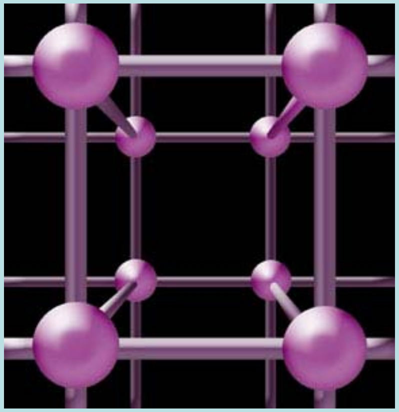
Conformality and The Lattice



Florence

September, 2012

Lattice Simulations

1. Collaboration with George Fleming and Ethan Neil

   1) arXiv:0712.0609, PRL 100, 171607 , 2008

   2) arXiv:0901.3766 PR D79, 076010, 2009

2. LSD collaboration **J. C. Osborn, R. Babich, R. C. Brower, M.I. Buchoff, M. A. Clark, C. Rebbi, D. Schaich, M. Cheng, J. Kiskis, R. Soltz, P. M. Vranas, T. Appelquist, G. T. Fleming, E. T. Neil, M. Lin, G. Voronov, J. Wasem**

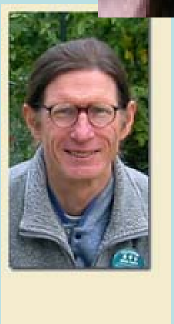
   1) **arXiv:0910.2224** PRL 104, 071601 (2010) 2→6 Flavors

   2) arXiv:1009.5967 PRL106:231601,2010 Parity Doubling and S

   3) arXiv: 1201.3977 PR D85, 074505 (2012) W-W scattering

   4) arXiv: 1204.6000 --> PRL 10 Flavors

   5) Underway: SU(2)



3. GF, EN, Meifeng Lin, David Schaich

   arXiv: 1106.2148 PRD D84, 054501, 2011

USQCD clusters, LLNL BG/L, NSF Teragrid

Disclaimer:

I am not a lattice gauge theorist.

OUTLINE

1. Introduction - Conformality

2. Degree-of Freedom Inequality Constraint
   ~ c-theorems , a-theorems

3. Conformal Window via the Lattice:
   SU(3) with N_f = 8, 12

4. Creeping toward the conformal window
   SU(3) with N_f = 2, 6, 8, 10,

5. A light dilaton in walking gauge theories?

New Strong Dynamics at TeV Energies?

- New, SM-singlet Sector

  Conformal behavior

- Electroweak breaking

  Near-conformal IR: walking technicolor (1980's + …)

  Composite Higgs models

5

Conformal Symmetry / Scale Invariance / Dilatation Symmetry

No fixed scales (masses)

Expected possibly at high energies or temperatures

Examples:

σ (e+e- $\rightarrow$ Hadrons) $\sim$ $1/ E_{cm}^2$

$F(T) \sim T^4$

Not Quite

Quantum field theories *typically* depend on a renormalization scale Λ.

$$\alpha \equiv g^2/4\pi = \alpha(q^2 / \Lambda^2)$$

For a Yang-Mills theory

$$\xrightarrow{q^2\to\infty} \sim 1/ \log(q^2 / \Lambda^2)$$

Asymptotic freedom

QCD Infrared Features

1. Quark and gluon confinement

   $\Lambda \sim 200$ MeV

2. Spontaneous chiral symmetry (nearly massless quarks) breaking $SU(2)_L x SU(2)_R \rightarrow SU(2)$

   3 Pions = PNGoldstoneB's

   F = 93 MeV

   $<\bar{\psi}\psi> \sim 4 \pi F^3$

8

Beyond QCD

- SU(N) Gauge Theories with N_f Massless Fermions (fundamental and other reps) Screening

Asymptotically-free $N_f < N_{af}$

Vary N_f and study how the infrared behavior changes

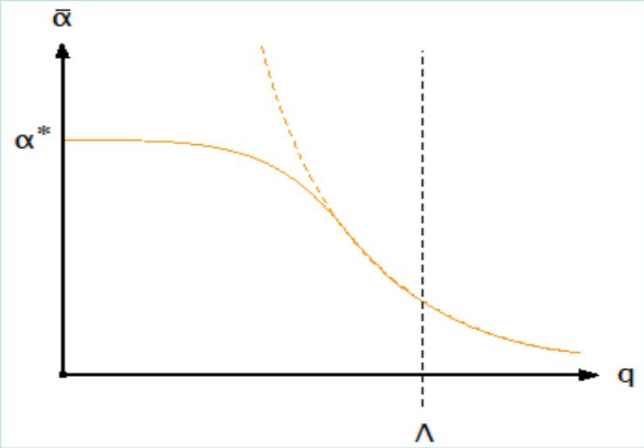
(Chiral symmetry breaking and confinement versus *infrared* conformal behavior.)

SUSY Theories?

9

Conformal Window

N_{af} > N_f > N_{fc} (Fermion screening)



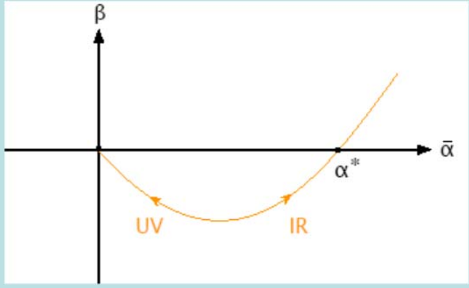
Perturbative for $N_f \rightarrow N_{af}$

Gross and Wilczek, antiquity

William Caswell, 1974 --→

Banks and Zaks, 1982

……….



α* increases as N_f decreases.

$N_f \rightarrow N_{fc}$ (Non-perturbative)

SU(3) Perturbation Theory

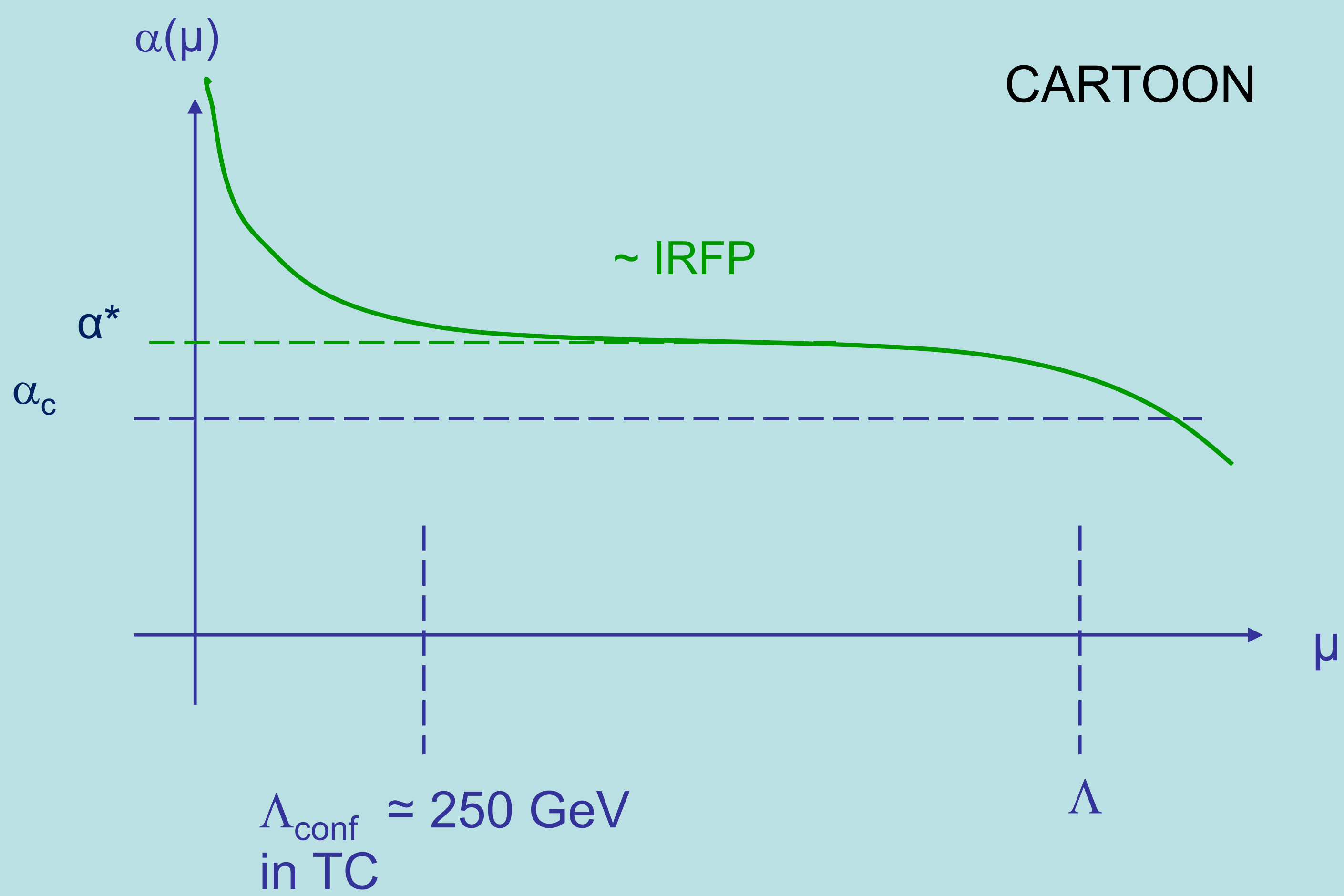
$$\mu \frac{\partial}{\partial \mu} \bar{g}^2(\mu) = \beta\left(\bar{g}^2(\mu)\right) = -b_1 \bar{g}^4(\mu) - b_2 \bar{g}^6(\mu) - b_3 \bar{g}^8(\mu) + \ldots.$$

$$b_1 = \frac{2}{(4\pi)^2}\left(11 - \frac{2}{3} N_f\right), \qquad b_2 = \frac{2}{(4\pi)^4}\left(102 - \frac{38}{3} N_f\right)$$

$$b_3^{\overline{MS}} = \frac{2}{(4\pi)^6}\left[\frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2\right]$$

NEAR-CONFORMAL BEHAVIOR

For N_f just below N_{fc} (Walking)



Questions

1. Value of N_{fc}? (Order of transition?)

2. Inside the conformal window $(N_f \; > \; N_{fc})$

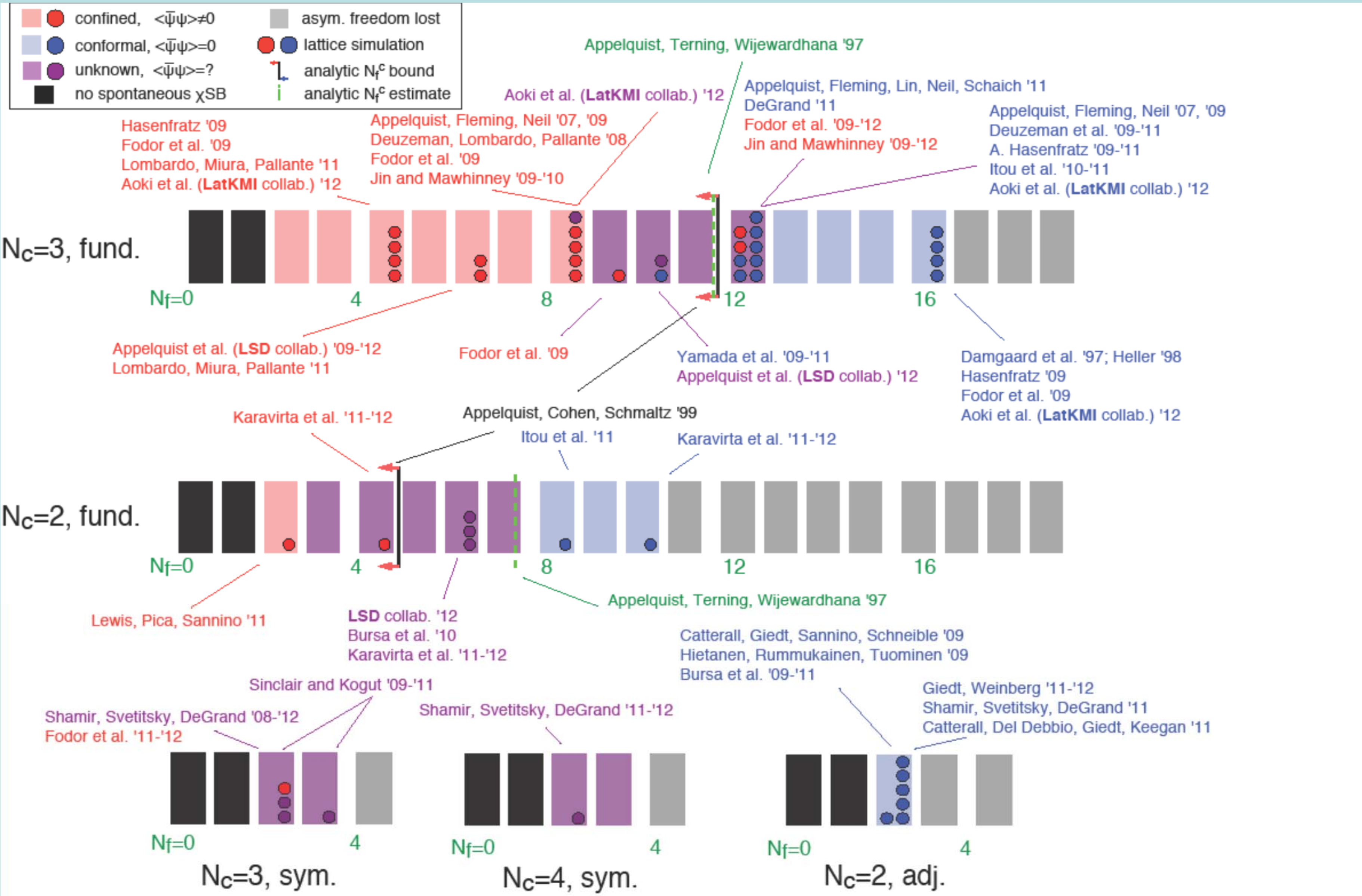
3. Below and near the transition $(N_f \; \lesssim \; N_{fc})$

Approximate IRFP (Walking)?

Condensate enhancement? Parity Doubling? EW precision studies (S parameter)?

Dilaton ? (Light 0^{++} State)

13



confined, <ψ̄ψ>≠0
conformal, <ψ̄ψ>=0
unknown, <ψ̄ψ>=?
no spontaneous χSB
asym. freedom lost
lattice simulation
analytic N_f^c bound
analytic N_f^c estimate
Appelquist, Terning, Wijewardhana '97
Aoki et al. (LatKMI collab.) '12
Appelquist, Fleming, Lin, Neil, Schaich '11
DeGrand '11
Fodor et al. '09-'12
Jin and Mawhinney '09-'12
Appelquist, Fleming, Neil '07, '09
Deuzeman, Lombardo, Pallante '09-'11
A. Hasenfratz '09-'11
Itou et al. '10-'11
Aoki et al. (LatKMI collab.) '12
Hasenfratz '09
Fodor et al. '09
Lombardo, Miura, Pallante '11
Aoki et al. (LatKMI collab.) '12
Appelquist, Fleming, Neil '07, '09
Deuzeman, Lombardo, Pallante '08
Fodor et al. '09
Jin and Mawhinney '09-'10
N_c=3, fund.
N_f=0
4
8
12
16
Appelquist et al. (LSD collab.) '09-'12
Lombardo, Miura, Pallante '11
Fodor et al. '09
Yamada et al. '09-'11
Appelquist et al. (LSD collab.) '12
Damgaard et al. '97; Heller '98
Hasenfratz '09
Fodor et al. '09
Aoki et al. (LatKMI collab.) '12
Karavirta et al. '11-'12
Appelquist, Cohen, Schmaltz '99
Itou et al. '11
Karavirta et al. '11-'12
N_c=2, fund.
N_f=0
4
8
12
16
Appelquist, Terning, Wijewardhana '97
Lewis, Pica, Sannino '11
LSD collab. '12
Bursa et al. '10
Karavirta et al. '11-'12
Catterall, Giedt, Sannino, Schneible '09
Hietanen, Rummukainen, Tuominen '09
Bursa et al. '09-'11
Sinclair and Kogut '09-'11
Shamir, Svetitsky, DeGrand '08-'12
Fodor et al. '11-'12
Shamir, Svetitsky, DeGrand '11-'12
Giedt, Weinberg '11-'12
Shamir, Svetitsky, DeGrand '11
Catterall, Del Debbio, Giedt, Keegan '11
N_f=0
4
N_c=3, sym.
N_f=0
4
N_c=4, sym.
N_f=0
4
N_c=2, adj.

I will not discuss:

Holographic Methods (gauge-gravity duality)

Holographic Walking Technicolor from D-branes
Lilia Anguelova, Peter Suranyi, L.C. Rohana Wijewardhana
arXiv:1105.4185 Nucl. Phys. B852 (2011) 39-60

Holographic Electroweak Symmetry Breaking from D-branes,
C. Carone, J. Erlich and M. Sher,
arXiv:0704.3084, Phys. Rev. D76 (2007) 0150

Holographic Technicolor Models and their S-parameter,
O. Mintakevich and J. Sonnenschein,
arXiv:0905.3284 JHEP 0907 (2009) 032,

A Negative S Parameter from Holographic Technicolor,
J. Hirn and V. Sanz,
Phys. Rev. Lett. 97 (2006) 121803, hep-ph/0606086;

Dynamical Electroweak Symmetry Breaking from Deformed AdS: Vector Mesons and Eective Couplings M. Fabbrichesi, M. Piai and L. Vecchi, ,
Phys. Rev. D78 (2008) 045009, arXiv:0804.0124

+ ......

N_{fc} in SU(N) Gauge Theory

- Degree-of-Freedom Inequality (Cohen, Schmaltz, TA 1999). Fundamental rep: ← Conjecture

$$N_{fc} < 4\,N[1 - 1/18N^2 + \ldots] \qquad N = 3, 4, 5, \ldots$$
$$< 5 \qquad N = 2$$

- Gap-Equation Studies, Instantons (1996 ….): ←Suspect methods

$N_{fc} \cong 4\,N$

Pioneers:
Maskawa & Nakajima,
Kugo & Fukuda 1970's

16

Thermal Inequality

Introduction and Discussion

Related to c-theorems, a-theorems

Komargodski and Schwimmer 1107.3987

Luty, Polchinski, and Rattazzi 1204.5221

+ …………

2+1 QED and Abelian Higgs model

(including recent lattice studies)

Inequality Constraint on Strongly Coupled Field Theories Based on the Thermodynamic Free Energy

A. Cohen
M. Schmaltz
T. Appelquist
hep-th/9901109

- Basic Idea

$$F(T) \sim T^4 \, f(T)$$

"Counts" Thermodynamic Degrees of Freedom

$$\boxed{f(T \to 0) \le f\,(T \to \infty)}$$

NATURE
"ABHORS"
MASSLESS
PARTICLES

FINITE TEMPERATURE

$$\boxed{Z = \mathrm{Tr}\, e^{-\beta H} \qquad \beta = T^{-1}}$$

$$A_\mu(\beta, \vec{x}) = A_\mu(0, \vec{x})$$

$$\psi(\beta, \vec{x}) = -\,\psi(0, \vec{x})$$

$$\boxed{F(T) = -\ln Z / \beta V}$$

$$P(T) = -\,F(T)$$

$$s = \partial P / \partial T > 0$$

$$C_v = T\, \partial s / \partial T > 0$$

$$F = \rho - Ts$$

FREE FIELD THEORY

Path Integral $\rightarrow$ Gaussian

$$\boxed{F(T) = \frac{-\pi^2 T^4}{90} \left[N_s + 2\, N_v + \frac{7}{8} \cdot 4\, N_f \right]}$$

Massless

+ Additive, T-Independent
Quartically Divergent
Zero-Point Energy
(Non-SUSY)

Perturbative Corrections

More Generally, Focus on Theories for Which

$$F(T \rightarrow \infty) \rightarrow \frac{-\pi^2 T^4}{90} f(\infty)$$

Finite Constant Counts Fundamental
Degrees of Freedom For A.F. Theory

$$F(T \rightarrow 0) \rightarrow \frac{-\pi^2 T^4}{90} f(0)$$

Finite Constant Counts Massless
L.E. Degrees of Freedom for I.F. Theory

Will apply to theories with non-trivial
IR fixed points

THE INEQUALITY

$$\boxed{f(0) \leq f(\infty)}$$

Natural Route to Proof:

Define: $f(T) \equiv \dfrac{-\pi^2 T^4}{90} F(T)$

Prove: $f(T)$ is Monotonic in T

A "c-theorem"

Problem: It ain't so.

SUSY SU(N)
N_f Matter Multiplets

Finite T Destroys SUSY

$$f(\infty) = \left(1 + \frac{7}{8}\right) [2(N^2 - 1) + 4\, N_f\, N]$$

$$f_{\substack{\text{Free}\\ \text{Mag}}}(0) \underset{\substack{\text{Duality}\\ N_f \geq N+2}}{=} \left(1 + \frac{7}{8}\right) \{2[(N_f - N)^2 - 1] + 4\, N_f (N_f - N) + 2\, N_f^2\}$$

Theorem $\Rightarrow$ $\boxed{N_f \leq \frac{3}{2} N}$

SU(N) (Vector) Gauge Theory
N_f Dirac Fermions

$$f(\infty) = 2(N^2 - 1) + \frac{7}{8} 4N_f N$$

$$f_{NG}(0) = N_f^2 - 1$$

IR Free

Inequality $\Rightarrow$

$$\boxed{N_f \leq 4N \left\{ 1 - \frac{1}{18N^2} + \cdots \right\}}$$

For N-G Phase

Interesting ?

SU(2) with N_f Dirac Fermions

$f(\infty) = 6 + 7\, N_f$

Broken Vacuum: $SU(2\, N_f) \rightarrow SP(2\, N_f)$

$f(0) = [4\, N_f^2 - 1] - [N_f\,(2N_f + 1)]$

$f(0) < f(\infty) \quad \rightarrow \quad N_f < 4.8$

$$\boxed{T \frac{\partial}{\partial T} f(T)}$$

$$(1)\quad T \frac{\partial}{\partial T} f(g(T)) = \beta(g)\, f'(g)$$

(One Coupling)

$$(2)\quad T \frac{\partial}{\partial T} f(T) = \frac{90}{\pi^2}\left\{\frac{s}{T^4} - 4\frac{P}{T^4}\right\}$$

$$= \frac{90}{\pi^2}\frac{\rho - 3P}{T^4}$$

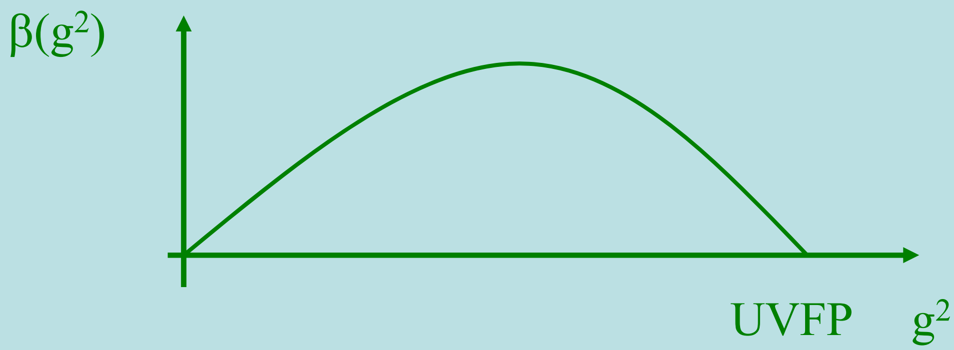
$$= \frac{90}{\pi^2}\frac{\Theta^\mu{}_\mu}{T^4}$$

$$= \frac{90}{\pi^2}\frac{\beta(g)}{g}\left\langle F_{\mu\nu}^2\right\rangle / T^4$$

Classically scale-invariant Gauge theory.

SUGGESTS HOW TO FIND A COUNTER-EXAMPLE:

Find a gauge theory for which



- NO 4D EXAMPLE
- THERE EXIST 3D EXAMPLES

$\boxed{\text{RESTRICT TO AF THEORIES}}$

SU(N) Theory, N_f Fermions

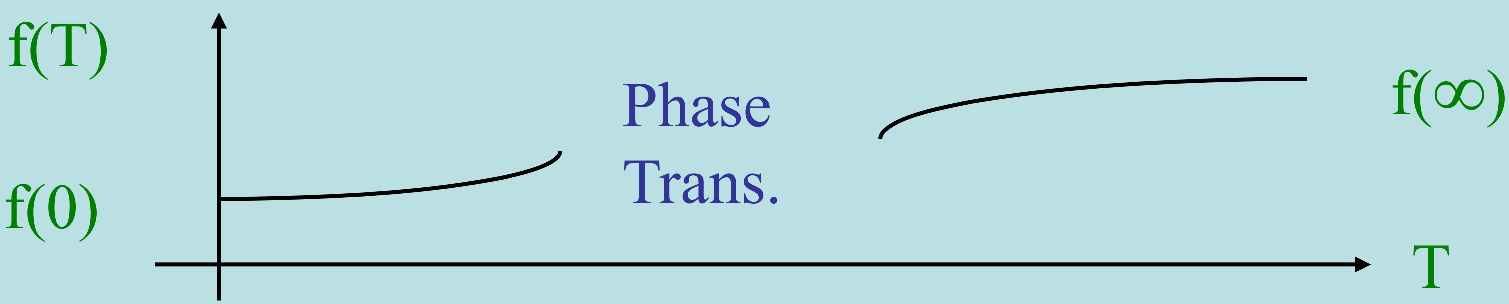
$$T \to \infty: \quad f(T) \to \quad 2(N^2-1) + \frac{7}{2} N_f N$$

$$-10\frac{g^2(T)}{16\pi^2}(N^2-1)(N+\frac{5}{4}N_f)$$

$$+0(g^3)$$

$$T \to 0: \quad f_{NG}(T \to 0) = N_f^2 - 1$$

$$+\frac{N_f^2-1}{36}\frac{(\pi T)^4}{(\pi F_\pi)^4}\ln\frac{F\pi}{T} + \cdots$$



$N_f < 4N$

MONOTONICITY COUNTER – EXAMPLE(S)

$$\boxed{\text{BUT} \quad f(0) \le f(\infty)}$$

SUSY SU(N) $\qquad N_f < 3/2 \; N$

$T \to \infty$:

$$f(T) \to (1+\frac{7}{8})[2(N^2-1)+4N_fN]$$

$$-(N^2-1)(N+3N_f)\frac{45g^2(T)}{32\pi^2}+\cdots$$

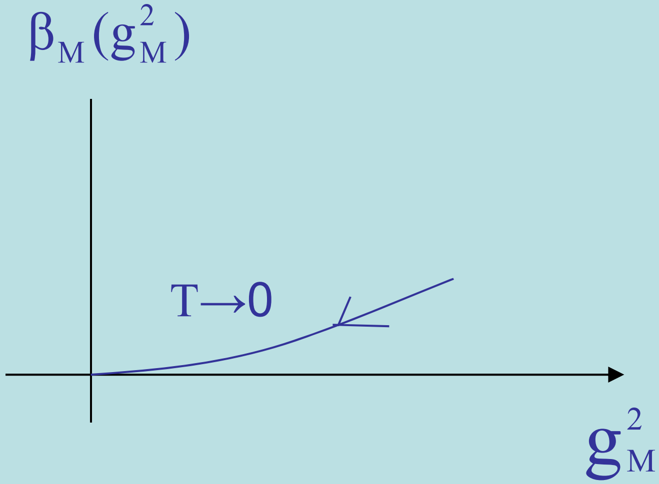
$T \to 0$:

$$f_{\substack{\text{Free}\\\text{Mag}}}(T) \to (1+\frac{7}{8})[2((N_f-N)^2-1)$$

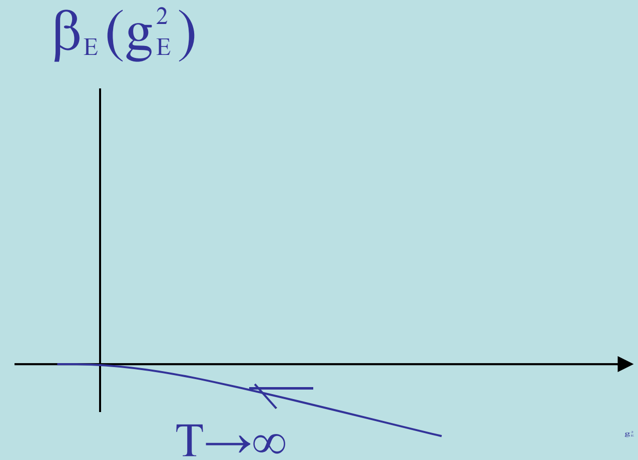
$$+4N_f(N_f-N)+2N_f^2]$$

$$-((N_f-N)^2-1)(4N_f-N)\frac{45g^2(T)}{32\pi^2}$$

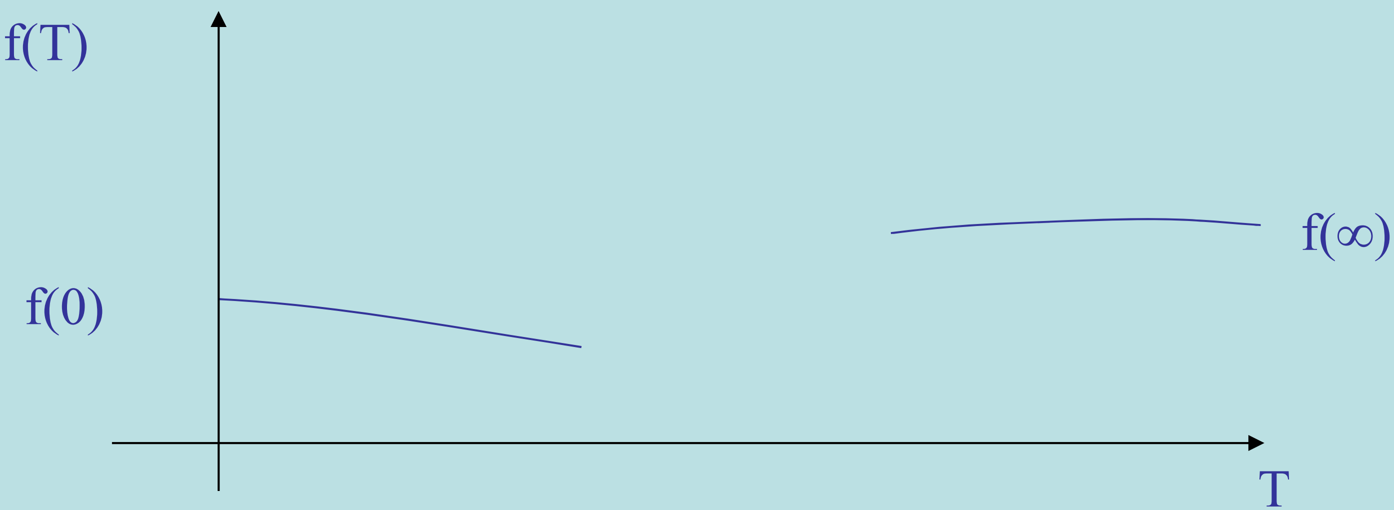
$$-3N_f^2N\frac{45y^2(T)}{32\pi^2}$$



$$N_f < \frac{3}{2}N$$



$$N_f < 3N$$



SUMMARY

1. Correct for all AF theories where it can be checked.

2. Interesting constraint on SU(N) theory with N_f fermions.

   Saturated?

3. Proof ?

   Homework assignment.

4. Counter examples with UV fixed points.

PHASE STRUCTURE
OF
NON-COMPACT QED3

With L.C.R. Wijewardhana
hep-ph/0403250

- An Old Problem (1988-present)

  "Simple", strongly-coupled QFT

- Planar condensed matter systems

  Anti-ferromagnet

  High-T_c Cuprates

- The Model

  Couple A_μ to N Massless Fermions

- The Question

  Spontaneous (dynamical) Fermion Mass Generation vs. N.
  (Chrial symmetry breaking)

- Condensed Matter Correspondence

  Antiferromagnetic (Neel) ordering (N = 2)
  But, it's compact QED3

POLYAKOV INSTANTONS?

THE ABELIAN HIGGS MODEL

$$L = -\frac{1}{4}(F_{\mu\nu})^2 + \sum_{j=1}^{N} \overline{\psi}_j (i\partial - eA)\psi_j$$

$$+\frac{1}{2} D_\mu \phi^* D^\mu \phi - \lambda(\phi^* \phi - Nv)^2$$

Remove Higgs

$\lambda \to \infty$ v fixed

- Dimensionful Coupling

$$\boxed{\alpha = e^2 N/8 \quad \text{(UV cutoff)}}$$

- Gauge Boson Mass

$$\boxed{M = e\sqrt{Nv} = \sqrt{8\alpha v}}$$

FERMIONS

- ψ_i = Four-Component Fermions

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

- Two further anticommuters

$$\gamma^3 = i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^5 = i\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- U(2) "chiral" symmetry for each spinor

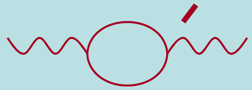
$$\mathbf{1},\ -i\gamma^3,\ \gamma^5,\ \frac{1}{2}\left[\gamma^3, \gamma^5\right] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\Rightarrow$ U(2N) chiral symmetry

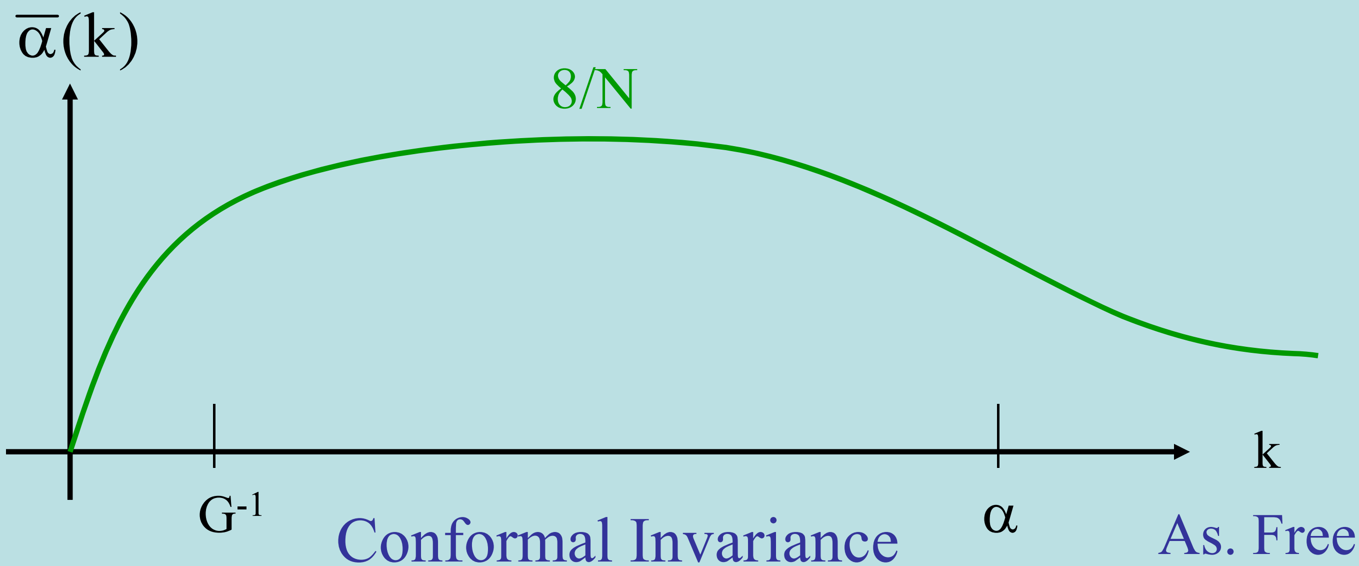
LARGE N

Dimensionless running coupling

$$\boxed{\bar{\alpha} \equiv \frac{e^2 k}{k^2 + M^2 + \alpha k} = \frac{8}{N}\,\frac{k}{k^2/\alpha + G^{-1} + k}}$$



$G = \alpha/M^2$



Finite N?

Dynamical Fermion Mass?

- Chiral Symmetry Breaking *(U(2N) → U(N) × U(N))*

$$m\,\bar{\psi}_j\psi_j = m\,\psi_j^{+}\begin{pmatrix}\sigma_3 & 0\\ 0 & -\sigma_3\end{pmatrix}\psi_j$$

Invariant under parity

$$P\,\psi_j\left(t,\vec{x}\right)P^{-1} = \begin{pmatrix}0 & \sigma_1\\ \sigma_1 & 0\end{pmatrix}\psi_j\left(t,\vec{x}_P\right)$$

- Alternate (Parity Violating) Mass

$$m\,\bar{\psi}_j\frac{\left[\gamma^3,\gamma^5\right]}{2}\psi_j = m\,\psi_j^{+}\begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix}\psi_j$$

U(2N) Invariant ⟷ Chern-Simons Term

VAFA-WITTEN: Not spontaneously

Consensus: Yes for $N \leq N_c(M/\alpha)$

QED3:

$M \to 0$, α fixed $(G^{-1} = M^2/\alpha \to 0)$: $N_c(0)$

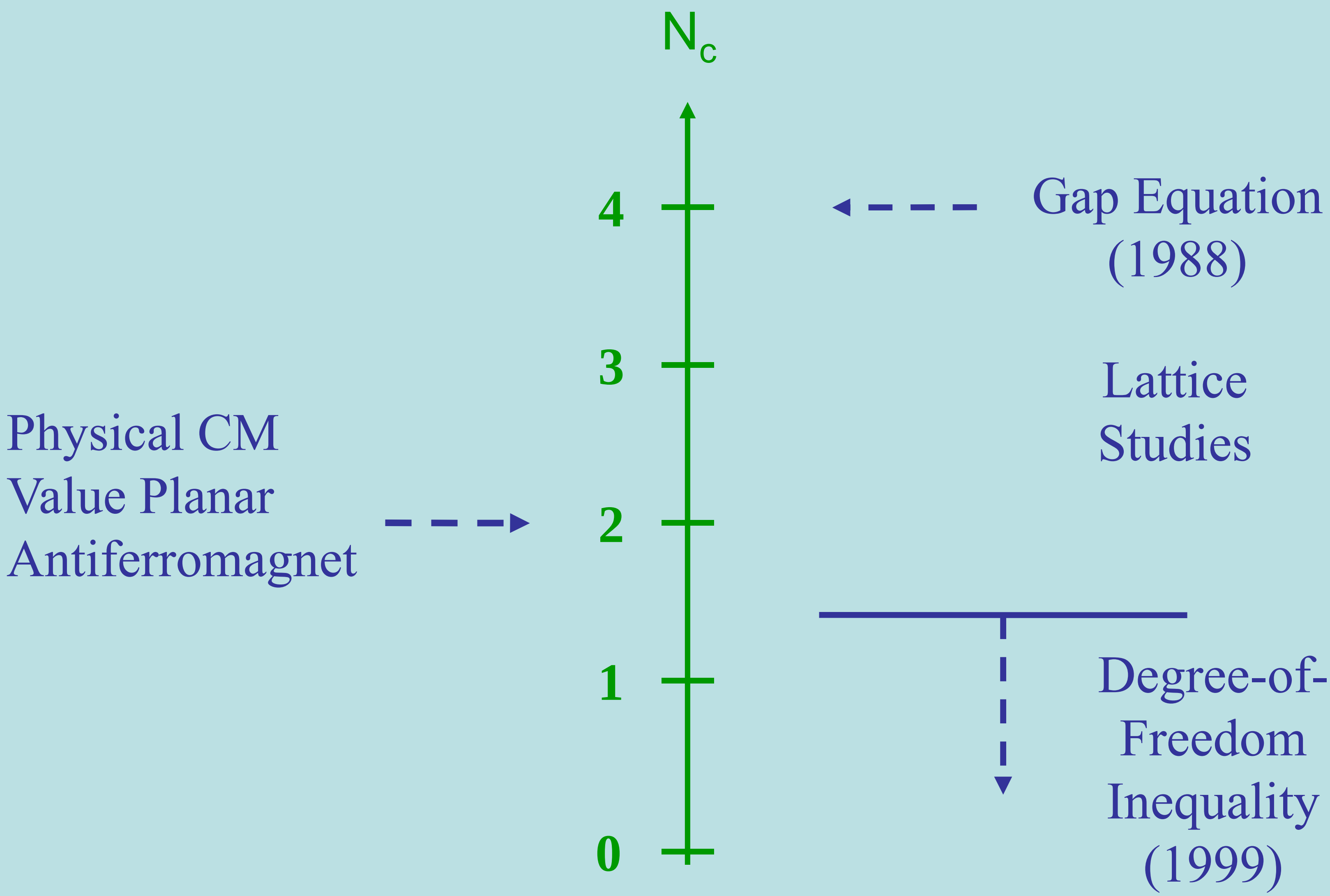
$N > N_c(0)$: IRFP

$N \lessapprox N_c(0)$: Approximate IRFP (walking)

Thirring Model:

$G^{-1} = M^2/\alpha$ fixed, $\alpha \to \infty$ $(M/\alpha \to 0)$: $N_c(0)$

QED3:



N_c
4
3
2
1
0
Gap Equation
(1988)
Lattice
Studies
Physical CM
Value Planar
Antiferromagnet
Degree-of-
Freedom
Inequality
(1999)

Degree-of-Freedom Inequality

Thermodynamic Free Energy

$$f(T) \equiv \frac{F(T)}{T^3}\,\Omega_{d=3} \qquad \left(\begin{array}{c} = 1 \text{ for a} \\ \text{free massless} \\ \text{boson} \end{array} \right)$$

Conjecture for AF Theories

$$\boxed{f(0) \leq f(\infty)}$$

CONCERN

Mermin-Wagner-Coleman Theorem

No NGB's in 2D
Finite-T 3D $\rightarrow$ 2D in IR

Rosenstein, Warr, Park (1990)

Symmetry unbroken at Finite-T

$$m_{NGB}^2 \sim T^2\, e^{-\alpha/T}$$

VANISHES MORE RAPIDLY THAN T AS T $\rightarrow$ 0

<u>NO</u> <u>EFFECT</u> <u>ON</u> <u>f(0)</u>

QED3 Ultraviolet

$$f(\infty) = 1 + \frac{3}{4}(4N)$$

QED3 Infrared (Broken Phase)

$$f(0) = 1 + 2N^2$$

(Broken Chiral Symmetry)

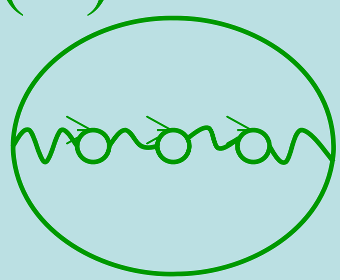
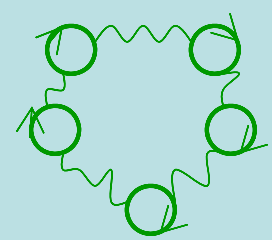
$$\Rightarrow \boxed{N_c \leq 3/2}$$

LARGE-N QED3

$$f(\infty) = \frac{3}{4}4N + 1$$

Weak
IRFP

$$f(0) \cong f(\infty)$$

$+$  $=$ 

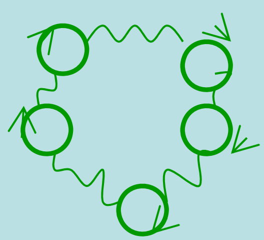
Expect Negative

LARGE-N THIRRING MODEL

$\alpha \to \infty$, FIXED $G^{-1} \equiv M^2/a$

$f(0) = 3N + 1$

Weak
UVFP

$f(\infty) = f(0) +$ 

Negative?   Then $f(0) > f(\infty)$

Recent Lattice work
Kogut and Strouthos

- **Chiral Symmetry breaking in Three Dimensional QED.**
  Costas Strouthos (Cyprus U.) , John B. Kogut (DOE, Wash., D.C. & Maryland U.) . Aug 2008.
  25th International Conference on Low Temperature Physics (LT25), Amsterdam, The Netherlands, 6-13 Aug 2008.
  J.Phys.Conf.Ser.150:052247,2009.
  arXiv:0808.2714
-

- **The Phases of Non-Compact QED(3).**
  Costas Strouthos (Lausanne) , John B. Kogut (DOE, Wash., D.C. & Maryland U.) . Apr 2008.
  Presented at 25th International Symposium on Lattice Field Theory, Regensburg, Germany, 30 Jul - 4 Aug 2007.
  PoS LAT2007:278,2007.
  arXiv:0804.0300