Interquark Potentials in Charmonium at Finite Temperature[a]

[a] Wynne Evans, Jon-Ivar Skullerud, CA

Interquark Potentials in Charmonium at Finite Temperature[a]

and related topics[b]

[a] Wynne Evans, Jon-Ivar Skullerud, CA

[b] Gert Aarts, Tim Harris, Seyong Kim, Maria Paola Lombardo, Mehmet Oktay, Sinead Ryan, Don Sinclair, Jon-Ivar Skullerud, CA

Outline

Interquark potential in charmonium at finite temperature

- Schrödinger Equation Approach
  - Nambu-Bethe-Salpeter wavefunction
- Conventional Exponential Fits
- Maximum Entropy Method

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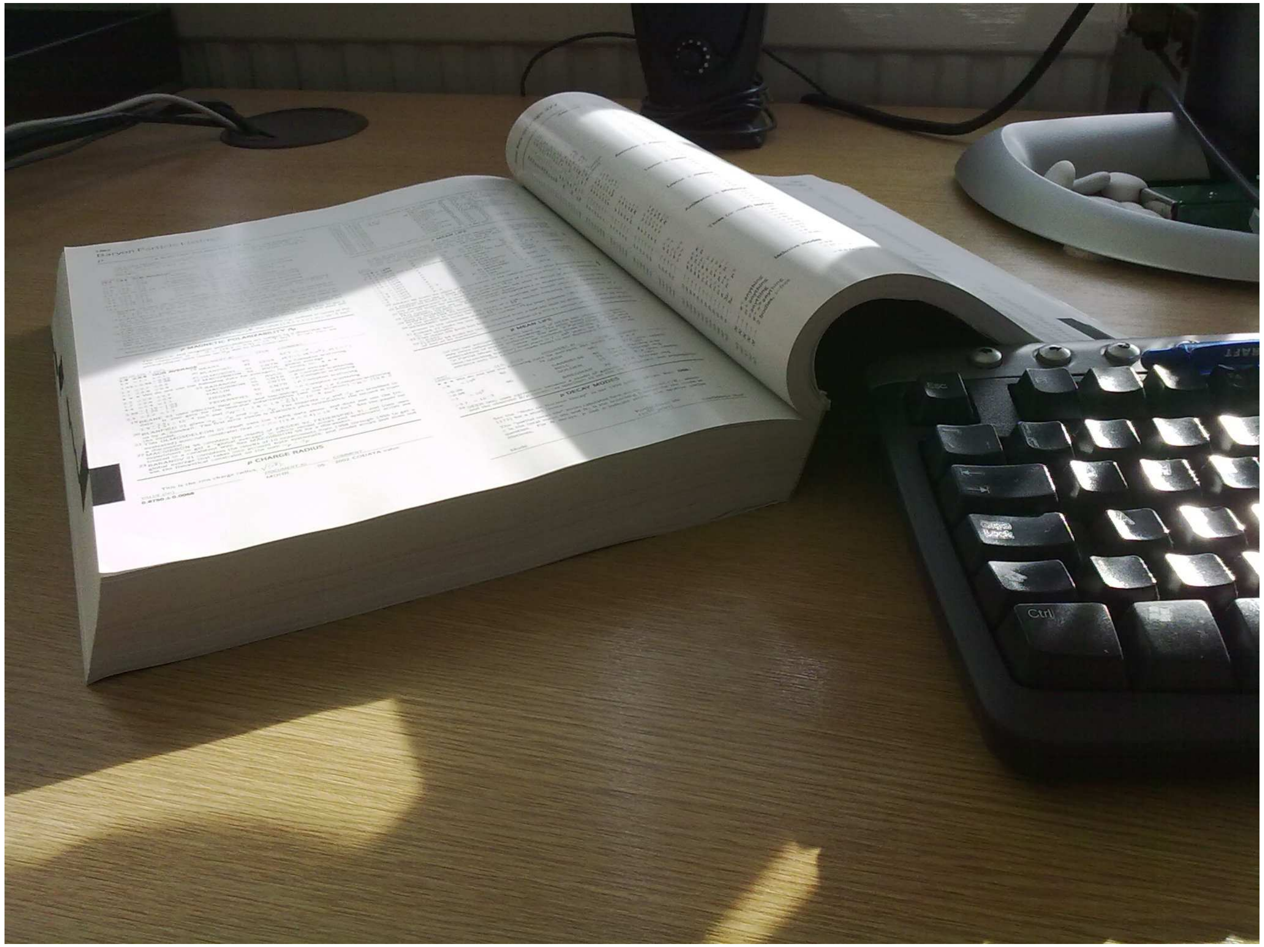
Related topics = bottomonium at finite temperature

- Spectral Functions
- Non-zero momenta
- “Ground state” mass
  - width

Particle Data Book

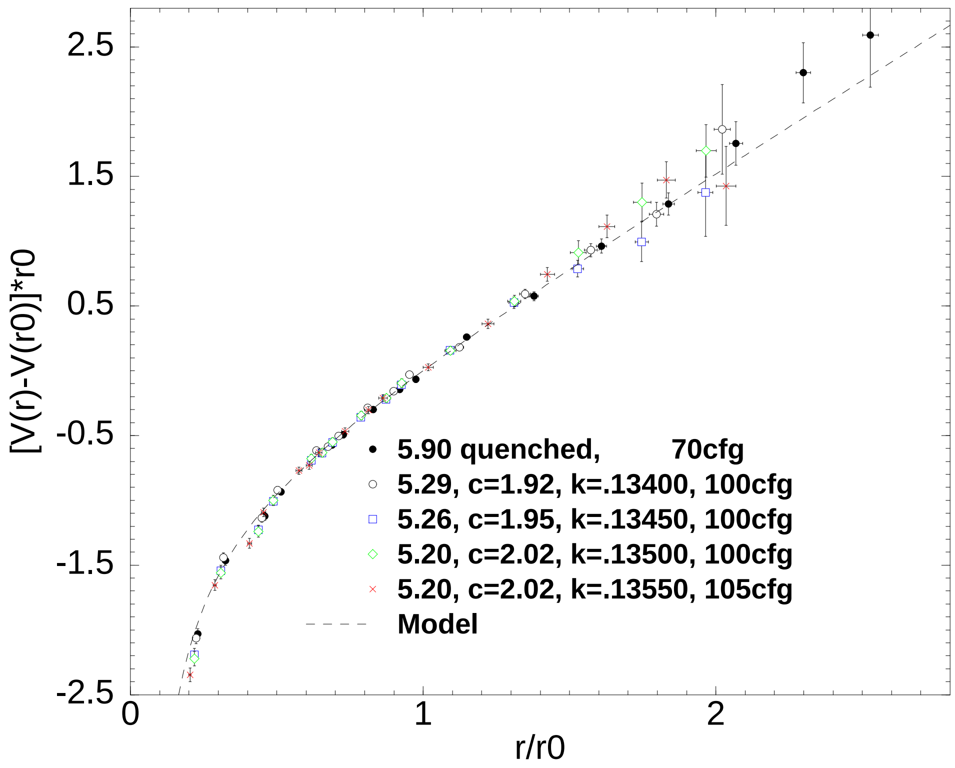


$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

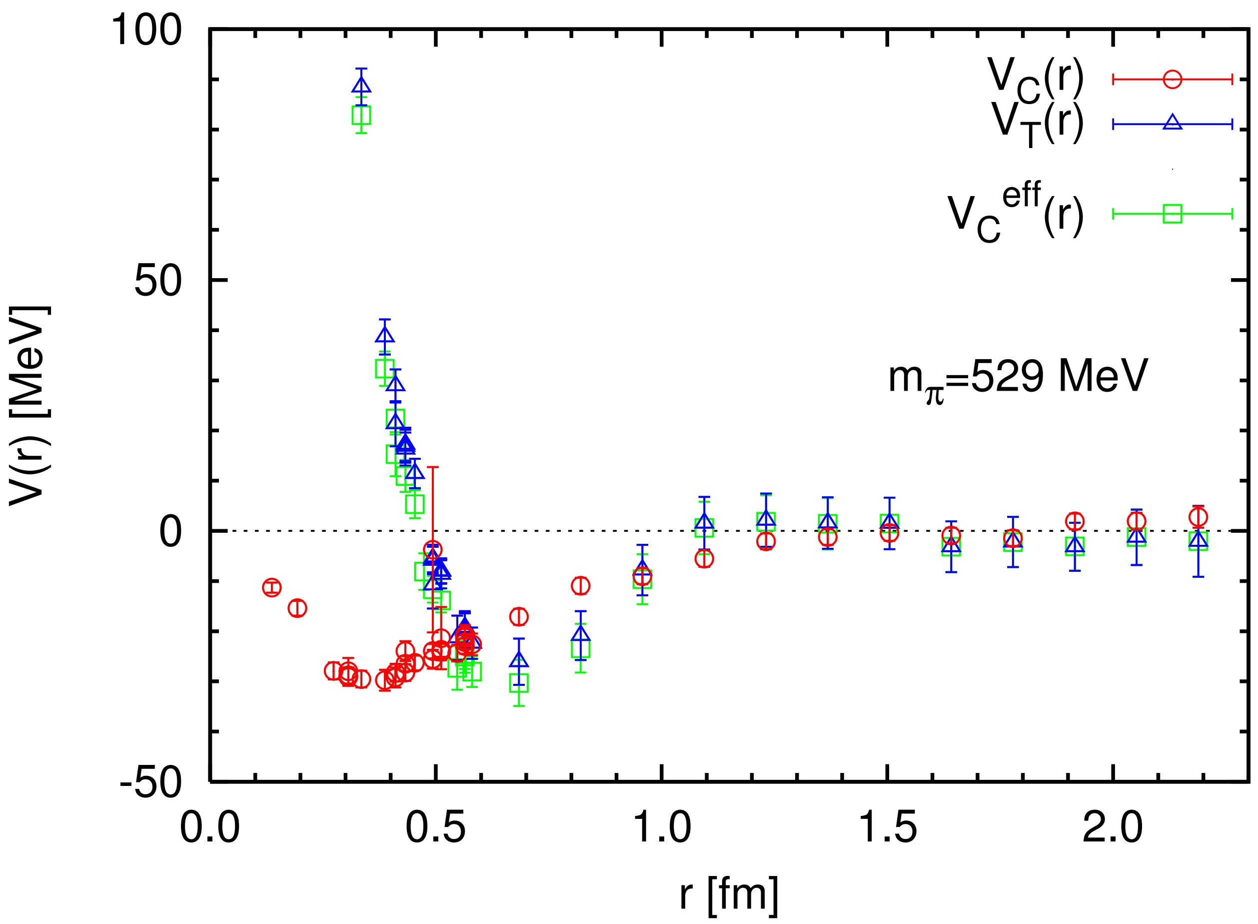
Static Quark Potential ($T = 0$)

UKQCD Collaboration [pre-history]



Lattice goes Nuclear

N-N potential



HAL QCD Collaboration, Aoki, Doi, Hatsuda, Ikeda, Inoue, Ishii, Murano, Nemura, Sasaki

Inter-quark potential from the Lattice

Kawanai, Sasaki, arXiv:1111.025
Ikeda, Iida, arXiv:1102.2097

- finite-quark mass
- quenched

We extend this by using:

- $2\ \ (+1)$ flavour
- finite temperature
- anisotropic lattices

Schrödinger Equation Approach

Hatsuda, PoS CD09 (2009) 068 use the Schrödinger equation to "reverse engineer" the potential, $V(r)$, given the Nambu-Bethe-Salpeter wavefunction, $\psi(r)$:

input input

$$\left(\frac{p^2}{2M} + \underset{\substack{\downarrow \\ \text{output}}}{V(r)}\right) \overset{\downarrow}{\psi(r)} = \overset{\downarrow}{E} \; \overset{\downarrow}{\psi(r)}$$

$\psi(r)$ is determined from a lattice simulation from correlators of *non-local* (point-split) operators, $J(x;\vec{r}) = q(x)\,\Gamma\,U(x, x+\vec{r})\overline{q}(x+\vec{r})$

$$\begin{aligned} C(\vec{r}, t) &= \sum_{\vec{x}} < J(0;\vec{r})\; J(x;\vec{r}) > \\ &\longrightarrow |\psi(r)|^2\, e^{-Et} \end{aligned}$$

Lattice Parameters

Dublin-Maynooth $N_f = 2$ configurations

N_s	N_τ	T(MeV)	T/T_c	$N_{\rm cfg}$
12	80	90	0.42	250
12	32	230	1.05	1000
12	28	263	1.20	1000
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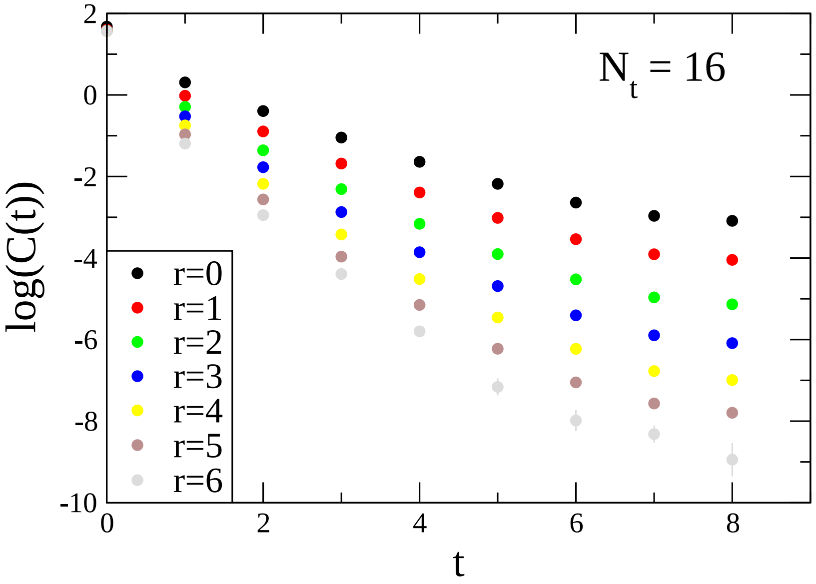
anisotropic lattice with $\xi = a_s/a_\tau \approx 6$

$a_s = 0.167$ fm

Vector and Pseudoscalar Channels
Charm treated relativistically

Correlation Functions (PS)



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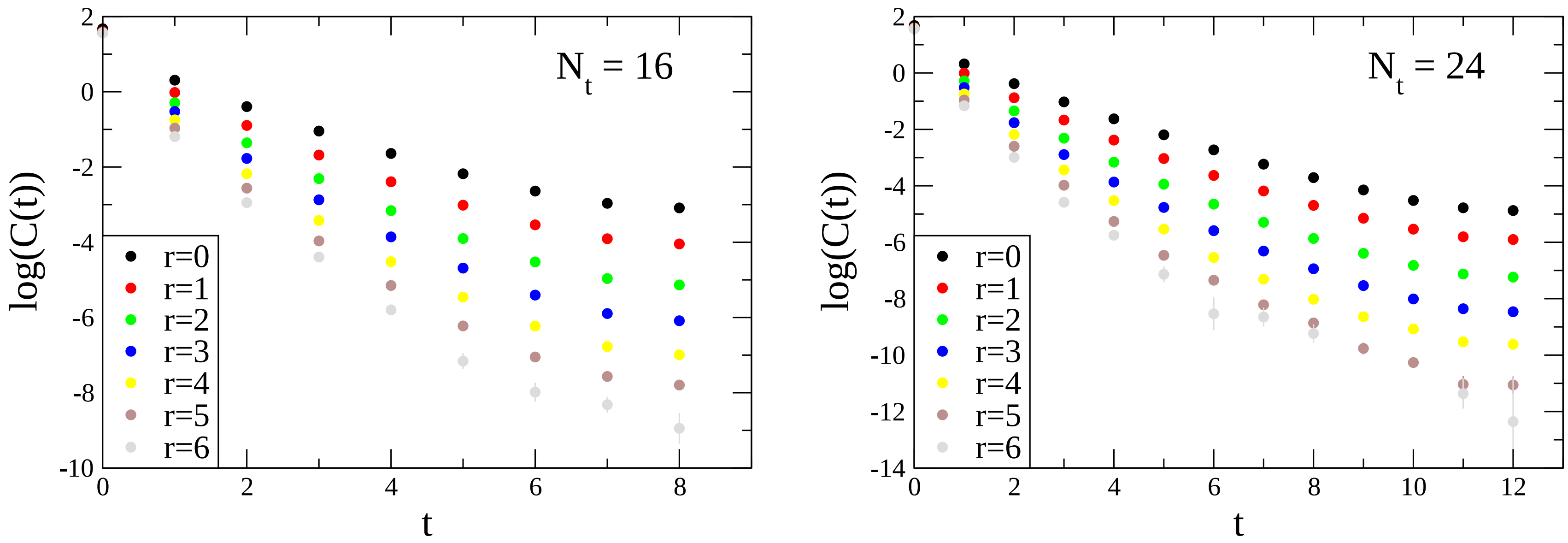

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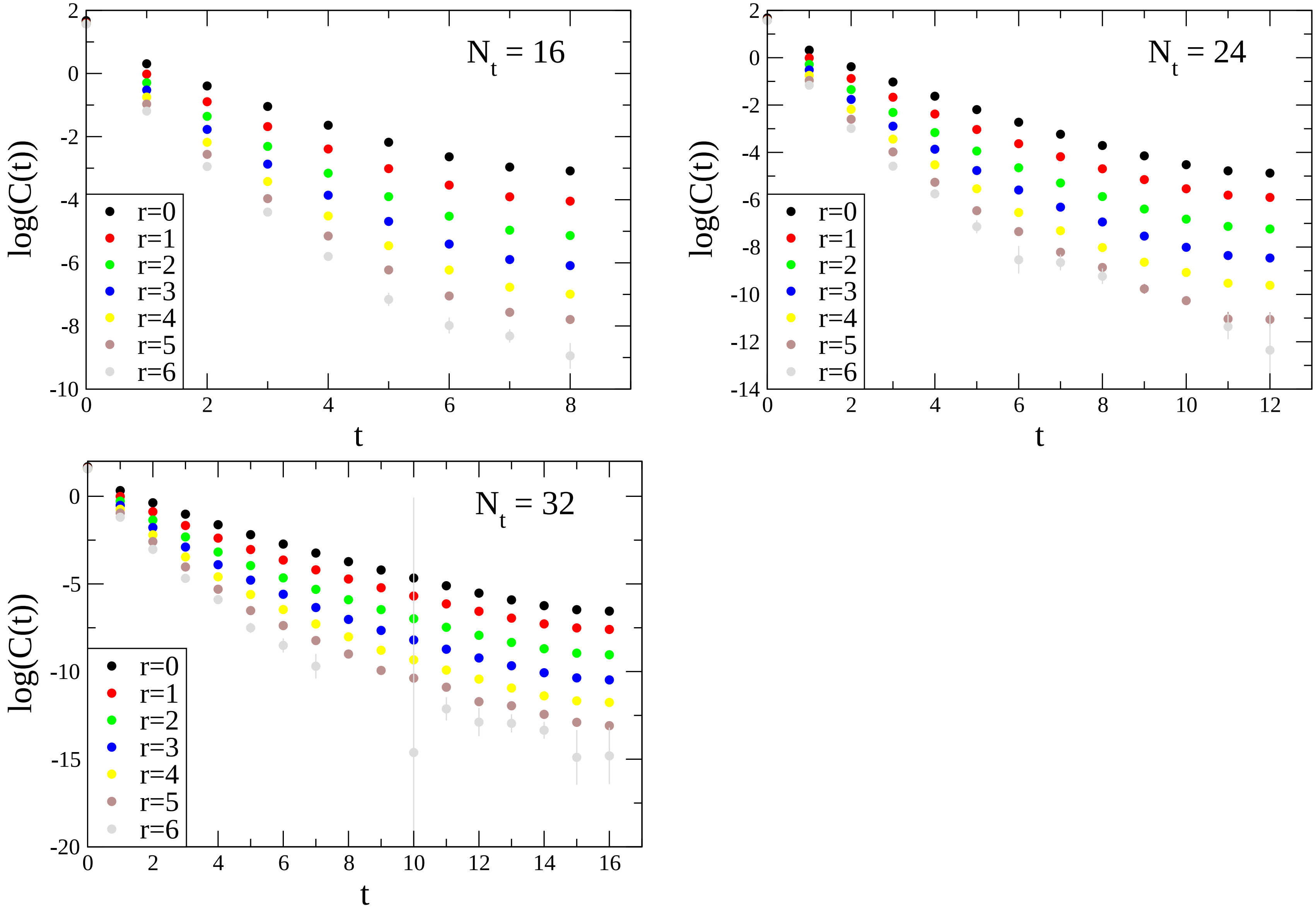

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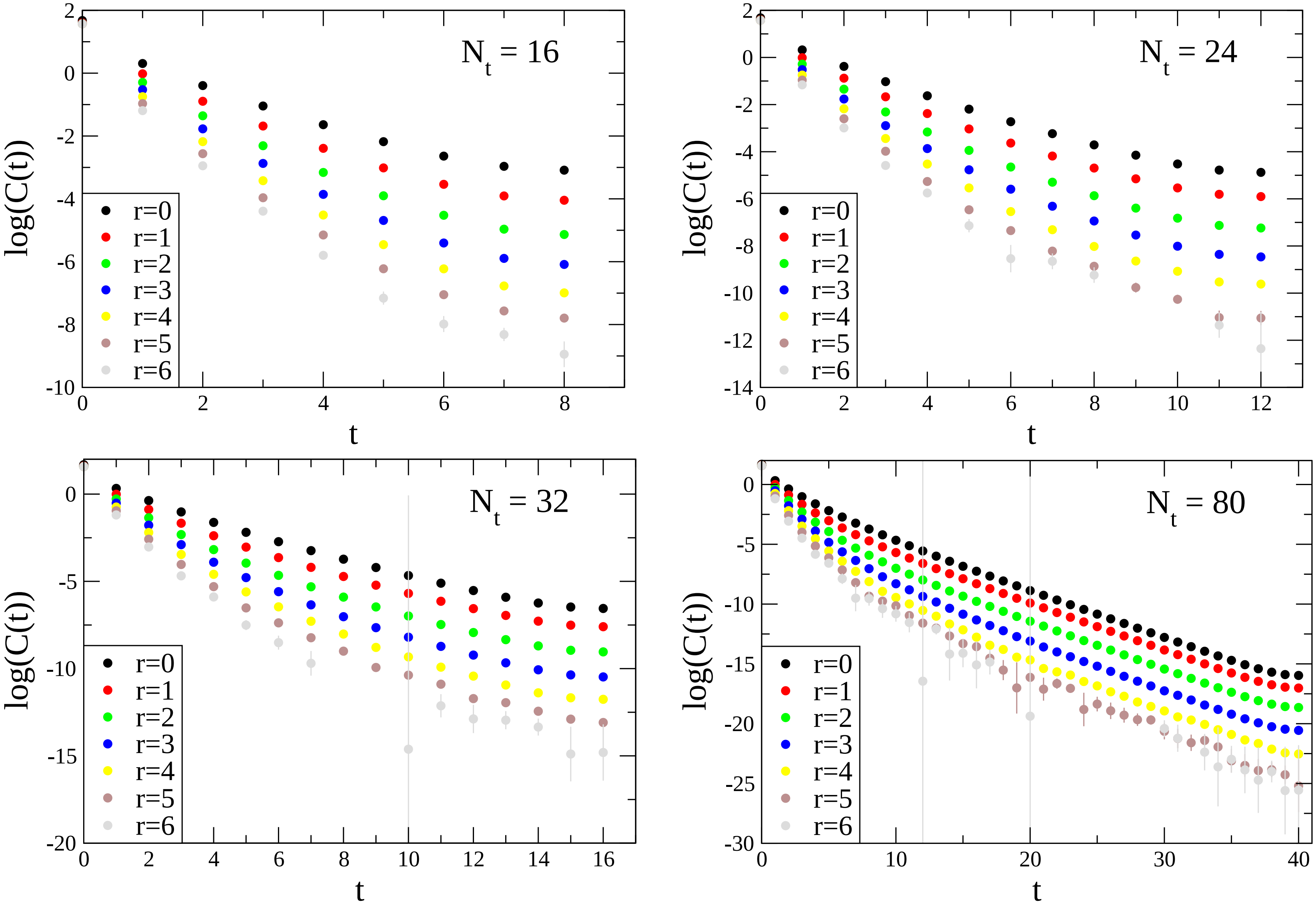
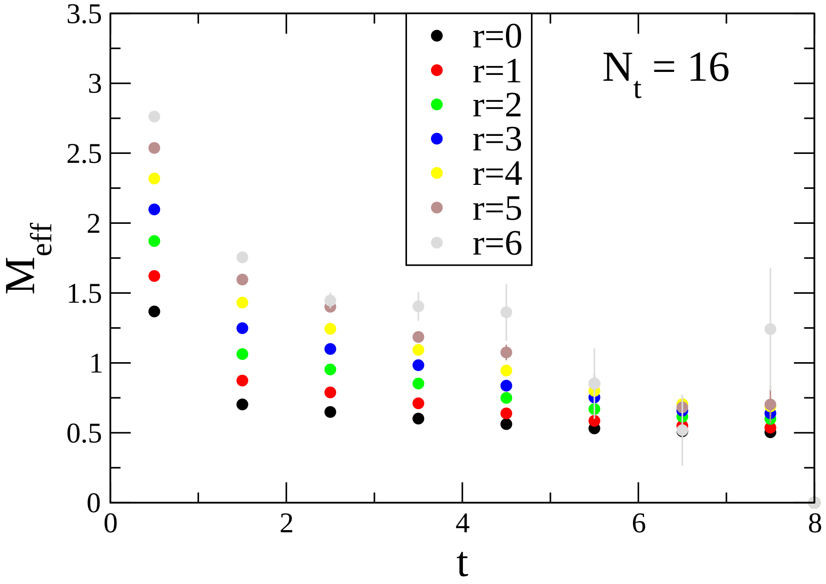


Effective Masses (PS)



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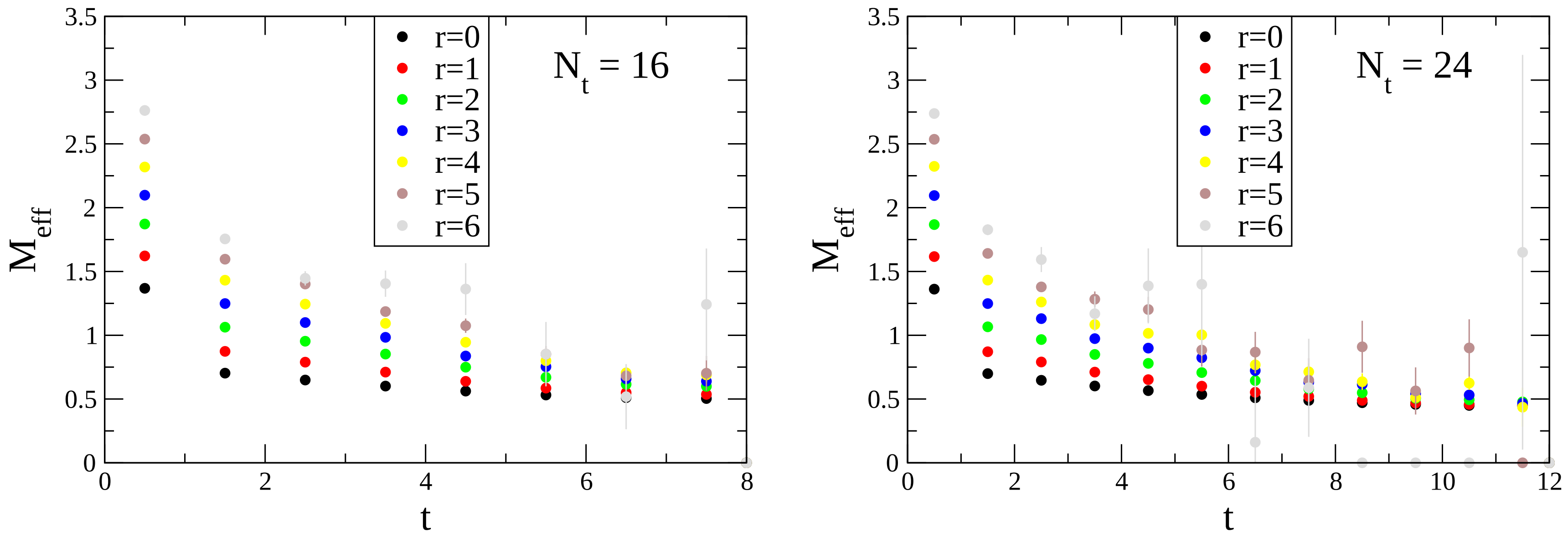

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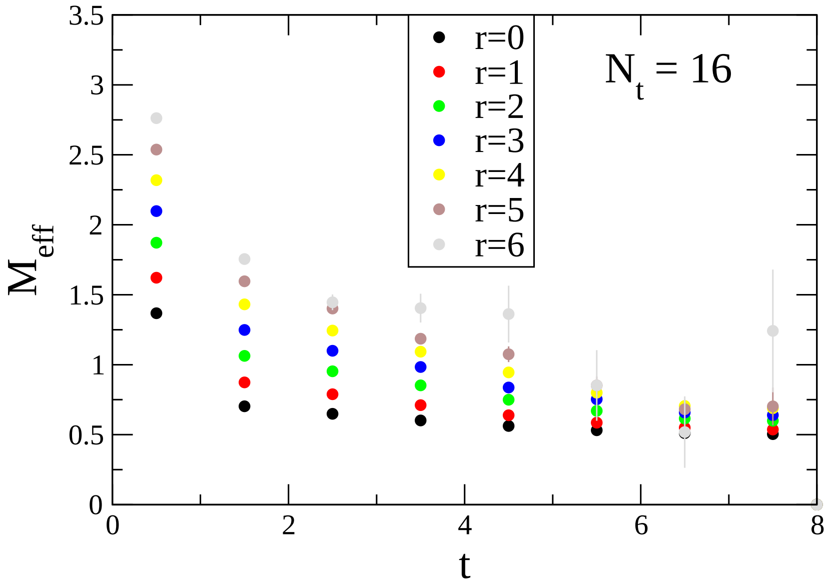

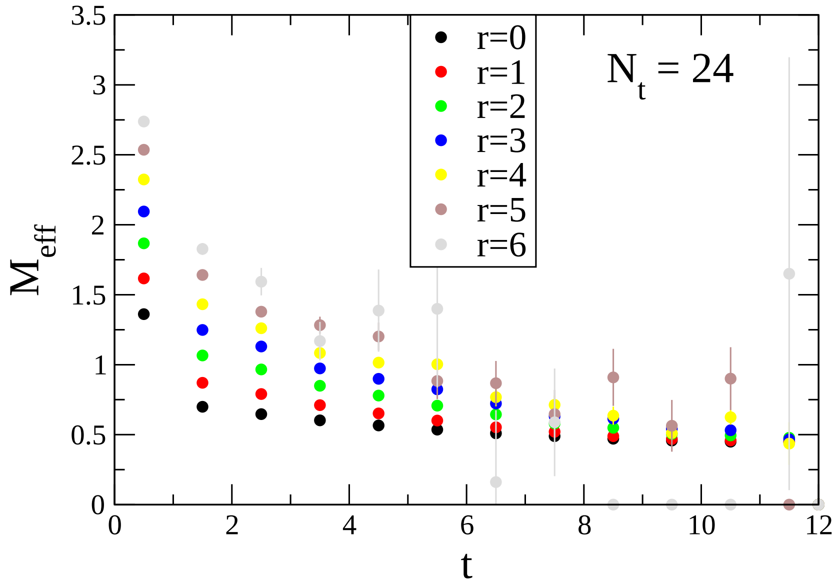

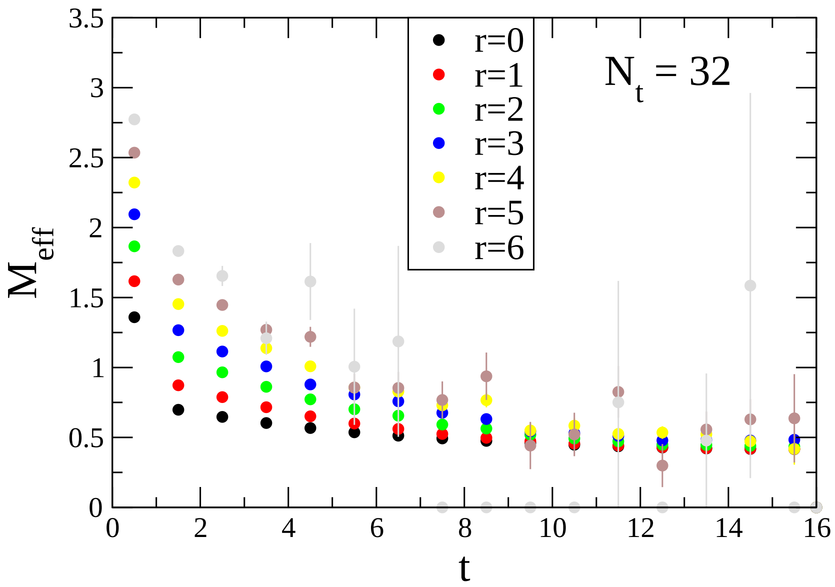

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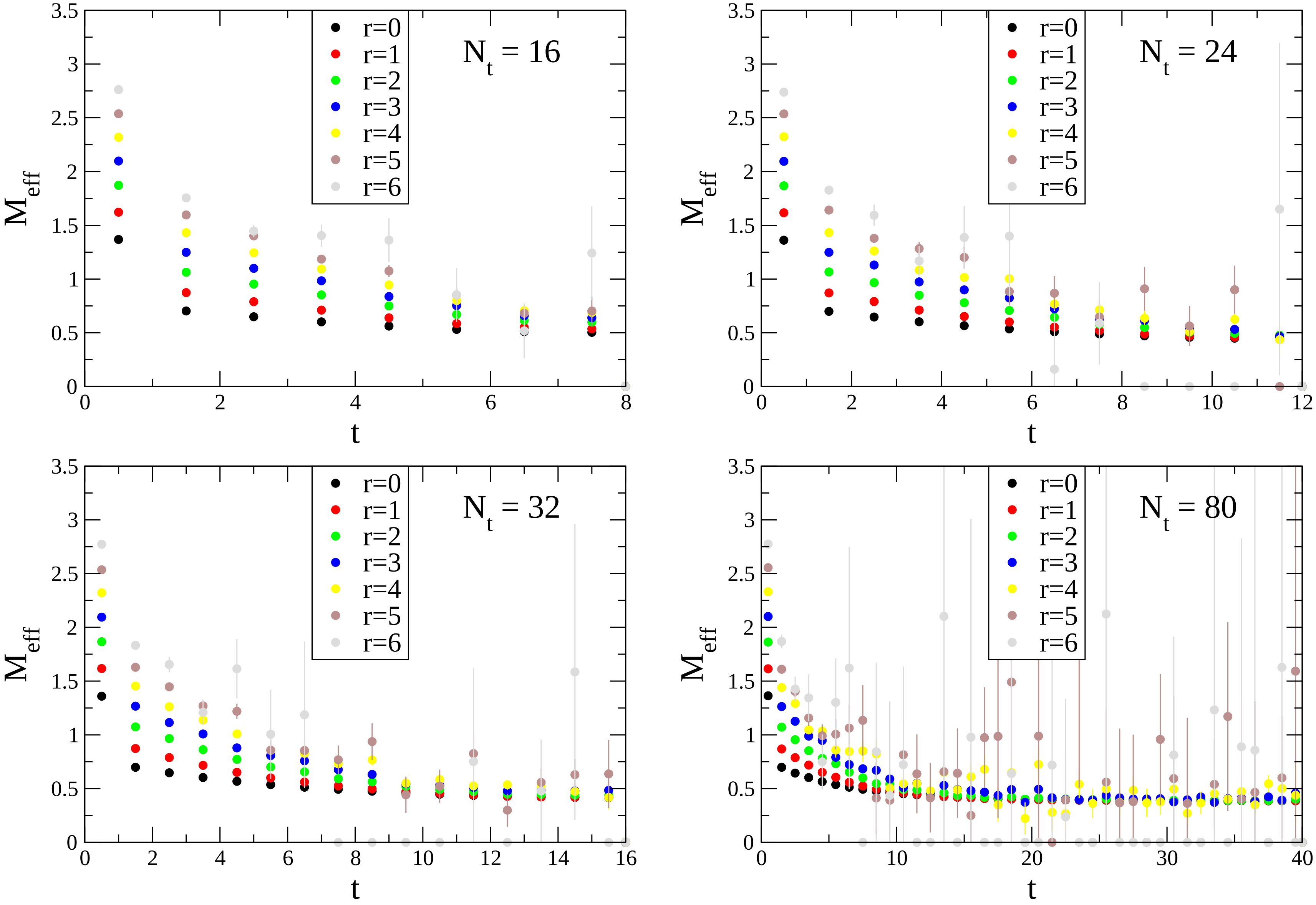
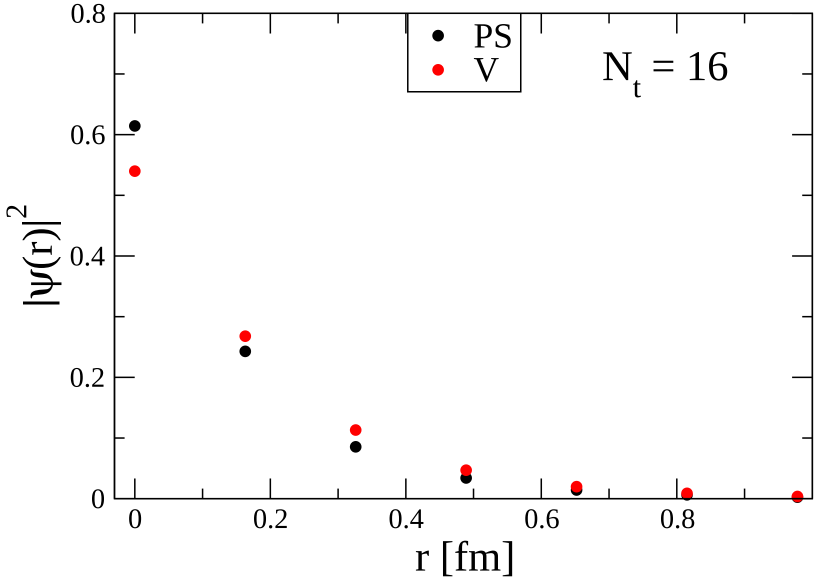


Wavefunctions (exp fitting) $C(t) \to |\psi(r)|^2 e^{-MT}$



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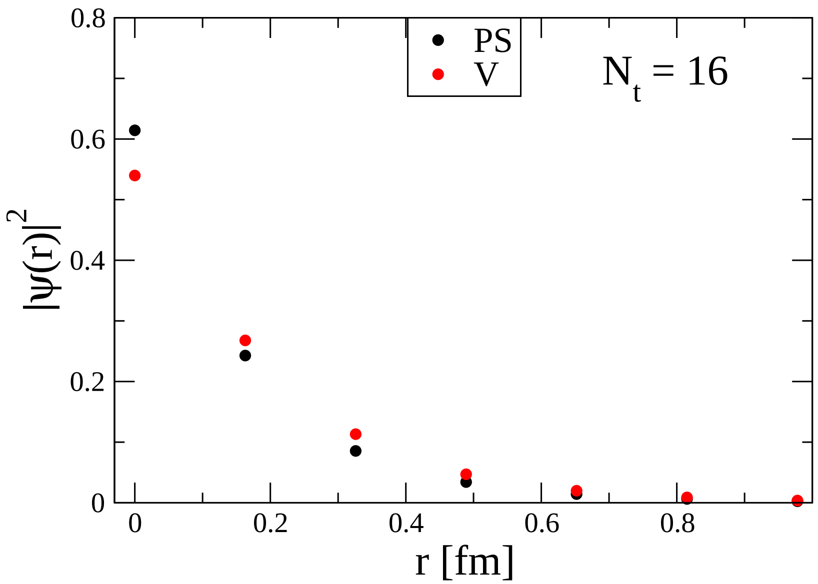

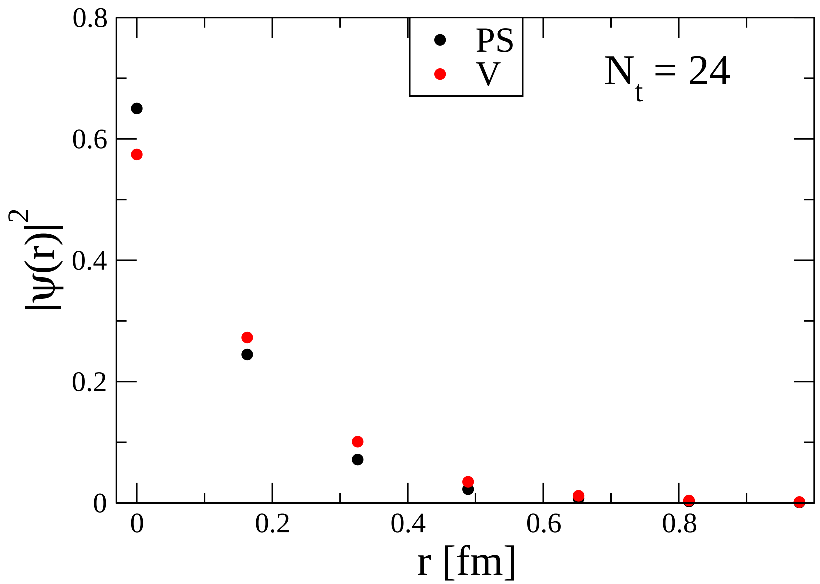

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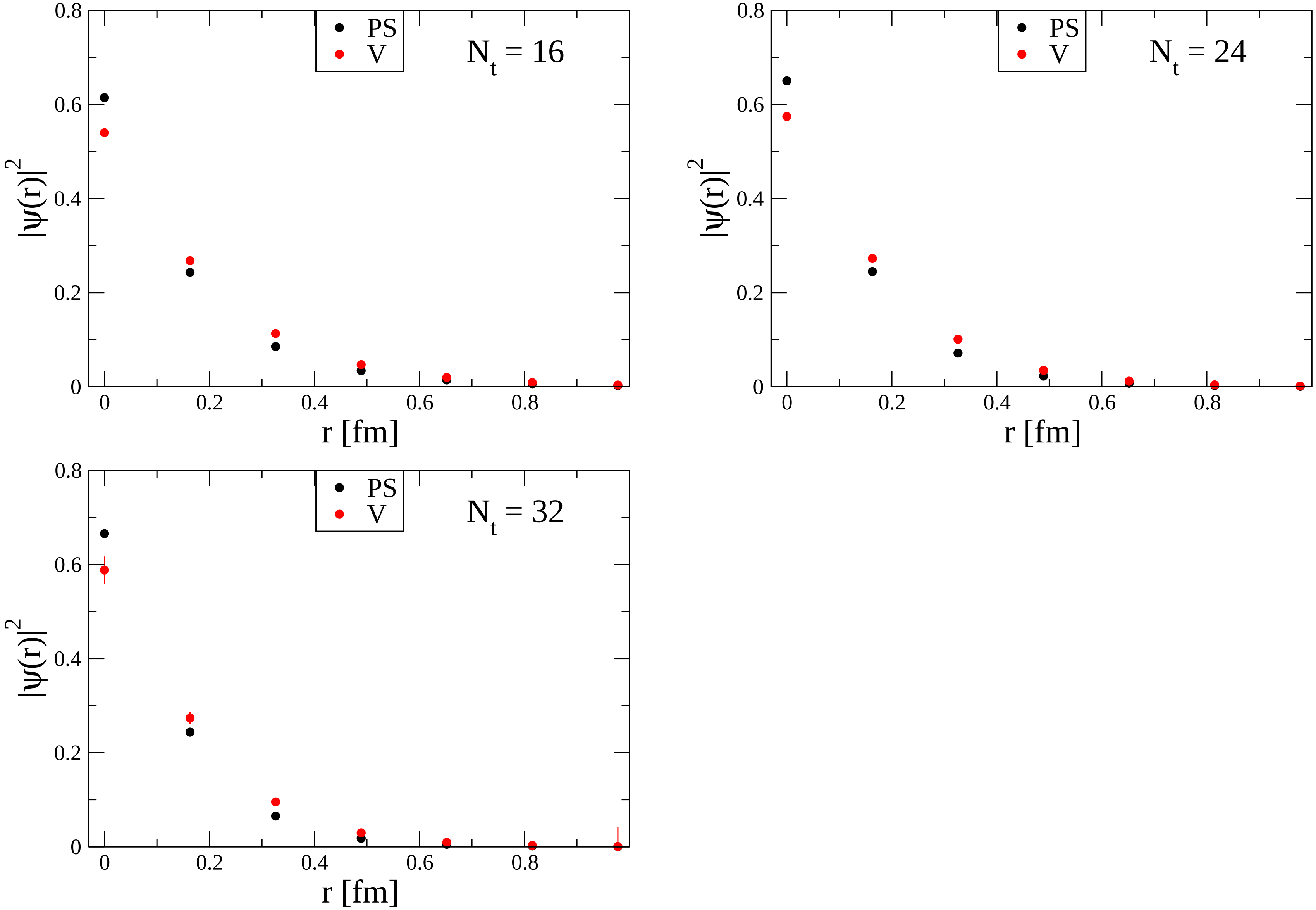

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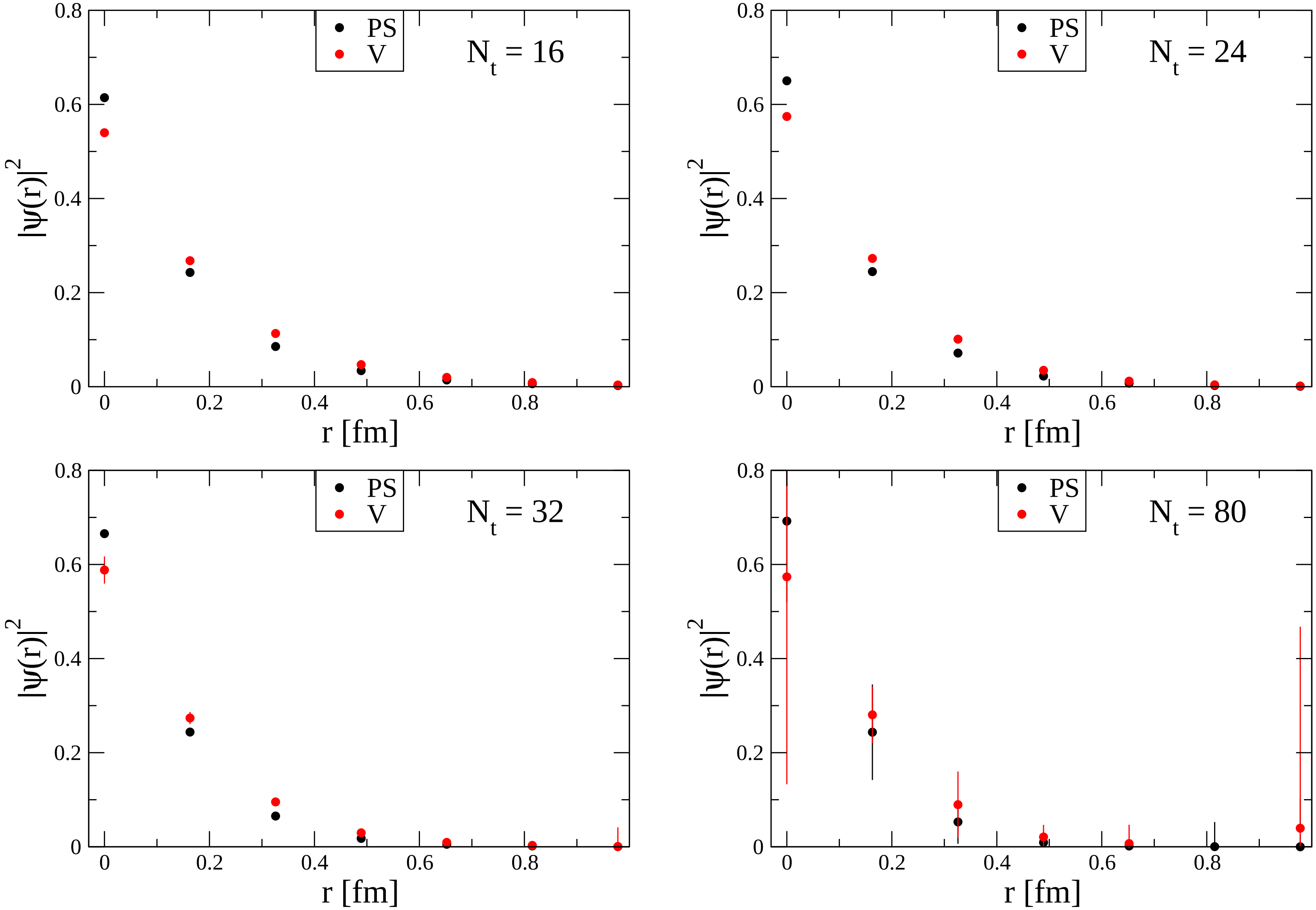


Spin Dependent Potential

$$V_{q\overline{q}} = \frac{1}{4}[V_{\mathsf{PS}}(r) + 3V_V(r)]$$

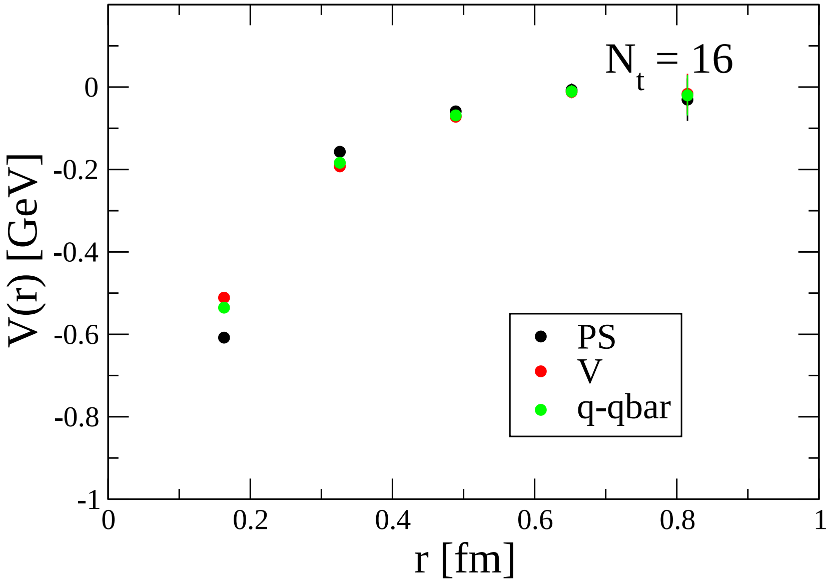
with the Schrödinger Eq'n used to define $V(r)$:

$$V_\Gamma(r) = E + \frac{1}{\psi(r)}\frac{\nabla^2}{2\mu}\psi(r)$$

where μ is the reduced mass:

$$\mu = \frac{1}{2}m_Q \qquad \text{where} \qquad m_Q \approx M_H/2$$

Potential (exp fitting) [Preliminary]



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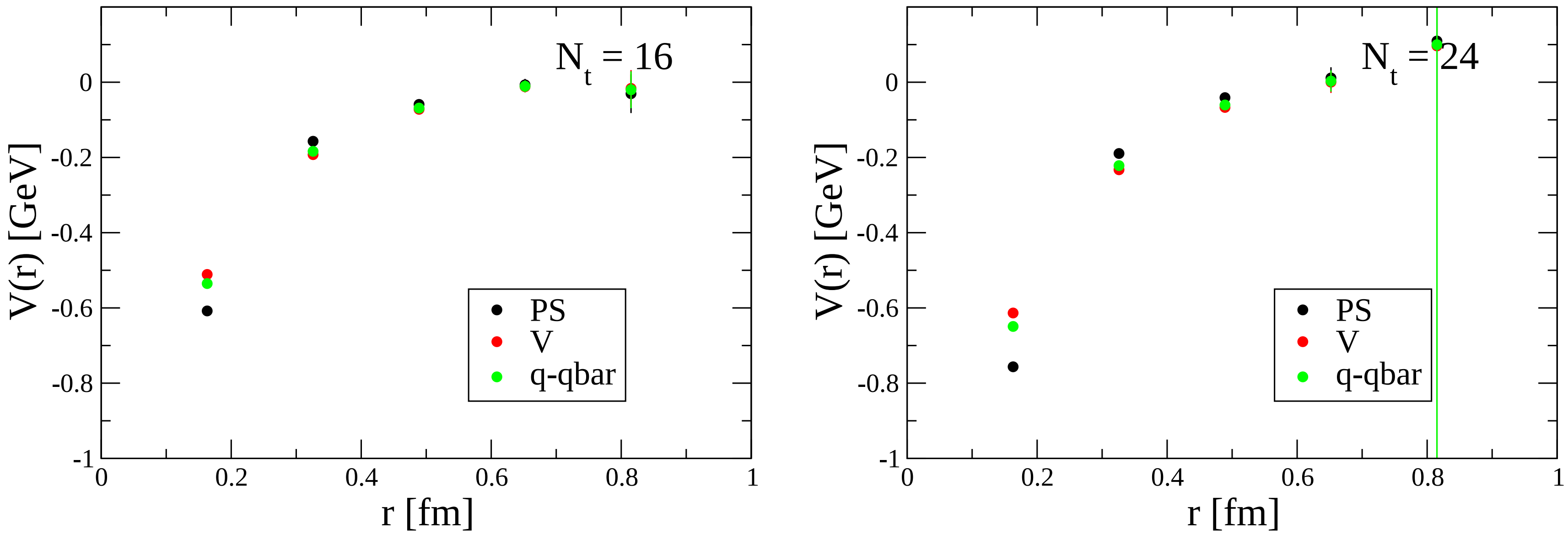

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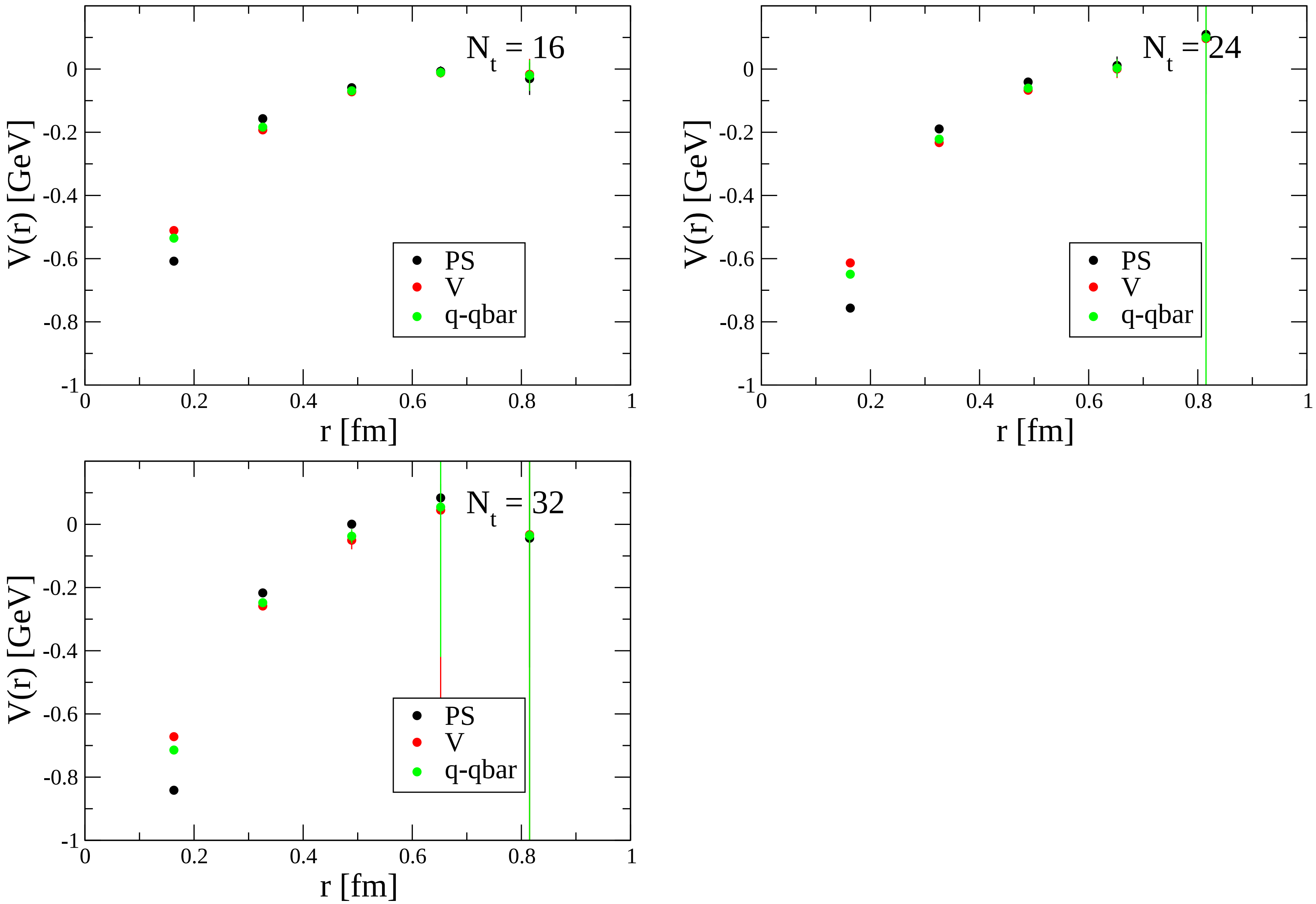

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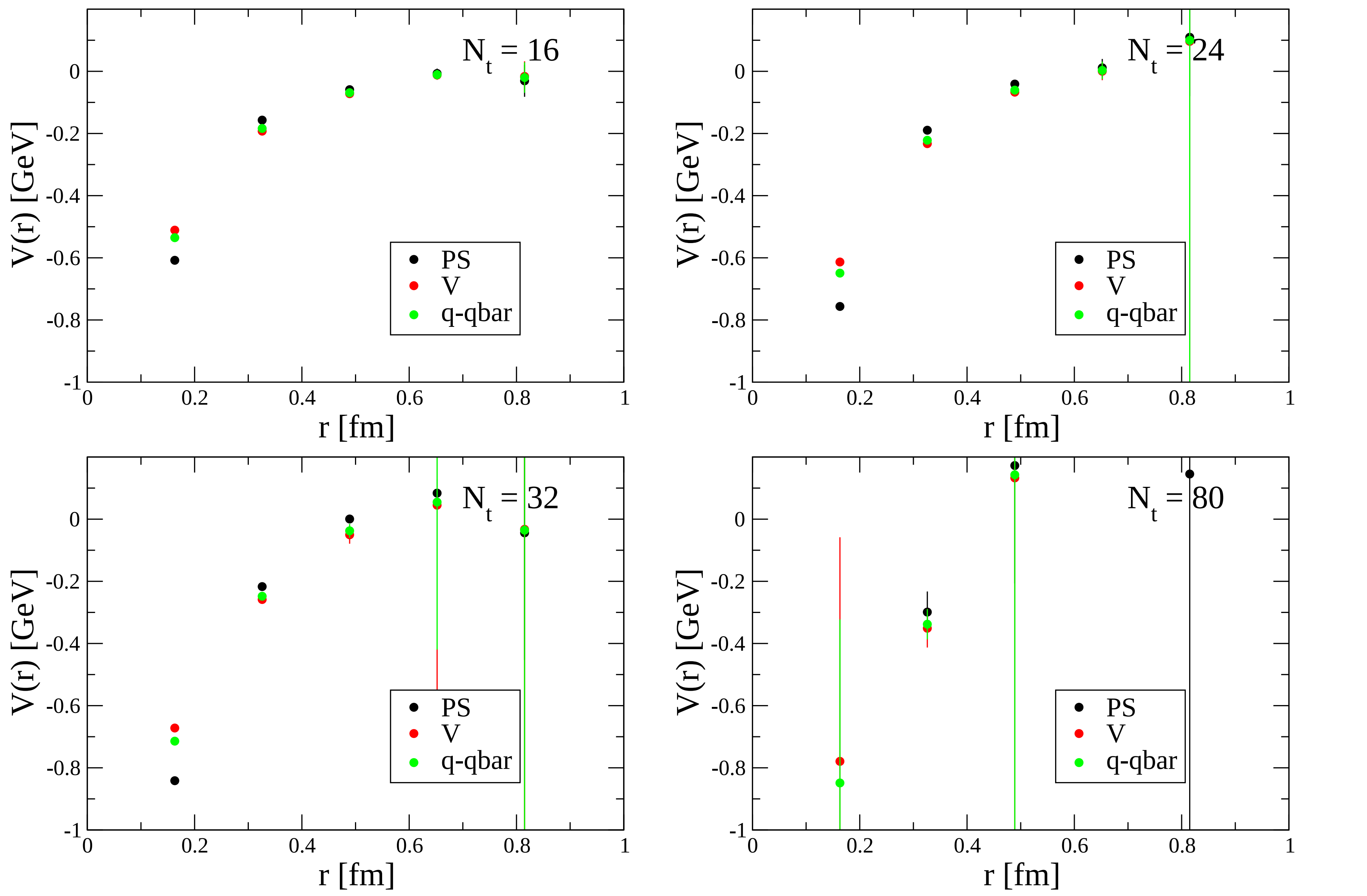
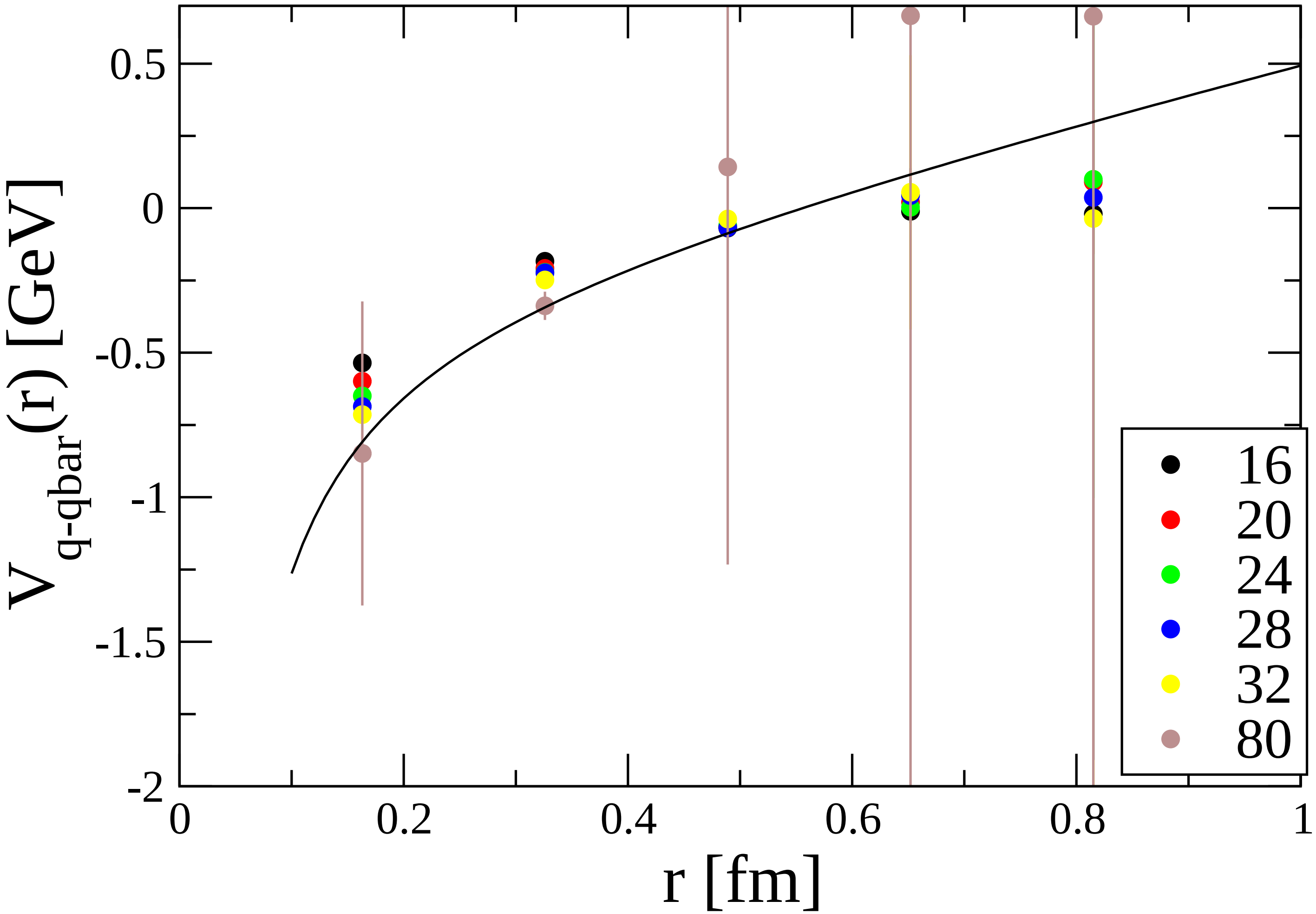


$V_{q-\bar{q}}$ Potential (exp fitting)



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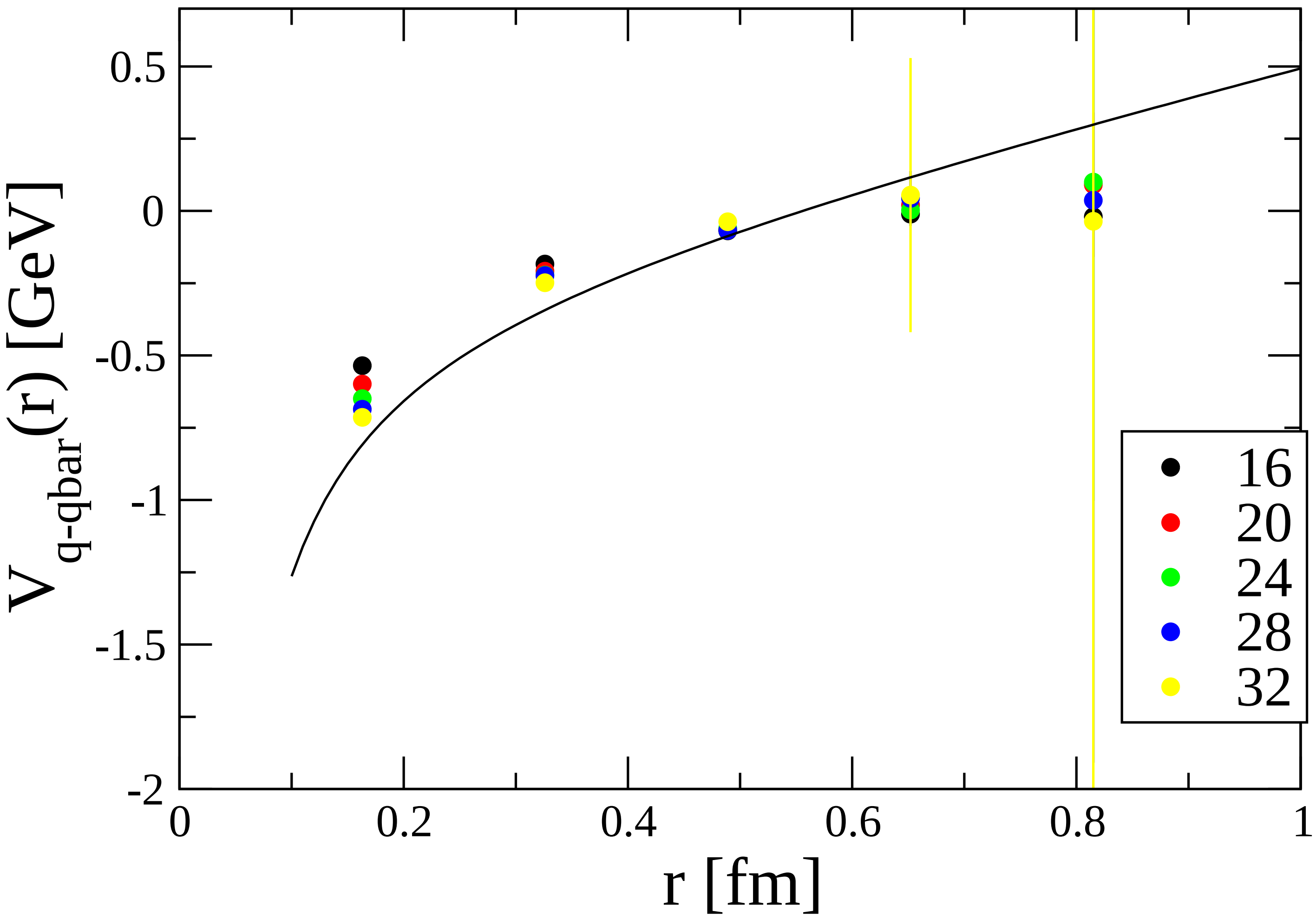


MEM

Motivation

Do bound hadronic states persist into the "quark-gluon" plasma phase?
How can we extract transport coefficients?

- *Spectral functions* can answer this!

$$C(t,\vec{p}) \quad = \quad \int \rho(\omega,\vec{p})\; K(t,\omega)\; d\omega$$

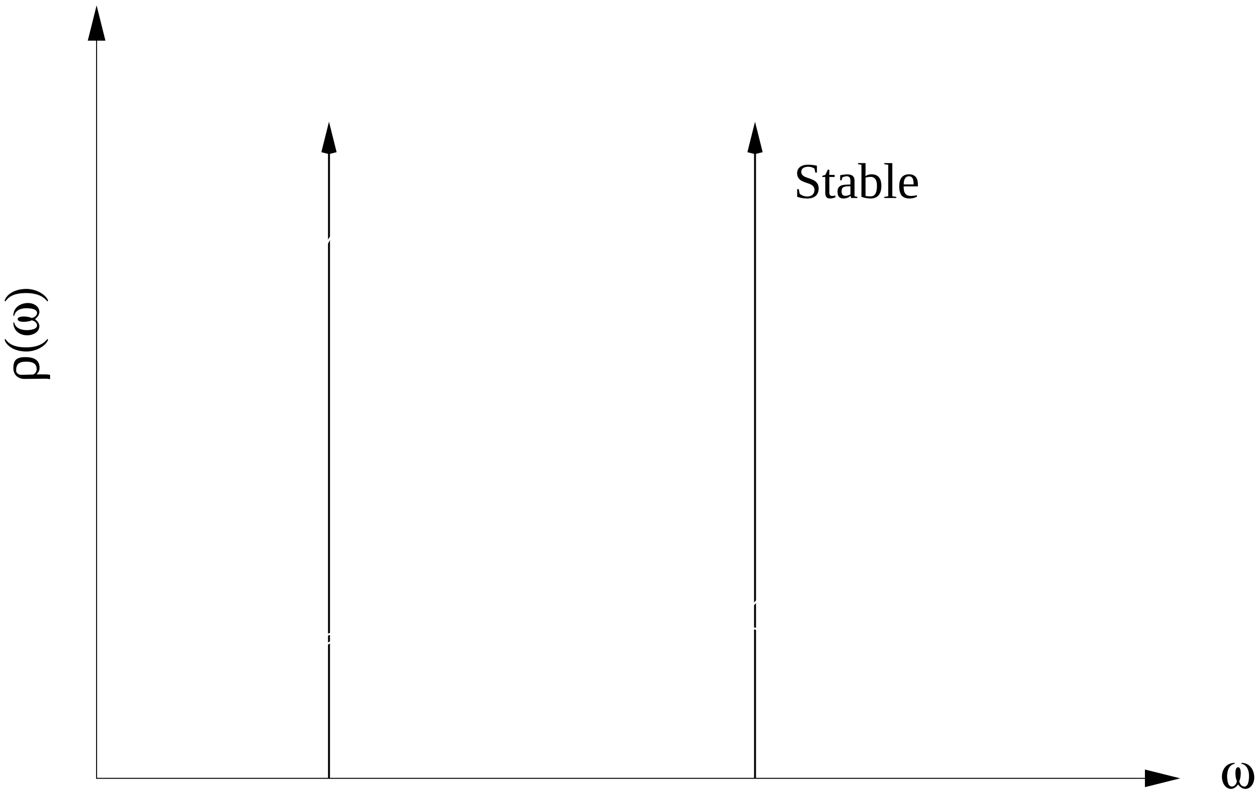
$\uparrow$	$\downarrow$	$\nwarrow$
Euclidean	Spectral	(Lattice)
Correlator	Function	Kernel

where the (lattice) Kernel is:

$$\begin{aligned} K(t,\omega) &= \frac{\cosh[\omega(t-N_t/2)]}{\sinh[\omega/(2T)]} \\ &\sim \exp[-\omega t] \end{aligned}$$

Example Spectral Functions

$$C(t) \sim \int \rho(\omega)\, e^{-\omega t}\, d\omega$$



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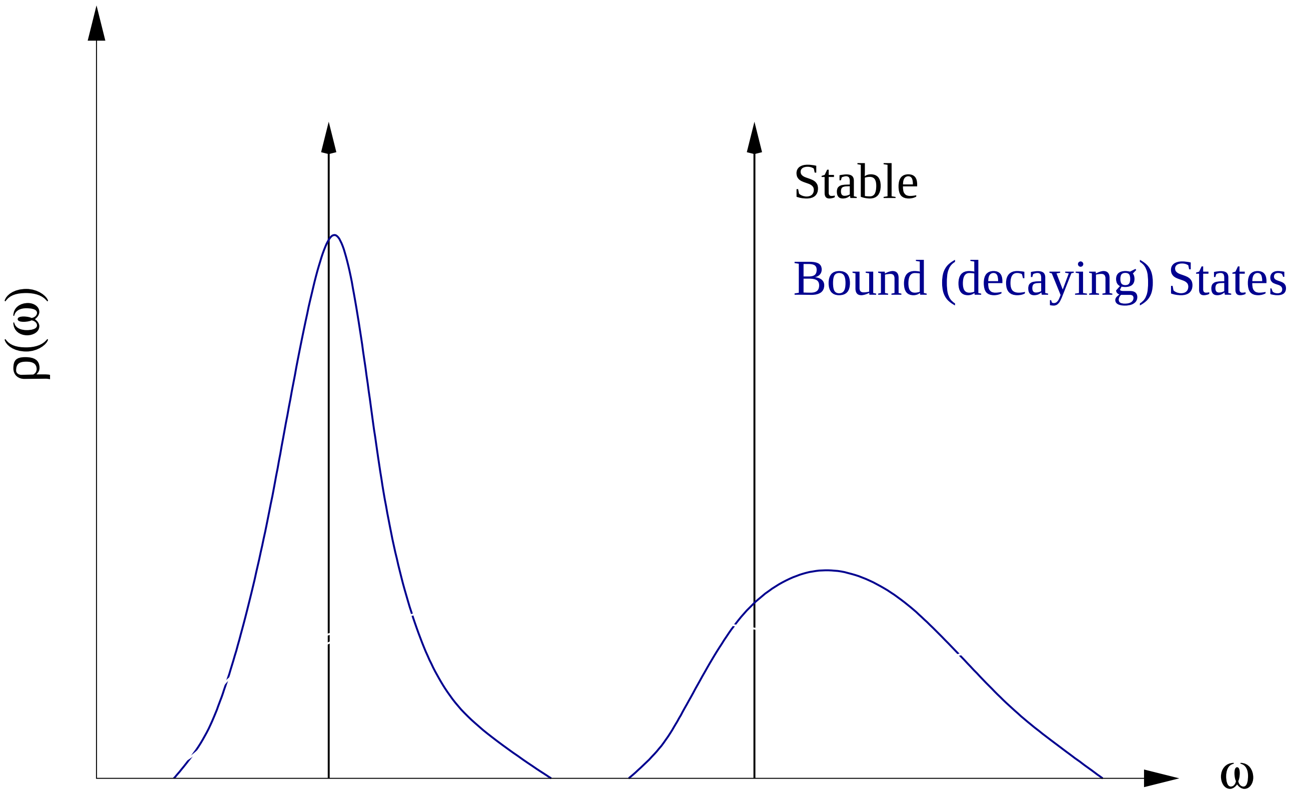

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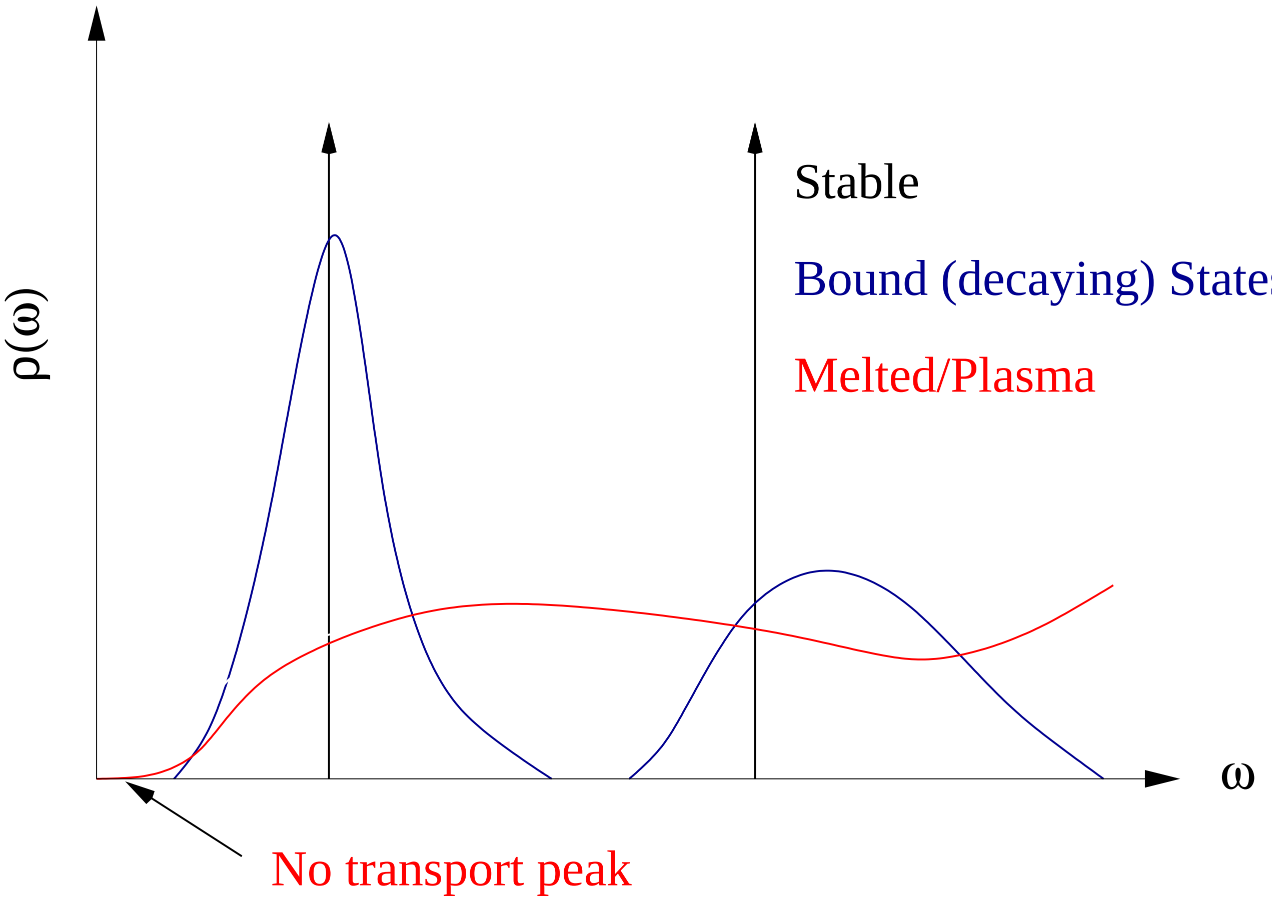

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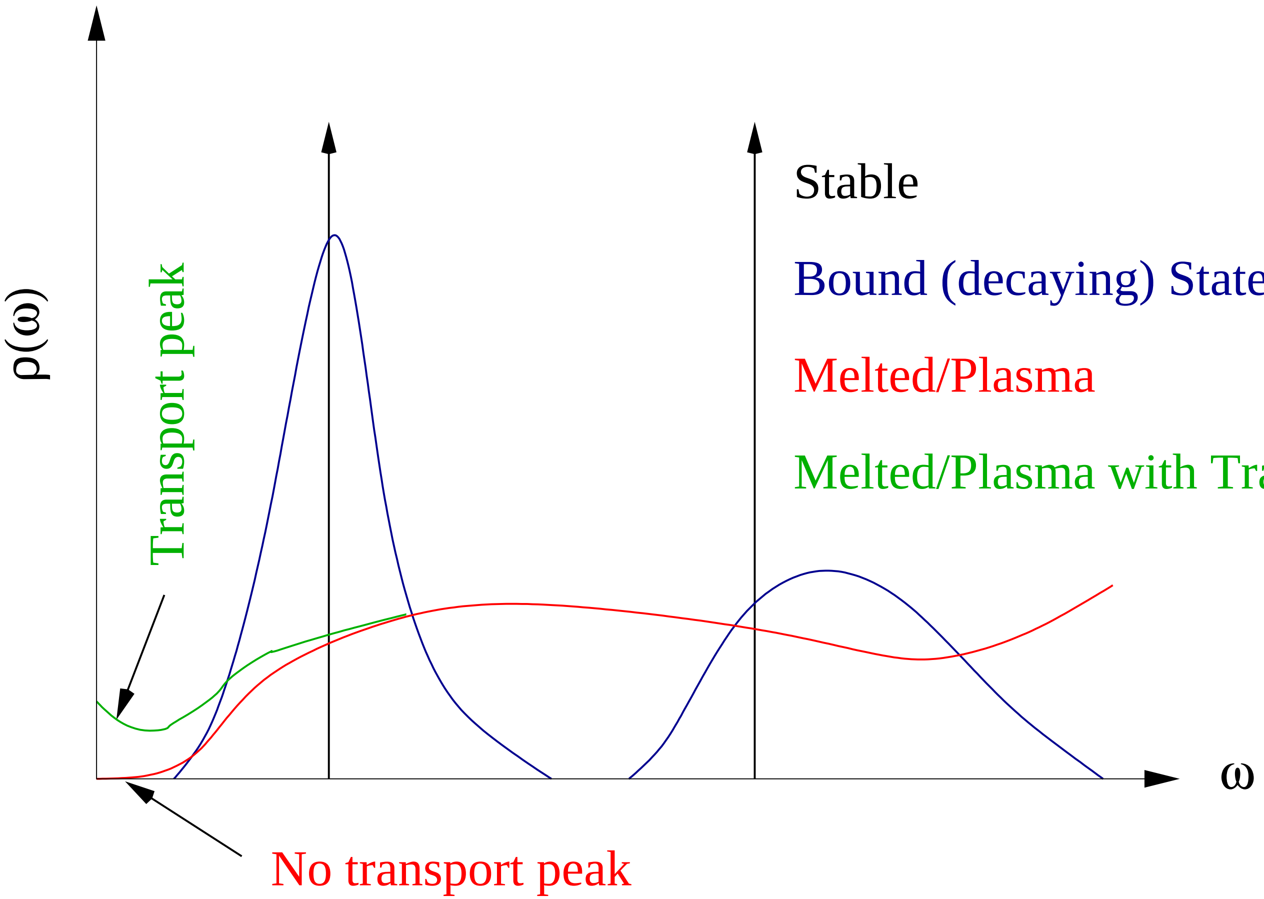
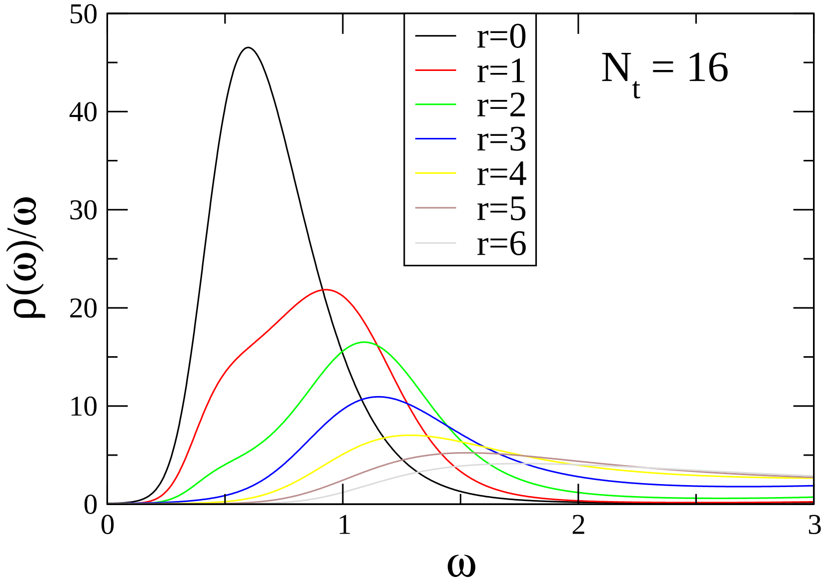


Spectral Functions via MEM

- Extraction of a spectral density from a lattice correlator is an ill-posed problem:
  - *Given* $C(t)$ *derive* $\rho(\omega)$
  - *More* ω *data points then* t *data points!*
- Requires the use of Bayesian analysis - Maximum Entropy Method (MEM)
  - Hatsuda, Asakawa et al
  - Commonly used in other areas...
- Need to check MEM output w.r.t. choice of:
  - Default model
  - Statistics
  - Energy range
  - Euclidean time range

Spectral Functions (PS)



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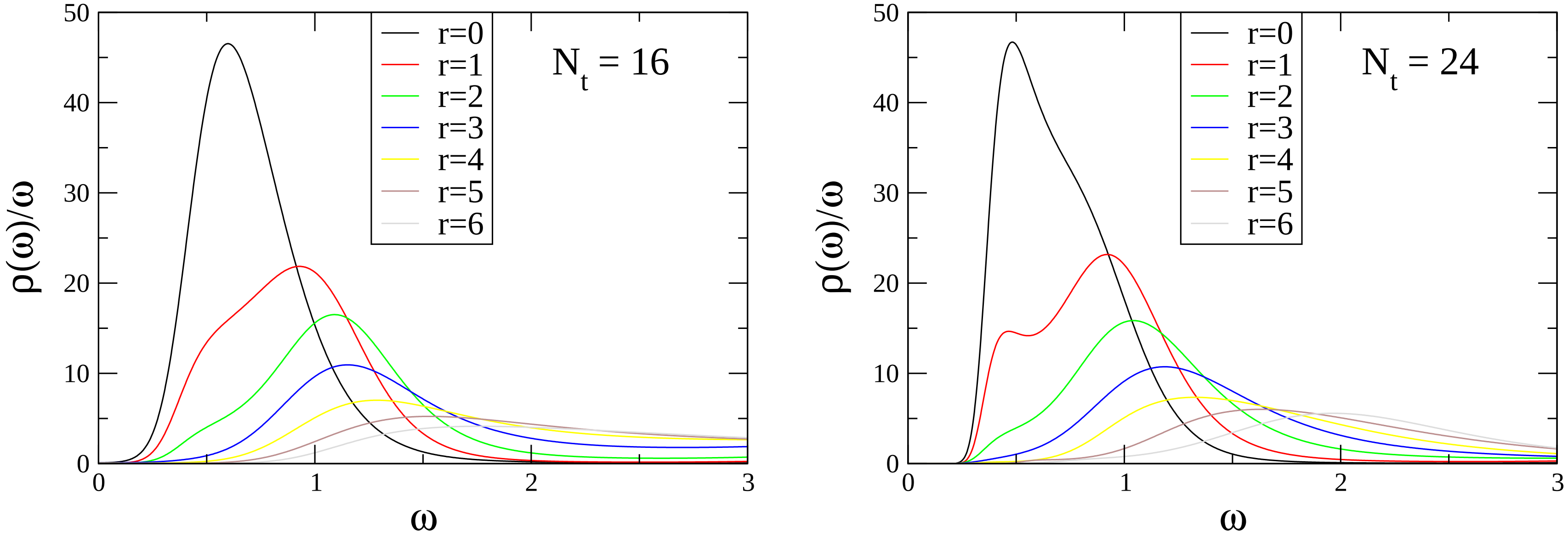

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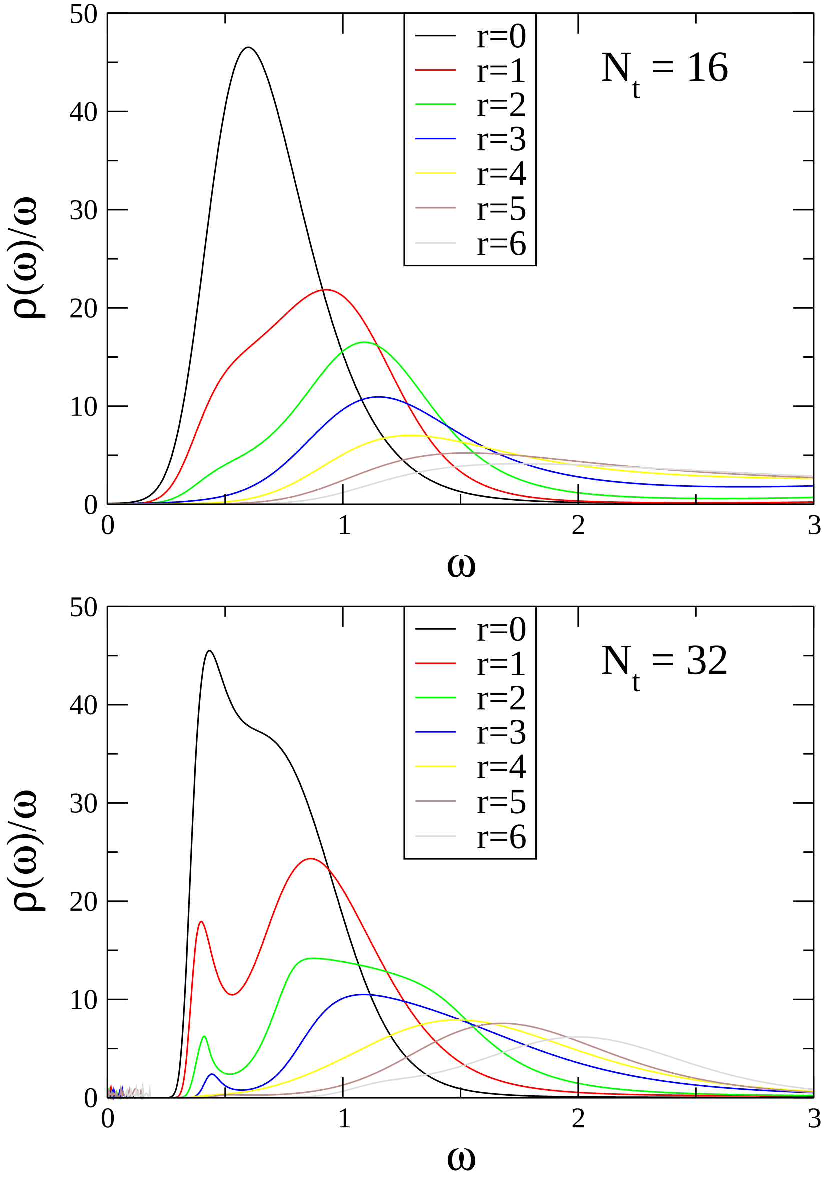

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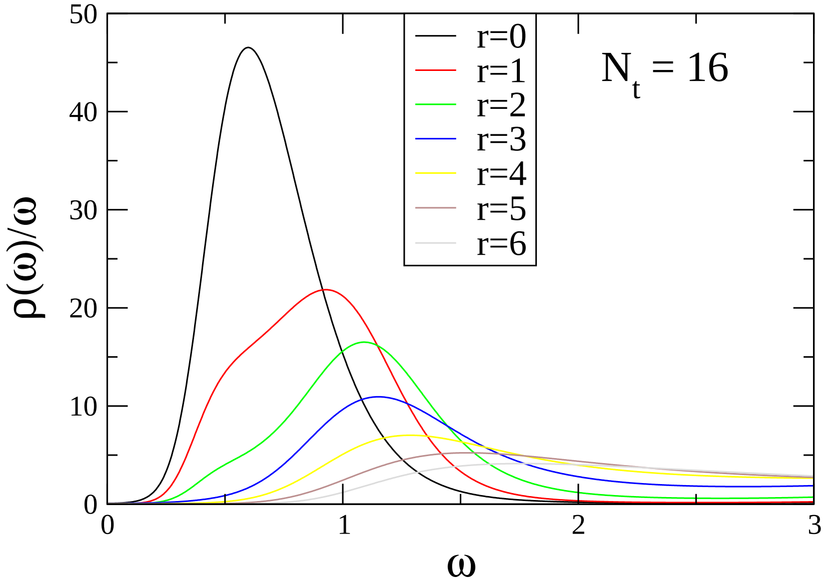
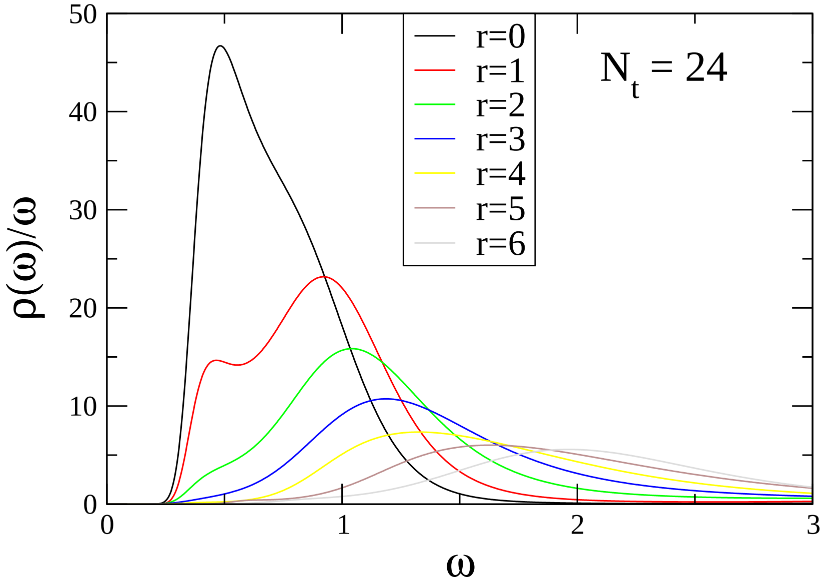
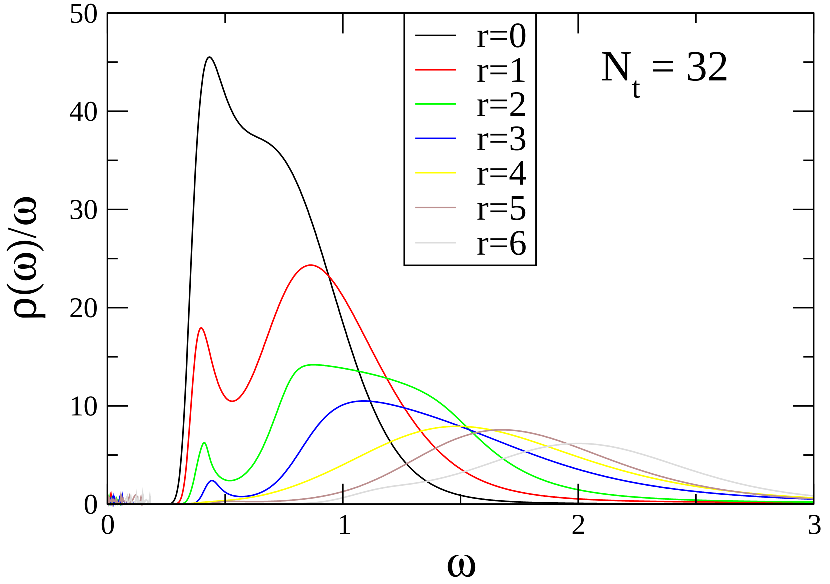


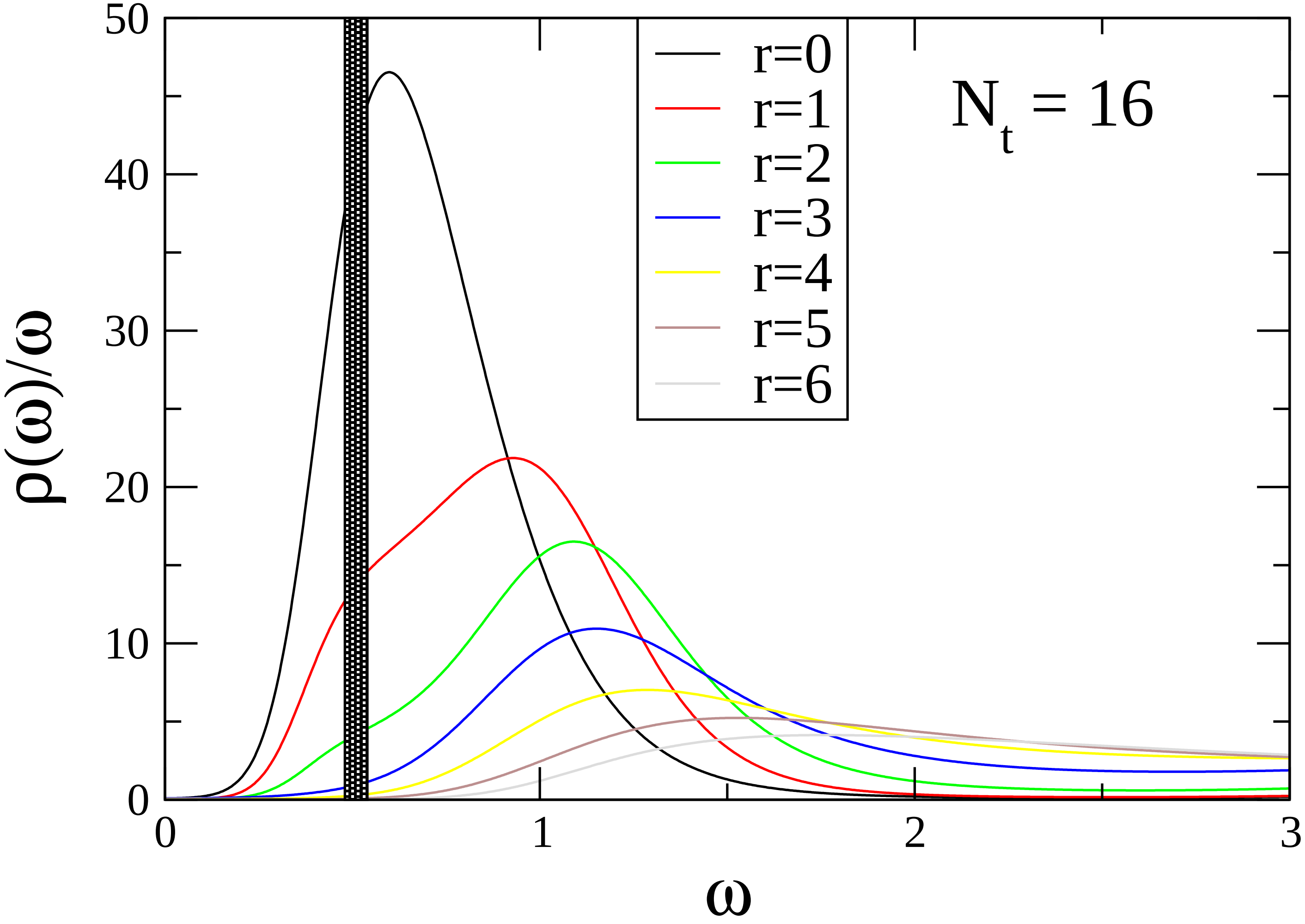




$$C(t) \sim \int \rho(\omega)\; e^{-\omega t}\; d\omega$$

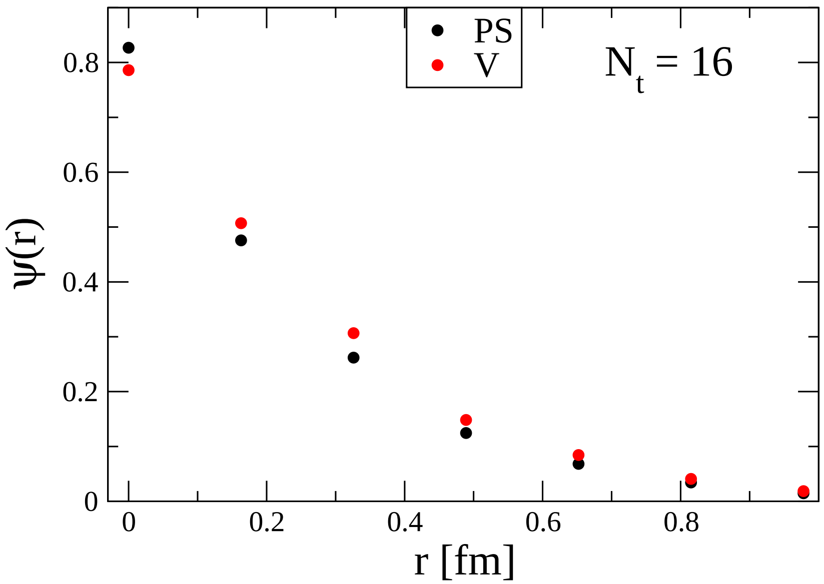
But $\rho(\omega) \sim |\psi(r)|^2$

Spectral Functions (PS)



Range in ω spans the ground state mass from exp fit

Wavefunctions (MEM)



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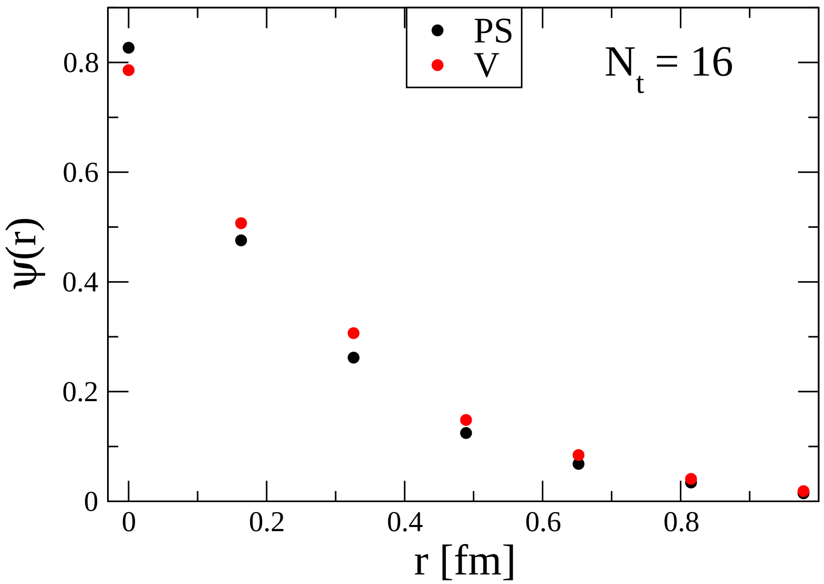

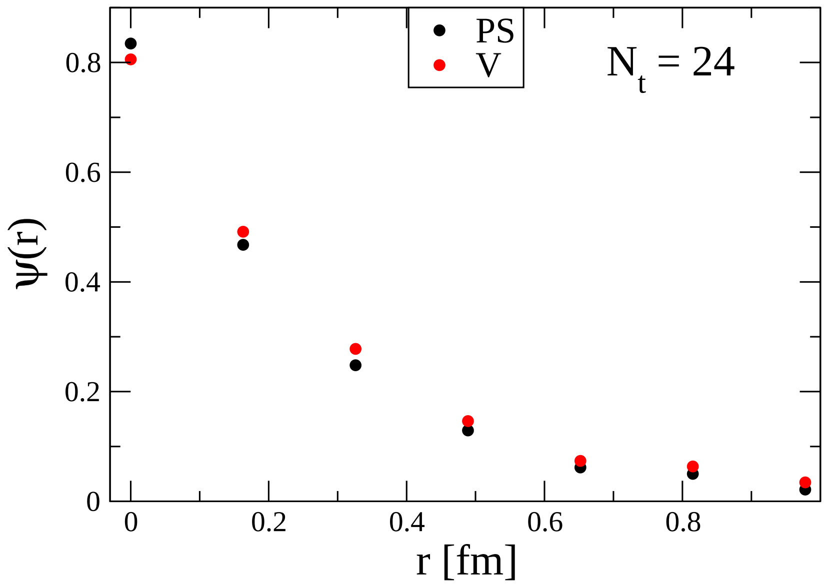

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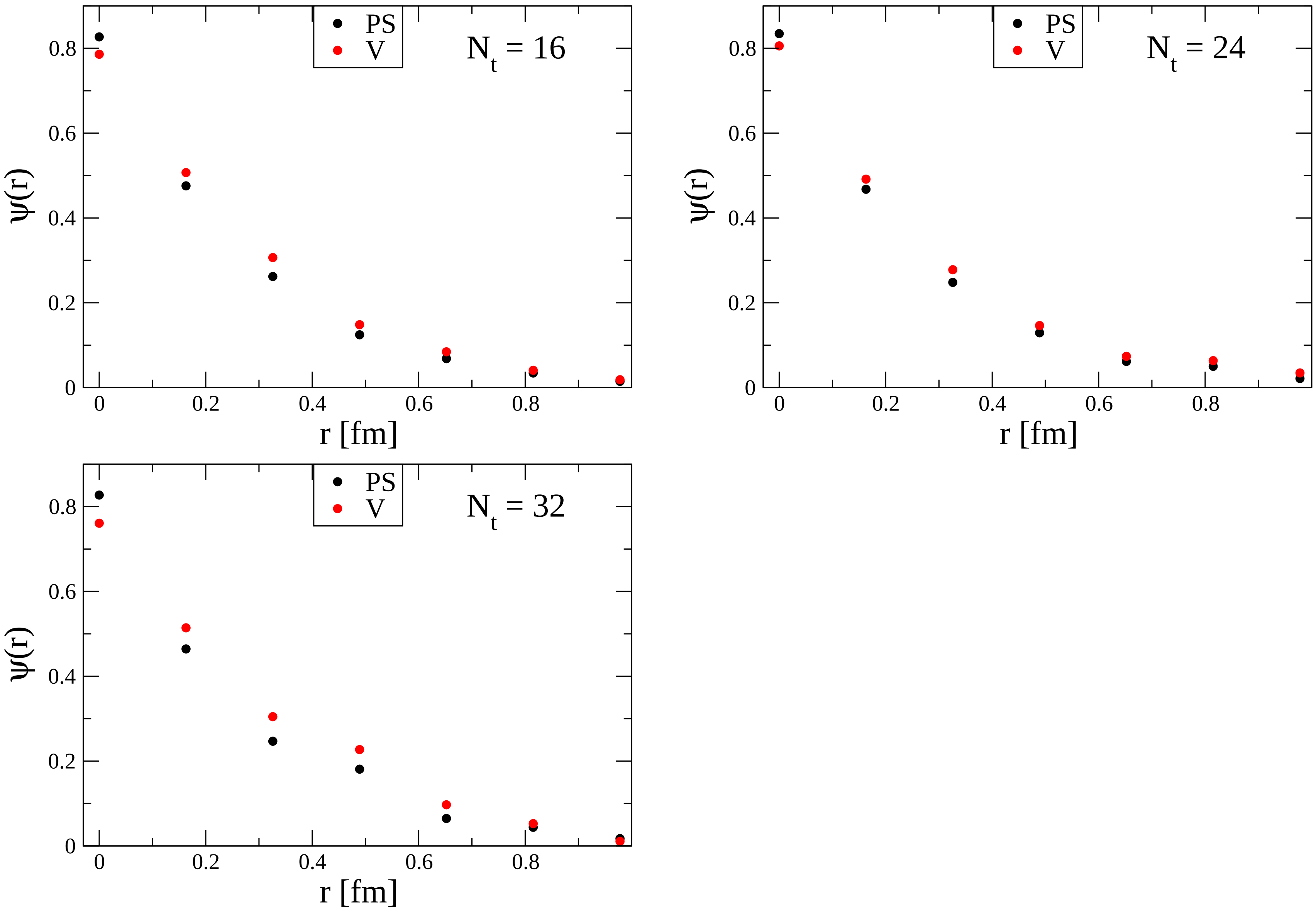

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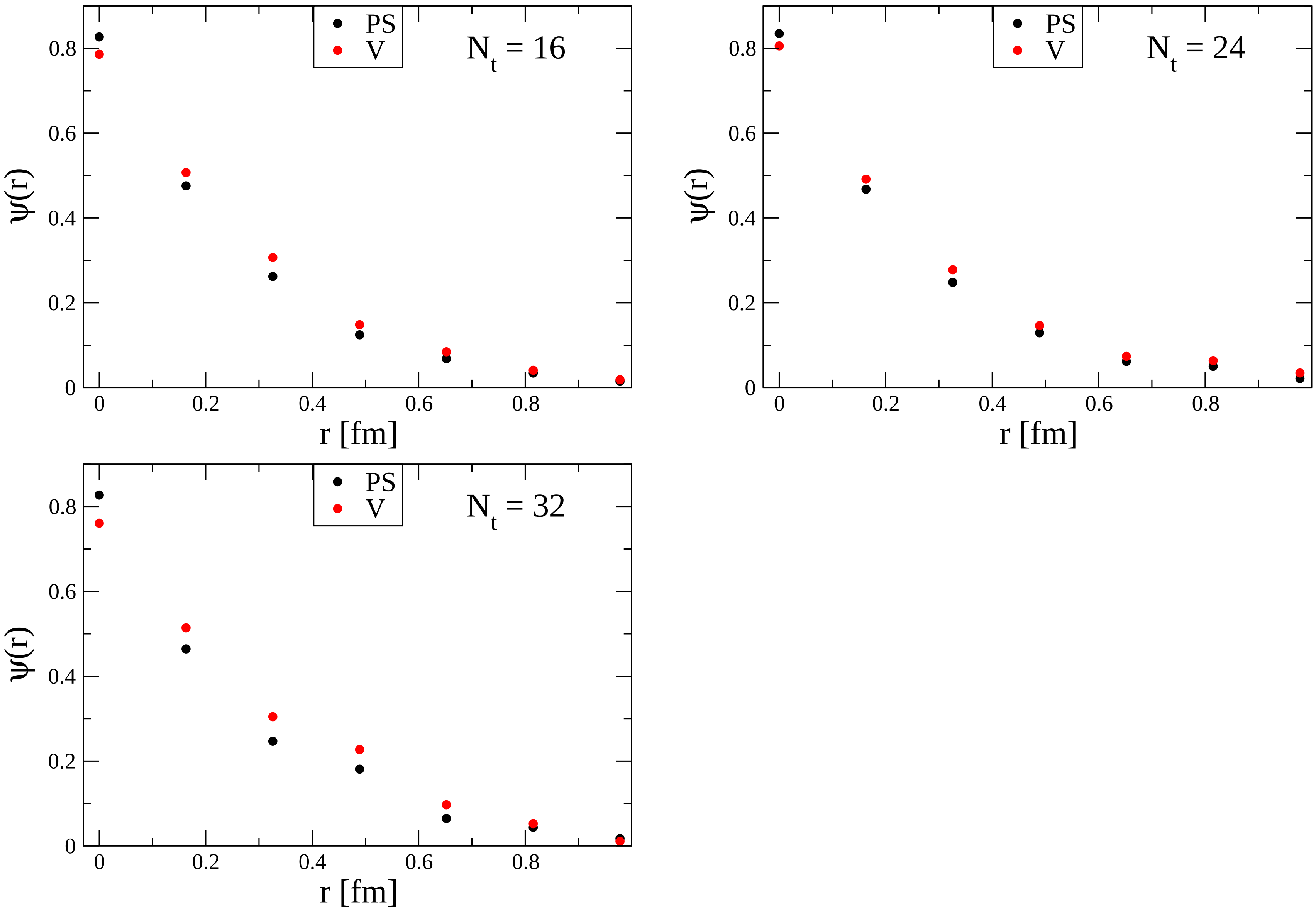
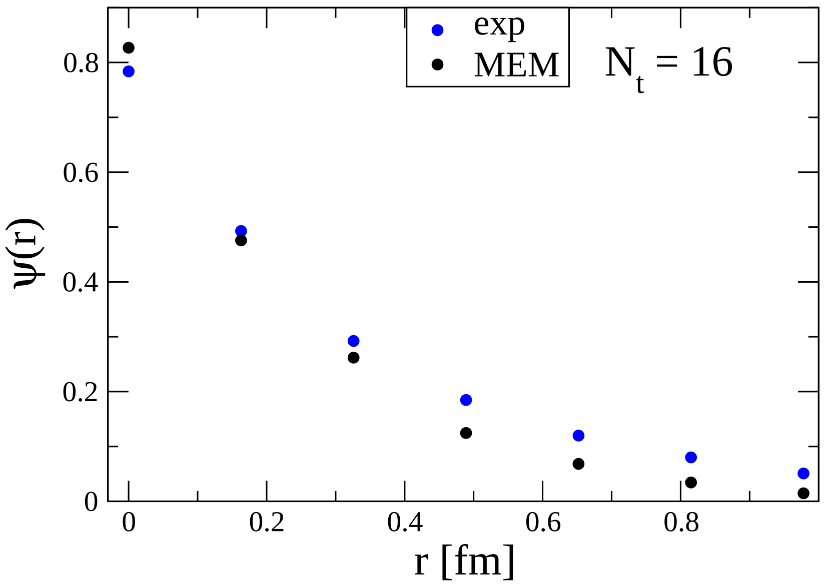


Wavefunctions: MEM v exp (PS)



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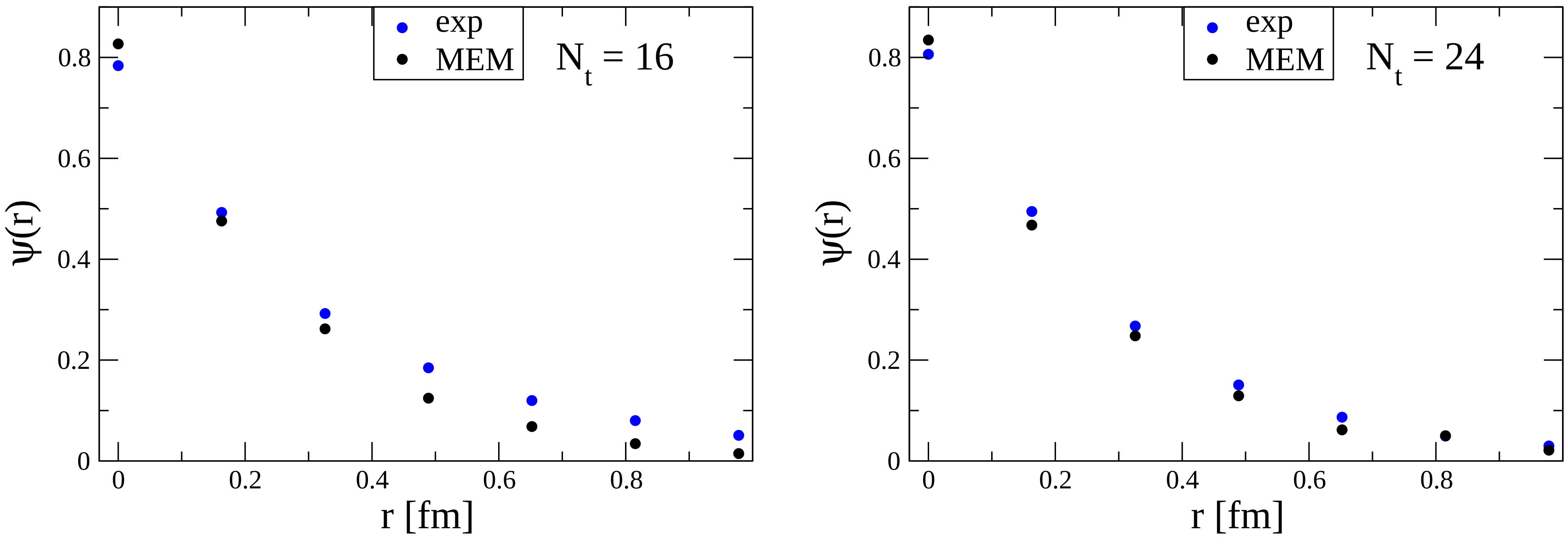

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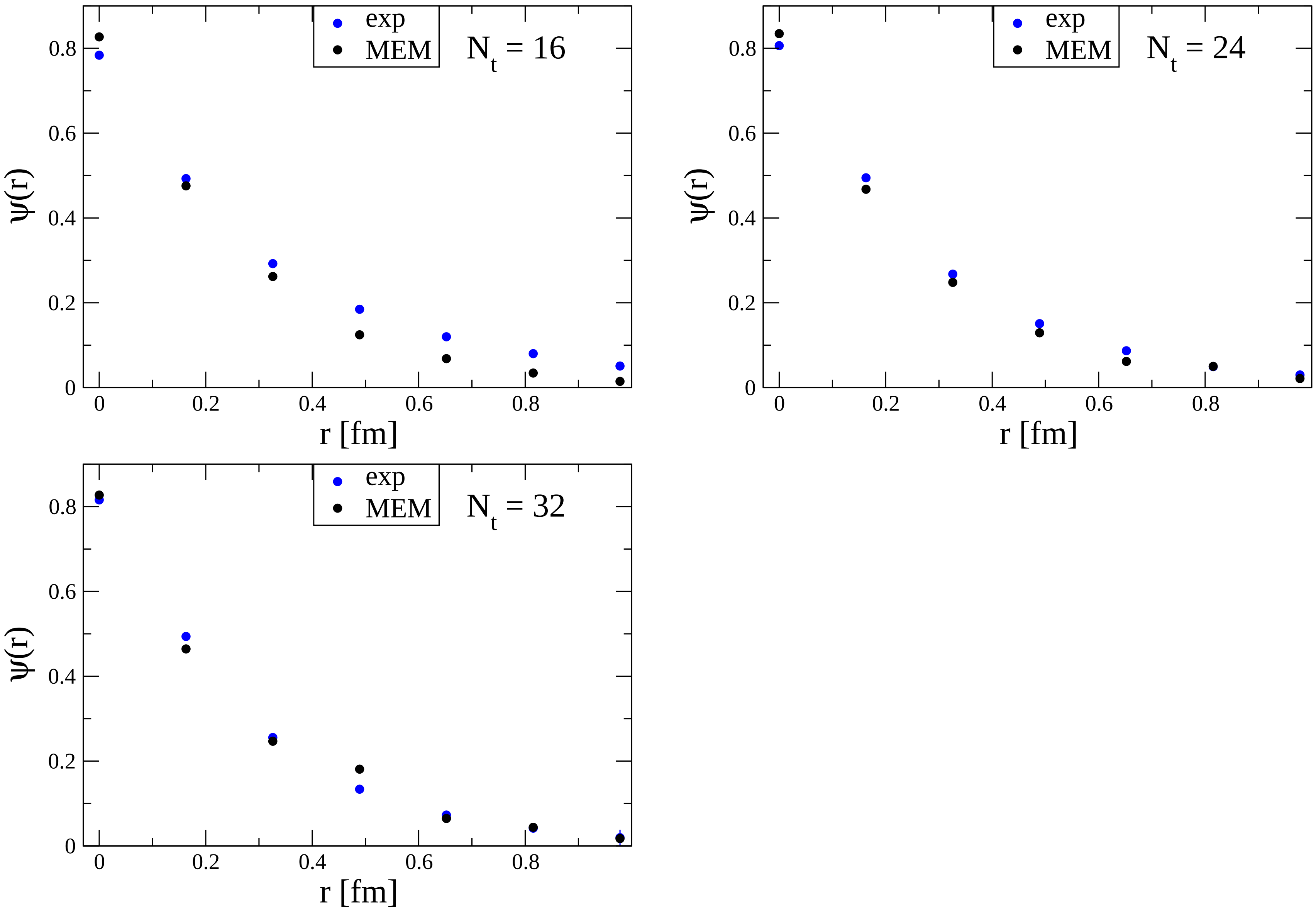

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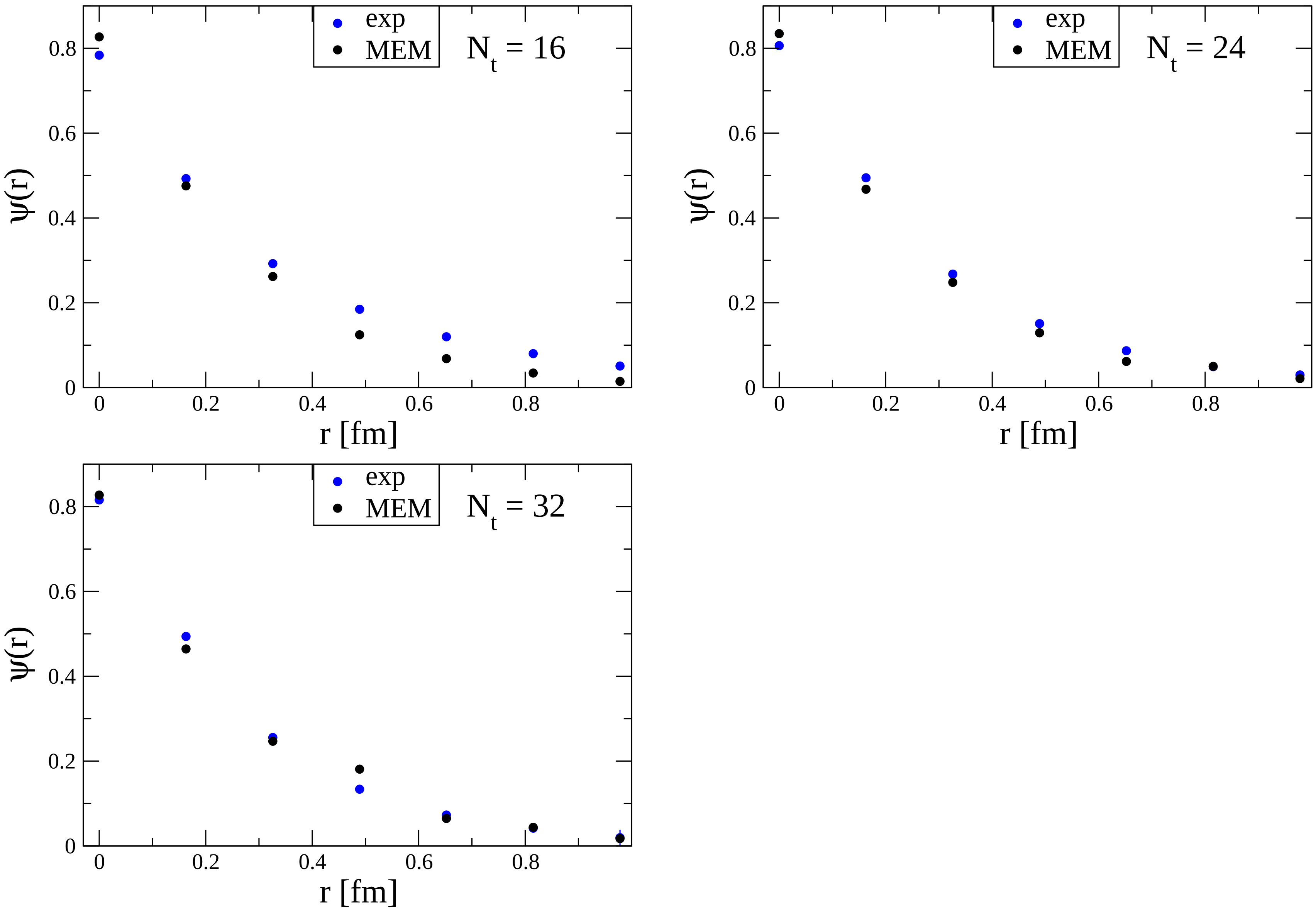
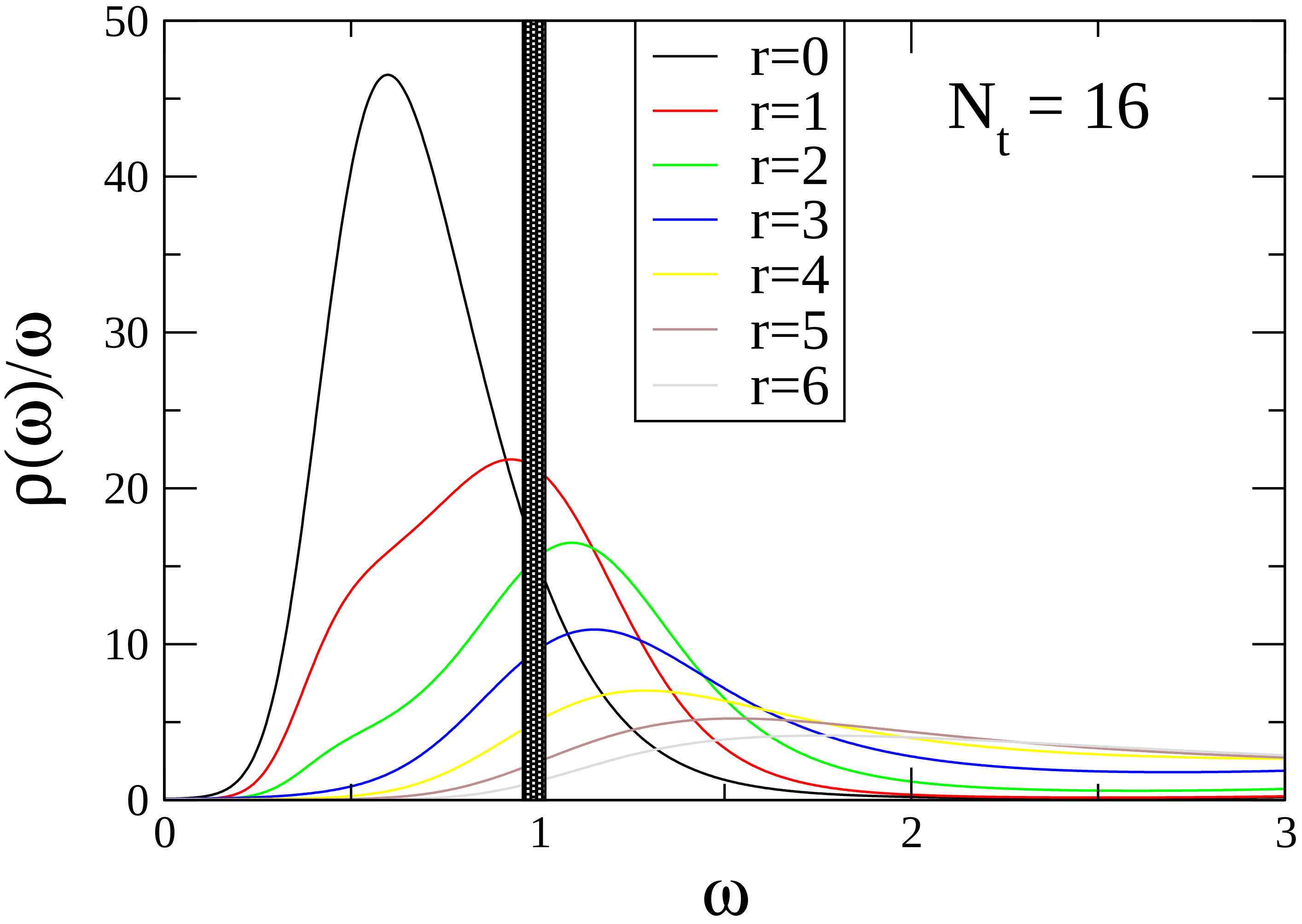


Spectral Functions: Excited State (PS)



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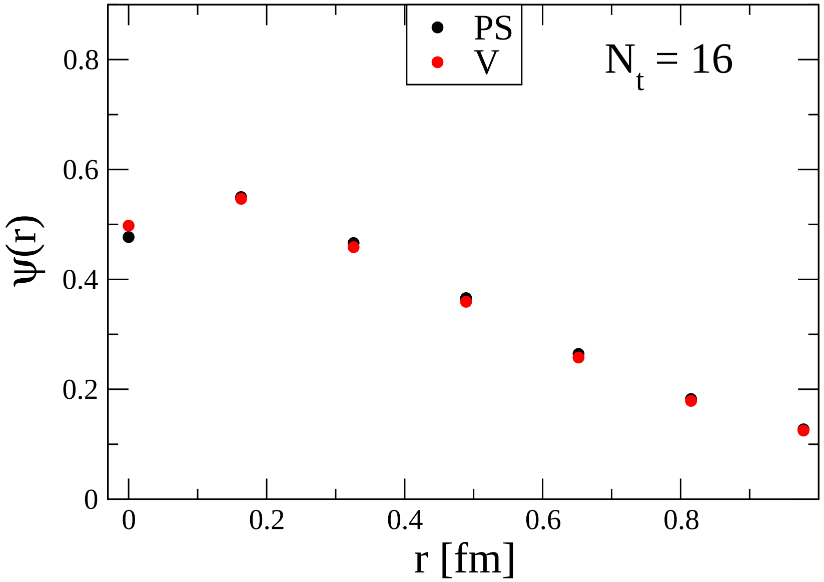

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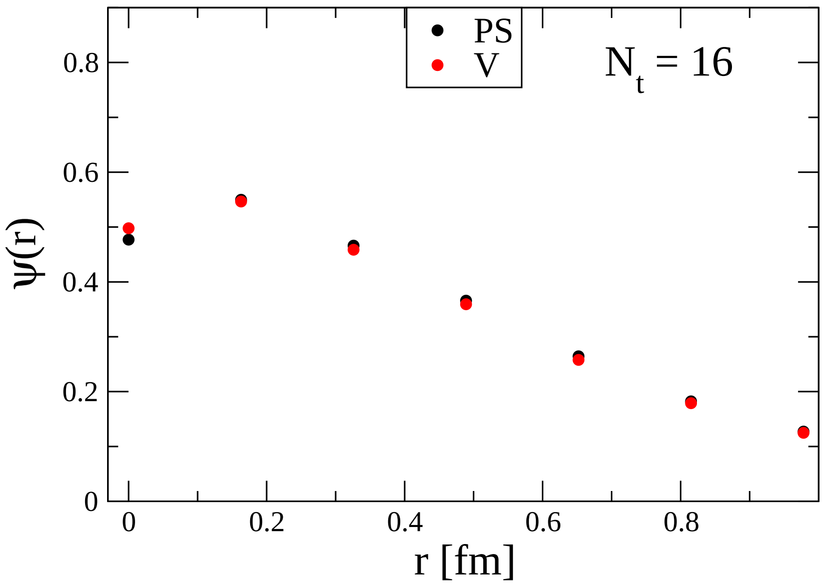

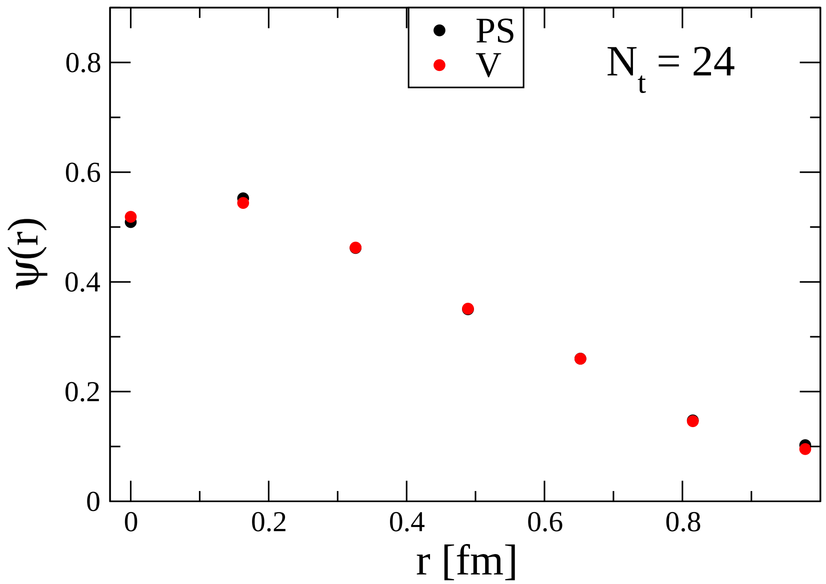

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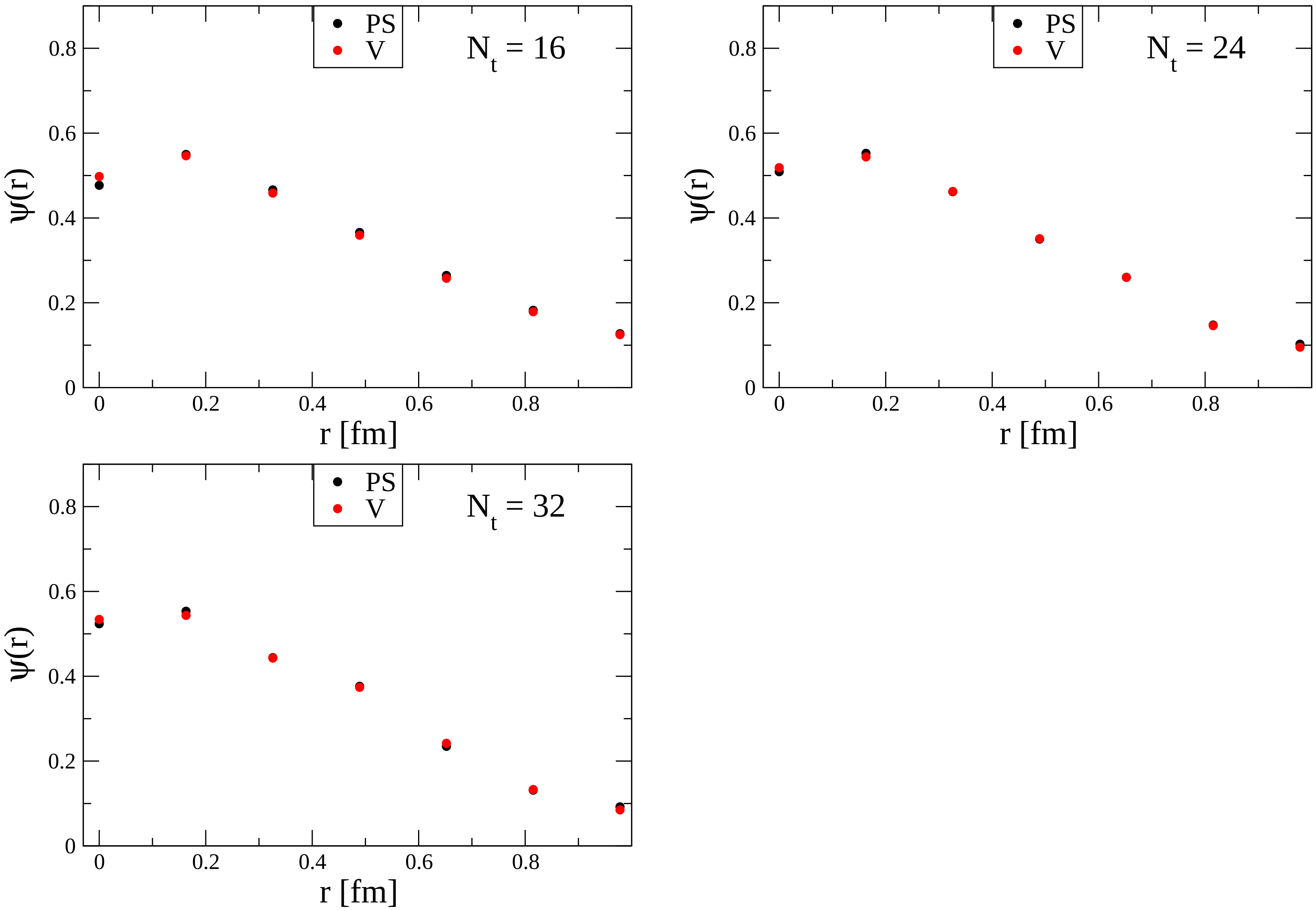

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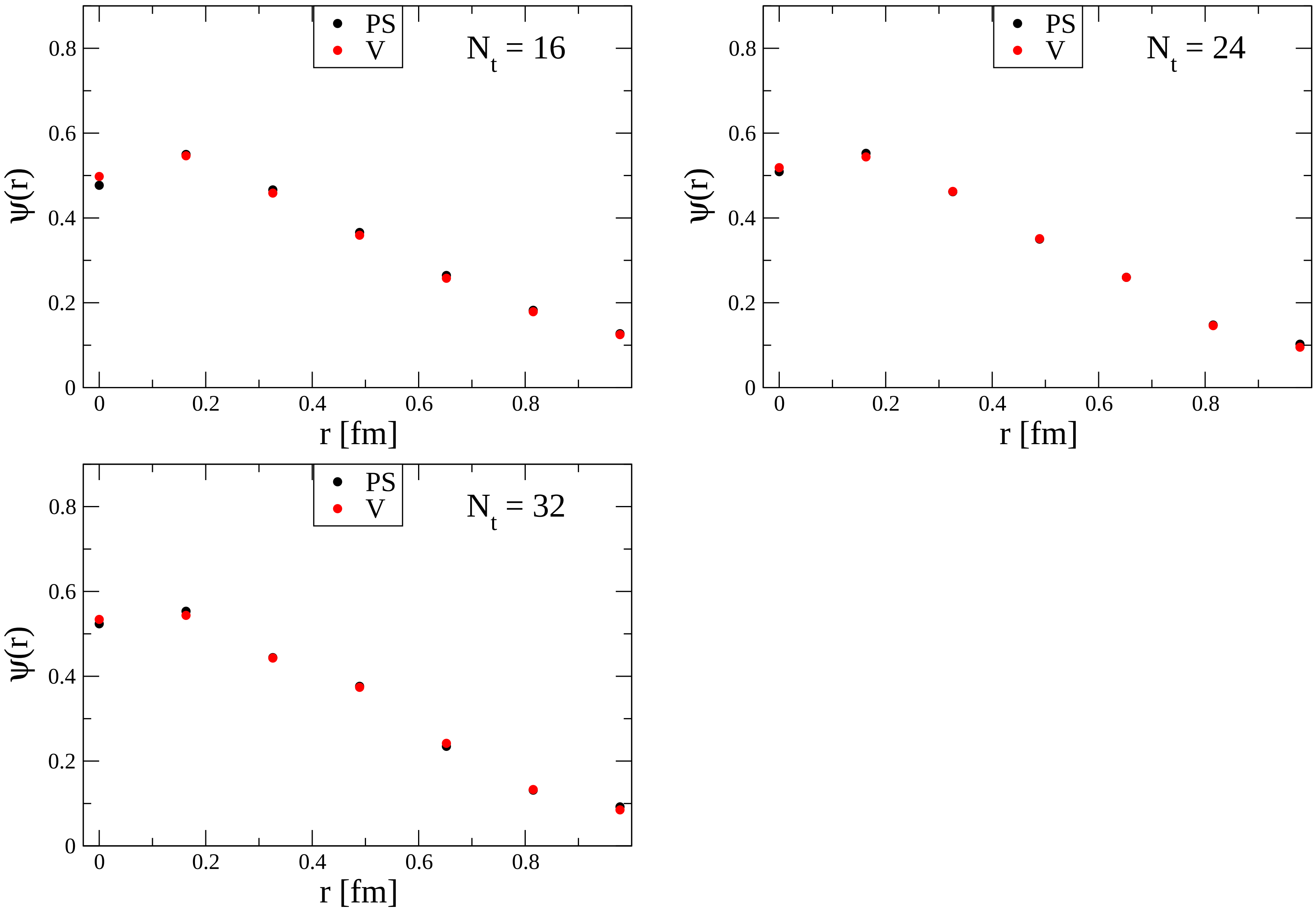


Outline & Future Plans

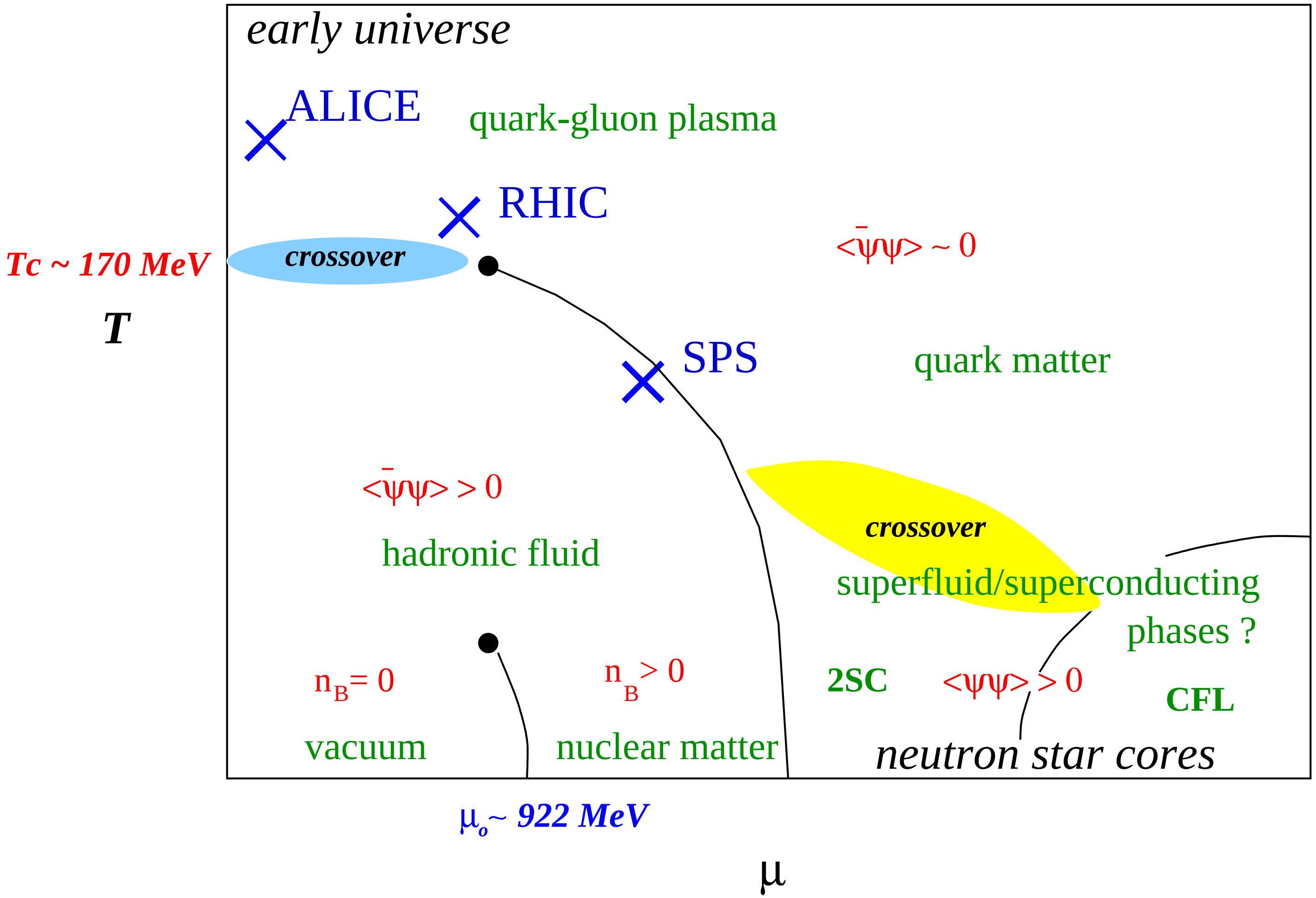
Successfully calculated the inter-quark potential in charmonium at finite temperature.

Future Plans:

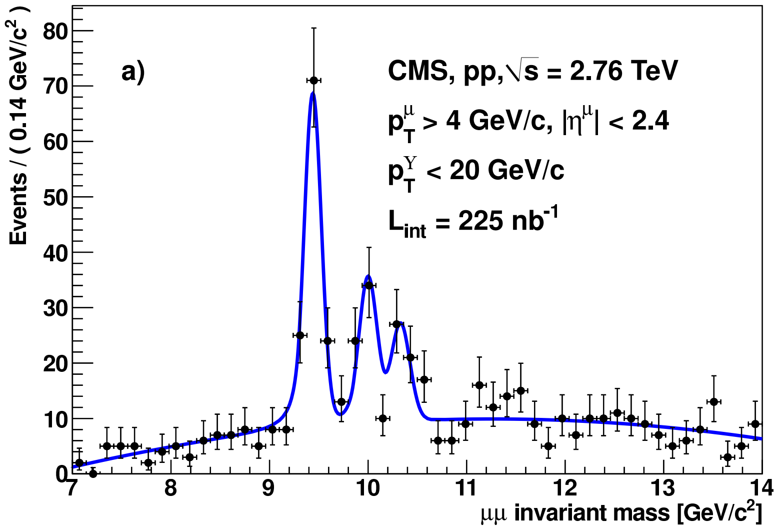
- Increase from 12^3 to 24^3 and 32^3 volumes with $N_f = 2 + 1$
- Will study P-wave states

Part B: “Related topics”

Related topics = bottomonium at finite temperature



CMS Results arXiv:1105.4894



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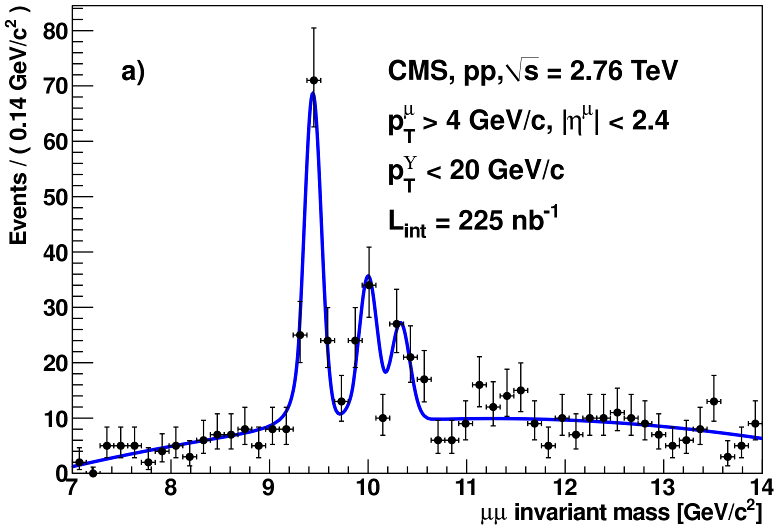

p-p collisions

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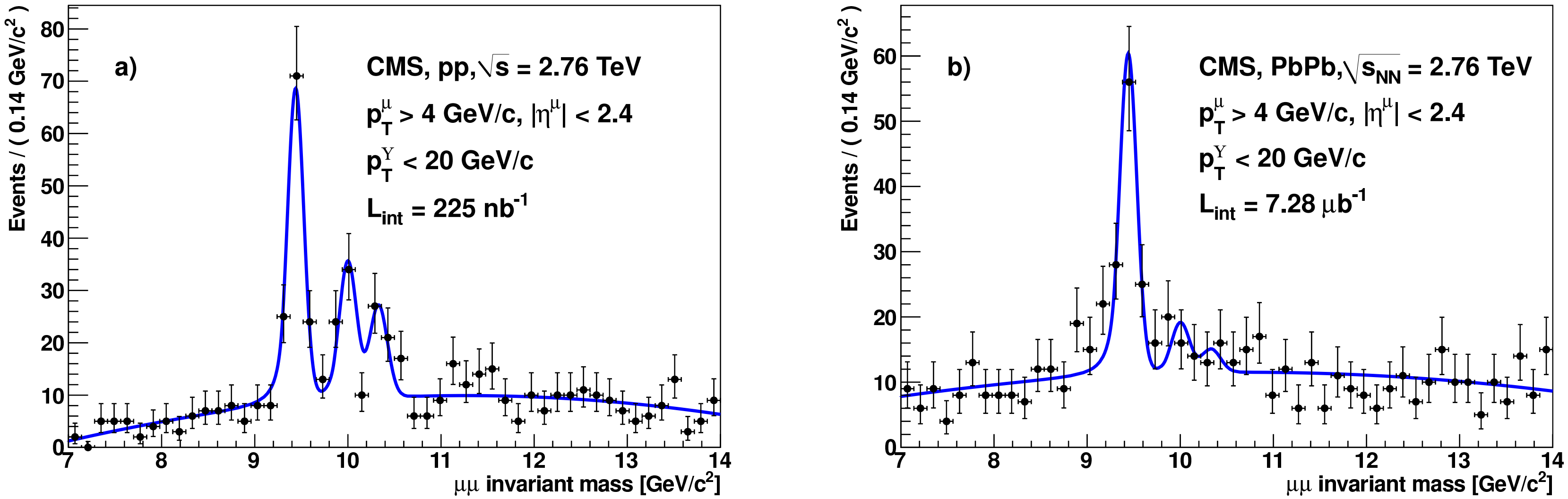

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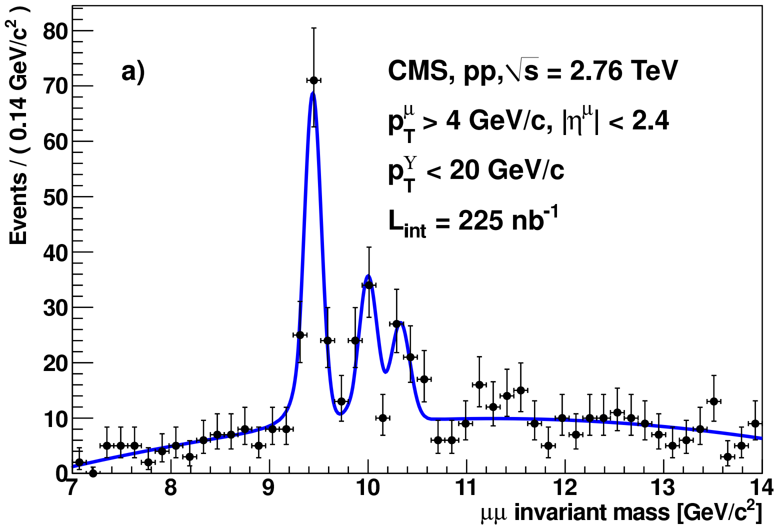

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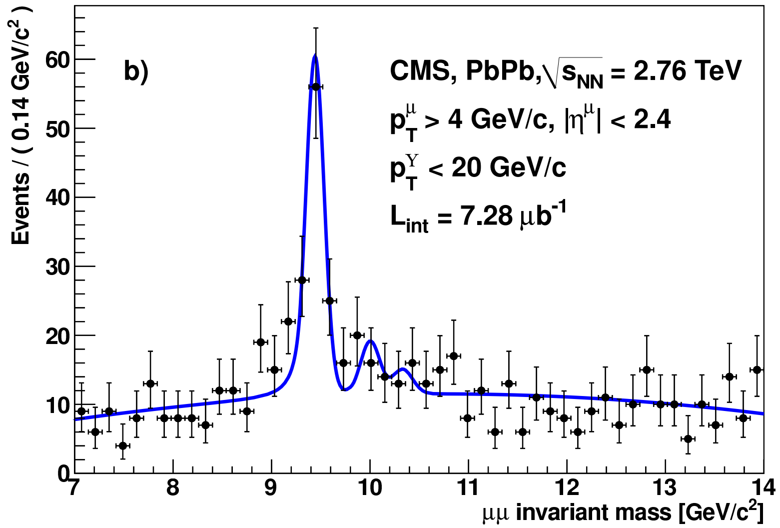

Pb-Pb collisions

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12	16	458	2.09	1000

anisotropic lattice with $\xi = a_s/a_\tau \approx 6$ ($a_s = 0.167$ fm)

Bottom quark = NRQCD

Have < one part per mille statistical error in correlators

$p_{\text{latt}} = (1,0,0), \ldots (2,2,0)$ i.e. $p = 0.634, \ldots 1.73$ GeV

P-wave Correlators

$$G_P(\tau) \sim \int \frac{d^3p}{(2\pi)^3}\, p^2 \exp(-2E\tau) \sim \tau^{-5/2}$$

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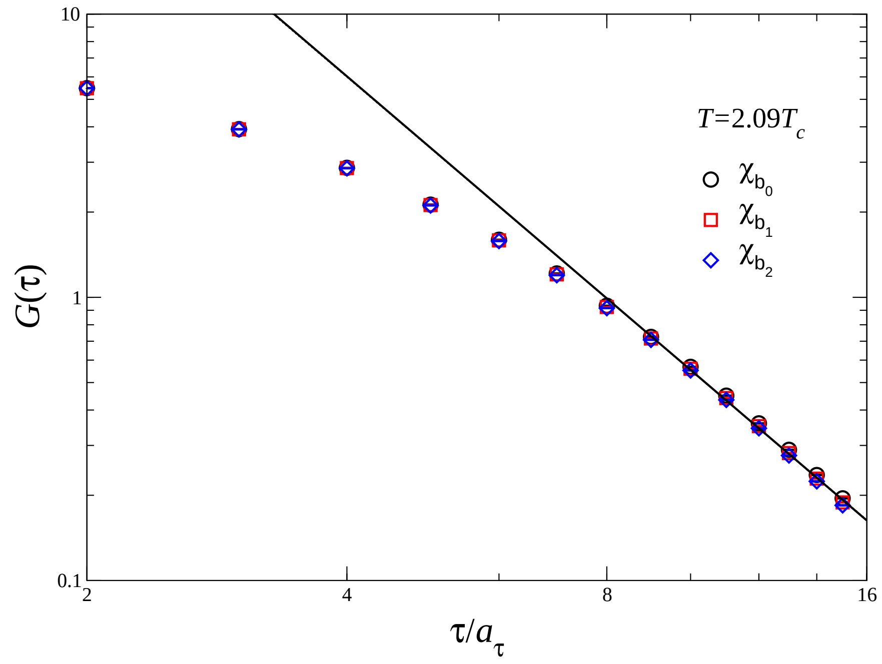

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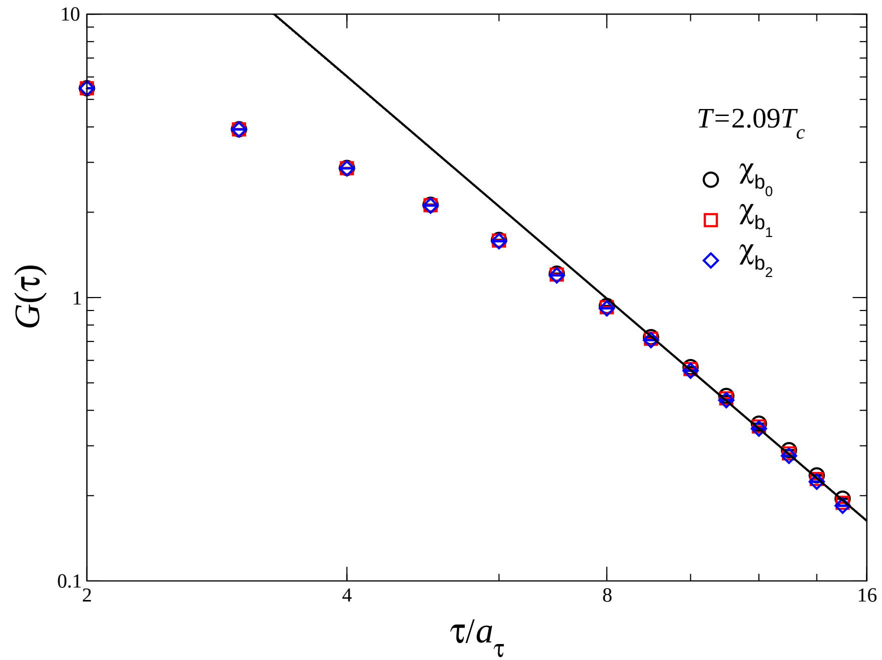

Confirmation that (high temp) P-wave state is “free”

Zero temperature spectrum results

state	$a_\tau \Delta E$	Mass (MeV)	Expt (MeV)
$1^1S_0(\eta_b)$	0.118(1)	9438(8)	9390.9(2.8)
$2^1S_0(\eta_b(2S))$	0.197(2)	10019(15)	-
$1^3S_1(\Upsilon)$	0.121(1)	9460*	9460.30(26)
$2^3S_1(\Upsilon')$	0.198(2)	10026(15)	10023.26(31)
$1^1P_1(h_b)$	0.178(2)	9879(15)	
$1^3P_0(\chi_{b0})$	0.175(4)	9857(29)	9859.44(42)(31)
$1^3P_1(\chi_{b1})$	0.176(3)	9864(22)	9892.78(26)(31)
$1^3P_2(\chi_{b2})$	0.182(3)	9908(22)	9912.21(26)(31)

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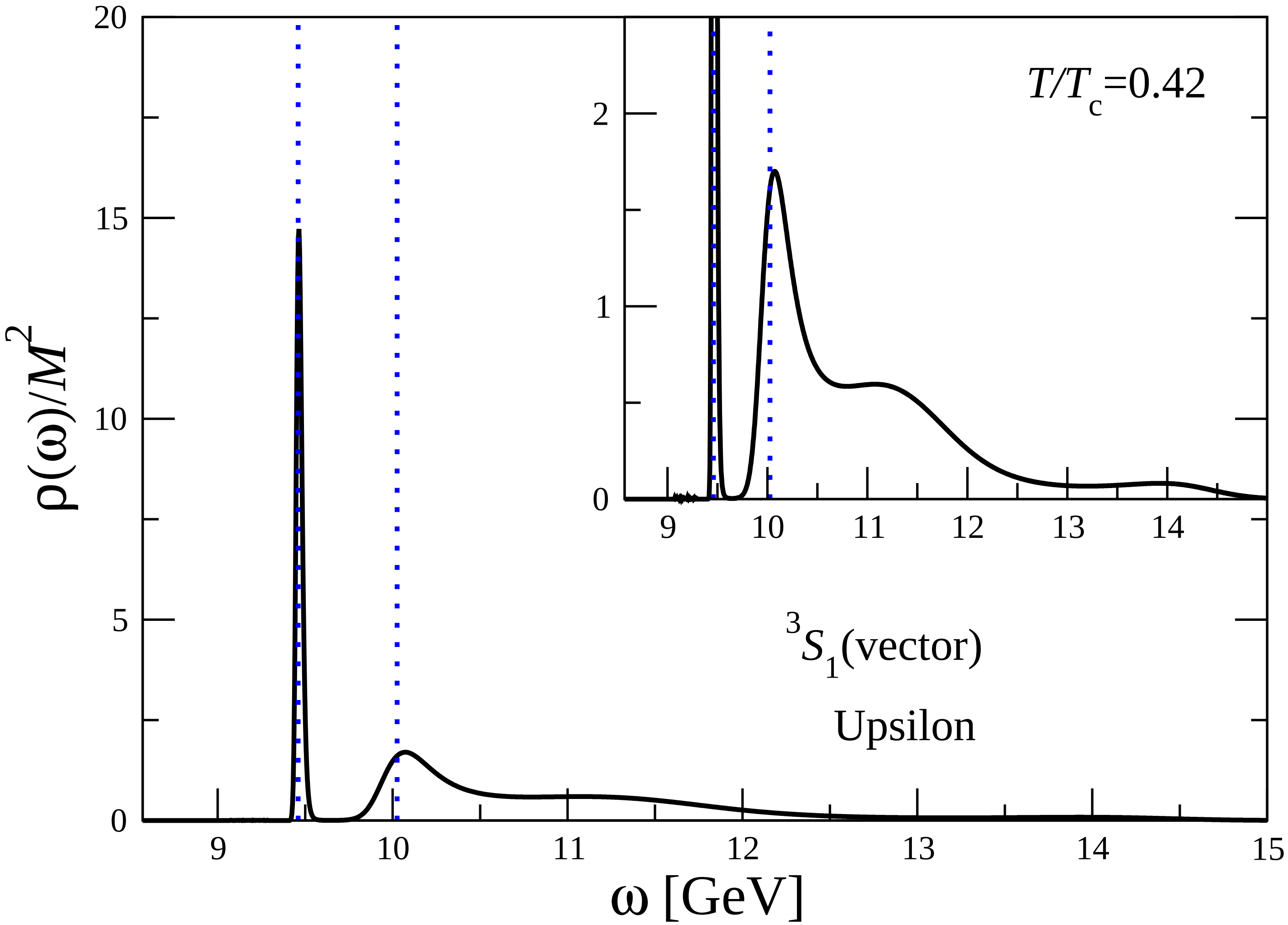
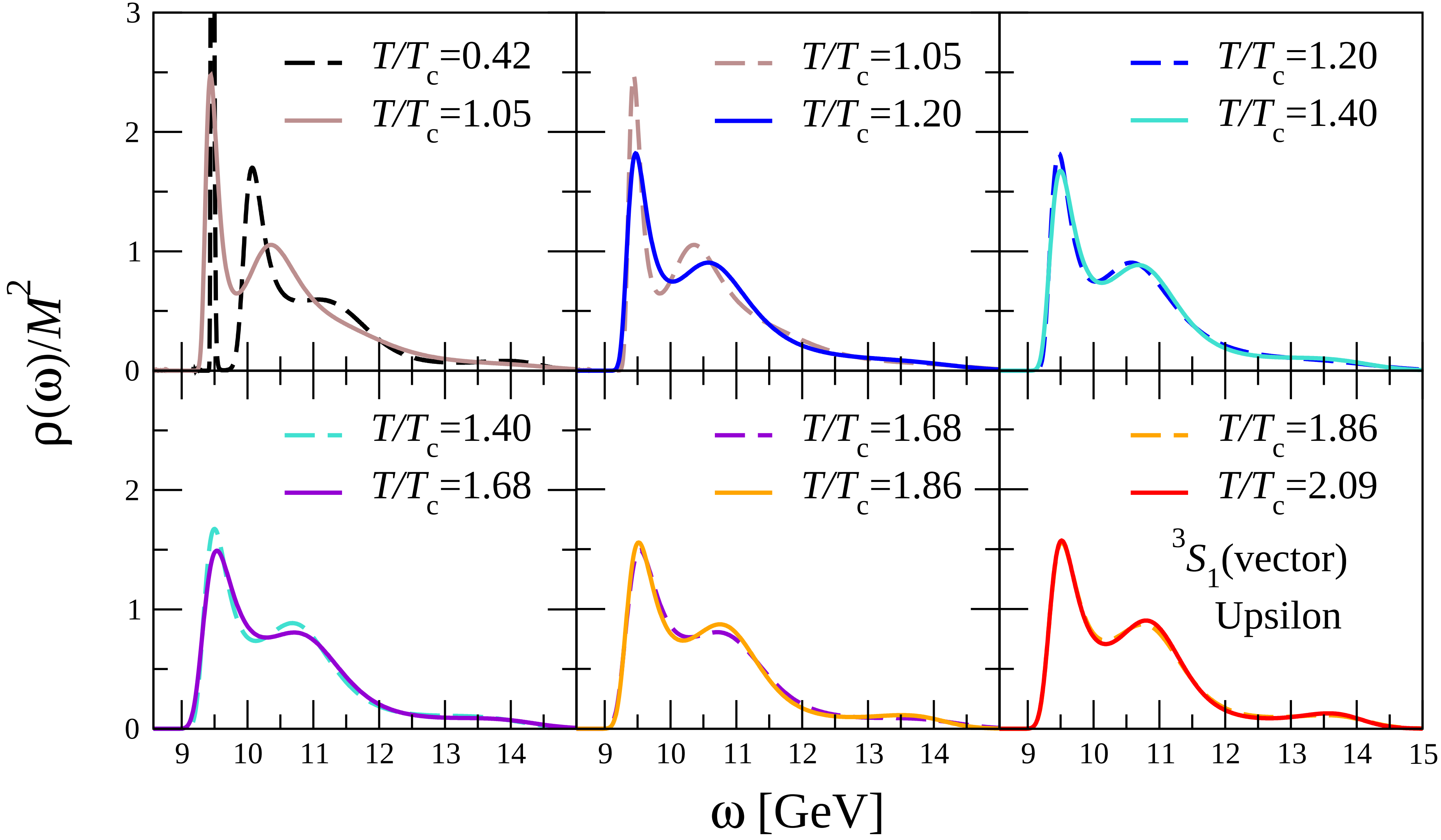
Belle Collaboration

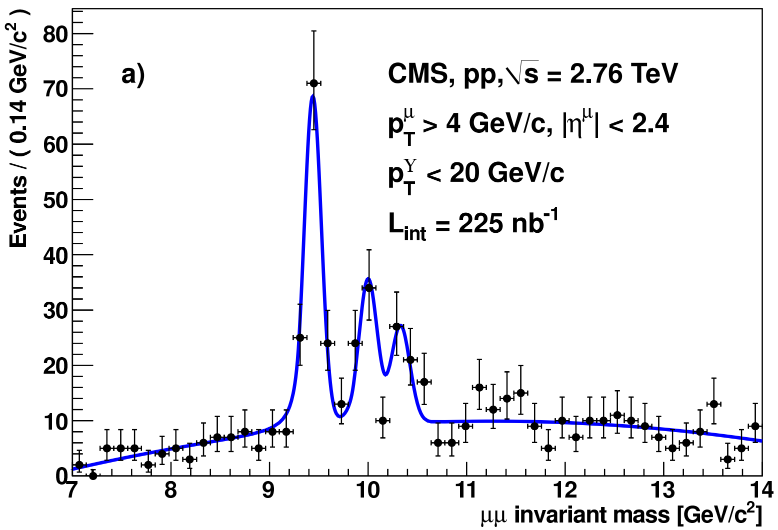
Zero temperature spectral functions, $p = 0$



Non-zero temperature spectral functions, $p = 0$



CMS Results arXiv:1105.4894



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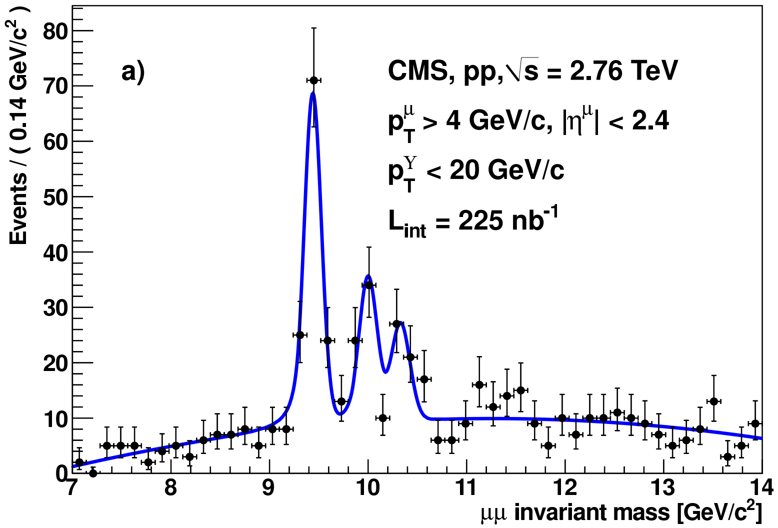
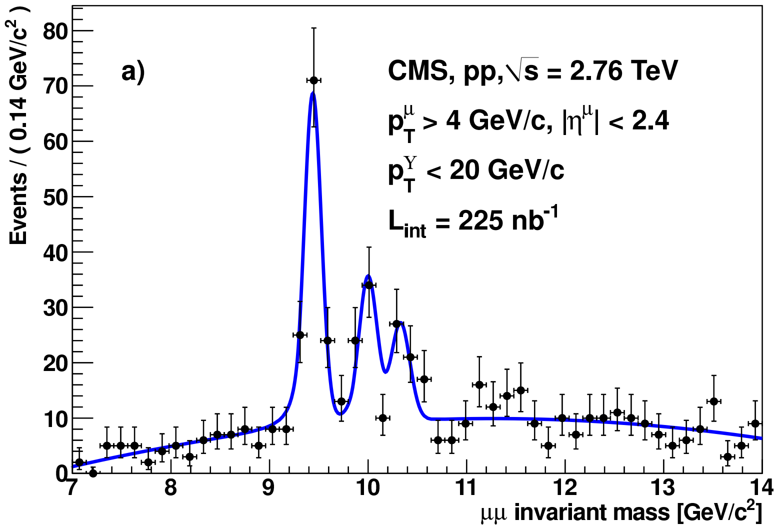
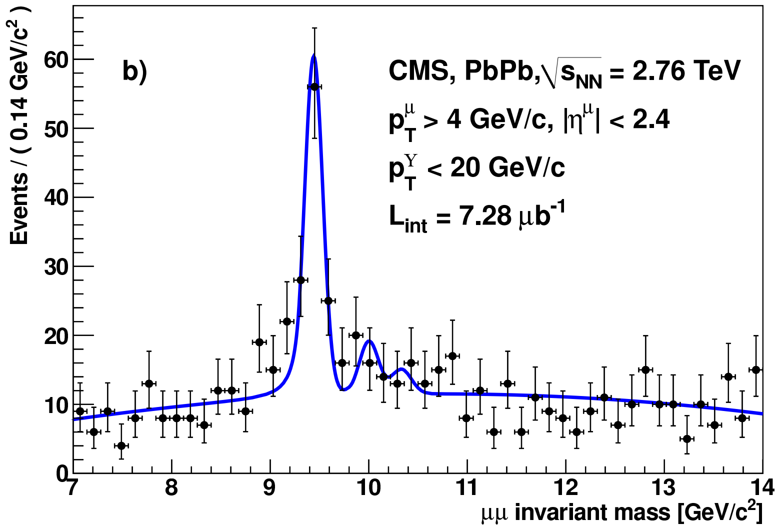


p-p collisions

CMS Results arXiv:1105.4894





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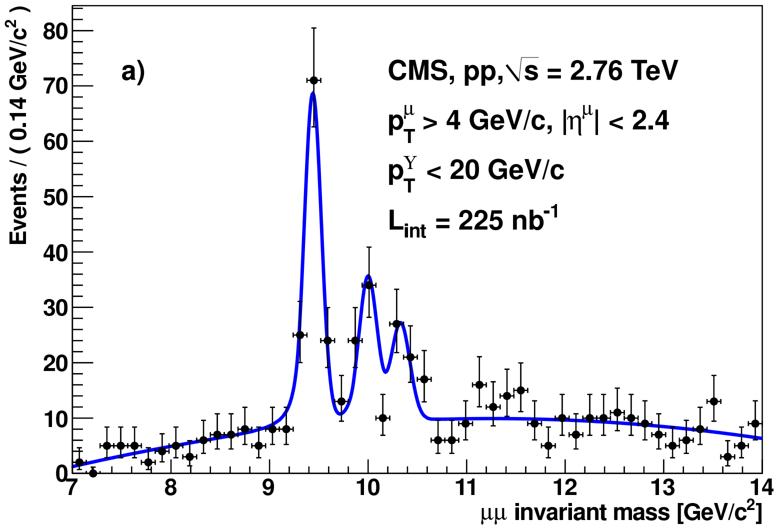
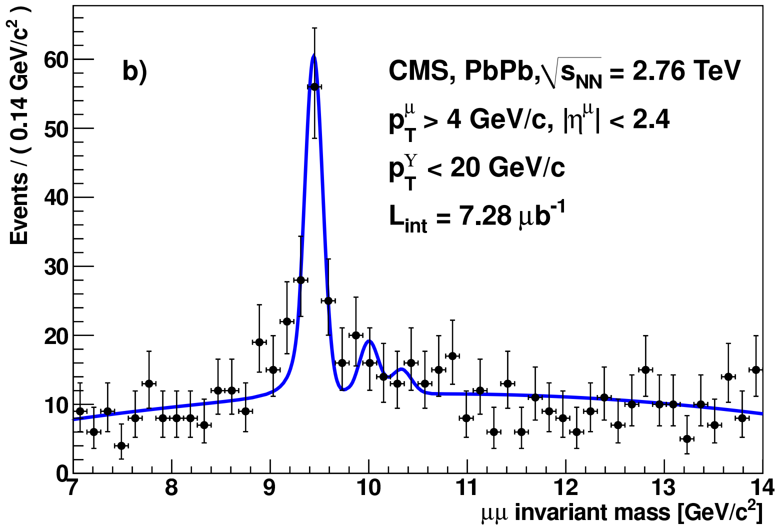
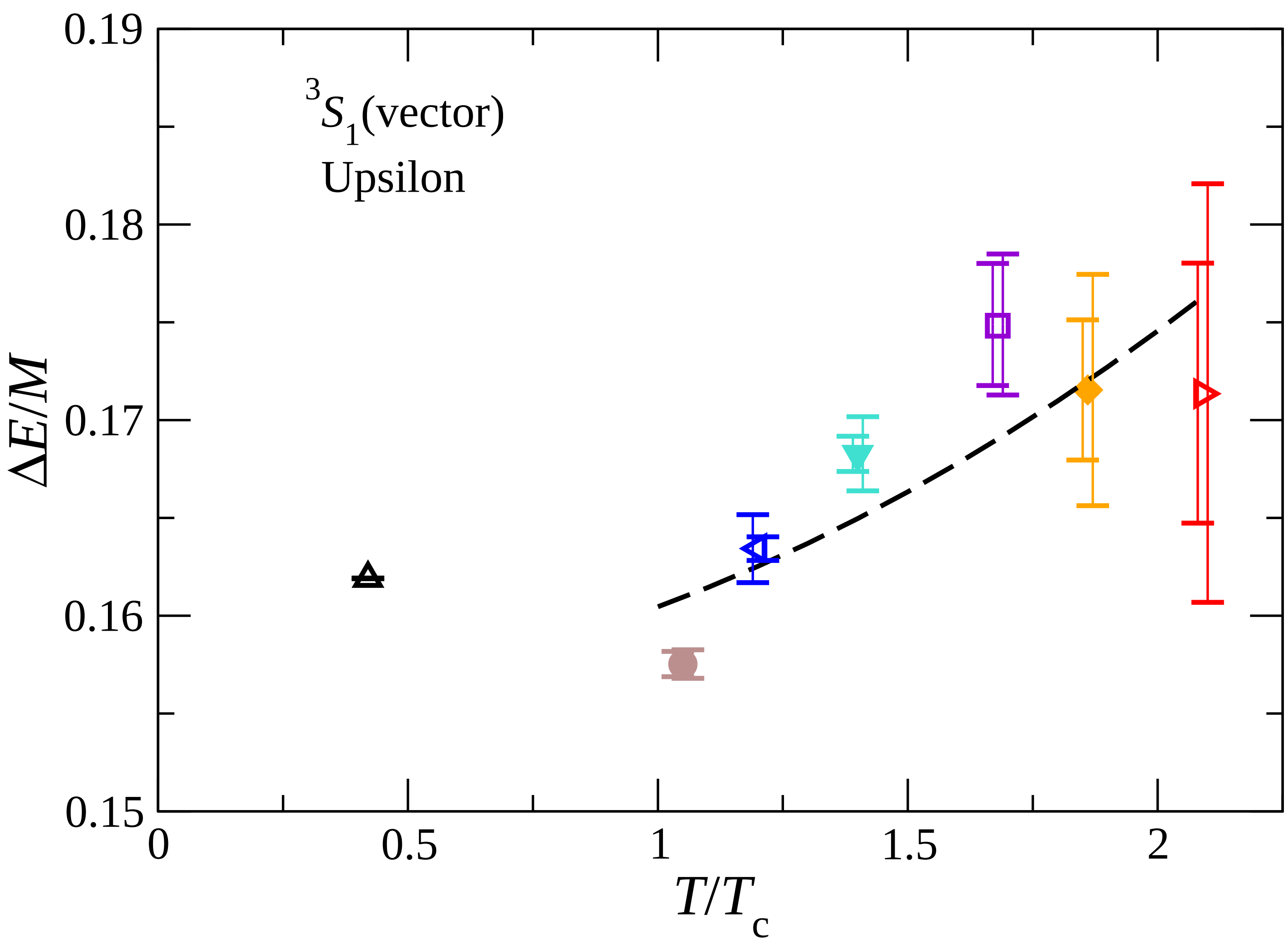




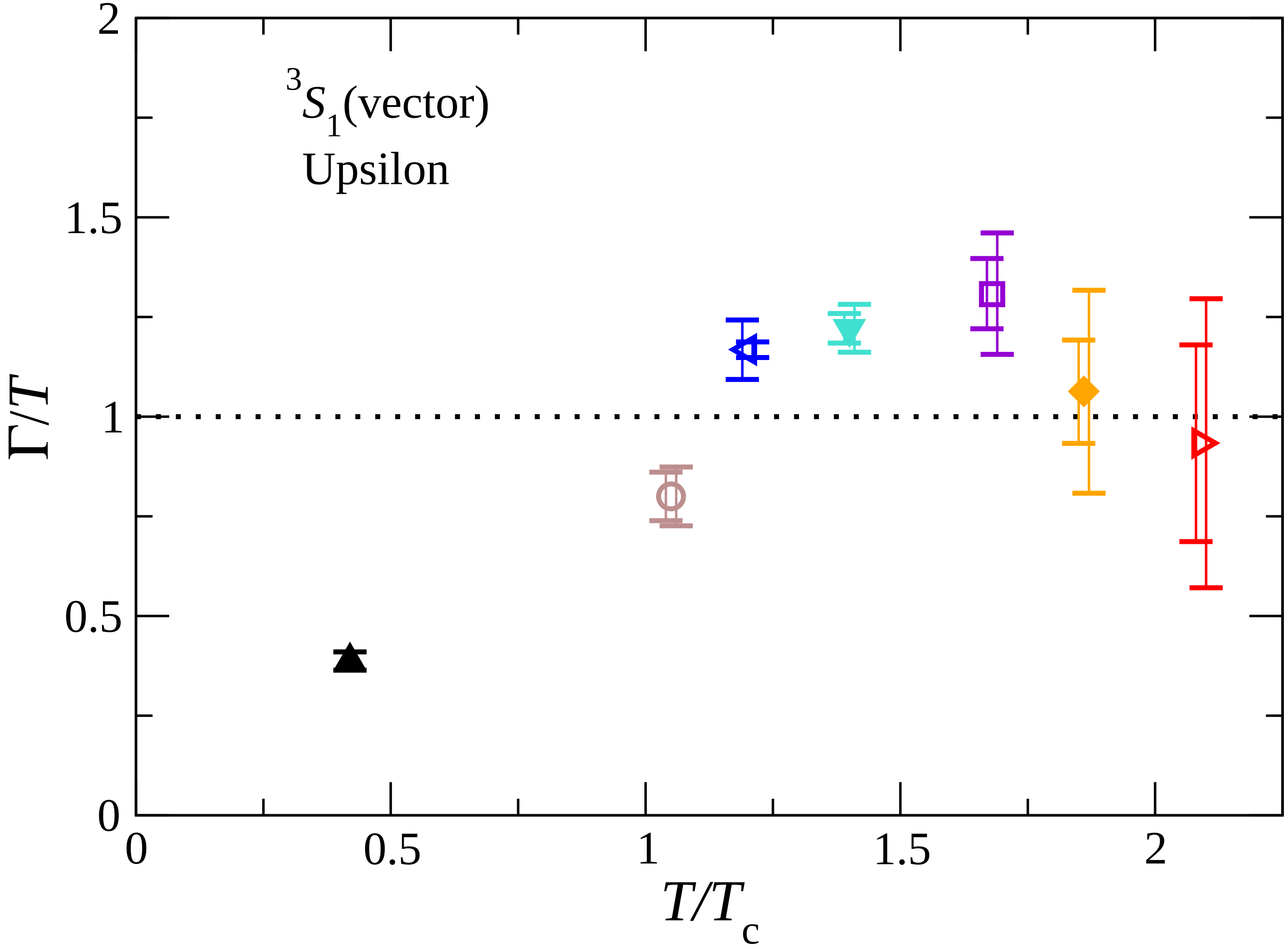
p-p collisions

Pb-Pb collisions

Non-zero temperature Mass, $p = 0$



Non-zero temperature Width, $p = 0$



Comparison with phenomenology

From Brambilla et al thermal contribution to the width is

$$\frac{\Gamma}{T} = \frac{1156}{81}\alpha_s^3 \simeq 14.27\alpha_s^3,$$

(at leading order in weak coupling and large mass expansion).

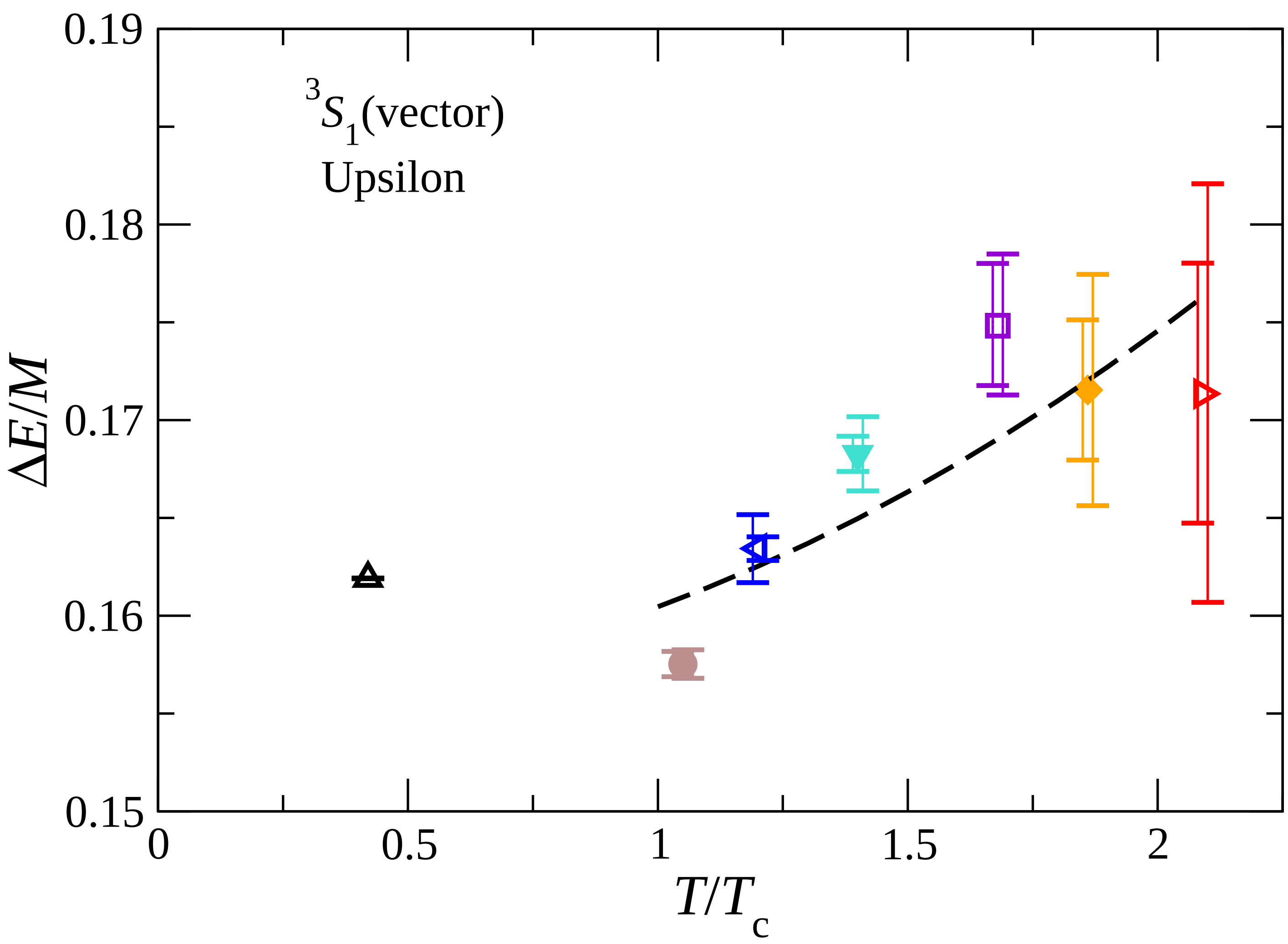
Our results $\longrightarrow \Gamma/T \sim 1$ so $\alpha_s \sim 0.4$.

Also from Brambilla et al thermal contribution to the mass is

$$\delta E_{\rm thermal} = \frac{17\pi}{9}\alpha_s\frac{T^2}{M} \simeq 5.93\alpha_s\frac{T^2}{M}$$

(see dashed line)

Non-zero temperature Mass, $p = 0$

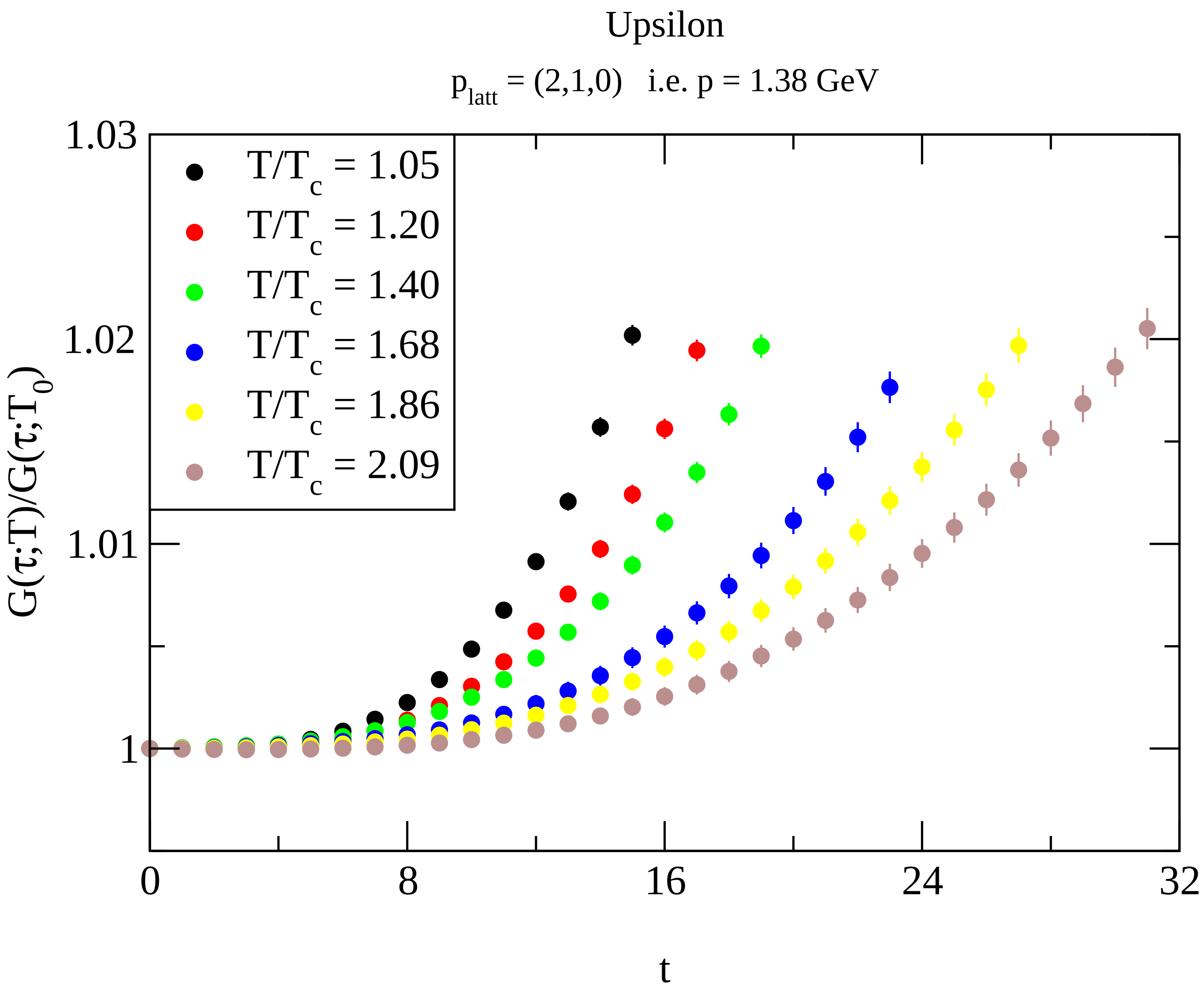


New results with non-zero momentum

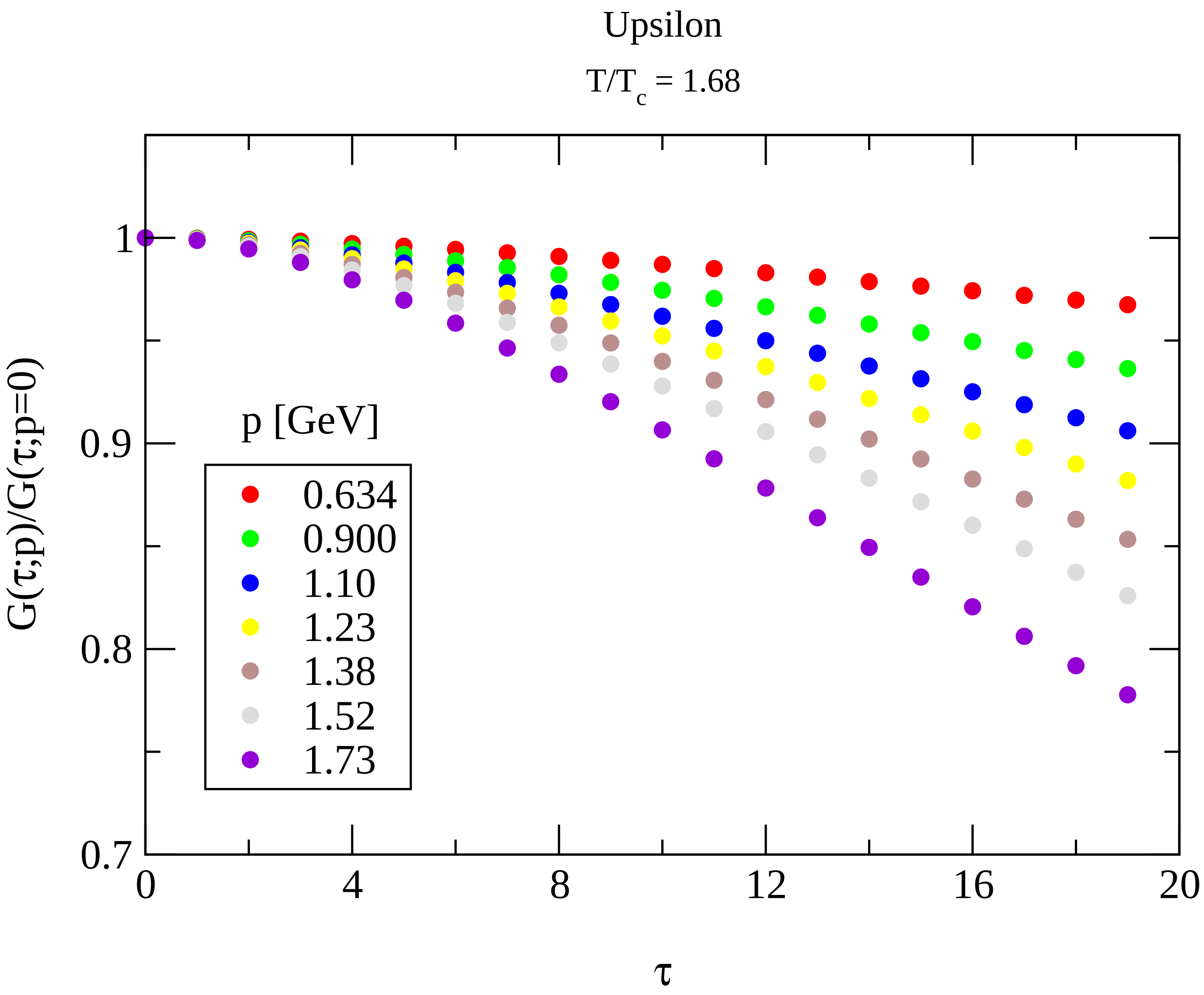
Preliminary...

$p_{\text{latt}} = (1,0,0), \ldots (2,2,0)$ i.e. $p = 0.634, \ldots 1.73$ GeV

Variation with T, fixed $p \neq 0$

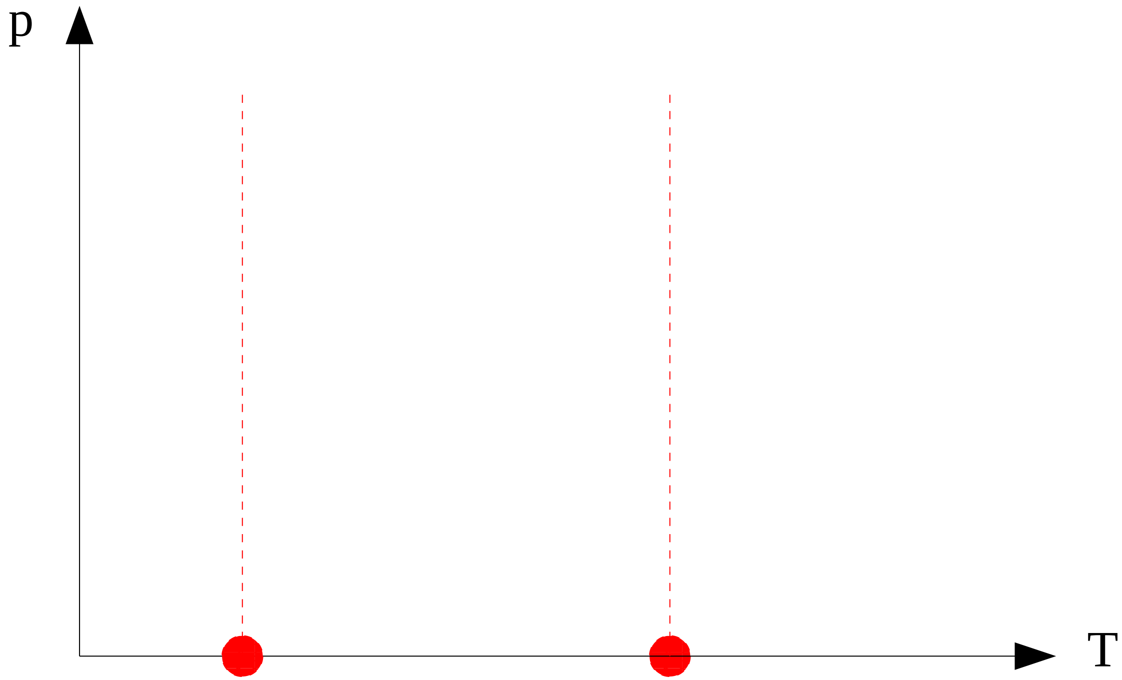


Variation with p, fixed T



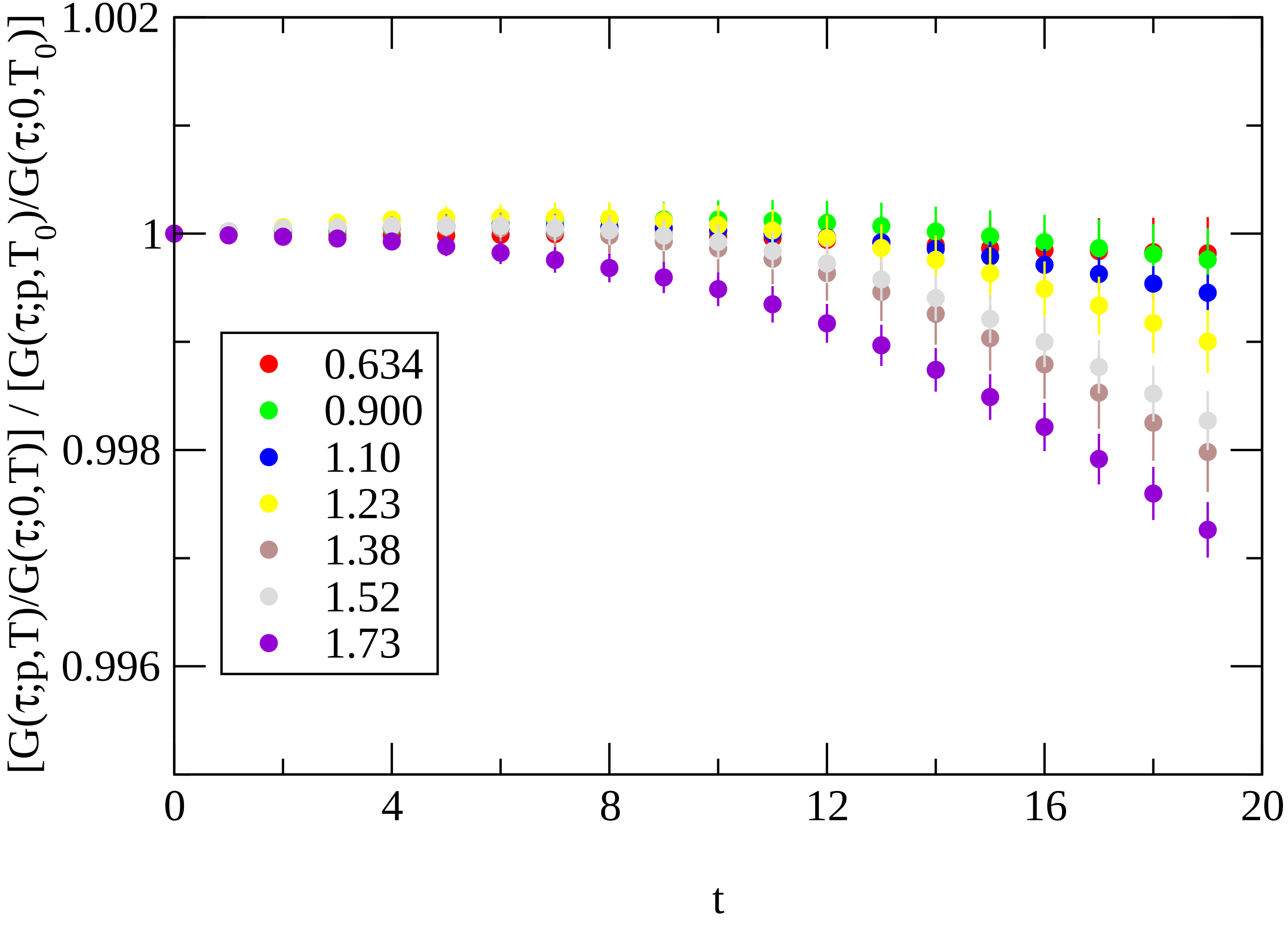
Single ratio

Double ratio

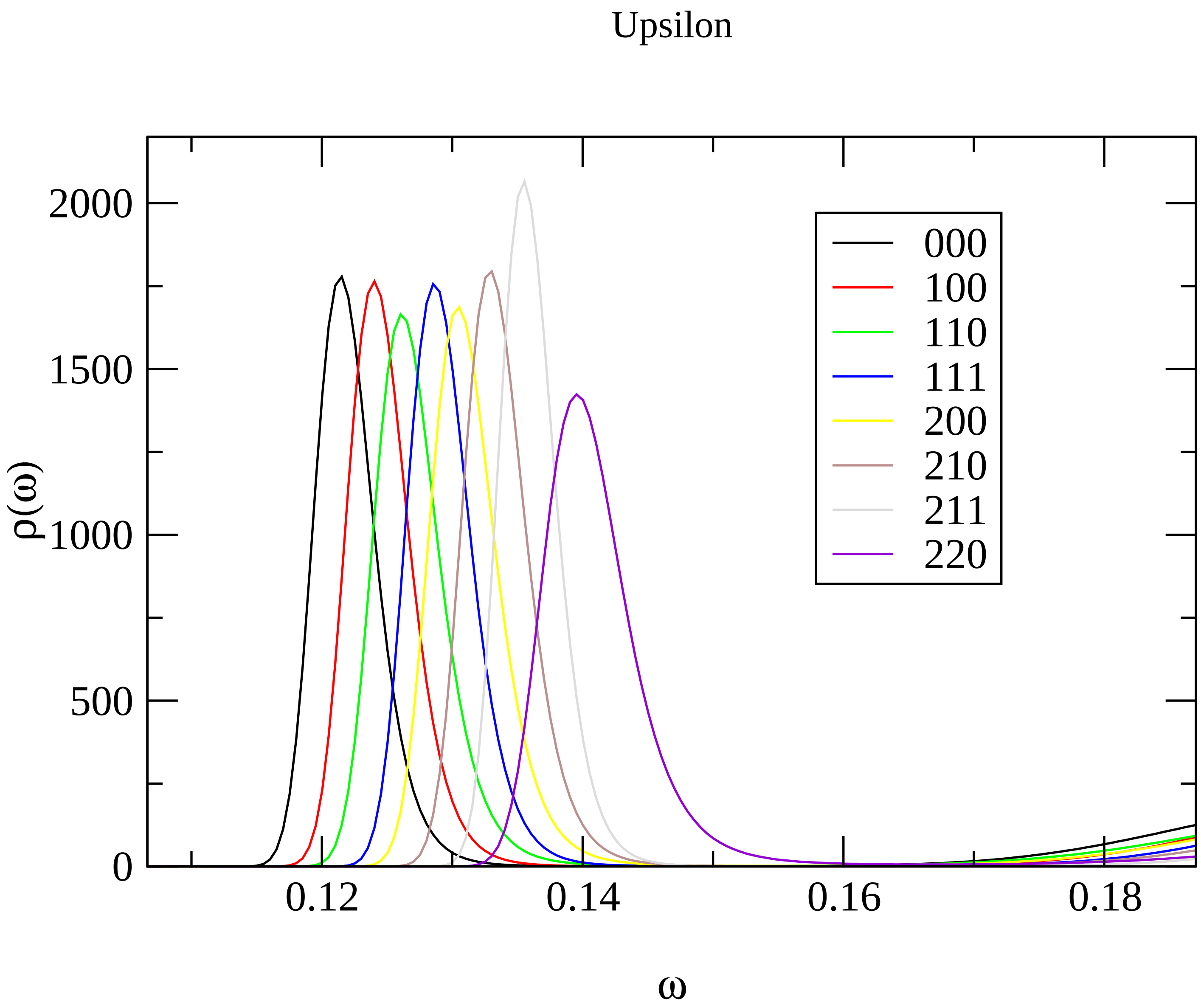


$$\frac{G(\tau;p,T)}{G(\tau;p=0,T)}\Big/\frac{G(\tau;p,T_0)}{G(\tau;p=0,T_0)}$$

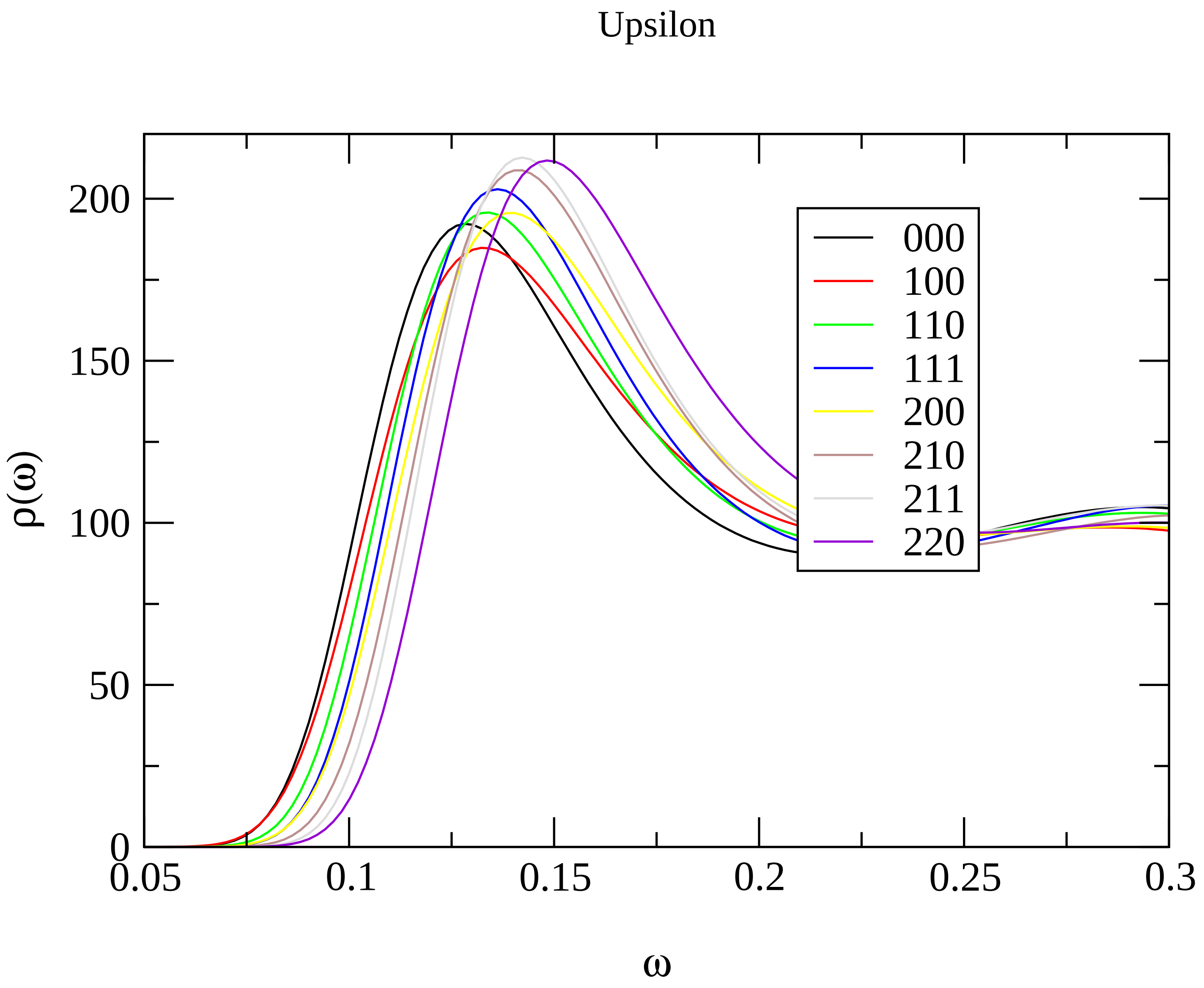
Double ratio



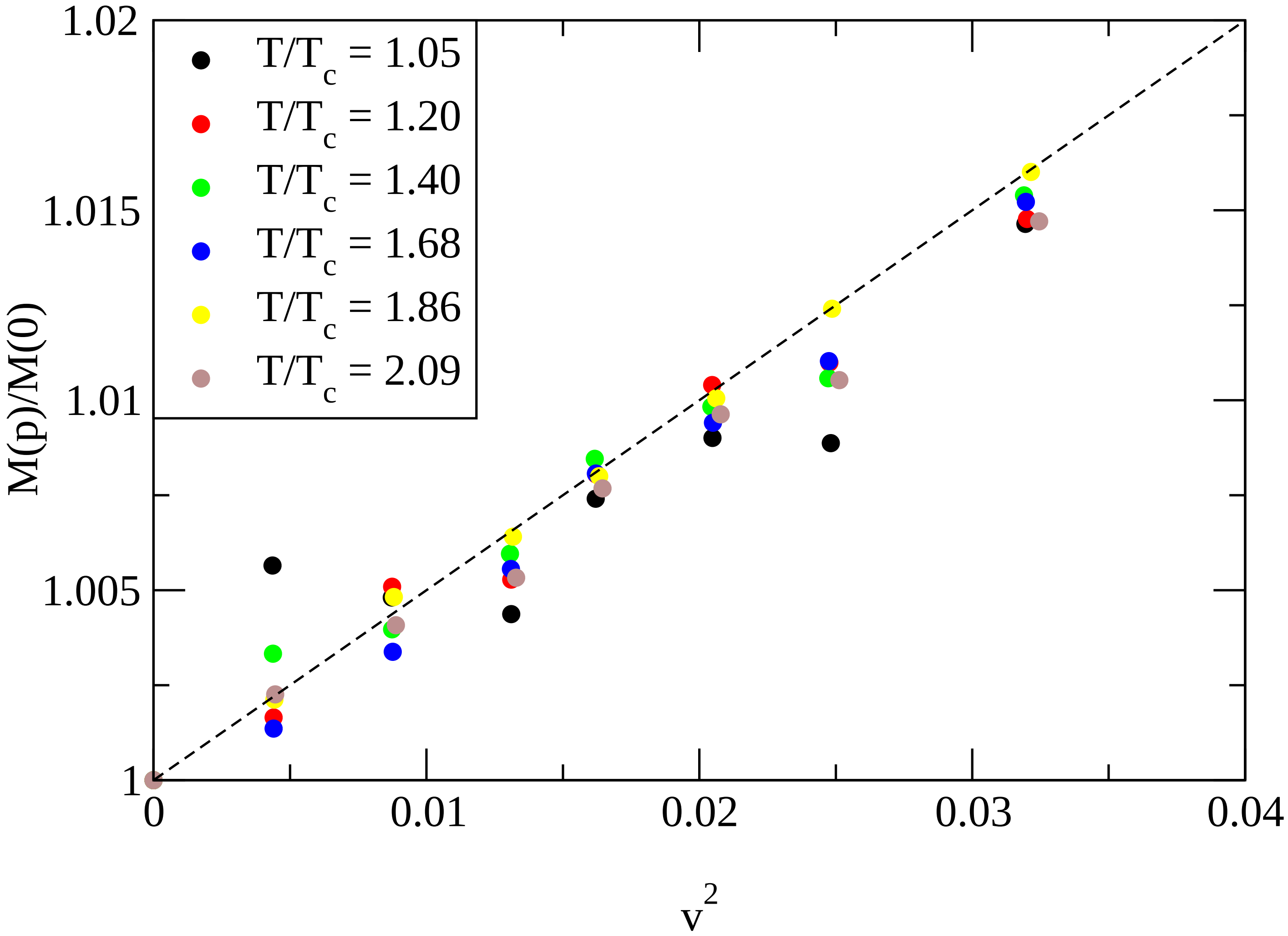
Non-zero momentum, spectral function, $T = 0$



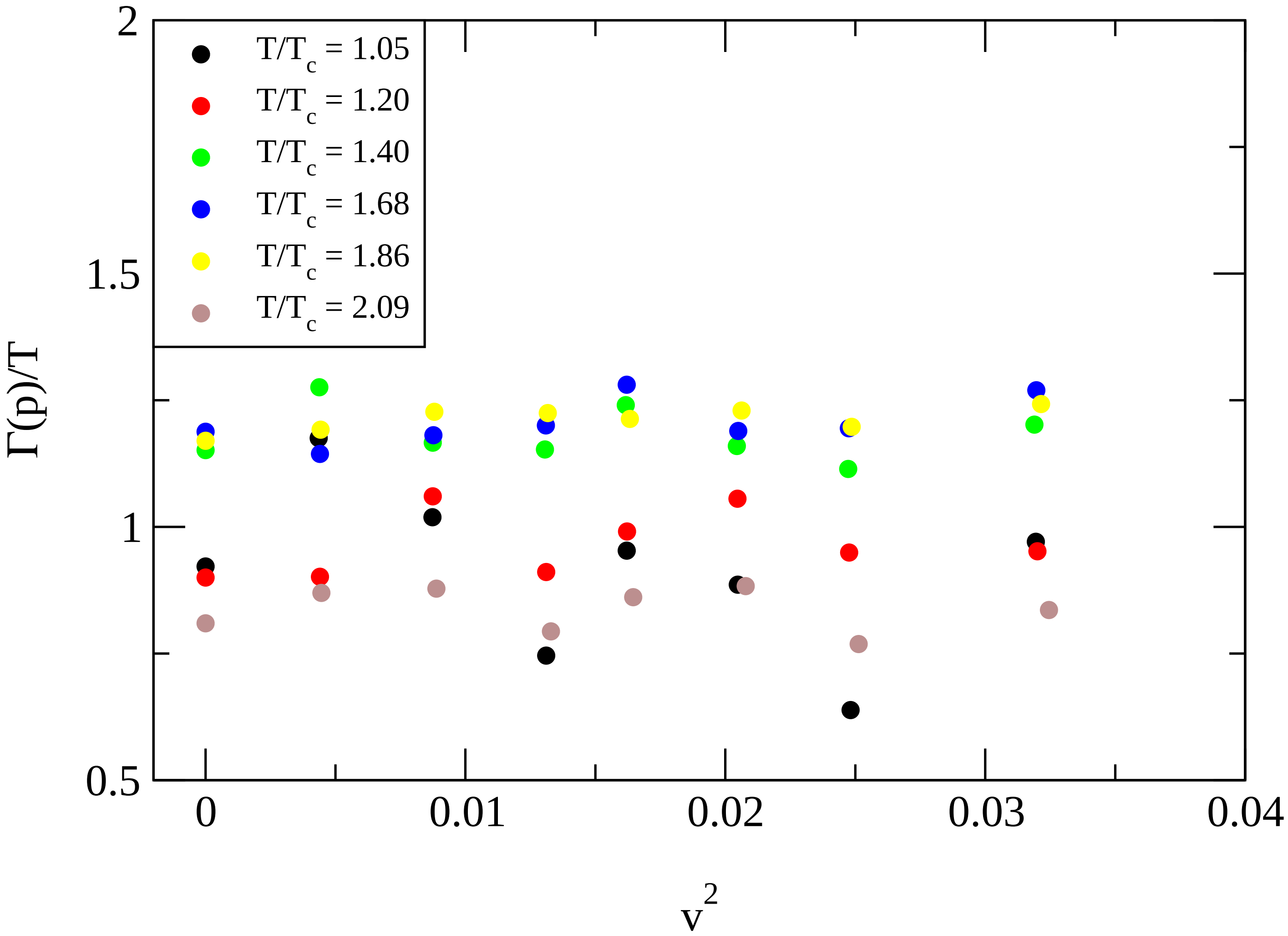
Non-zero momentum, spectral function, $T \neq 0$



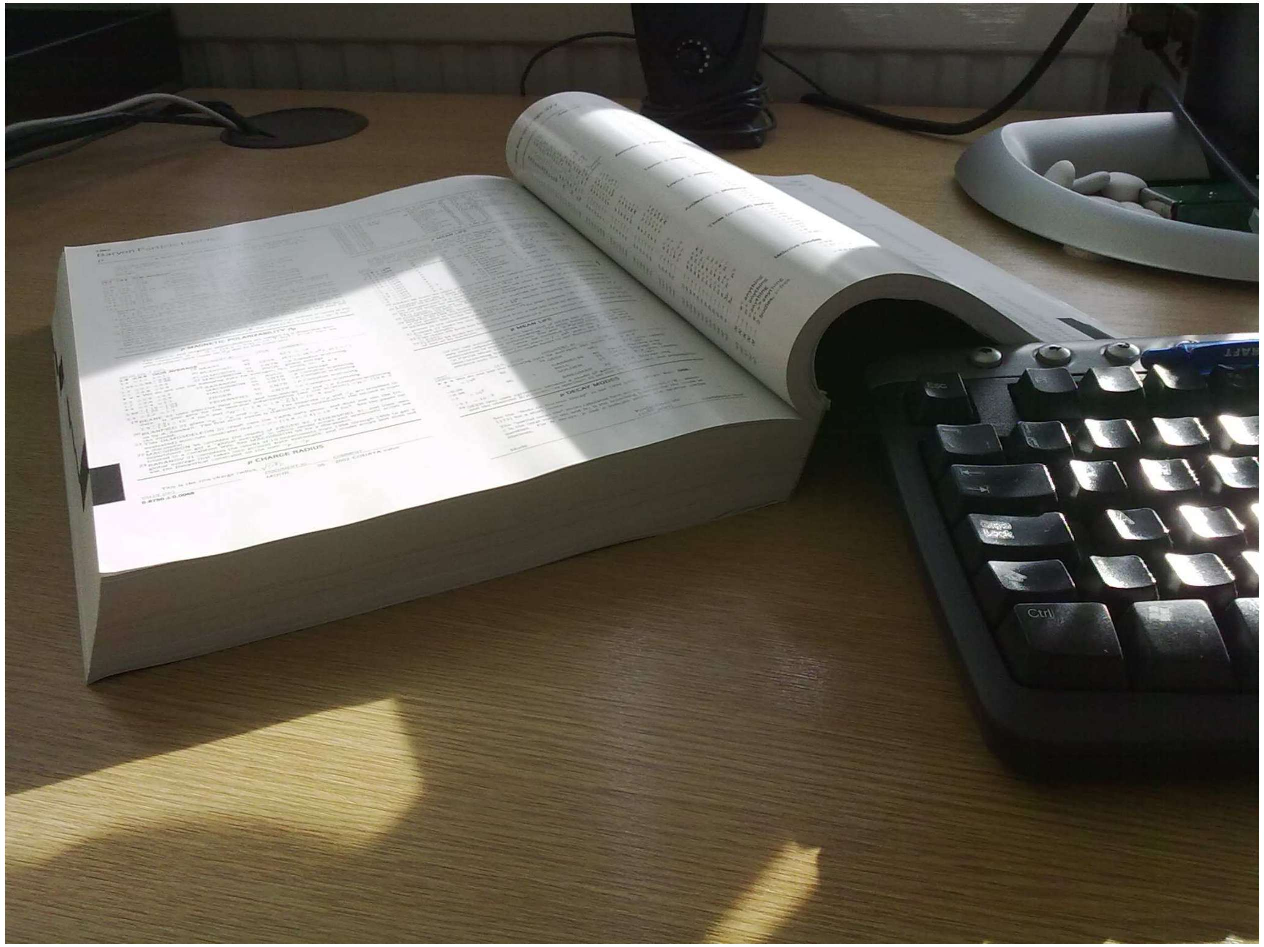
Mass as function of speed



Width as function of speed



Particle Data Book



$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

the end

Slides to help me answer difficult questions