Kaon mixing beyond the SM from $N_f = 2$ tmQCD

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K^0–$\bar{K}^0$ oscillations and constraints on new physics (NP)

Flavour physics processes vanishing at tree level in the SM (possibly also CKM- or chirality-suppressed) are a key tool to search for NP virtual particle effects. FCNC $\Delta F = 2$ transitions provided most stringent constraints on NP (e.g. technicolor) models.

Here: parameters describing K^0–$\bar{K}^0$ mixing in the framework of

$$\mathcal{H}_{\rm eff}^{\Delta S=2} = \sum_{i=1}^{5} c_i(\Lambda/\mu)\mathcal{O}_i(x\mu) + \sum_{i=1}^{3} \tilde{c}_i(\Lambda/\mu)\tilde{\mathcal{O}}_i(x\mu)\,,$$

$$\begin{aligned}
\mathcal{O}_1 &= [\bar{s}^\alpha\gamma_\mu(1-\gamma_5)d^\alpha][\bar{s}^\beta\gamma_\mu(1-\gamma_5)d^\beta]\\
\mathcal{O}_2 &= [\bar{s}^\alpha(1-\gamma_5)d^\alpha][\bar{s}^\beta(1-\gamma_5)d^\beta]\\
\mathcal{O}_3 &= [\bar{s}^\alpha(1-\gamma_5)d^\beta][\bar{s}^\beta(1-\gamma_5)d^\alpha]\\
\mathcal{O}_4 &= [\bar{s}^\alpha(1-\gamma_5)d^\alpha][\bar{s}^\beta(1+\gamma_5)d^\beta]\\
\mathcal{O}_5 &= [\bar{s}^\alpha(1-\gamma_5)d^\beta][\bar{s}^\beta(1+\gamma_5)d^\alpha]\\
\tilde{\mathcal{O}}_1 &= [\bar{s}^\alpha\gamma_\mu(1+\gamma_5)d^\alpha][\bar{s}^\beta\gamma_\mu(1+\gamma_5)d^\beta]\\
\tilde{\mathcal{O}}_2 &= [\bar{s}^\alpha(1+\gamma_5)d^\alpha][\bar{s}^\beta(1+\gamma_5)d^\beta]\\
\tilde{\mathcal{O}}_3 &= [\bar{s}^\alpha(1+\gamma_5)d^\beta][\bar{s}^\beta(1+\gamma_5)d^\alpha]
\end{aligned}$$

Bag parameters and ratios thereof relevant for K^0–$\bar{K}^0$ oscillations

Only the parity-even part of $\mathcal{O}_i$ ($\tilde{\mathcal{O}}_i$), $i = 1, 2, 3, 4, 5,$ matters i.e.

$$
\begin{aligned}
O_1 &= [\bar{s}^\alpha \gamma_\mu d^\alpha][\bar{s}^\beta \gamma_\mu d^\beta] + [\bar{s}^\alpha \gamma_\mu \gamma_5 d^\alpha][\bar{s}^\beta \gamma_\mu \gamma_5 d^\beta] \\
O_2 &= [\bar{s}^\alpha d^\alpha][\bar{s}^\beta d^\beta] + [\bar{s}^\alpha \gamma_5 d^\alpha][\bar{s}^\beta \gamma_5 d^\beta] \\
O_3 &= [\bar{s}^\alpha d^\beta][\bar{s}^\beta d^\alpha] + [\bar{s}^\alpha \gamma_5 d^\beta][\bar{s}^\beta \gamma_5 d^\alpha] \\
O_4 &= [\bar{s}^\alpha d^\alpha][\bar{s}^\beta d^\beta] - [\bar{s}^\alpha \gamma_5 d^\alpha][\bar{s}^\beta \gamma_5 d^\beta] \\
O_5 &= [\bar{s}^\alpha d^\beta][\bar{s}^\beta d^\alpha] - [\bar{s}^\alpha \gamma_5 d^\beta][\bar{s}^\beta \gamma_5 d^\alpha]
\end{aligned}
$$

K-$\bar{K}$ matrix elements in units of vacuum saturation approximation

$$
\begin{aligned}
\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle &= \xi_1 \, B_1(\mu) \; m_K^2 f_K^2 \\
\langle \bar{K}^0 | O_i(\mu) | K^0 \rangle &= \xi_i \, B_i(\mu) \; \left[\frac{m_K^2 f_K}{m_s(\mu) + m_d(\mu)} \right]^2 \qquad i = 2, 3, 4, 5 \; ,
\end{aligned}
$$

$\xi_i = (8/3,\ -5/3,\ 1/3,\ 2,\ 2/3)$. For accurate determinations define

$$R_i = \langle \bar{K}^0 | O_i | K^0 \rangle / \langle \bar{K}^0 | O_1 | K^0 \rangle \qquad i = 2, 3, 4, 5$$

Pioneering quenched lattice QCD studies (with two a's each):

$\star$ Donini et al., Phys.Lett. B470 (1999) 233 (clover Wilson fermions)

$\star$ Babich at al., Phys.Rev. D74 (2006) 073009 (overlap fermions)

ETMC (arXiv:1207.1287) continuum $N_f = 2$ results for B_i, R_i

i	1	2	3	4	5
$\overline{\mathrm{MS}}$ (3 GeV)					
B_i	0.51(02)	0.51(02)	0.85(07)	0.82(04)	0.66(07)
R_i	1	-16.3(06)	5.5(04)	30.6(13)	8.2(05)
RI-MOM (3 GeV)					
B_i	0.50(02)	0.63(03)	1.07(09)	0.95(06)	0.75(09)
R_i	1	-15.4(06)	5.3(03)	26.9(12)	7.1(05)

[$\overline{\mathrm{MS}}$-scheme as in Buras, Misiak, Urban, Nucl.Phys. B586 (2000) 397]

Quenching of s-quark: from comparison with $N_f = 2+1$ results for B_1 ($a \to 0$) $\Rightarrow$ systematic quenching error $\lesssim 1-2\%$.

Lattice artifacts are typically 5-10 times larger - depending on O_i and action details $\Rightarrow$ continuum limit crucial

At one a (~ 0.086 fm): $N_f = 2+1$ results from RBC+UKQCD, arXiv:1206.5737

Update of the SM+NP UTfit-'08 analysis [JHEP 0803 (2008) 049] ...

... in arXiv:1207.1287 – triggered by our unquenched B_i-estimates

• Input: experimental and/or phenomenological determinations of heavy meson masses, decay widths and leptonic decay constants, CKM parameters, heavy quark masses, $B_{K,D,B}$-parameters, ...

• NP in $\Delta F = 2$ processes via $\underline{N_f = 3}$ effective weak Hamiltonian

$$\mathcal{H}^{\Delta F=2}_{\mathrm{eff;LO}} = \sum_{f=s,c,b} \left[\sum_{i=1}^{5} c_i \mathcal{O}_i^{fd} + \sum_{i=1}^{3} \tilde{c}_i \tilde{\mathcal{O}}_i^{fd} \right]$$

neglecting non-local contributions and subleading local ones.

• SM+NP UTfit results provide bounds on C_i (of P-even O_i)

$C_i \sim F_i L_i / \Lambda^2$, with F_i the (complex) NP coupling and

L_i a loop factor specific to the interaction that generates O_i.

• $|\epsilon_K| \propto \mathrm{Im}[\langle K^0 | \mathcal{H}^{\Delta F=2;P-even}_{\mathrm{eff;LO}} | \bar{K}^0 \rangle] \quad \Rightarrow \quad$ bounds on $\mathrm{Im}[C_i]$

Switching on one C_i at the time (with $L_i = F_i = 1$) yields ...

	95% allowed range (GeV^{-2})	same from UTfit-2008	lower bound on Λ (TeV)	same from UTfit-2008
$\mathrm{Im}\, C_1^K$	$[-2.8, 2.6] \cdot 10^{-15}$	$[-4.4, 2.8] \cdot 10^{-15}$	$1.9 \cdot 10^4$	$1.5 \cdot 10^4$
$\mathrm{Im}\, C_2^K$	$[-1.6, 1.8] \cdot 10^{-17}$	$[-5.1, 9.3] \cdot 10^{-17}$	$24 \cdot 10^4$	$10 \cdot 10^4$
$\mathrm{Im}\, C_3^K$	$[-6.7, 5.9] \cdot 10^{-17}$	$[-3.1, 1.7] \cdot 10^{-16}$	$12 \cdot 10^4$	$5.7 \cdot 10^4$
$\mathrm{Im}\, C_4^K$	$[-4.1, 3.6] \cdot 10^{-18}$	$[-1.8, 0.9] \cdot 10^{-17}$	$49 \cdot 10^4$	$24 \cdot 10^4$
$\mathrm{Im}\, C_5^K$	$[-1.2, 1.1] \cdot 10^{-17}$	$[-5.2, 2.8] \cdot 10^{-17}$	$29 \cdot 10^4$	$14 \cdot 10^4$

⋆ models with tree-level FCNC from NP excluded up to 10^5 TeV

⋆ gluinos exchange in MSSM $\Rightarrow L_{i>1} \sim \alpha_s^2(\Lambda) \sim 0.01$ $(\Lambda_{min} = \Lambda_{min}^{tab}/10)$

⋆ loop-mediated weak FCNC $\Rightarrow L_{i>1} \sim \alpha_w^2(\Lambda) \sim 10^{-3}$ $(\Lambda_{min} = \Lambda_{min}^{tab}/30)$

⋆ warped 5dim model with flavour hierarchy (RS scenario)

$$F_4 = \frac{2 m_d m_s}{Y_*^2 v^2},\ L_4 = (g^*_{KKs})^2,\ \Lambda = M_{KKG} \Rightarrow F_4 L_4 \sim 10^{-8} \quad (\Lambda_{min} = \Lambda_{min}^{tab}/10^4)$$

ETMC lattice computation of (renormalized) $\langle K|O_i|\bar{K}\rangle$ ...

... based on a lattice regularization of the correlator

$\sum_{\vec{y},\vec{z}}\langle \mathcal{P}_K(y) O_i(x) \mathcal{P}_{\bar{K}}(z)\rangle$ that guarantees

- continuum-like renormalization pattern of O_i's
- O(a) improvement of physical quantities (no artefacts $\sim a^{2k+1}$)
- numerical efficiency ($\Rightarrow$ data at several a's, $a^2 \to 0$ feasible)

Mixed Action setup of maximally twisted mass (Mtm) lattice QCD

$$S = S_{sea}^{\mathrm{Mtm}} + S_{val}^{OS} + S_{ghost}^{OS}\,, \qquad \psi = (u_{sea}, d_{sea}) \text{ \& valence } q_f\text{'s}$$

$$S_{sea}^{\mathrm{Mtm}} = a^4 \sum_x \bar{\psi}(x)\Big\{\tfrac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - i\gamma_5\tau^3 r_{\mathrm{sea}} W_{cr} + \mu_{sea}\Big\}\psi(x)$$

$$S_{val}^{OS} = a^4 \sum_{x,f} \bar{q}_f(x)\Big\{\tfrac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - i\gamma_5 r_f W_{cr} + \mu_f\Big\}q_f(x)$$

$$W_{cr} \equiv M_{cr} - \tfrac{a}{2}\nabla^*_\mu\nabla_\mu \qquad M_{cr} \equiv \text{optimal critical } m_0$$

MA setup of MtmLQCD (R.F., G.C. Rossi, JHEP10 (2004) 070)

Two degenerate sea quarks with $\mu_{sea} = \mu_\ell$ & four valence quarks:

$q_1,\, q_3$ with $\mu_1 = \mu_3 \equiv \mu_{\text{“}s\text{”}}$, $\qquad q_2,\, q_4$ with $\mu_2 = \mu_4 \equiv \mu_\ell$

and valence Wilson parameters $\quad r_1 = r_2 = r_3 = -r_4\,, \qquad |r_f| = 1$

Evaluate two- and three-point correlators involving the fields

$$P^{12} = \bar{q}_1\gamma_5 q_2\,, \quad P^{34} = \ldots\,, \qquad A^{12}_\mu = \bar{q}_1\gamma_\mu\gamma_5 q_2\,, A^{34}_\mu = \ldots$$

$$O^{MA}_{1[\pm]} = 2\big\{\big([\bar{q}_1^\alpha\gamma_\mu q_2^\alpha][\bar{q}_3^\beta\gamma_\mu q_4^\beta] + [\bar{q}_1^\alpha\gamma_\mu\gamma_5 q_2^\alpha][\bar{q}_3^\beta\gamma_\mu\gamma_5 q_4^\beta]\big) \pm \big(2 \leftrightarrow 4\big)\big\}$$

$$O^{MA}_{2[\pm]} = 2\big\{\big([\bar{q}_1^\alpha q_2^\alpha][\bar{q}_3^\beta q_4^\beta] + [\bar{q}_1^\alpha\gamma_5 q_2^\alpha][\bar{q}_3^\beta\gamma_5 q_4^\beta]\big) \pm \big(2 \leftrightarrow 4\big)\big\}$$

$$O^{MA}_{3[\pm]} = 2\big\{\big([\bar{q}_1^\alpha q_2^\beta][\bar{q}_3^\beta q_4^\alpha] + [\bar{q}_1^\alpha\gamma_5 q_2^\beta][\bar{q}_3^\beta\gamma_5 q_4^\alpha]\big) \pm \big(2 \leftrightarrow 4\big)\big\}$$

$$O^{MA}_{4[\pm]} = 2\big\{\big([\bar{q}_1^\alpha q_2^\alpha][\bar{q}_3^\beta q_4^\beta] - [\bar{q}_1^\alpha\gamma_5 q_2^\alpha][\bar{q}_3^\beta\gamma_5 q_4^\beta]\big) \pm \big(2 \leftrightarrow 4\big)\big\}$$

$$O^{MA}_{5[\pm]} = 2\big\{\big([\bar{q}_1^\alpha q_2^\beta][\bar{q}_3^\beta q_4^\alpha] - [\bar{q}_1^\alpha\gamma_5 q_2^\beta][\bar{q}_3^\beta\gamma_5 q_4^\alpha]\big) \pm \big(2 \leftrightarrow 4\big)\big\}$$

in particular

$$C_i(x_0) = \left(\frac{a}{L}\right)^3 \sum_{\vec{x}} \langle \mathcal{P}^{43}_{y_0+\frac{T}{2}}\, O^{MA}_{i[+]}(\vec{x}, x_0)\, \mathcal{P}^{21}_{y_0}\rangle\,, \quad i = 1, \ldots, 5$$

In such a MA setup one finds (JHEP10 (2004) 070, arXiv:1207.1287)

• the op.s $O^{MA}_{i[+]}$ renormalize as in the formal QCD:

$$\begin{pmatrix} O^{MA}_{1[+]} \\ O^{MA}_{2[+]} \\ O^{MA}_{3[+]} \\ O^{MA}_{4[+]} \\ O^{MA}_{5[+]} \end{pmatrix} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} O^{MA}_{1[+]} \\ O^{MA}_{2[+]} \\ O^{MA}_{3[+]} \\ O^{MA}_{4[+]} \\ O^{MA}_{5[+]} \end{pmatrix}^{(b)}$$

[mass-independent Z_{ij} related to plain Wilson 4-fermion op. RC's]

• the relevant quark bilinear operators renormalize according to

$$[P^{12/34}] = Z_{S/P}[P^{12/34}]^{(b)}\,, \qquad [A_\mu^{12/34}] = Z_{A/V}[A_\mu^{12/34}]^{(b)}$$

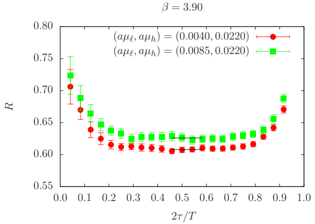
• if $\mu_{1,3} = \mu_s$ and $\mu_{2,4} = \mu_{u/d}$ the m.e. $\langle P^{43}|O^{MA}_{i[+]}|P^{12}\rangle$ extracted from the correlators with insertion of $O^{MA}_{i[+]}$ as $a \to 0$ approaches (with rate a^2) the continuum QCD m.e. $\langle \bar{K}^0|O_i|K^0\rangle$

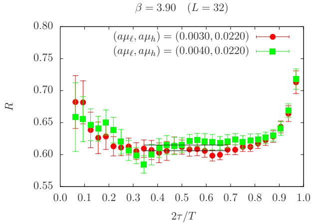
Lattice parameters for correlators at $\beta = 3.80,\ 3.90$ and 4.05.

$a\mu_\ell = a\mu_{sea}$	$a^{-4}(L^3 \times T)$	$a\mu_{\text{“}s\text{”}}$	N_{stat}
$\beta = 3.80$, $a \sim 0.10$ fm			
0.0080	$24^3 \times 48$	0.0165, 0.0200, 0.0250	170
0.0110	“	“	180
$\beta = 3.90$, $a \sim 0.09$ fm			
0.0040	$24^3 \times 48$	0.0150, 0.0220, 0.0270	400
0.0064	“	“	200
0.0085	“	“	200
0.0100	“	“	160
0.0030	$32^3 \times 64$	“	300
0.0040	“	“	160
$\beta = 4.05$, $a \sim 0.07$ fm			
0.0030	$32^3 \times 64$	0.0120, 0.0150, 0.0180	190
0.0060	“	“	150
0.0080	“	“	220

To improve signal-to-noise ratio: stochastic spatial-wall sources used for $\mathcal{P}^{21}_{y_0}$, $\mathcal{P}^{43}_{y_0+T/2}$ and sum over spatial location of O_i.

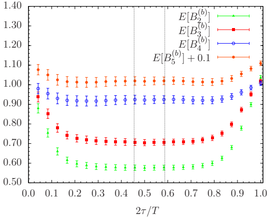
Time-plateaux for bare estimators of B_1 at $\beta = 3.90$, $L/a = 24\&32$

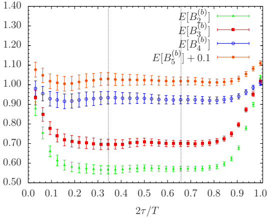




Bare bag-parameter estimators vs. $2\tau/T \equiv 2(x_0 - y_0)/T$, $T = 2L$.

Time-plateaux for bare estimators of $B_{2,\ldots,5}$ at $\beta = 3.90$, $L/a = 24\&32$





Renormalization constants (RC) of 4– & 2—quark operators evaluated in the RI-MOm scheme (Martinelli et al. Nucl.Phys. B445 (1995) 81)
following the implementation in JHEP 1008 (2010) 068 , with details specific to O_i given in Phys.Rev. D83 (2011) 014505, arXiv:1207.1287

A convenient basis for RC of the relevant 4-quark operators is

$$
\begin{aligned}
Q_{1[\pm]}^{MA} &= 2\{([\bar{q}_1\gamma_\mu q_2][\bar{q}_3\gamma_\mu q_4] + [\bar{q}_1\gamma_\mu\gamma_5 q_2][\bar{q}_3\gamma_\mu\gamma_5 q_4]) \pm (2 \leftrightarrow 4)\} \\
Q_{2[\pm]}^{MA} &= 2\{([\bar{q}_1\gamma_\mu q_2][\bar{q}_3\gamma_\mu q_4] - [\bar{q}_1\gamma_\mu\gamma_5 q_2][\bar{q}_3\gamma_\mu\gamma_5 q_4]) \pm (2 \leftrightarrow 4)\} \\
Q_{3[\pm]}^{MA} &= 2\{([\bar{q}_1 q_2][\bar{q}_3 q_4] - [\bar{q}_1\gamma_5 q_2][\bar{q}_3\gamma_5 q_4]) \pm (2 \leftrightarrow 4)\} \\
Q_{4[\pm]}^{MA} &= 2\{([\bar{q}_1 q_2][\bar{q}_3 q_4] + [\bar{q}_1\gamma_5 q_2][\bar{q}_3\gamma_5 q_4]) \pm (2 \leftrightarrow 4)\} \\
Q_{5[\pm]}^{MA} &= 2\{([\bar{q}_1\sigma_{\mu\nu} q_2][\bar{q}_3\sigma_{\mu\nu} q_4]) \pm (2 \leftrightarrow 4)\} \ \ (\text{for } \mu > \nu),
\end{aligned}
$$

with q_f the valence quarks in our MA setup and $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$

In fact the following renormalization formulae hold

$$O^{MA}_{i[+]}\Big|^{\rm ren} = Z_{ij} O^{MA}_{j[+]}\Big|^{(b)} ,$$
$$Z = \Lambda^{[+]} Z_Q (\Lambda^{[+]})^{-1} ,$$

$$Z_Q = \begin{pmatrix} \mathcal{Z}^{[+]}_{11} & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Z}^{[-]}_{22} & -\mathcal{Z}^{[-]}_{23} & 0 & 0 \\ 0 & -\mathcal{Z}^{[-]}_{32} & \mathcal{Z}^{[-]}_{33} & 0 & 0 \\ 0 & 0 & 0 & \mathcal{Z}^{[+]}_{44} & \mathcal{Z}^{[+]}_{45} \\ 0 & 0 & 0 & \mathcal{Z}^{[+]}_{54} & \mathcal{Z}^{[+]}_{55} \end{pmatrix}$$

To extract RC compute quark propagators $S_{q_f}(p)$ and correlators

$$G_i(p,\, p,\, p,\, p)^{a\,b\,c\,d}_{\alpha\,\beta\,\gamma\,\delta} = a^{16} \sum_{x_1,x_2,x_3,x_4} e^{-ip(x_1-x_2+x_3-x_4)} \langle\, [q_1(x_1)]^a_\alpha \, [\bar{q}_2(x_2)]^b_\beta \; Q_i(0) \; [q_3(x_3)]^c_\gamma \, [\bar{q}_4(x_4)]^d_\delta \,\rangle \, .$$

... impose the standard RI-MOM renormalization conditions at finite quark mass and for a suitable set of p's and proceed to the analysis of the resulting RC-estimators along the following steps:

$\star$ valence and sea chiral extrapolation

$\star$ removal of $\mathrm{O}(a^2\tilde{g}^2)$ artefacts

$\star$ NLO evolution of $Z_{ij}^{\mathrm{RI}'}(\tilde{p}^2; a^2\tilde{p}^2; 0; 0)$ to a reference scale μ_0^2

$\star$ from $Z_{ij}^{\mathrm{RI}'}(\mu_0^2; a^2\tilde{p}^2; 0; 0)$ RC are evaluated
− either extrapolating to $\tilde{p}^2 = 0$ (M1-method)
− or taking $\tilde{p}^2$ fixed in physical units (M2: here $\tilde{p}^2 = 9$ GeV^2)

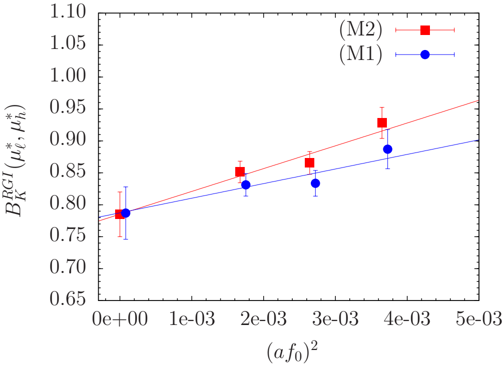
I refer to arXiv:1207.1287 (app.s A and B) for technical details ...
... see backup slides for typical results (RI-MOM, 2 GeV scheme)

Extraction of B_i with partial cutoff effect cancellation

$$\xi_1 B_1 = \frac{Z_{11}}{Z_A Z_V} \frac{\langle K^{34}|O_1|K^{21}\rangle}{\langle K^{34}|A_0^{34}|0\rangle\langle 0|A_0^{21}|K^{21}\rangle}$$

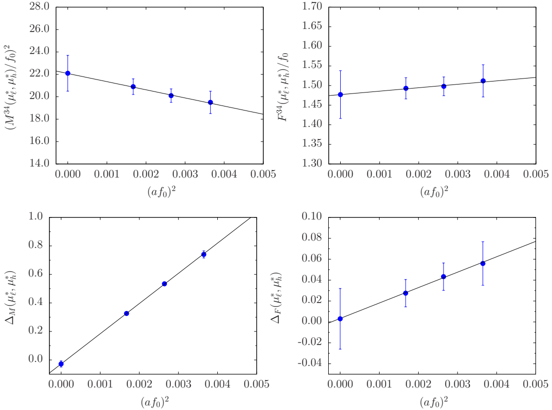
$$\xi_i B_i = \frac{Z_{ij}}{Z_A Z_V} \frac{\langle K^{34}|O_j|K^{21}\rangle}{\langle K^{34}|P^{34}|0\rangle\langle 0|P^{21}|K^{21}\rangle}, \quad i = 2,3,4,5$$

$B_{K,\mathrm{lat}}^{\mathrm{RGI}}$ vs. $(af_0)^2$ at fixed quark masses $\hat{\mu}_\ell^* \sim 40$ MeV, $\hat{\mu}_h^* \sim 90$ MeV



Scaling test, at fixed $\hat{\mu}_\ell^*$, $\hat{\mu}_h^*$ as above, for $(M^{34}/f_0)^2$, F^{34}/f_0 and

$$\Delta_M = [(M^{12})^2 - (M^{34})^2](M^{34})^{-2}\,, \qquad \Delta_F = -[F^{12} - F^{34}](F^{34})^{-1}$$



... this suggested an estimator of R_i with reduced lattice artefacts

$$\tilde{R}_i = \left(\frac{f_K}{m_K}\right)^2_{\rm expt.} \left[\frac{M^{12}M^{34}}{F^{12}F^{34}}\, \frac{Z_{ij}\langle K^{34}|O_j|K^{21}\rangle}{Z_{11}\langle K^{34}|O_1|K^{21}\rangle}\right]_{\rm Lat.}, \qquad i,j = 2,3,4,5$$

Continuum and chiral extrapolation (at fixed $\hat{\mu}_s = 95(6)$ MeV)

[Symbol ^ denotes renormalization in the $(\overline{MS}, 2GeV)$-scheme; $f_0 = 121.0(1)$ MeV, $\hat{B}_0 = 2.84(11)$ GeV]

- $O(a^2)$ artefacts happen to have negligible μ_ℓ-dependence

- choose a (standard) fit ansatz based on SU(2) χPT

$$\hat{B}_i = B_i^\chi(r_0\hat{\mu}_s)\left[1 + b_i(r_0\hat{\mu}_s) \mp \frac{2\hat{B}_0\hat{\mu}_\ell}{2(4\pi f_0)^2}\log\frac{2\hat{B}_0\hat{\mu}_\ell}{(4\pi f_0)^2}\right] + D_{Bi}^\chi(r_0\hat{\mu}_s)\left[\frac{a}{r_0}\right]^2$$

with sign $\pm$ being $-$ for $i = 1,2,3$ and $+$ for $i = 4,5$

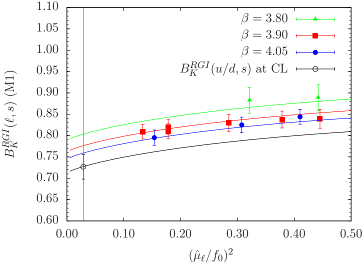
and fit formulae with 1st & 2nd order polynomial μ_ℓ-dependence

$$\hat{B}_i = B_i^\chi(r_0\hat{\mu}_s)\left[1 + P_1((r_0\hat{\mu}_s)[r_0\hat{\mu}_\ell] + P_2(r_0\hat{\mu}_s)[r_0\hat{\mu}_\ell]^2\right] + D_{Bi}^{pol}(r_0\hat{\mu}_s)\left[\frac{a}{r_0}\right]^2$$

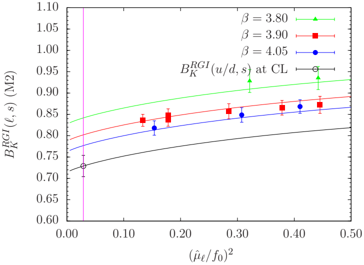
fit ansatz for R_i's follow from those for B_i (taking $M_{sl}^2/(\hat{\mu}_s + \hat{\mu}_\ell) \sim \hat{B}_0$)

- spread of results from different ansatz included in the systematic error [for $B_1^{RGI} = 0.729(25)(17)$, with 0.017 from 0.014(chiral-fit), 0.009(latt. artefacts), 0.004(ren.)]

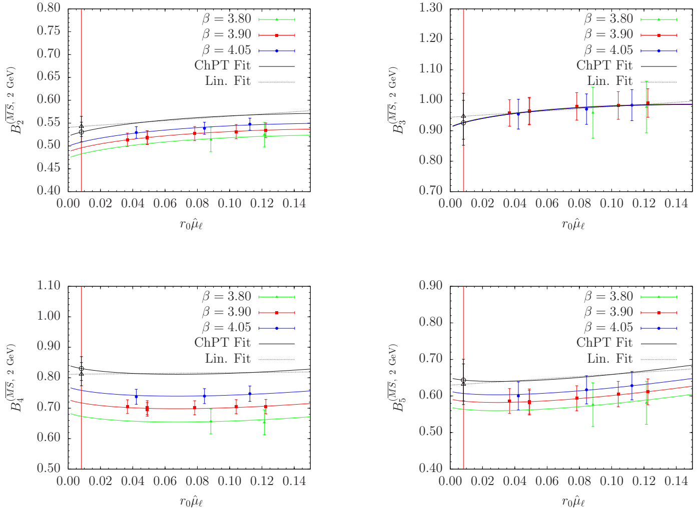
$B_1 = B_K$: combined chiral and continuum extrapolation (χPT ansatz)



M1 or M2 refer to the RI-MOM evaluation method for Z_{VA+AV}^{RGI} and Z_A.

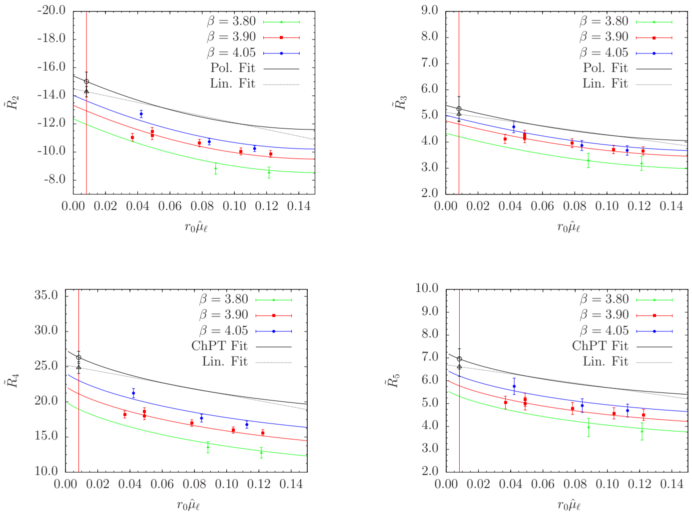


$B_{2,3,4,5}$: combined chiral and continuum extrapolation (χPT ansatz)



RC from M1-method. The dashed black line represents the continuum limit in case of linear fit in $\hat{\mu}_\ell$

$R_{2,3,4,5}$: combined chiral and continuum extrapolation (χPT ansatz)



RC from M1-method. The dashed black line represents the continuum limit in case of linear fit in $\hat{\mu}_\ell$

Thanks for your attention!

Results for RC-matrix Z_Q above: RI-MOM (M1-def), 2 GeV scale

$$Z_Q^{\mathrm{M1}}|_{\beta=3.80} = \begin{pmatrix} 0.415(12) & 0 & 0 & 0 & 0 \\ 0 & 0.503(13) & 0.237(08) & 0 & 0 \\ 0 & 0.016(01) & 0.190(08) & 0 & 0 \\ 0 & 0 & 0 & 0.236(08) & -0.013(02) \\ 0 & 0 & 0 & -0.239(08) & 0.572(14) \end{pmatrix}$$

$$Z_Q^{\mathrm{M1}}|_{\beta=3.90} = \begin{pmatrix} 0.432(07) & 0 & 0 & 0 & 0 \\ 0 & 0.517(07) & 0.237(05) & 0 & 0 \\ 0 & 0.018(01) & 0.212(05) & 0 & 0 \\ 0 & 0 & 0 & 0.259(05) & -0.014(01) \\ 0 & 0 & 0 & -0.241(05) & 0.591(08) \end{pmatrix}$$

$$Z_Q^{\mathrm{M1}}|_{\beta=4.05} = \begin{pmatrix} 0.486(06) & 0 & 0 & 0 & 0 \\ 0 & 0.566(08) & 0.256(07) & 0 & 0 \\ 0 & 0.019(01) & 0.241(06) & 0 & 0 \\ 0 & 0 & 0 & 0.294(05) & -0.012(01) \\ 0 & 0 & 0 & -0.256(07) & 0.659(10) \end{pmatrix}$$

Results for RC-matrix Z_Q above: RI-MOM (M2-def), 2 GeV scale

$$Z_Q^{\mathrm{M2}}|_{\beta=3.80} = \begin{pmatrix} 0.433(08) & 0 & 0 & 0 & 0 \\ 0 & 0.527(07) & 0.318(05) & 0 & 0 \\ 0 & 0.034(01) & 0.324(04) & 0 & 0 \\ 0 & 0 & 0 & 0.338(04) & -0.011(02) \\ 0 & 0 & 0 & -0.149(04) & 0.522(09) \end{pmatrix}$$

$$Z_Q^{\mathrm{M2}}|_{\beta=3.90} = \begin{pmatrix} 0.441(04) & 0 & 0 & 0 & 0 \\ 0 & 0.528(05) & 0.304(04) & 0 & 0 \\ 0 & 0.031(01) & 0.307(04) & 0 & 0 \\ 0 & 0 & 0 & 0.332(03) & -0.012(01) \\ 0 & 0 & 0 & -0.169(03) & 0.550(05) \end{pmatrix}$$

$$Z_Q^{\mathrm{M2}}|_{\beta=4.05} = \begin{pmatrix} 0.487(05) & 0 & 0 & 0 & 0 \\ 0 & 0.570(05) & 0.306(05) & 0 & 0 \\ 0 & 0.026(01) & 0.291(03) & 0 & 0 \\ 0 & 0 & 0 & 0.331(03) & -0.011(01) \\ 0 & 0 & 0 & -0.212(04) & 0.632(08) \end{pmatrix}$$