Lattice Study of the Extent of the Conformal window in an SU(3) Gauge Theory with N_f Fermions in the Fundamental Representation

George Fleming, Ethan Neil, TA

1) arXiv:0712.0609, PRL 100, 171607 , 2008

2) arXiv:0901.3766 PR D79, 076010, 2009

Conformality violated by a, L !!

Focus: Gauge Invariant and Non-Perturbative Definition of the Running Coupling from the Schroedinger Functional of the Gauge Theory

ALPHA Collaboration: Luscher, Sommer, Weisz, Wolff, Bode, Heitger, Simma, …

Transition amplitude from a prescribed state at t=0 to one at t=T= L± a (Dirichlet BC).(m = 0)

At three loops

$N_f = 16$ IRFP at $g^{*2}_{SF} = 0.47$ $(g^{*2}_{SF}/4\pi \approx .04)$

$N_f = 12$ IRFP at $g^{*2}_{SF} = 5.18$ $(g^{*2}_{SF}/4\pi \approx 0.4)$

$N_f \leq 8$ No perturbative IRFP

Using Staggered Fermions as in

U. Heller, Nucl. Phys. B504, 435 (1997)
Miyazaki & Kikukawa

Focus on N_f = multiples of 4:

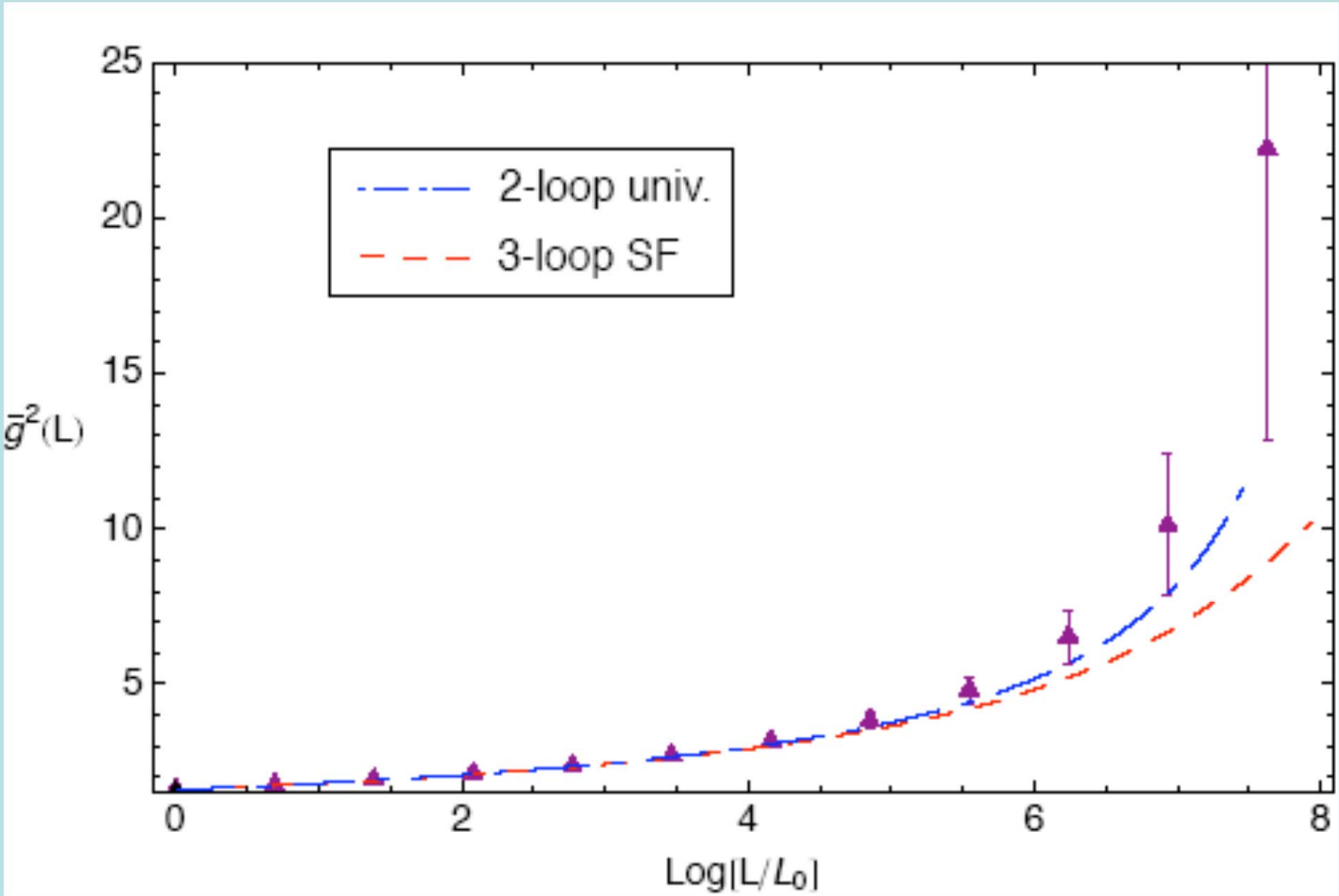
16: Perturbative IRFP

12: IRFP “expected”, Simulate

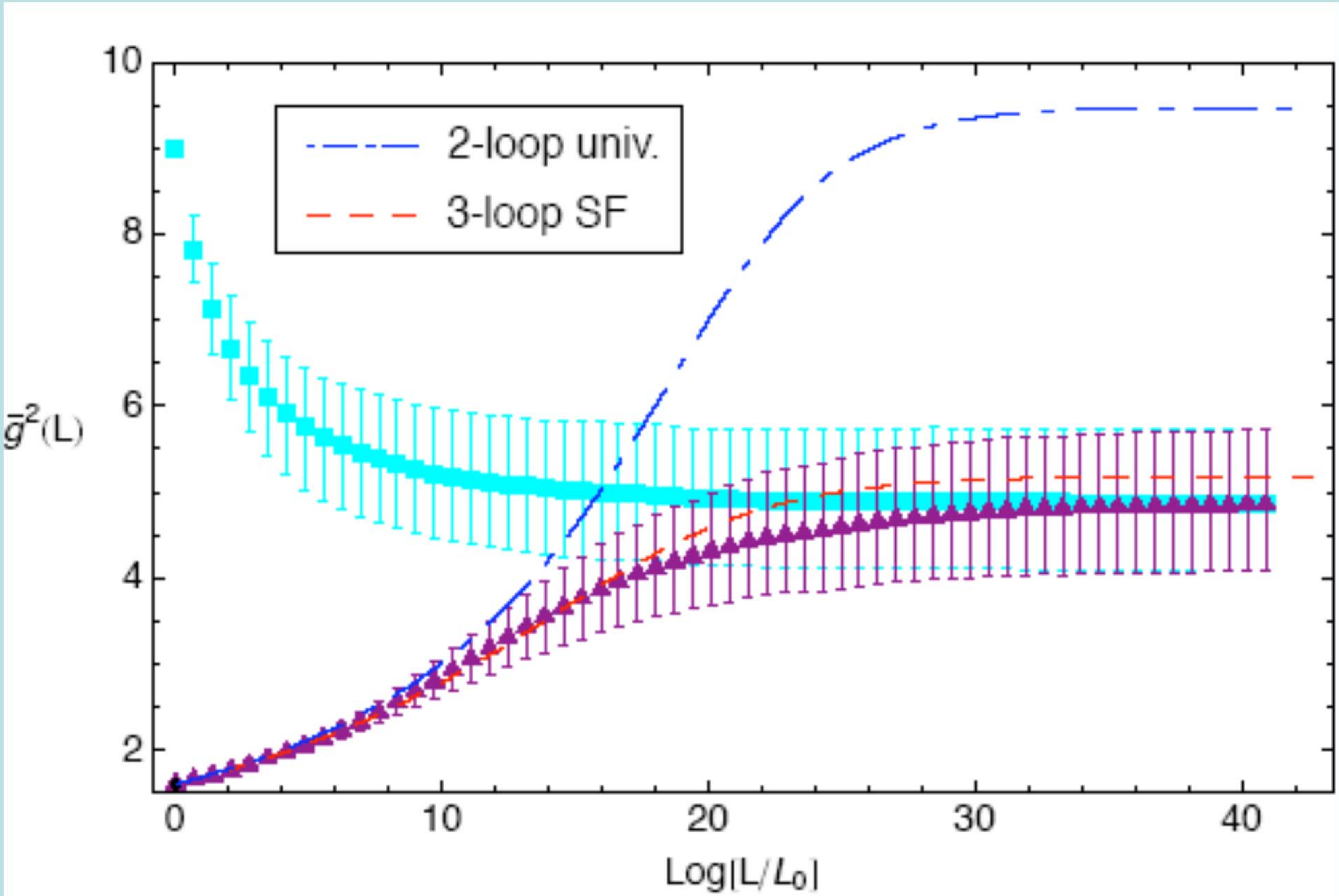
8 : IRFP uncertain , Simulate

4 : Confinement, ChSB

N_f = 8 Continuum Running



N_f = 12 Continuum Running



Approach to Fixed Point

$$\overline{\beta}\left(\overline{g}^2(L)\right) \simeq \gamma\left[\overline{g}^2_* - \overline{g}^2(L)\right]$$

$$\overline{g}^2(L) \to \overline{g}^2_* - \frac{\text{const}}{L^\gamma}$$

$$Fit : \gamma = 0.13 \pm 0.03$$

$$3 - loop : \gamma = 0.296$$

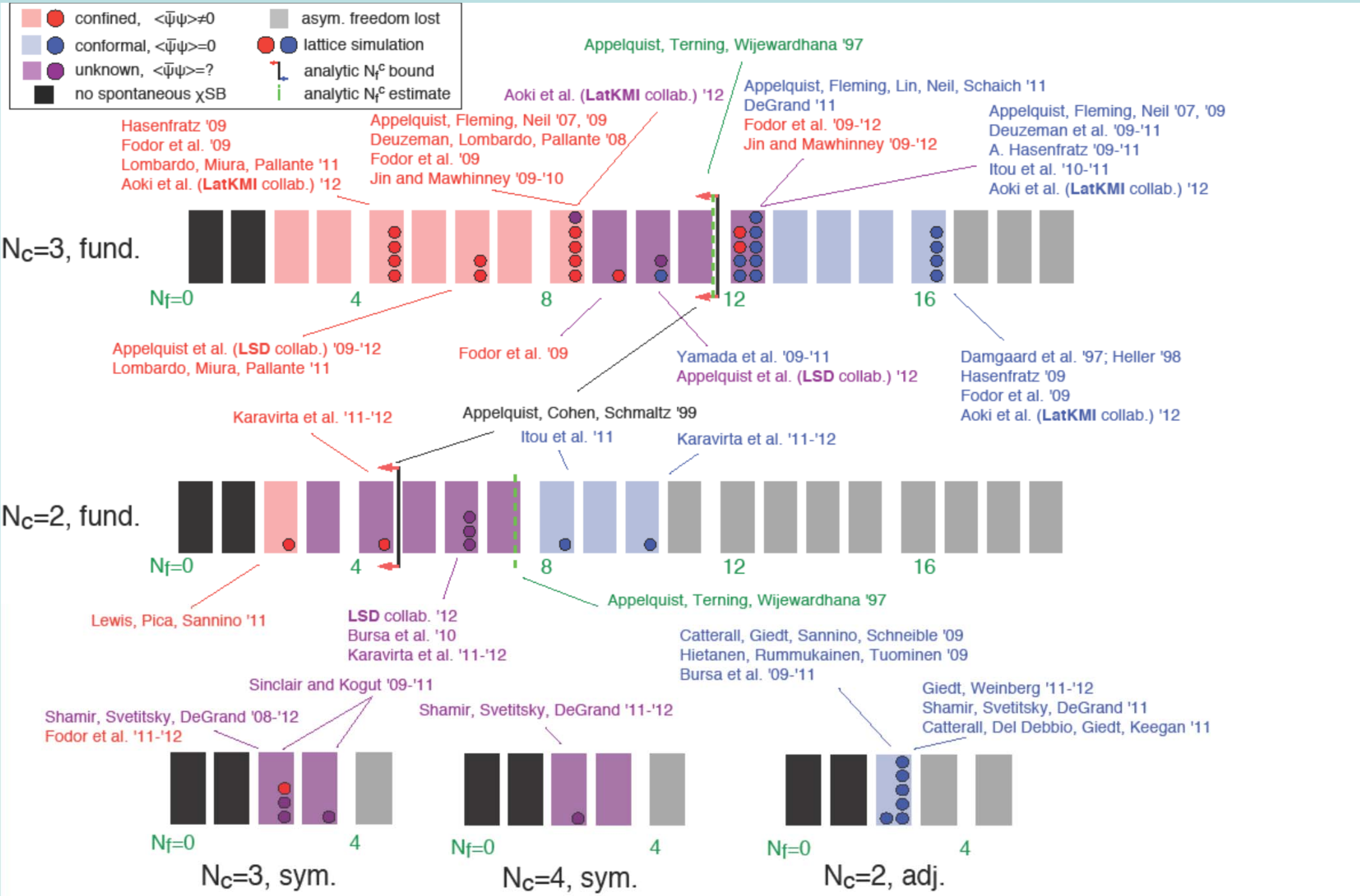
Our Conclusions

1. Lattice evidence that for an SU(3) gauge theory with N_f Dirac fermions in the fundamental representation
   $8 < N_{fc} < 12$

2. N_f=12: Relatively weak IRFP

3. N_f=8: Confinement → chiral symmetry breaking.

Employing the Schroedinger-functional running coupling defined at the box boundary L


confined, <ψ̄ψ>≠0
conformal, <ψ̄ψ>=0
unknown, <ψ̄ψ>=?
no spontaneous χSB
asym. freedom lost
lattice simulation
analytic N_f^c bound
analytic N_f^c estimate
Appelquist, Terning, Wijewardhana '97
Aoki et al. (LatKMI collab.) '12
Appelquist, Fleming, Lin, Neil, Schaich '11
DeGrand '11
Fodor et al. '09-'12
Jin and Mawhinney '09-'12
Appelquist, Fleming, Neil '07, '09
Deuzeman et al. '09-'11
A. Hasenfratz '09-'11
Itou et al. '10-'11
Aoki et al. (LatKMI collab.) '12
Hasenfratz '09
Fodor et al. '09
Lombardo, Miura, Pallante '11
Aoki et al. (LatKMI collab.) '12
Appelquist, Fleming, Neil '07, '09
Deuzeman, Lombardo, Pallante '08
Fodor et al. '09
Jin and Mawhinney '09-'10
Nc=3, fund.
Nf=0
4
8
12
16
Appelquist et al. (LSD collab.) '09-'12
Lombardo, Miura, Pallante '11
Fodor et al. '09
Yamada et al. '09-'11
Appelquist et al. (LSD collab.) '12
Damgaard et al. '97; Heller '98
Hasenfratz '09
Fodor et al. '09
Aoki et al. (LatKMI collab.) '12
Karavirta et al. '11-'12
Appelquist, Cohen, Schmaltz '99
Itou et al. '11
Karavirta et al. '11-'12
Nc=2, fund.
Nf=0
4
8
12
16
Appelquist, Terning, Wijewardhana '97
Lewis, Pica, Sannino '11
LSD collab. '12
Bursa et al. '10
Karavirta et al. '11-'12
Catterall, Giedt, Sannino, Schneible '09
Hietanen, Rummukainen, Tuominen '09
Bursa et al. '09-'11
Sinclair and Kogut '09-'11
Shamir, Svetitsky, DeGrand '08-'12
Fodor et al. '11-'12
Shamir, Svetitsky, DeGrand '11-'12
Giedt, Weinberg '11-'12
Shamir, Svetitsky, DeGrand '11
Catterall, Del Debbio, Giedt, Keegan '11
Nf=0
4
Nc=3, sym.
Nf=0
4
Nc=4, sym.
Nf=0
4
Nc=2, adj.

Physics with SU(3), Nf = 2 and 6.
Toward IR conformality

(LSD) arXiv:0910.2224
PRL 104, 071601 (2010)

Walking Idea:

As the conformal window is approached ($N_f \rightarrow N_{fc}$),
$\langle \psi \; \psi \rangle$ is enhanced relative to its nominal value $4\pi \, F^3$.

LSD Program:

Search for enhancement of $\langle \bar{\psi} \; \psi \rangle / F^3$ by starting at $N_f = 2$,
then $\rightarrow N_f = 6$. (Creeping Toward the Conformal Window)

($\Lambda = a^{-1}$)

Some Details

- Domain-wall fermions, Iwasaki improved action
- USQCD: Chroma, CPS

- 32^3 x 64 lattice ($L_s = 16$)
- m_f = .005, .01, .015, .02, .025 , $m = m_f + m_{res}$

- $N_f^2 - 1$ PNGB's

- Simulate: M_P , F, $<\overline{\psi}\, \psi>$, M_V $M_P L > 4$

- Extrapolate to m=0 with Chiral Perturbation Theory

Extrapolate to m=0 with Chiral Perturbation Theory

- $M^2_{Pm} = 2m <\bar{\psi}\,\psi> / F^2 \{1 + zm\,[\alpha_{M1} + (1/N_f)\,\log(zm)] + \ldots\}$

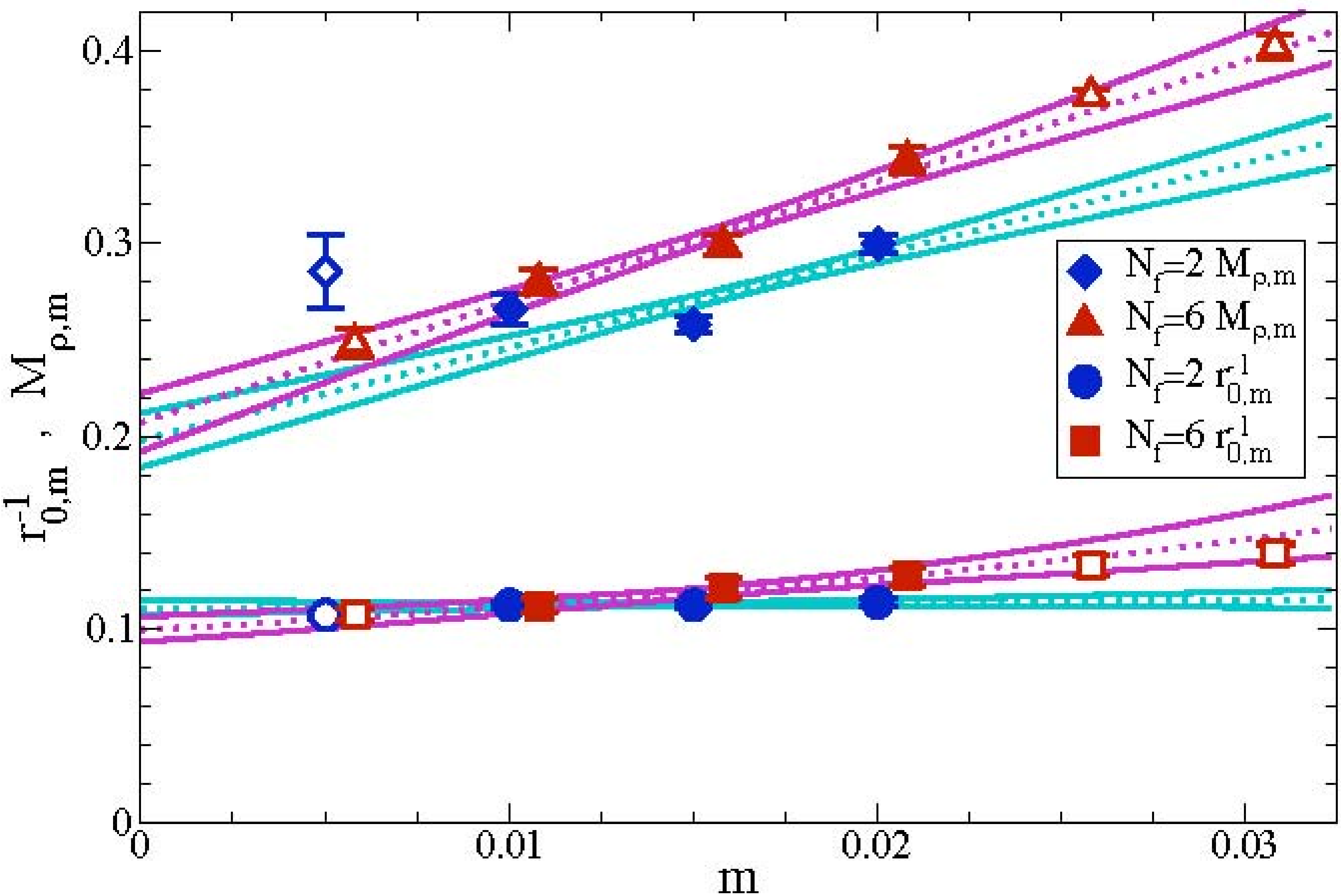
$z \equiv 2 <\bar{\psi}\,\psi> / (4\pi)^2 F^4$

- $F_m = F\{\,1 + zm\,[\alpha_{F1} - (N_f/2)\,\log(zm)] + \ldots\}$

- $<\bar{\psi}\,\psi>_m = <\bar{\psi}\,\psi>\{1 + zm\,[\alpha_{C1} - ((N_f^2 - 1)/N_f)\,\log(zm)] + \ldots\}$

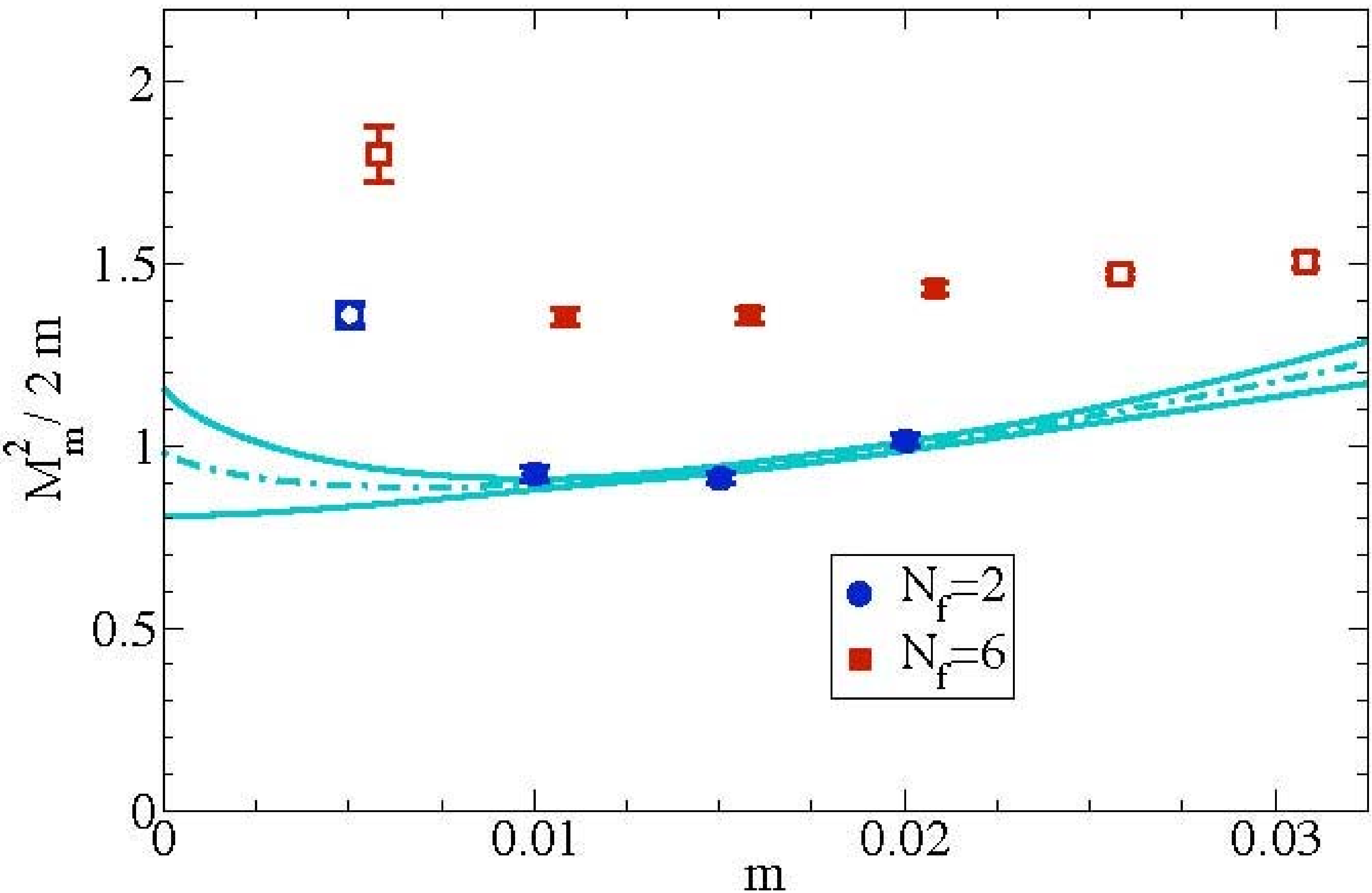
$M_{Vm} = M_V\{\,1 + \alpha_{R1}\,zm + \alpha_{R3/2}\,(zm)^{3/2} + \ldots\}$

$M_{Am} = M_A\{\,1 + \alpha_{A1}\,zm + \alpha_{A3/2}\,(zm)^{3/2} + \ldots\}$

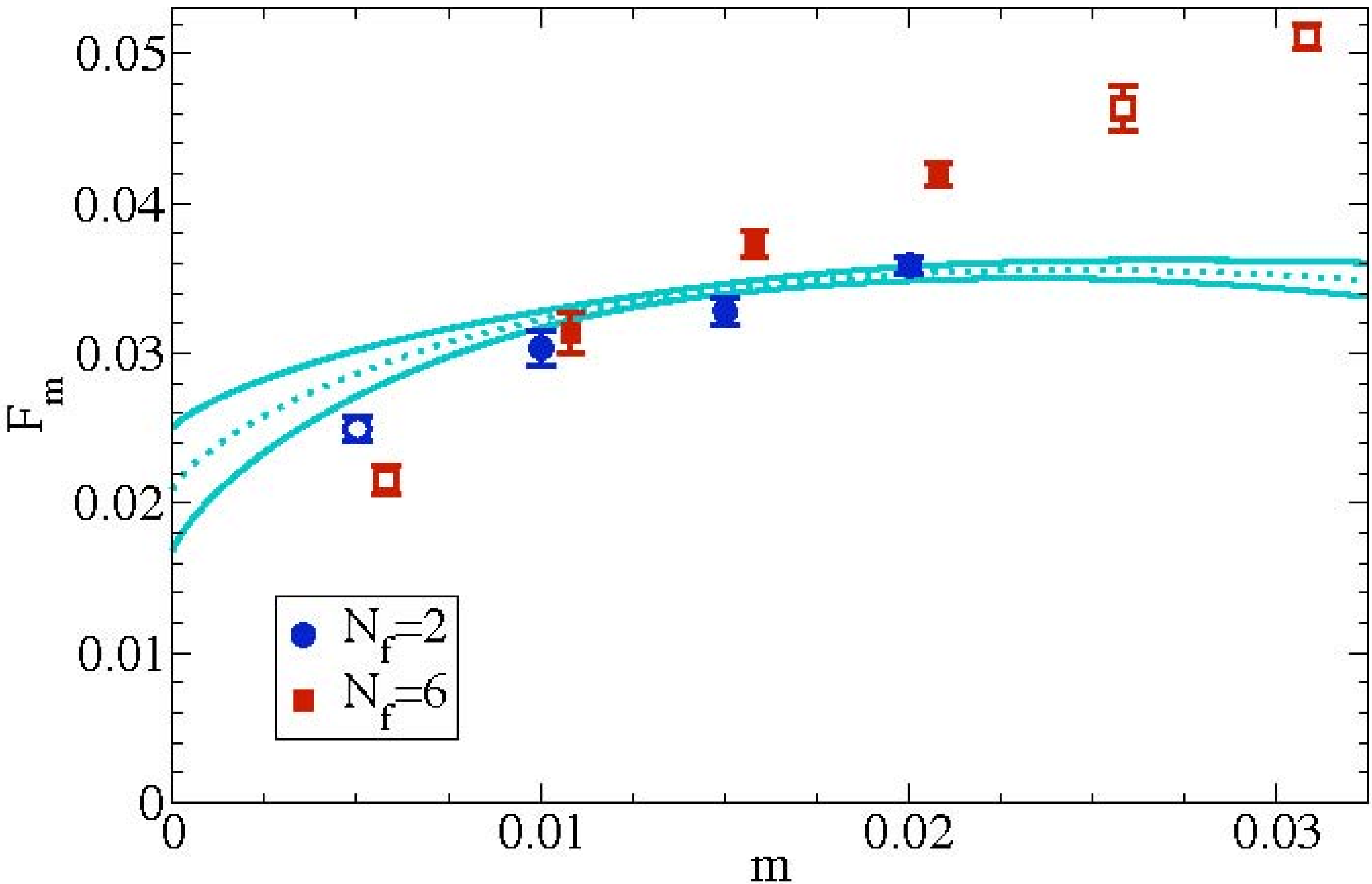


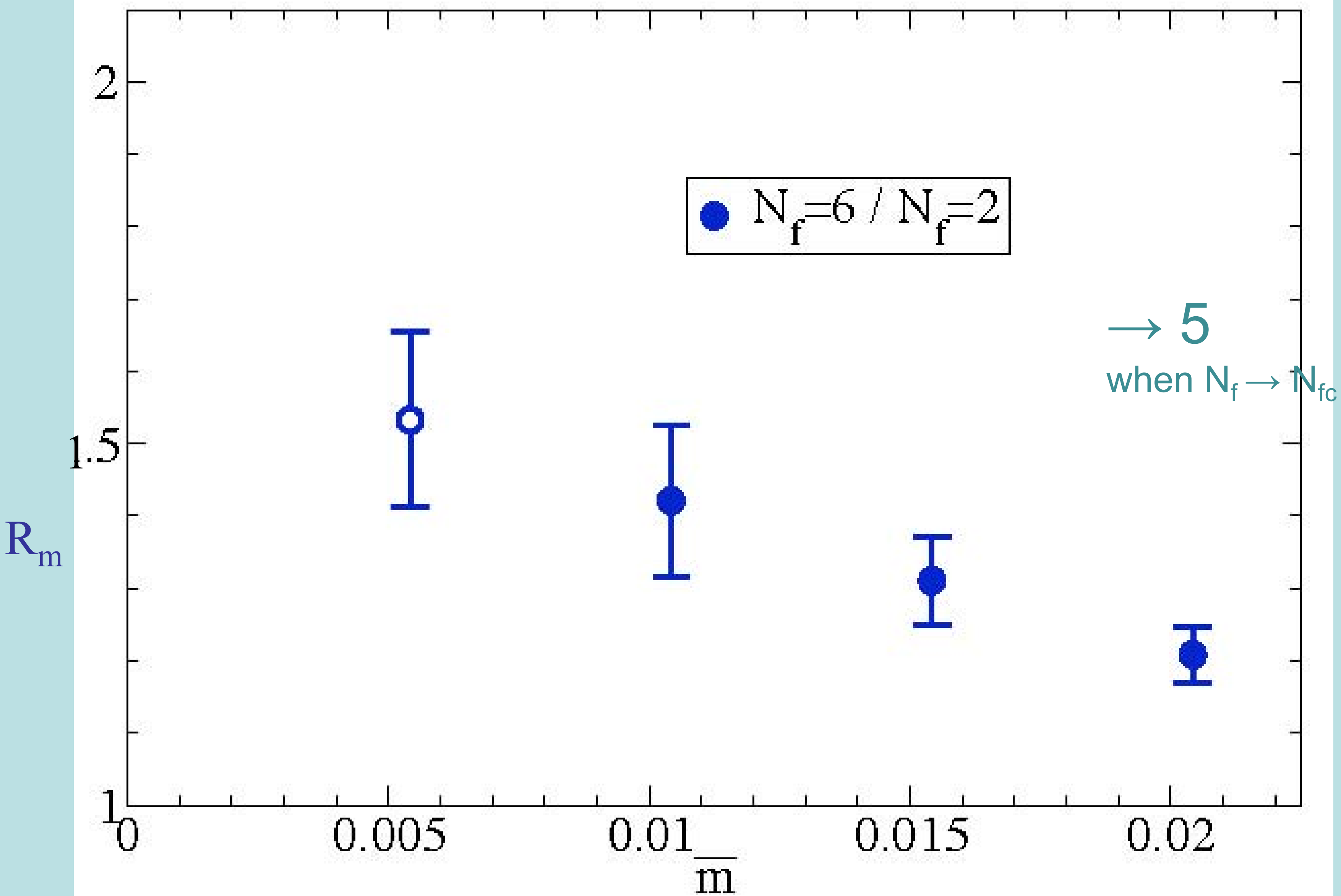
$N_f = 2$: $\beta = 2.7$ $N_f = 6$: $\beta = 2.1$

4



5


0.05
0.04
0.03
0.02
0.01
0
F_m
0
0.01
0.02
0.03
m
$N_f=2$
$N_f=6$



$$R_m = [\langle\bar{\psi}\,\psi\rangle_m / F_m^3]_{6f} \,/\, [\langle\bar{\psi}\,\psi\rangle_m / F_m^3]_{2f}$$

7

$N_f = 2$

- Chiral perturbation theory extrapolation:

$$\langle \bar{\psi} \psi \rangle / F^3 = 47.1\ (17.6)$$

QCD Experimental Value: (renormalized to our lattice scheme - Aoki et al hep-lat/0206013)

$$\langle \bar{\psi} \psi \rangle / F^3 = 36.2\ (6.5)$$

8

$N_f = 6$

Linear Extrapolation $\longrightarrow$

Conservative Lower Bound on $\langle\bar{\psi}\,\psi\rangle / F^2$

Conservative Upper Bound on F

Thus $\langle\bar{\psi}\,\psi\rangle / F^3 \geq 60.0\ (8.0)$

9

Resonance Spectrum and the S Parameter

- Parity Doubling?
- Diminished S parameter?

$$S\left(m_{H,ref}\right)=4\int_0^\infty \frac{ds}{s}\left\{\left[\operatorname{Im}\Pi_{VV}(s)-\operatorname{Im}\Pi_{AA}(s)\right]-\frac{1}{48\pi}\left[1-\left(1-\frac{m_{H,ref}}{s}\right)^3\theta\left(s-m_{H,ref}^2\right)\right]\right\}$$

~Same Details

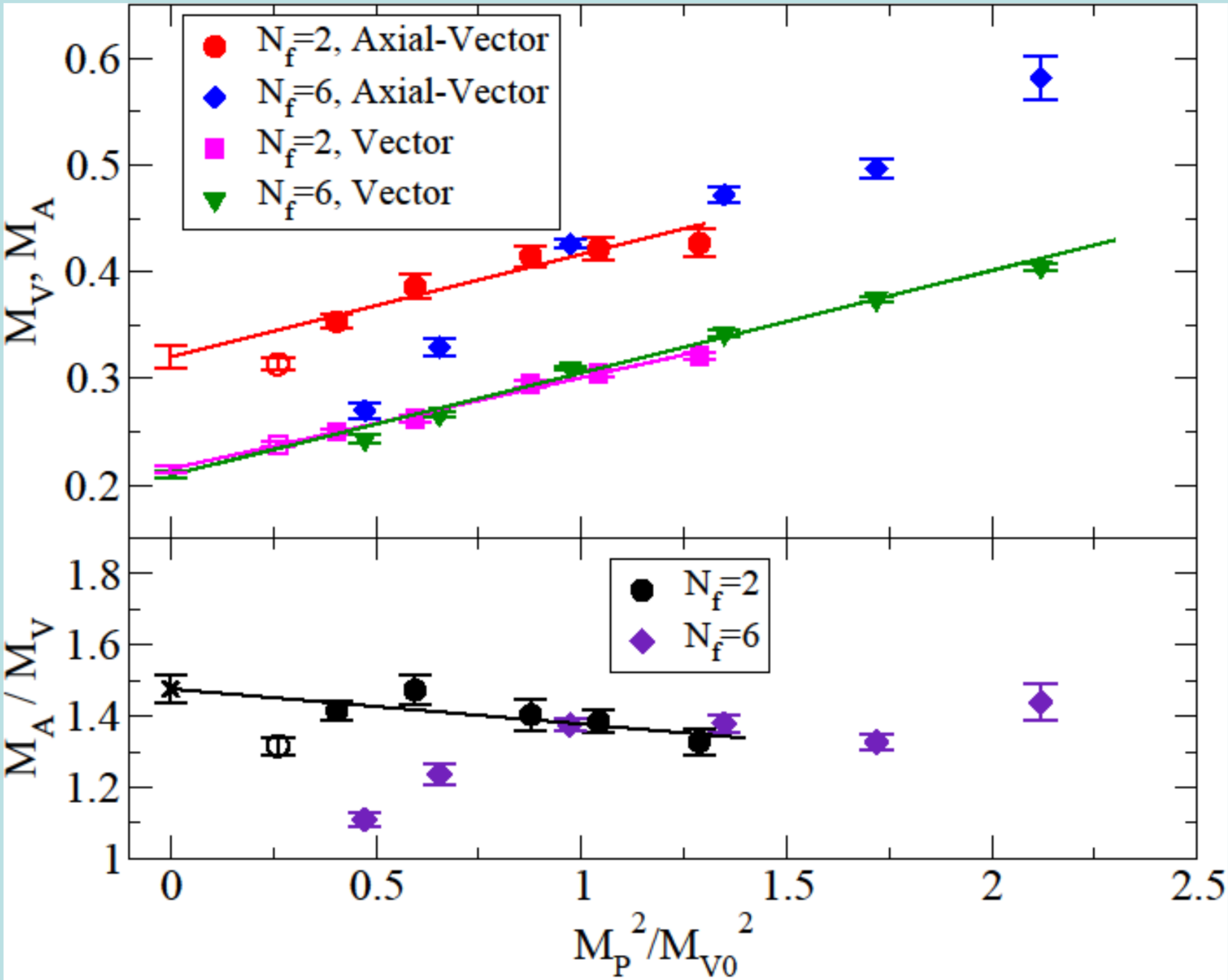
- Domain-wall fermions, Iwasaki improved action
- USQCD: Chroma, CPS

- 32^3 x 64 lattice (L$_s$ = 16)
- m_f = .005, .01, .015, .02, .025 , $m = m_f + m_{res}$

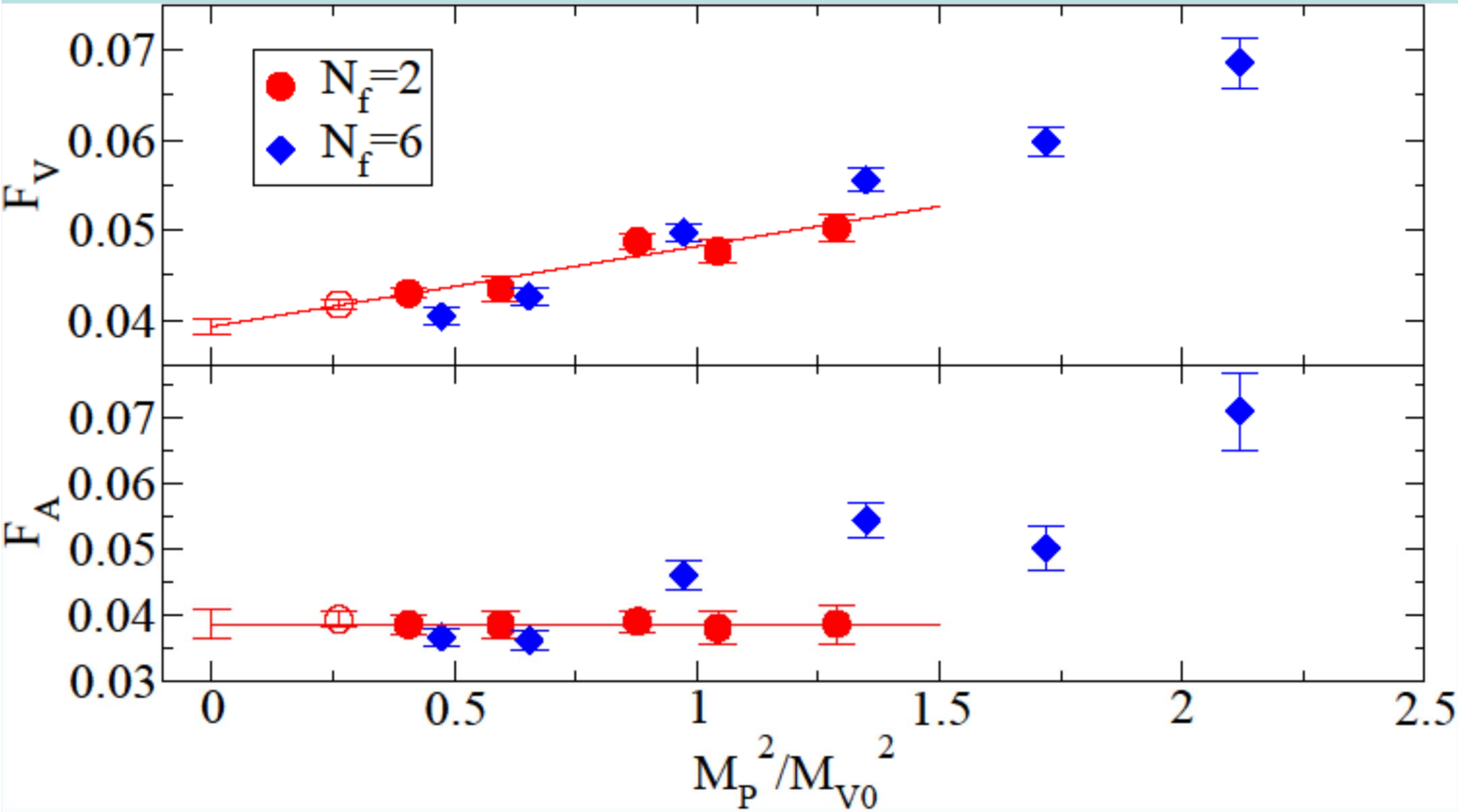
$M_P L > 4$

11

Vector and Axial-Vector Masses



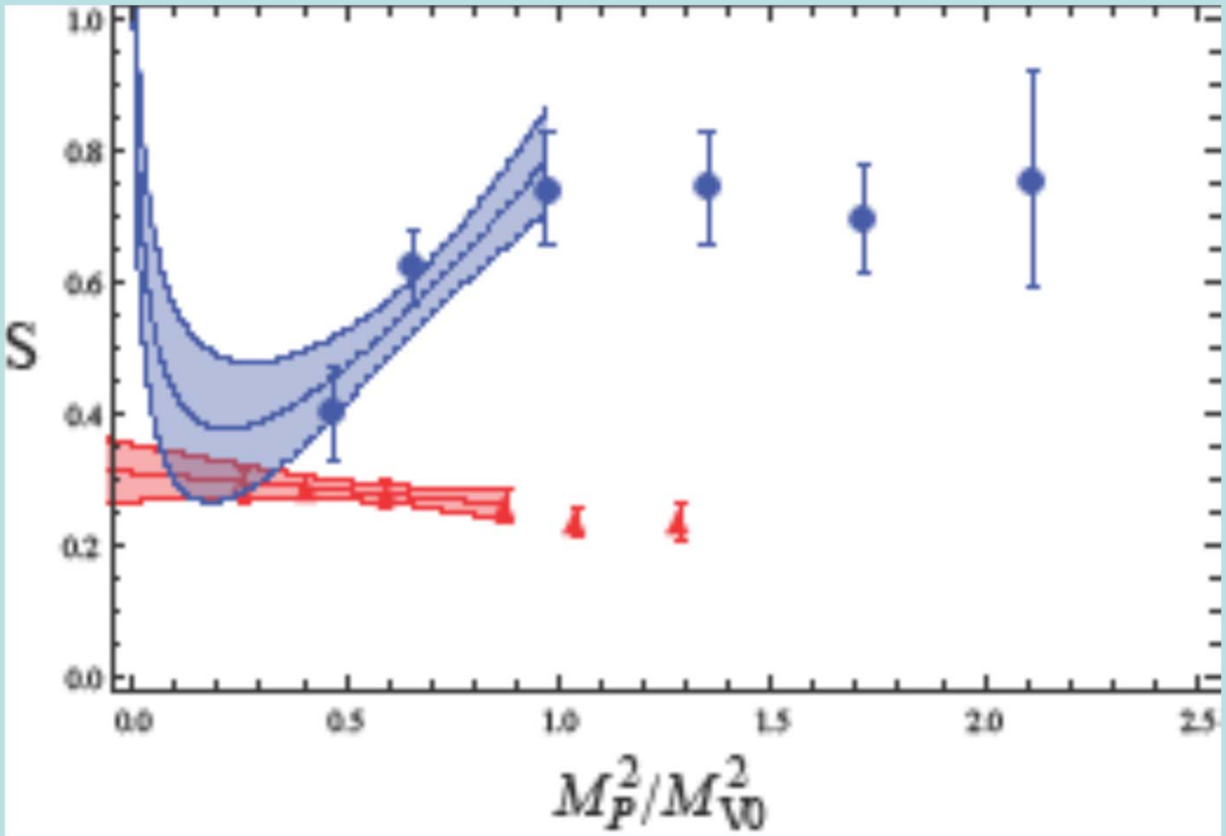
12



N_f=2
N_f=6
F_V
F_A
0.07
0.06
0.05
0.04
0.07
0.06
0.05
0.04
0.03
0
0.5
1
1.5
2
2.5
M_P^2/M_{V0}^2

13

S Parameter



3 EW doublets

DelDebbio et al
arXiv:0909.4931
Shintani et al
arXiv:0806.4222

Extrapolation:

Nf = 2: S is smooth

Nf = 6: S ~ 1/12π [N_f^2/4 – 1] log (1/m)

Cut off by PNGB masses

14

Features

When N_f is increased from 2 to 6:

1. The lightest vector and axial states become more parity doubled.

2. The S parameter per electroweak doublet decreases
(In the chiral limit m → 0, the full answer will depend logarithmically on PNGB masses.)

Single pole dominance ($S = 4\pi [F_V^2 / M_V^2 - F_A^2 / M_A^2]$) works to within 20% at $N_f = 2$ and at least as well at $N_f = 6$, showing the relative decrease of S per electroweak doublet.

15

Current Projects

1. SU(3) $N_f = 10$ LSD arXiv: 1204.6000

   Consistent with Conformality $\gamma^* = 1.10 \pm 0.17$
   But finite-volume, topology, …

2. SU(2) LSD coming soon

   $N_f = 6$ Looking broken

3. Big question: Light 0^{++} State ?

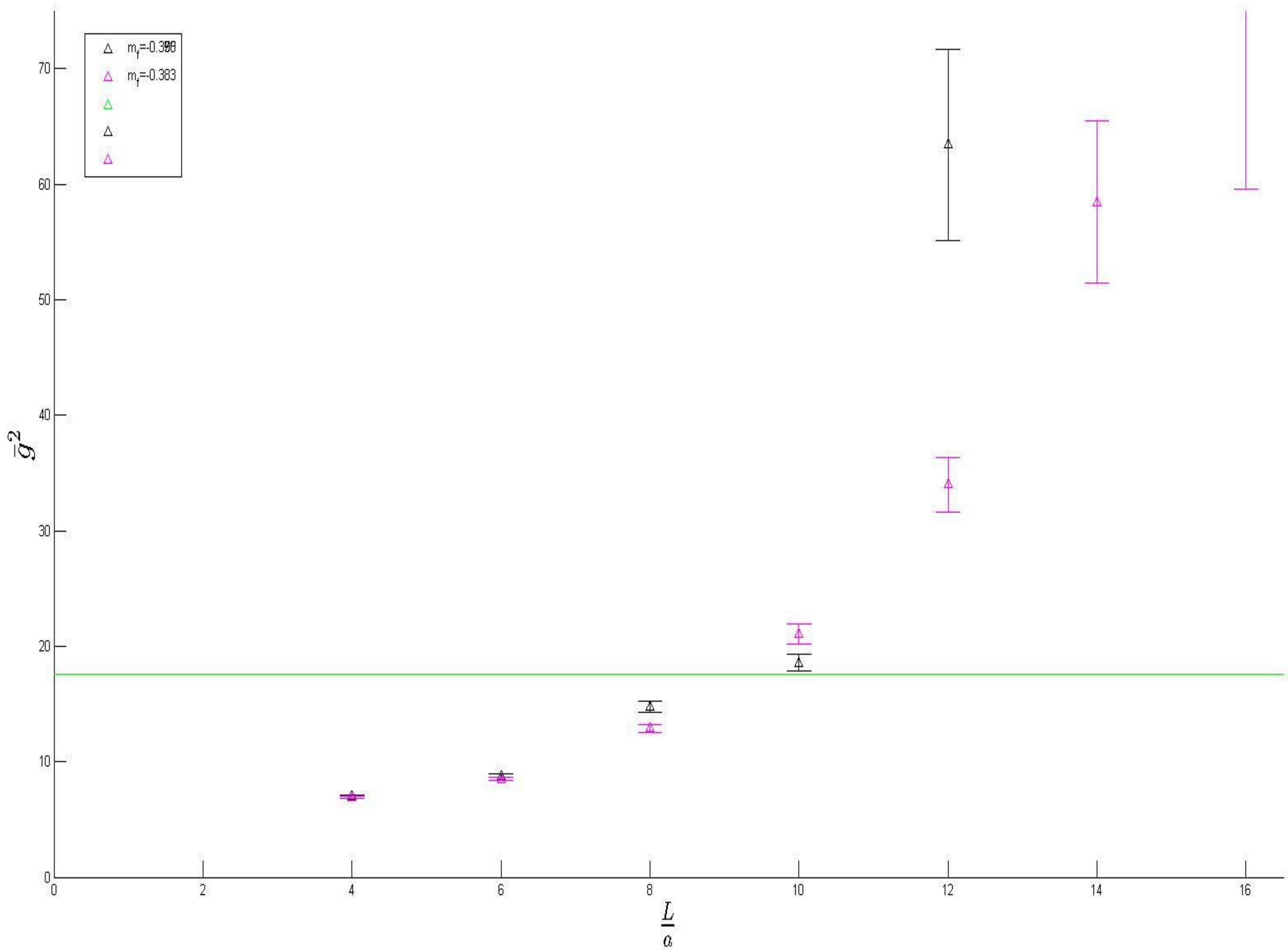
SU(3) $N_f = 10$ LSD arXiv: 1204.6000

Topology : Ordered and Disordered starts

Finite-Volume Effects

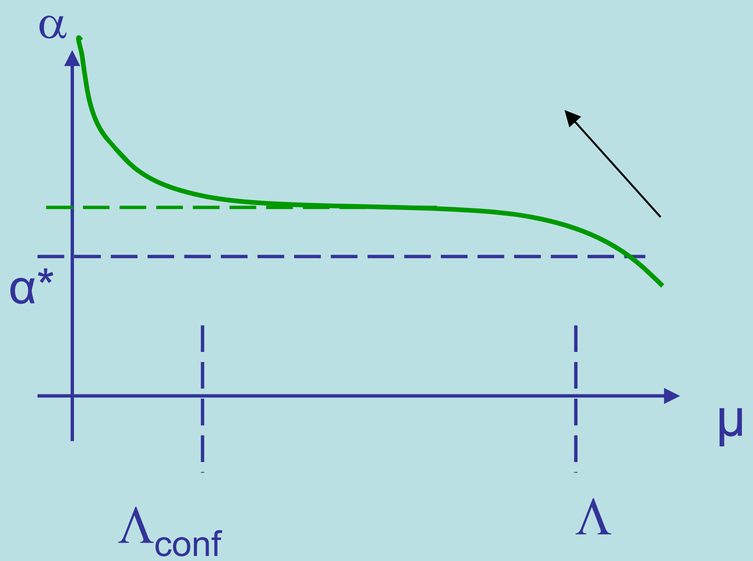
Consistent with Conformality $\gamma^* = 1.10 \pm 0.17$

……



Running Coupling SU(2) , Nf = 6

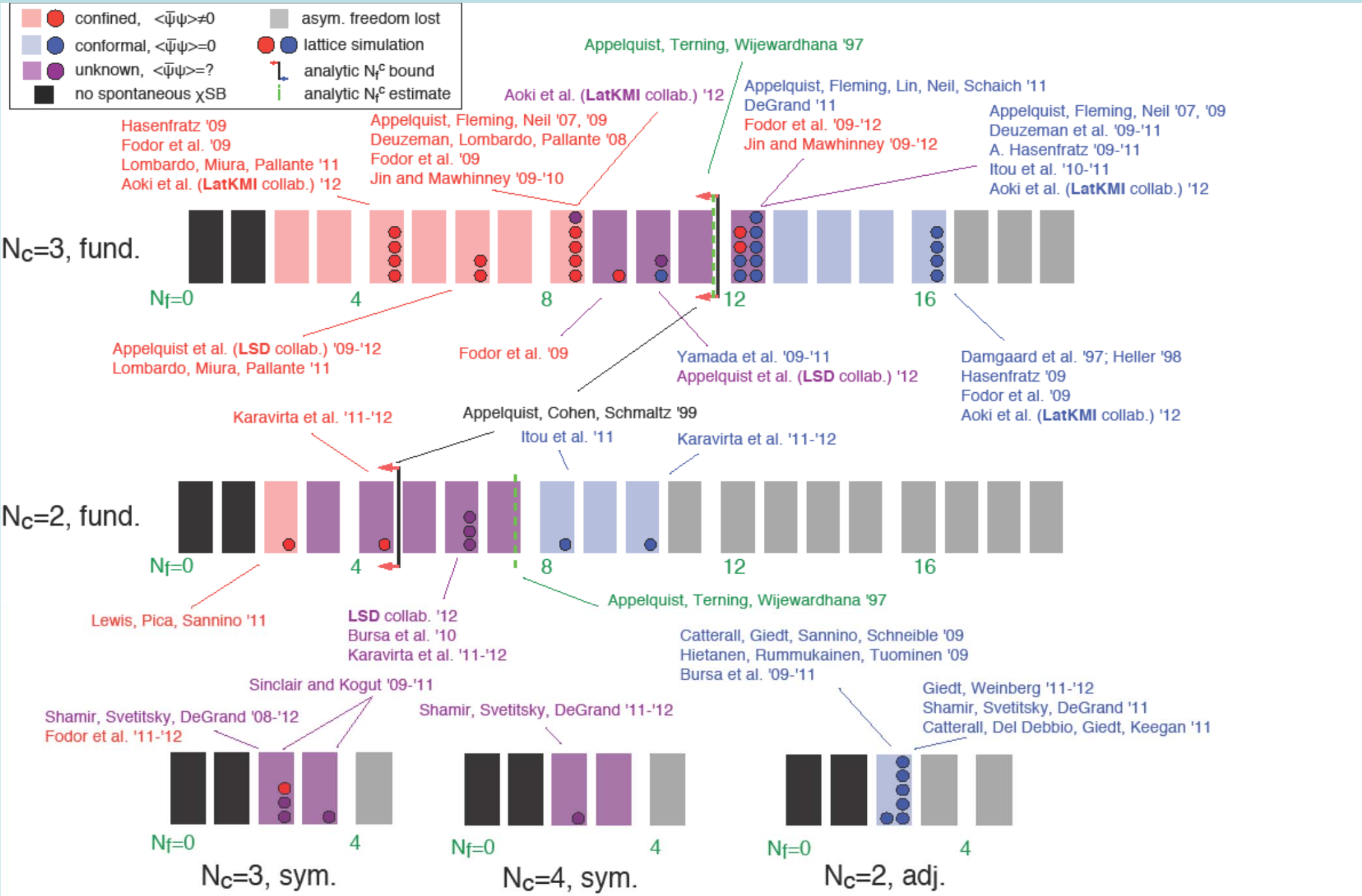
Dilaton ?



An (approximate) NGB (a PNGB) associated with the spontaneous breaking of (approximate) scale symmetry

Yang Bai and TA arXiv: 1006.4375
PRL 104:071601, 2010

Dilaton Phenomenology:
Goldberger, Grinstein, Skiba PRL 2008


confined, <ψ̄ψ>≠0
conformal, <ψ̄ψ>=0
unknown, <ψ̄ψ>=?
no spontaneous χSB
asym. freedom lost
lattice simulation
analytic N_f^c bound
analytic N_f^c estimate
N_c=3, fund.
N_c=2, fund.
N_c=3, sym.
N_c=4, sym.
N_c=2, adj.
N_f=0
4
8
12
16
Appelquist, Terning, Wijewardhana '97
Appelquist, Fleming, Lin, Neil, Schaich '11
DeGrand '11
Fodor et al. '09-'12
Jin and Mawhinney '09-'12
Aoki et al. (LatKMI collab.) '12
Appelquist, Fleming, Neil '07, '09
Deuzeman, Lombardo, Pallante '08
Fodor et al. '09
Jin and Mawhinney '09-'10
Hasenfratz '09
Fodor et al. '09
Lombardo, Miura, Pallante '11
Aoki et al. (LatKMI collab.) '12
Appelquist, Fleming, Neil '07, '09
Deuzeman, Lombardo, Pallante '08-'11
A. Hasenfratz '09-'11
Itou et al. '10-'11
Aoki et al. (LatKMI collab.) '12
Appelquist et al. (LSD collab.) '09-'12
Lombardo, Miura, Pallante '11
Fodor et al. '09
Yamada et al. '09-'11
Appelquist et al. (LSD collab.) '12
Damgaard et al. '97; Heller '98
Hasenfratz '09
Fodor et al. '09
Aoki et al. (LatKMI collab.) '12
Appelquist, Cohen, Schmaltz '99
Karavirta et al. '11-'12
Itou et al. '11
Karavirta et al. '11-'12
Lewis, Pica, Sannino '11
LSD collab. '12
Bursa et al. '10
Karavirta et al. '11-'12
Appelquist, Terning, Wijewardhana '97
Catterall, Giedt, Sannino, Schneible '09
Hietanen, Rummukainen, Tuominen '09
Bursa et al. '09-'11
Sinclair and Kogut '09-'11
Shamir, Svetitsky, DeGrand '08-'12
Fodor et al. '11-'12
Shamir, Svetitsky, DeGrand '11-'12
Giedt, Weinberg '11-'12
Shamir, Svetitsky, DeGrand '11
Catterall, Del Debbio, Giedt, Keegan '11