Finite size scaling for 4-flavor QCD with finite chemical potential

Shinji Takeda

Kanazawa University

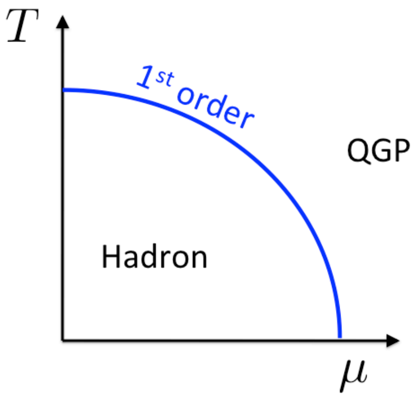
in collaboration with

X-Y. Jin, Y. Kuramashi, Y. Nakamura & A. Ukawa

GGI workshop “New frontiers in Lattice Gauge Theory’’

Why 4-flavor ?

- Good testing ground before 3-flavor
- Expected to have
  1st order phase transition line
  even for relatively larger mass

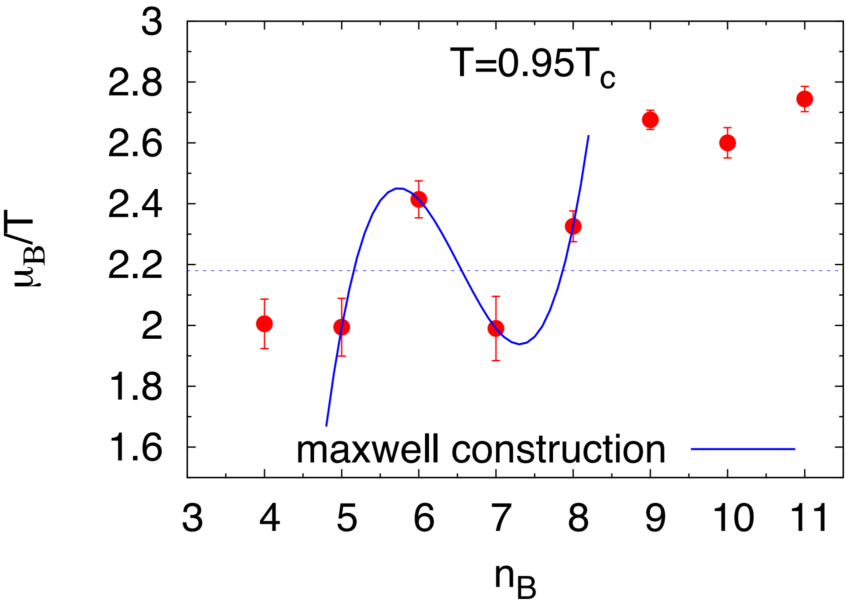


- Lattice study so far
  - Multi-parameter reweighting Fodor & Katz 01
  - Imaginary chemical potential D'Elia & Lombardo 02
  - Canonical approach de Forcrand 06, Kentucky 10

Kentucky group's result

PRD82.054502(2010)

- Canonical approach
- Nf=4
- Wilson type fermions
- T=150MeV
- pion mass=830MeV
- 6^3x4



It is likely to be 1st order, but need a finite size scaling study to pin down the order of transition!

What we do here

- Grand canonical approach with Wilson type fermions

$$\mathcal{Z}_{\rm QCD}(T,\mu) = \int [dU] e^{-S_{\rm g}[U]} \det D(\mu;U)$$

Complex

- Phase reweighting

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{iN_{\rm f}\theta} \rangle_{||}}{\langle e^{iN_{\rm f}\theta} \rangle_{||}}$$

Phase can be controlled for larger temporal size

ST, Kuramashi & Ukawa (2011)

$$\mathcal{Z}_{||}(T,\mu) = \int [dU] e^{-S_{\rm g}[U]} |\det D(\mu;U)|$$

- Reduction technique Danzer & Gattringer (2008)
  - exact phase & quark number
  - Many-core & GPU machine

Details of simulation parameters

- Clover fermions and Iwasaki gauge
- Parameters:
  - a=0.33fm (a^{-1}=610MeV)
  - pion mass=830MeV
  - T=150MeV
  - Chemical potential

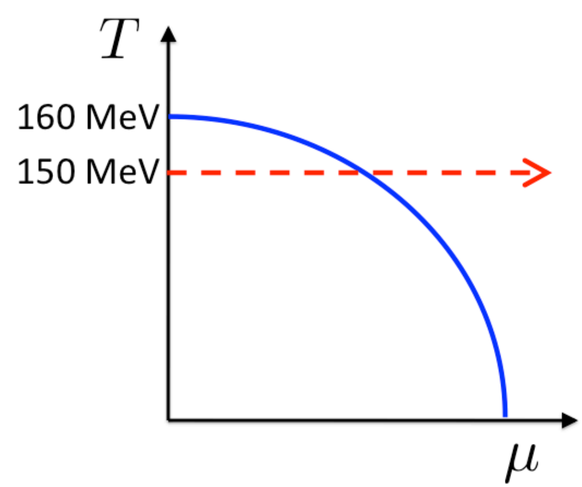
$$a\mu = 0.1 - 0.35$$

  - Spatial Volume

$$6^3,\ 668,\ 688,\ 8^3$$

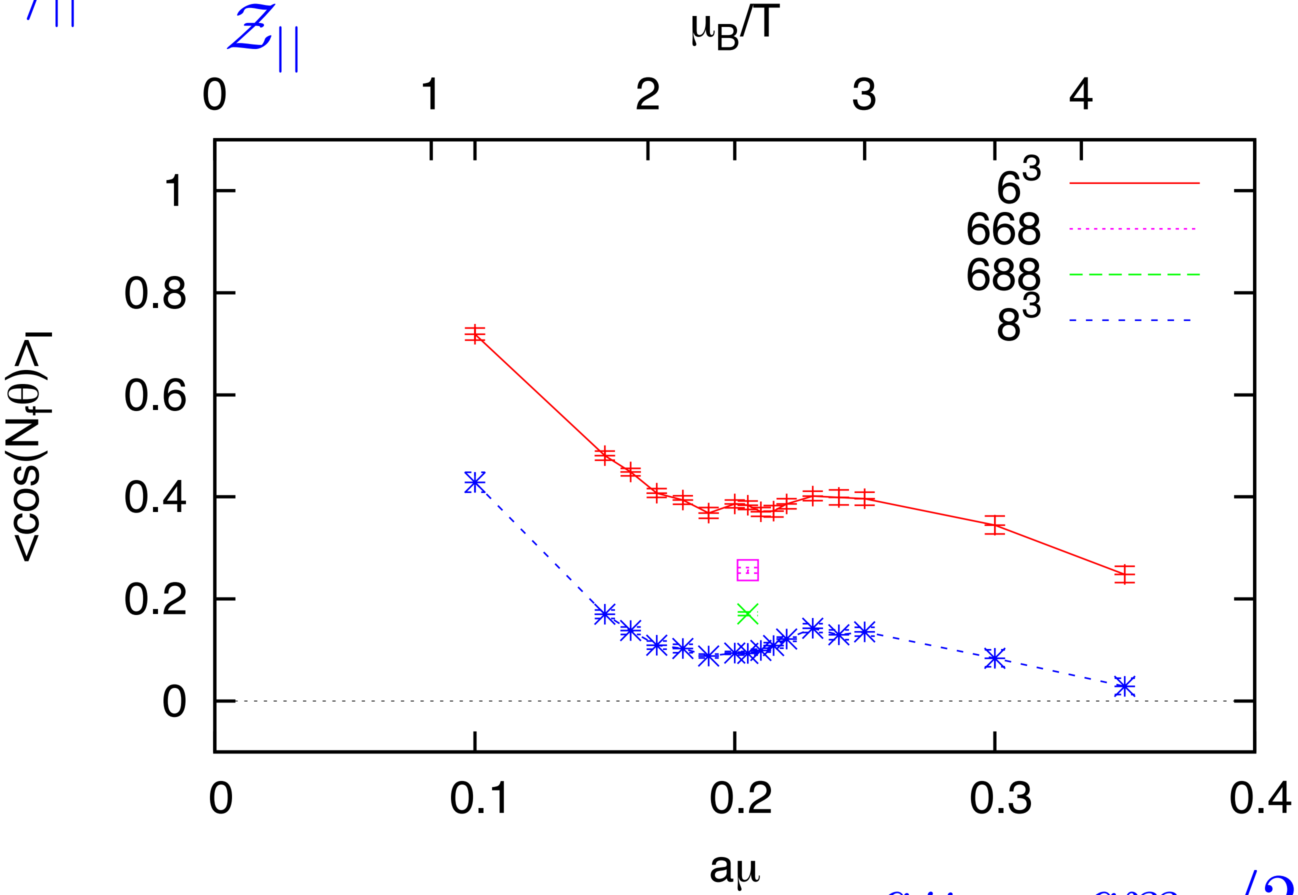
Kentucky group PRD82.054502(2010)

$$\beta = 1.6 \qquad \kappa = 0.1371$$
$$N_{\mathrm{T}} = 4 \qquad c_{\mathrm{sw}} = 1.9655$$



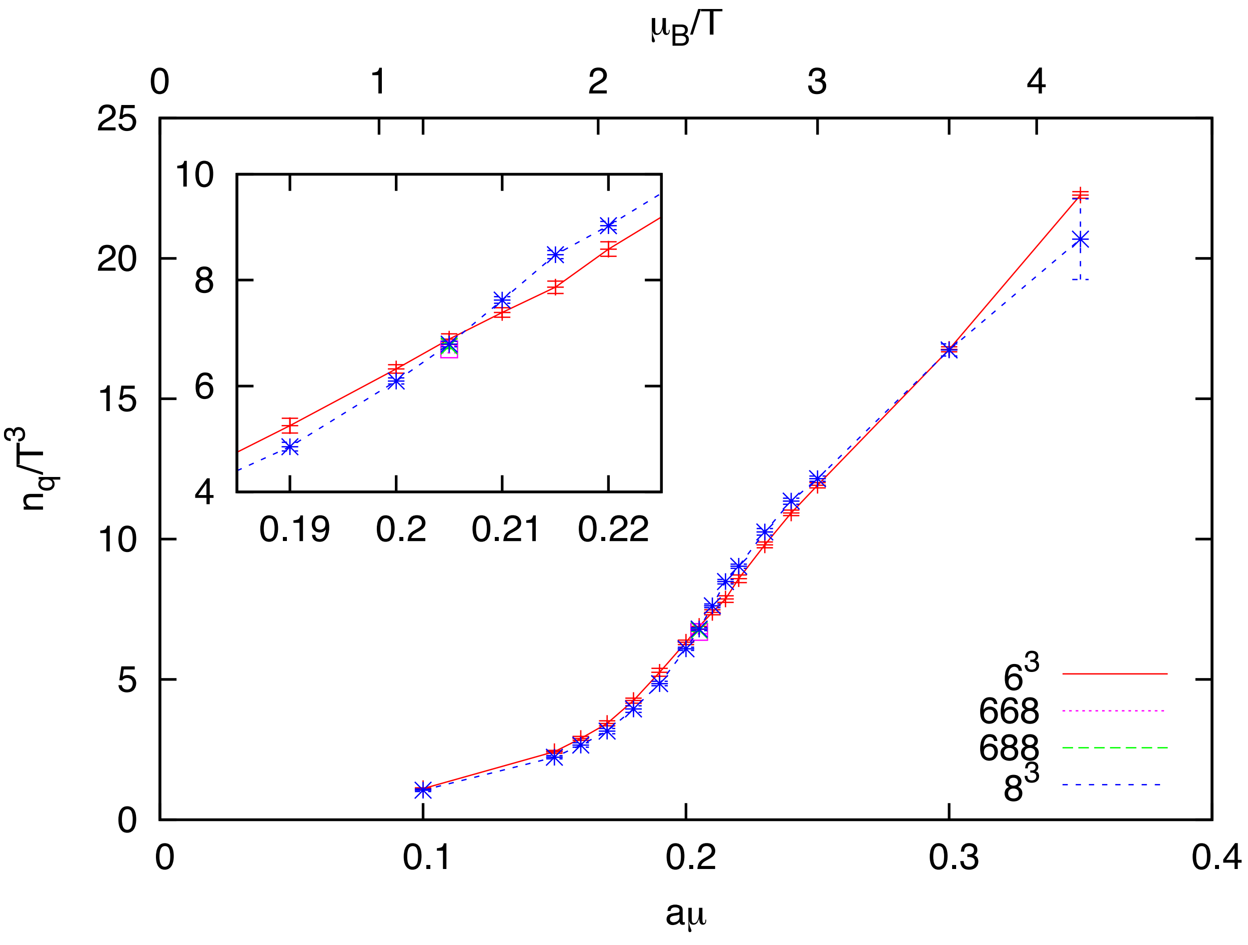
Phase-reweighting factor

$$\langle e^{i4\theta} \rangle_{||} = \frac{\mathcal{Z}_{\rm QCD}}{\mathcal{Z}_{||}}$$

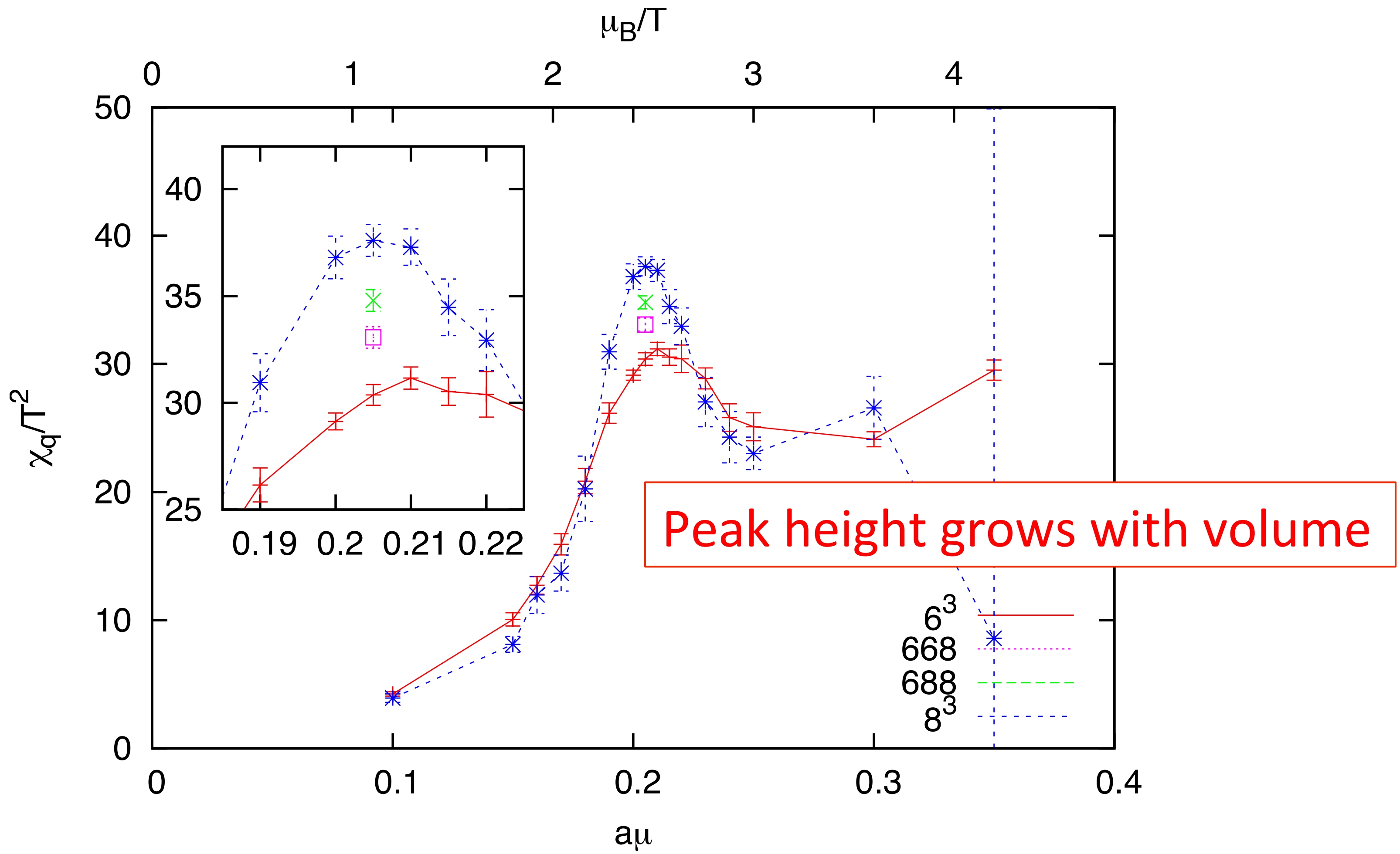


$$a\mu_c = am_\pi/2 \sim 0.7$$

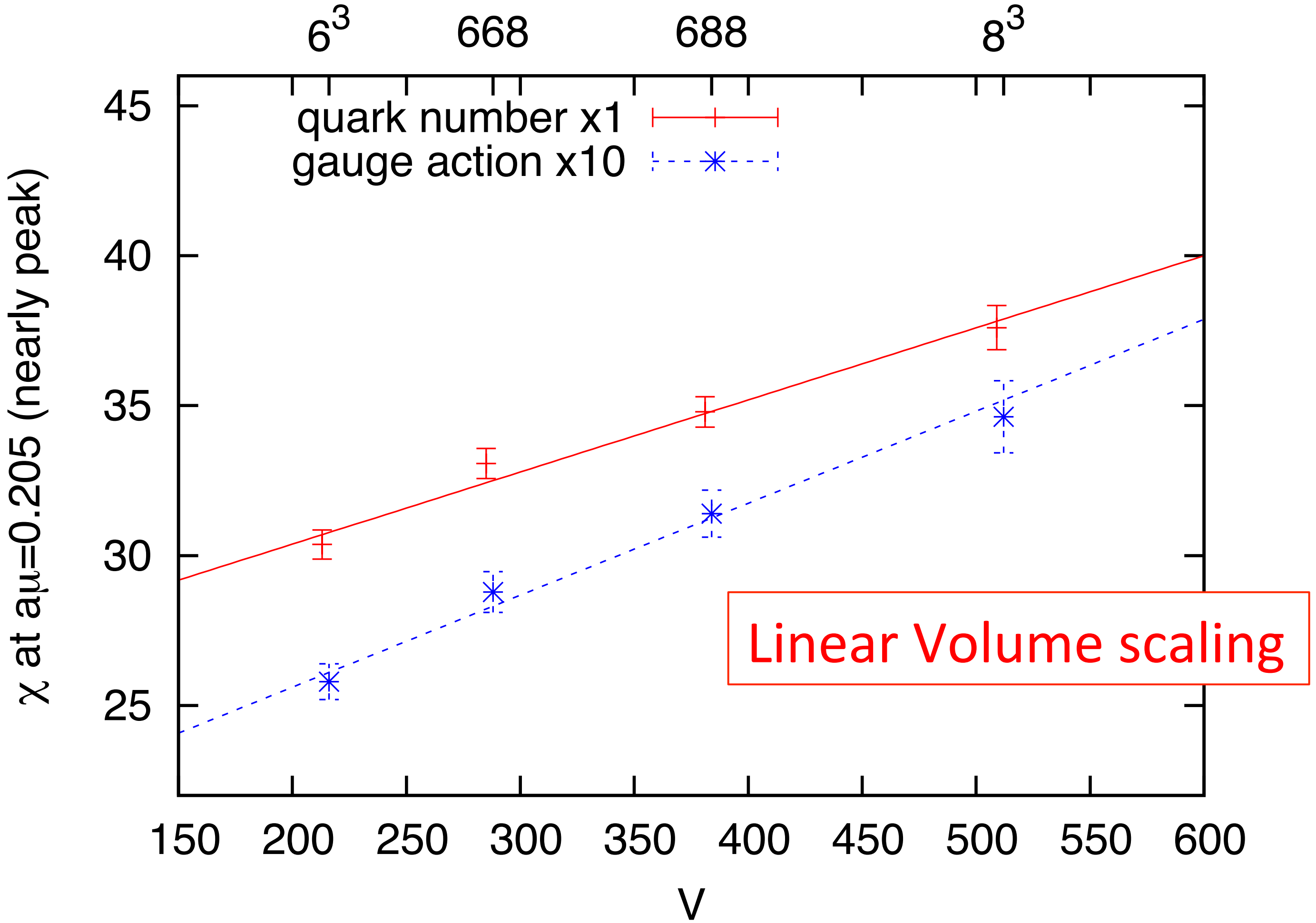
Quark Number density



Susceptibility of quark number

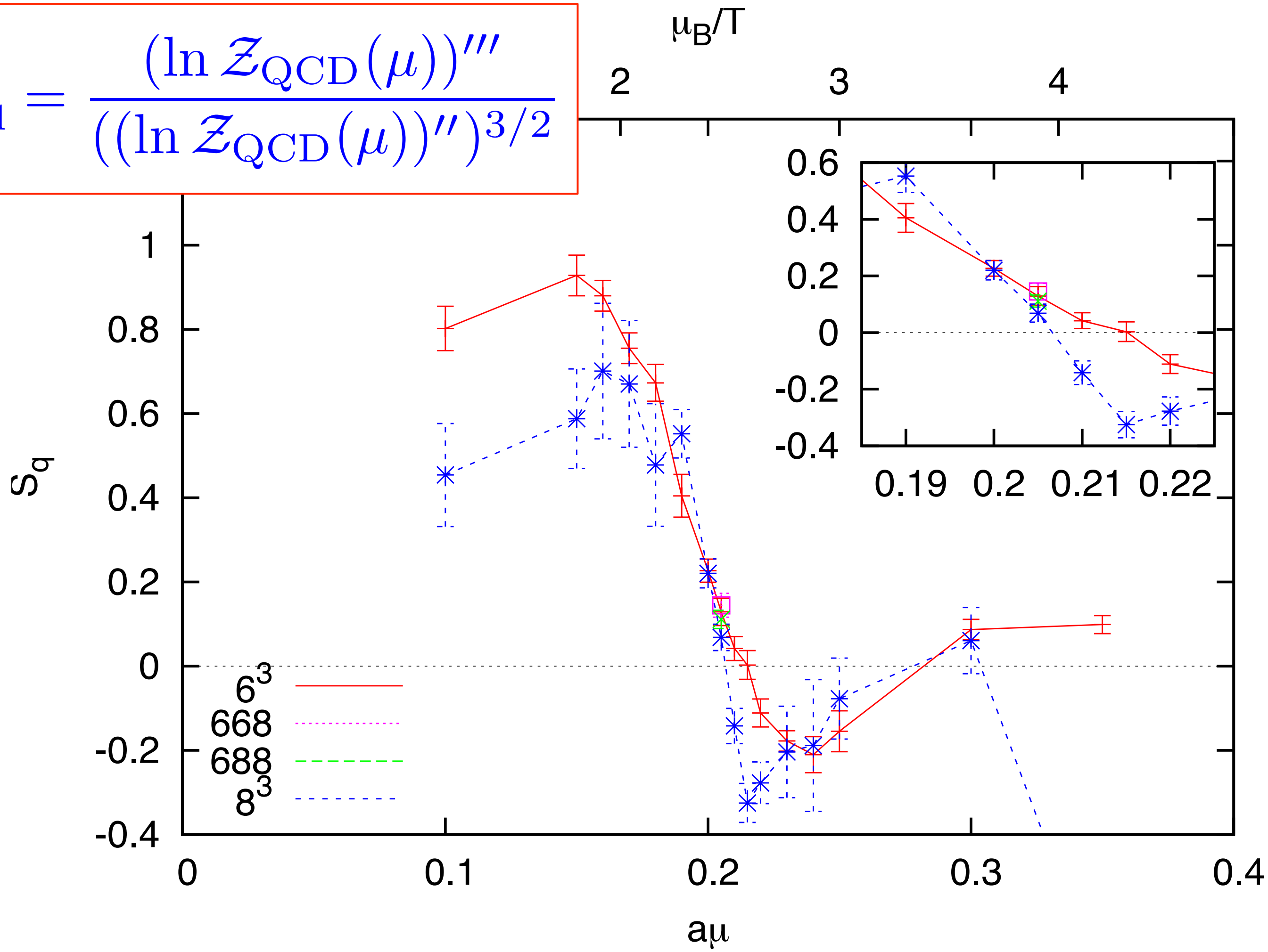


Volume scaling for peak of susceptibility



Skewness for quark number

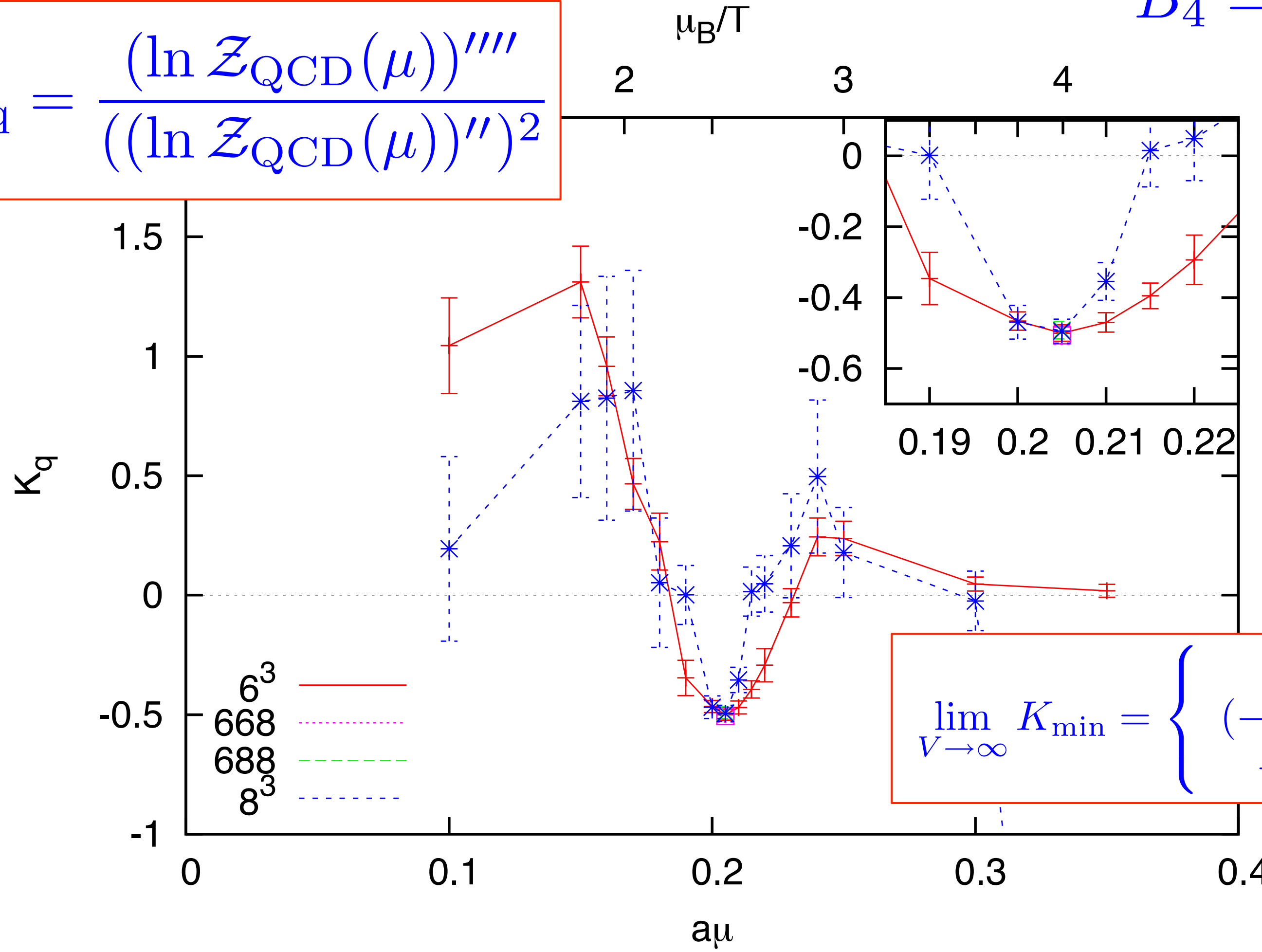
$$S_{\rm q} = \frac{(\ln \mathcal{Z}_{\rm QCD}(\mu))'''}{((\ln \mathcal{Z}_{\rm QCD}(\mu))'')^{3/2}}$$



Kurtosis for quark number

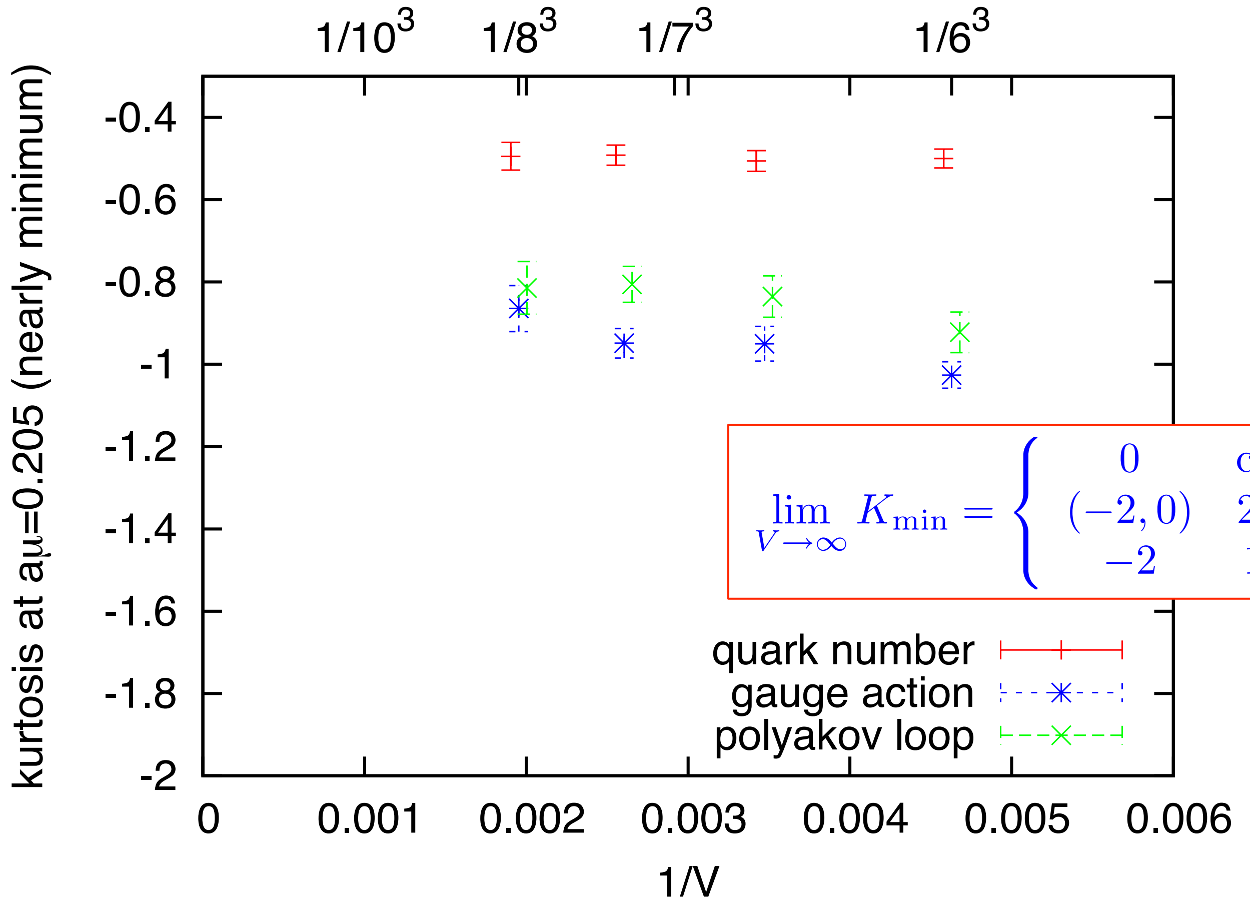
$$B_4 = K + 3$$

$$K_{\mathrm{q}} = \frac{(\ln \mathcal{Z}_{\mathrm{QCD}}(\mu))''''}{((\ln \mathcal{Z}_{\mathrm{QCD}}(\mu))'')^2}$$



$$\lim_{V\to\infty} K_{\min} = \begin{cases} 0 & \text{cross over} \\ (-2, 0) & \text{2nd order} \\ -2 & \text{1st order} \end{cases}$$

Scaling for the minimum of kurtosis

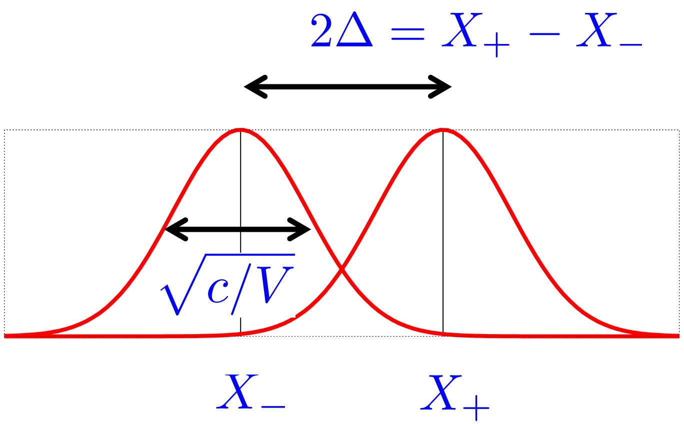


$$\lim_{V \to \infty} K_{\min} = \begin{cases} 0 & \text{cross over} \\ (-2, 0) & \text{2nd order} \\ -2 & \text{1st order} \end{cases}$$

Distribution argument

- For a double peak distribution

$$P(x) \propto e^{-\frac{(x-X_-)^2}{2c/V}} + e^{-\frac{(x+X_+)^2}{2c/V}}$$



- Susceptibility & Kurtosis are given by

For our case, $\frac{c}{\Delta^2} \sim V$

$$\chi = c + \Delta^2 V$$

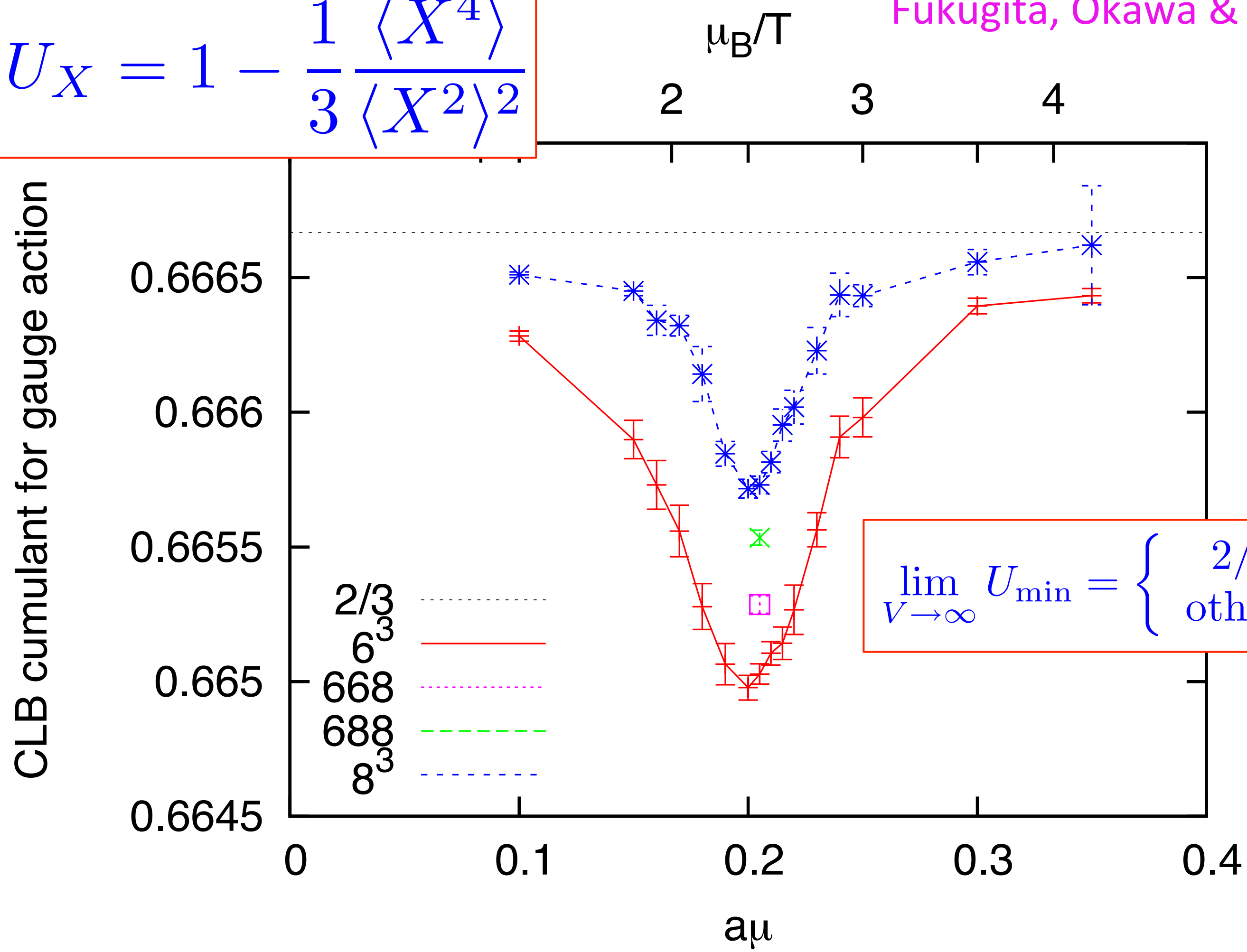
$$K = \frac{-2}{(1+\frac{c}{\Delta^2}\frac{1}{V})^2} = -2\left[1 - \frac{2c}{\Delta^2}\frac{1}{V} + O(V^{-2})\right]$$

Challa Landau Binder cumulant

Challa, Landau & Binder 86

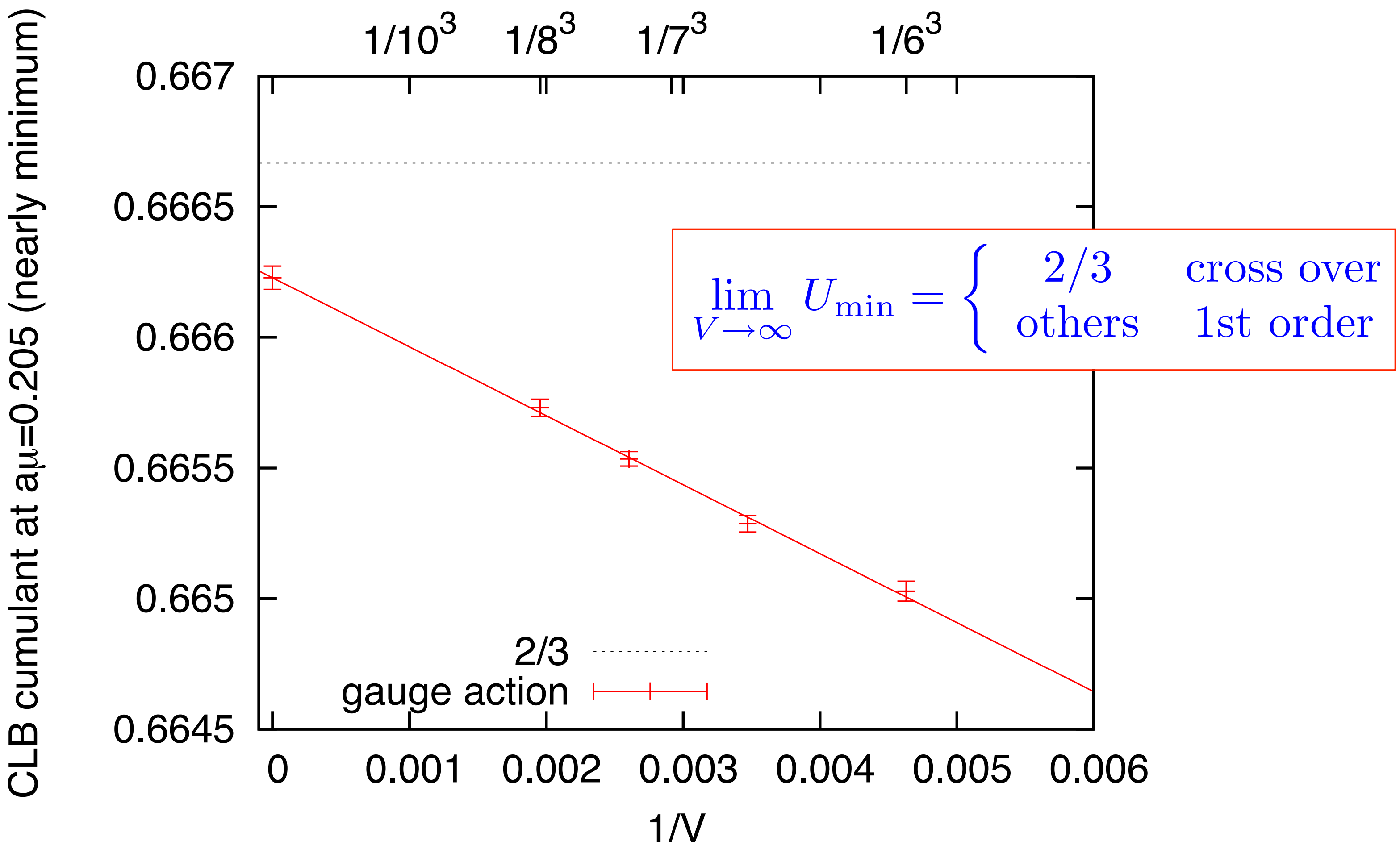
Fukugita, Okawa & Ukawa 89

$$U_X = 1 - \frac{1}{3}\frac{\langle X^4 \rangle}{\langle X^2 \rangle^2}$$



$$\lim_{V\to\infty} U_{\min} = \begin{cases} 2/3 & \text{cross over} \\ \text{others} & \text{1st order} \end{cases}$$

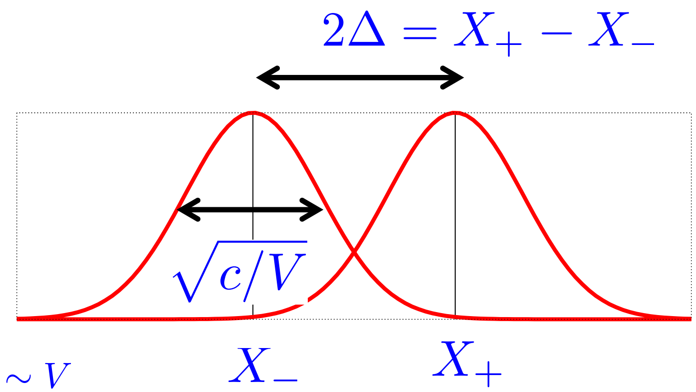
Scaling for the minimum of CLB cumulant



Distribution argument

- For a double peak distribution

$$P(x) \propto e^{-\frac{(x-X_-)^2}{2c/V}} + e^{-\frac{(x+X_+)^2}{2c/V}}$$



$\frac{c}{\Delta^2} \sim V$

- Kurtosis & CLB are given by

$$K \equiv \frac{\langle (x-\langle x \rangle)^4 \rangle}{\langle (x-\langle x \rangle)^2 \rangle^2} - 3 = -2\left[1 - \frac{2c}{\Delta^2}\frac{1}{V} + O\left(\left(\frac{c}{\Delta^2}\frac{1}{V}\right)^2\right)\right]$$

$$X = \frac{1}{2}(X_+ + X_-)$$

$$U \equiv 1 - \frac{\langle x^4 \rangle}{3\langle x^2 \rangle^2} = \frac{2}{3}\frac{X^4 + \Delta^4}{(X^2+\Delta^2)^2}\left[1 - \frac{2c}{X^2+\Delta^2}\frac{1}{V} + O\left(\left(\frac{c}{X^2+\Delta^2}\frac{1}{V}\right)^2\right)\right]$$

For an observable like gauge action $X \gg \Delta$

Summary

Distribution argument can explain them

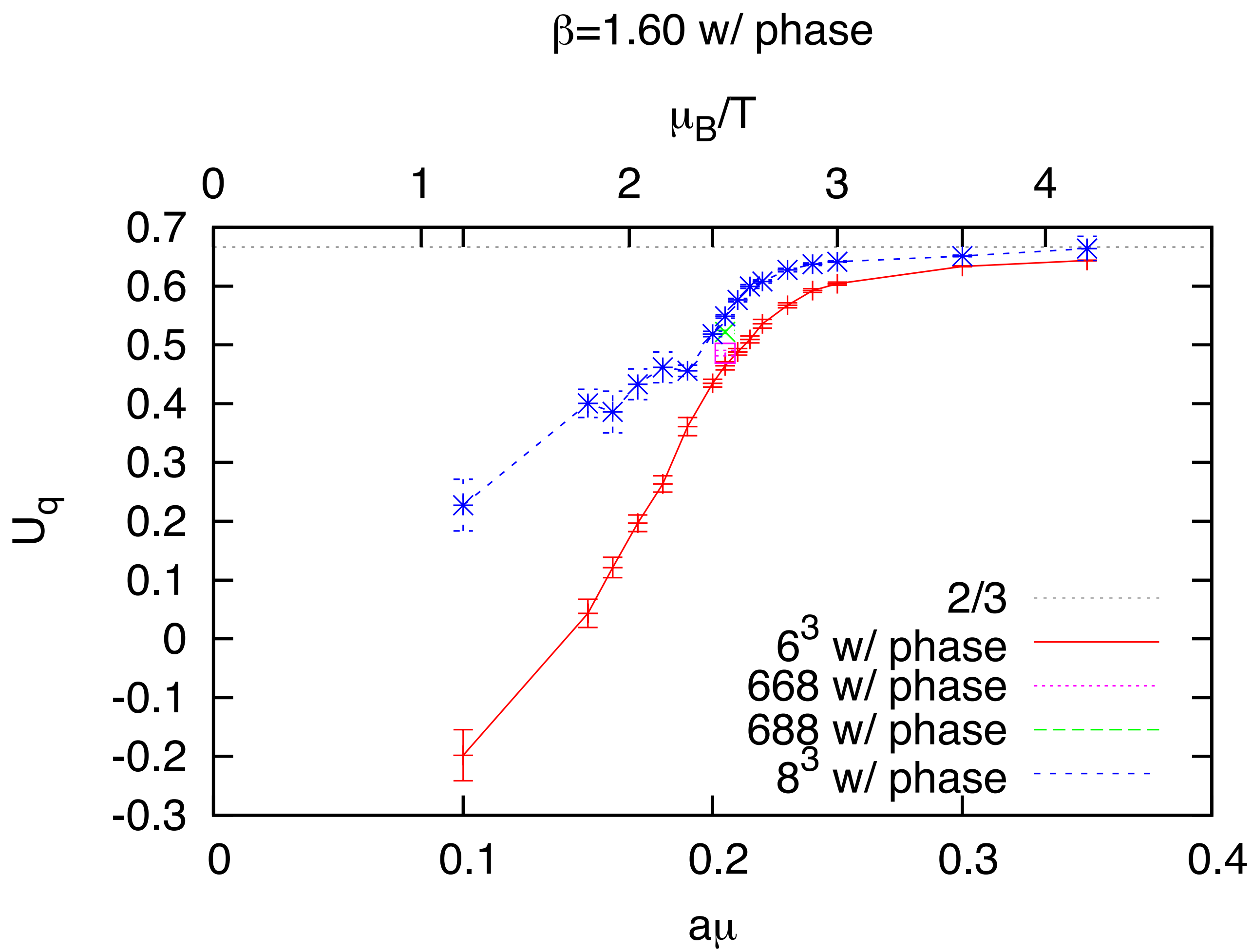
- Volume scaling
  - Susceptibility : linear volume scaling is observed
  - Kurtosis : volume is too small to see 1/V scaling
  - CLB cumulant : 1/V scaling is clearly observed
- At this parameter set (pion mass=830MeV, T=150MeV, μ_B=300MeV), transition is consistent with 1st order

Future

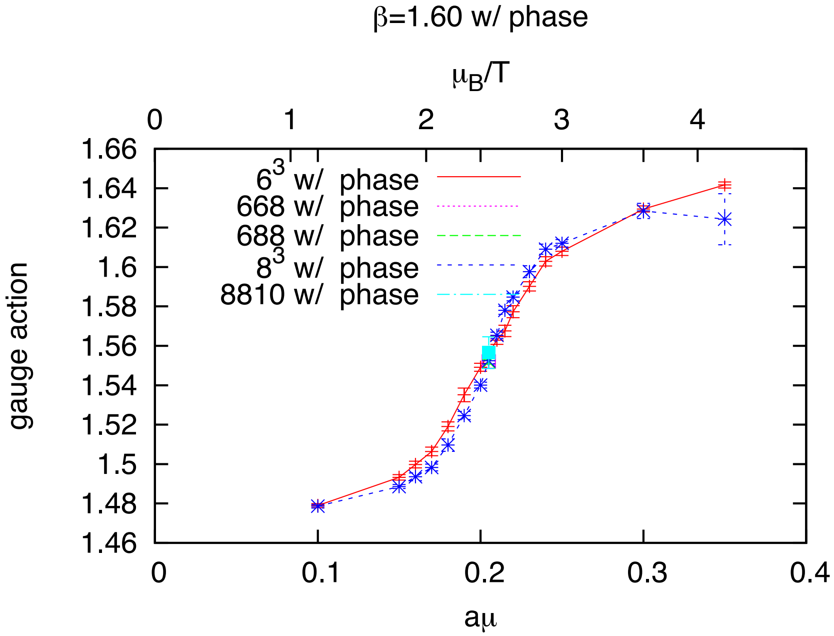
- Further refined analysis (identifying a transition point precisely by μ-reweighting) to consolidate the above conclusion
- Lee-Yang zero analysis
- 3-flavor / 2+1-flavor study

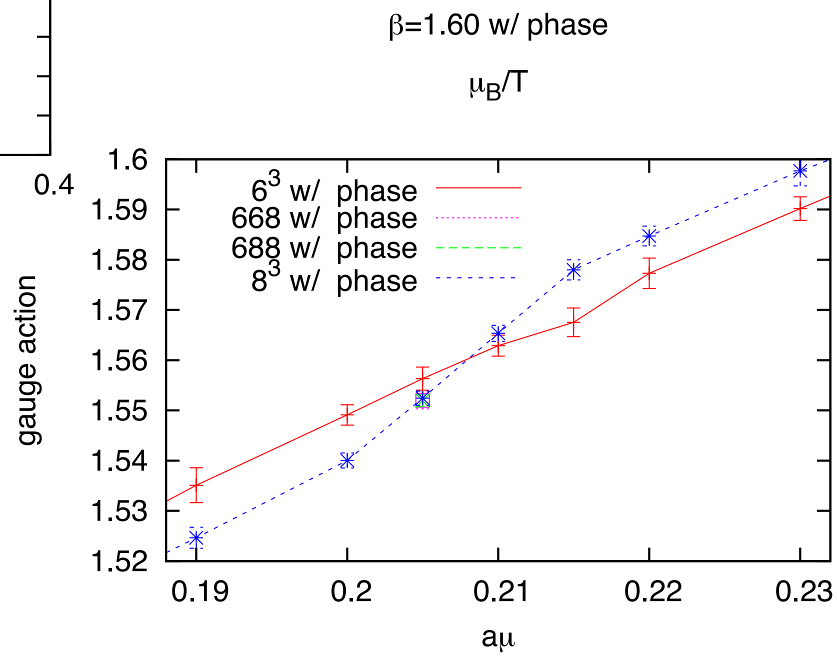
BACK UP SLIDES

CLB cumulant for quark number

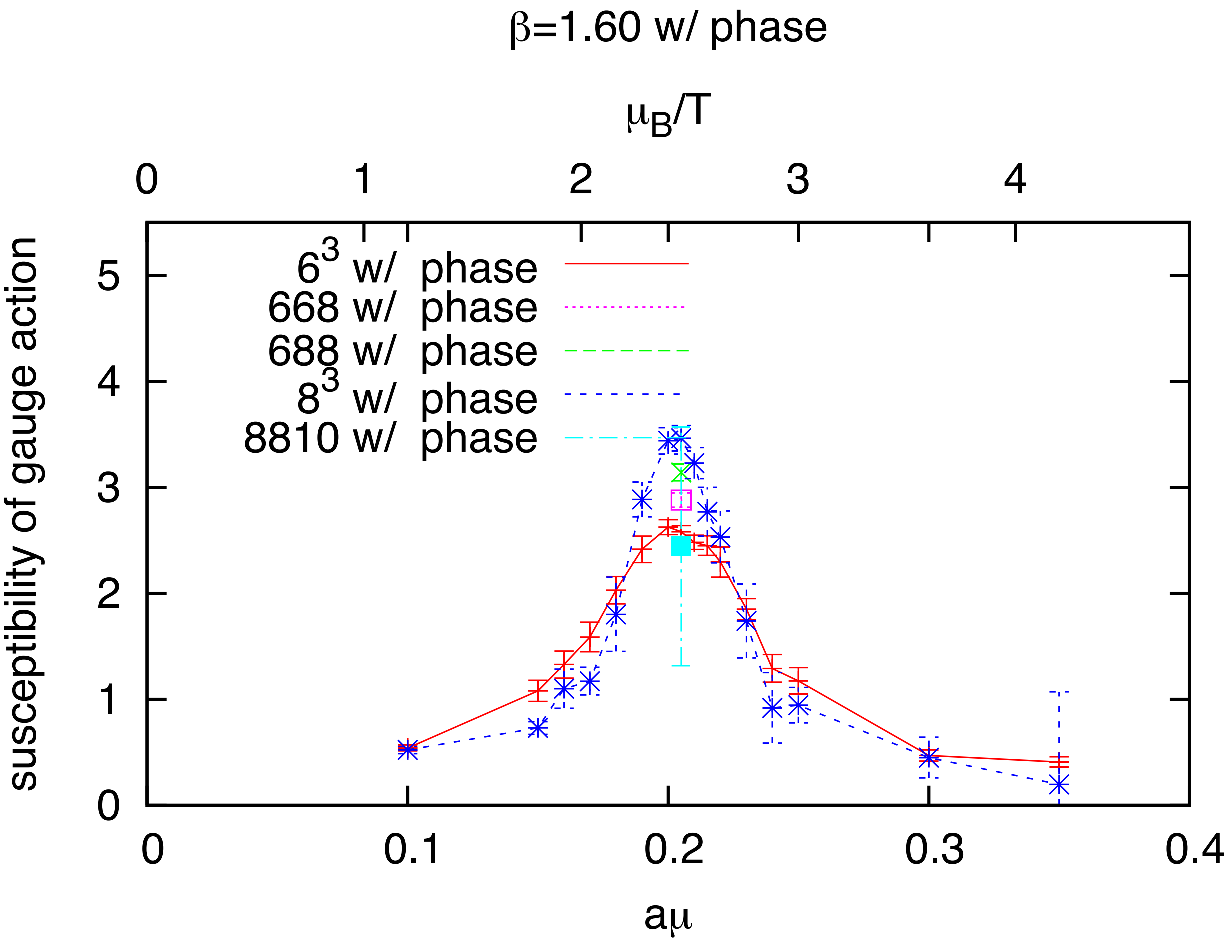


Gauge action

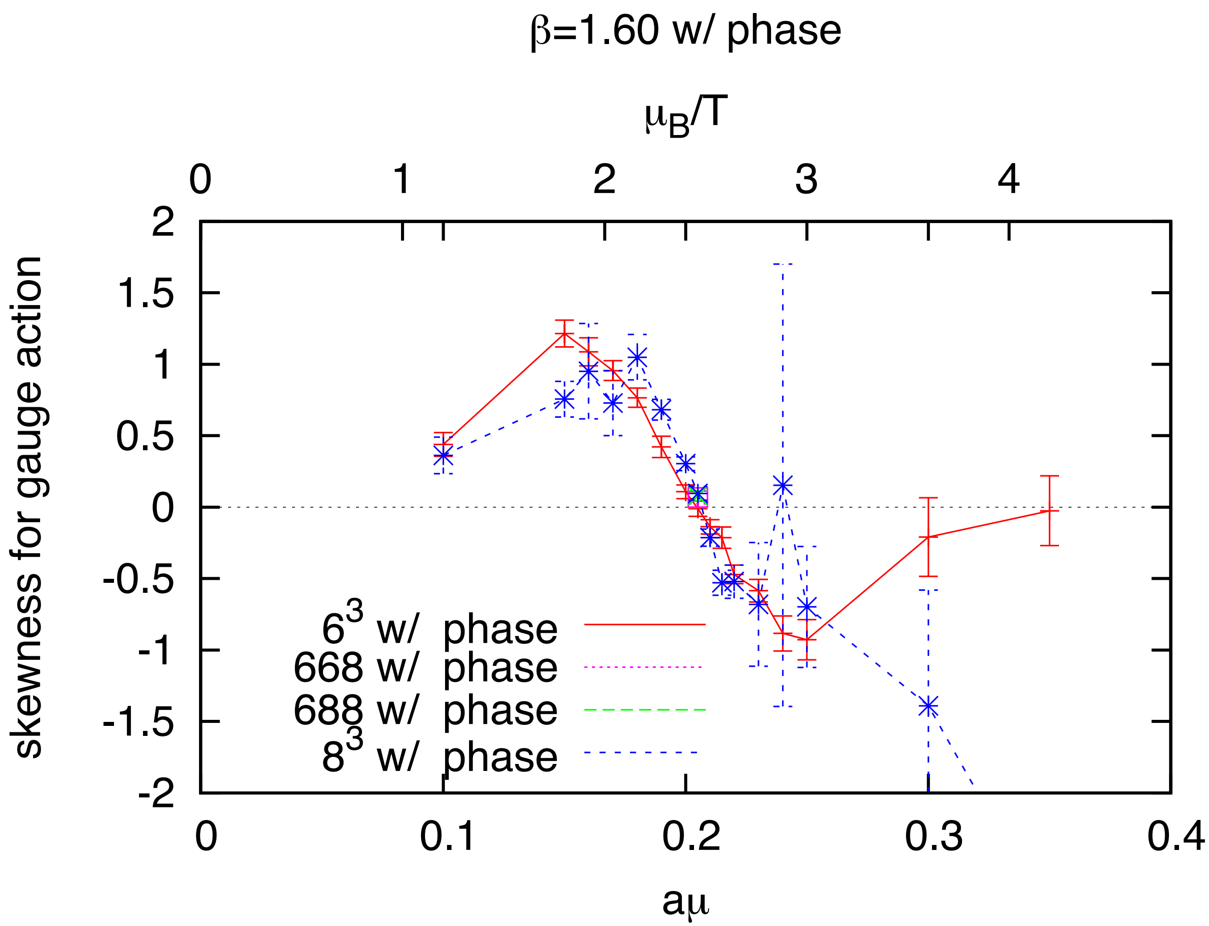




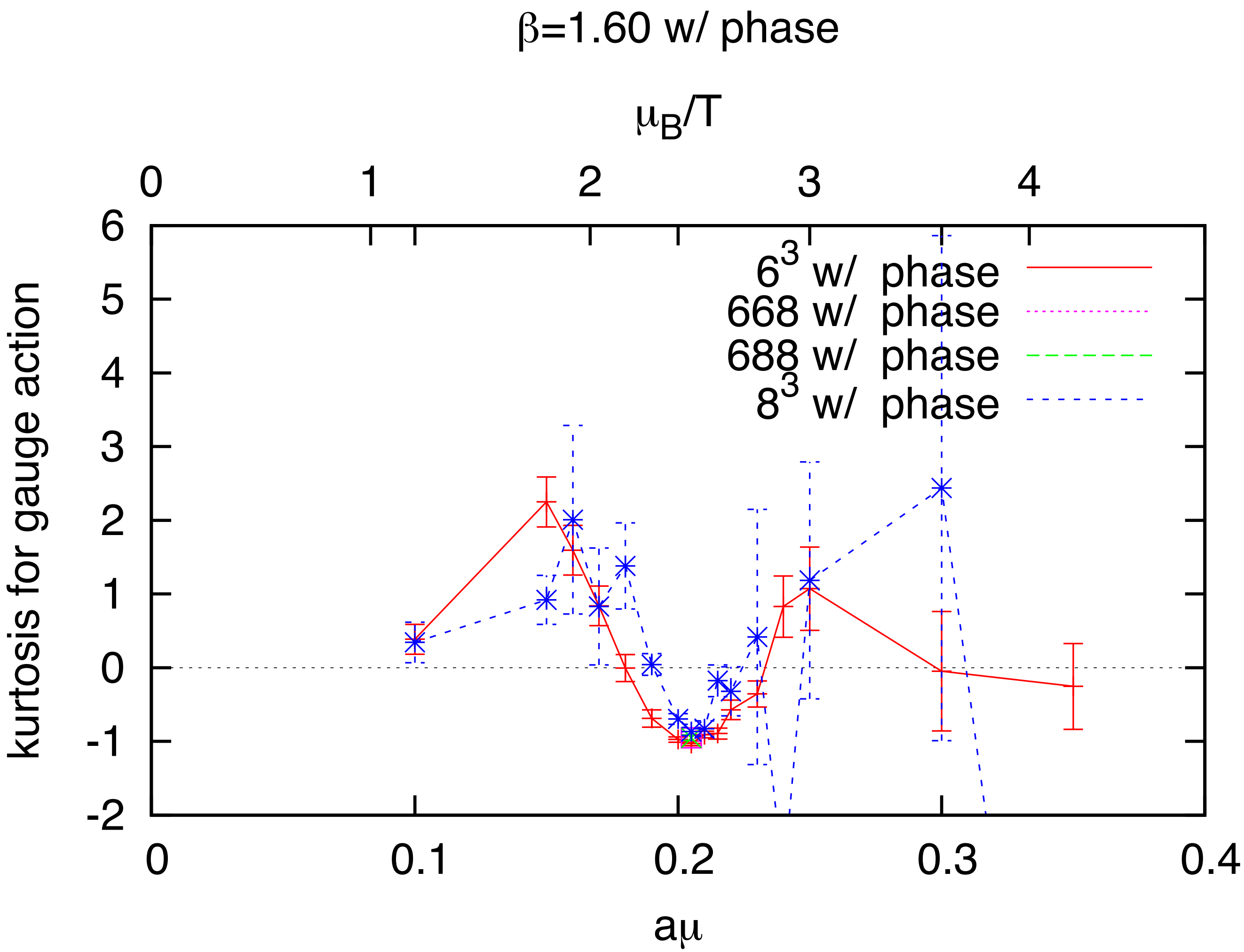
Susceptibility of gauge action



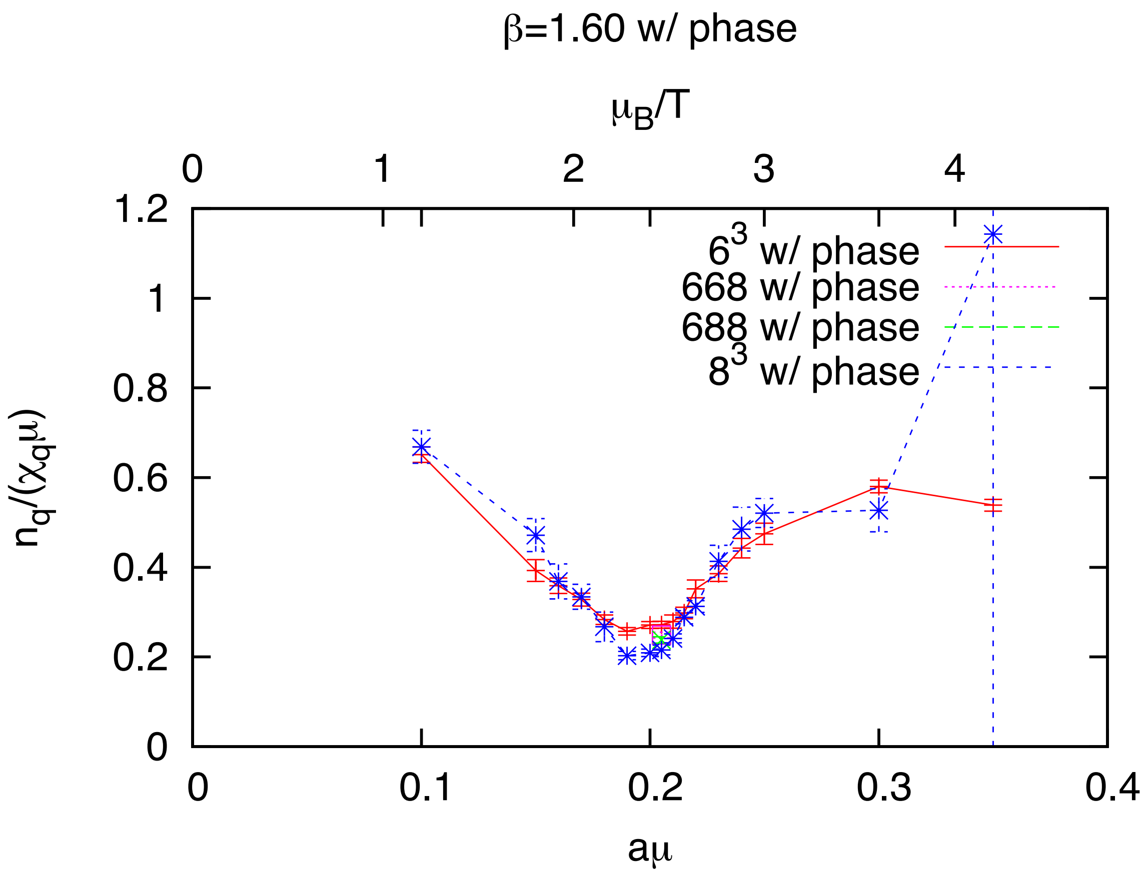
Skewness of gauge action



Kurtosis of gauge action

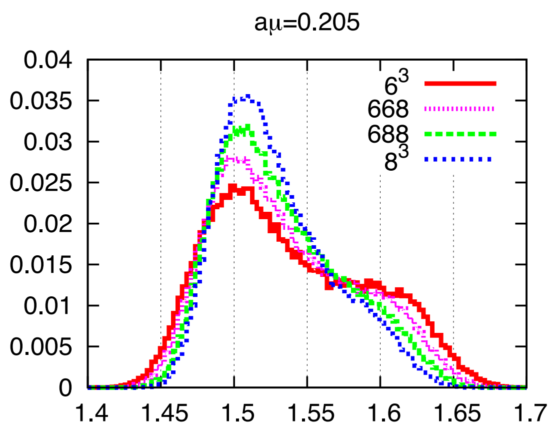


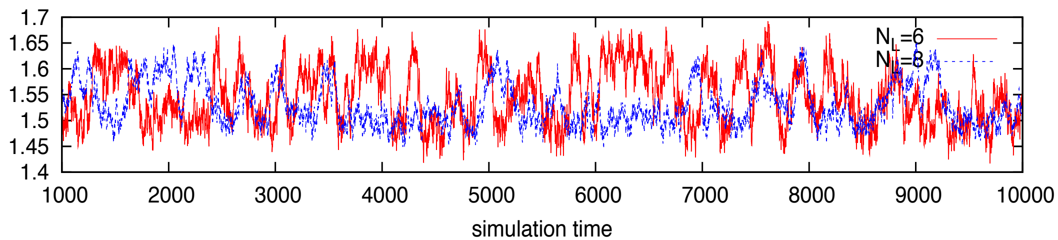
Ratio

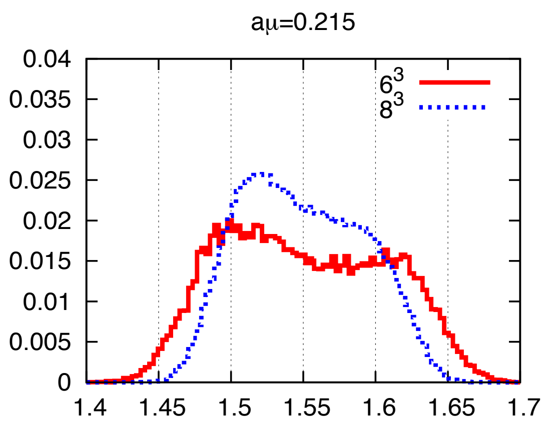


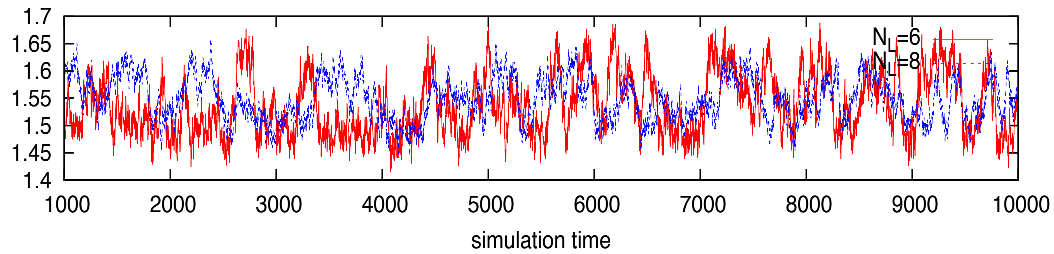
β=1.60 w/ phase
μB/T
0
1
2
3
4
1.2
1
0.8
0.6
0.4
0.2
0
6^3 w/ phase
668 w/ phase
688 w/ phase
8^3 w/ phase
nq/(χqμ)
0
0.1
0.2
0.3
0.4
aμ

History & histogram of gauge action on the phase quenched configuration



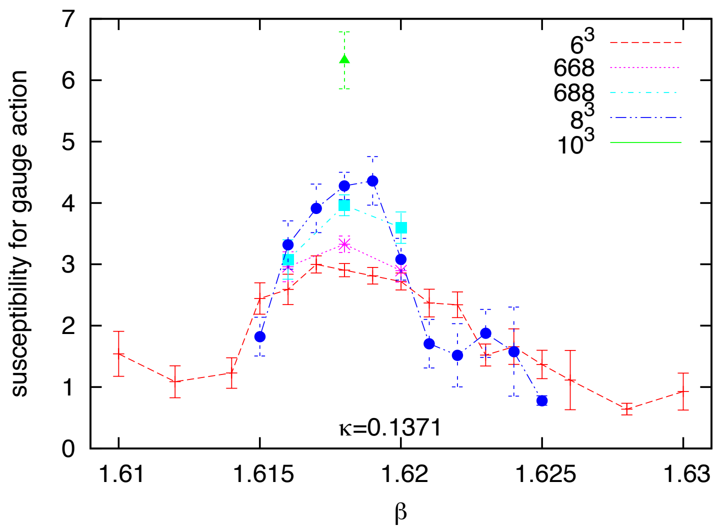


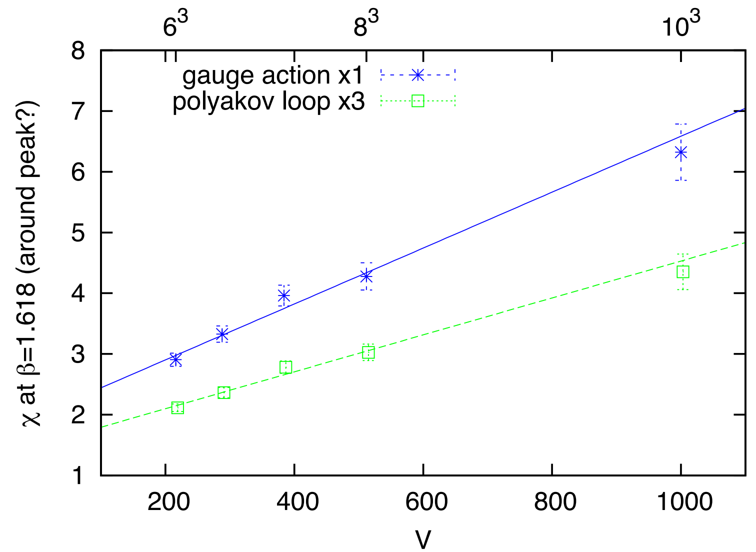




Finite temperature transition (μ=0)

N_T=4





Preliminary!

Constant physics with rescaling cut off

Constant physics

$$\begin{aligned} T &= 1/aN_{\rm T} \\ \mu & \\ V &= L^3 = (aN_{\rm L})^3 \\ m & \end{aligned}$$

Rescaling by factor b

$$\begin{aligned} a &\longrightarrow a/b \\ a\mu &\longrightarrow a\mu/b \\ N_{\rm T} &\longrightarrow bN_{\rm T} \\ N_{\rm L} &\longrightarrow bN_{\rm L} \\ \kappa &\longrightarrow \kappa' \approx \kappa(am \longrightarrow am/b) \end{aligned}$$

$$\frac{\text{Bound of } |\theta|_{\rm after}}{\text{Bound of } |\theta|_{\rm before}} = \frac{12(bN_{\rm L})^3(2\kappa')^{bN_{\rm T}}\sinh(\mu/T)}{12(N_{\rm L})^3(2\kappa)^{N_{\rm T}}\sinh(\mu/T)} = b^3(2\kappa)^{N_{\rm T}(b-1)}$$

Reduction

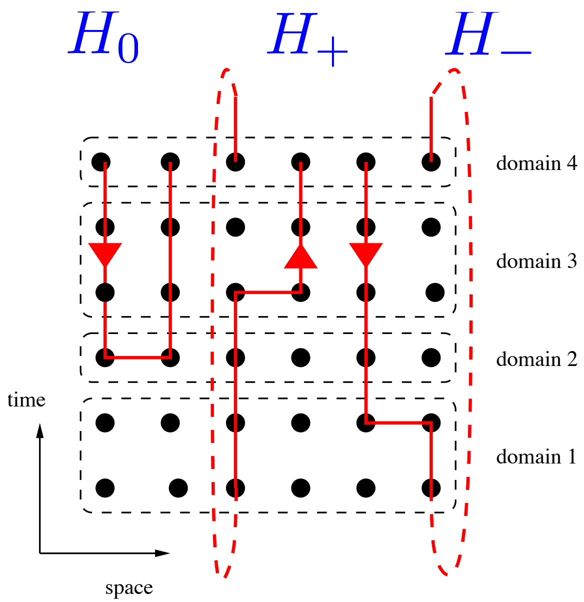
$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det[A]\det[D]\det[1 - D^{-1}CA^{-1}B],$$

$$\det D(\mu) = A_0 \det[1 - H_0 - e^{\mu/T} H_+ - e^{-\mu/T} H_-]$$

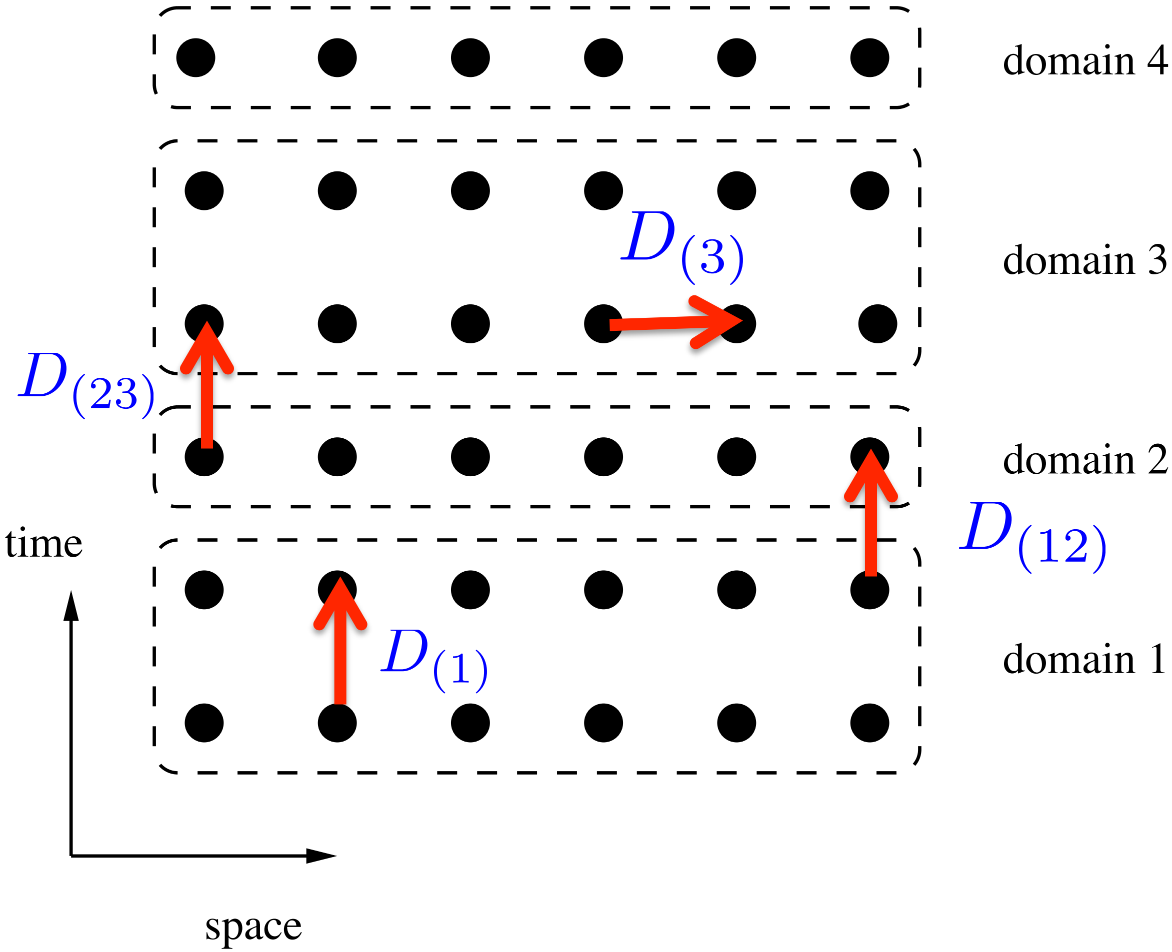
3-d operator size

$$\begin{aligned}
A_0 &= \det D_{(1)} \det D_{(3)} \cdots \\
H_+ &= (D_{(4)} - D_{(41)} D^{-1}_{(1)} D_{(14)} - D_{(43)} D^{-1}_{(3)} D_{(34)})^{-1} \\
&\times\ D_{(41)} D^{-1}_{(1)} D_{(12)} \\
&\times\ (D_{(2)} - D_{(21)} D^{-1}_{(1)} D_{(12)} - D_{(23)} D^{-1}_{(3)} D_{(32)})^{-1} \\
&\times\ D_{(23)} D^{-1}_{(3)} D_{(34)} \\
H_- &= \ldots \\
H_0 &= \ldots
\end{aligned}$$

μ-independent



Domain decomposition



Exact calculation of Quark Number

$$\det D(\mu) \propto \det\left(1 - \sum_{i=\pm,0} e^{i\mu/T} H_i\right)$$

$$n_q \propto \frac{\partial}{\partial\mu} \ln Z_{\mathrm{QCD}}(\mu)$$
$$\propto \left\langle \mathrm{tr}\left[\left(1 - \sum_{i=\pm,0} e^{i\mu/T} H_i\right)^{-1} \left(e^{\mu/T} H_+ - e^{-\mu/T} H_-\right)\right]\right\rangle$$

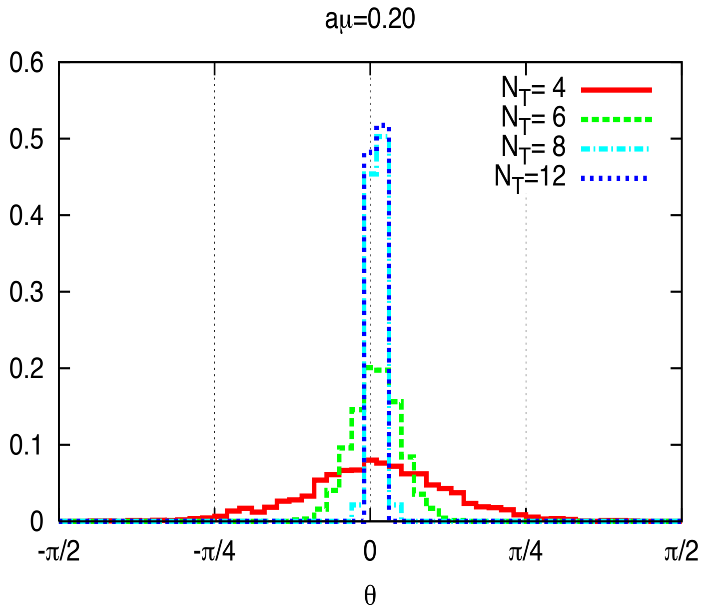
- Once $H_{+,-,0}$ are calculated, we can obtain phase, Quark Number & Susceptibility exactly
- For the computation of $H_{+,-,0}$ Matrix-Matrix product and inverse matrix whose rank is $12N_L^3$ are required LAPACK&OpenMP
- Cost $\propto (N_L^3)^3 \times N_T$
- Memory size $\propto (N_L^3)^2$ independent of N_T

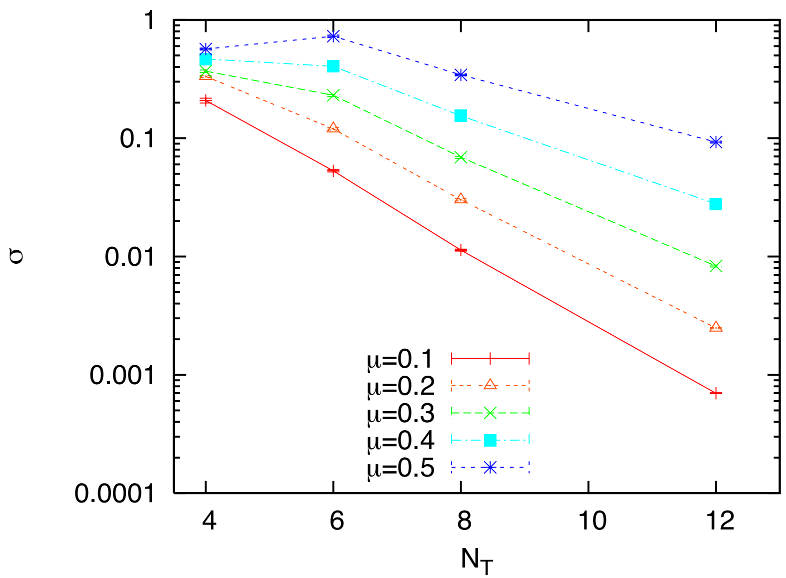
Distribution of the exact phase

$$\theta = \arg[\det(1 - H_0 - e^{\mu/T}H_+ - e^{-\mu/T}H_-)]$$

On the same configurations as before

$$\text{distribution} \propto \exp\left(-\frac{\theta^2}{2\sigma^2}\right)$$

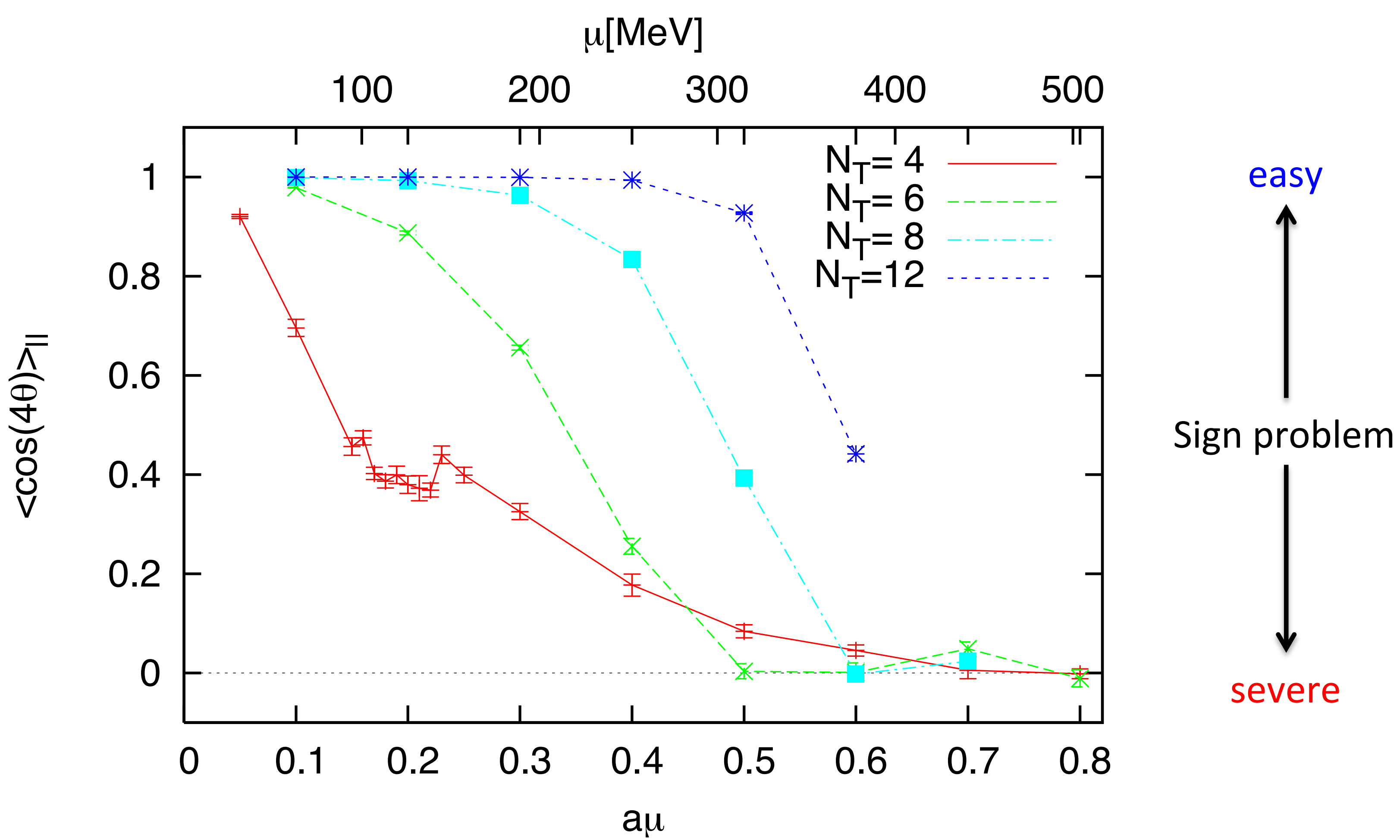




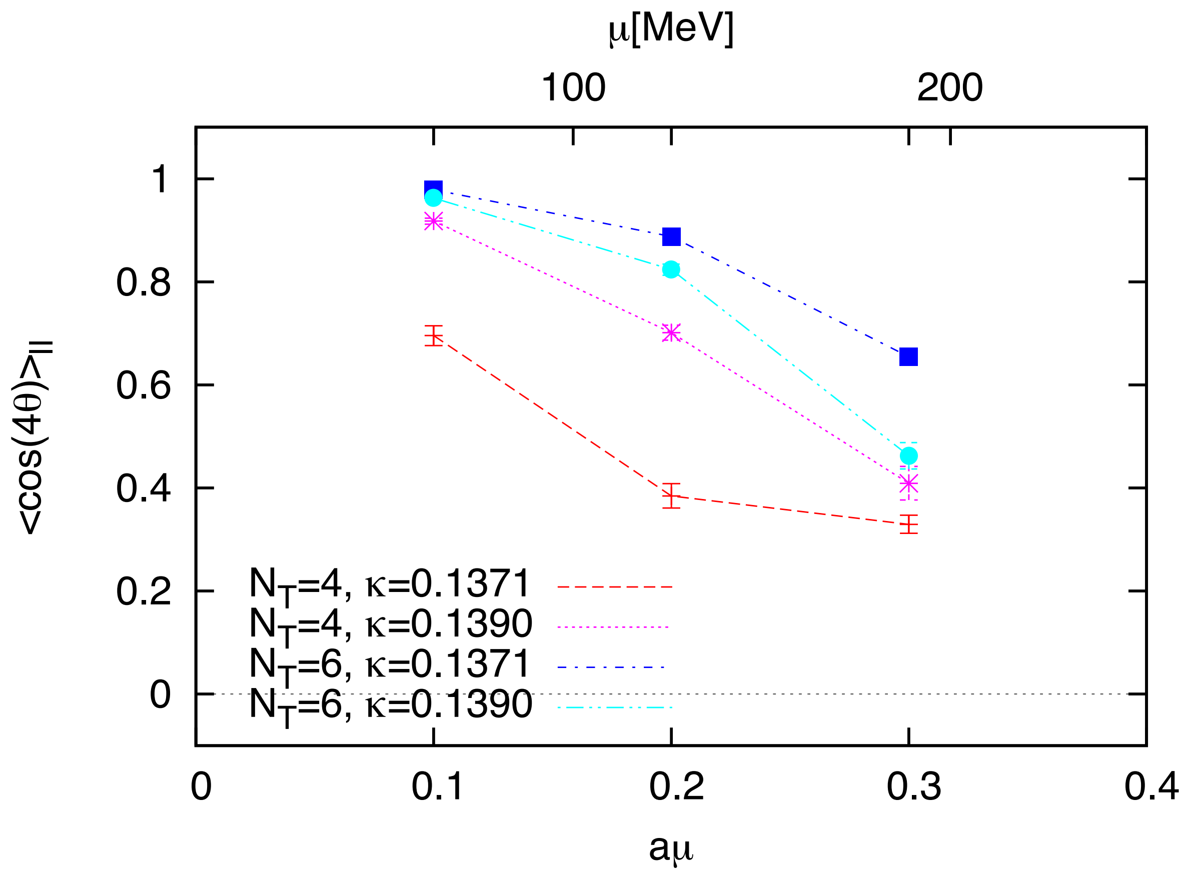
Fluctuation of the phase gets small for large N_T

Reweighting factor

On the phase quenched configurations

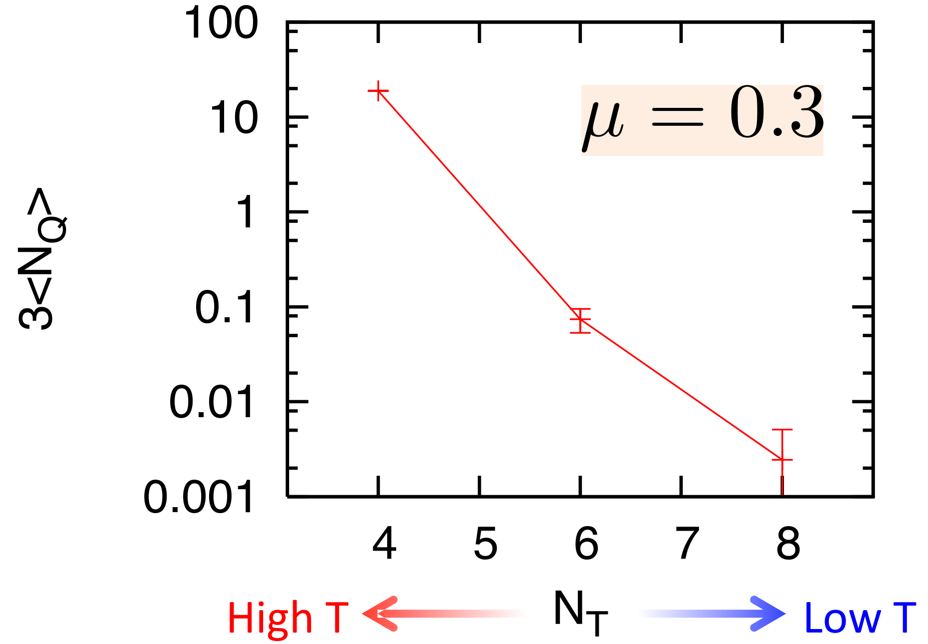


Quark mass dependence



N_T dependence of quark number

$$\langle N_{\mathrm{Q}} \rangle = \frac{\partial \ln \mathcal{Z}_{\mathrm{QCD}}}{\partial(\mu/T)}$$



At low T, quarks are not excited
therefore sign problem gets weak?