# THE HIGGS BOSON FROM ITS DISCOVERY TO PRECISION PHENOMENOLOGY

Babis Anastasiou
ETH Zurich

# OUTLINE

- First impressions from the Higgs discovery
- Theory input for measurements of Higgs decay widths and couplings
- Solidity of perturbative computations
- New methods for further progress in pertubative predictions

# THE HIGGS BOSON DISCOVERY

- July $4^{th}$ is a historic day for science.
- The most difficult discovery in modern particle physics.
- Search for very rare events. Cross-sections of a 2-50 fb.
- Clever and mature analyses with very high signal efficiencies.
- The mass is where electroweak precision tests like a Higgs boson to be in the Standard Model.

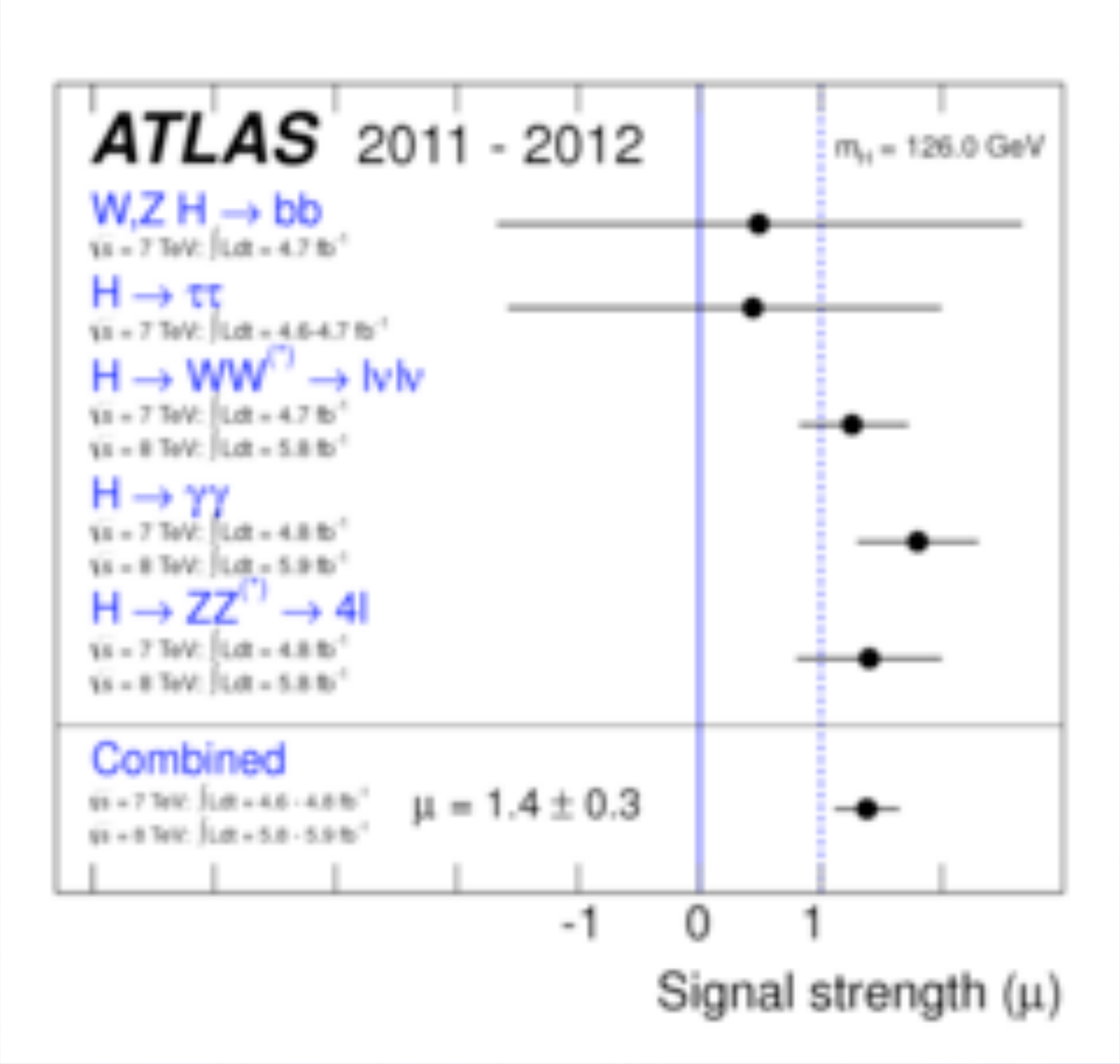
# ATLAS DISCOVERY

- Three channels with 8 TeV data:
  $H \to ZZ \to llll$,
  $H \to$ gamma gamma,
  $H \to WW$

- Results: a convincing excess in all three channels.

- Consistent with Standard Model

- However, a stronger production is quite possible.



# CMS DISCOVERY

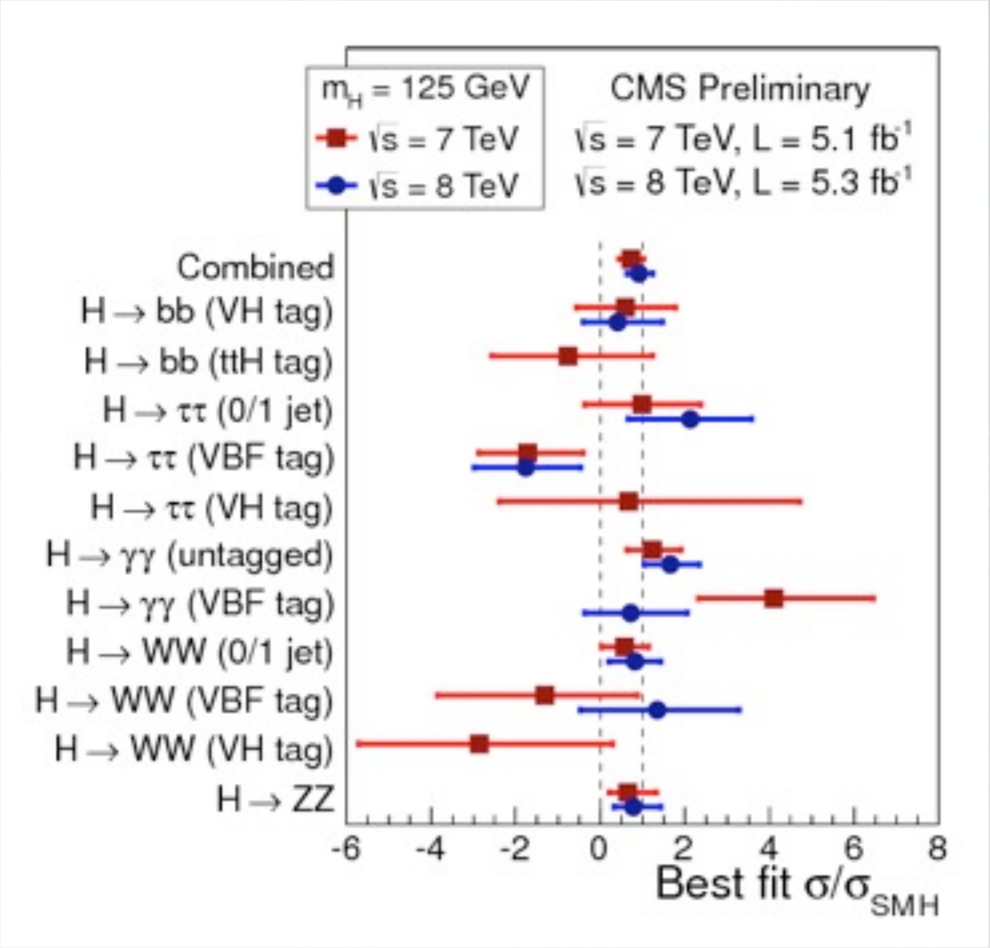
- H→gamma,gamma,
  H→ZZ,
  H→WW,
  H→tau tau,
  H→bb with 8 TeV data

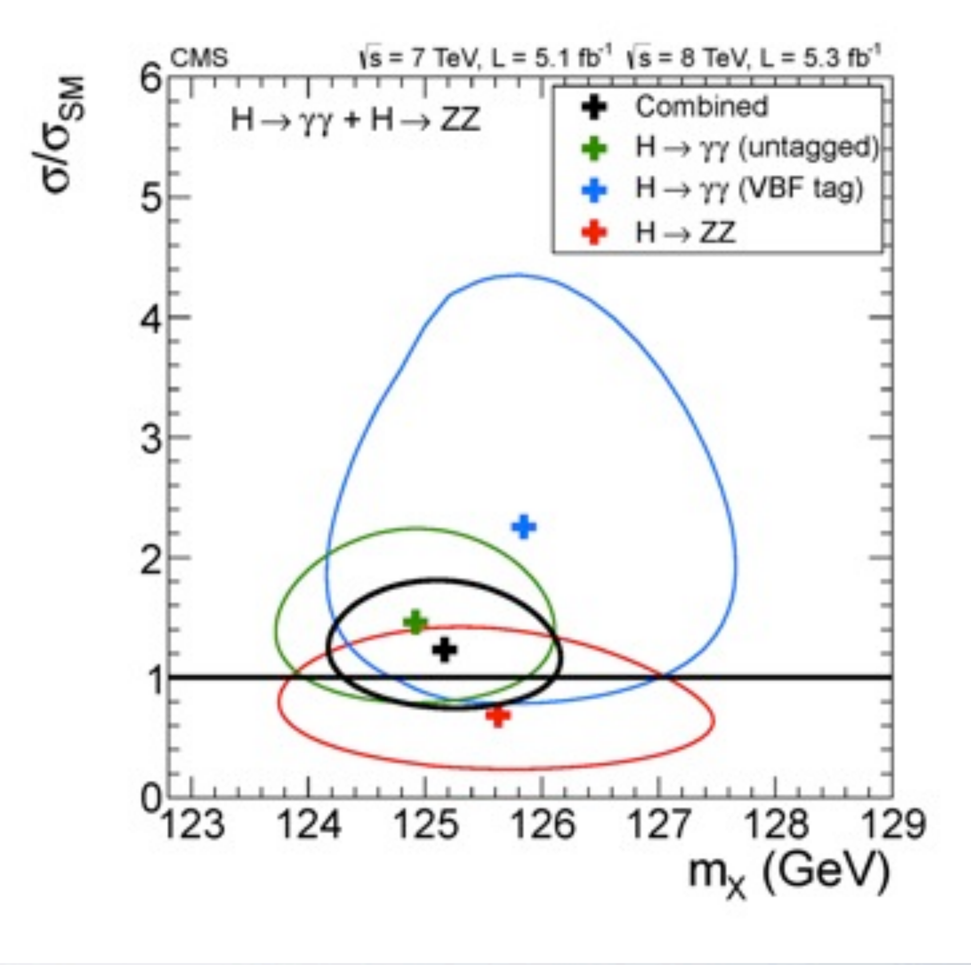
- A convincing excess in H→ZZ,WW,gamma-gamma

- H→tau tau has a very small S/B (~3%).
  e-hadronic seems in tension with a SM Higgs at 125 GeV. mu-hadronic seems consistent.

- In WW, the 0-jet mu-e category seems to be consistent with SM or higher rate.

- Other categories seem to favor a smaller cross-section than SM, but are consistent with one times SM.



# READING OF GENERAL EXPERIMENTAL PICTURE

- We have a Higgs boson which is consistent with the Standard Model.
- But we rely on a very small number of events to draw conclusions.
- We cannot distinguish clearly between data fluctuations and a new physics phenomenon.
- Is the di-photon branching ratio enhanced? Is the tau-tau coupling reduced? Are the WW and ZZ fine or bigger than the Standard Model?
- Measurements leave a lot of room for new physics manifesting itself as atypical Higgs interactions.
- We will know with a better precision (factor of two?) by the end of this LHC run. The picture will be much clearer with the 14TeV run.

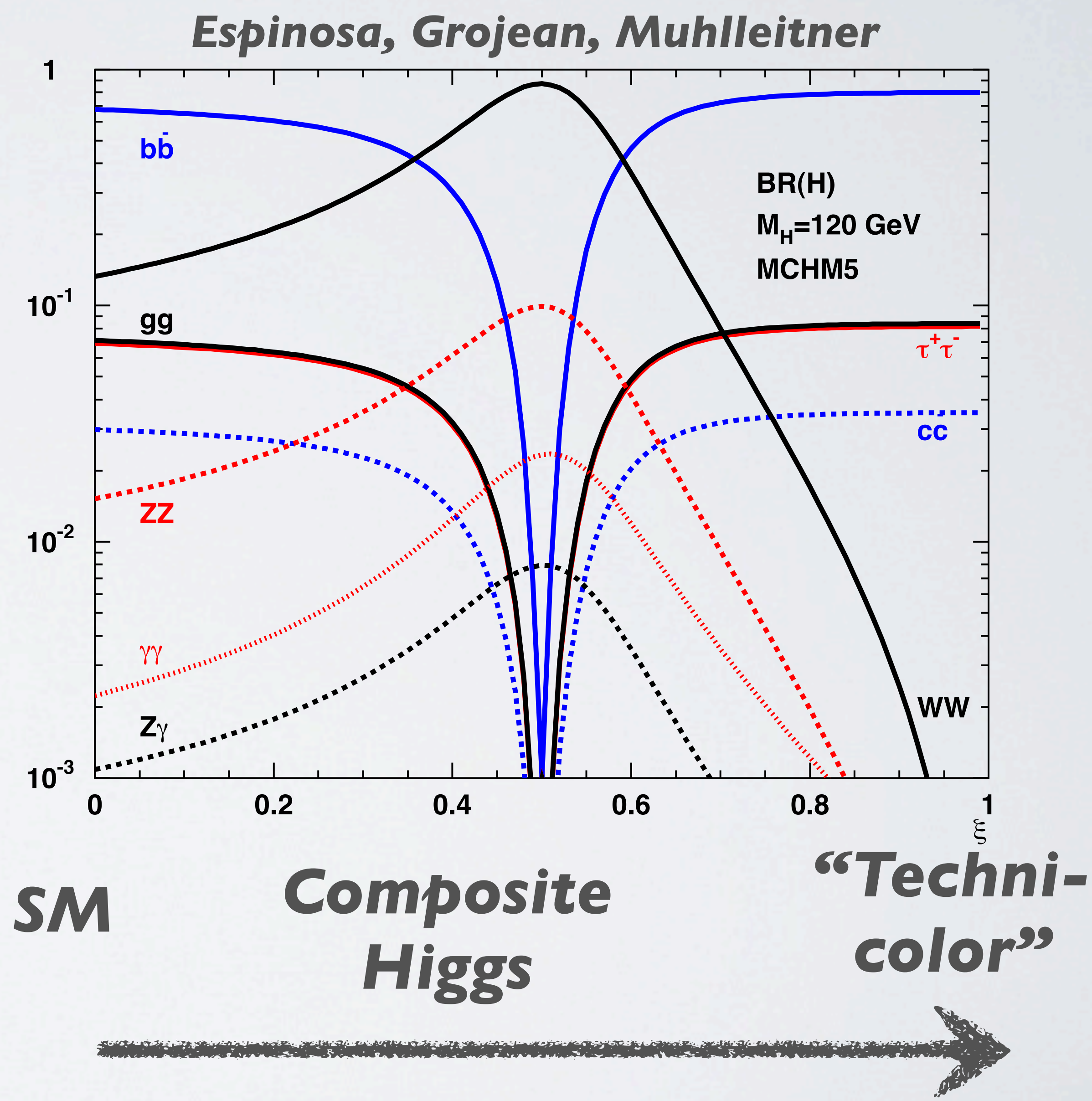
# BEFORE JULY 4

- Theorists had various expectations about how a Higgs discovery would come.
- Three years back we had a grand belief and a grand hope
- Grand belief: physics beyond the Standard Model is inevitable
- Grand hope: physics beyond the Standard Model at the LHC is inevitable.

***Higgs physics could be different than in the Standard Model***

# COMPOSITE HIGGS BOSON PHENOMENOLOGY

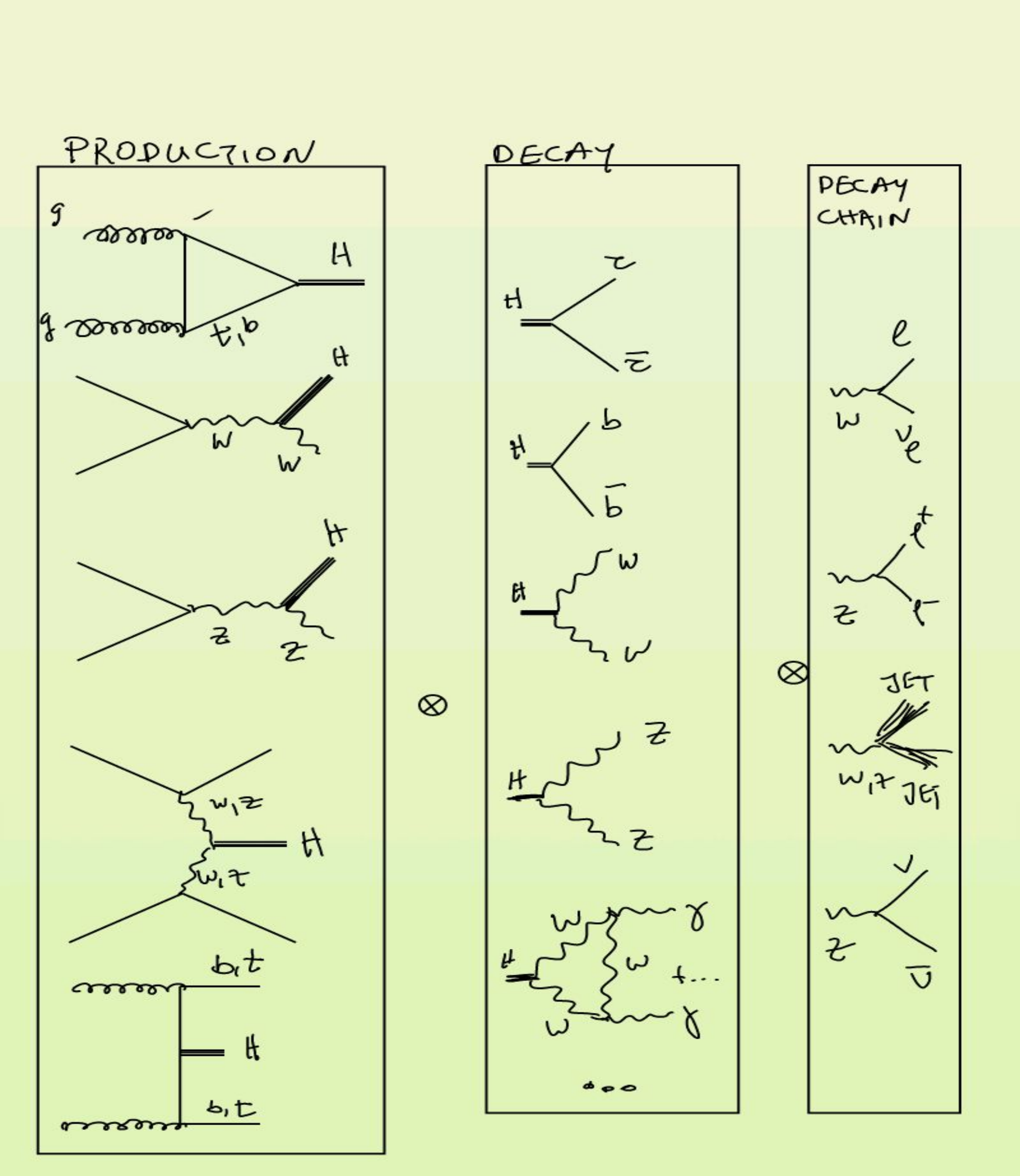
- Higgs production cross-sections and decay widths are typically smaller than in the SM.
- Some room for enhanced branching ratios (by reducing the $H \to bb$ width or enhancing directly the loop induced decay widths).
- Deviations from SM rates for Higgs signals are not meant to be the "smoking guns" of these theories which have a rich spectrum of light new particles.
- Large excesses over Standard Model rates for Higgs signals are difficult to accommodate.




*An example of a natural theory*

# HIGGS COUPLING EXTRACTION AT THE LHC

- A prime goal of particle physics after the Higgs discovery will be the determination of the couplings for its interactions.
- A precise determination will be one of the strongest consistency checks of the Standard Model or any other model which replaces it.
- The LHC is a very good Higgs boson factory.
- The Higgs mass of 125 GeV allows the observation and cross-section measurements for many of its signatures.



# PRECISE COUPLING EXTRACTIONS

- Excellent detectors (exp)
- Energy and luminosity at the LHC,
  leading to bigger cross-sections and small statistical uncertainties (exp)
- Experimental ingenuity to reduce backgrounds and to control systematic uncertainties (exp)
- Reducing as much as possible the dependence on the modeling of QCD and BSM effects (th)
- Improving perturbative QCD uncertainties

# REDUCING BSM AND QCD DEPENDENCE

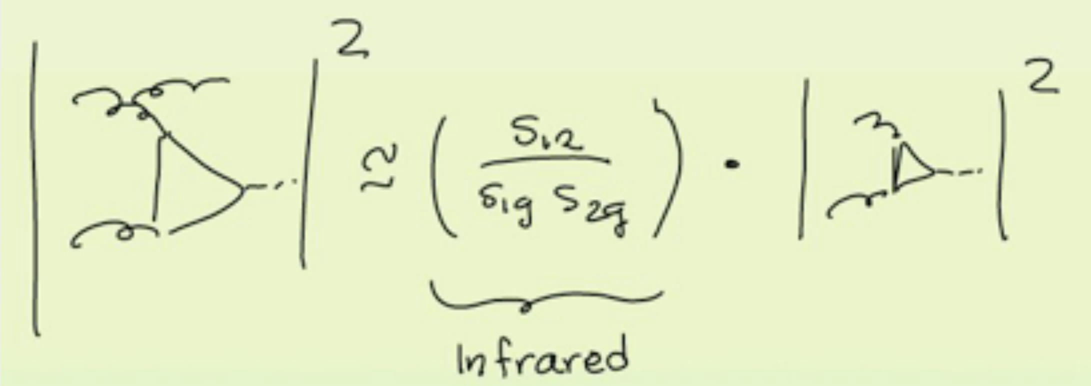
- The Higgs boson is a narrow resonance: we can separate production from decay
- QCD is diagonal to electroweak and BSM physics. Perturbative QCD corrections have a good degree of universality.
- BSM particle spectrum is above the Higgs mass, allowing for effective theory description of new physics in Higgs observables.
- Various production and decay channels can be efficiently disentangled kinematically.
- Cross-sections for many Higgs signals can be measured

$$\sigma_{Y,\mathrm{exp}} = \sigma_p \times BR(H \to Y) = \left(\frac{\sigma_p}{\Gamma_p}\right) \times \left(\frac{\Gamma_p \Gamma_Y}{\Gamma}\right)$$

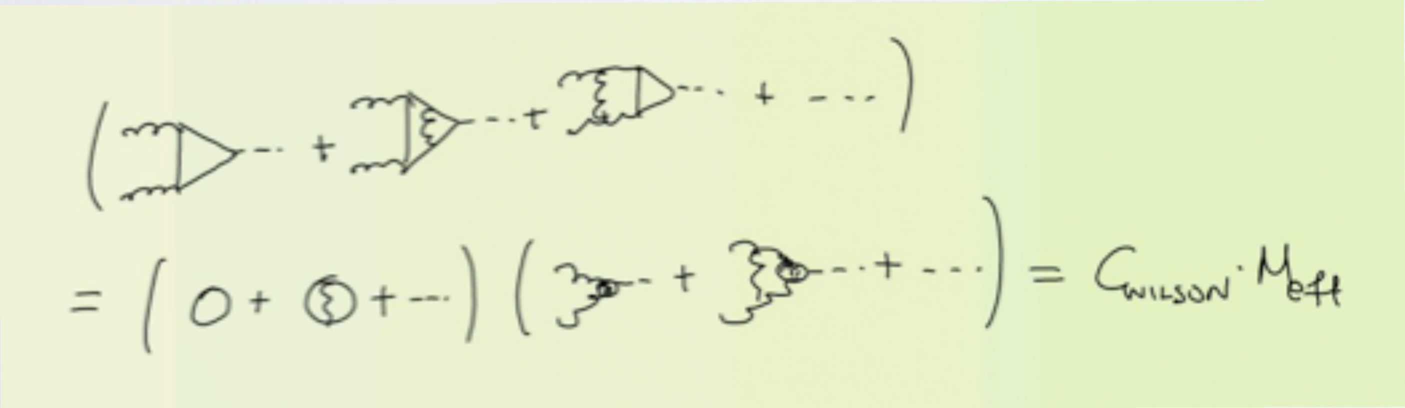
***Duhrssen et al***

# FACTORIZATION OF BSM EFFECTS FROM QCD CORRECTIONS

- Fairly model independent ratio: $\left(\frac{\sigma_p}{\Gamma_p}\right) \approx \left(\frac{\sigma_p}{\Gamma_p}\right)_{SM}$
- in enhanced phase-space regions of QCD infrared radiation




- Fast decoupling of new physics




$$\mathcal{L} \to \mathcal{L}_{eff}$$

$$\mathcal{M}_{gg\to H} \to C_{\text{Wilson}}\, \mathcal{M}^{\text{eff}}_{gg\to H}$$

- Possible exception: enhanced Higgs couplings to light quarks

# AN EXAMPLE OF THEORY INPUT

known to NNLO

$$\sigma^{SM}_{gg\to H} = \cancel{C}^2 \alpha_s(\cancel{\mu_r})^2 \left[X_0\left(\mu_r,\mu_f\right) + \alpha_s(\mu_r)X_1\left(\mu_r,\mu_f\right) + \alpha_s(\mu_r)^2 X_2\left(\mu_r,\mu_f\right) + \ldots\right]$$

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$$\Gamma^{SM}_{H\to gg} = \cancel{C}^2 \alpha_s(\cancel{\mu_r})^2 \left[Y_0\left(\mu_r\right) + \alpha_s(\mu_r)Y_1\left(\mu_r\right) + \alpha_s(\mu_r)^2 Y_2\left(\mu_r\right) + \alpha_s(\mu_r)^3 Y_3\left(\mu_r\right) + \ldots\right]$$

known to NNNLO

***Harlander,Kilgore; CA, Melnikov; Neerven,Ravindran,Smith***

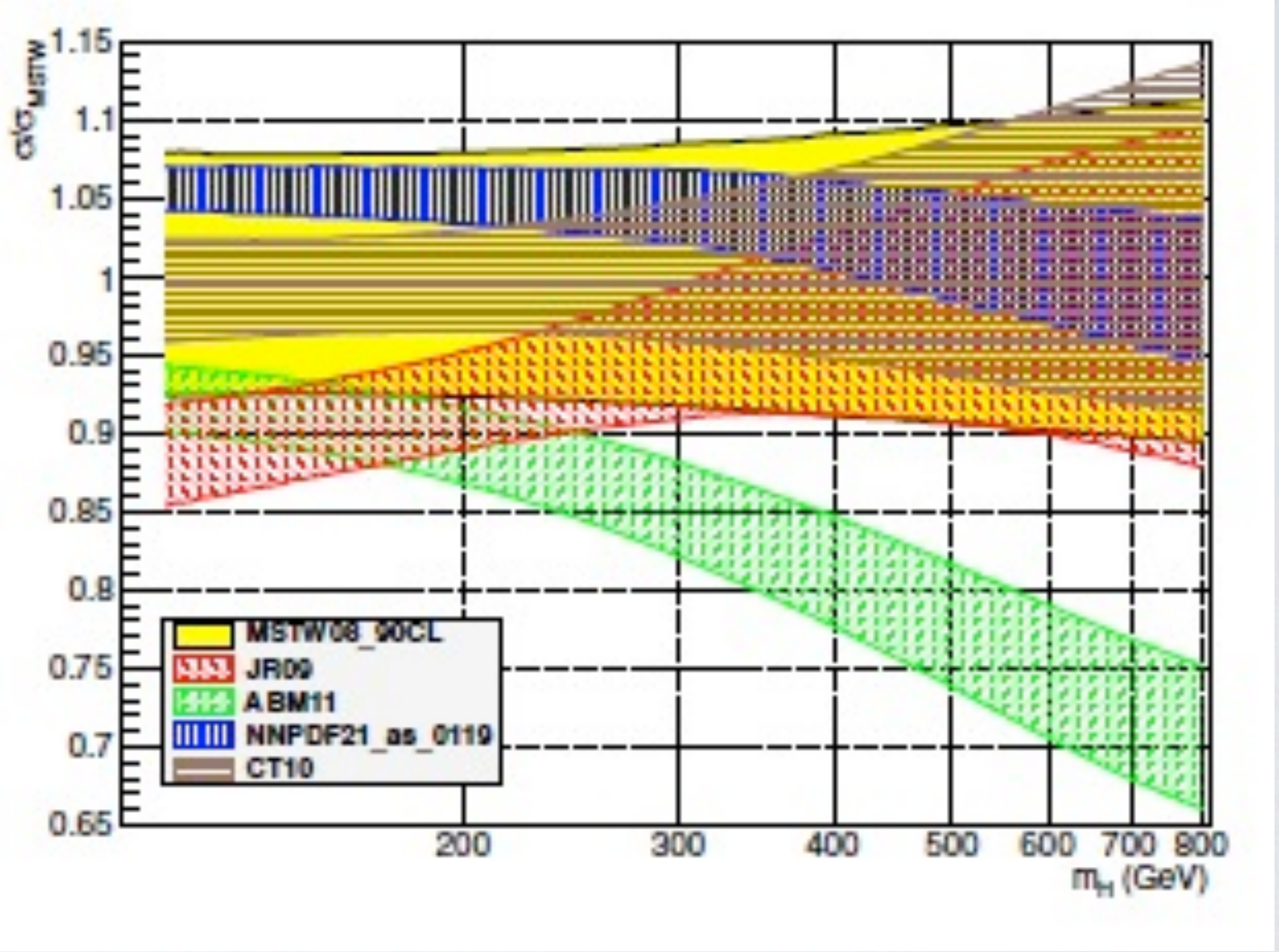
$$\frac{\sigma^{SM}_{gg\to H}}{\Gamma^{SM}_{H\to gg}} = \frac{1 + 0.72 + 0.28 + \ldots}{1 + 0.65 + 0.20 + 0.002 + \ldots}$$

***Baikov, Chetyrkin***

- Slow perturbative convergence
- but many orders in perturbation theory are known
- ***How much do we trust our perturbative QCD computations?***

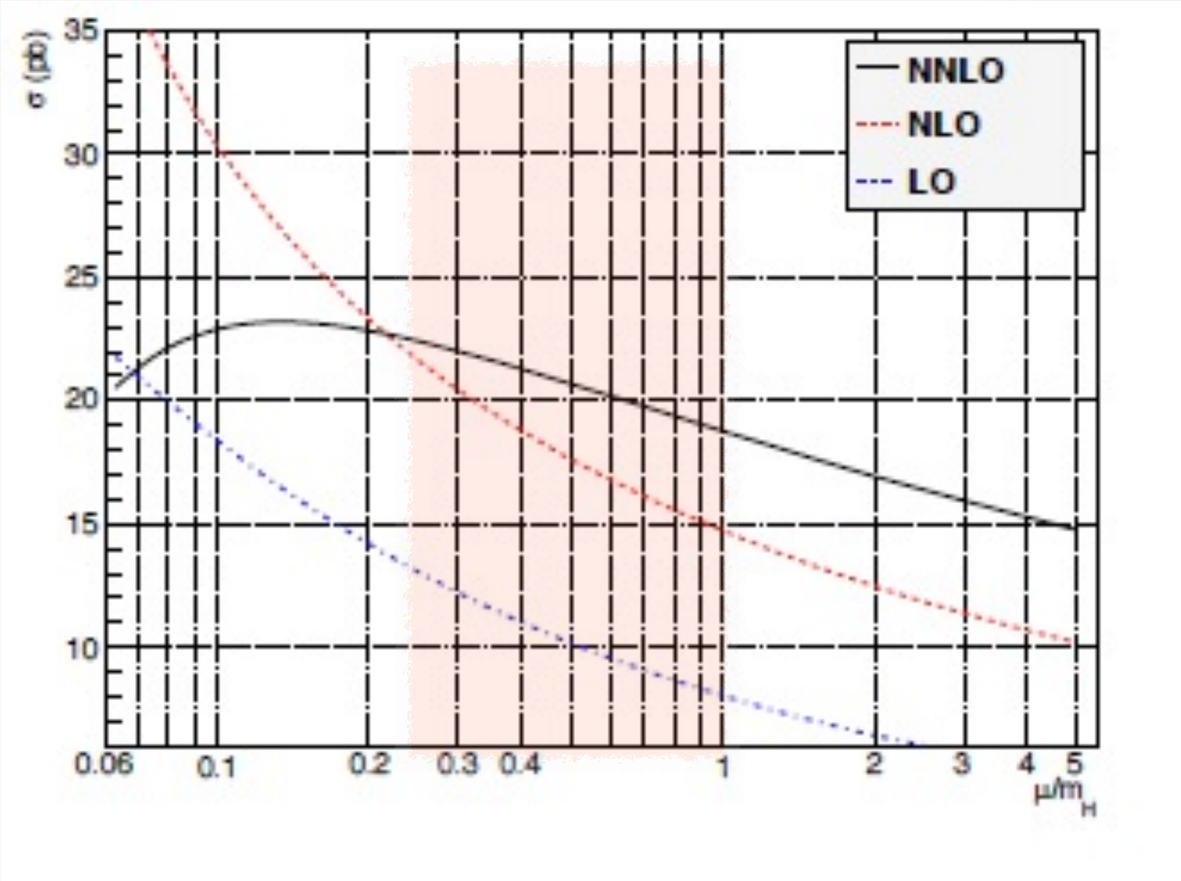
# PDF UNCERTAINTIES

- Five NNLO pdf sets
- 68% confidence level uncertainties show discrepancies
- Situation can be ameliorated by adopting the 90%CL uncertainty of MSTW
- Still, ABM11 set is quite different.
  ABM11 finds a lower value of alpha_s, relies on less data, but not yet shown to disagree with LHC data. Their alpha_s value is in tension with measurements of the Z and W decay widths as well as LEP data and tau decays.
- Important: high precision measurements of top and other SM cross-sections at the LHC.



# SCALE VARIATIONS

- The Higgs cross-section has worried us for a long time about its slow perturbative convergence.
- perturbative series converges slowly for scales around half the Higgs mass
- and very slowly for higher scales.
- should we trust the NNLO computations?
- Let's dissect them



# NLO QCD CORRECTIONS

cross-section for gluon fusion via a heavy (top) quark:

$$\sigma \sim \mathcal{L}_{gg}(\mu) \times \left(\frac{\alpha_s(\mu)}{\pi}\right)^2$$

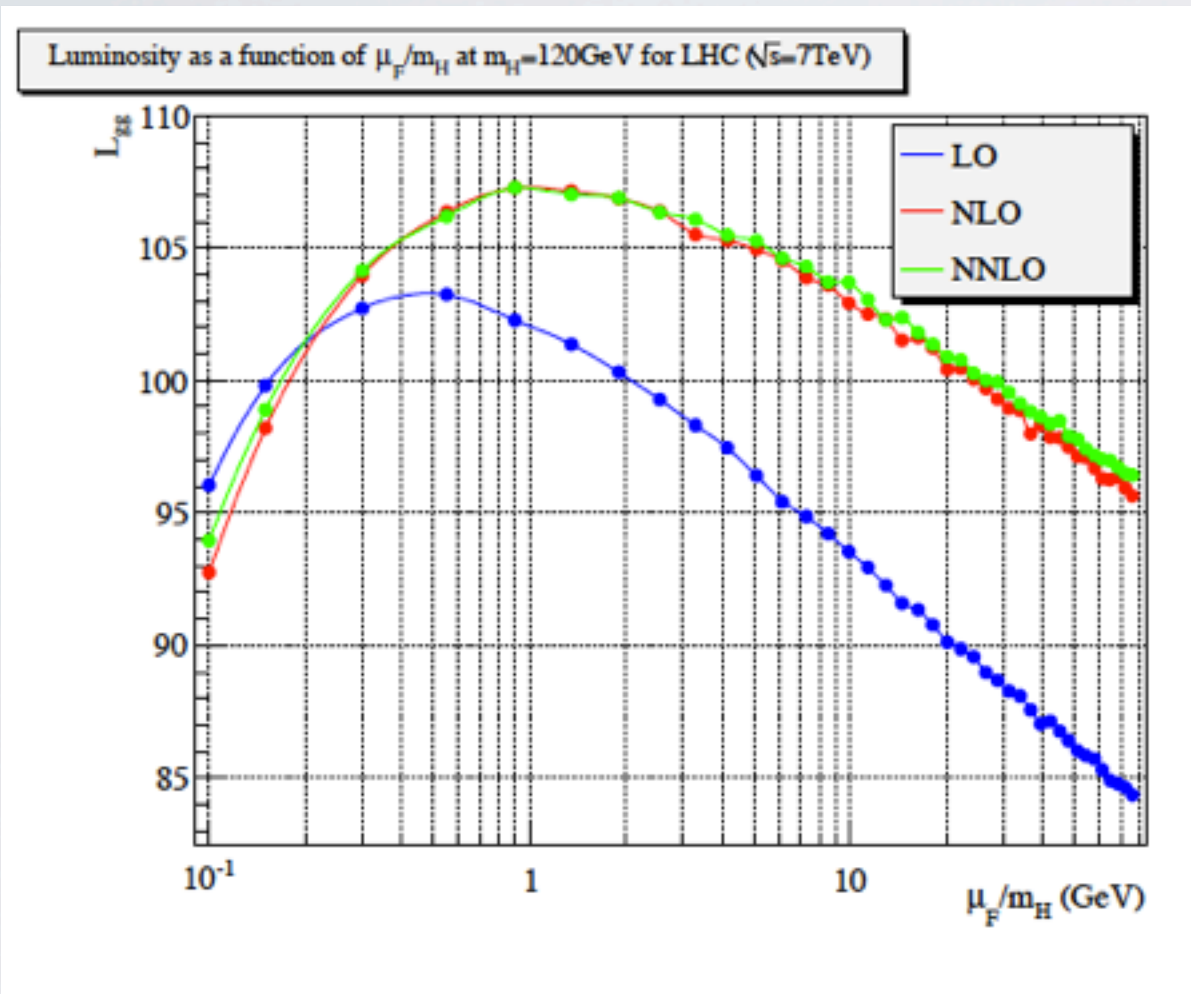
$$\left\{1 + \frac{\alpha_s(\mu)}{\pi}\left[N_c \frac{\pi^2}{3} + \frac{11}{2}\right] + 2\log\left(\frac{\mu^2}{p_T^2}\right) N_c \mathrm{Coll}\left(\frac{p_t^2}{M_h^2}\right) + \mathrm{Reg}\left(\frac{p_t^2}{M_h^2}, \theta\right)\right\}$$

Soft real and virtual corrections $\pi^2, \log\left(\frac{\mu^2}{p_T^2}\right)$

$\frac{11}{2} = 2\,C_1$ Wilson coefficient of Heavy Quark Effective Theory (~ UV nature)

$\mathrm{Reg}\left(\frac{p_t^2}{M_h^2}, \theta\right) \to 0,$ hard, vanishes in $p_t, \theta, \pi - \theta \to 0$

# GLUON-GLUON LUMINOSITY




Lgg(Mh=120GeV, LHC7, MSTW08)

- Very stable from NLO to NNLO
- Within 5% from LO for a light Higgs boson at the LHC for reasonable factorization scales.
- ~ 20% higher than LO for very very large factorization scales

# LARGE K-FACTORS

$$\frac{\text{NLO}}{\text{LO}} \sim (80\% - 105\%)\left\{1 + 4\%\Big[\underbrace{\overbrace{9.876}^{\pi^2} + \underbrace{5.5}_{\text{Wilson coefficient}}}\Big] + \dots\right\}$$

*NLO/LO gluons and alpha_s*

Bound to have a large K-factor of at least 1.5-1.6 due to pi's and the Wilson coefficient

Milder K-factor if gluon fusion is mediated through a light quark (bottom) as, for example, in large tan(beta) MSSM.

$$\frac{\text{NLO}}{\text{LO}} \sim (80\% - 105\%)\left\{1 + 4\%\Big[\underbrace{9.876}_{\pi^2} + \overbrace{0.9053}^{\text{Two-loop bottom amplitude.}}\Big] + \dots\right\}$$

# LARGE K-FACTORS (II)

$$\frac{\mathrm{NLO}}{\mathrm{LO}} \sim (80\% - 105\%) \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ \ldots + 6 \log \left( \frac{\mu^2}{p_T^2} \right) + \ldots \right] \right\}$$

*NLO/LO gluons and alpha_s*

- Logarithmic enhancement at small transverse momentum
- Integrable: reliable perturbative expansion for inclusive cross-sections.
- The mu scale is arbitrary, but no need to be senseless.
- Choices very different than pt can spoil the perturbative expansion.

$$M_H = 120\,\mathrm{GeV}\ @\mathrm{LHC7} \rightsquigarrow < p_t > \sim 35\,\mathrm{GeV}$$

$$\left\{ 1 + 4\% \left[ \underbrace{9.876}_{\pi^2} + \underbrace{5.5}_{\text{Wilson coefficient}} + \underbrace{\mathcal{O}(15.)}_{\text{Pt-Log}} \right] + \ldots \right\}_{\mu = M_h}$$

$$\frac{\mathrm{NLO}}{\mathrm{LO}} \sim (80\% - 105\%)$$

*NLO/LO gluons and alpha_s*

$$\left\{ 1 + 4\% \left[ 9.876 + 5.5 + \mathcal{O}(1.) \right] + \ldots \right\}_{\mu = \frac{M_h}{4}}$$

# PERTURBATIVE CONVERGENCE?

- Three main worries from the NLO calculation:
  - Large NLO Wilson coefficient ~15-20%
  - Pi^2 = 2 x Nc x (Pi^2/6) term ~ 30-40%
  - Large logs (2 x Nc x Log(pt^2/mu^2)) of transverse momentum (sensitive to mu) ~1% - 80%

- Comforting that the NNLO corrections are mild. The Wilson coefficient has a regular perturbative expansion.

*At NNLO:*

*Wilson coefficient*

$$C \sim 1 + (4\%) \cdot 5.5 + (4\%)^2 \cdot 10.$$

*Chetyrkin, Kniehl, Steinhauser*

# PERTURBATIVE CONVERGENCE?

- Half of Pi^2 belongs to a different Wilson coefficient when matching to SCET. It ``exponentiates''. We are left to explain the other half, which is a smaller (half) concern.

*At NNLO and beyond:*

*Ahrens, Becher, Neubert*

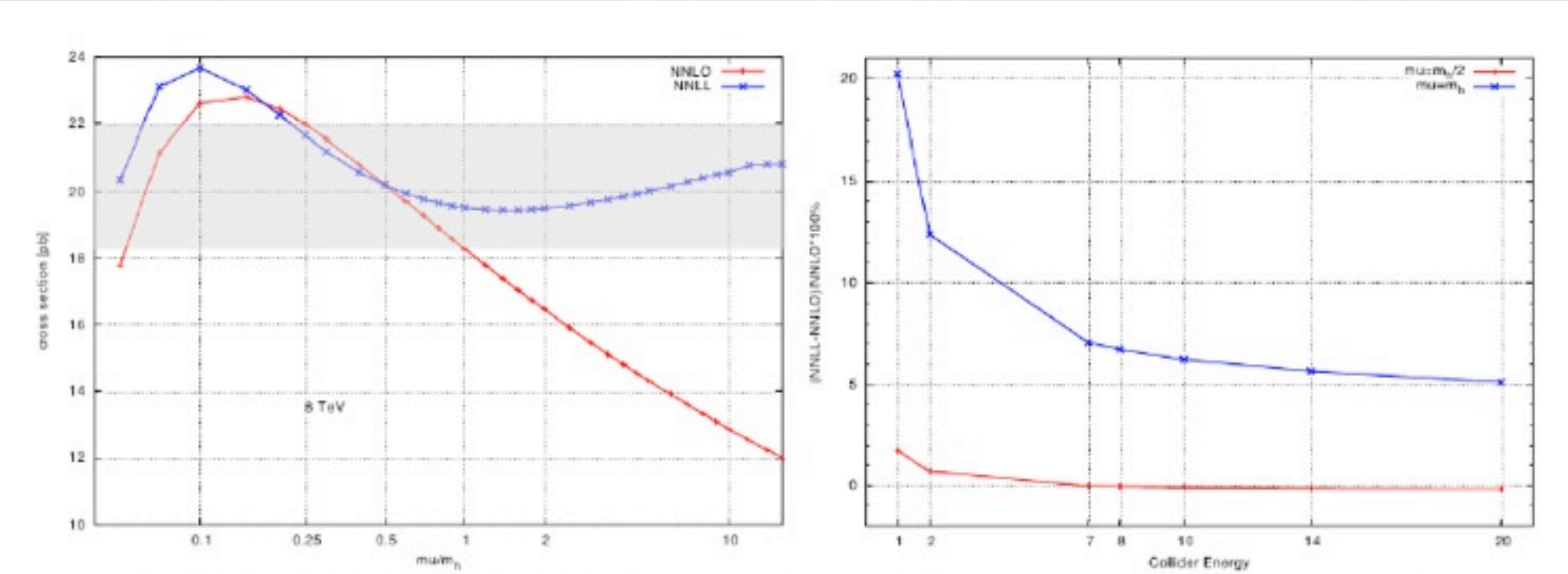
$$1 + \frac{\alpha_s}{\pi} \cdot (\pi^2) + \ldots \sim e^{\frac{\alpha_s}{\pi} \cdot \left(\frac{\pi^2}{2}\right)} \left(1 + \frac{\alpha_s}{\pi} \left[\frac{\pi^2}{2}\right] \ldots\right)$$

- Logs due to soft radiation exponentiate and can be resummed with NNLL accuracy at all orders.

*Catani, de Florian, Grazzini*

- Luckily, they yield small corrections beyond NNLO

# CHECKS AGAINST KNOWN BEYOND NNLO EFFECTS

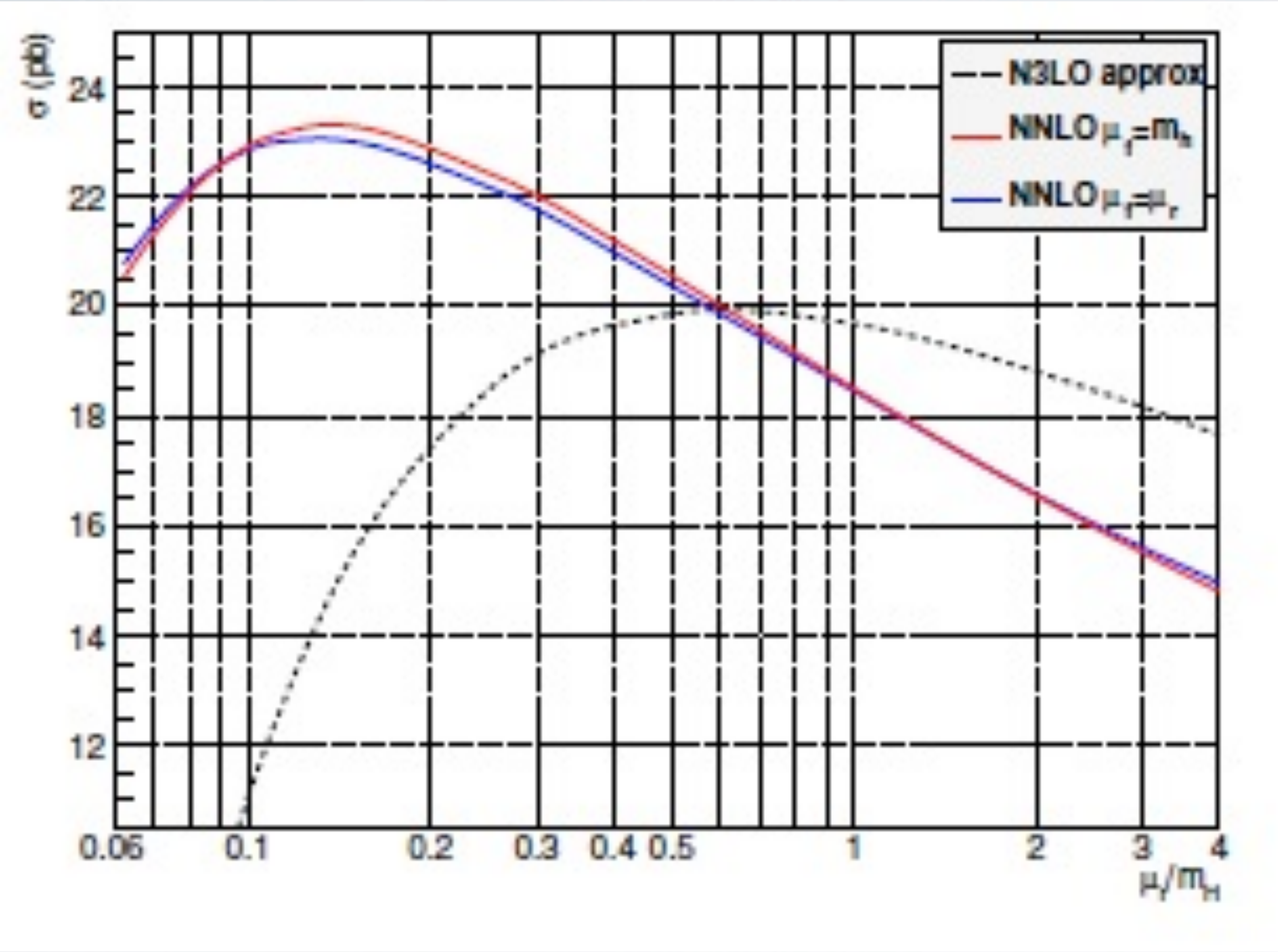



All numbers for these plots are calculated with the online calculator by M. Grazzini, see http://theory.fi.infn.it/grazzini/hcalculators.html

- NNLO vs NNLL resummation (*Catani, Grazzini, de Florian)* agree very well, over a vast range of collider energies

- Similar observations for SCET-type threshold resummation (Ahrens, Becher, Neubert)

# SOFT LOGS AT NNNLO




The NNLO logarithmic terms are also known.

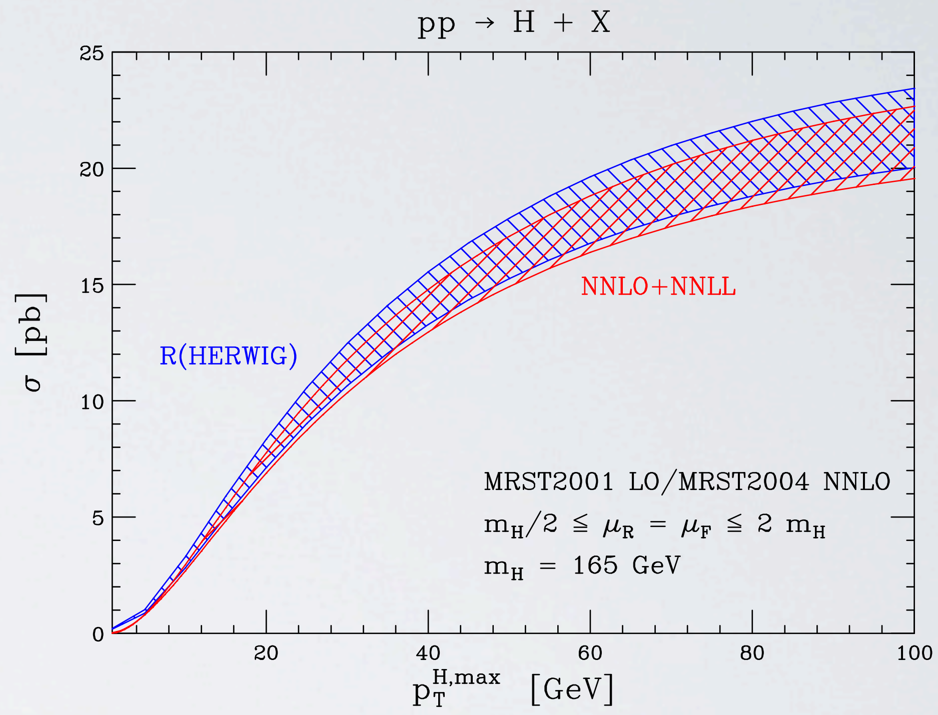
*Moch,Vogt*

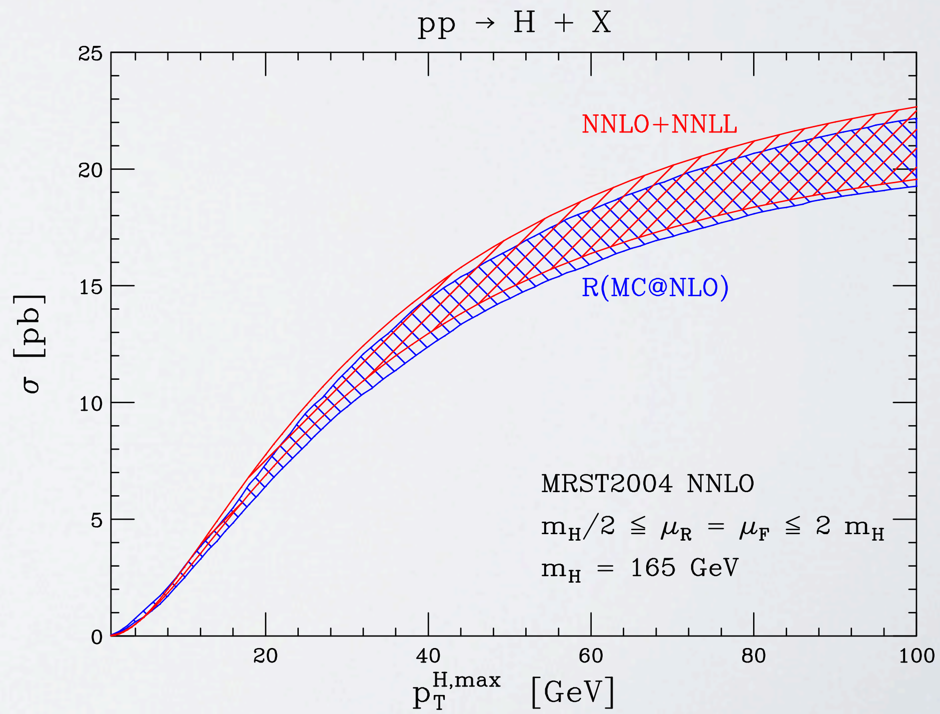
Consistent with NNLO

***We have reshuffled/resummed perturbation theory in all sensible ways that we can think of with very consistent results. inspires confidence that we have achieved a very good accuracy which we can trust for the inclusive cross-section***

# CHECK ON EFFICIENCIES

- Exhaustive comparisons between parton-shower, resummation and fixed order already five years ago.
- Showing a very good agreement in efficiencies for jet vetoes and other cuts. ***CA,Dissertori,Grazzini,Stoeckli,Webber***
- The question of jet vetoes tantalized theorists for quite some time
- Could the success of the NNLO vs parton-shower comparison and pt-resummation be an accident?

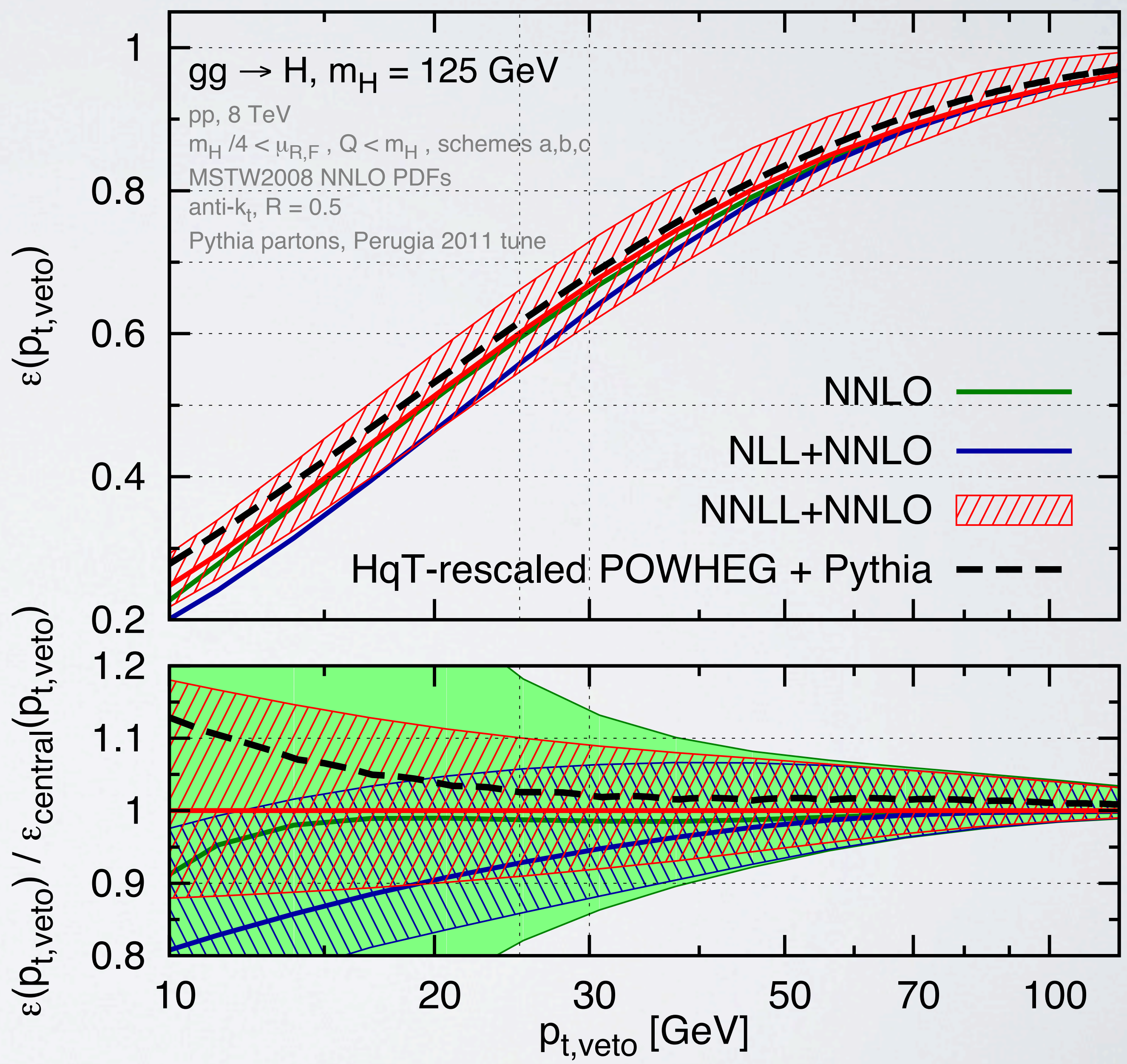


# RESUMED JET-VETO EFFICIENCIES

***Banfi, Monni, Salam, Zanderighi***

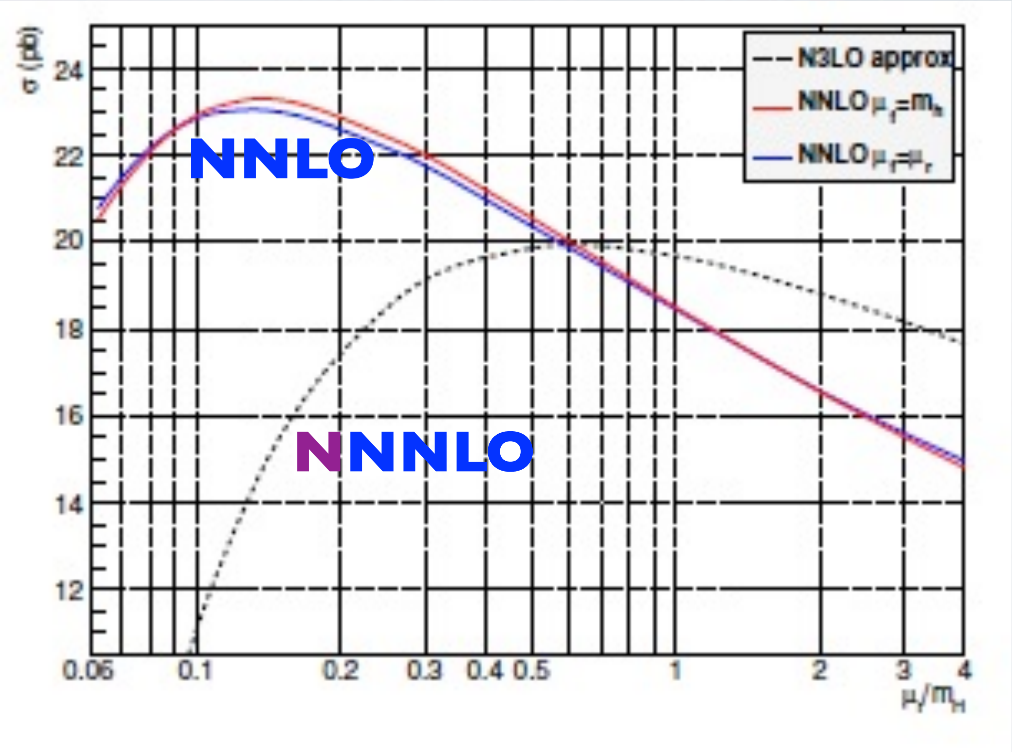
- Explicit Jet-veto resummation at NNLL matched to NNLO.
  ***Banfi, Monni, Salam, Zanderighi***
- Excellent agreement with fixed order NNLO down to very low veto values
- Lesson 1: caution is needed when the matching and resummation are not at the same level of accuracy (NLL-NNLO differs from NNLL-NNLO)
- Lesson II: A poor man's solution to rescale bad Monte-Carlo such that it matches a precisely known distribution is indeed poor!




*(Similar studies with a SCET formalism by **Becher, Neubert**)*

# EVEN BETTER PRECISION?

- The cross-section for gluon fusion is a very important ingredient for Higgs coupling extractions,
- causing the largest theoretical uncertainty.
- shall we go to an NNNLO precision?
- We can know already the precision which we can claim at the next order.
- surprisingly, we can only reduce the scale uncertainty from a 8% at NNLO to 5% at NNNLO.




***NNNLO is necessary to instill more confidence in our existing predictions.***

# A PATH TO NNNLO

*REVERSE UNITARITY AND THRESHOLD EXPANSIONS*

# EXISTING NNLO METHODS

- Pioneering work by van Neerven et al in Drell-Yan (~1990).

  - computing the inclusive cross-section in the soft limit $z \equiv \frac{M_V^2}{\hat{s}} \to 1.$
  - followed by complete calculation for arbitrary partonic energy.

- Additional techniques for Higgs production

  - Soft limit (*Catani, de Florian, Grazzini; Harlander,Kilgore*)
  - Systematic method for threshold expansion and resuming of the series (*Harlander,Kilgore*)

$$\hat{\sigma}_{RR} = (1-z)^{-1-4\epsilon}\left[a_1 + a_2(1-z) + a_3(1-z)^2 + \ldots\right]$$

$$\begin{aligned}\hat{\sigma}_{RV} = &(1-z)^{-1-4\epsilon}\left[b_1 + b_2(1-z) + b_3(1-z)^2 + \ldots\right] \\ &+(1-z)^{-1-2\epsilon}\left[c_1 + c_2(1-z) + c_3(1-z)^2 + \ldots\right]\end{aligned}$$

# REVERSE UNITARITY

*Melnikov, CA*

- Convert phase-space integrals into loop integrals.

$$\delta\left(p^2 - M^2\right) \to \frac{i}{p^2 - M^2} - c.c.$$

can almost forget about it

- Use IBP identities and the Laporta algorithm to reduce phase-space integrals into master integrals

$$\int d^d k \frac{\partial}{\partial k_\mu} \frac{q^\mu}{k^2 \dots} = 0 \rightsquigarrow I_1 + I_2 + \dots = 0$$

- Simplification for cut propagators.

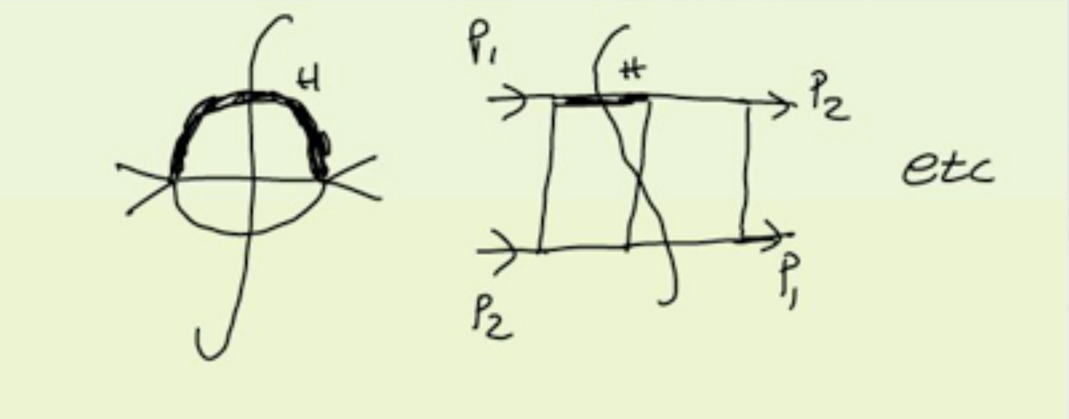
$$\left(\cancel{\frac{i}{k^2}}\right)^n \to 0, \quad n = 0, -1, 2, \dots$$

- Few remaining master integrals. Solved using differential equations, derived and solved in the same way as for loop master integrals (*Kotikov; Gehrmann, Remiddi, Smirnov, Veretin, ...*)

# REVERSE UNITARITY

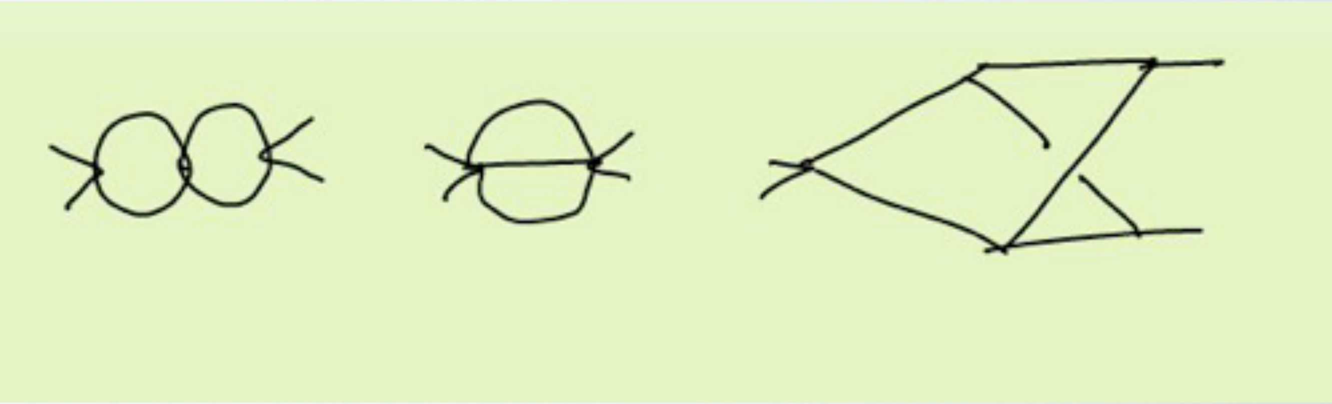
*Melnikov, CA*

- 18 double real-radiation master integrals




- 7 real-virtual master integrals


- 3 double-virtual master integrals for the two-loop form factor



# FROM NNLO TO NNNLO

| | NLO | NNLO | NNNLO |
|---|---|---|---|
| topologies | 1 | 11 | 217 |
| master integrals per topology | 1 | ~5 | ~25 |
| total number of master integrals | 1 | 18 | ~1000 |
| | integrations over real radiation tree-level graphs | | |

- *Sheer magnitude of such a calculation is frightening*
- *But, we can hope in sharpening our methods*

# IN THIS TALK

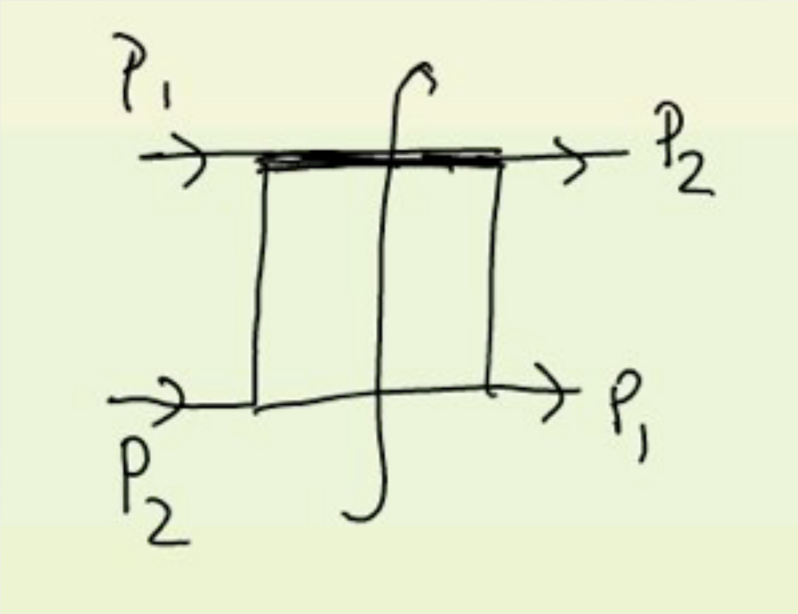
- Threshold series expansion with the "reverse unitarity" method
  - $\mathbf{z = 1}$ *limit is extremely useful as a first step towards a complete calculation*
  - *necessary boundary condition for solving master integral differential equations*
  - *important contribution to the cross-section*

- The method allows for a systematic expansion around the soft limit, acquiring as many terms in the series as computer power permits us to do so.

- Enormous simplification permitting the use of IBP identities directly in the soft limit.

# THE NLO REAL RADIATION EXAMPLE

Consider the NLO real radiation topology:




$$I\left[\nu_1,\nu_2\right]=\int d^d k \frac{\delta\left((p_{12}-k)^2-M_V^2\right)\delta\left(k^2\right)}{\left[(k-p_1)^2\right]^{\nu_1}\left[(k-p_2)^2\right]^{\nu_2}}$$

**two-scale integral**

$$\nu_1,\nu_2=\ldots,-2,-1,0,1,2,\ldots$$

*Scaling of the gluon momentum:* $k=\bar{z}\quad l,\quad \bar{z}\equiv 1-z=1-\frac{M_V^2}{\hat{s}}$

**(no approximation made)**

$$I\left[\nu_1,\nu_2\right]=\bar{z}^{1-\nu_{12}-2\epsilon}I_{reg}\left[\nu_1,\nu_2\right]$$

**exponent X divergent**

**= LOGS**

$$I_{reg}\left[\nu_1,\nu_2\right]=\int d^d l \frac{\delta\left((l-p_{12})^2\right)\delta\left(l^2\right)}{\left[(l-p_1)^2\right]^{\nu_1}\left[(l-p_2)^2\right]^{\nu_2}}$$

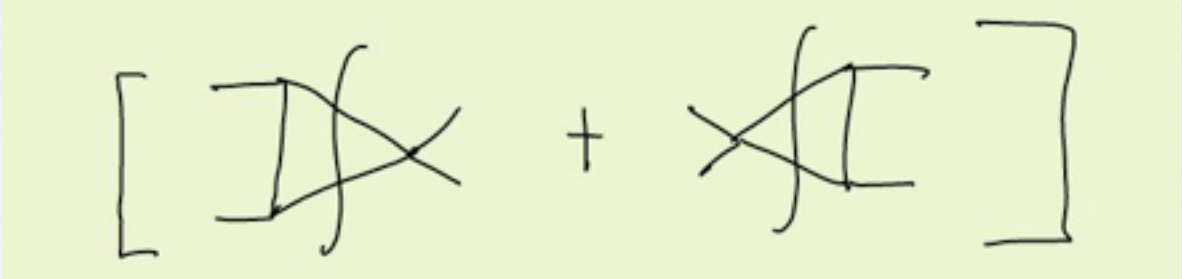
**one-scale integral**

# THE NLO REAL RADIATION EXAMPLE

$$I_{reg}\left[\nu_1,\nu_2\right]=\int d^d l\frac{\delta\left((l-p_{12})^2\right)\delta\left(l^2\right)}{\left[(l-p_1)^2\right]^{\nu_1}\left[(l-p_2)^2\right]^{\nu_2}}$$

Trivial to perform the integration over the rescaled momentum.
But, let's resist the temptation.

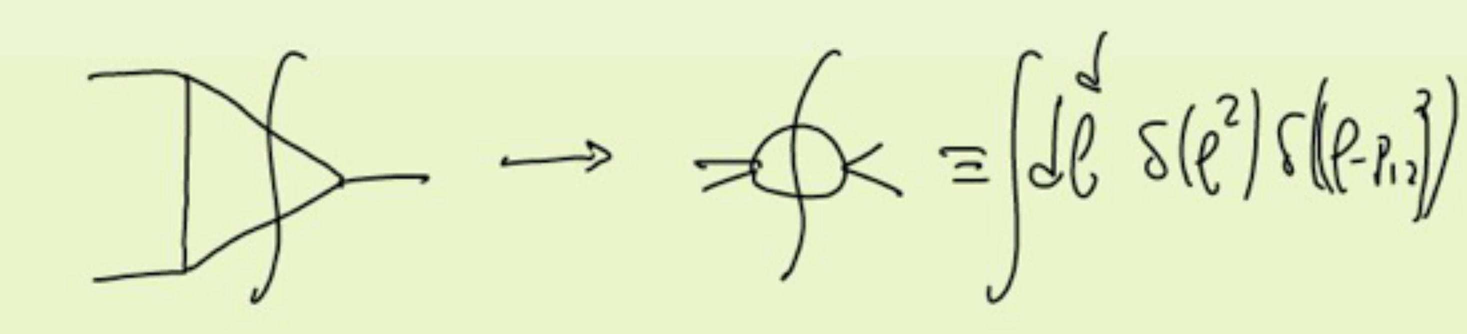
$$(l-p_1)^2+(l-p_2)^2=(l-\cancel{p_{12}})^2+\cancel{l^2}-p_{12}^2 \rightsquigarrow$$



*Double cut of one-loop form factor integrals*

## REVERSE UNITARITY:

$$\delta\left(l^2\right),\delta\left((l-p_{12})^2\right)\to\frac{i}{\cancel{l^2}},\frac{i}{(l-\cancel{p_{12}})^2} \quad \rightsquigarrow$$



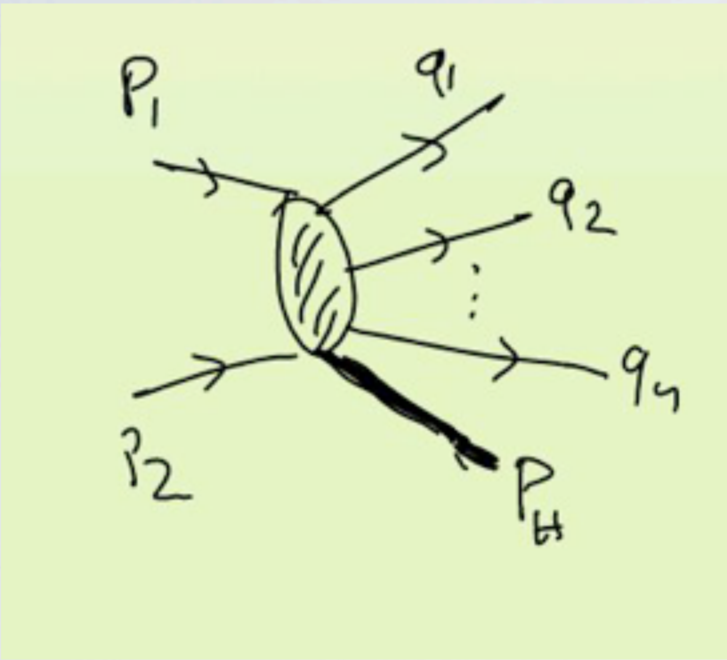
*One master integral:*
*two massless particle phase-space measure*

# FIRST LESSONS

- A rescaling of gluon momenta which captures their behavior in the soft limit leads to phase-space integrals which depend only on a single kinematic scale (at NLO).
- Reverse unitarity and integration by parts minimize the amount of integrations (down to one integral).
- Calculation is almost entirely algebraic (=algorithmic).

# MULTIPLE REAL EMISSION

$$I = \int d^d q_1 \ldots d^d q_N \delta(q_1^2) \ldots \delta(q_N^2) \delta\left((p_{12} - q_{12\ldots N})^2 - M_V^2\right) \left|\mathcal{M}^2\right|^2$$




T

**reverse unitarity**

↓

$$I = \int \frac{d^d q_1 \ldots d^d q_N}{\cancel{q_1^2} \ldots \cancel{q_N^2} \, \left((p_{12} - q_{12\ldots N})^{\cancel{2}} - M_V^2\right)} \left|\mathcal{M}^2\right|^2$$

SCALING: $q_i \to \bar{z} q_i$

***(no approximation made yet)***

$$I = \bar{z}^{N(d-2)-1} \int \frac{d^d q_1 \ldots d^d q_N}{\cancel{q_1^2} \ldots \cancel{q_N^2} \, \left((p_{12} - q_{12\ldots N})^{\cancel{2}} - z q_{12\ldots N}^2\right)} \left|\mathcal{M}\right|^2 (\bar{z} q_i, p_1, p_2)$$

***Correct asymptotic behavior***

***New integral depends on z. But it is regular at z=1.***
***Can be expanded INSIDE the integration sign.***

# MULTIPLE REAL RADIATION

Taylor expanding the integrand:

$$I = \bar{z}^{2N\epsilon-1}\left[I_{\text{reg}}^{(0)} + I_{\text{reg}}^{(1)}\bar{z} + I_{\text{reg}}^{(2)}\bar{z}^2 + \ldots\right]$$

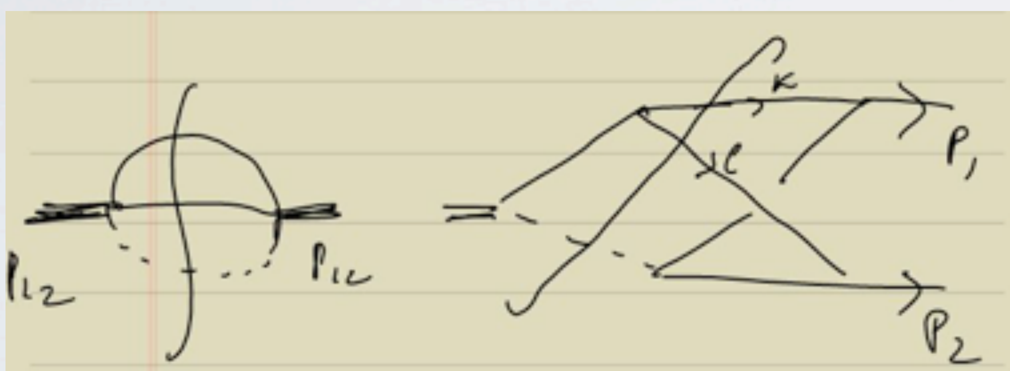
**IBP REDUCTION**

$\boxed{\textbf{MASTERS}}$

- Integrals of sub-leading terms reduce to the *same master integrals* as the ones making up the strict soft limit!
- Computing more terms in the series expansion is an algebraic problem
- no new master integrals emerge.

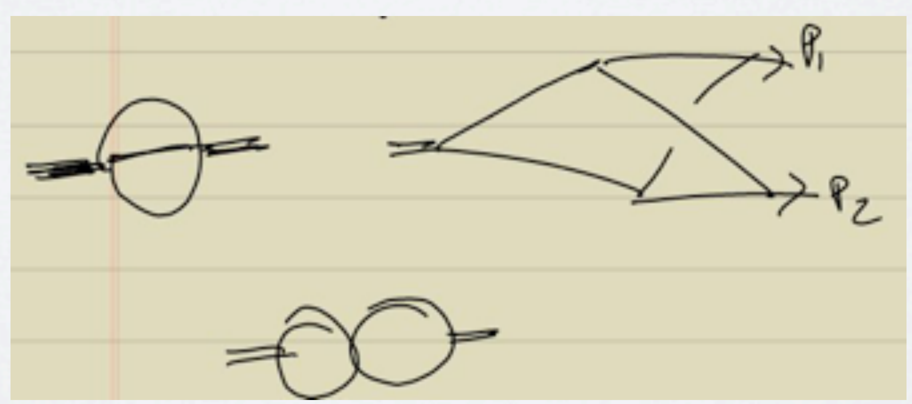
# DOUBLE REAL RADIATION AT NNLO

- 18 master integrals for a generic value of z.
- Two master integrals for the expansion around the soft limit:



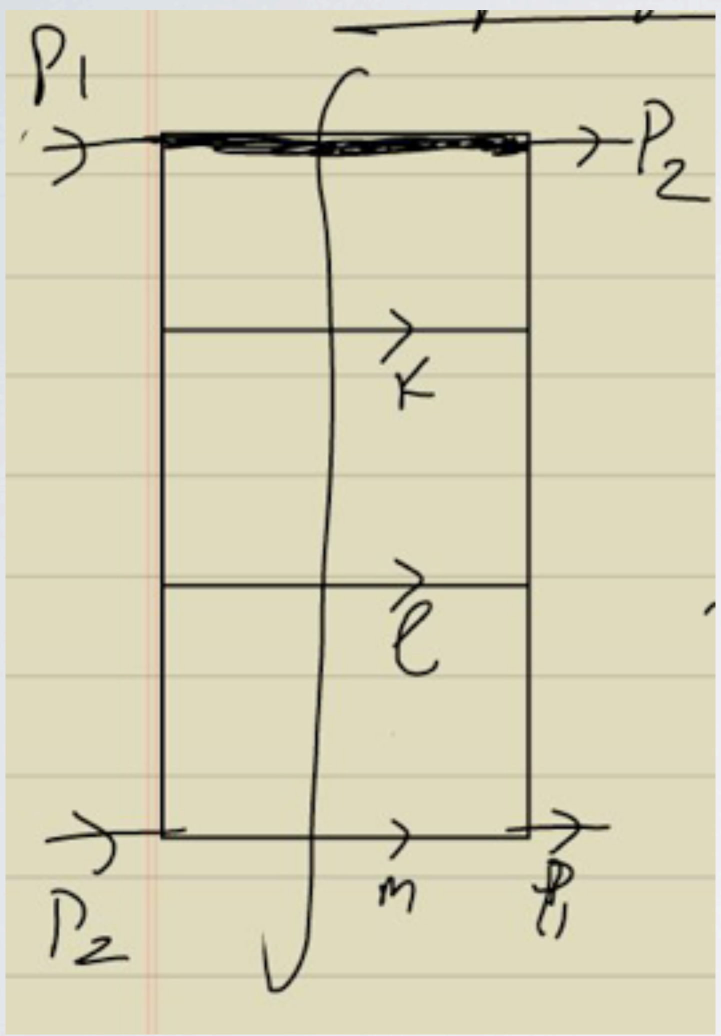

$$- - - - \equiv \frac{1}{(p_{12}-k+l)^2 - z(k+l)^2}\Bigg|_{z=1}$$

- Recall the master integrals for the two-loop form factor:

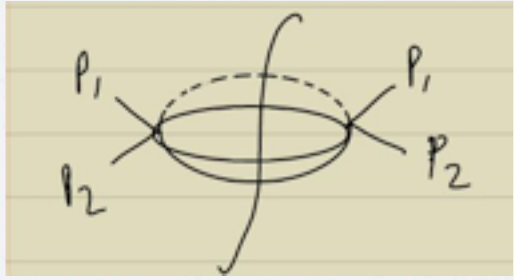



- They are of similar nature (coincide in the "wrong" limit $z=0$).

# TRIPLE REAL RADIATION AT NNNLO




- Looked at some of the 215 topologies which appear at NNNLO.
- A verified example of a topology is shown here.
- 23 master integrals for generic z.
- These collapse to one very simple master integral, the phase-space measure, when expanding around threshold.




- Total number of master integrals
~ master integrals for the three-loop form factor with a quadruple cut (< 10).

# WORK IN HAPPY PROGRESS

- Identifying and reducing to master integrals all triple real-radiation topologies
- Further steps:
  - extend this method to combinations of real and virtual radiation
  - requires scalings of loop-momenta in the soft limit and it is conceptually harder.
  - success for real-virtual master integrals at NNLO *(Dulat, Mistlberger)*
  - a lot more inventiveness is needed for RVV and RRV at NNNLO, but we hope to get guidance from the two-loop master integral computations for Higgs+1jet production *(Gehrmann,Remiddi)*
- Watch this space

# CONCLUSIONS

- Higgs discovery came with excitement, a feeling of relief, but also puzzles.
- Soon we will move to an era of Higgs boson precision phenomenology
- This era for theorists has started since long time ago, with precise calculations for inclusive and differential Higgs cross-sections and decay rates at very high orders in perturbation theory.
- Further progress can come with even harder calculations at even higher orders and the development of new methods in perturbative QCD.
- First attempt to improve on gluon fusion inclusive cross-section, performing an expansion in the soft limit.
- Extended the application of the reverse unitarity method to threshold expansions of phase-space integrals.
- Attainable NNNLO precision 5%.