Progress on B-physics lattice calculations by ETMC

Petros Dimopoulos
University of Rome Tor Vergata

on behalf of ETM Collaboration

September 19, 2012

"New Frontiers in Lattice Gauge Theory"

Galileo Galilei Institute for Theoretical Physics

Outline

- ETMC computation
  - Method based on Ratios of heavy-light $(h, \ell/s)$ observables using relativistic quarks and exact knowledge of static limit for the appropriate ratios
  - Interpolation of $(h, \ell/s)$ observables to the b-region from the charm region and the static limit
- Application of Ratio method to b-quark mass, decay constants and Bag parameters.
- Summary

ETMC, JHEP **1201** (2012) 046; JHEP **1004** (2010) 049; N. Carrasco and A. Shindler @ LAT2012

ETMC-b : B. Blossier, N. Carrasco, P. Dimopoulos, R. Frezzotti, V. Gimenez, G. Herdoiza, K. Jansen, V. Lubicz, G. Martinelli, C. Michael, D. Palao, G. C. Rossi, F. Sanfilippo, A. Shindler, S. Simula, C. Tarantino, M. Wagner

ETMC – $\mathrm{N_f} = 2$ twisted-mass formulation

- Mtm lattice regularization of $N_f = 2$ QCD action is

[Frezzotti, Grassi, Sint, Weisz, JHEP 2001; Frezzotti, Rossi, JHEP 2004]

$$S^{\mathrm{ph}}_{N_f=2} = S^{\mathrm{YM}}_L + a^4 \sum_x \bar\psi(x) \left[\gamma\cdot\widetilde\nabla - i\gamma_5\tau^3 \left(-\frac{a}{2} r\nabla^*\nabla + M_{\mathrm{cr}}(r)\right) + \mu_q\right] \psi(x)$$

- ψ is a flavour doublet, $M_{\mathrm{cr}}(r)$ is the critical mass and τ^3 acts on flavour indices

- From the "physical" basis (where the quark mass is real), the non-anomalous

$$\psi = \exp(i\pi\gamma_5\tau^3/4)\chi\,, \quad \bar\psi = \bar\chi \exp(i\pi\gamma_5\tau^3/4)$$

transformation brings the lattice action in the so-called "twisted" basis

$$S^{\mathrm{tw}}_{N_f=2} = S^{\mathrm{YM}}_L + a^4 \sum_x \bar\chi(x) \left[\gamma\cdot\widetilde\nabla - \frac{a}{2} r\nabla^*\nabla + M_{\mathrm{cr}}(r) + i\mu_q\gamma_5\tau^3\right] \chi(x)$$

- Unlike the standard Wilson regularization, in Mtm Wilson case the subtracted Wilson operator $-\frac{a}{2} r\nabla^*\nabla + M_{\mathrm{cr}}(r)$ is "chirally rotated" w.r.t. the quark mass $\Longrightarrow$ offers important advantages...

ETMC – $N_f = 2$ twisted-mass formulation

- Automatic $O(a)$ improvement for the physical quantities
- Dirac-Wilson matrix determinant is positive

  and (lowest eigenvalue)^2 bounded from below by ${\mu_q}^2$

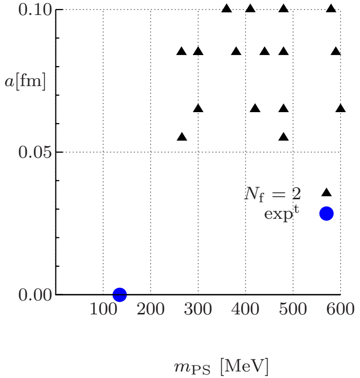
- Simplified (operator) renormalization ...

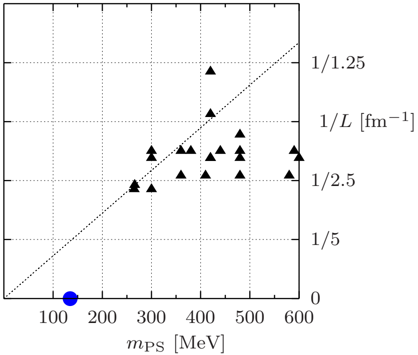
  - Multiplicative quark mass renormalization
  - No RC for pseudoscalar decay constant (PCAC)

- $O(a^2)$ breaking of parity and isospin

Frezzotti, Rossi, JHEP 2004;
ETMC, Phys.Lett.B 2007

ETMC $N_f = 2$ simulations





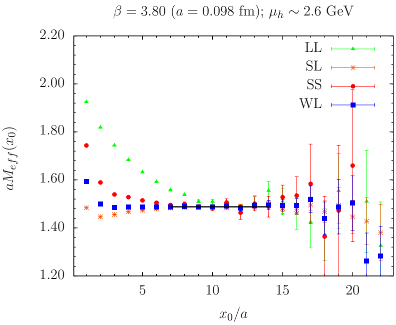
- $a \;=\; \{0.054,\; 0.067,\; 0.085,\; 0.098\}$ fm
- $m_{ps} \;\in\; \{270,\; 600\}$ MeV
- $L \;\in\; \{1.7,\;\; 2.8\}$ fm , $m_{ps}L \geq 3.5$

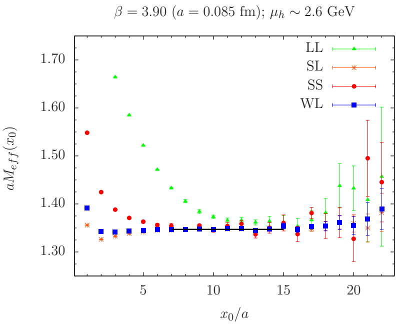
ETMC $N_f = 2$ simulations

β	$a\mu_\ell$	$a\mu_s$ (valence)	$a\mu_h$ (valence)
3.80	0.0080, 0.0110	0.0175, 0.0194, 0.0213	0.1982, . . ., 0.8536
3.90	0.0030, 0.0040, 0.0064, 0.0085, 0.0100	0.0159, 0.0177, 0.0195	0.1828, . . ., 0.7873
4.05	0.0030, 0.0060, 0.0080	0.0139, 0.0154, 0.0169	0.1572, . . ., 0.6771
4.20	0.0020, 0.0065	0.0116, 0.0129, 0.0142	0.13315, . . ., 0.4876

- $\mu_\ell \in [\sim m_s/6, \sim m_s/2]$
- $\mu_h \in [\sim m_c, \sim 3m_c]$

$\mathbf{M_{eff}^{(hl)}}$ plateau quality





- Smearing techniques improve signal; reduce the coupling between the ground and excired states; safe good plateaux at earlier times; absolutely necessary for obtaining safe plateau in the calculation of 3-point correlation functions (when large heavy quark mass ($>$ 1 GeV) are employed).
- Employ "optimal" source: $\Phi_W^{\rm source}(w) = w\Phi_S + (1-w)\Phi_L$;
  Check *vs.* GEVP – when two states matter – seems OK (in progress).

Ratio method

- we use correlators with relativistic quarks
- c-mass region computations are reliable ('small' discr. errors)
- construct HQET-inspired ratios of the observable of interest at successive (nearby) values of the heavy quark mass ($\mu_h^{(n)} = \lambda \mu_h^{(n-1)}$)
- ratios show smooth chiral and continuum limit behaviour
- ratios at the ∞-mass (static) point are exactly *known* ($= 1$)
- physical values of the observable at the b-mass point is related to its c-like value by a *chain* of the ratios ending up at the static point: use HQET-inspired interpolation

b-quark mass computation - 1

- observing that $\lim_{\mu_h^{\mathrm{pole}}\to\infty}\left(\frac{M_{h\ell}}{\mu_h^{\mathrm{pole}}}\right) = \text{constant}$ (HQET)
- construct (taking $\frac{\bar{\mu}_h^{(n)}}{\bar{\mu}_h^{(n-1)}} = \lambda$):

$$
\begin{aligned}
y(\bar{\mu}_h^{(n)}, \lambda; \bar{\mu}_\ell, a) \quad &\equiv \quad \frac{M_{h\ell}(\bar{\mu}_h^{(n)}; \bar{\mu}_\ell, a)}{M_{h\ell}(\bar{\mu}_h^{(n-1)}; \bar{\mu}_\ell, a)} \cdot \frac{\bar{\mu}_h^{(n-1)}}{\bar{\mu}_h^{(n)}} \cdot \frac{\rho(\bar{\mu}_h^{(n-1)}, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)} = \\
&= \quad \lambda^{-1} \frac{M_{h\ell}(\bar{\mu}_h^{(n)}; \bar{\mu}_\ell, a)}{M_{h\ell}(\bar{\mu}_h^{(n)}/\lambda; \bar{\mu}_\ell, a)} \cdot \frac{\rho(\bar{\mu}_h^{(n)}/\lambda, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)}\,, \qquad n = 2, \ldots, N
\end{aligned}
$$

$\mu_h^{\mathrm{pole}} = \rho(\bar{\mu}_h, \mu^*)\,\bar{\mu}_h(\mu^*)$ (with $\bar{\mu}_h \leftarrow \overline{\mathrm{MS}}$ scheme)

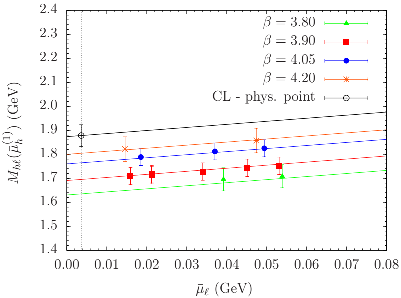
$\rho(\bar{\mu}_h, \mu^*)$ known up to N^{3}LO – relevant only for the '$1/\bar{\mu}_h$' interpolation

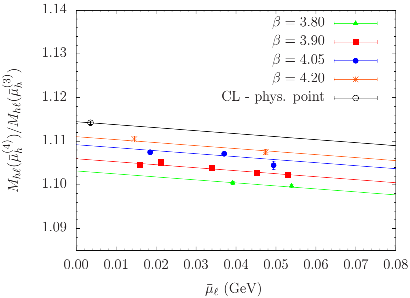
$\rightarrow$ In the static limit (and in CL) obviously:

$$\boxed{\lim_{\bar{\mu}_h\to\infty} y(\bar{\mu}_h, \lambda; \bar{\mu}_\ell, a = 0) = 1}$$

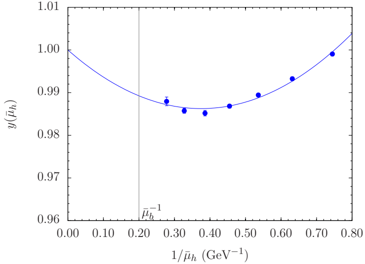
b-quark mass computation - 2

- *Triggering* input mass: $M_{h\ell}(\bar{\mu}_h^{(1)})$ PS meson mass (at $\bar{\mu}_h^{(1)} \sim m_c$) affected by (tolerably) small cutoff effects.
- Ratios $y(\bar{\mu}_h^{(n)}, \lambda; \bar{\mu}_\ell, a)$ have small discretisation errors





b-quark mass computation - 3



- fit ansatz $y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$ (inspired by HQET)
- Determine K (integer) such that $M_{hu/d}(\bar{\mu}_h^{(K+1)}) \equiv M_B^{expt}$:

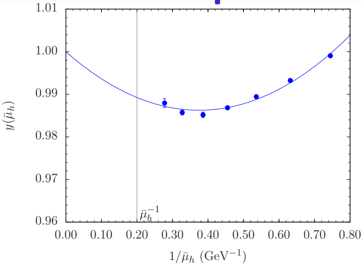
  $$y(\bar{\mu}_h^{(2)})\, y(\bar{\mu}_h^{(3)}) \ \ldots \ y(\bar{\mu}_h^{(K+1)}) = \lambda^{-K} \frac{M_{hu/d}(\bar{\mu}_h^{(K+1)})}{M_{hu/d}(\bar{\mu}_h^{(1)})} \cdot \left[\frac{\rho(\bar{\mu}_h^{(1)}, \mu^*)}{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)} \right]$$

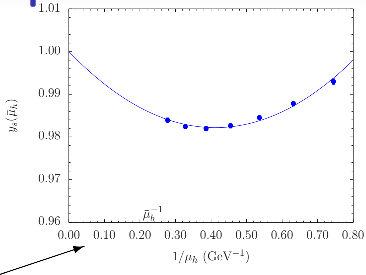
  (strong cancellations of perturbative factors in the ratios)
- One adjusts $(\lambda, \bar{\mu}_h^{(1)})$ such that K integer
- our calculation: $\lambda = 1.1784$ and $\bar{\mu}_h^{(1)} = 1.14$ GeV (in $\overline{\mathrm{MS}}$, 2 GeV)

$\rightarrow$ $\bar{\mu}_b = \lambda^K \bar{\mu}_h^{(1)}$ $(K = 9)$

- y deviates from its static value $\sim 1\%$ for $\bar{\mu_h} \leq m_b$.
- curvature denotes a large $1/\bar{\mu_h}^2$ contribution to ratios y.

b-quark mass computation - 4





(similar results if M_{hs} data is used as input)

- $y(\bar\mu_h) = 1 + \frac{\eta_1}{\bar\mu_h} + \frac{\eta_2}{\bar\mu_h^2}$
- Determine K (integer) such that $M_{hu/d}(\bar\mu_h^{(K+1)}) \equiv M_B^{expt}$:

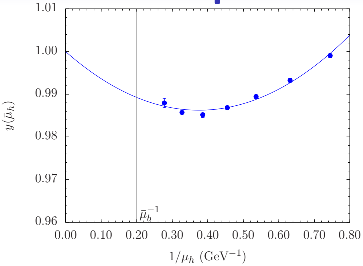
  $$y(\bar\mu_h^{(2)})\, y(\bar\mu_h^{(3)}) \ \ldots \ y(\bar\mu_h^{(K+1)}) = \lambda^{-K} \frac{M_{hu/d}(\bar\mu_h^{(K+1)})}{M_{hu/d}(\bar\mu_h^{(1)})} \cdot \left[\frac{\rho(\bar\mu_h^{(1)}, \mu^*)}{\rho(\bar\mu_h^{(K+1)}, \mu^*)}\right]$$

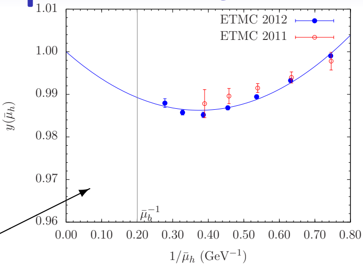
  (strong cancellations of perturbative factors in the ratios)
- One adjusts $(\lambda, \bar\mu_h^{(1)})$ such that K integer
- our calculation: $\lambda = 1.1784$ and $\bar\mu_h^{(1)} = 1.14$ GeV (in $\overline{\mathrm{MS}}$, 2 GeV)

$\rightarrow$ $\bar\mu_b = \lambda^K \bar\mu_h^{(1)}$ $(K = 9)$

- y deviates from its static value $\sim 1\%$ for $\bar\mu_h \leq m_b$.
- curvature denotes a large $1/\bar\mu_h{}^2$ contribution to ratios y.

b-quark mass computation - 5





- $y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$ (Comparison with less precise data from ETMC-2011 paper)
- Determine K (integer) such that $M_{hu/d}(\bar{\mu}_h^{(K+1)}) \equiv M_B^{expt}$:

  $$y(\bar{\mu}_h^{(2)})\, y(\bar{\mu}_h^{(3)}) \, \ldots \, y(\bar{\mu}_h^{(K+1)}) = \lambda^{-K} \frac{M_{hu/d}(\bar{\mu}_h^{(K+1)})}{M_{hu/d}(\bar{\mu}_h^{(1)})} \cdot \left[\frac{\rho(\bar{\mu}_h^{(1)}, \mu^*)}{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)}\right]$$

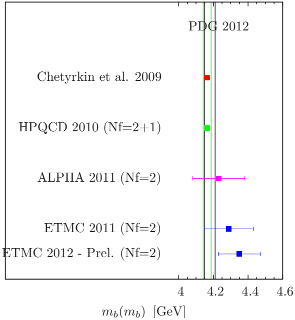
  (strong cancellations of perturbative factors in the ratios)
- One adjusts $(\lambda, \bar{\mu}_h^{(1)})$ such that K integer
- our calculation: $\lambda = 1.1784$ and $\bar{\mu}_h^{(1)} = 1.14$ GeV (in $\overline{\mathrm{MS}}$, 2 GeV)

$\rightarrow$ $\bar{\mu}_b = \lambda^K \bar{\mu}_h^{(1)}$ $(K = 9)$

- y deviates from its static value $\sim 1\%$ for $\bar{\mu}_h \leq m_b$.
- curvature denotes a large $1/\bar{\mu}_h{}^2$ contribution to ratios y.

b-quark mass - results

- $m_b(m_b, \overline{\mathrm{MS}})|_{N_f=2} = 4.35(12)$ GeV (*PRELIMINARY!*)
- compatible result for m_b *when* (*hs*)-data and M_{Bs}^{expt} as input are used



- Main source of uncertainty of the ETMC result is due to quark mass RC and scale setting uncertainties; stat & fit errors very small.

f_B and f_{Bs}

- HQET behaviour $\lim_{\mu_h^{\rm pole}\to\infty} f_{h\ell(s)}\sqrt{\mu_h^{\rm pole}} = \text{constant}$
- construct

$$z_\ell(\bar\mu_h,\lambda;\bar\mu_\ell,a) \equiv \lambda^{1/2}\frac{f_{h\ell}(\bar\mu_h,\bar\mu_\ell,a)}{f_{h\ell}(\bar\mu_h/\lambda,\bar\mu_\ell,a)}\cdot\frac{C_A^{stat}(\mu^*,\bar\mu_h/\lambda)}{C_A^{stat}(\mu^*,\bar\mu_h)}\frac{[\rho(\bar\mu_h,\mu^*)]^{1/2}}{[\rho(\bar\mu_h/\lambda,\mu^*)]^{1/2}}$$

$$z_s(\bar\mu_h,\lambda;\bar\mu_\ell,\bar\mu_s,a) \equiv \lambda^{1/2}\frac{f_{hs}(\bar\mu_h,\bar\mu_\ell,\bar\mu_s,a)}{f_{hs}(\bar\mu_h/\lambda,\bar\mu_\ell,\bar\mu_s,a)}\cdot\frac{C_A^{stat}(\mu_b^*,\bar\mu_h/\lambda)}{C_A^{stat}(\mu_b^*,\bar\mu_h)}\frac{[\rho(\bar\mu_h,\mu^*)]^{1/2}}{[\rho(\bar\mu_h/\lambda,\mu^*)]^{1/2}}$$

$C_A^{stat}(\mu^*,\bar\mu_h)$ known up to N^{2}LO – relevant for the interpolation in '$1/\bar\mu_h$' fit

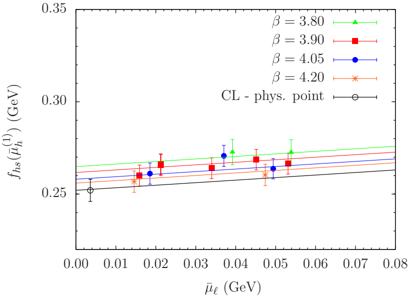
$\rightarrow$ In the static limit (and in CL) obviously:

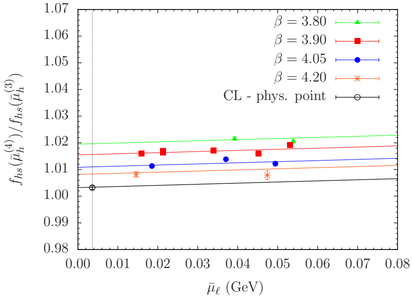
$$\lim_{\bar\mu_h\to\infty} z_{\ell/s}(\bar\mu_h,\lambda;\bar\mu_\ell,a=0)=1$$

$$\lim_{\bar\mu_h\to\infty}\frac{z_s(\bar\mu_h,\lambda;\bar\mu_\ell,a=0)}{z_\ell(\bar\mu_h,\lambda;\bar\mu_\ell,a=0)}=1$$

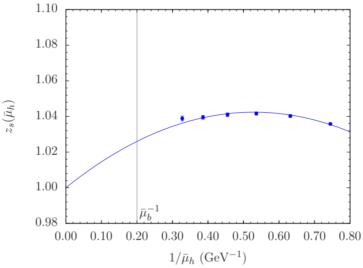
f_{Bs}

- Ratios $z_s(\bar{\mu}_h^{(n)}, \lambda; \bar{\mu}_\ell, a)$ have small discretisation effects (from $\sim 1\%$ for the smallest to $\sim 3\%$ for the largest heavy quark masses)
- At *Triggering point* $f_{hs}(\bar{\mu}_h^{(1)})$ pseudoscalar decay constant (with $\bar{\mu}_h^{(1)} \sim m_c$) is affected by (tolerably) small cutoff effects.





f_{Bs}

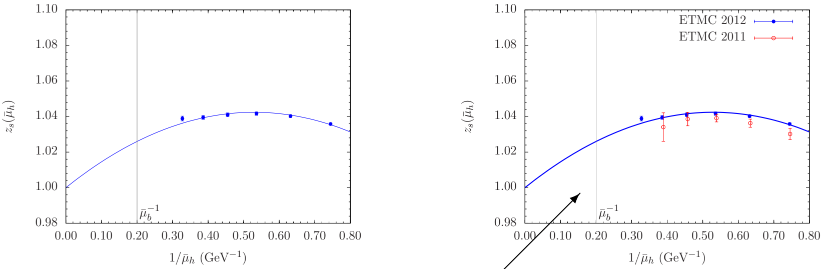


- fit ansatz $z_s(\bar{\mu}_h) = 1 + \frac{\zeta_1}{\bar{\mu}_h} + \frac{\zeta_2}{\bar{\mu}_h^2}$
- use $z_s(\bar{\mu}_h^{(2)})\, z_s(\bar{\mu}_h^{(3)}) \, \ldots \, z_s(\bar{\mu}_h^{(K+1)}) =$
  $\lambda^{K/2} \frac{f_{hs}(\bar{\mu}_h^{(K+1)})}{f_{hs}(\bar{\mu}_h^{(1)})} \cdot \left[\frac{C_A^{stat}(\bar{\mu}_h^{(1)}, \mu^*)}{C_A^{stat}(\bar{\mu}_h^{(K+1)}, \mu^*)}\right] \left[\frac{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)}{\rho(\bar{\mu}_h^{(1)}, \mu^*)}\right]^{1/2}$
- for $\bar{\mu}_h^{(K+1)} = \lambda^K \bar{\mu}_h^{(1)} = m_b$, identify $f_{hs}(\bar{\mu}_h^{(K+1)})$ with f_{Bs}

$\rightarrow$ $f_{Bs} = 234(6)$ MeV *(PRELIMINARY)*

(principal uncertainty from scale setting; other errors (stat+fit) < 1 %)

f_{Bs}



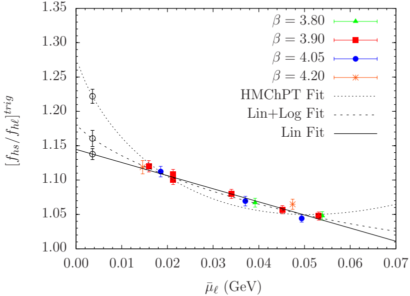
(comparison with ETMC-2011 results)

- fit ansatz $z_s(\bar{\mu}_h) = 1 + \frac{\zeta_1}{\bar{\mu}_h} + \frac{\zeta_2}{\bar{\mu}_h^2}$
- use $z_s(\bar{\mu}_h^{(2)})\, z_s(\bar{\mu}_h^{(3)}) \ldots z_s(\bar{\mu}_h^{(K+1)}) =$
  $$\lambda^{K/2} \frac{f_{hs}(\bar{\mu}_h^{(K+1)})}{f_{hs}(\bar{\mu}_h^{(1)})} \cdot \left[\frac{C_A^{stat}(\bar{\mu}_h^{(1)}, \mu^*)}{C_A^{stat}(\bar{\mu}_h^{(K+1)}, \mu^*)}\right] \left[\frac{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)}{\rho(\bar{\mu}_h^{(1)}, \mu^*)}\right]^{1/2}$$
- for $\bar{\mu}_h^{(K+1)} = \lambda^K \bar{\mu}_h^{(1)} = m_b$, identify $f_{hs}(\bar{\mu}_h^{(K+1)})$ with f_{Bs}

$\rightarrow$ $f_{Bs} = 234(6)$ MeV *(PRELIMINARY)*

(principal uncertainty from scale setting; other errors (stat+fit) < 1 %)

f_{Bs}/f_B @ triggering point

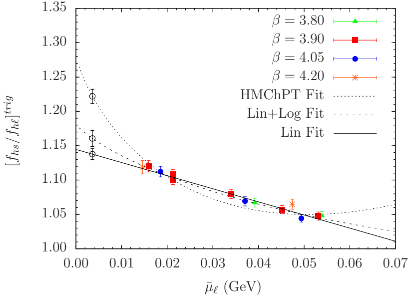


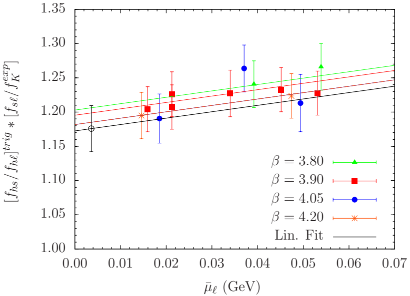
- $f_{hs}/f_{h\ell}$ @ triggering point ($\sim m_c$)

  Linear fit, Linear + Log and HMChPT *vs.* light quark mass

  $\rightarrow$ increase systematic uncertainty: $f_{hs}/f_{hu/d} = 1.18(1)_{stat}(4)_{syst}$

f_{Bs}/f_B @ triggering point





- $f_{hs}/f_{h\ell}$ @ triggering point ($\sim m_c$)

  Linear fit, Linear + Log and HMChPT *vs.* light quark mass → increase systematic uncertainty: $f_{hs}/f_{hu/d} = 1.18(1)_{stat}(4)_{syst}$

- Try to smooth out the light quark dependence in the $f_{hs}/f_{h\ell}$ ratio:

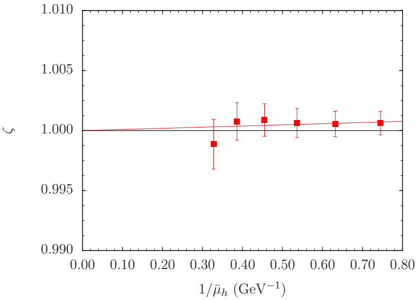
  fit $(f_{hs}/f_{h\ell})^{trig} * (f_{s\ell}/f_K^{exp})$ vs. $\bar{\mu}_\ell$ using linear fit ansatz (consistent with SU(2) ChPT; A. Roessl NPB 1999.)

  → datapoints with larger error due to the scale uncertainty necessary for converting $f_{s\ell}$ in phys. units: $f_{hs}/f_{hu/d} = 1.18(3)$

f_{Bs}/f_B

- double ratio $\zeta = z_s/z_\ell$ shows no significant μ_ℓ dependence at successive values of the heavy quark mass up to 3 GeV and small discr. effects

- double ratio $\zeta = z_s/z_\ell$ *vs.* $(1/\bar{\mu}_h)$: very weak dependence

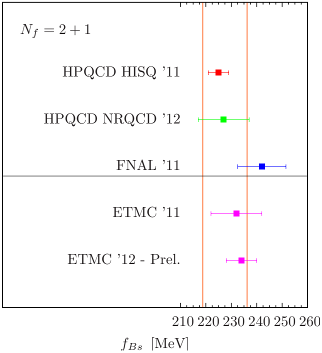


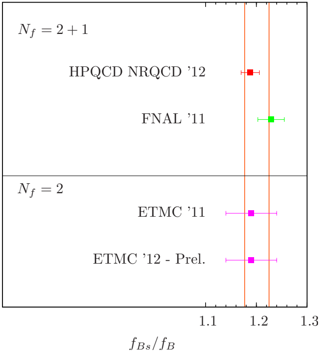
- apply similar method for z_s/z_ℓ to reach the b-quark mass point ...

$\rightarrow$ $f_{Bs}/f_B = 1.19(05)$ *(PRELIMINARY!)*

(principal source of uncertainty due to systematics in the fit ansatz @ triggering mass point)

Results & Comparisons - I





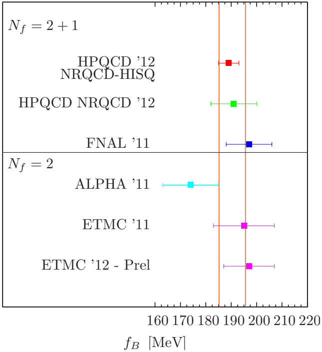
- $f_{Bs}(\mathrm{ETMC}-2012) = 234(06)$ MeV *(PRELIMINARY!)*
- $f_{Bs}/f_B(\mathrm{ETMC}-2012) = 1.19(05)$ MeV *(PRELIMINARY!)*

  ⋆ vertical lines show average over $N_f = 2+1$ results ...

  no significant dependence on dynamical strange degree of freedom

  (within the present precision)

Results & Comparisons - II



- $f_B(\mathrm{ETMC} - 2012) = 197(10)$ MeV

  (PRELIMINARY!)

  $(f_B = f_{Bs}/(f_{Bs}/f_B))$

Ratio method for the $\Delta B = 2$ operators

- QCD

$$\begin{aligned}
O_1 &= [\bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha][\bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta] \\
O_2 &= [\bar{b}^\alpha (1-\gamma_5) q^\alpha][\bar{b}^\beta (1-\gamma_5) q^\beta] \\
O_3 &= [\bar{b}^\alpha (1-\gamma_5) q^\beta][\bar{b}^\beta (1-\gamma_5) q^\alpha] \\
O_4 &= [\bar{b}^\alpha (1-\gamma_5) q^\alpha][\bar{b}^\beta (1+\gamma_5) q^\beta] \\
O_5 &= [\bar{b}^\alpha (1-\gamma_5) q^\beta][\bar{b}^\beta (1+\gamma_5) q^\alpha]
\end{aligned}$$

- HQET

$$\begin{aligned}
\tilde{O}_1 &= [\bar{h}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha][\bar{h}^\beta \gamma_\mu (1-\gamma_5) q^\beta] \\
\tilde{O}_2 &= [\bar{h}^\alpha (1-\gamma_5) q^\alpha][\bar{h}^\beta (1-\gamma_5) q^\beta] \\
\tilde{O}_3 &= -\tilde{O}_2 - (1/2)\tilde{O}_1 \\
\tilde{O}_4 &= [\bar{h}^\alpha (1-\gamma_5) q^\alpha][\bar{h}^\beta (1+\gamma_5) q^\beta] \\
\tilde{O}_5 &= [\bar{h}^\alpha (1-\gamma_5) q^\beta][\bar{h}^\beta (1+\gamma_5) q^\alpha]
\end{aligned}$$

⋆ Matching between QCD and HQET operators:

$$[\mathbf{W}^T_{QCD}(\mu_h, \mu)]^{-1} \langle \vec{O}(\mu) \rangle_{\mu_h} = C(\mu_h)\, [\mathbf{W}^T_{HQET}(\mu_h, \tilde{\mu})]^{-1} \langle \vec{\tilde{O}}(\tilde{\mu}) \rangle + \mathcal{O}(1/\mu_h) + \ldots$$

$[\mathbf{W}^T_{\ldots}(\mu_1, \mu_2)]^{-1}$: evolution 5x5 matrices

$C(\mu_h)$: matching matrix

(e.g. D.Becirevic, V.Gimenez, G.Martinelli, M.Papinutto, J.Reyes, JHEP 2002)

Calculation of $B_{Bd/s}$

- B-bag parameters encode the non-perturbative QCD contribution to the $B_{d/s} - \bar{B}_{d/s}$ mixing amplitude: $\Delta M_q \propto |V_{tq}^* V_{tb}|^2 M_{B_q} f_{B_q}^2 \hat{B}_{B_q}$
- ETMC Calculation: [Frezzotti and Rossi, JHEP 2004] use mixed action; Osterwalder-Seiler valence quarks; suitable combinations of maximally twisted valence quarks ensure both

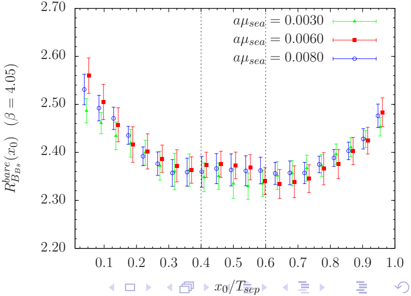
$\rightarrow$ continuum-like renormalisation pattern for the 4-fermion operators

$\rightarrow$ automatic O(a)-improvement.

(application to the K-sector: ETMC, Phys.Rev.D 2011; ETMC, 1207.1287)

(see talk by R. Frezzotti)

- $R_{B_B} = \dfrac{C_{PO_1P}(x_0)}{8/3 C_{PA}(x_0) C_{AP}(x_0)} \rightarrow B_B$
- We present results at three values of lattice spacing $a \in [0.1, 0.067]$ fm; a finer lattice spacing $a = 0.054$ fm in progress.



Ratio method for the $\Delta B = 2$ operators

- set:

  $\vec{\Theta}(\mu_h, \mu, \tilde{\mu}) \equiv (\mathbf{W}^T_{QCD}(\mu_h, \mu) C(\mu_h)[\mathbf{W}^T_{HQET}(\mu_h, \tilde{\mu})]^{-1})^{-1} \langle \vec{O}(\mu) \rangle_{\mu_h} \equiv$

  $[C_B(\mu_h, \mu, \tilde{\mu})]^{-1} \langle \vec{O}(\mu) \rangle_{\mu_h} = \langle \vec{\tilde{O}}(\tilde{\mu}) \rangle + \mathcal{O}(1/\mu_h) + \ldots$

- $[\mathbf{W}^T_{\ldots}(\mu_1, \mu_2)]^{-1}$ and $C(\mu_h)$ are $(3 \times 3 \oplus 2 \times 2)$ block-diagonal matrices

- For B_{Bq} case, calculate ratios at successive values of $\mu_h^{(n)} = \lambda \mu_h^{(n-1)}$

  (need only 3×3 matrices):

  $w_\Theta^{(n)} = \dfrac{\Theta_j(\mu_h^{(n)}, \mu, \tilde{\mu})}{\Theta_j(\mu_h^{(n-1)}, \mu, \tilde{\mu})}$ for $j = 1, 2, 3$

  and construct the appropriate ratio chain.

- up to LL order O_1 and $\tilde{O}_1$ renormalise multipicatively; need only $j = 1$

Ratio method for B_{B_s}, B_{B_d} and their ratio

- HQET predicts: $\lim_{\mu_h^{\rm pole}\to\infty} B_{B_{d/s}} = \text{constant}$
- $\lim_{\bar\mu_h\to\infty} w_{B_{d/s}}(\bar\mu_h, \lambda; \bar\mu_\ell, a=0) = 1$
- form the *chain* for B_{B_s} and B_{B_d}:

$$w_{B_{d/s}}(\bar\mu_h^{(2)})\, w_{B_{d/s}}(\bar\mu_h^{(3)})\,\ldots\, w_{B_{d/s}}(\bar\mu_h^{(K+1)}) = \frac{B_{B_{d/s}}(\bar\mu_h^{(K+1)},\mu,\tilde\mu)}{B_{B_{d/s}}(\bar\mu_h^{(1)},\mu,\tilde\mu)} \cdot \left[\frac{C_B(\bar\mu_h^{(1)},\mu,\tilde\mu)}{C_B(\bar\mu_h^{(K+1)},\mu,\tilde\mu)}\right]$$

- work in a similar way; form the double ratios for B_{B_s}/B_{B_d}:

$$\zeta_w(\bar\mu_h^{(2)})\,\zeta_w(\bar\mu_h^{(3)})\,\ldots\,\zeta_w(\bar\mu_h^{(K+1)}) = \frac{w_{B_s}(\bar\mu_h^{(K+1)})\,w_{B_d}^{-1}(\bar\mu_h^{(K+1)})}{w_{B_s}(\bar\mu_h^{(1)})\,w_{B_d}^{-1}(\bar\mu_h^{(1)})}$$

- for $\bar\mu_h^{(K+1)} = \lambda^K \bar\mu_h^{(1)} = m_b$ get the physical values for the Bag parameters and of their ratio

B_{Bs}/B_{Bd}

- compute the double ratio

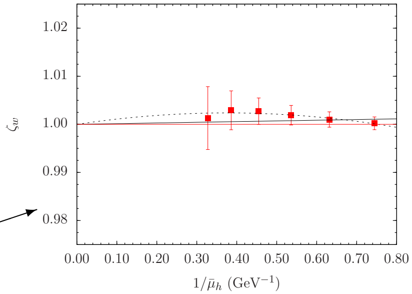
  $\zeta_w = w_{B_s}/w_{B_d}(\bar{\mu}_h^{(n)}, \bar{\mu}_\ell)$

  for values of $\bar{\mu}_\ell$ and extrapolate to

  CL @ u/d at each $\bar{\mu}_h^{(n)}$

- $\zeta_w(\bar{\mu}_h^{(n)}, \mu_\ell)$ values show no significant μ_ℓ dependence and cutoff effects are small

- very weak dependence on heavy quark mass



- fit ansatze

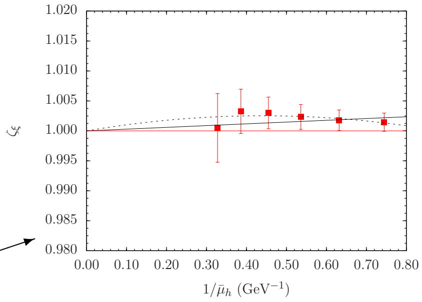
  $\zeta_w(\bar{\mu}_h) = 1 + c_1/\bar{\mu}_h \; (+ c_2/\bar{\mu}_h^2)$

  $\rightarrow$ $B_{B_s}/B_{B_d} = 1.03(2)$ (at $\bar{\mu}_h^{(K+1)} = \lambda^K \bar{\mu}_h^{(1)} = m_b$) (*PRELIMINARY!*)

  - **For Comparison**
    $B_{B_s}/B_{B_d}(\mathrm{FNAL/MILC} - 2012) = 1.06(11)$;
    $B_{B_s}/B_{B_d}(\mathrm{HPQCD} - 2009) = 1.05(07)$

ξ

- $\left|\frac{V_{td}}{V_{ts}}\right| = \xi\left(\frac{\Delta M_d M_{Bs}}{\Delta M_s M_{Bd}}\right)^{1/2}$
- $\xi = (f_{Bs}/f_{Bd})\sqrt{B_{B_s}/B_{B_d}}$
- form the ratio $\zeta_\xi(\bar{\mu}_h^{(n)}, \bar{\mu}_\ell)$ at successive values of $\bar{\mu}_h^{(n)} = \lambda\bar{\mu}_h^{(n-1)}$; determine CL @ u/d at each $\bar{\mu}_h^{(n)}$
- $\zeta_\xi(\bar{\mu}_h^{(n)}, \mu_\ell)$ show no significant μ_ℓ dependence and small cutoff effects
- ζ_ξ *vs.* heavy quark mass $(1/\bar{\mu}_h)$: very weak dependence



- fit ansatze $\zeta_\xi(\bar{\mu}_h) = 1 + c_1'/\bar{\mu}_h \; (+\, c_2'/\bar{\mu}_h^2)$

  $\rightarrow$ $\xi = 1.21(06)$ (at $\bar{\mu}_h^{(K+1)} = \lambda^K \bar{\mu}_h^{(1)} = m_b$) *(PRELIMINARY!)*

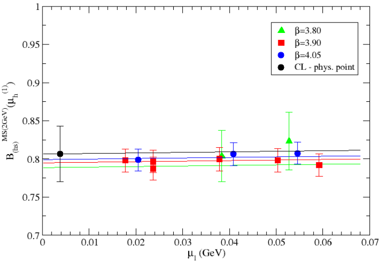
  (syst. error $\sim 4\%$ due to fit ansatze @ triggering mass point)

  - For comparison:

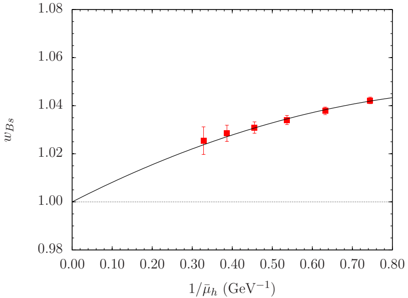
    $\xi(\mathrm{FNAL/MILC}-2012) = 1.268(63)$; $\xi(\mathrm{HPQCD}-2009) = 1.258(33)$

B_{Bs}

- smooth behaviour in the CL @ triggering point value ($\sim m_c$)



- ratios $w_{B_s}(\bar{\mu}_h^{(n)}, \bar{\mu}_\ell)$ show smooth behaviour with $\bar{\mu}_\ell$ and small discr. effects
- fit ansatz $w_{B_s}(\bar{\mu}_h) = 1 + w_1/\bar{\mu}_h \; + \; w_2/\bar{\mu}_h^2$
- *Similar analysis for B_{B_d}*



Results for B-Bag Parameters

- $B_{B_s}/B_{B_d} = 1.03(2)$
- $\xi = 1.21(06)$
- $B_{B_d}(m_b, \overline{\mathrm{MS}}) = 0.87(05)$
- $B_{B_s}(m_b, \overline{\mathrm{MS}}) = 0.90(05)$

(PRELIMINARY!)

Summary

- Ratio method uses relativistic quarks and an *obvious* value of the static limit. No static calculation needed.
- Ratio method can be used for all observables whose static limit behaviour is known from HQET.
- ETMC ($N_f = 2$) results for

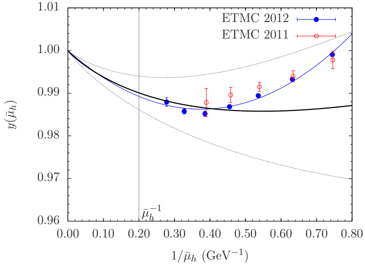
  m_b, f_{Bs}, f_B, f_{Bs}/f_B, B_{Bs}, B_{Bd}, B_{Bs}/B_{Bd} and ξ

  are in the same ballpark of results from other collaborations.
- Repeat/extend the study to $N_f = 2 + 1 + 1$ ensembles; work in progress; results available very soon using three lattice spacings $a \in [0.09, 0.06]$ fm.

Thank you for your attention !

backup slides

b-quark mass ratios - phenom. indications



$$y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$$

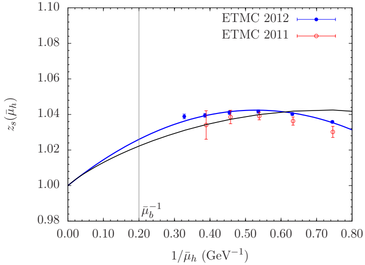
- consider (HQET) $M_{h\ell} = \mu_h^{\rm pole} + \bar{\Lambda} - \frac{(\lambda_1+3\lambda_2)}{2}\frac{1}{\mu_h^{\rm pole}} + \mathcal{O}\left(\frac{1}{(\mu_h^{\rm pole})^2}\right)$
- and get $y = 1 - \bar{\Lambda}\frac{\lambda^{\rm pole}-1}{\mu_h^{\rm pole}} + \left(\frac{(\lambda_1+3\lambda_2)}{2}(\lambda^{\rm pole}+1) + \bar{\Lambda}^2\lambda^{\rm pole}\right)\frac{\lambda^{\rm pole}-1}{(\mu_h^{\rm pole})^2}$

  with $\lambda^{\rm pole} = \mu_h^{\rm pole}(\bar{\mu}_h)/\ \mu_h^{\rm pole}(\bar{\mu}_h/\lambda) = \lambda\,\rho(\bar{\mu}_h)/\rho(\bar{\mu}_h/\lambda)$
- use phenomenological estimates for HQET parameters, as e.g.

  $\bar{\Lambda} = 0.39(11)\ \mathrm{GeV}\,, \quad \lambda_1 = -0.19(10)\ \mathrm{GeV}^2\,, \quad \lambda_2 = 0.12(2)\ \mathrm{GeV}^2$

  [M. Gremm, A. Kapustin, Z. Ligeti, M.B. Wise, PhysRevLett 1996]

Ratios for f_{B_s} - Phenomenological Indications



$$z_s(\bar{\mu}_h) = 1 + \frac{\zeta_1}{\bar{\mu}_h} + \frac{\zeta_2}{\bar{\mu}_h^2}$$

(for $1/\bar{\mu}_h > 0.60$ estimated uncertainty on the black curve ~ 0.03)

- consider (HQET)

$$\Phi_{hs}(\bar{\mu}_h, \mu_b^*) = \frac{(f_{hs}\sqrt{M_{hs}})^{\mathrm{QCD}}}{C_A^{stat}(\bar{\mu}_h, \mu_b^*)} = \Phi_0(\mu_b^*)\left(1 + \frac{\Phi_1(\mu_b^*)}{\mu_h^{\mathrm{pole}}} + \frac{\Phi_2(\mu_b^*)}{(\mu_h^{\mathrm{pole}})^2}\right) + \mathcal{O}\left(\frac{1}{(\mu_h^{\mathrm{pole}})^3}\right)$$

- and get

$$y_s^{1/2}\, z_s = \frac{\Phi_{hs}(\bar{\mu}_h)}{\Phi_{hs}(\bar{\mu}_h/\lambda)} = 1 - \Phi_1 \frac{\lambda^{\mathrm{pole}} - 1}{\mu_h^{\mathrm{pole}}} - \left(\Phi_2(\lambda^{\mathrm{pole}} + 1) - \Phi_1^2 \lambda^{\mathrm{pole}}\right) \frac{\lambda^{\mathrm{pole}} - 1}{(\mu_h^{\mathrm{pole}})^2}$$

- use phenomenological values for HQET parameters
  $\bar{\Lambda}_s = \bar{\Lambda} + M_{B_s} - M_B$, $\lambda_{1s} = \lambda_1$, $\lambda_{2s} = \lambda_2$, $\Phi_0 = 0.60\ \mathrm{GeV}^{3/2}$ and the estimates
  $\Phi_1 = -0.48\ \mathrm{GeV}$, $\Phi_2 = 0.08\ \mathrm{GeV}^2$ ($\rightarrow$ values obtained from inputs at B_s and D_s.)