Ferrara–Zumino supermultiplet and the energy-momentum tensor in the lattice formulation of 4D $\mathcal{N} = 1$ SYM

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- [H.S., arXiv:1209.5155 [hep-lat]]

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- I address an issue of the above kind, in the context of the lattice formulation of 4D $\mathcal{N} = 1$ SYM (lattice breaks SUSY!)

4D $\mathcal{N} = 1$ SYM

- classical continuum action

$$S = \int d^4x \left[\frac{1}{2} \operatorname{tr}(F_{\mu\nu} F_{\mu\nu}) + \operatorname{tr}(\bar{\psi} \not{D} \psi) \right], \qquad \bar{\psi} = \psi^T (-C^{-1})$$

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- translational invariance (and the rotational invariance)
- notation

global: $\bar{\delta}$ local: δ

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- (symmetric) energy-momentum tensor

$$\check{T}_{\mu\nu}(x) = 2\operatorname{tr}\left[F_{\mu\rho}(x)F_{\nu\rho}(x)\right] - \frac{1}{2}\delta_{\mu\nu}\operatorname{tr}\left[F_{\rho\sigma}(x)F_{\rho\sigma}(x)\right]$$
$$+ \frac{1}{4}\operatorname{tr}\left[\bar{\psi}(x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\psi(x)\right] - \frac{1}{2}\delta_{\mu\nu}\operatorname{tr}\left[\bar{\psi}(x)\overleftrightarrow{\not{D}}\psi(x)\right]$$

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- Ferrara–Zumino (FZ) supermultiplet ($\bar{\delta}_\xi$: global SUSY, ξ: parameter)
$$\bar{\delta}_\xi\breve{\jmath}_{5\mu}(x) = \bar{\xi}\gamma_5\breve{S}_\mu(x)$$
$$\bar{\delta}_\xi\breve{S}_\mu(x) = 2\gamma_\nu\xi\Big\{\breve{T}_{\mu\nu}(x) + \frac{3}{4}\delta_{\mu\nu}\operatorname{tr}\left[\bar{\psi}(x)\not{D}\psi(x)\right] + (\text{terms anti-symmetric in } \mu \text{ and } \nu)\Big\} + (\text{terms proportional to } \gamma_5\gamma_\nu\xi,\ \xi,\ \gamma_5\xi,\ \sigma_{\nu\rho}\xi)$$
$$\bar{\delta}_\xi\breve{T}_{\mu\nu}(x) = \cdots$$
- This is a sort of the “current algebra” in SUSY theory

SUSY WT relation on the lattice

- Under the localized SUSY transformation

$$\delta_\xi U_\mu(x) = iag\frac{1}{2}\left[\bar{\xi}(x)\gamma_\mu\psi(x)U_\mu(x) + \bar{\xi}(x+a\hat{\mu})\gamma_\mu U_\mu(x)\psi(x+a\hat{\mu})\right]$$

$$\delta_\xi \psi(x) = -\frac{1}{2}\sigma_{\mu\nu}\xi(x)\left[F_{\mu\nu}\right]^L(x)$$

$\left[F_{\mu\nu}\right]^L(x)$: lattice field strength

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- We have an identity ($\partial_\mu^S f(x) \equiv (1/2a)[f(x+a\hat{\mu}) - f(x-a\hat{\mu})]$)

$$\left\langle \partial_\mu^S S_\mu(x)\mathcal{O}\right\rangle = \left\langle [M\chi(x) + X_S(x)]\,\mathcal{O}\right\rangle - \left\langle \frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\delta_\xi\mathcal{O}\right\rangle$$

where

$$S_\mu(x) \equiv -\sigma_{\rho\sigma}\gamma_\mu \operatorname{tr}\left\{\psi(x)\left[F_{\rho\sigma}\right]^L(x)\right\}, \qquad \chi(x) \equiv \sigma_{\mu\nu}\operatorname{tr}\left\{\psi(x)\left[F_{\mu\nu}\right]^L(x)\right\}$$

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- $X_S(x)$ is an $O(a)$ symmetry breaking attributed to the lattice regularization

Renormalization of $X_S(x)$ (Curci–Veneziano (1987), Taniguchi (1999), Farchioni–Feo–Galla–Gebert–Kirchner–Montvay–Münster–Vladikas (2001), H.S. (2012))

- Assuming the locality and the hypercubic symmetry of the lattice action,

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\begin{aligned}
X_S(x) = (1 - \mathcal{Z}_S)\partial_\mu^S S_\mu(x) - \mathcal{Z}_T \partial_\mu^S T_\mu(x) \\
- \frac{1}{a}\mathcal{Z}_\chi \chi(x) \\
- \mathcal{Z}_{3F}\,\mathrm{tr}\left[\psi(x)\bar{\psi}(x)\psi(x)\right] \\
- \mathcal{Z}_{\mathrm{EOM}}\sigma_{\mu\nu}\,\mathrm{tr}\{[F_{\mu\nu}]^L(x)(D+M)\psi(x)\} \\
+ a\mathcal{E}(x),
\end{aligned}
$$

where

$$T_\mu(x) = 2\gamma_\nu\,\mathrm{tr}\left\{\psi(x)\,[F_{\mu\nu}]^L(x)\right\}$$

and the dimension $11/2$ operator $\mathcal{E}(x)$ is a linear combination of renormalized operators with logarithmically divergent coefficients

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- Plugging this $X_S(x)$ into the original identity,. . .

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- we have

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\begin{aligned}
&\left\langle \partial_\mu^S \left[\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x)\right] \mathcal{O} \right\rangle \\
&= \left(M - \frac{1}{a}\mathcal{Z}_\chi \right) \langle \chi(x)\mathcal{O} \rangle \\
&\qquad - \mathcal{Z}_{3F} \left\langle \mathrm{tr} \left[\psi(x)\bar{\psi}(x)\psi(x) \right] \mathcal{O} \right\rangle \\
&\qquad\qquad - \left\langle \frac{1}{a^4} \frac{\partial}{\partial \bar{\xi}(x)} \delta_\xi \mathcal{O} \right\rangle - \mathcal{Z}_{\mathrm{EOM}} \left\langle \sigma_{\mu\nu}\, \mathrm{tr}\{[F_{\mu\nu}]^L(x)(D + M)\psi(x)\} \mathcal{O} \right\rangle \\
&\qquad\qquad\qquad + \langle a\mathcal{E}(x)\mathcal{O} \rangle
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$$\left\langle \partial_\mu^S \left[\mathcal{Z}_S S_\mu(x) + \mathcal{Z}_T T_\mu(x)\right] \mathcal{O} \right\rangle$$

$$= \left(M - \frac{1}{a}\mathcal{Z}_\chi\right) \langle \chi(x)\mathcal{O}\rangle \longleftarrow \text{additive mass renormalization}$$

$$- \mathcal{Z}_{3F} \left\langle \mathrm{tr}\left[\psi(x)\bar{\psi}(x)\psi(x)\right] \mathcal{O}\right\rangle$$

$$- \left\langle \frac{1}{a^4} \frac{\partial}{\partial\bar{\xi}(x)} \delta_\xi \mathcal{O} \right\rangle - \mathcal{Z}_{\mathrm{EOM}} \left\langle \sigma_{\mu\nu}\, \mathrm{tr}\{[F_{\mu\nu}]^L(x)(D + M)\psi(x)\}\mathcal{O} \right\rangle$$

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$$\longleftarrow \text{modification of super transformation}$$

$$+ \langle a\mathcal{E}(x)\mathcal{O}\rangle$$

Renormalized SUSY WT relation

- We tune M so that (Donini–Guagnelli–Hernandez–Vladikas (1997))

$$M - \frac{1}{a}\mathcal{Z}_\chi = 0$$

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- In terms of this,

$$\left\langle \partial_\mu^S \mathcal{S}_\mu(x)\mathcal{O}\right\rangle = \left\langle \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x)\right]\mathcal{O}\right\rangle,$$

where

$$\Delta_\xi \equiv \delta_\xi + \mathcal{Z}_{\text{EOM}}\delta_{F\xi}$$

and

$$\delta_{F\xi}U_\mu(x) = 0, \qquad \delta_{F\xi}\psi(x) = \delta_\xi\psi(x), \qquad \delta_{F\xi}\bar{\psi}(x) = \delta_\xi\bar{\psi}(x)$$

Implication of the renormalized SUSY WT relation

- From the relation

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for a renormalized operator $\mathcal{O}$, we have the conservation law

$$\left\langle \partial_\mu^S \mathcal{S}_\mu(x)\mathcal{O} \right\rangle \xrightarrow{a\to 0} 0, \qquad \text{for } x \leftrightsquigarrow \text{supp}(\mathcal{O})$$

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- Moreover, we see that the operation

$$\mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x) \right]\mathcal{O}, \qquad \text{for } x \in \text{supp}(\mathcal{O})$$

should define another renormalized operator

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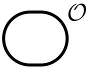

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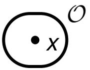

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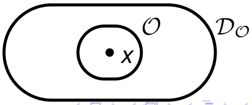

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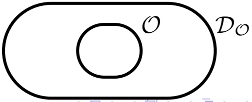

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$$\left. + \text{(terms anti-symmetric in } \mu \text{ and } \nu) \right\}$$
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- Thus, we make an ansatz ($\bar{\Delta}_\xi$ is the global version of Δ_ξ):

$$\mathcal{Z}\bar{\Delta}_\xi\mathcal{S}_\mu(x) \equiv 2\gamma_\nu\xi\{\mathcal{T}_{\mu\nu}(x) + c\delta_{\mu\nu}\,\mathrm{tr}\left[\bar{\psi}(x)(D+M)\psi(x)\right] + (\text{terms anti-symmetric in } \mu \text{ and } \nu)\} + (\text{terms proportional to } \gamma_5\gamma_\nu\xi,\ \xi,\ \gamma_5\xi,\ \sigma_{\nu\rho}\xi)$$

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- According to (Caracciolo–Curci–Menotti–Pelissetto (1989)), such a conserved symmetric energy-momentum tensor is, if it exists, unique, up to the overall normalization and the constant c

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- We know that the ratio $\mathcal{Z}_T/\mathcal{Z}_S$ has actually been measured (DESY-Münster-Rome Collaboration (2000–present)) by

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- Although c is just a choice of the origin of the energy, there exists a natural choice in the present SUSY theory, that is

$$\langle\mathcal{T}_{00}(x)\rangle_{\text{periodic boundary conditions}} = 0$$

(cf. Kanamori–Sugino–H.S. (2007)). This fixes

$$c = -\frac{a^4}{2(N_c^2-1)}\langle\Theta_{00}(x)\rangle$$

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is realized on the lattice? Understanding of the anomaly puzzle?

Proof of the conservation law of $\mathcal{T}_{\mu\nu}(x)$

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- After some rearrangements (α, β: spinor indices),

$$\begin{aligned}
&\left\langle \frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}_\beta(y)}\left[\mathcal{Z}\Delta_\xi\partial_\mu^S\mathcal{S}_\mu(x)\right]_\alpha \mathcal{O}\right\rangle \\
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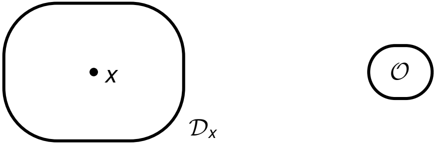
- We then sum this relation over y within a finite region $\mathcal{D}_x$, that contains the operator $\partial^S_\mu\mathcal{S}_\mu(x)$, but $\mathcal{D}_x \cap \mathrm{supp}(\mathcal{O}) = \emptyset$



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\begin{aligned}
&\left\langle \partial^S_\mu \Theta_{\mu\nu}(x) \mathcal{O} \right\rangle \\
&= \frac{1}{8}(C^{-1}\gamma_\nu)_{\alpha\beta} \left\langle \mathcal{Z}\left[a\mathcal{E}(x)\right]_\alpha a^4 \sum_{y\in\mathcal{D}_x} \left[\partial^S_\nu \mathcal{S}_\nu(y)\right]_\beta \mathcal{O} \right\rangle \\
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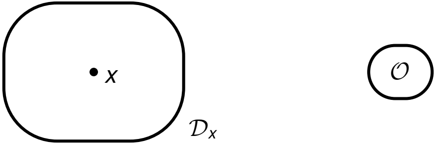

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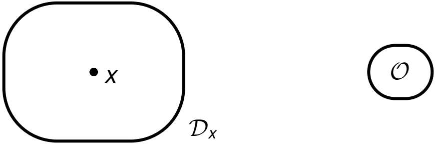

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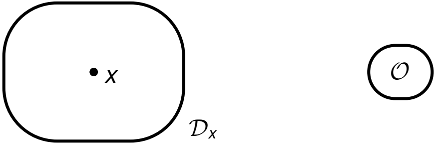
&\quad - \frac{1}{8}(C^{-1}\gamma_\nu)_{\alpha\beta} \left\langle a^4 \sum_{y\in\mathcal{D}_x} \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x)\right]_\alpha \mathcal{Z}\left[a\mathcal{E}(y)\right]_\beta \mathcal{O} \right\rangle
\end{aligned}
$$



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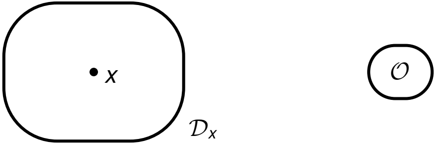
Conservation of $\Theta_{\mu\nu}(x)$

- Then, we have

$$\left\langle \partial_\mu^S \Theta_{\mu\nu}(x)\mathcal{O}\right\rangle$$

$$= \frac{1}{8}(C^{-1}\gamma_\nu)_{\alpha\beta}\left\langle \mathcal{Z}\left[a\mathcal{E}(x)\right]_\alpha a^4 \sum_{y\in\mathcal{D}_x}\left[\partial_\nu^S \mathcal{S}_\nu(y)\right]_\beta \mathcal{O}\right\rangle \xrightarrow{a\to 0} 0$$

$$-\frac{1}{8}(C^{-1}\gamma_\nu)_{\alpha\beta}\left\langle a^4 \sum_{y\in\mathcal{D}_x} \mathcal{Z}\left[-\frac{1}{a^4}\frac{\partial}{\partial\bar{\xi}(x)}\Delta_\xi + a\mathcal{E}(x)\right]_\alpha \mathcal{Z}\left[a\mathcal{E}(y)\right]_\beta \mathcal{O}\right\rangle$$



- The first term of r.h.s. is a correlation function of renormalized operators with no mutual overlap with an overall factor *a*
- The second term is, according to our argument, also a correlation function of renormalized operators with no mutual overlap with an overall factor *a*

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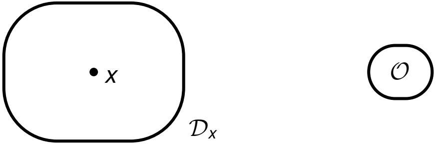

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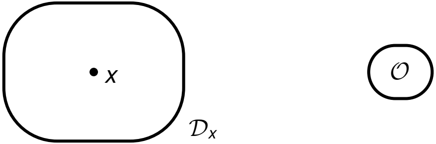

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- Step 2: Conservation of the anti-symmetric part of $\Theta_{\mu\nu}(x)$:

$$\mathcal{A}_{\mu\nu}(x) \equiv \frac{1}{2}\left[\Theta_{\mu\nu}(x) - \Theta_{\nu\mu}(x)\right]$$

Conservation of the anti-symmetric part of $\Theta_{\mu\nu}(x)$

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- It turns out that (using the constraint $\bar{\psi}(x) = \psi^T(x)(-C^{-1})$),

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- In conclusion, we have

$$\left\langle \partial_\mu^S \mathcal{T}_{\mu\nu}(x)\mathcal{O}\right\rangle \xrightarrow{a\to 0} 0, \qquad \text{for } x \leftrightsquigarrow \operatorname{supp}(\mathcal{O})$$

□