Open issues in Hadron Structure

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with
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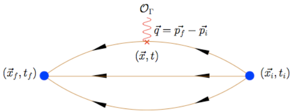
Workshop on New Frontiers in Lattice Gauge Theory
Galileo Galilei Institute for Theoretical Physics, Florence

Outline

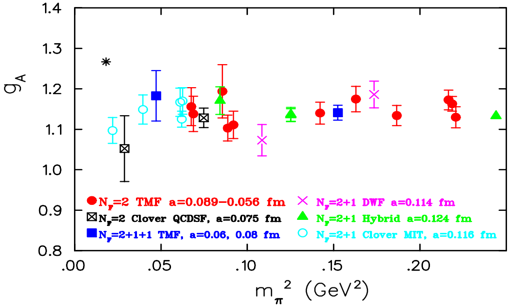
1. **Nucleon structure**
   - Axial charge
   - Excited state contributions
   - Nucleon spin
   - Form factors
   - σ-terms
   - Isoscalar axial charge including disconnected contributions
2. **Hyperons and Charmed baryons**
   - Masses
   - Axial charge
   - σ-terms
3. $N\gamma^* \to \Delta$
4. **Conclusions**

Nucleon Structure: axial charge

- Many lattice studies down to lowest pion mass of $m_\pi \sim 300$ MeV
  $\Longrightarrow$ Lattice data in general agreement
- Axial-vector FFs: $A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5 \frac{\tau^a}{2}\psi(x)$
  $\Longrightarrow \frac{1}{2}\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m} G_p(q^2)\right]$



Axial charge is well known experimentally, straight forward to compute in lattice QCD



- Agreement among recent lattice results - all use non-perturbative Z_A
- Weak light quark mass dependence

- $N_F = 2+1$ Clover: J. R. Green *et al.*, Lattice2012
- $N_F = 2$ and $N_F = 2+1+1$ TMF: C. A. *et al.* (ETMC), PRD 83 (2011) 045010, and in preparation
- DWF: T. Yamazaki *et al.*, (RBC-UKQCD), PRD 79 (2009) 14505; S. Ohta, arXiv:1011.1388
- Hybrid: J. D. Bratt *et al.* (LHPC),PRD 82 (2010) 094502
- $N_F = 2$ Clover:D. Pleiter *et al.* (QCDSF), arXiv:1101.2326

Results on the Nucleon $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$

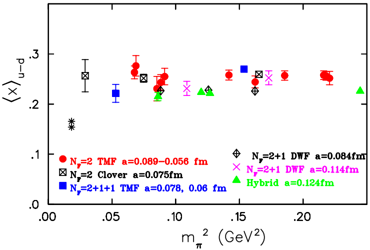
Moments of parton distributions:

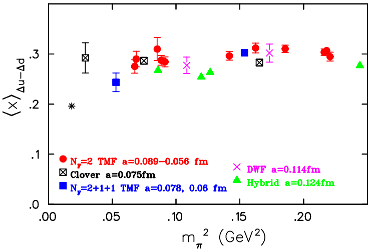
$$\langle x \rangle_q = \int_0^1 dxx\,[q(x) + \bar{q}(x)]\ , \qquad \langle x \rangle_{\Delta q} = \int_0^1 dxx\,\left[\Delta q(x) - \Delta\bar{q}(x)\right]$$

$q = q_\downarrow + q_\uparrow,\, \Delta q = q_\downarrow - q_\uparrow$

Extracted from nucleon matrix elements of $\mathcal{O}_q^{\mu_1\mu_2} = \bar{\psi}\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2\}}\psi$ and $\mathcal{O}_{\Delta q}^{\mu_1\mu_2} = \bar{\psi}\gamma^{\{\mu_1}\gamma_5 i \overleftrightarrow{D}^{\mu_2\}}\psi$

Summary of $N_F = 2$ and $N_F = 2+1+1$ results in the $\overline{MS}$ scheme at $\mu = 2$ GeV.

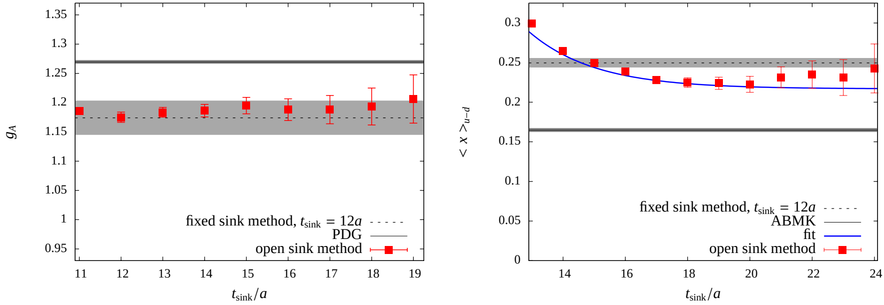




Study of excited state contributions

$N_F = 2 + 1 + 1$ with $m_\pi \sim 390$ MeV and $a = 0.08$ fm

Vary source- sink separation:



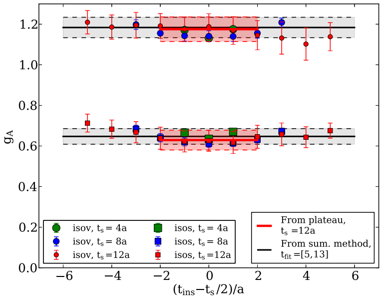
g_A unaffected, $\langle x \rangle_{u-d}$ 10% lower

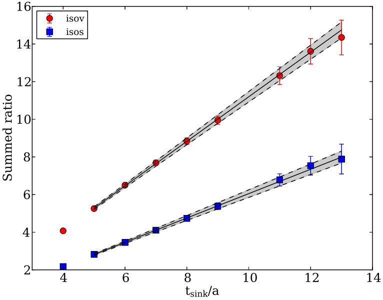
$\Longrightarrow$ Excited contributions are operator dependent

S. Dinter, C.A., M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076

g_A with the summation method

Contamination due to excited states $\sim e^{-\Delta E t_s}$ instead of $\sim e^{-\Delta E t_{ins}}$. However need to extract the slope.
One twisted mass ensemble, a = 0.08 fm, m_π = 390 MeV,
iso-scalar (only connected) and iso-vector g_A



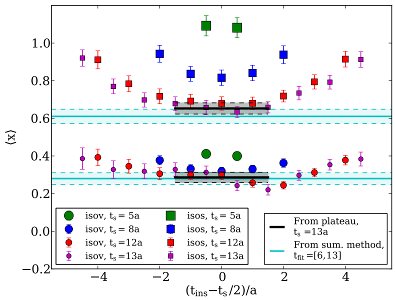


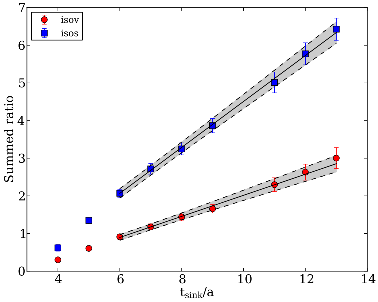
- Use of incremental eigCG algorithm, A. Stathopoulos and K. Orginos, arXiv:0707.0131
  - One sequential inversion for each t_{sink}
  - $\sim 3\times$ cheaper
- Comparable error between summation and standard method
- See also S. Capitani *et al.*, arXiv:1205.0180

- No detectable excited states contamination, agrees with high precision study S. Dinter *et al.*, arXiv:1108.1076 and C. Alexandrou *et al.*, arXiv:1112.2931
  - Same plateau for multiple t_{sink}s
  - No curvature in summed ratio, consistent results for various fit-ranges

$\langle x \rangle_{u-d}$ with the summation method

One twisted mass ensemble, a = 0.08 fm, m_π = 390 MeV,
iso-scalar (only connected) and iso-vector $\langle x \rangle$





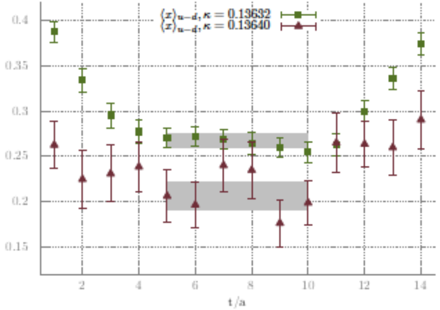
- Noticeable contamination, especially for the iso-scalar
- Summation method uses 7 sink-source time separations
- For the plateau method one needs to show convergence by varying the sink-source time separation $\rightarrow$ also requires a number of sequential inversions

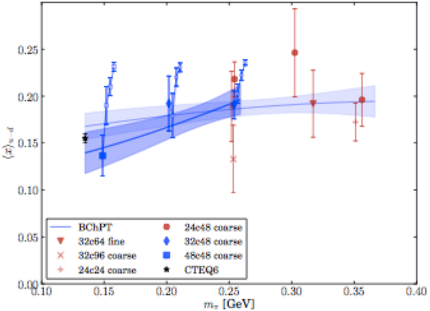
$\Longrightarrow$ Plateau and summation method give consistent results.

Results at almost physical pion mass

Very recent results claim correct value of $\langle x \rangle_{u-d}$:

- $N_F = 2$ Clover at $m_\pi = 157(6)$ MeV, $a = 0.07$ fm and $m_\pi L = 2.74$ using time separation ~ 1 fm, G. Bali *et al.* arXiv:1207.1110
- $N_F = 2 + 1$ BMW configurations at $m_\pi = 149$ MeV, $a = 0.116$ fm and $m_\pi L = 4.2$ using 3 time separations up to 1.4 fm in combination with summation method, J. R. Green *et al.* arXiv:1209.1687





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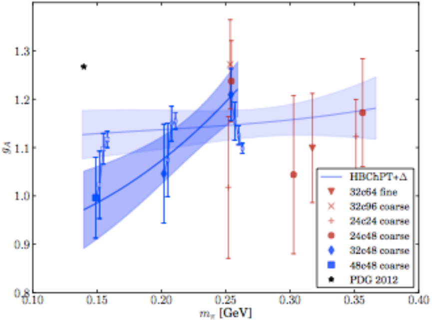
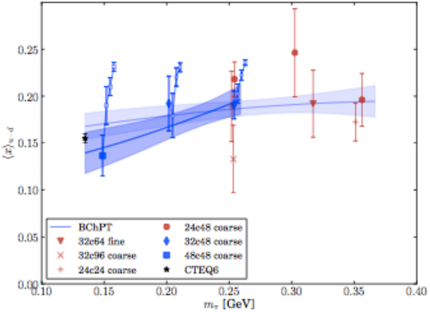




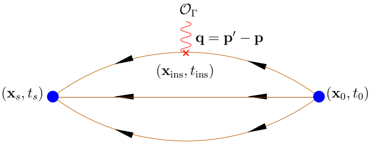
Nucleon spin

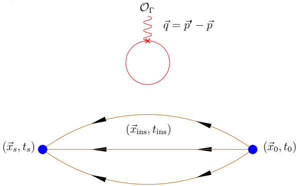
Contributions to the spin of the nucleon
Spin sum: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_G$

Non-relativistic quark model:

If $\Delta\Sigma_{u,d} = \Delta u + \Delta d = 1 \Rightarrow L_q = 0$ and $J_G = 0$, as well as $\Delta s = 0$, where Δq contains both the spin of q and $\bar{q}$.
Lattice QCD: Need both connected and disconnected contributions to evaluate contributions to spin

Bali *et al.* (QCDSF), Phys.Rev.Lett. 108 (2012) 222001 : $\Delta u + \Delta d + \Delta s$ = 0.45 (4)(9) with $\Delta s = -0.020(10)(4)$ at $\mu = \sqrt{7.4}$ GeV
$\Longrightarrow$ Small strangeness (disconnected) contribution to the nucleon spin





Lattice results on the nucleon spin

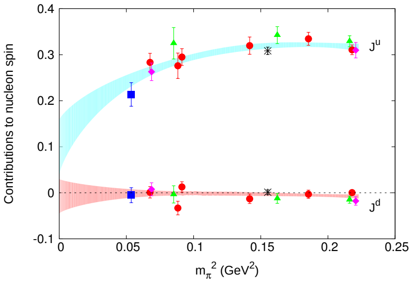
$J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma + L_q$
$\Delta\Sigma = \tilde{A}_{10}$

Only connected contribution

Results using $N_F = 2$ TMF for 270 MeV $< m_\pi < 500$ MeV, C. Alexandrou *et al.* (ETMC), arXiv:1104.1600 and $N_F = 2+1+1$ at $m_\pi \sim 230$ MeV and 390 MeV

In agreement with A. Sternbeck *et al.* (QCDSF) arXiv:1203.6579

In qualitative agreement with J. D. Bratt *et al.* (LHPC), PRD82 (2010) 094502



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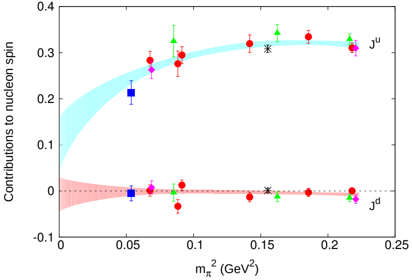

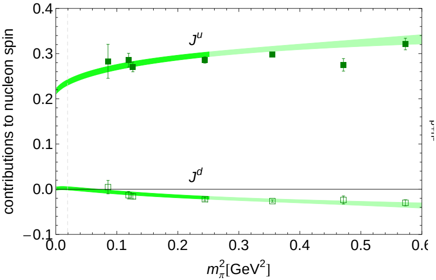

$\Longrightarrow$ Total spin for u-quarks $J^u \sim 0.25$ and for d-quark $J^d \sim 0$

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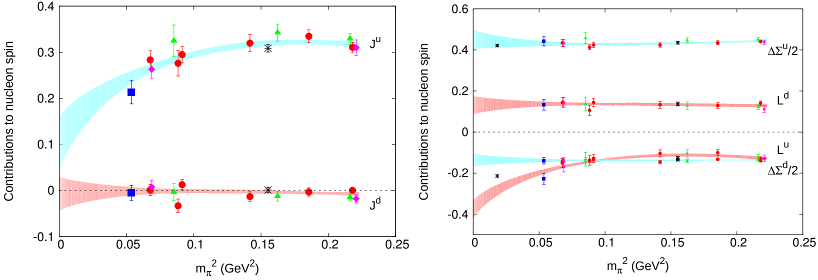
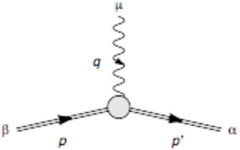


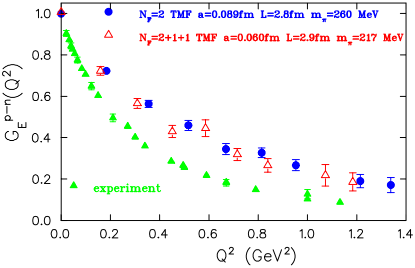
- Lattice results for $\Delta\Sigma^{u-d}$ and L^{u-d} in good agreement
- $L^{u+d} \sim 0$ at physical point.
- How about the disconnected contributions to L_q and contributions from J_g? K.-F. Liu *et al.* (χQCD), arXiv:1203.6388 claim they are large $\Longrightarrow$ Need to be confirmed using dynamical quarks, larger volumes and lighter quark masses

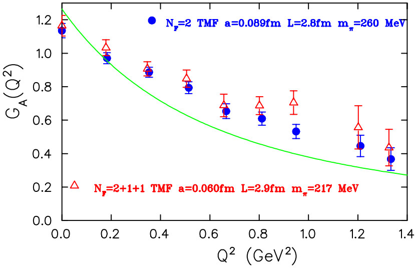
Nucleon EM and axial form factors



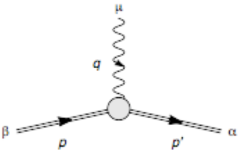
$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$

New results for $N_F = 2+1+1$ twisted mass fermions, $48^3 \times 96$, $\beta = 2.1$ ($a = 0.063$ fm from nucleon mass), $m_\pi \sim 230$ MeV, 950 configurations





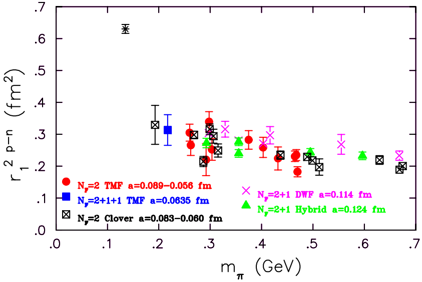
Nucleon Dirac and Pauli isovector radii

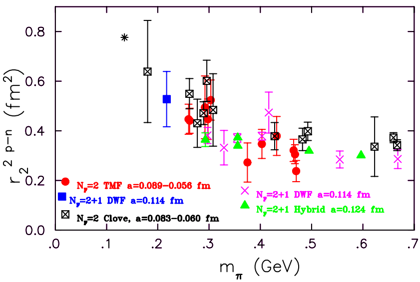


$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$

Dirac and Pauli radii: $r^2_{1,2} = -\frac{6}{F_{1,2}(0)}\frac{dF_{1,2}}{dq^2}|_{q^2=0}$

Use a dipole Ansatz to fit the q^2-dependence of F_1 and F_2.





- TMF: C. A. *et al.* (ETMC), PRD83 (2011) 094502
- Clover: S. Collins *et al.* (QCDSF), Phys.Rev. D84 (2011) 074507
- DWF: S. N. Syritsyn *et al.* (LHPC), PRD 81, 034507 (2010); T. Yamazaki *et al.* (RBC-UKQCD), PRD 79, 114505 (2009)
- Hybrid: J. D. Bratt *et al.* (LHPC), Phys. Rev. D82, 094502 (2010)

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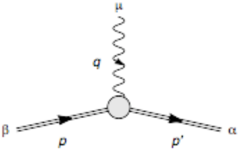
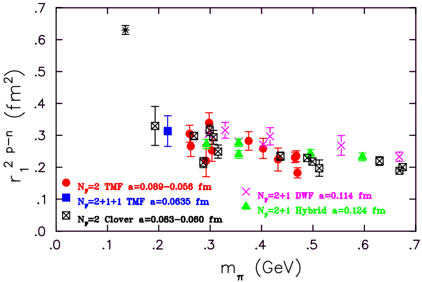
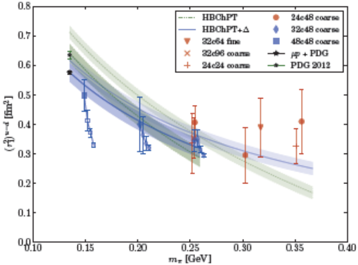


$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$





J. R. Green *et al.* 1209.1687

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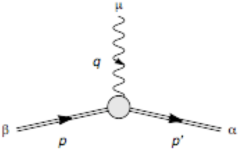

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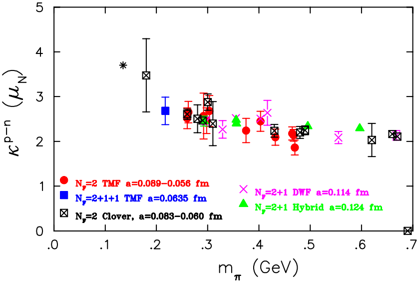
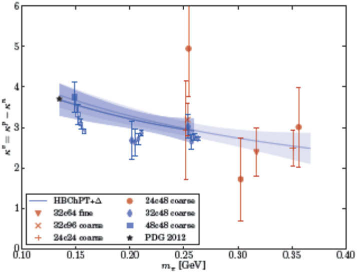
Anomalous magnetic moment: $F_2(0)\,\frac{m_N^{\rm phys}}{m_N^{\rm lat}}$





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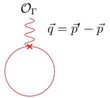
Nucleon σ-terms

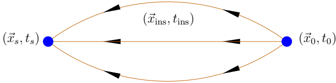
- $\sigma_l \equiv m_l\langle N|\bar{u}u + \bar{d}d|N\rangle$: measures the explicit breaking of chiral symmetry
  Extracted from analysis of low-energy pion-proton scattering data
- In lattice QCD it can be obtained via the Feynman-Hellman theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$
- Similarly $\sigma_s \equiv m_s\langle N|\bar{s}s|N\rangle >= m_s \frac{\partial m_N}{\partial m_s}$
- The strange quark content of the nucleon: $y_N = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle}$
- A number of groups have use the spectral method to extract the σ-terms.
- Can also be calculated directly.

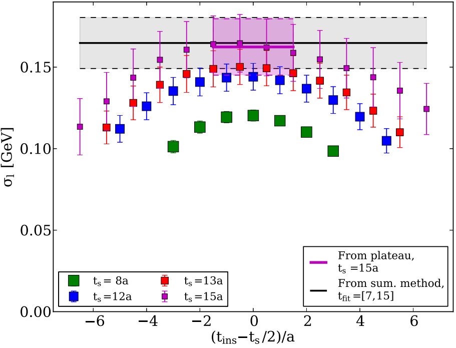
Nucleon σ-terms

Advantages for twisted mass fermions:

- In the twisted basis the scalar $\bar{u}u + \bar{d}d$ becomes: $i(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$
  From the TM action: $D_u - D_d = 2i\mu\gamma_5$
  $\rightarrow D_u^{-1} - D_d^{-1} = -2i\mu D_d^{-1}\gamma_5 D_u^{-1}$ with noise suppression due to small value of μ
- The light quark loops can be computed by calculating stochastically $D_u^{-1}\,(2i\mu\gamma_5)\,D_d^{-1}$ using the **one-end-trick** to further improve the signal, S. Dinter *et al.* 1202.1480
- Renormalization straight forward







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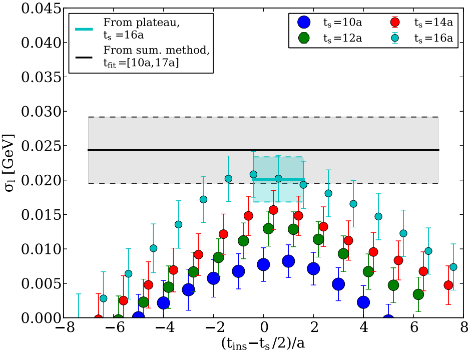

Disconnected

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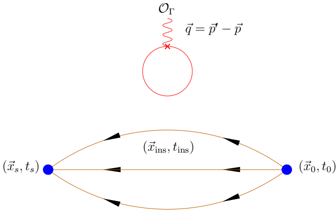

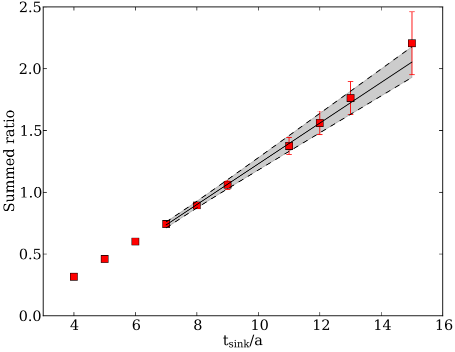

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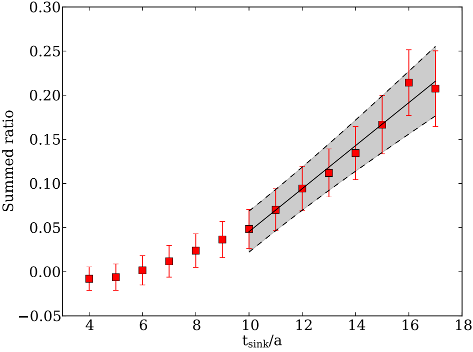

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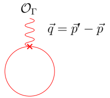
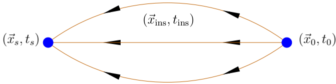
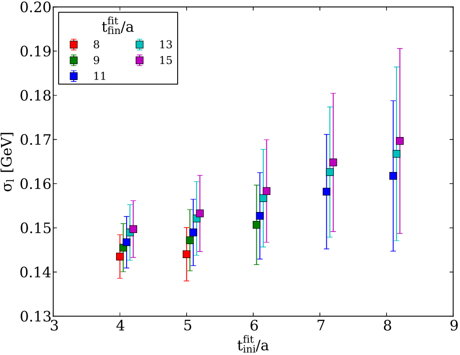
- The light quark loops can be computed by calculating stochastically $D_u^{-1}\,(2i\mu\gamma_5)\,D_d^{-1}$ using the **one-end-trick** to further improve the signal, S. Dinter *et al.* 1202.1480
- Renormalization straight forward







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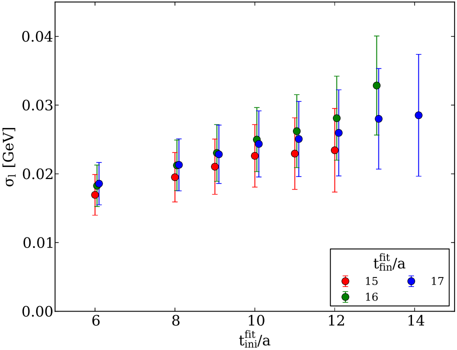

Disconnected

Nucleon σ-terms

Advantages for twisted mass fermions:

- In the twisted basis the scalar $\bar{u}u + \bar{d}d$ becomes: $i(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$
  From the TM action: $D_u - D_d = 2i\mu\gamma_5$
  $\rightarrow D_u^{-1} - D_d^{-1} = -2i\mu D_d^{-1}\gamma_5 D_u^{-1}$ with noise suppression due to small value of μ
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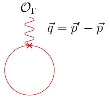
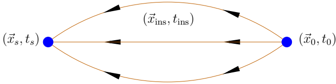
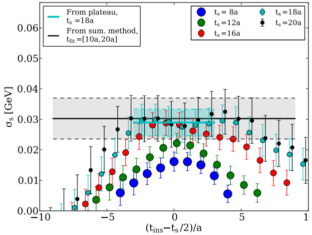
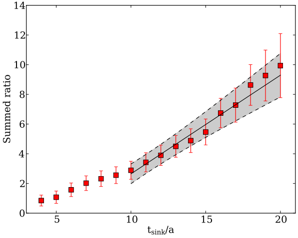
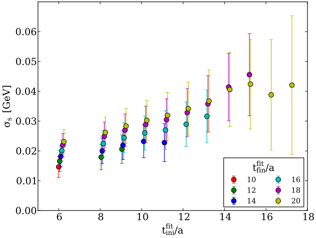










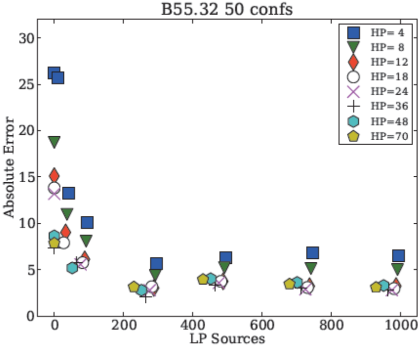
We use Osterwalder-Seiler s-quarks and compute $\frac{1}{2}\,(\bar{s}_+ s_+ + \bar{s}_- s_-)$
At $m_\pi = 390$ MeV:

$$y_N = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle} = \begin{cases} 0.098(23) & \text{(sum)} \\ 0.097(17) & \text{(plat)} \end{cases}$$

Nucleon isoscalar axial charge

- If one has the combination $\bar{u}u + \bar{d}d$ then one can use: $D_u + D_d = 2D_W$ and the loop is given by $2D_u^{-1} D_W D_d^{-1}$. One applies the one-end-trick to this combination $\rightarrow$ **generalized one-end-trick**
- Stochastic noise larger $\rightarrow$ combine with truncated solver method (TSM), G. Bali, S. Collins and A. Schäffer, PoSLat2007, 141
- Need to tune in addition to the high precision noise vectors $N_{\rm HP}$ and number of low precision vectors $N_{\rm LP}$

Bilinear (Physical basis)	Transforms to (Twisted basis)	Standard One-end Trick
$\bar{\psi}\psi$	$i\bar{\psi}\gamma_5\tau_3\psi$	✓
$\bar{\psi}\tau_3\psi$	$i\bar{\psi}\gamma_5\psi$	×
$i\bar{\psi}\gamma_5\psi$	$-\bar{\psi}\tau_3\psi$	✓
$i\bar{\psi}\gamma_5\tau_3\psi$	$-\bar{\psi}\psi$	×
$\bar{\psi}\gamma_\mu\psi$	$\bar{\psi}\gamma_\mu\psi$	×
$\bar{\psi}\gamma_\mu\tau_3\psi$	$\bar{\psi}\gamma_\mu\tau_3\psi$	✓
$\bar{\psi}\gamma_5\gamma_\mu\psi$	$\bar{\psi}\gamma_5\gamma_\mu\psi$	×
$\bar{\psi}\gamma_5\gamma_\mu\tau_3\psi$	$\bar{\psi}\gamma_5\gamma_\mu\tau_3\psi$	✓
$i\bar{\psi}\gamma_\mu D_\nu\psi$	$i\bar{\psi}\gamma_\mu D_\nu\psi$	×
$i\bar{\psi}\gamma_\mu D_\nu\tau_3\psi$	$i\bar{\psi}\gamma_\mu D_\nu\tau_3\psi$	✓
$i\bar{\psi}\gamma_5\gamma_\mu D_\nu\psi$	$i\bar{\psi}\gamma_5\gamma_\mu D_\nu\psi$	×
$i\bar{\psi}\gamma_5\gamma_\mu D_\nu\tau_3\psi$	$i\bar{\psi}\gamma_5\gamma_\mu D_\nu\tau_3\psi$	✓



Results for $\langle x \rangle_{u+d}$

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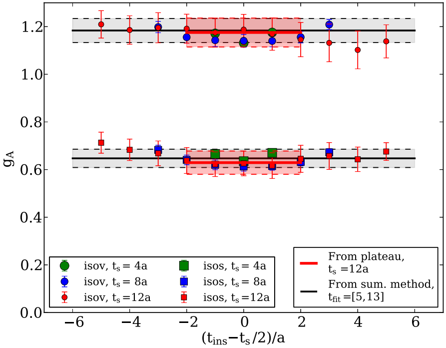
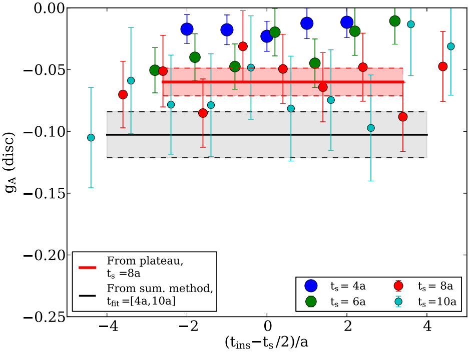
- If one has the combination $\bar{u}u + \bar{d}d$ then one can use: $D_u + D_d = 2D_W$ and the loop is given by $2D_u^{-1} D_W D_d^{-1}$. One applies the one-end-trick to this combination → **generalized one-end-trick**
- Stochastic noise larger → combine with truncated solver method (TSM), G. Bali, S. Collins and A. Schäffer, PoSLat2007, 141
- Need to tune in addition to the high precision noise vectors $N_{\rm HP}$ and number of low precision vectors $N_{\rm LP}$
- Use to compute isoscalar g_A using $N_{\rm HP} = 24$ and $N_{\rm LP} = 300$
- Since the LP sources don't require an accurate inversion, we can take advantage of the half precision algorithms for GPUs - use the QUDA library



Connected



Disconnected

Nucleon isoscalar axial charge

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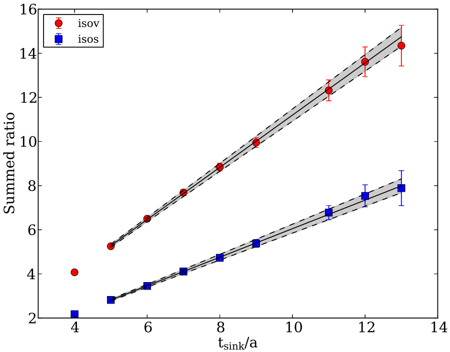

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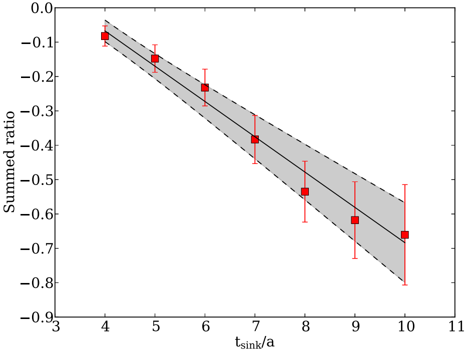


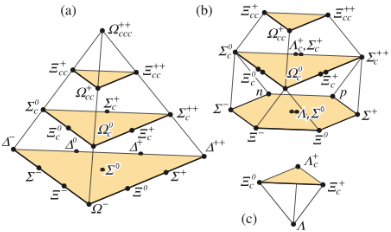
Disconnected, in qualitative agreement with Bali *et al.* (QCDSF), Phys.Rev.Lett. 108 (2012) 222001

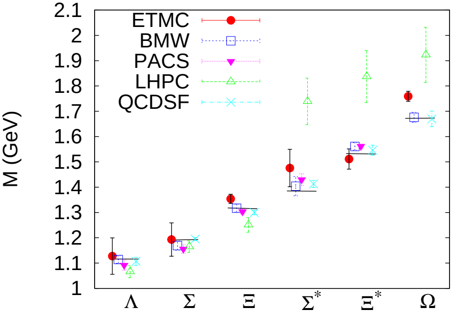
Hyperons and Charmed baryons

SU(4) representations:

$$4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus \overline{4}$$

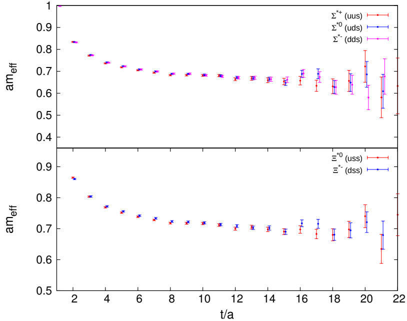
$$\square \otimes \square \otimes \square = \square\square\square \oplus \begin{array}{l}\square\square\\\square\end{array} \oplus \begin{array}{l}\square\square\\\square\end{array} \oplus \begin{array}{l}\square\\\square\\\square\end{array}$$





$N_F = 2+1$ clover, hybrid and $N_F = 2$ twisted mass fermions.

C. A., J. Carbonell, D. Christaras, V. Drach, M. Gravina, M. Papinutto, arXiv:1205.6856

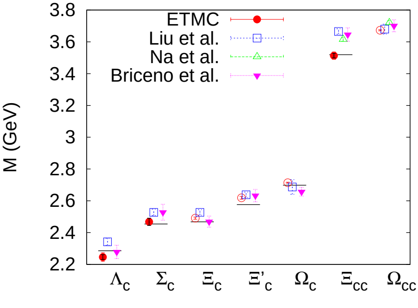


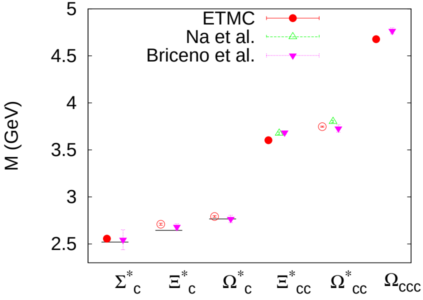
New results using $N_F = 2+1+1$ are in the pipeline. The Δ, Σ^* and Ξ^* show no isospin breaking at $a \sim 0.078$ fm

Mass of charmed baryons

All use a mixed action approach:

- ETMC: TM $N_F = 2$ fermions gauge configurations
- Other collaborations use staggered $N_F = 2 + 1$ quarks, and a relativistic heavy quark or clover action for the charm quark
- $N_F = 2 + 1$ Clover quarks, G. Bali *et al.* at CIPANP2012.





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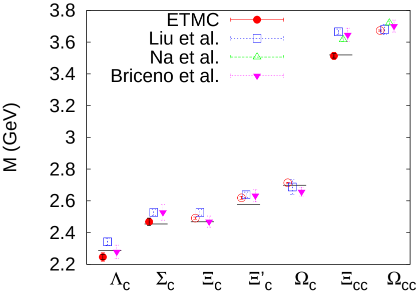
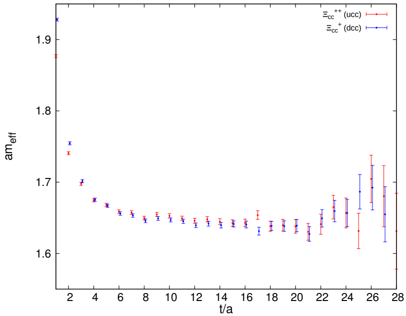
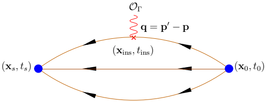




$N_F = 2 + 1 + 1$ at $m_\pi \sim 390$ MeV and $a = 0.078$ fm

Axial charge for hyperons

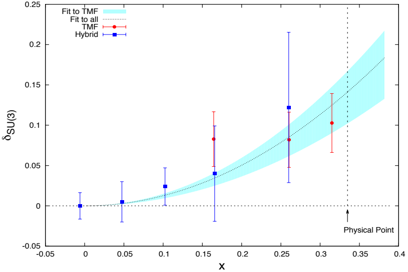
- Given by the hadron matrix element at zero momentum transfer: $\langle h|\bar{\psi}\gamma_\mu\gamma_5\psi|h\rangle|_{q^2=0}$
- Efficient to calculate (connected contribution with fixed current method) - computational cost for all hadrons about twice that required for one hadron (cost of additional contractions)



If exact SU(3) flavor symmetry:

- $g_A^N = F + D$, $g_A^\Sigma = 2F$, $g_A^\Xi = -D + F \Longrightarrow g_A^N - g_A^\Sigma + g_A^\Xi = 0$

Probe deviation: $\delta_{\rm SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ versus $x = (m_K^2 - m_\pi^2)/4\pi^2 f_\pi^2$, H.- W. Lin and K. Orginos, PRD 79, 034507 (2009)



Breaking $\sim x^2$ leads to about 15% at the physical point

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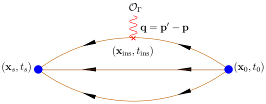

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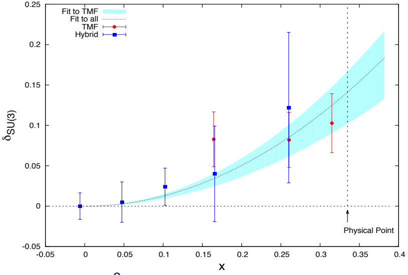
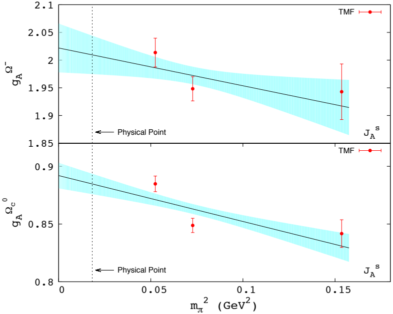


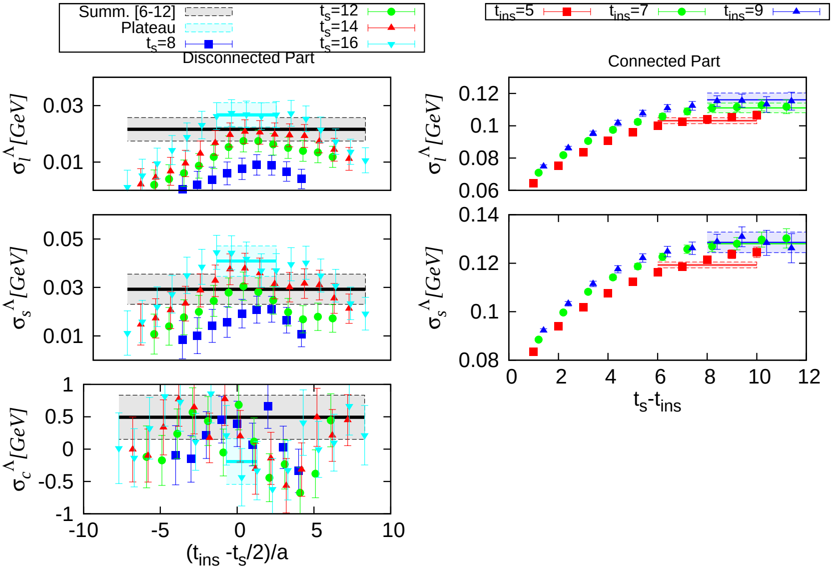


Breaking $\sim x^2$ leads to about 15% at the physical point $x_{\rm phy} = 0.33$

σ-terms for hyperons and charmed baryons

Need both connected and disconnected pieces

First results using $N_F = 2 + 1 + 1$ TMF at $m_\pi = 390$ MeV



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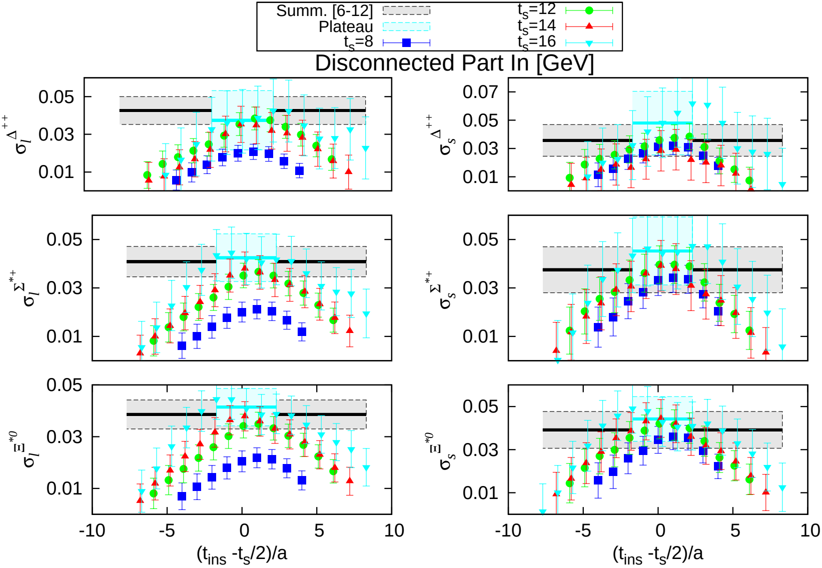

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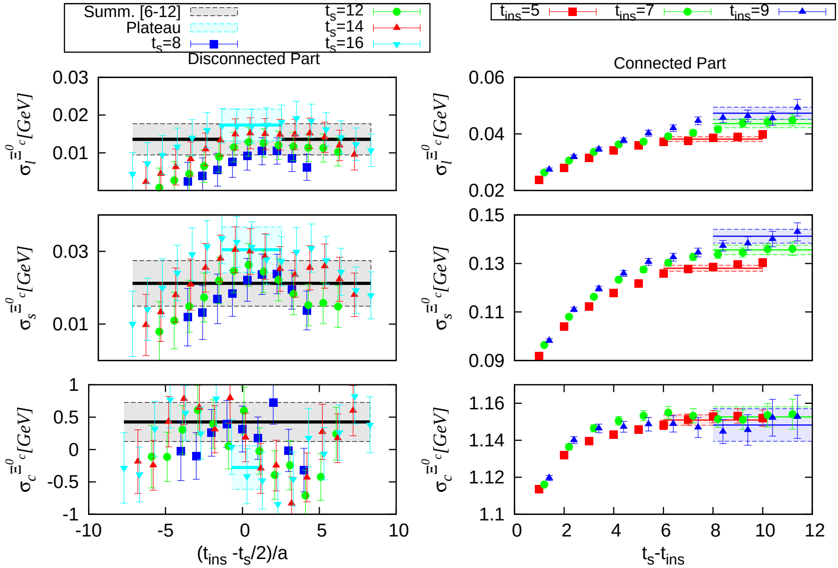


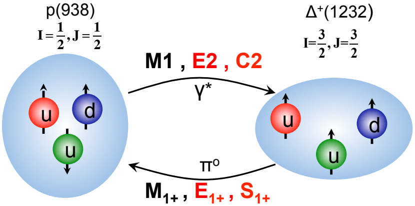
$N\gamma^* \to \Delta$ form factors

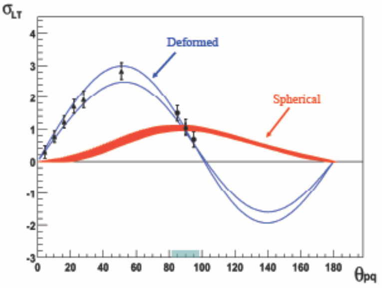
- A dominant magnetic dipole, M1
- An electric quadrupole, E2 and a Coulomb, C2 signal a deformation in the nucleon/Δ

  $$R_{EM}(\mathrm{EMR}) = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)},$$

  $$R_{SM}(\mathrm{CMR}) = -\frac{|\vec{q}|}{2m_\Delta}\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)},$$

  in lab frame of the Δ.
- Used for probing nucleon shape since 1/2-spin particles have vanishing quadrupole moment in the lab-frame
- Difficult to measure/calculate since quadrupole amplitudes are sub-dominant





C. N. Papanicolas, Eur. Phys. J. A18 (2003); N. Sparveris *et al.*, PRL **94**, 022003 (2005)

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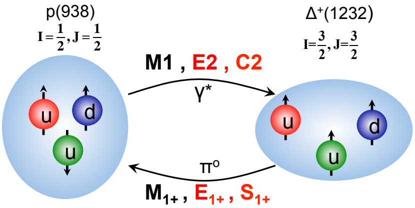

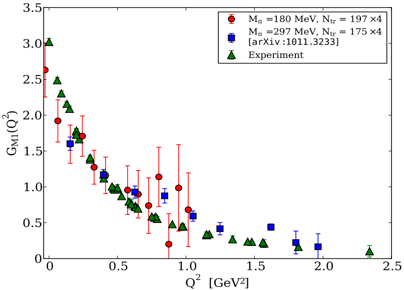

- $N_F = 2+1$ with DWF, slope at $m_\pi \sim 300$ MeV smaller than experiment, underestimate G_{M1} i.e. like for nucleon form factors
- New results with $N_F = 2+1$ DWF at $m_\pi \sim 180$ MeV require more statistics

Conclusions

- Nucleon structure is a benchmark for the LQCD approach
  Some puzzles remain like g_A. Others need to be confrim by other groups like $\langle x \rangle_{u-d}$
  $\Longrightarrow$ simulations of the full theory at near physical parameters will eliminate umbiguities due chiral extrapolations
- Evaluation of quark loop diagrams has become feasible
- Predictions for other hadron observables are beginning to emerge e.g. axial charge of hyperons and charmed baryons
- Studying baryon resonances is also beginning $\rightarrow$ provide insight into the structure of hadrons providing information that is difficult to extract experimentally.

As simulations at the physical pion mass and more computer are becoming available we expect many physical results on these key hadron observables