On the effective string theory of confining flux tubes

Michael Teper (Oxford) - GGI 2012

- Flux tubes and string theory :

  effective string theories - recent analytic progress

  fundamental flux tubes in D=2+1

  fundamental flux tubes in D=3+1

  higher representation flux tubes

- Concluding remarks

gauge theory and string theory

$\leftrightarrow$

A long history ...

- Veneziano amplitude
- 't Hooft large-N – genus diagram expansion
- Polyakov action
- Maldacena ... AdS/CFT/QCD ...

at large N, flux tubes and perhaps the whole gauge theory can be described by a weakly-coupled string theory calculate the spectrum of closed flux tubes

$\longrightarrow$ close around a spatial torus of length l :

- flux localised in 'tubes'; long flux tubes, $l\sqrt{\sigma} \gg 1$ look like 'thin strings'
- at $l = l_c = 1/T_c$ there is a 'deconfining' phase transition: 1st order for $N \geq 3$ in $D = 4$ and for $N \geq 4$ in $D = 3$
- so may have a simple string description of the closed string spectrum for all $l \geq l_c$
- most plausible at $N \to \infty$ where scattering, mixing and decay, e.g string $\to$ string + glueball, go away
- in both D=2+1 and D=3+1

Note: the static potential $V(r)$ describes the transition in r between UV (Coulomb potential) and IF (flux tubes) physics; potentially of great interest as $N \to \infty$.

Some References

recent analytic work:

Luscher and Weisz, hep-th/0406205; Drummond, hep-th/0411017.

Aharony with Karzbrun, Field, Klinghoffer, Dodelson, arXiv:0903.1927; 1008.2636; 1008.2648; 1111.5757; 1111.5758

recent numerical work:

closed flux tubes:
Athenodorou, Bringoltz, MT, arXiv:1103.5854, 1007.4720, ... ,0802.1490, 0709.0693

open flux tubes and Wilson loops:
Caselle, Gliozzi, et al ..., arXiv:1202.1984, 1107.4356, ...

also

Brandt, arXiv:1010.3625; Lucini,..., 1101.5344; ......

historical aside:

for the ground state energy of a long flux tube, not only

$$E_0(l) \stackrel{l\to\infty}{=} \sigma l$$

but also the leading correction is 'universal'

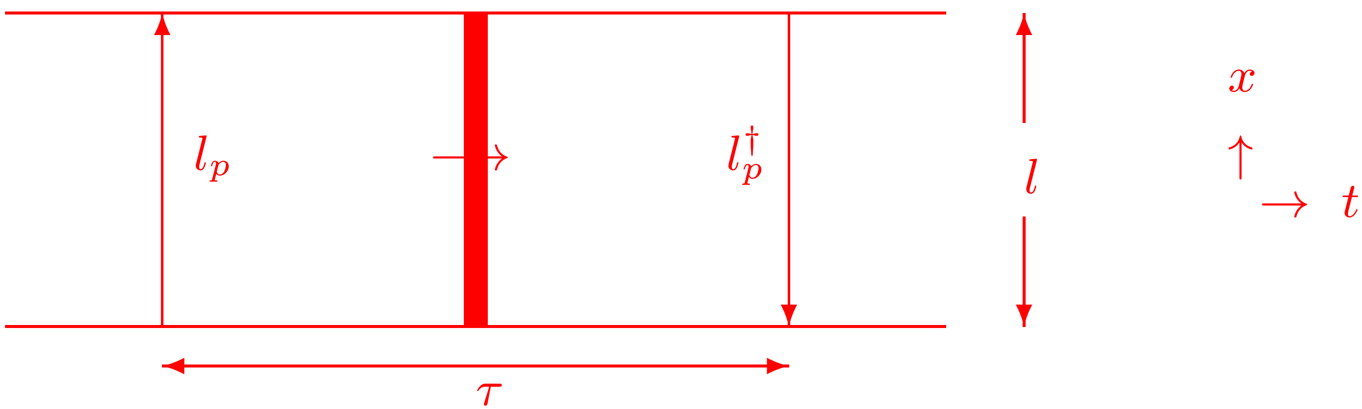
$$E_0(l) = \sigma l - \frac{\pi(D-2)}{6}\frac{1}{l} + O(1/l^3)$$

the famous Luscher correction (1980/1)

calculate the energy spectrum of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops (Wilson lines):

$$\langle l_p^\dagger(\tau) l_p(0)\rangle = \sum_{n,p_\perp} c_n(p_\perp, l) e^{-E_n(p_\perp,l)\tau} \overset{\tau\to\infty}{\propto} \exp\{-E_0(l)\tau\}$$

in pictures



a flux tube sweeps out a cylindrical $l \times \tau$ surface $S \cdots$ integrate over these world sheets with an effective string action $\propto \int_{cyl=l\times\tau} dS e^{-S_{eff}[S]}$

$\Rightarrow$

$$\langle l_p^\dagger(\tau) l_p(0)\rangle = \sum_{n,p_\perp} c_n(p_\perp, l) e^{-E_n(p_\perp,l)\tau} = \int_{cyl=l\times\tau} dS e^{-S_{eff}[S]}$$

where $S_{eff}[S]$ is the effective string action for the surface S

$\Rightarrow$

the string partition function will predict the spectrum $E_n(l)$ – just a Laplace transform – but will be constrained by the Lorentz invariance encoded in $E_n(p_\perp, l)$

Luscher and Weisz; Meyer

this can be extended from a cylinder to a torus (Aharony)

$$Z^{w=1}_{torus}(l,\tau) = \sum_{n,p} e^{-E_n(p,l)\tau} = \sum_{n,p} e^{-E_n(p,\tau)l} = \int_{T^2=l\times\tau} dS e^{-S_{eff}[S]}$$

where p now includes both transverse and longitudinal momenta

$\leftrightarrow$

'closed-closed string duality'

Example: Gaussian approximation:

$$S_{G,eff} = \sigma l \tau + \int_0^\tau dt \int_0^l dx \tfrac{1}{2} \partial_\alpha h \partial_\alpha h$$

$\Rightarrow$

$$Z_{cyl}(l,\tau) = \sum_n e^{-E_n(\tau) l} = \int_{cyl=l\times\tau} dS e^{-S_{G,eff}[S]} = e^{-\sigma l \tau} |\eta(q)|^{-(D-2)} \quad : \; q = e^{-\pi l/\tau}$$

in terms of the Dedekind eta function: $\eta(q) = q^{\frac{1}{24}} \prod_{n=1}^\infty (1 - q^n)$

$\Rightarrow$ open string energies and degeneracies

$$E_n(\tau) = \sigma\tau + \tfrac{\pi}{\tau} \left\{ n - \tfrac{1}{24}(D-2) \right\}$$

– the famous universal Luscher correction(1981)

Also : modular invariance of $\eta(q) \rightarrow$ closed string energies,

$$\hat{E}_n(l) = \sigma l + \tfrac{4\pi}{l} \{ n - \tfrac{1}{24}(D-2) \} + O(1/l^3)$$

So what do we know today?

any S_{eff} $\Rightarrow$ ground state energy

$$E_0(l) \stackrel{l\to\infty}{=} \sigma l - \frac{\pi(D-2)}{6l} - \frac{\{\pi(D-2)\}^2}{72}\frac{1}{\sigma l^3} - \frac{\{\pi(D-2)\}^3}{432}\frac{1}{\sigma^2 l^5} + O\left(\frac{1}{l^7}\right)$$

with universal terms:

- $\circ\ O\left(\frac{1}{l}\right)$ Luscher correction, $\sim$ 1980
- $\circ\ O\left(\frac{1}{l^3}\right)$ Luscher, Weisz; Drummond, $\sim$ 2004
- $\circ\ O\left(\frac{1}{l^5}\right)$ Aharony et al, $\sim$ 2009-10

and similar results for $E_n(l)$, but only to $O(1/l^3)$ in $D = 3 + 1$

$\sim$ simple free string theory : Nambu-Goto in flat space-time up to $O(1/l^7)$

Nambu-Goto free string theory

$$\int \mathcal{D}S e^{-\kappa A[S]}$$

spectrum (Arvis 1983, Luscher-Weisz 2004):

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma \left(\frac{N_L + N_R}{2} - \frac{D-2}{24} \right) + \left(\frac{2\pi q}{l} \right)^2 .$$

$p = 2\pi q/l =$ total momentum along string;

$N_L, N_R =$ sum left and right 'phonon' momentum:

$$N_L = \sum_{k>0} n_L(k)\, k, \quad N_R = \sum_{k>0} n_R(k)\, k, \quad N_L - N_R = q$$

so the ground state energy is:

$$E_0(l) = \sigma l \left(1 - \frac{\pi(D-2)}{3} \frac{1}{\sigma l^2} \right)^{1/2}$$

$$\text{state} = \prod_{k>0} a_k^{n_L(k)} a_{-k}^{n_R(k)} |0\rangle \quad , \quad P = (-1)^{number\ phonons}$$

lightest $p = 0$ states:

$|0\rangle$

$a_1 a_{-1}|0\rangle$

$a_2 a_{-2}|0\rangle,\ a_2 a_{-1} a_{-1}|0\rangle,\ a_1 a_1 a_{-2}|0\rangle,\ a_1 a_1 a_{-1} a_{-1}|0\rangle$

$\ldots$

lightest $p \neq 0$ states:

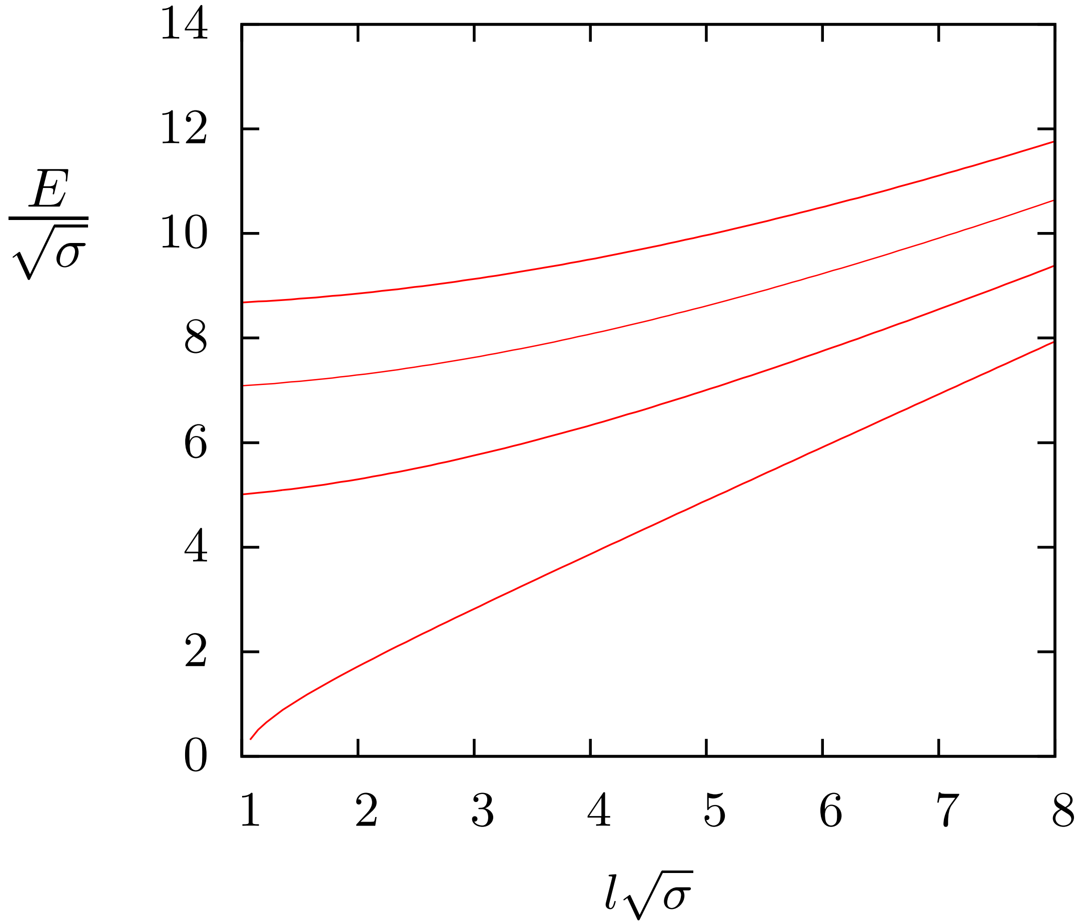
$a_1|0\rangle$ $\qquad P = -,\ p = 2\pi/l$

$a_2|0\rangle$ $\qquad P = -,\ p = 4\pi/l$

$a_1 a_1|0\rangle$ $\qquad P = +,\ p = 4\pi/l$

$\Rightarrow$

$\Rightarrow$ lightest states with $p = 0$ solid lines: Nambu-Goto



gs : P=+ . ex1 : P=+. ex2: 2$\times$ P=+ and 2$\times$ P=-

So what does one find numerically?

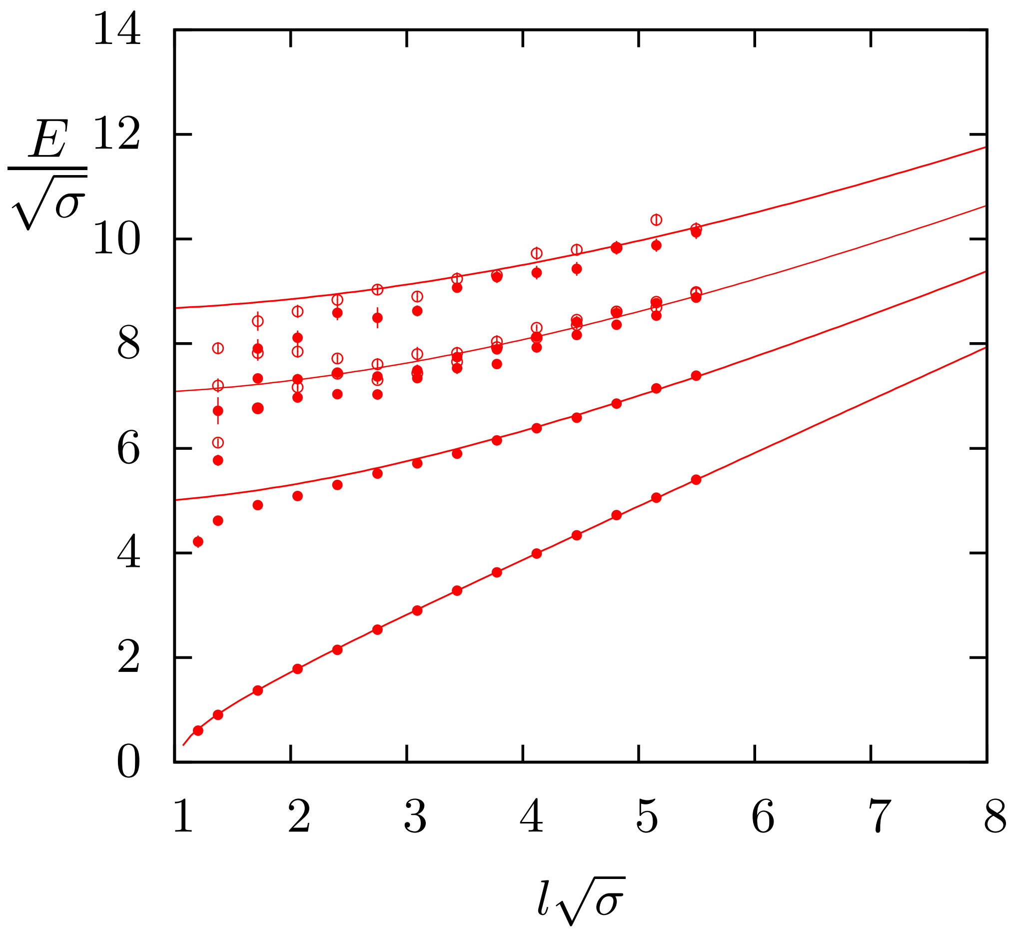
results here are from:

- $D = 2+1$ Athenodorou, Bringoltz, MT, arXiv:1103.5854, 0709.0693
- $D = 3+1$ Athenodorou, Bringoltz, MT, arXiv:1007.4720
- higher rep Athenodorou, MT, in progress

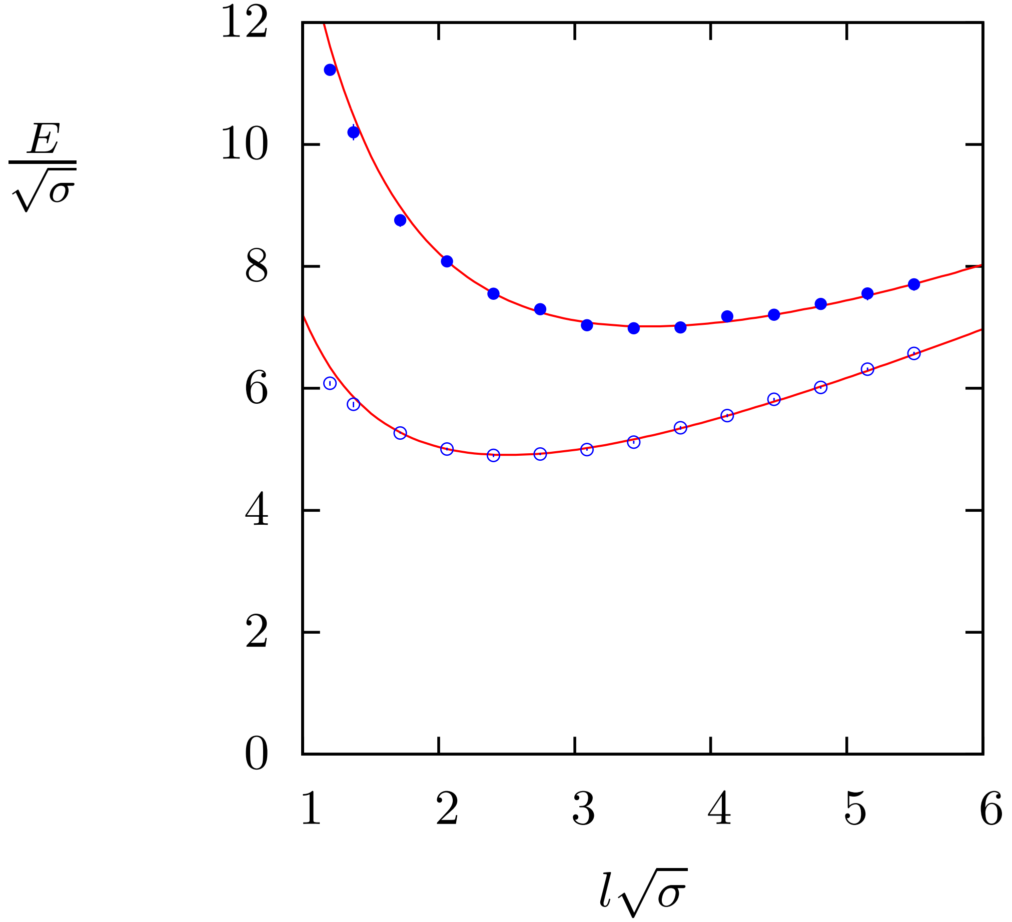
and we start with:

$D = 2+1,\ SU(6),\ a\sqrt{\sigma} \simeq 0.086$ i.e $N \sim \infty,\ a \sim 0$

lightest 8 states with $p = 0$ $P = +(\bullet), P = -(\circ)$



solid lines: Nambu-Goto ground state $\rightarrow$ σ: only parameter lightest levels with $p = 2\pi q/l,\ 4\pi q/l$ $P = -$



Nambu-Goto : solid lines

Now, when Nambu-Goto is expanded the first few terms are universal e.g. ground state

$$
\begin{aligned}
E_0(l) &= \sigma l \left(1 - \frac{\pi(D-2)}{3\sigma l^2}\right)^{\frac{1}{2}} \\
&\stackrel{l \geq l_0}{=} \sigma l - \frac{\pi(D-2)}{6l} - \frac{\{\pi(D-2)\}^2}{72}\frac{1}{\sigma l^3} - \frac{\{\pi(D-2)\}^3}{432}\frac{1}{\sigma^2 l^5} + O\left(\frac{1}{l^7}\right)
\end{aligned}
$$

where $l_0\sqrt{\sigma} = \sqrt{3/\pi(D-2)}$; and also for excited states for $l\sqrt{\sigma} > l_n\sqrt{\sigma} \sim \sqrt{8\pi n}$

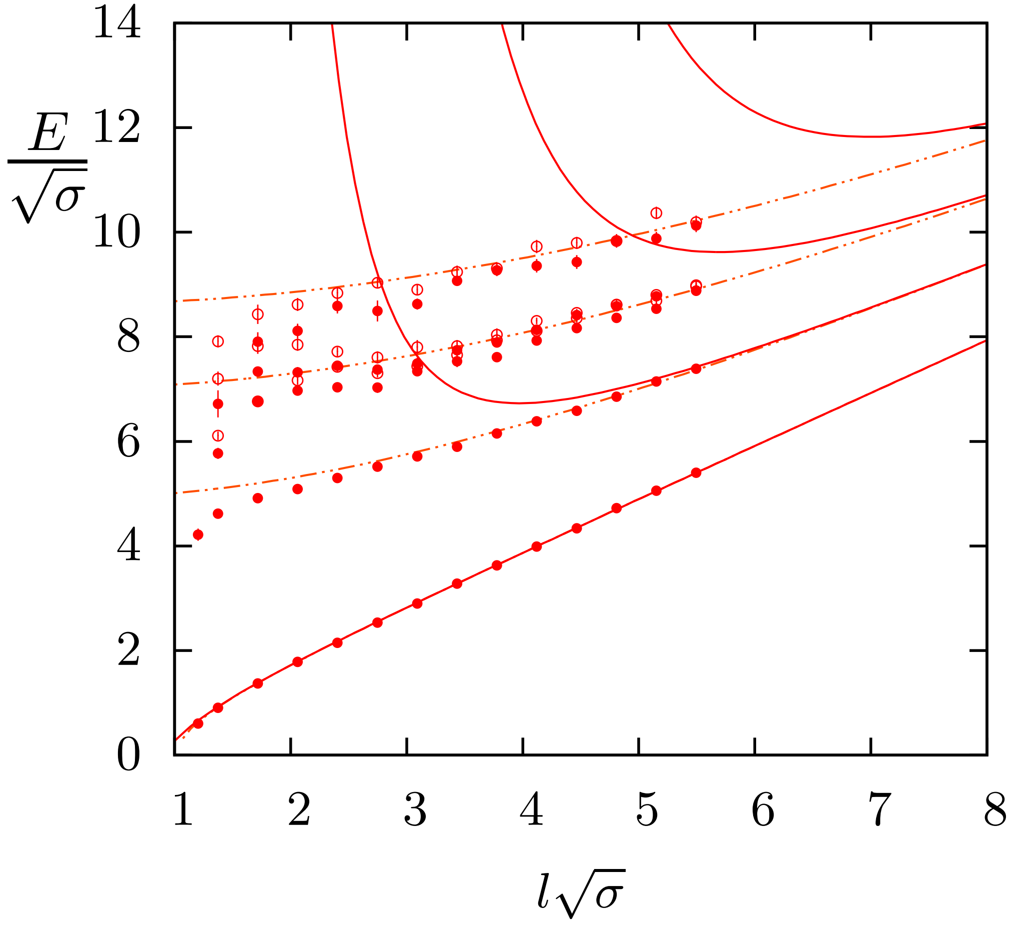
$\Rightarrow$

is the striking numerical agreement with Nambu-Goto no more than an agreement with the sum of the known universal terms?

NO!

universal terms: solid lines

Nambu-Goto : dashed lines



$\Longrightarrow$

• NG very good down to $l\sqrt{\sigma} \sim 2$, i.e energy

fat short flux 'tube' $\sim$ ideal thin string

• NG very good far below value of $l\sqrt{\sigma}$ where the power series expansion diverges, i.e. where all orders are important $\Rightarrow$

universal terms not enough to explain this agreeement ...

• no sign of any non-stringy modes, e.g.

$$E(l) \simeq E_0(l) + \mu \quad \text{where e.g.} \quad \mu \sim M_G/2 \sim 2\sqrt{\sigma}$$

$\Longrightarrow$

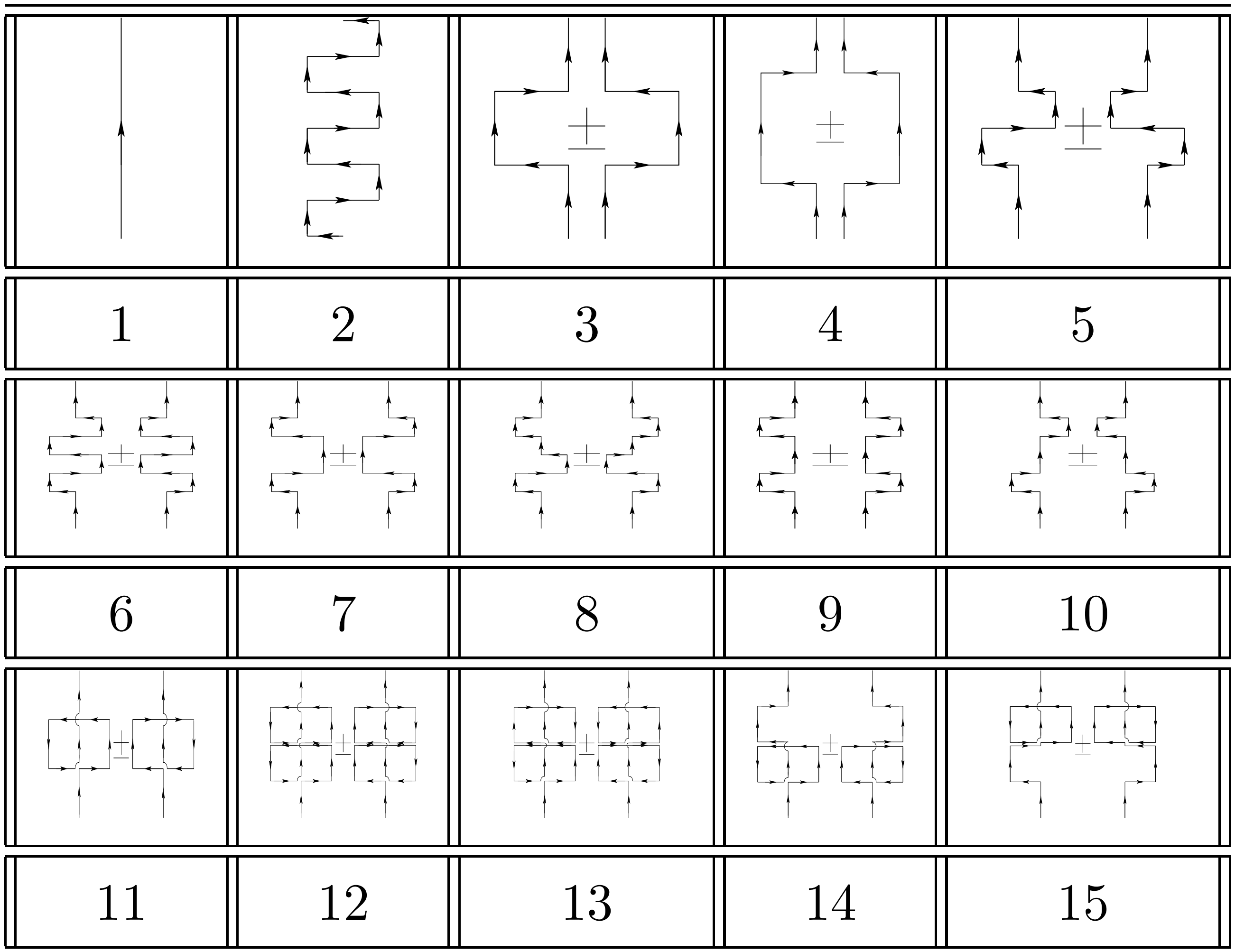
... in more detail ...

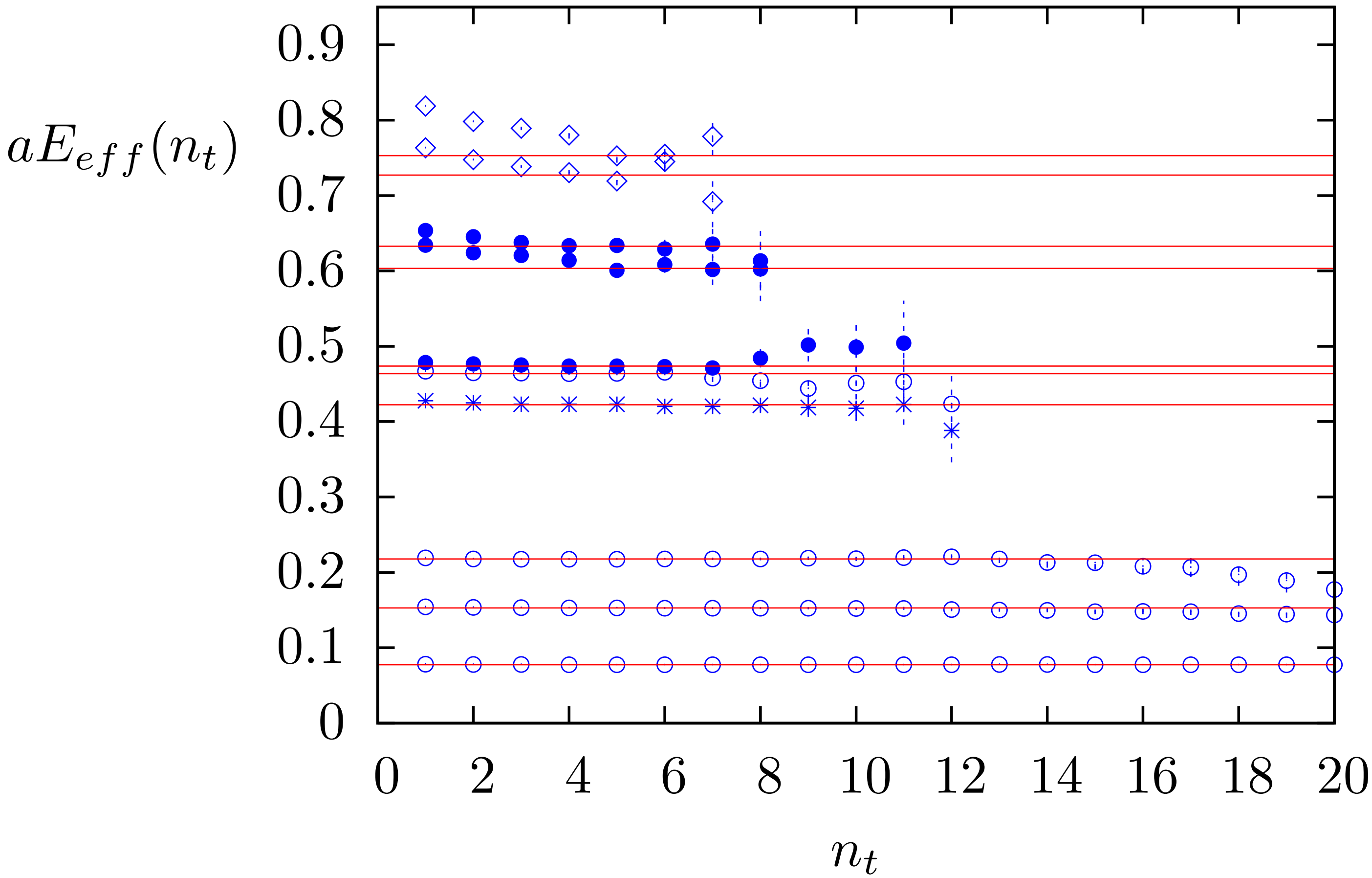
but first an 'algorithmic' aside – calculating energies

- deform Polyakov loops to allow non-trivial quantum numbers
- block or smear links to improve projection on physical excitations
- variational calculation of best operator for each energy eigenstate
- huge basis of loops for good overlap on a large number of states
- i.e. $C(t) \simeq c_n e^{-E_n(l)t}$ already for small t

for example:

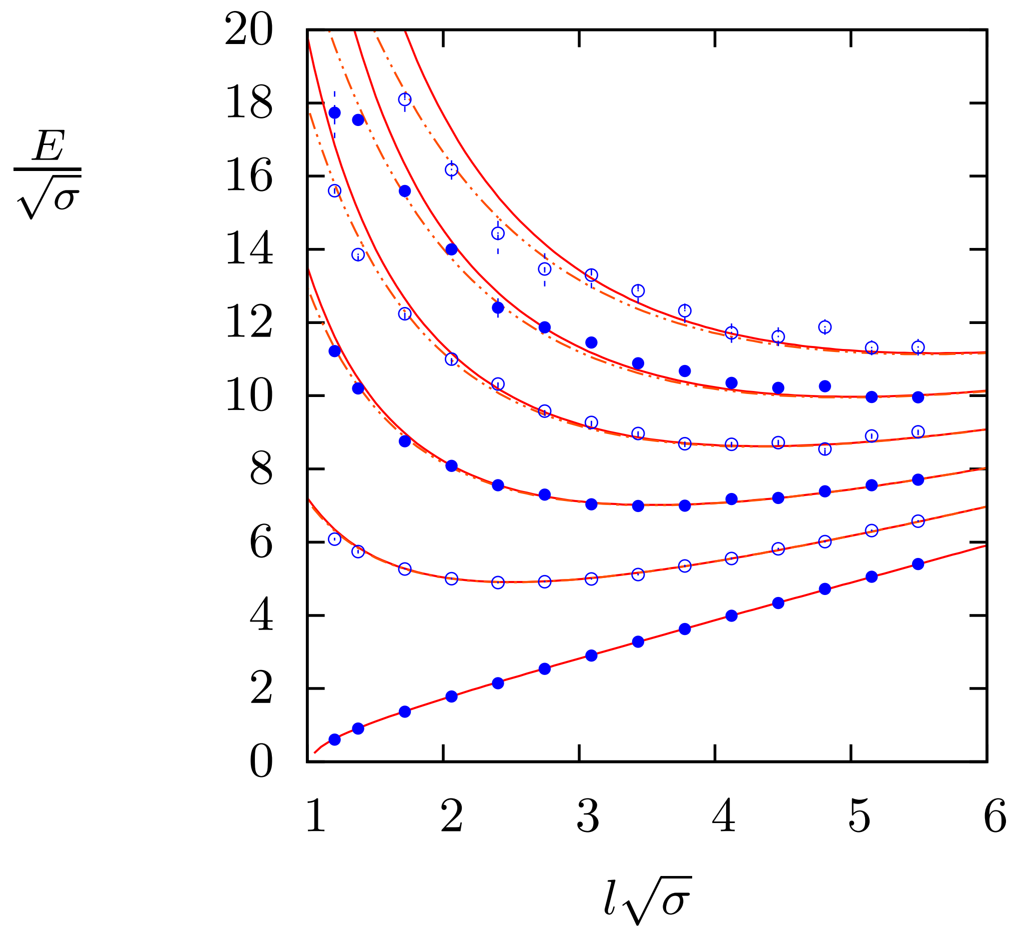
Operators in D=2+1:



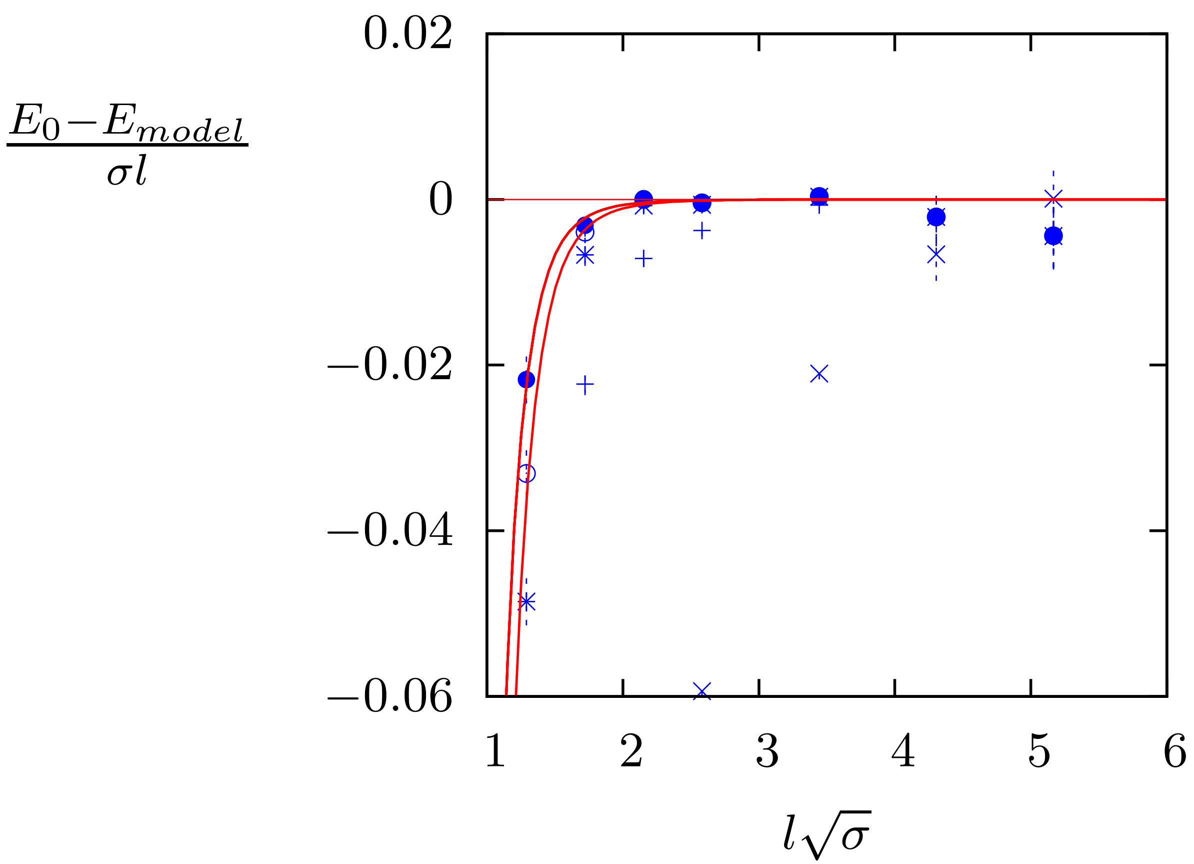


abs gs $l = 16, 24, 32, 64a$ ($\circ$); es p=0 P=+ ($\bullet$); gs $p = 2\pi/l,\ \ P = -$ ($\star$); gs, es $p = 0,\ \ P = -$ ($\diamond$)

lightest $P = -$ states with $p = 2\pi q/l$: $q = 0, 1, 2, 3, 4, 5$ $a_q|0\rangle$



Nambu-Goto : solid lines $(ap)^2 \to 2 - 2\cos(ap)$: dashed lines ground state deviation from various 'models' $D = 2 + 1$



model = Nambu-Goto, $\bullet$, universal to $1/l^5$, $\circ$, to $1/l^3$, $\star$, to $1/l$, $+$, just σl, $\times$

lines = plus $O(1/l^7)$ correction

$\Longrightarrow$

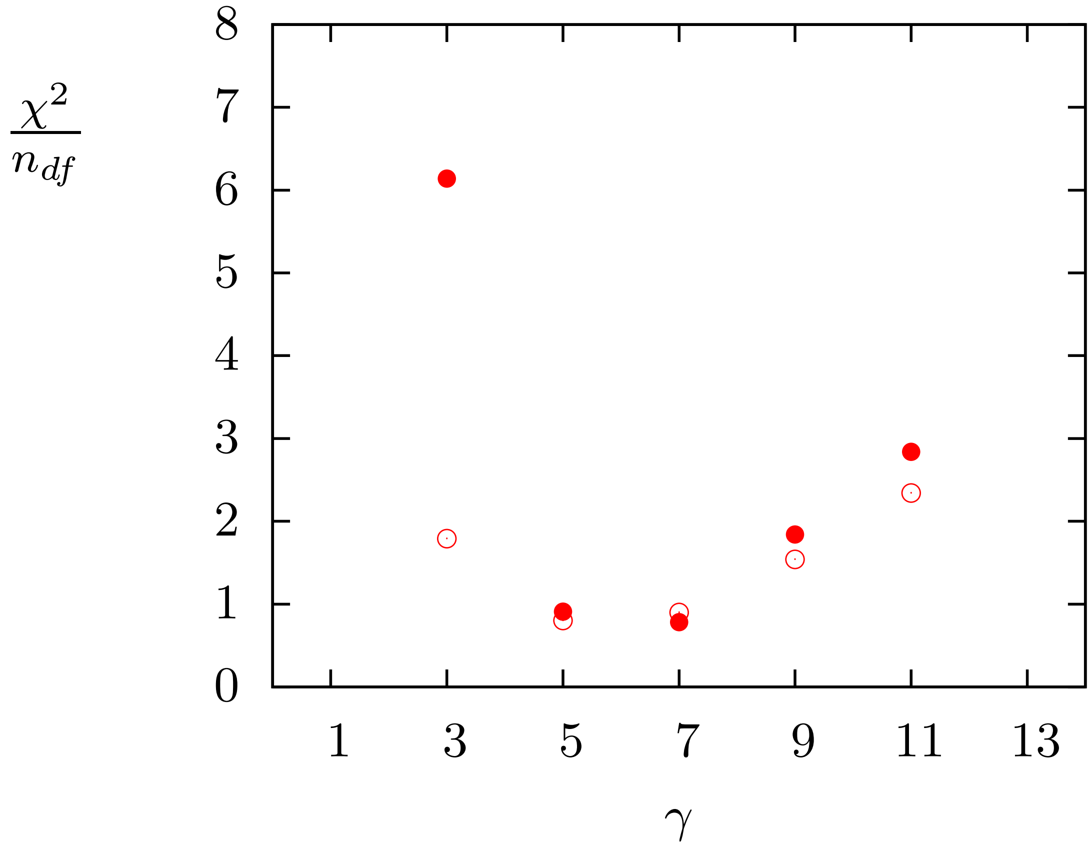
∘ for $l\sqrt{\sigma} \gtrsim 2$ agreement with NG to $\lesssim 1/1000$

moreover

∘ for $l\sqrt{\sigma} \sim 2$ contribution of NG to deviation from σl is $\gtrsim 99\%$

despite flux tube being short and fat

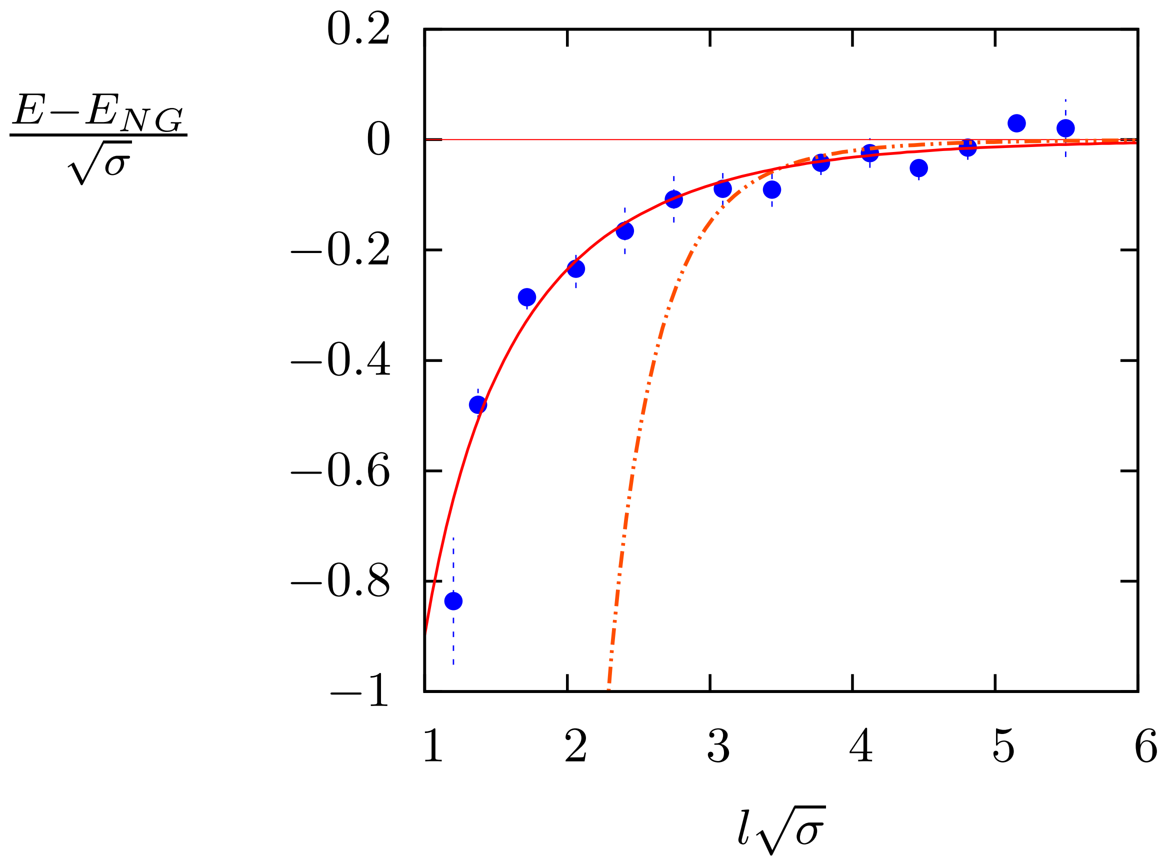
∘ and leading correction to NG consistent with $\propto 1/l^7$ as expected from current universality results



χ^2 per degree of freedom for the best fit

$$E_0(l) = E_0^{NG}(l) + \frac{c}{l^\gamma}$$

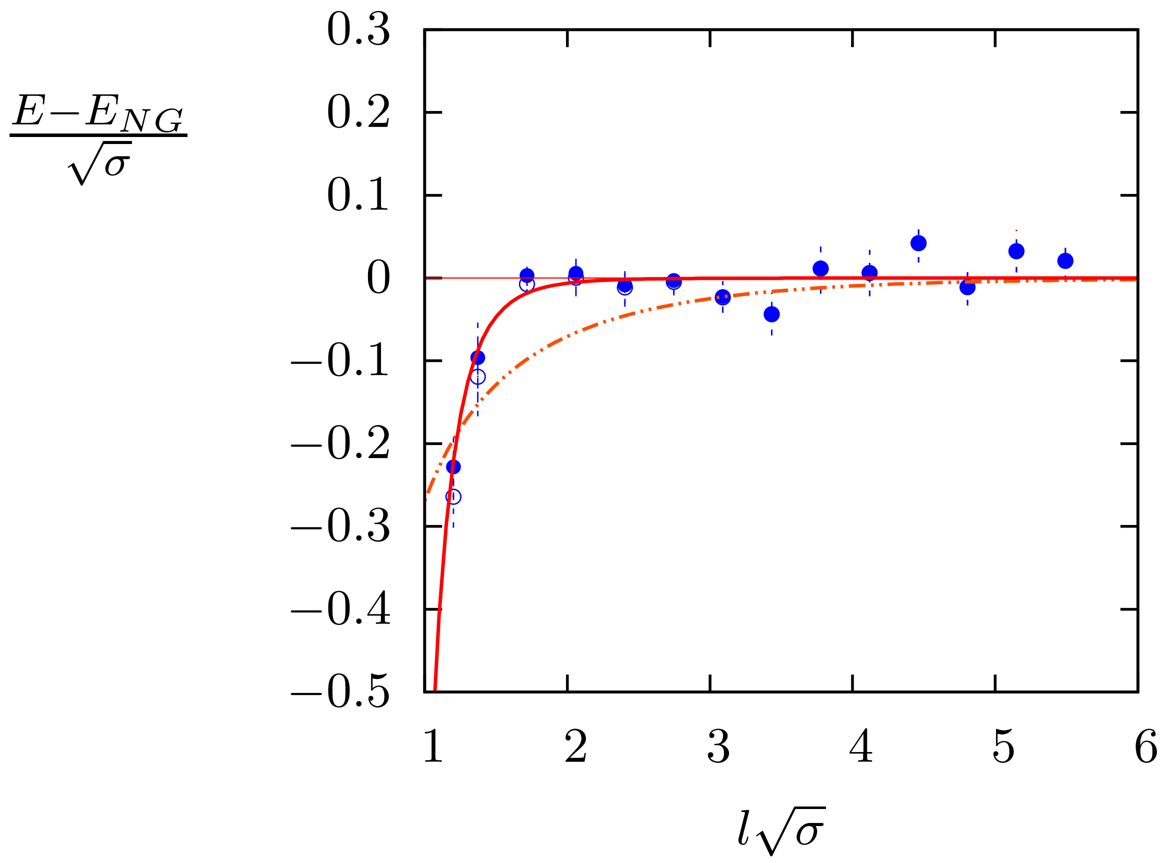
first excited $q = 0$, $P = +$ state $\qquad D = 2+1$



fits:

$\frac{c}{(l\sqrt{\sigma})^7}$ - dotted curve; $\qquad \frac{c}{(l\sqrt{\sigma})^7}\left(1+\frac{25.0}{l^2\sigma}\right)^{-2.75}$ - solid curve

$q = 1, P = -$ ground state $\qquad SU(6),\ D = 2+1$



fits:

$\frac{c}{(l\sqrt{\sigma})^7}$ solid curve; $\frac{c}{(l\sqrt{\sigma})^7}\left(1+\frac{25.0}{l^2\sigma}\right)^{-2.75}$: dashed curve

$D = 2 + 1$: some conclusions

as a few slides earlier $+$

• multi-phonon states with all phonons having $s_{ij} = 0$ have minimal corrections comparable to absolute ground state

$\stackrel{?}{\leftrightarrow}$

derivative interactions means such phonons have zero interactions and corrections

• other excited states have modest corrections, and only at small $l\sqrt{\sigma}$ $\stackrel{?}{\leftrightarrow}$ the corrections to Nambu-Goto resum to a small correction term at small l

$$D = 2 + 1 \qquad \longrightarrow \qquad D = 3 + 1$$

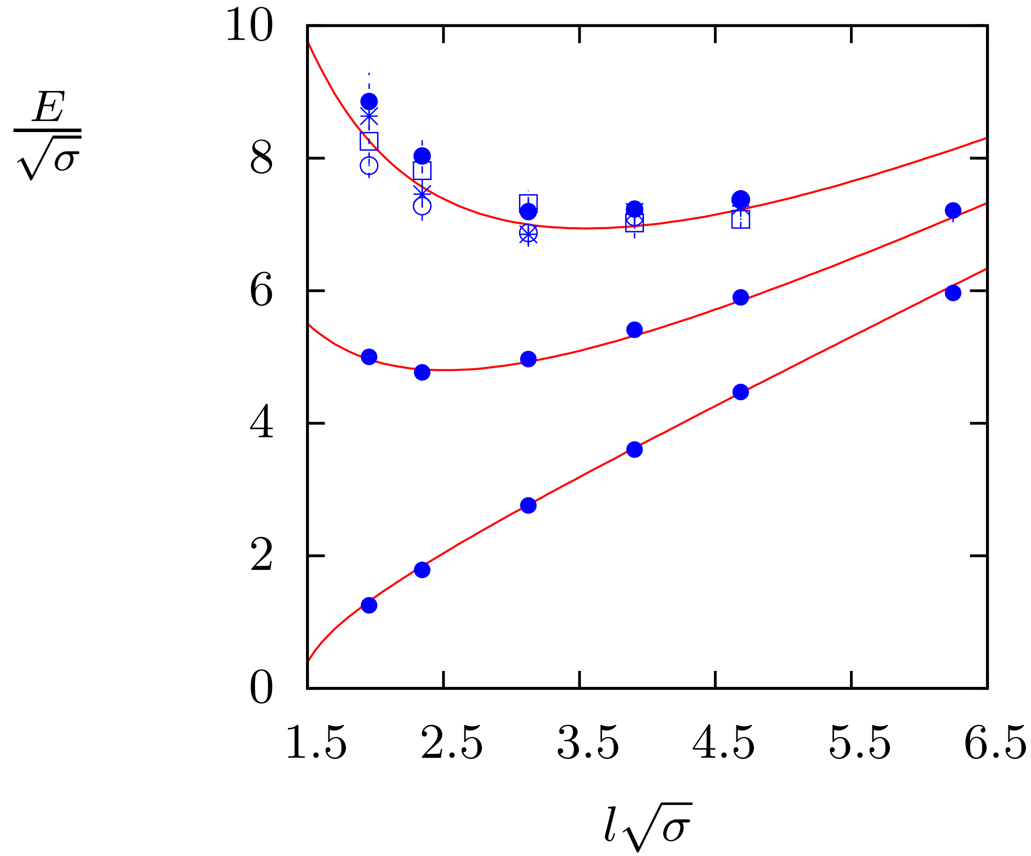
- additional rotational quantum number: phonon carries spin 1

- Nambu-Goto again remarkably good for most states

- BUT now there are some candidates for non-stringy (massive?) mode excitations ...

however in general results are considerably less accurate

$p = 2\pi q/l$ for $q = 0, 1, 2$ $D = 3+1,\ SU(3),\ l_c\sqrt{\sigma} \sim 1.5$

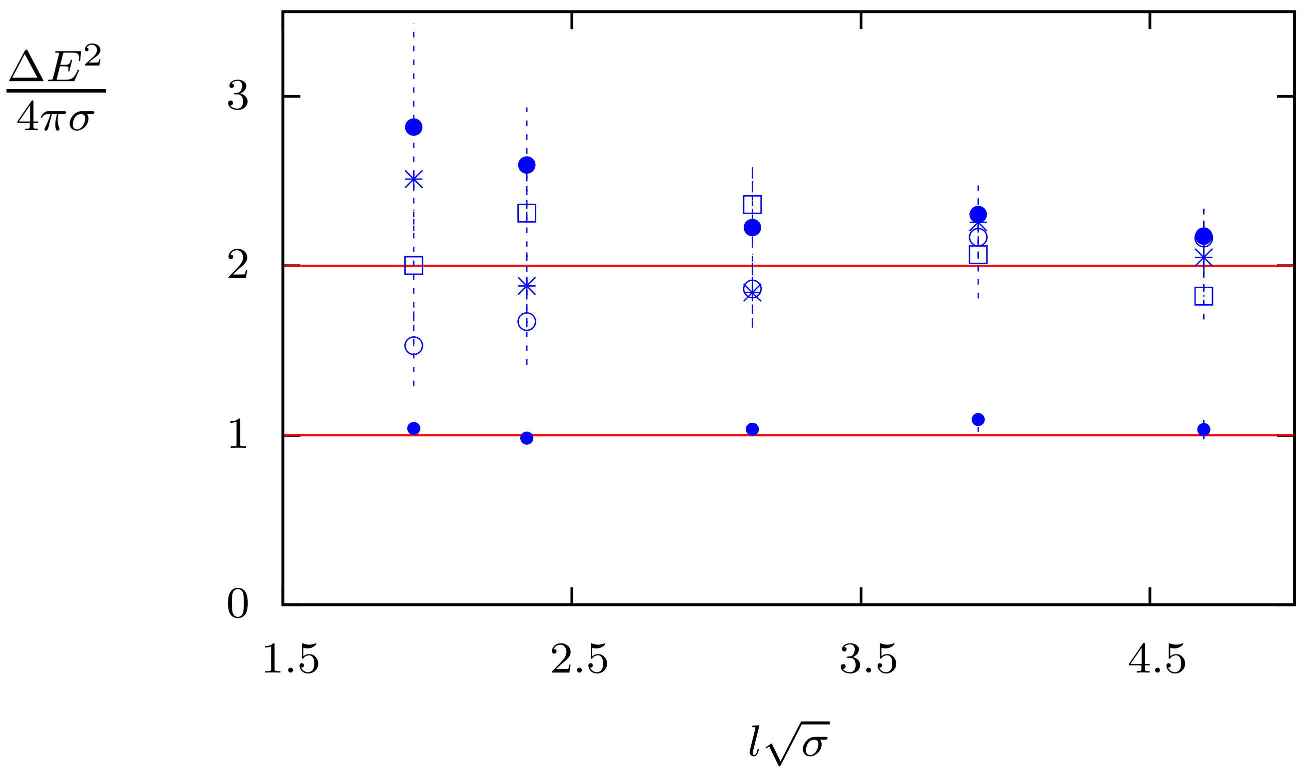


The four $q = 2$ states are: $J^{P_t} = 0^+(\star),\ 1^{\pm}(\circ),\ 2^+(\Box),\ 2^-(\bullet)$.
Lines are Nambu-Goto predictions.

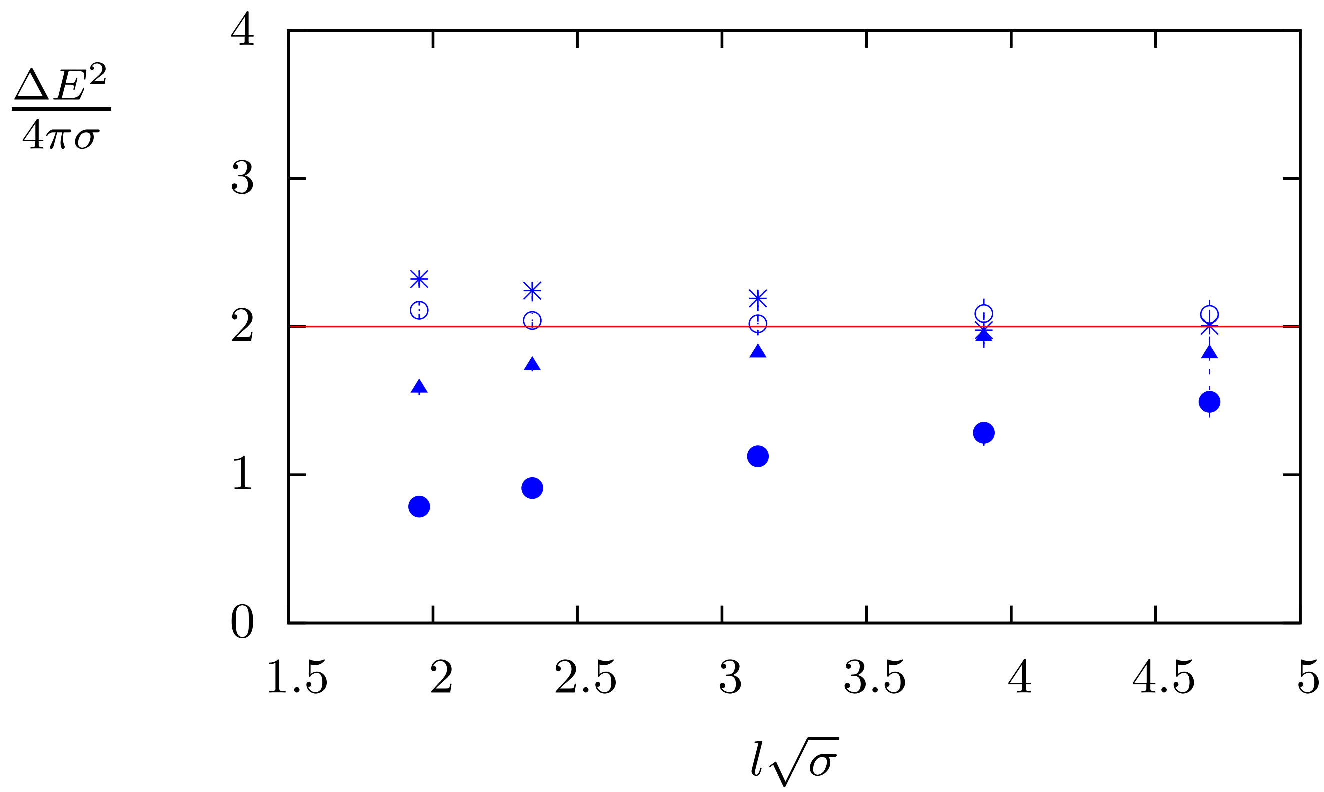
for a precise comparison with Nambu-Goto, define:

$$\Delta E^2(q,l) = E^2(q;l) - E_0^2(l) - \left(\frac{2\pi q}{l}\right)^2 \stackrel{NG}{=} 4\pi\sigma(N_L + N_R)$$

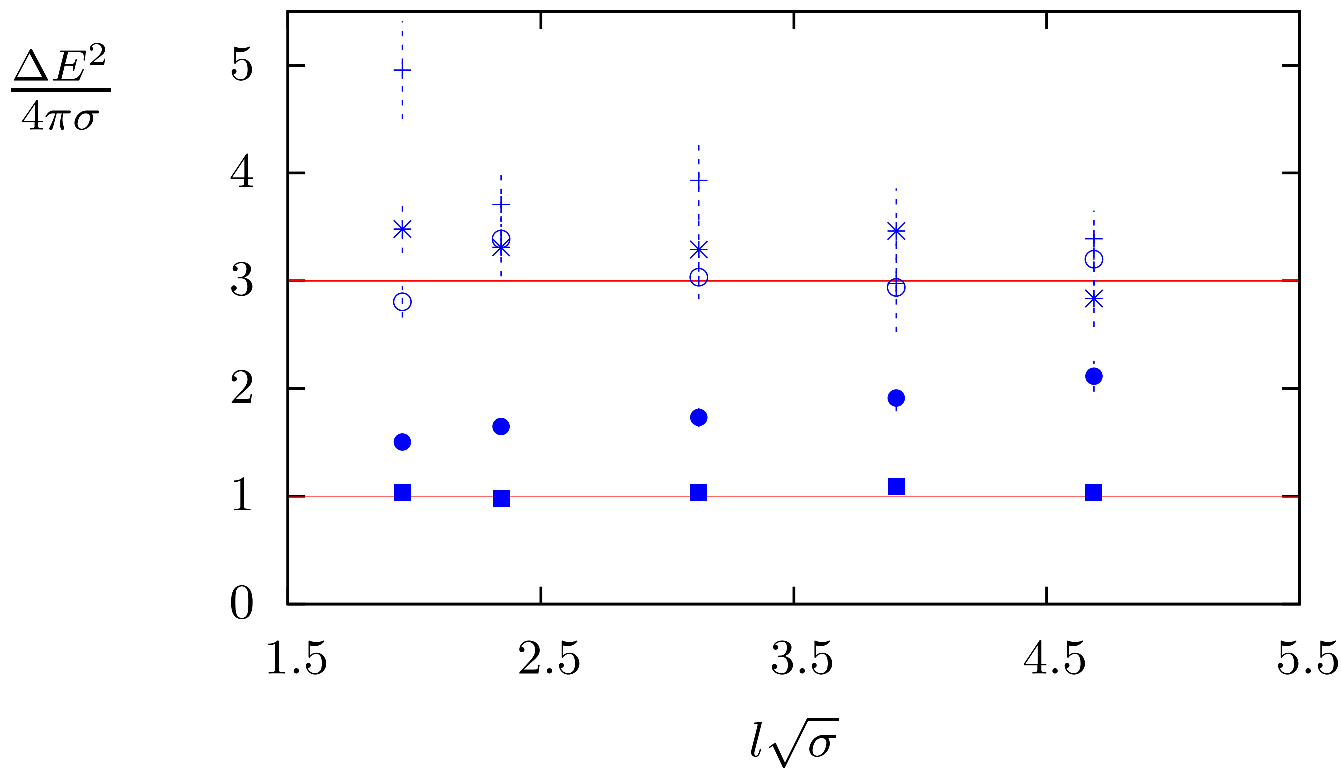
$\Longrightarrow$ lightest $q = 1, 2$ states:



lightest few $p = 0$ states



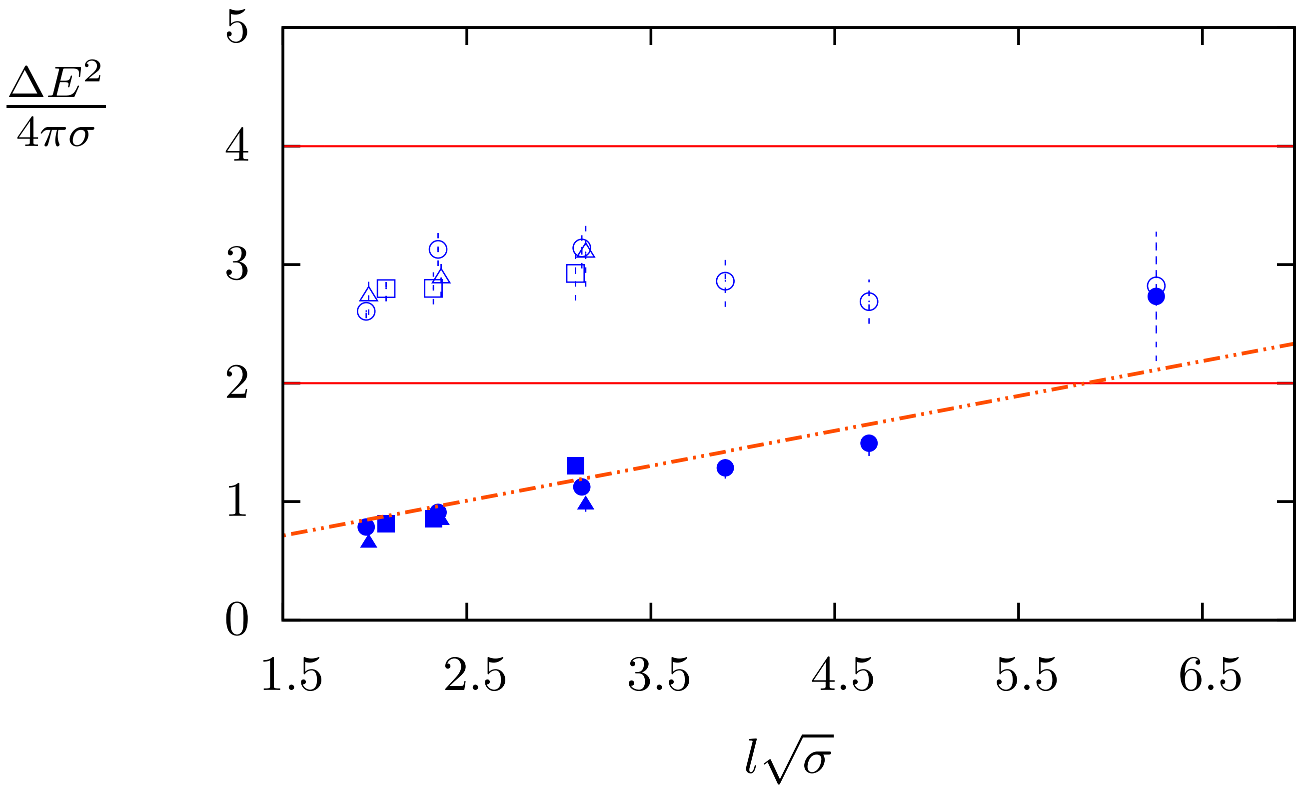
$\Longrightarrow$ anomalous 0^{--} state and also for $p = 2\pi/l$ states



states: $J^{P_t} = 0^+(\circ), 0^-(\bullet), 2^+(*), 2^-(+)$

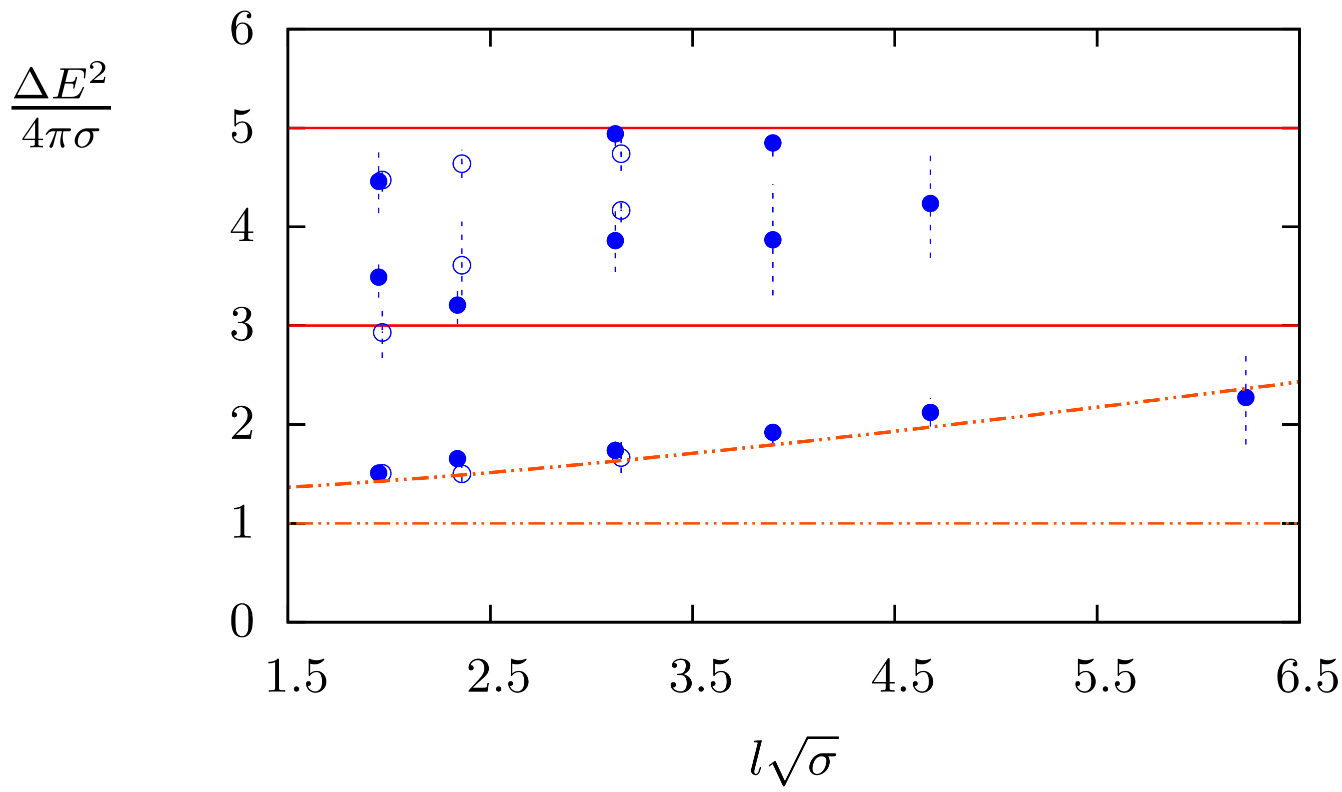
$\Longrightarrow$ anomalous 0^- state

$p = 0,\ 0^{--}$: is this an extra state – is there also a stringy state?



ansatz: $E(l) = E_0(l) + m \quad ; m = 1.85\sqrt{\sigma} \sim m_G/2$

similarly for $p = 1,\ 0^-$: SU(3), •; SU(5), ∘



ansatz: $E(l) = E_0(l) + (m^2 + p^2)^{1/2}$; $m = 1.85\sqrt{\sigma} \sim m_G/2$

fundamental flux $\longrightarrow$ higher representation flux

- k-strings: $f \otimes f \otimes \ldots$ k times, e.g.

$$\phi_{k=2A,S} = \frac{1}{2}\left(\{Tr_f \phi\}^2 \pm Tr_f\{\phi^2\}\right)$$

lightest flux tube for each $k \leq N/2$ is absolutely stable if $\sigma_k < k\sigma_f$ etc.

- binding energy $\Rightarrow$ mass scale $\Rightarrow$ massive modes?

- higher reps at fixed k, e.g. for $k = 1$ in SU(6)

$$f \otimes f \otimes \bar{f} \rightarrow f \oplus f \oplus \underline{84} \oplus \underline{120}$$

- $N \rightarrow \infty$ is not the 'ideal' limit that it is for fundamental flux:

– most 'ground states' are not stable (for larger l)

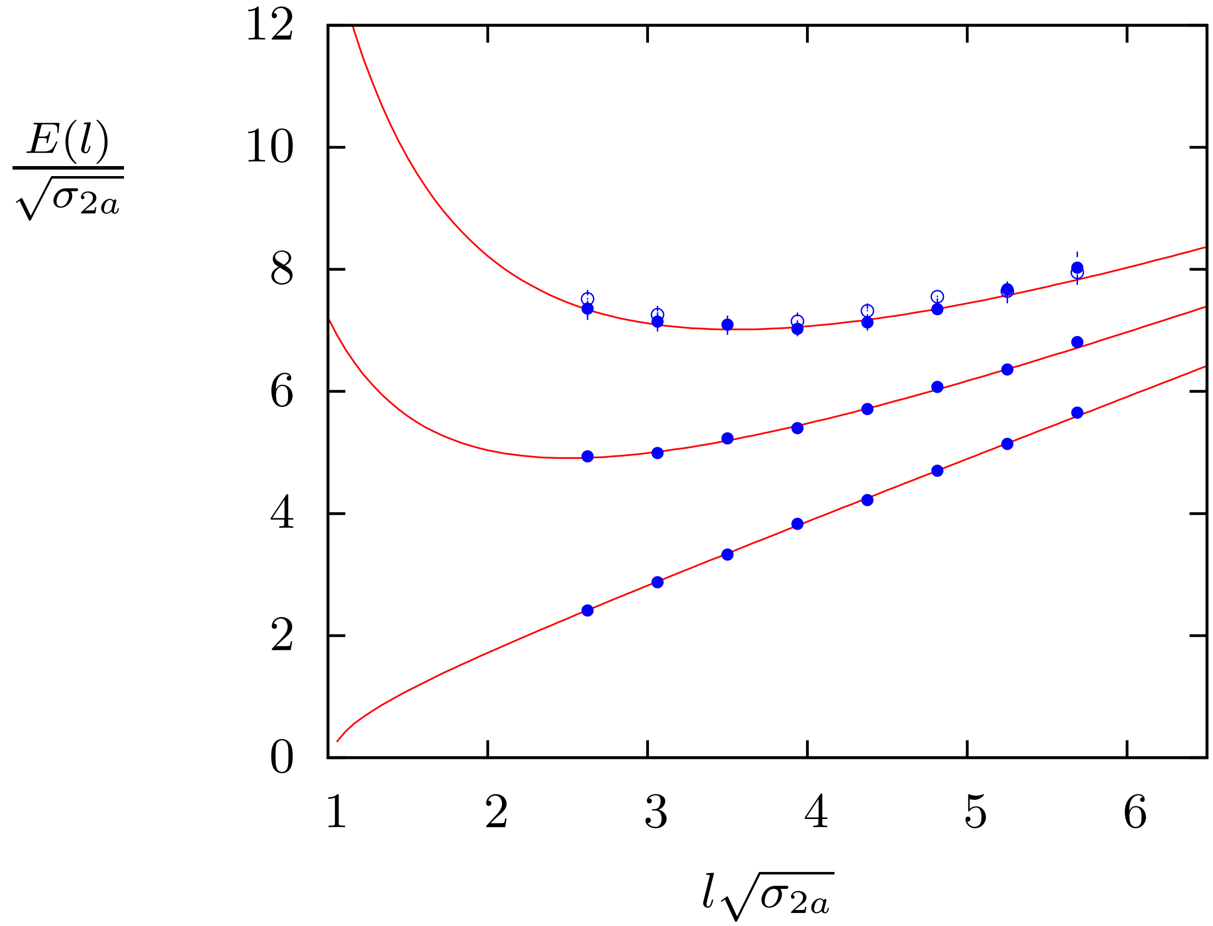
– typically become stable as $N \rightarrow \infty$, but

– $\sigma_k \rightarrow k\sigma_f$: states unbind?

$\longrightarrow$ some $D = 2 + 1$, SU(6) calculations ...

k=2A

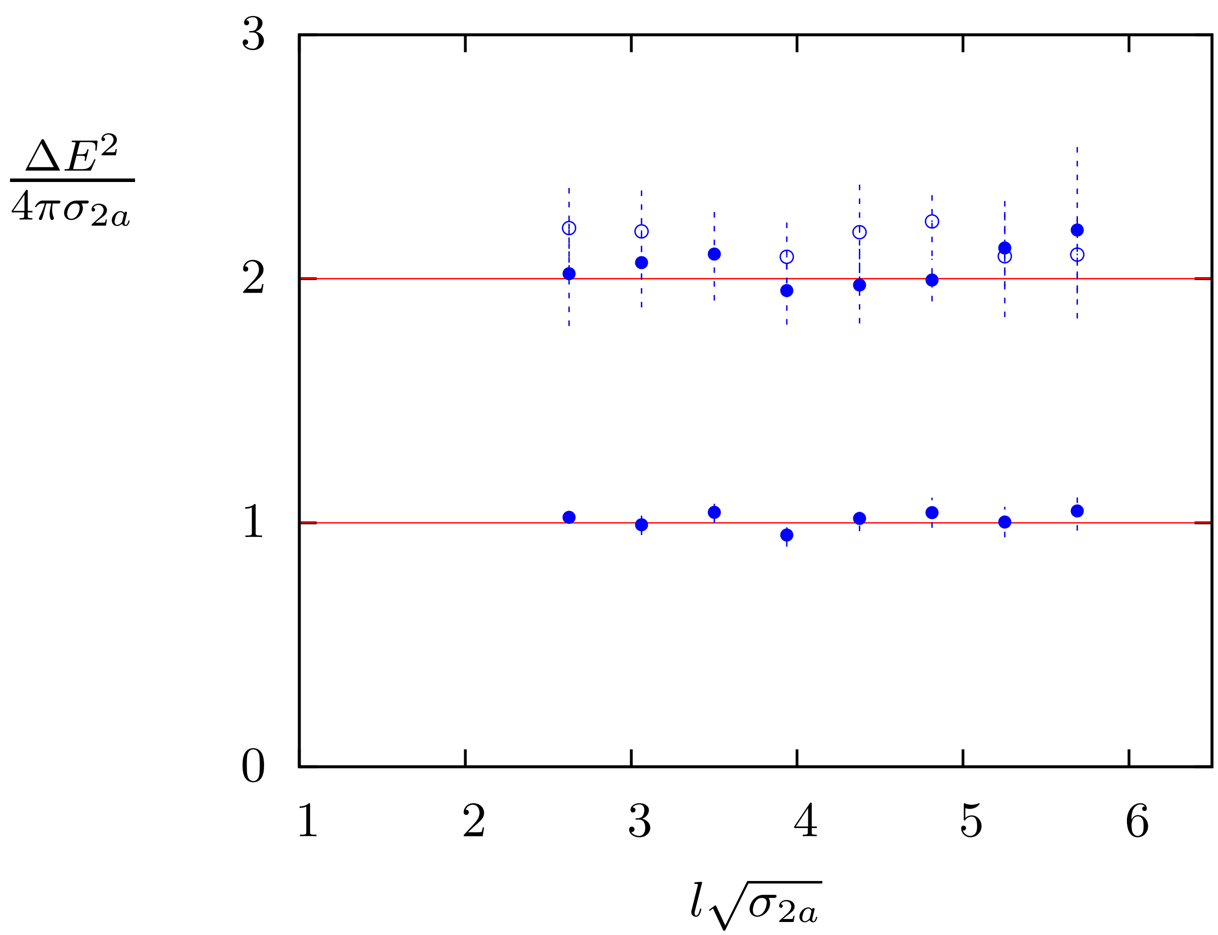
lightest $p = 2\pi q/l$ states with q=0,1,2



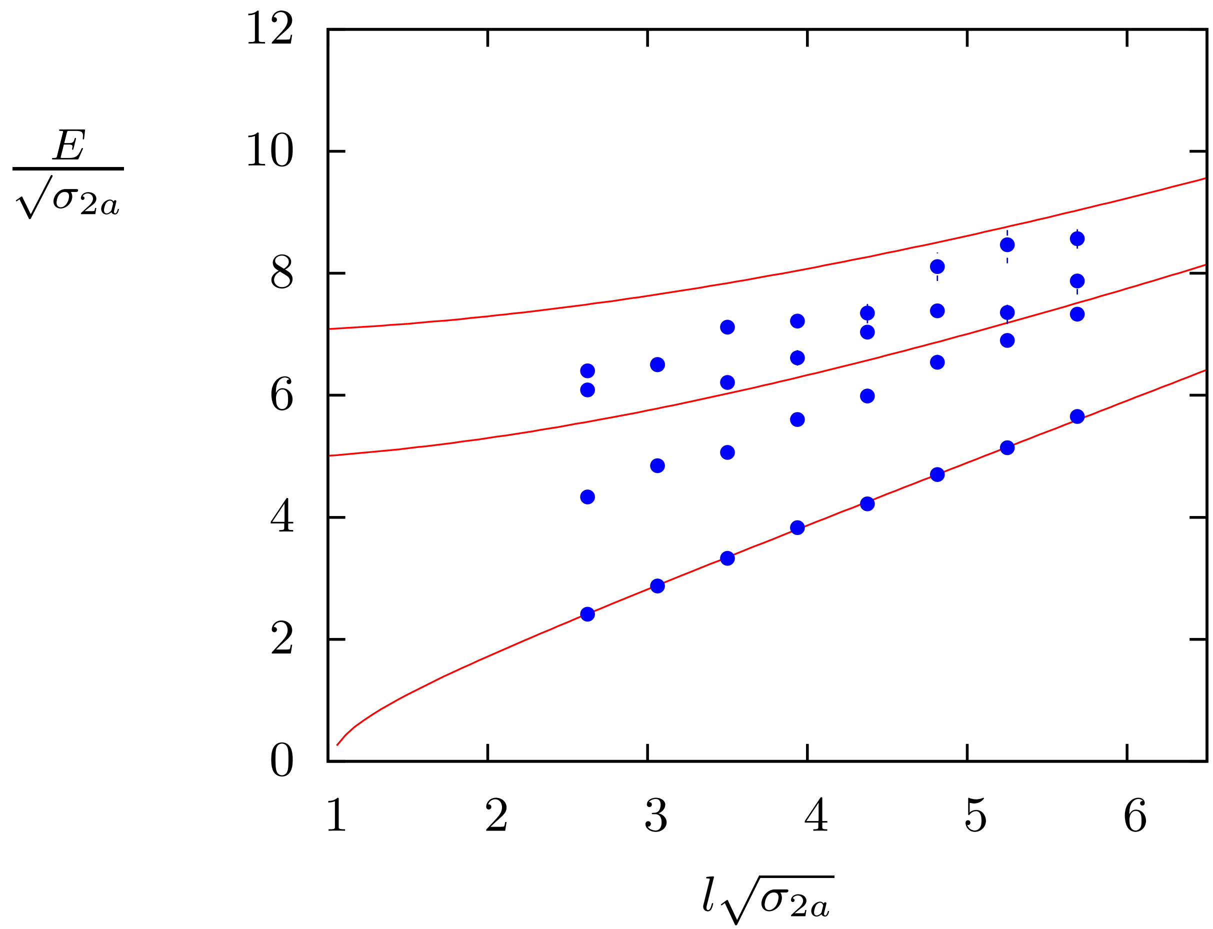
lines are NG

P=- (●), P=+ (○)

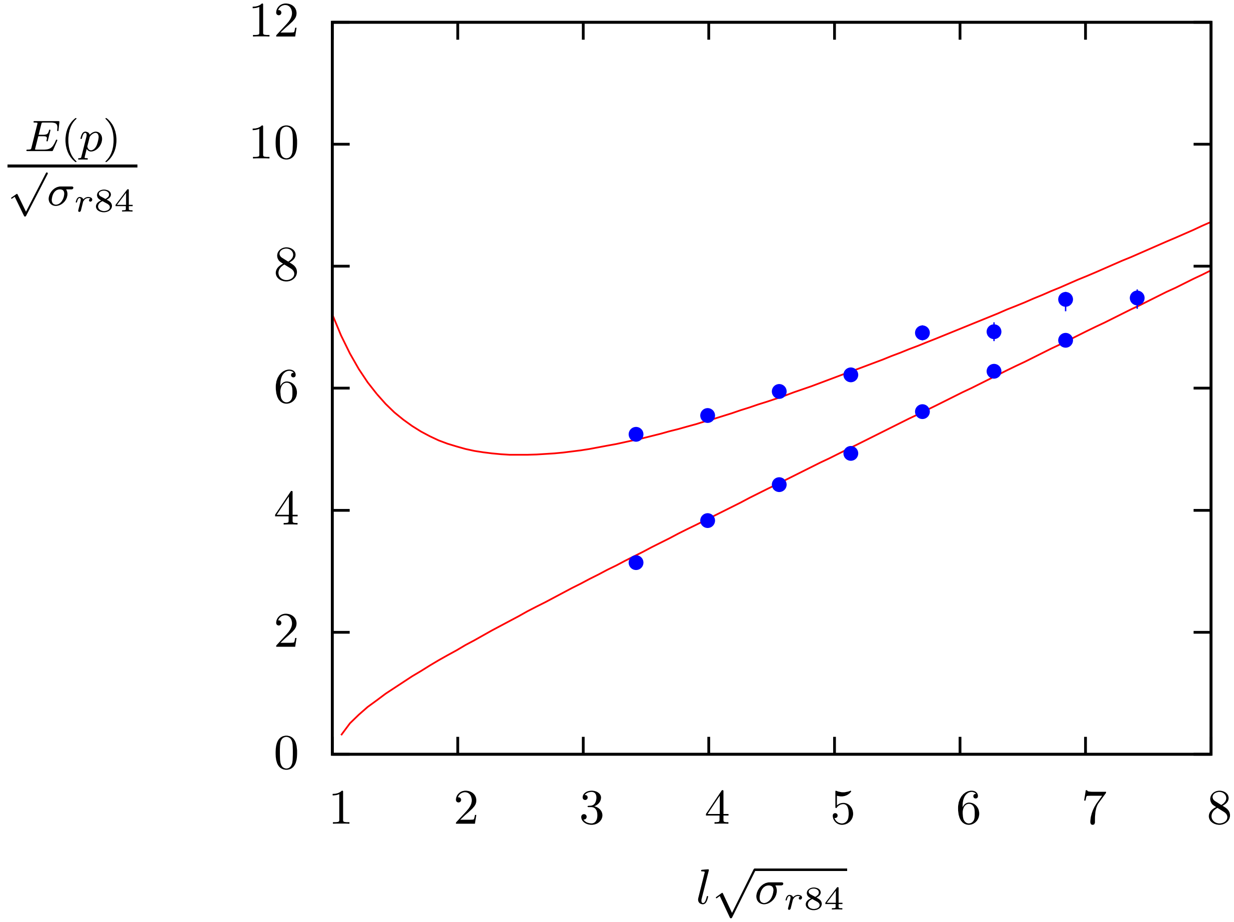
k=2A: versus Nambu-Goto, lightest $p = 2\pi/l,\ 4\pi/l$ states



$\Rightarrow$ here very good evidence for NG k=2A: lightest p=0, P=+ states



$\Rightarrow$ large deviations from Nambu-Goto for excited states k=1, R=<u>84</u>: lightest $p = 0, 2\pi/l$ states



$\Rightarrow$ all reps come with Nambu-Goto towers of states

Some conclusions on confining flux tubes and strings

- flux tubes are very like free Nambu-Goto strings, even when they are not much longer than they are wide

- this is so for all light states in $D = 2 + 1$ and most in $D = 3 + 1$

- ground state and states with one 'phonon' show corrections to NG only at *very* small l, consistent with $O(1/l^7)$

- most other excited states show small corrections to NG consistent with a resummed series starting with $O(1/l^7)$ and reasonable parameters

- in $D = 3 + 1$ we appear to see extra states consistent with the excitation of massive modes

- in $D = 2 + 1$, despite the much greater accuracy, we see no extra states

- we also find 'towers' of Nambu-Goto-like states for flux in other representations, even where flux tubes are not stable, but with much larger corrections – reflecting binding mass scale?

- theoretical analysis is complementary (in l) but moving forward rapidly, with possibility of resummation of universal terms and of identifying universal terms not seen in 'static gauge'

there is indeed a great deal of simplicity in the behaviour of confining flux tubes and in their effective string description — much more than one would have imagined ten years ago ...