Pion form factors in the ε regime

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1. Introduction

JLQCD (& TWQCD) project [2006-2012]

= Lattice QCD with dynamical overlap quarks.

1/a ~ 1.8 GeV, L ~ 1.8 fm

p regime lattices : m_π = 290-780 MeV

ε regime lattice : m_π ~ 100 MeV

$$m_\pi L \sim 0.9$$

(New project with Hitachi SR16K & IBM BG/Q started.)

1. Introduction

What is the ε regime ?

In the p regime, the vacuum is fixed:

$$U(x) = 1 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right), \quad \in SU(N_f)$$

but near the chiral limit at finite V, $M_\pi L < 1$

vacuum= zero-mode = dynamical variable

$$U(x) = U_0 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right),$$

U_0 should be non-perturbatively treated.

$\rightarrow$ ε regime

1. Introduction

ε expansion of Chiral Lagrangian

$$\mathcal{L} = -\frac{\Sigma}{2}\mathrm{Tr}\left[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}\right] + \frac{1}{2}\mathrm{Tr}(\partial_\mu \xi)^2 + \frac{\Sigma}{2F^2}\mathrm{Tr}[\mathcal{M}^\dagger U_0 \xi^2 + \xi^2 U_0^\dagger \mathcal{M}] + \cdots,$$

Zero-mode = SU(N) (or U(N)) matrix model

Non-zero-mode = massless bosons

(perturbative) interactions

= a hybrid system of
matrix model and massless bosonic fields

1. Introduction

2pt function in the ε regime

(w/ periodic boundary)

$$\int d^3x \langle P^a(x) P^b(0) \rangle =$$

ε -regime $$\delta^{ab}\left[C\left(t-\frac{T}{2}\right)^2+E\right]$$

(p-regime $$\delta^{ab} B \frac{\cosh(M_\pi^V(t-T/2))}{\sinh(M_\pi^V T/2)}$$)

[Bernardoni, Damgaard, HF, Hernandez, 2008]

1. Introduction

My talk at LAT2011:
Interpolation between ε & p regimes

keeping non-perturbative treatment

of the zero mode even in the p regime.

[Damgaard & HF 2009, Aoki & HF, 2011]

$$C_{PP}\frac{\cosh(M_\pi^{NLO}(t-T/2))}{\sinh(M_\pi^{NLO}T/2)}+D_{PP}$$

$$\delta^{ab}\left[C\left(t-\frac{T}{2}\right)^2+E\right]$$

$$\delta^{ab}B\frac{\cosh(M_\pi^{V}(t-T/2))}{\sinh(M_\pi^{V}T/2)}$$

1. Introduction

But no one followed us…

1) **Interpolation between the epsilon and p regimes.**
Sinya Aoki, (Tsukuba U., GSPAS & Tsukuba U.) , Hidenori Fukaya, (Osaka U.) . UTHEP-626, OU-HET-700-2011, May 2011. (Published Jul 1, 2011). 49pp.
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1. Introduction

Probably because ···

$$C_{PP} = \frac{\Sigma^2}{F^2}\frac{(Z_M^{vv}Z_F^{vv})^4}{(Z_F^{vv})^2}\frac{1}{2}\left(1+\mathcal{D}_{vv}^{\rm eff}+\frac{Q^2}{(\mu_v^{\rm eff})^2}-\frac{\partial\mathcal{S}_v^{\rm eff}}{\partial\mu_v^{\rm eff}}\right),$$

$$\mathcal{S}_v = -\frac{1}{(\mu^2-\mu_v^2)^2(\mu_s^2-\mu_v^2)} \times \frac{\det\begin{pmatrix} \partial_{\mu_v}K_Q(\mu_v) & I_Q(\mu_v) & I_Q(\mu) & \mu^{-1}I_{Q-1}(\mu) & I_Q(\mu_s) \\ -\partial_{\mu_v}(\mu_v K_{Q+1}(\mu_v)) & \mu_v I_{Q+1}(\mu_v) & \mu I_{Q+1}(\mu) & I_Q(\mu) & \mu_s I_{Q+1}(\mu_s) \\ \partial_{\mu_v}(\mu_v^2 K_{Q+2}(\mu_v)) & \mu_v^2 I_{Q+2}(\mu_v) & \mu^2 I_{Q+2}(\mu) & \mu I_{Q+1}(\mu) & \mu_s^2 I_{Q+2}(\mu_s) \\ -\partial_{\mu_v}(\mu_v^3 K_{Q+3}(\mu_v)) & \mu_v^3 I_{Q+3}(\mu_v) & \mu^3 I_{Q+3}(\mu) & \mu^2 I_{Q+2}(\mu) & \mu_s^3 I_{Q+3}(\mu_s) \\ \partial_{\mu_v}(\mu_v^4 K_{Q+4}(\mu_v)) & \mu_v^4 I_{Q+4}(\mu_v) & \mu^4 I_{Q+4}(\mu) & \mu^3 I_{Q+3}(\mu) & \mu_s^4 I_{Q+4}(\mu_s) \end{pmatrix}}{\det\begin{pmatrix} I_Q(\mu) & \mu^{-1}I_{Q-1}(\mu) & I_Q(\mu_s) \\ \mu I_{Q+1}(\mu) & I_Q(\mu) & \mu_s I_{Q+1}(\mu_s) \\ \mu^2 I_{Q+2}(\mu) & \mu I_{Q+1}(\mu) & \mu_s^2 I_{Q+2}(\mu_s) \end{pmatrix}}$$

$$\mathcal{D}_{vv} = -\frac{1}{(\mu^2-\mu_v^2)^2(\mu_s^2-\mu_v^2)} \times \frac{\det\begin{pmatrix} \partial_{\mu_v}K_Q(\mu_v) & \partial_{\mu_v}I_Q(\mu_v) & I_Q(\mu) & \mu^{-1}I_{Q-1}(\mu) & I_Q(\mu_s) \\ -\partial_{\mu_v}(\mu_v K_{Q+1}(\mu_v)) & \partial_{\mu_v}(\mu_v I_{Q+1}(\mu_v)) & \mu I_{Q+1}(\mu) & I_Q(\mu) & \mu_s I_{Q+1}(\mu_s) \\ \partial_{\mu_v}(\mu_v^2 K_{Q+2}(\mu_v)) & \partial_{\mu_v}(\mu_v^2 I_{Q+2}(\mu_v)) & \mu^2 I_{Q+2}(\mu) & \mu I_{Q+1}(\mu) & \mu_s^2 I_{Q+2}(\mu_s) \\ -\partial_{\mu_v}(\mu_v^3 K_{Q+3}(\mu_v)) & \partial_{\mu_v}(\mu_v^3 I_{Q+3}(\mu_v)) & \mu^3 I_{Q+3}(\mu) & \mu^2 I_{Q+2}(\mu) & \mu_s^3 I_{Q+3}(\mu_s) \\ \partial_{\mu_v}(\mu_v^4 K_{Q+4}(\mu_v)) & \partial_{\mu_v}(\mu_v^4 I_{Q+4}(\mu_v)) & \mu^4 I_{Q+4}(\mu) & \mu^3 I_{Q+3}(\mu) & \mu_s^4 I_{Q+4}(\mu_s) \end{pmatrix}}{\det\begin{pmatrix} I_Q(\mu) & \mu^{-1}I_{Q-1}(\mu) & I_Q(\mu_s) \\ \mu I_{Q+1}(\mu) & I_Q(\mu) & \mu_s I_{Q+1}(\mu_s) \\ \mu^2 I_{Q+2}(\mu) & \mu I_{Q+1}(\mu) & \mu_s^2 I_{Q+2}(\mu_s) \end{pmatrix}} + \left(\frac{4\mu_v}{\mu^2-\mu_v^2}+\frac{2\mu_v}{\mu_s^2-\mu_v^2}\right)\mathcal{S}_v.$$

1. Introduction

This talk :

How to make the ε regime simple

1. non-zero momenta
2. take an appropriate ratio

-> We can eliminate LO finite V effects

(= we can forget about Bessel functions)

even in the ε regime.

=> Pion form factors

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3. Preliminary lattice results
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2. 3pt functions in the ε regime

2pt functions in ε expansion of ChPT

Before d^3x integral

$$\langle P(x)P(0)\rangle = A + B\sum_{p'\neq 0}\frac{1}{V}\frac{e^{ip'x}}{p'^2} + C\sum_{p'\neq 0}\frac{e^{ip'x}}{p'^4} + \cdots$$

$A, B, C\cdots$: zero mode contribution

(Bessel functions of $m\Sigma V$)

$\sum_{p'\neq 0}(\cdots)$: non-zero mode's

2. 3pt functions in the ε regime

2pt functions in ε expansion of ChPT

After d^3x integral → power function of t.

$$\frac{1}{L^3}\int d^3x\langle P(x)P(0)\rangle = A + B\frac{1}{V}\left[\frac{1}{2}\left(t-\frac{T}{2}\right)^2-\frac{1}{24}\right]+C\left[\cdots\right]+\cdots$$

cf. p regime result:

$$B'\cosh(m_\pi(t-T/2))$$

2. 3pt functions in the ε regime

2pt functions in ε expansion of ChPT

After d^3x integral with $\mathbf{p} \neq 0$

$$\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x}\langle P(x)P(0)\rangle = \frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x} A + B\sum_{p_0'}\frac{1}{V}\frac{e^{-ip_0't}}{p_0'^2+\mathbf{p}^2} + \cdots$$

$$= 0 + \frac{B}{2E(\mathbf{p})L^3\sinh(E(\mathbf{p})T/2)}\cosh(E(\mathbf{p})(t-T/2)) + \cdots,$$

where $E(\mathbf{p}) = |\mathbf{p}|$. the same form as the p-regime :

$$B'\cosh(\sqrt{|\mathbf{p}|^2+m_\pi^2}(t-T/2))$$

But Bessel functions are still contained in B .

2. 3pt functions in the ε regime

2pt functions in ε expansion of ChPT

Ratio of 2pt functions with $\mathbf{p} \neq 0$

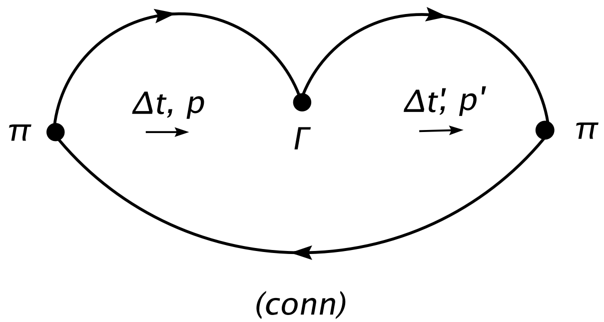
$$\frac{\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x}\langle P(x)P(0)\rangle}{\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}'x}\langle P(x)P(0)\rangle} = \frac{\cosh(E(\mathbf{p})(t-T/2))}{\cosh(E(\mathbf{p}')(t-T/2))} + \cdots,$$

LO finite V effect is eliminated !

NLO $\sim \dfrac{1}{4\pi F^2 L^2}$

2. 3pt functions in the ε regime

3pt functions in ε regime



$$C^{3\mathrm{pt}}_{PV_0P}(\Delta t, \Delta t'; \mathbf{p}_i, \mathbf{p}_f)$$

$$= B^{3\mathrm{pt}}(m\Sigma V)\left[E(\mathbf{p}_i) + E(\mathbf{p}_f)\right] F_V(q^2)$$

$$\times \cosh(E(\mathbf{p}_i)(\Delta t - T/2)) \cosh(E(\mathbf{p}_f)(\Delta t' - T/2)) + \cdots$$

$$R_V(\Delta t, \Delta t'; |\mathbf{p}_i|, |\mathbf{p}_f|, q^2) \equiv \frac{\frac{1}{N^{3\mathrm{pt}}_{|\mathbf{p}_i|,|\mathbf{p}_f|}} \sum_{\mathrm{fixed}|\mathbf{p}_i|,|\mathbf{p}_f|,q^2} C^{3\mathrm{pt}}_{PVP}(\Delta t, \Delta t'; \mathbf{p}_i, \mathbf{p}_f)}{\left(\frac{1}{N^{2\mathrm{pt}}_{|\mathbf{p}_i|}} \sum_{\mathrm{fixed}|\mathbf{p}_i|} C^{2\mathrm{pt}}_{PP}(\Delta t; \mathbf{p}_i)\right)\left(\frac{1}{N^{2\mathrm{pt}}_{|\mathbf{p}_f|}} \sum_{\mathrm{fixed}|\mathbf{p}_f|} C^{2\mathrm{pt}}_{PP}(\Delta t'; \mathbf{p}_f)\right)},$$

$$= B^{3\mathrm{pt}/2\mathrm{pt}}(m\Sigma V)\left[E(\mathbf{p}_i) + E(\mathbf{p}_f)\right] F_V(q^2) + \cdots$$

2. 3pt functions in the ε regime

Ratio of 3pt functions

$$F'_V(\Delta t, \Delta t'; q^2) = \frac{2E_\pi(1)}{E_\pi(|\mathbf{p}_i|) + E_\pi(|\mathbf{p}_f|)} \times \frac{R_V(\Delta t, \Delta t'; |\mathbf{p}_i|, |\mathbf{p}_f|, q^2)}{R_V(\Delta t, \Delta t'; 1, 1, 0)}$$

$$= F_V(q^2) + \mathcal{O}\left(\frac{1}{4\pi F^2 L^2}\right) \qquad (1 \equiv 2\pi/L)$$

LO finite V effect is eliminated !

(Cf. in the p regime, this ratio method is conventionally used for canceling the smearing effect, renormalization and so on.

$$F_V(\Delta t, \Delta t'; q^2) = \frac{2m_\pi}{E_\pi(|\mathbf{p}_i|)+E_\pi(|\mathbf{p}_f|)} \times \frac{R_V(\Delta t,\Delta t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2)}{R_V(\Delta t,\Delta t';0,0,0)}$$

$$= F_V(q^2) + \mathcal{O}\left(e^{-m_\pi L}\right)$$

)

[Hashimoto et al. 2000]

2. 3pt functions in the ε regime

Special case with zero momentum

PVP 3pt function → the constant doesn't appear even if $\mathbf{p}_i = 0,\ \mathbf{p}_f \neq 0 \ \text{ or } \ \mathbf{p}_i \neq 0,\ \mathbf{p}_f = 0$

2pt function → the constant term is known:

$$C_{PP}^{\text{2ptsub}}(\Delta t; 0) = C_{PP}^{\text{2pt}}(\Delta t; 0) - D_{PP}(m\Sigma V)$$

➡ $R_V(\Delta t, \Delta t'; |\mathbf{p}_i|, 0, q^2),$

$R_V(\Delta t, \Delta t'; 0, |\mathbf{p}_f|, q^2)$

are also good quantities to extract $F_V(q^2)$

3. Preliminary lattice results

Summary of numerical simulations

Iwasaki gauge action with β=2.30

2+1 dynamical overlap quarks

1/a ~ 1.759 GeV, L=$16^3 48$ (L~1.8 fm)

m_{ud} =0.002(~3 MeV), m_s=0.08

Q=0 fixed

Smearing is used for PS operators $\phi_s(\mathbf{r}) = e^{-0.4r}$

All-to-all propagators with 120 exact low-modes and noise method for the high modes.

Dispersion relation is used for E with $m_\pi = 98(5)\mathrm{MeV}$

Preliminary results with 68 samples from 2500 trj.

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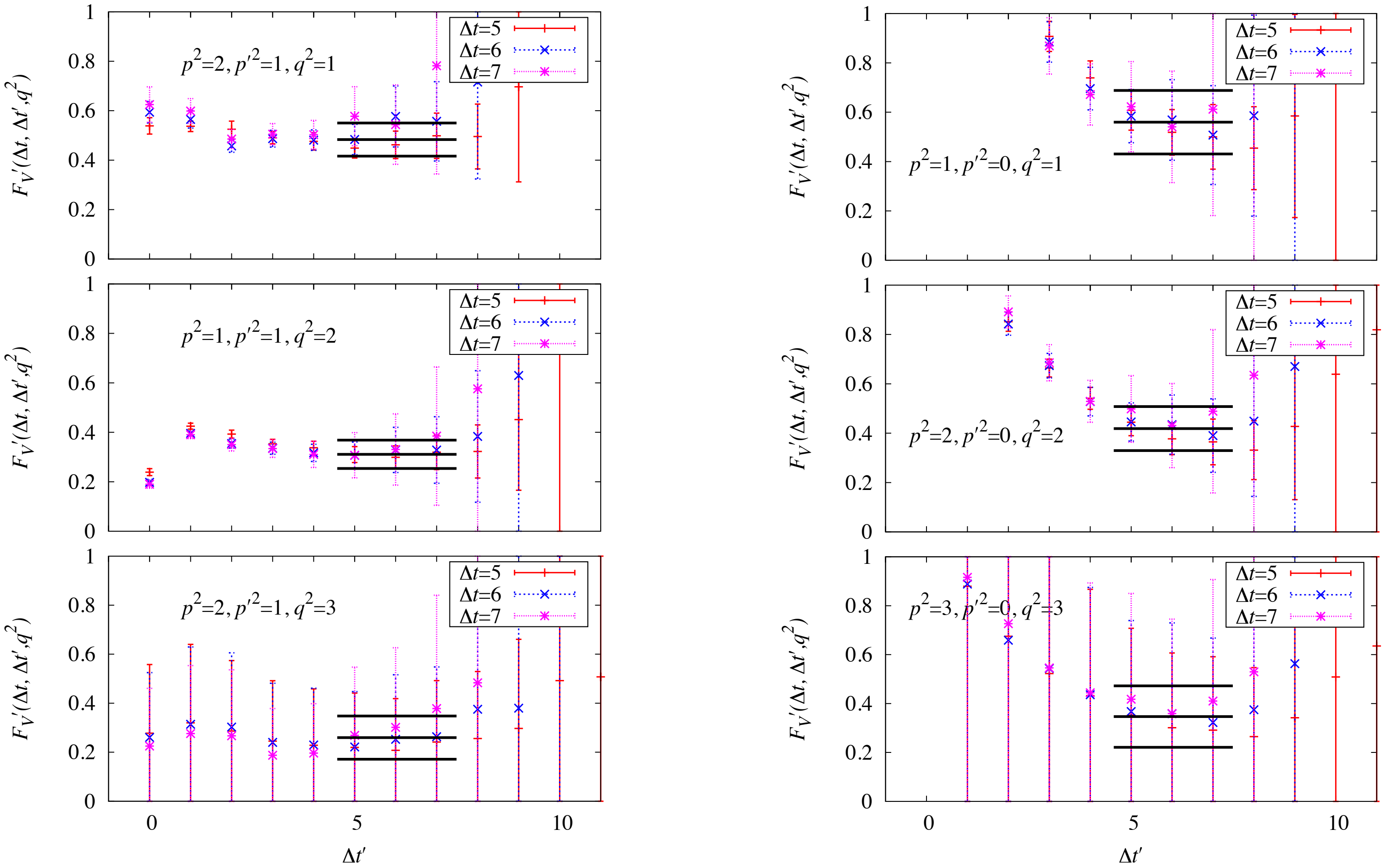
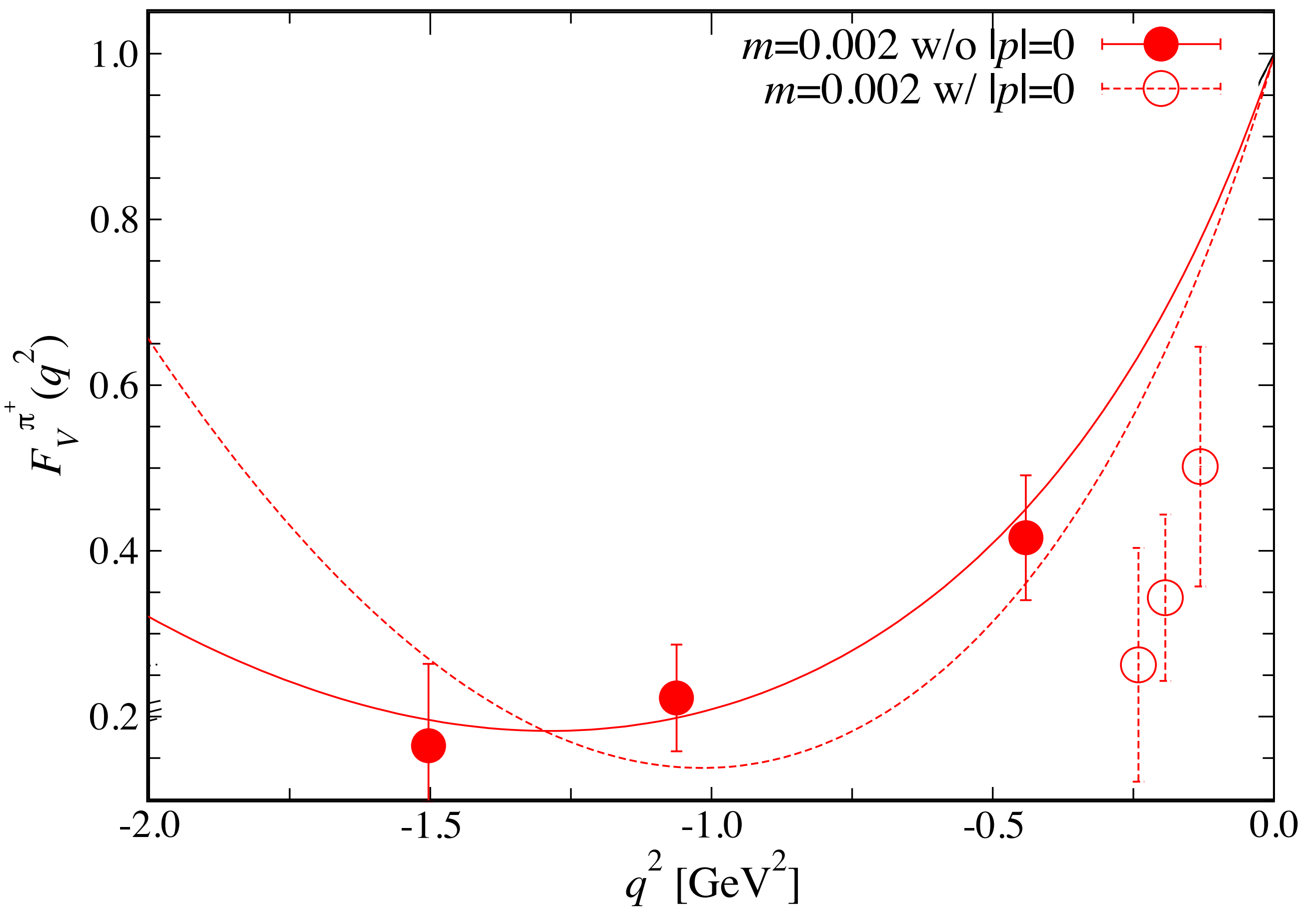
3. Preliminary lattice results

$$F'_V(\Delta t, \Delta t'; q^2) = \frac{2E_\pi(1)}{E_\pi(|\mathbf{p}_i|) + E_\pi(|\mathbf{p}_f|)} \times \frac{R_V(\Delta t, \Delta t'; |\mathbf{p}_i|, |\mathbf{p}_f|, q^2)}{R_V(\Delta t, \Delta t'; 1, 1, 0)}$$



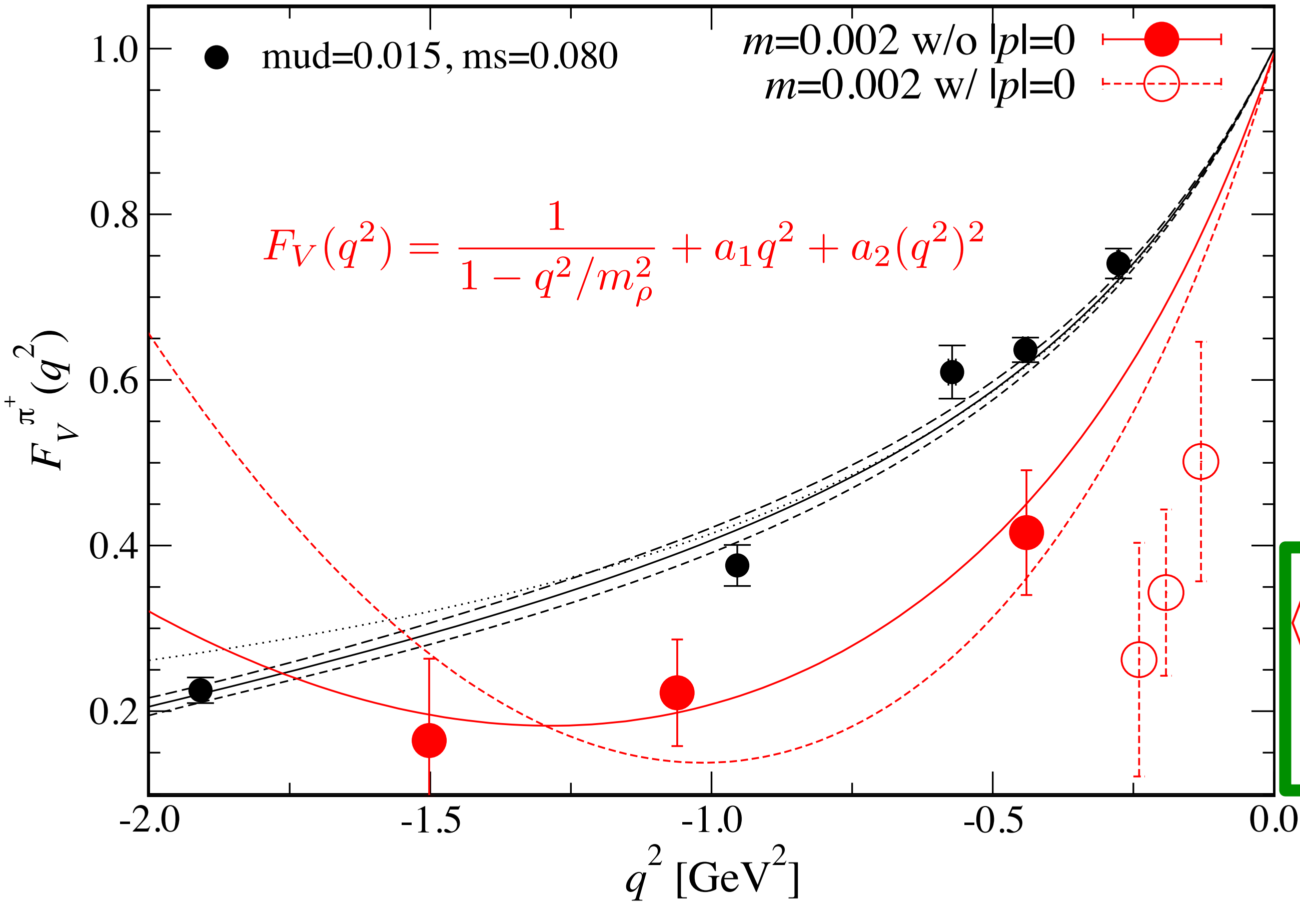
3. Preliminary lattice results

$F_V(q^2)$



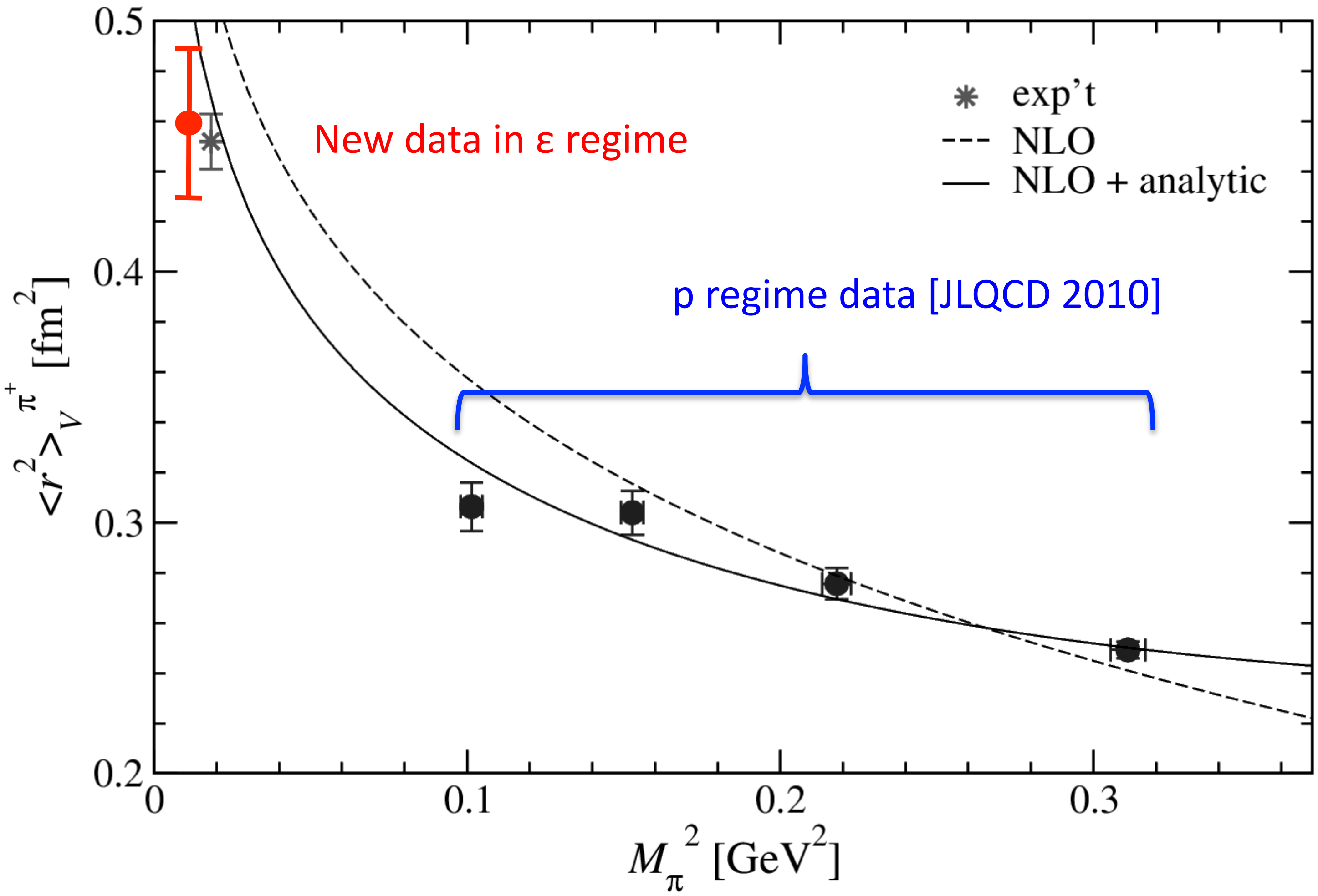
3. Preliminary lattice results

$F_V(q^2)$



$$\langle r^2 \rangle_V = 6 \frac{\partial F_V(q^2)}{\partial q^2} \bigg|_{q^2=0} = 0.46(3)\ \text{fm}^2.$$

3. Preliminary lattice results



3. Preliminary lattice results

NOTES

1. We DON'T need Bessel functions in the analysis.
2. The result still contain NLO $\sim \frac{1}{4\pi F^2 V^{1/2}} \sim 7\%$ finite V effects.

To do list :

1. Increase the statistics.
2. Check the stability of q^2 fit.
3. Check the dispersion relation.
4. 1-loop ChPT corrections.
5. Twisted boundary conditions.
6. Expand the volume in our new project.

3. Preliminary lattice results

Results by other collaborations

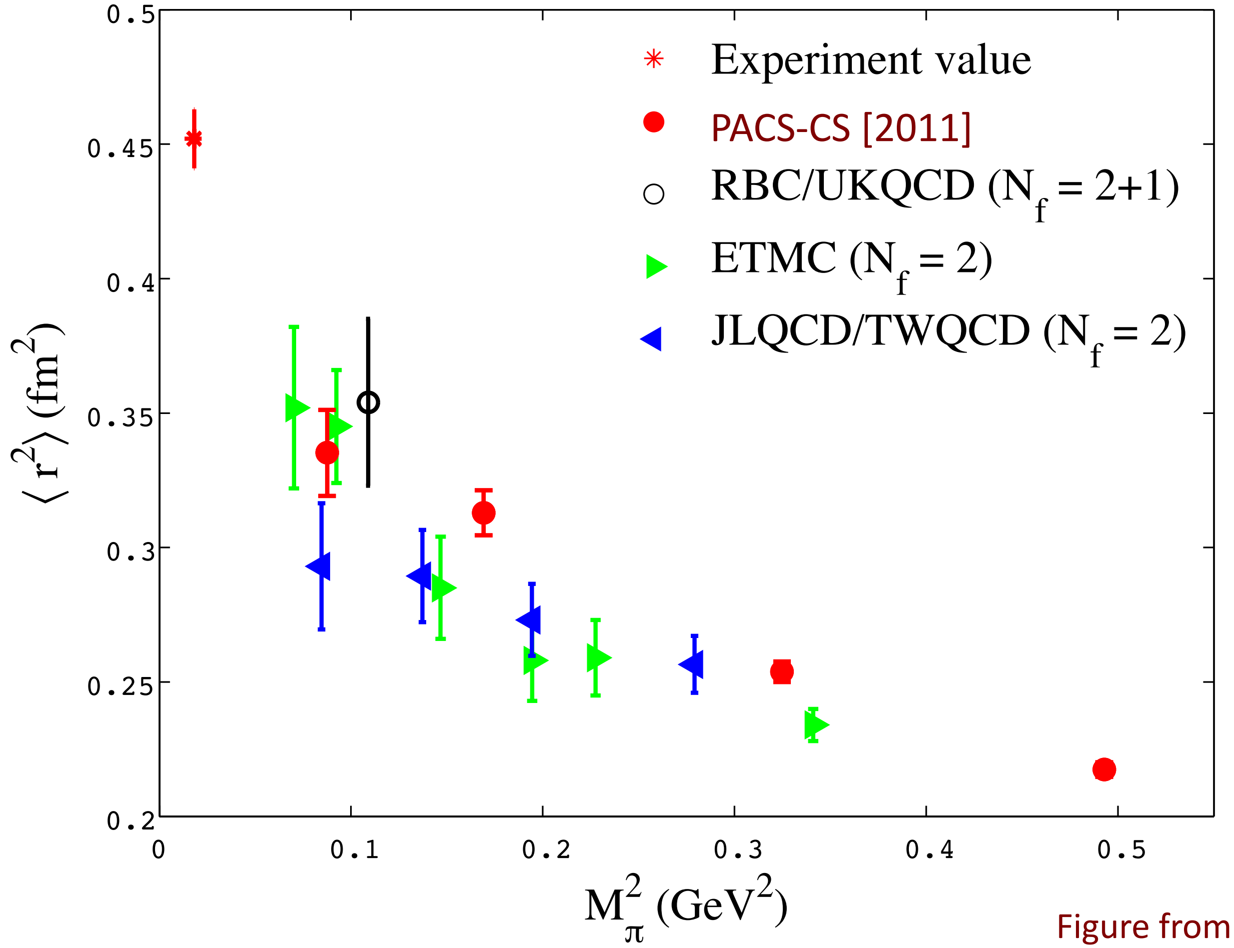


Figure from PACS-CS [2011]

3. Preliminary lattice results

Physical point simulation by PACS-CS [2011]

The PACS-CS gauge configurations has one more set corresponding to $M_\pi \approx 156$ MeV. We tried to calculate the form factor on this set, and found that the pion two- and three-point correlators exhibit very large fluctuations, to the extent that taking a meaningful statistical average is difficult. This trend becomes more pronounced as the twist carried by quarks becomes larger. Since $LM_\pi \approx 2.3$ at this pion mass for $L = 32$, we suspect that this phenomenon is caused by a small size of the lattice relative to the pion mass, and consequent increase of large fluctuations.

The use of $\mathbf{p} \neq 0$ correlators :

$$F'_V(\Delta t, \Delta t'; q^2) = \frac{2E_\pi(1)}{E_\pi(|\mathbf{p}_i|) + E_\pi(|\mathbf{p}_f|)} \times \frac{R_V(\Delta t, \Delta t'; |\mathbf{p}_i|, |\mathbf{p}_f|, q^2)}{R_V(\Delta t, \Delta t'; 1, 1, 0)}$$

may be helpful ?

4. Summary

This talk :

How to make the ε regime simple

1. non-zero momenta
2. take an appropriate ratio

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(= we can forget about Bessel functions)
even in the ε regime.

=> Pion form factors