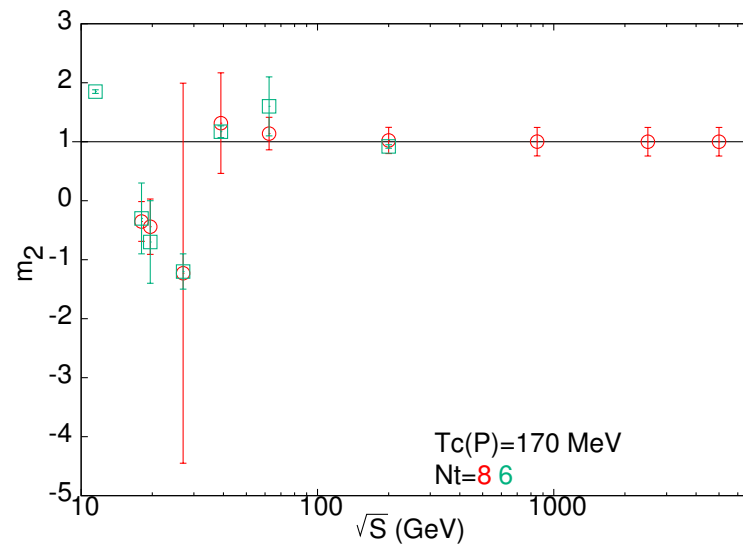
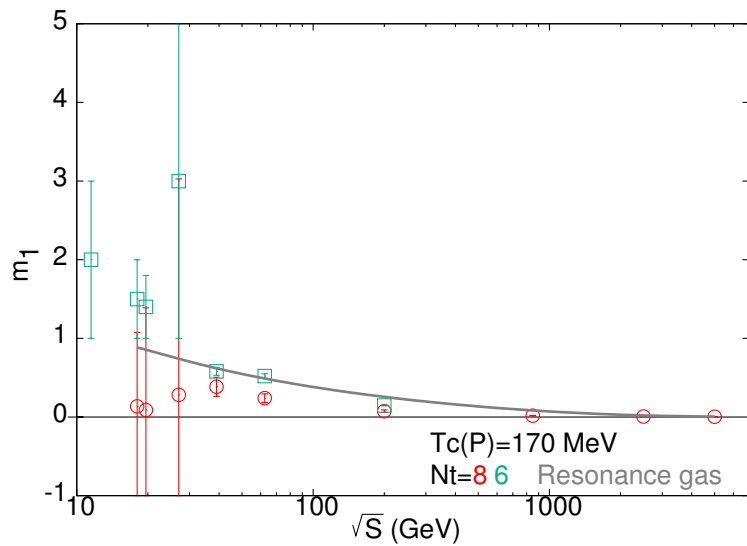
- Define  $m_1 = \frac{T\chi^{(3)}(T, \mu_B)}{\chi^{(2)}(T, \mu_B)}$ ,  $m_3 = \frac{T\chi^{(4)}(T, \mu_B)}{\chi^{(3)}(T, \mu_B)}$ , and  $m_2 = m_1 m_3$  and use the Padè method to construct them.



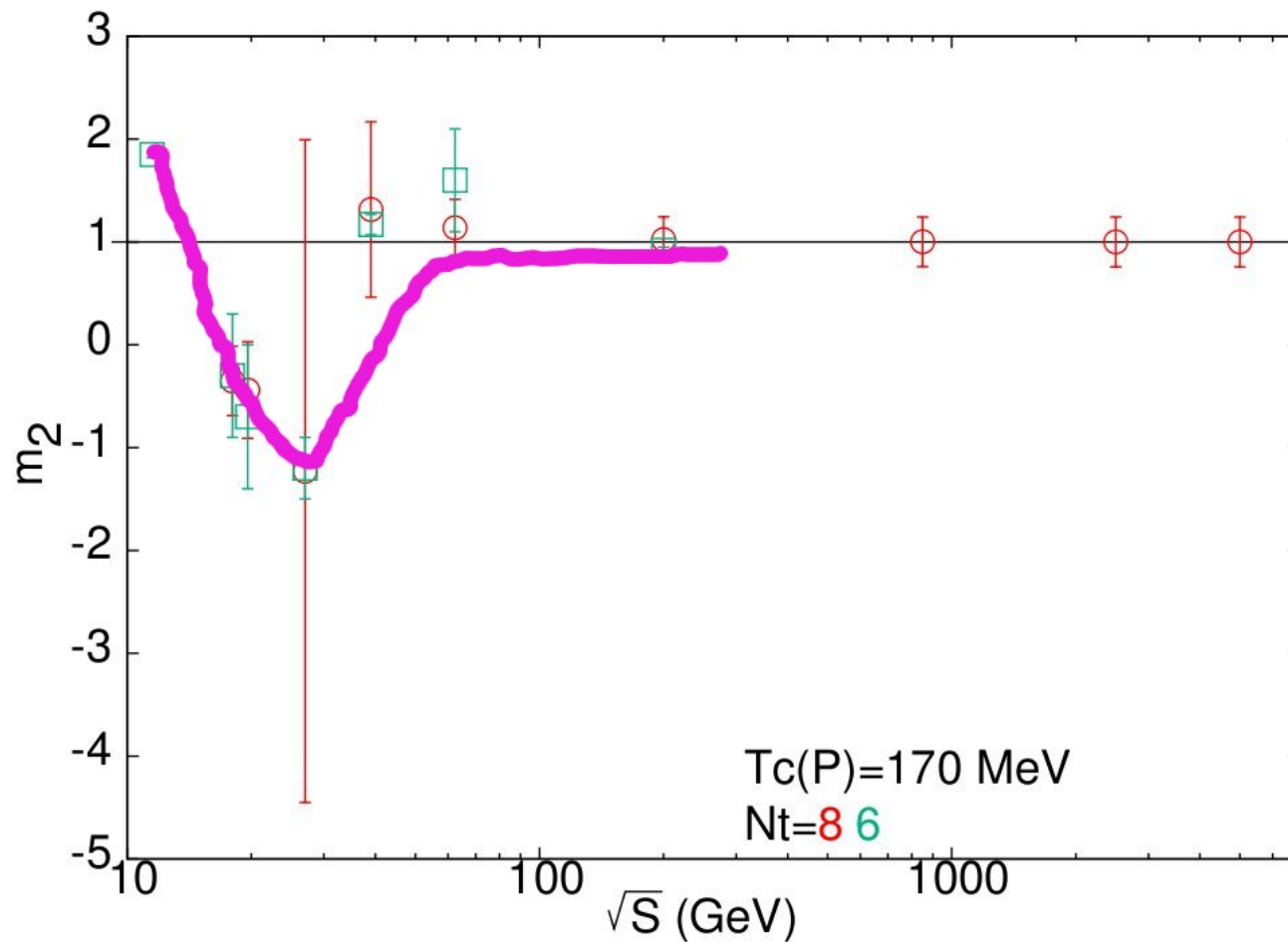
♠ Gavai & Gupta, arXiv: 1001.3796.



♠ Used  $T_c(\mu = 0) = 170$  MeV (Gavai & Gupta, arXiv: 1001.3796).



♠ Marginal change if  $T_c = 175$  MeV (Datta, Gavai & Gupta, QM '12).



Gavai-Gupta, '10 & Datta-Gavai-Gupta, QM '12

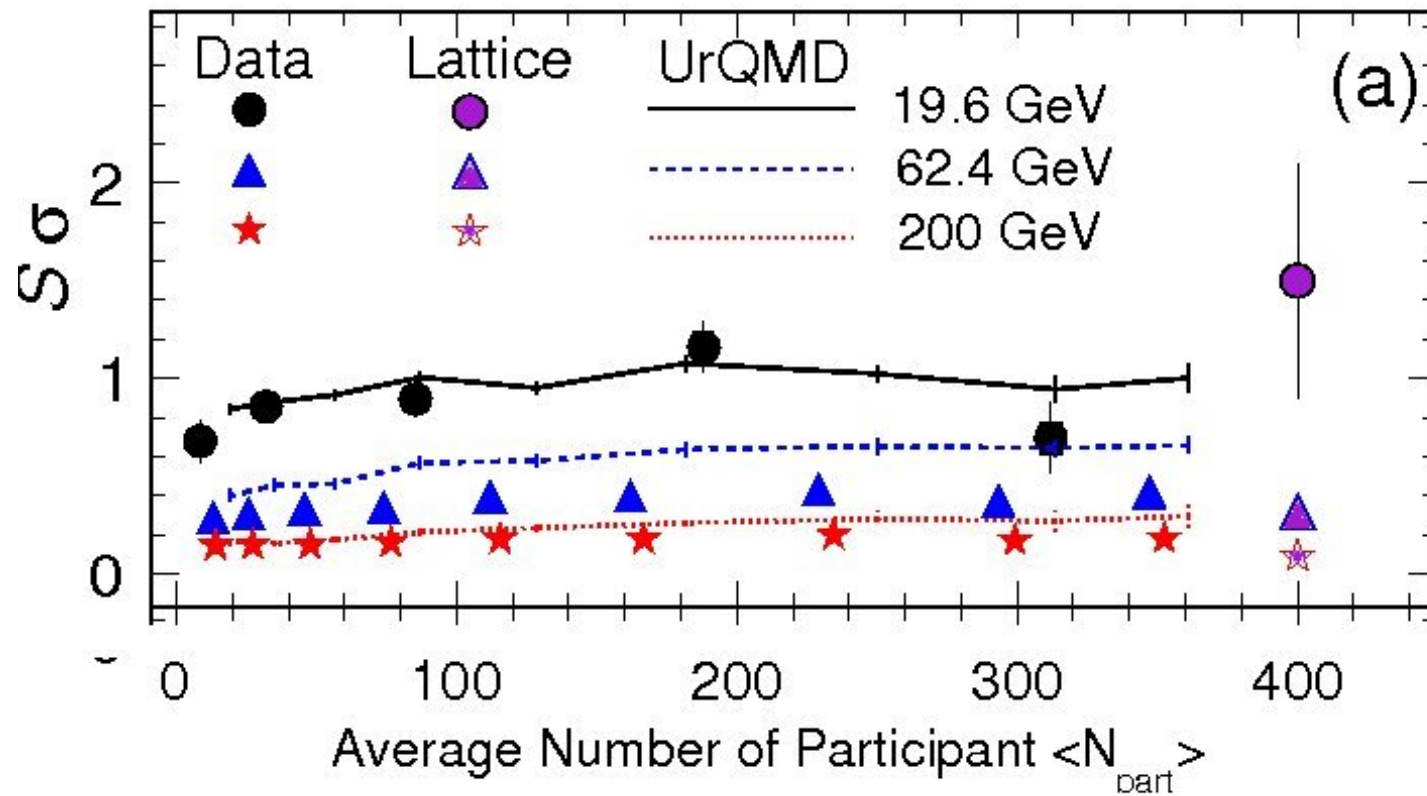
- Smooth & monotonic behaviour for large  $\sqrt{s}$  :  $m_1 \downarrow$  and  $m_3 \uparrow$ .
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Direct Non-Perturbative test of QCD in hot and dense environment.



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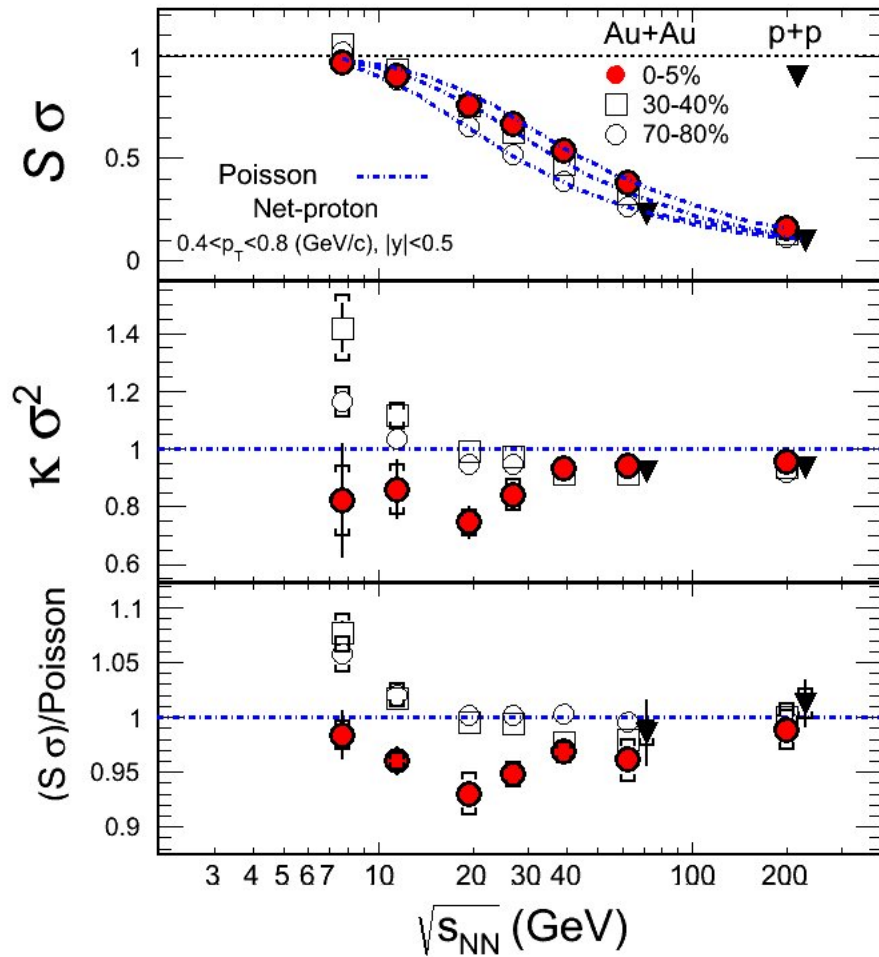
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- Leads to a ratio  $\chi_Q:\chi_I:\chi_B = 1:0:4$
- Assuming protons, neutrons, pions to dominate, both  $\chi_Q$  and  $\chi_B$  can be shown to be proton number fluctuations only.

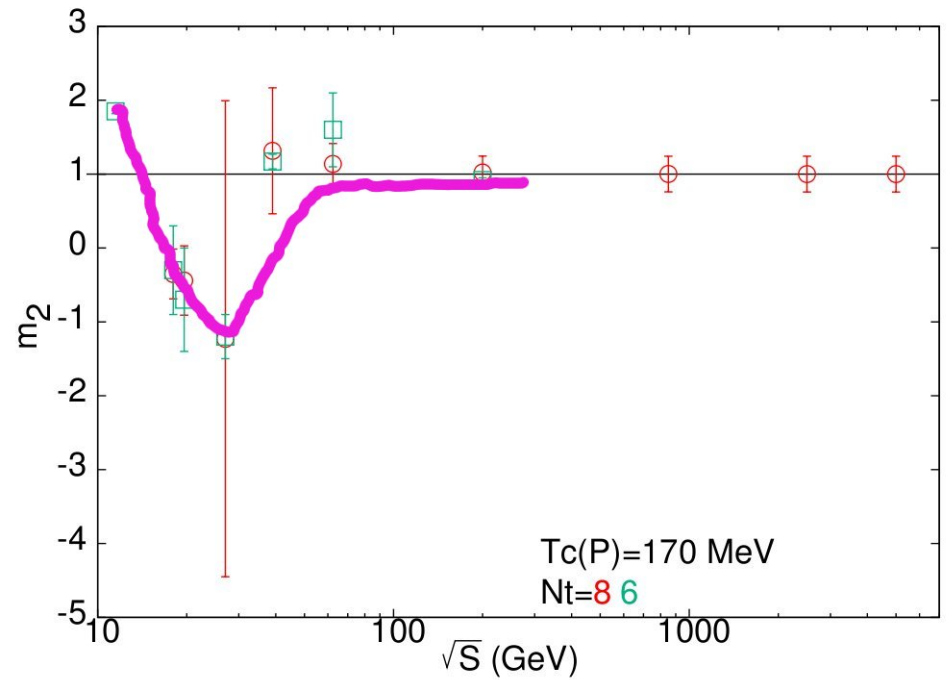


Aggarwal et al., STAR Collaboration, arXiv : 1004.4959

- Reasonable agreement with our lattice results. Where is the critical point ?



Xiaofeng Luo, QM'12  
From STAR Collaboration



Gavai-Gupta, '10  
Datta-Gavai-Gupta, QM '12

# Summary

- Phase diagram in  $T - \mu$  has begun to emerge: Different methods,  $\rightsquigarrow$  similar qualitative picture. Critical Point at  $\mu_B/T \sim 1 - 2$ .
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- Our results for  $N_t = 8$  first to begin the inching towards continuum limit.
- Critical Point leads to structures in  $m_i$  on the Freeze-Out Curve.
- STAR results appear to agree with our Lattice QCD predictions.

