A simple, yet subtle, invariance of the two-body decay kinematics

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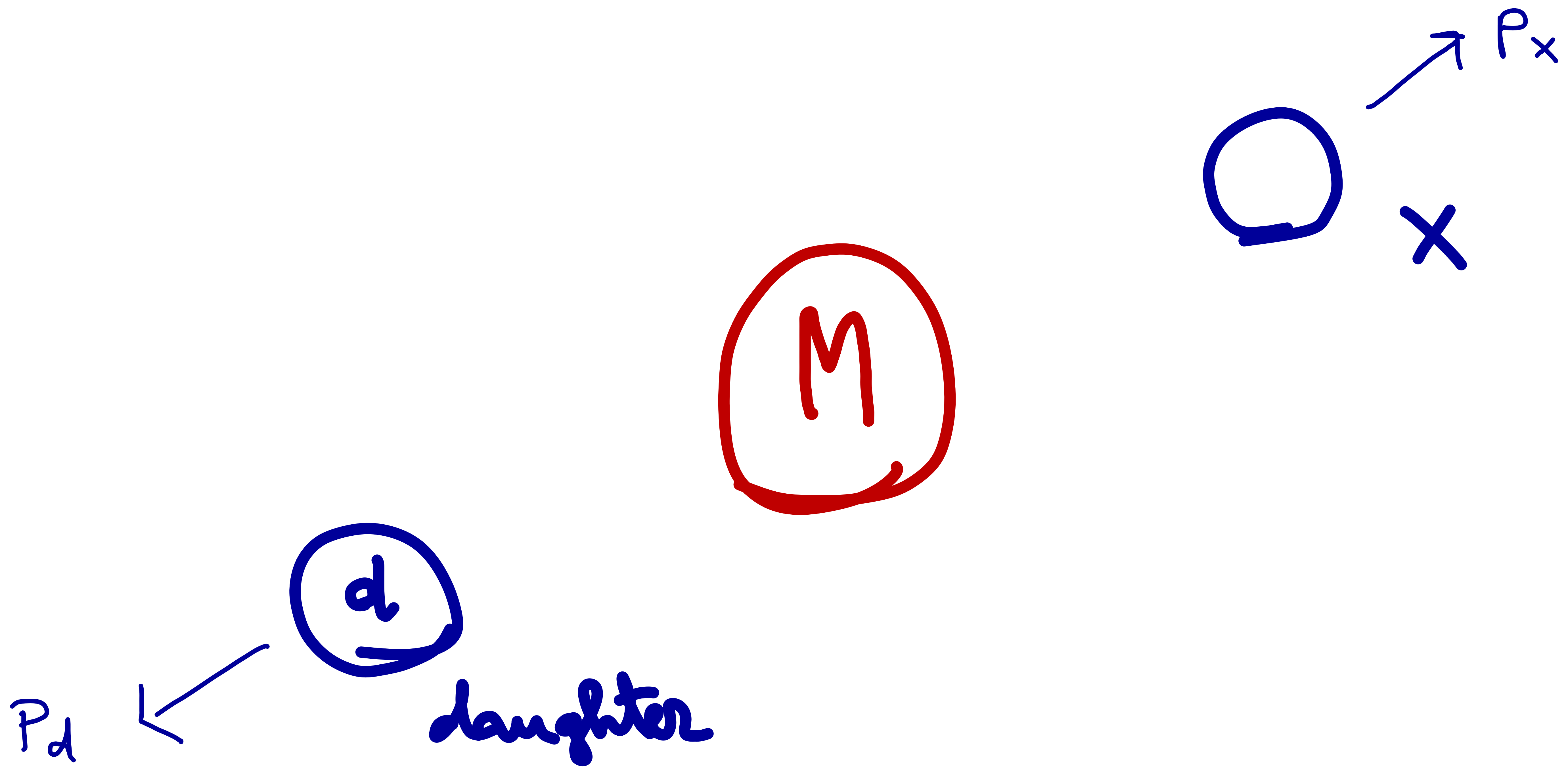
arXiv:1209.0772
with K.Agashe and D.Kim

TWO-BODY DECAY : $M \to d\,X$

$$P_M^{\mu} = P_d^{\mu} + P_x^{\mu}$$

KINEMATICS FULLY FIXED BY THE MASSES



$$E_d =: E_d^* = \frac{m_M^2 + m_d^2 - m_x^2}{2m_M} \qquad \bar{p}_d + \bar{p}_x = 0$$

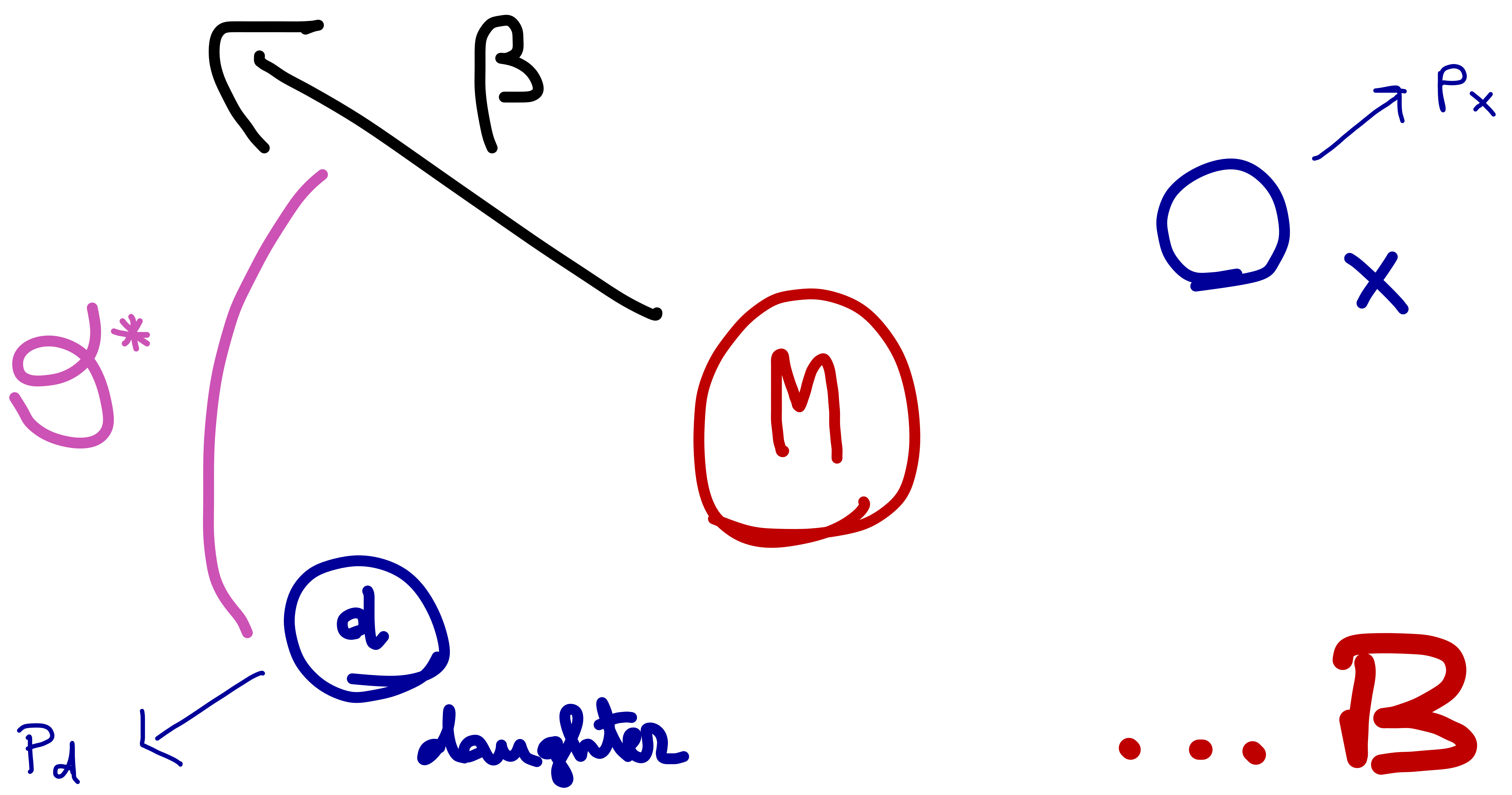
IN THE REST FRAME OF M

WHAT DOES IT LOOK LIKE IN ANOTHER FRAME?

IN GENERAL WE KNOW THE ANSWER

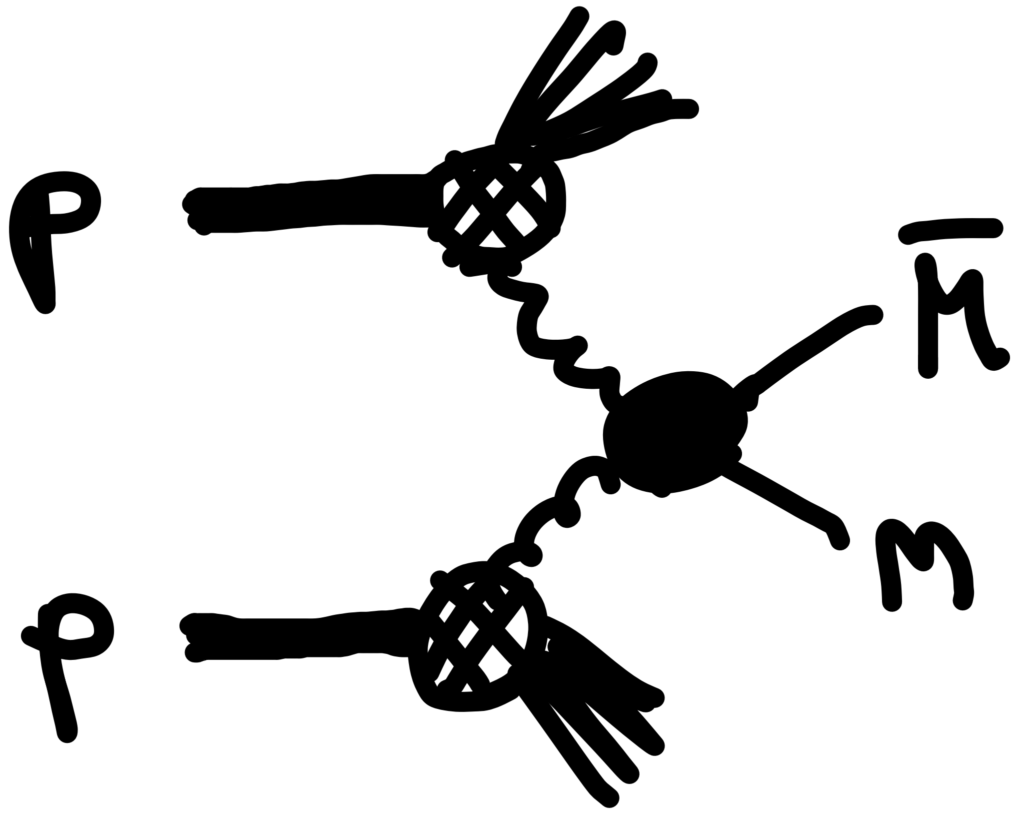
IF THE FRAME OF THE OBSERVER AND THAT OF REST OF THE MOTHER ARE CONNECTED BY A BOOST β

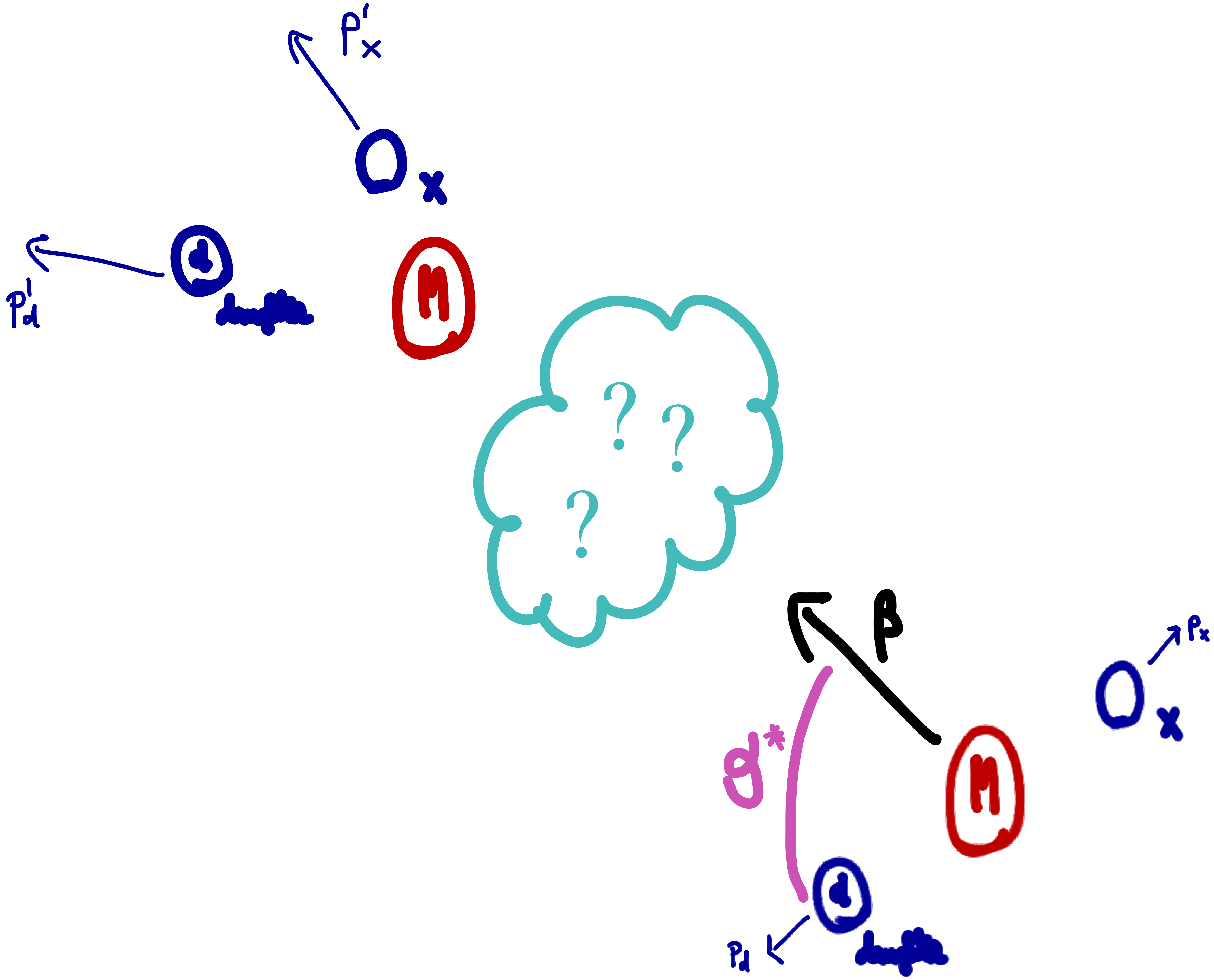
$$E'_d = E^*_d \gamma + p^*_d \gamma\beta \cos\vartheta^*$$

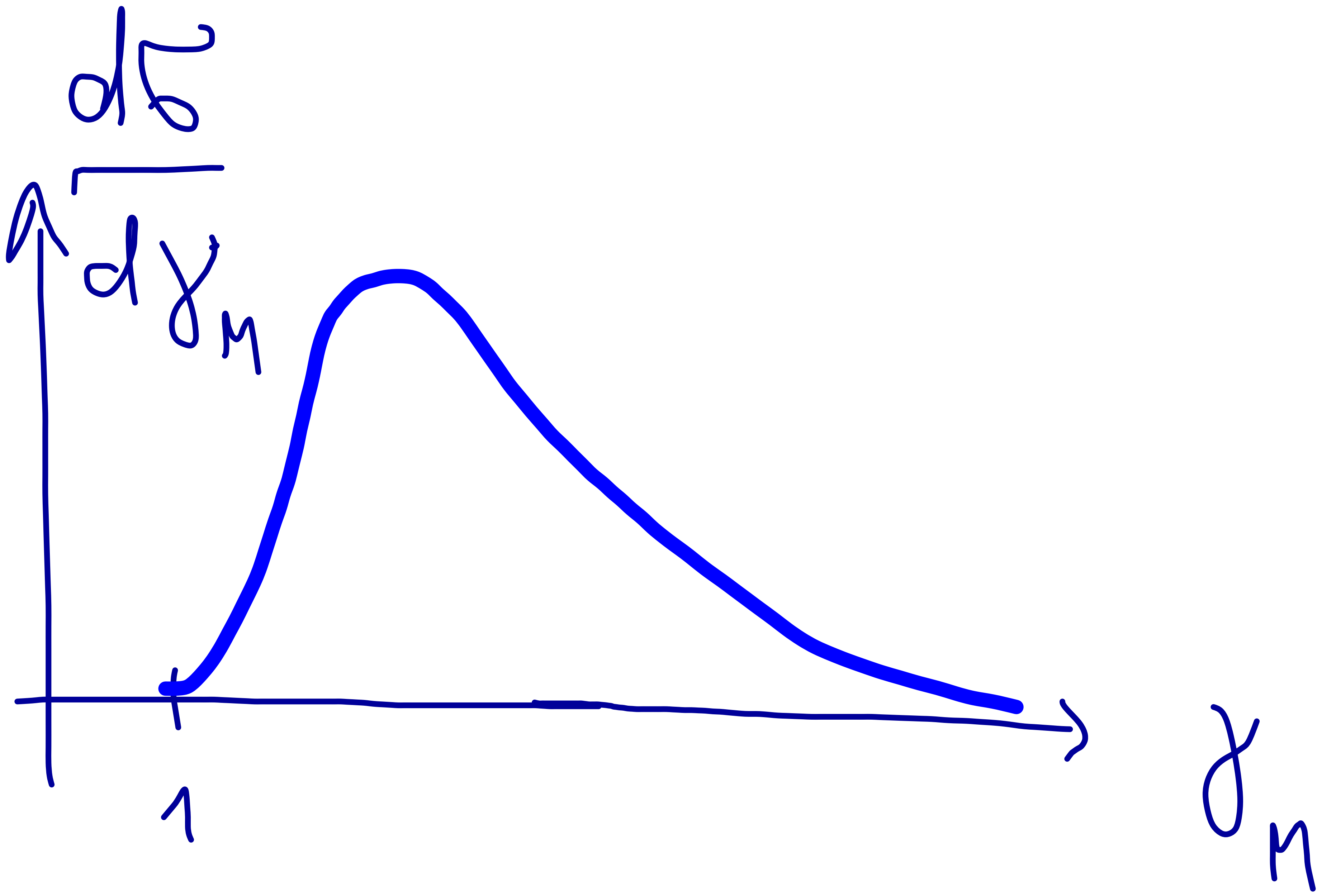


... BUT

BUT IN MOST CASES WE DO NOT KNOW THE BOOST OF THE MOTHER







SOLUTION TO OVERCOME THE UNKNOWN BOOST
USE BOOST INVARIANT QUANTITIES

- CONSERVED EVENT BY EVENT
- SIMPLE TO UNDERSTAND
- UNIVERSAL (SPECIAL RELATIVITY IS THE SAME FOR ALL PARTICLES)

- DEMANDING
  - GENERICALLY THEY ARE FUNCTION OF SEVERAL QUANTITIES
    TO MAKE AN INVARIANT MASS YOU NEED <u>TWO FOUR-VECTORS</u>
    WITH BOTH <u>ENERGY</u> <u>AND</u> <u>ANGLES</u>

IN THIS TALK :

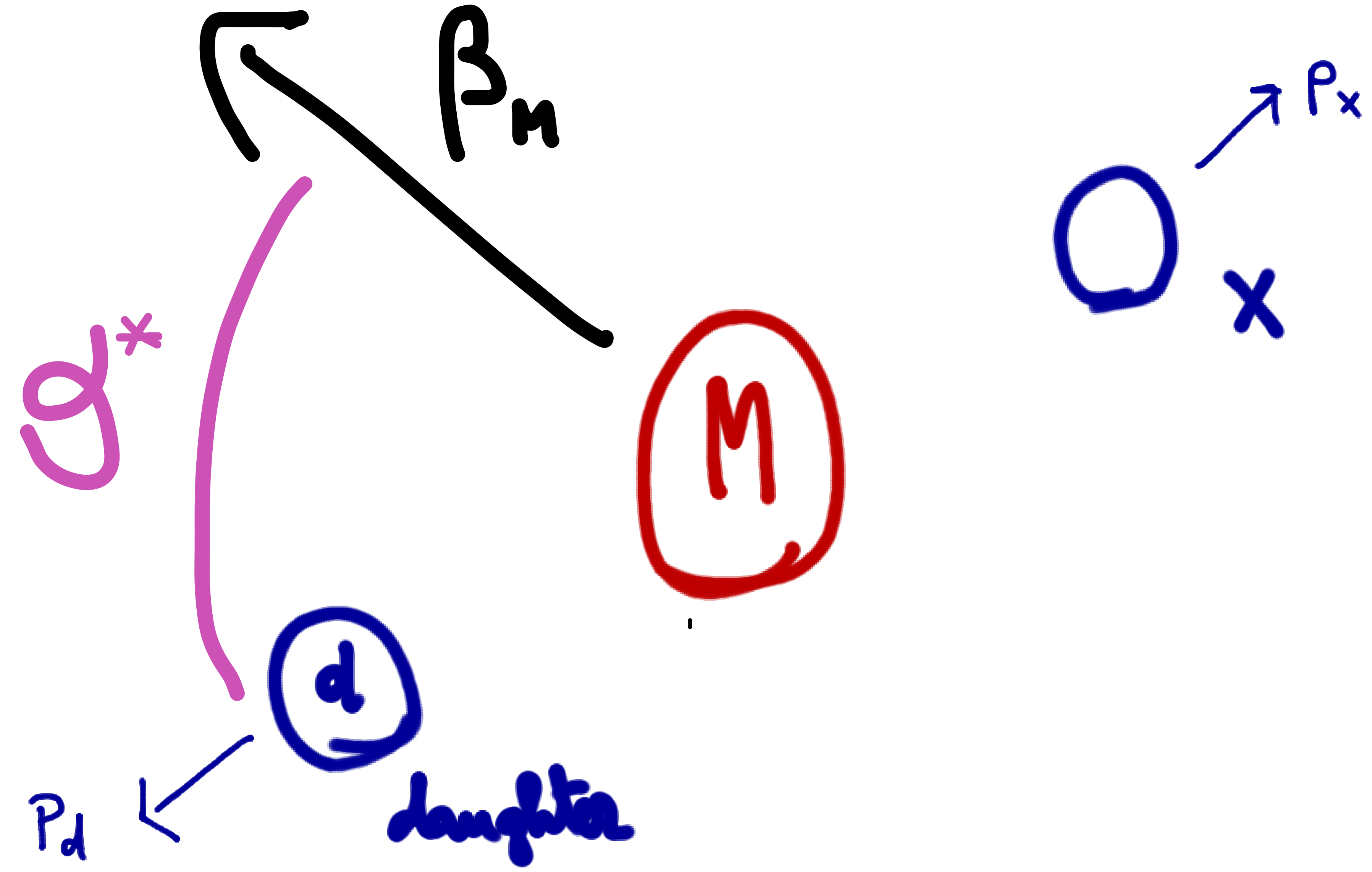
LORENTZ VARIANT QUANTITIES
WITH SOME KIND OF "PHENOMENOLOGICAL INVARIANCE"
TO ACCESS INVARIANTS OF THE DECAY

FOR INSTANCE:

THE OBSERVED ENERGY DEPENDS ON THE FRAME

THE ENERGY DISTRIBUTION IN PHENOMENOLOGICALLY RELEVANT SITUATIONS HAS SOME INVARIANCE

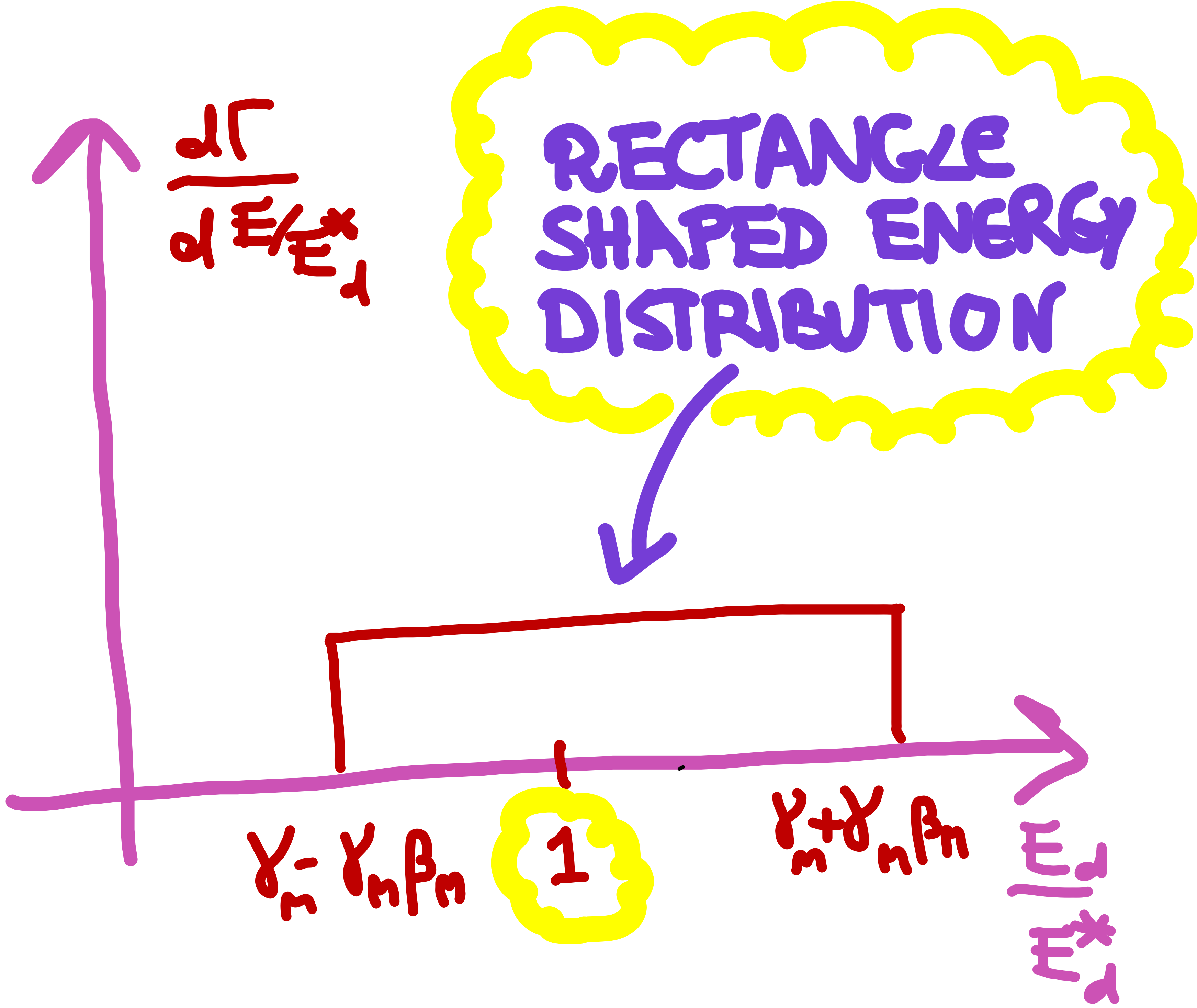
- DAUGHTER d IS MASSLESS (for now)
- IMAGINE THE MOTHER HAS A BOOST β_M IN THE LAB FRAME
- THE DAUGHTER MOMENTUM IS AT AN ANGLE ϑ W.R.T. β_M

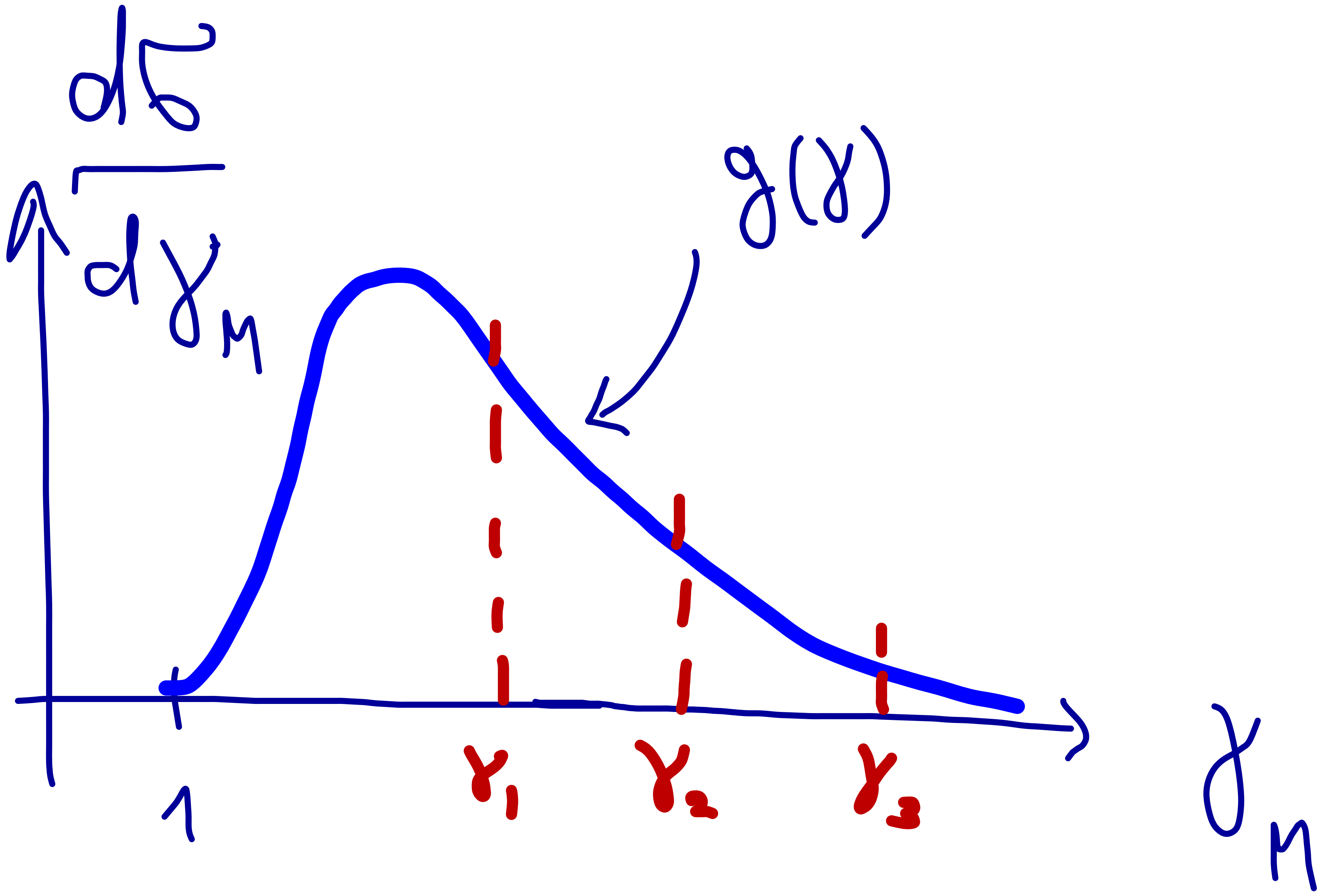


IN THE LAB

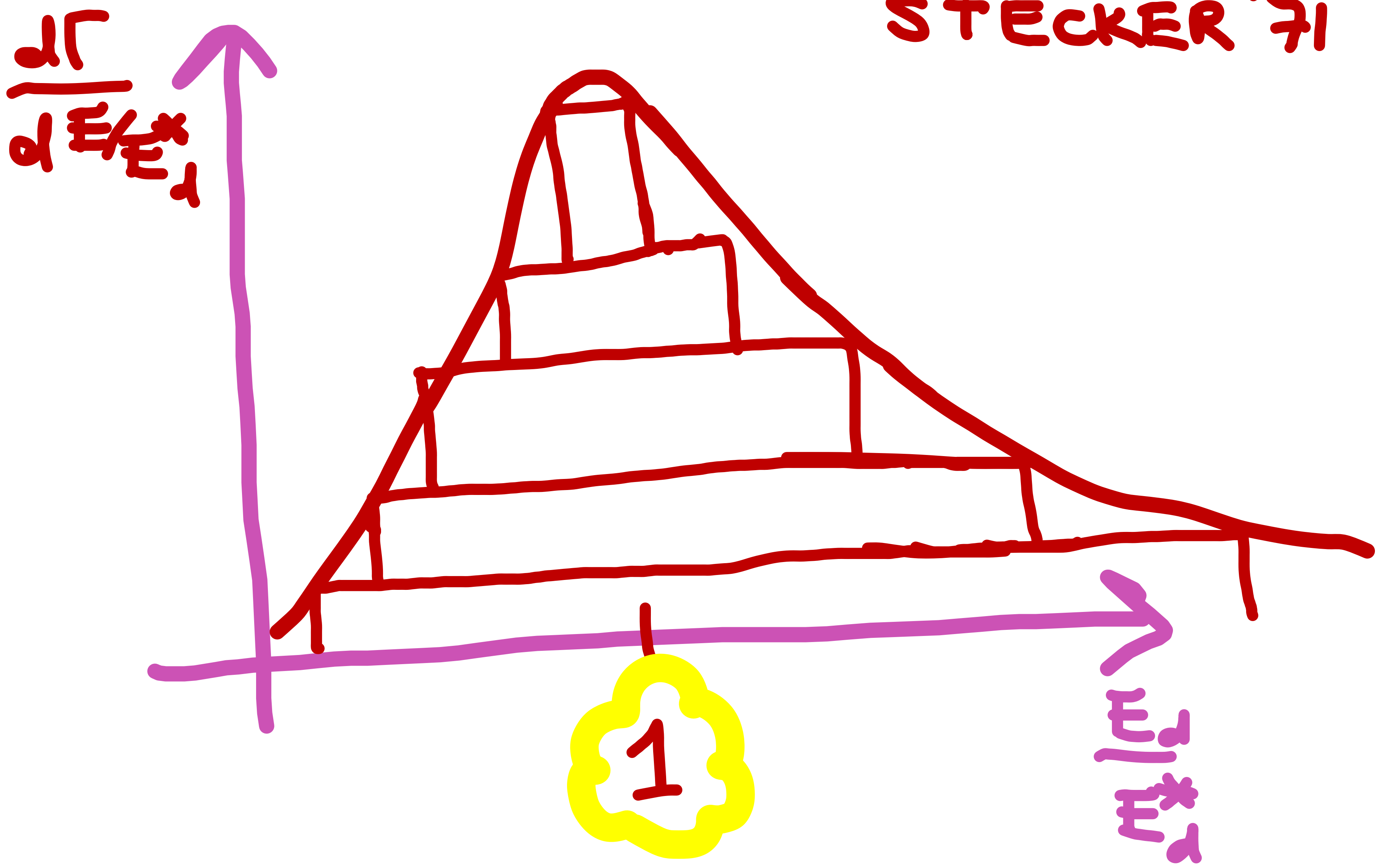
$$E_d = E_d^* \left(\gamma_M + \cos\vartheta^* \beta_M \gamma_M \right)$$

IF THE MOTHER IS A SCALAR
ϑ IS FLAT FROM -1 TO 1

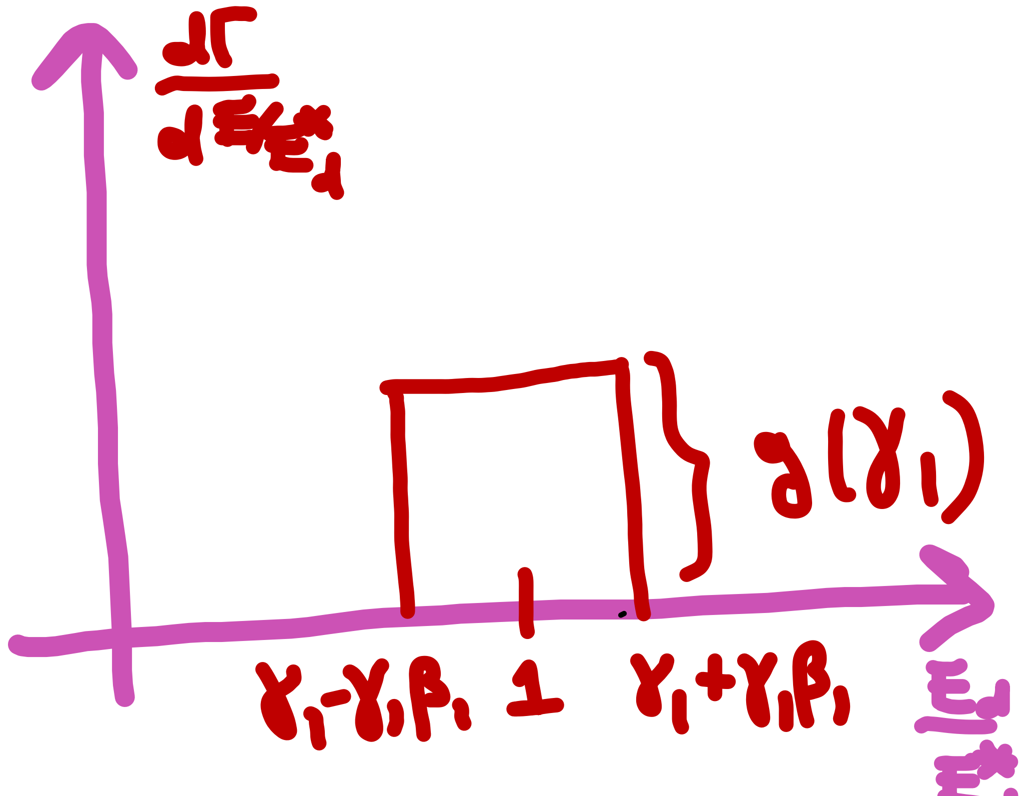


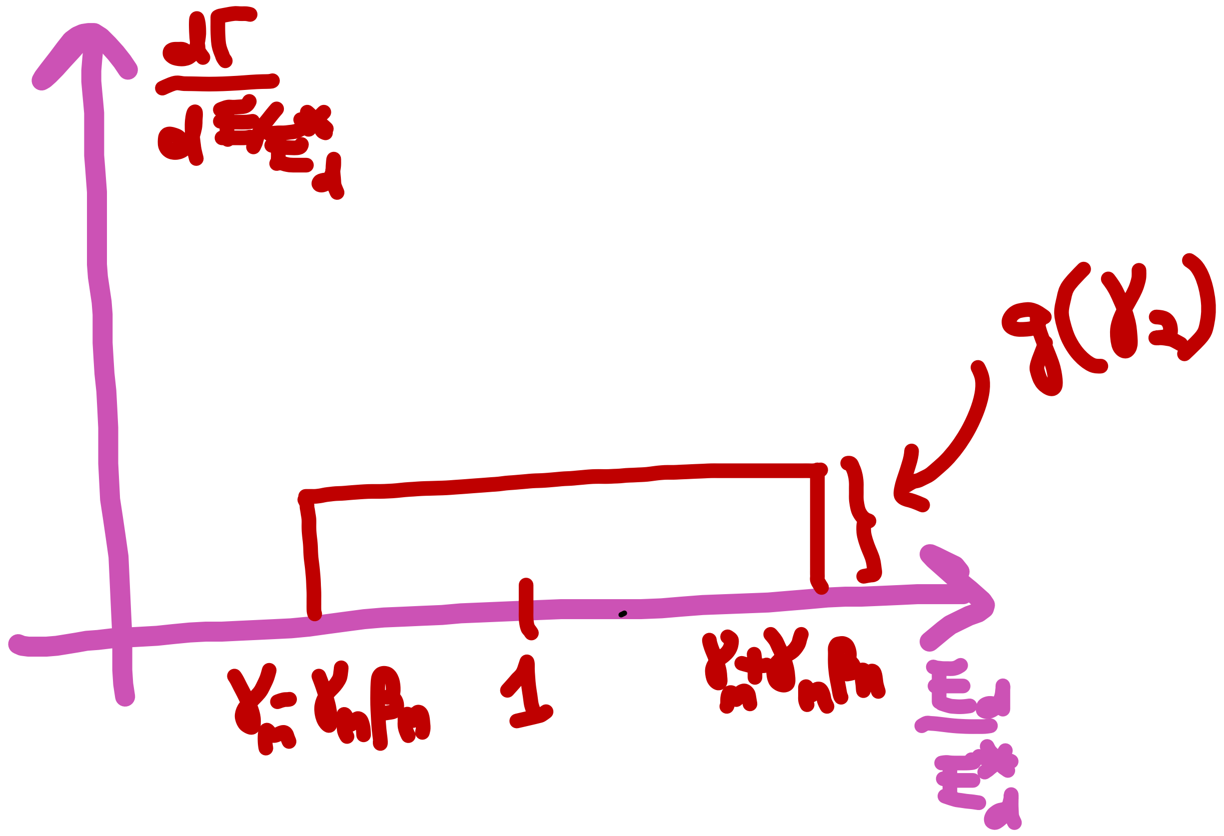


STECKER '71



THE ENERGY DISTRIBUTION IN THE LAB IS THE SUM OF ALL THE RECTANGLES





GENERALIZATIONS:

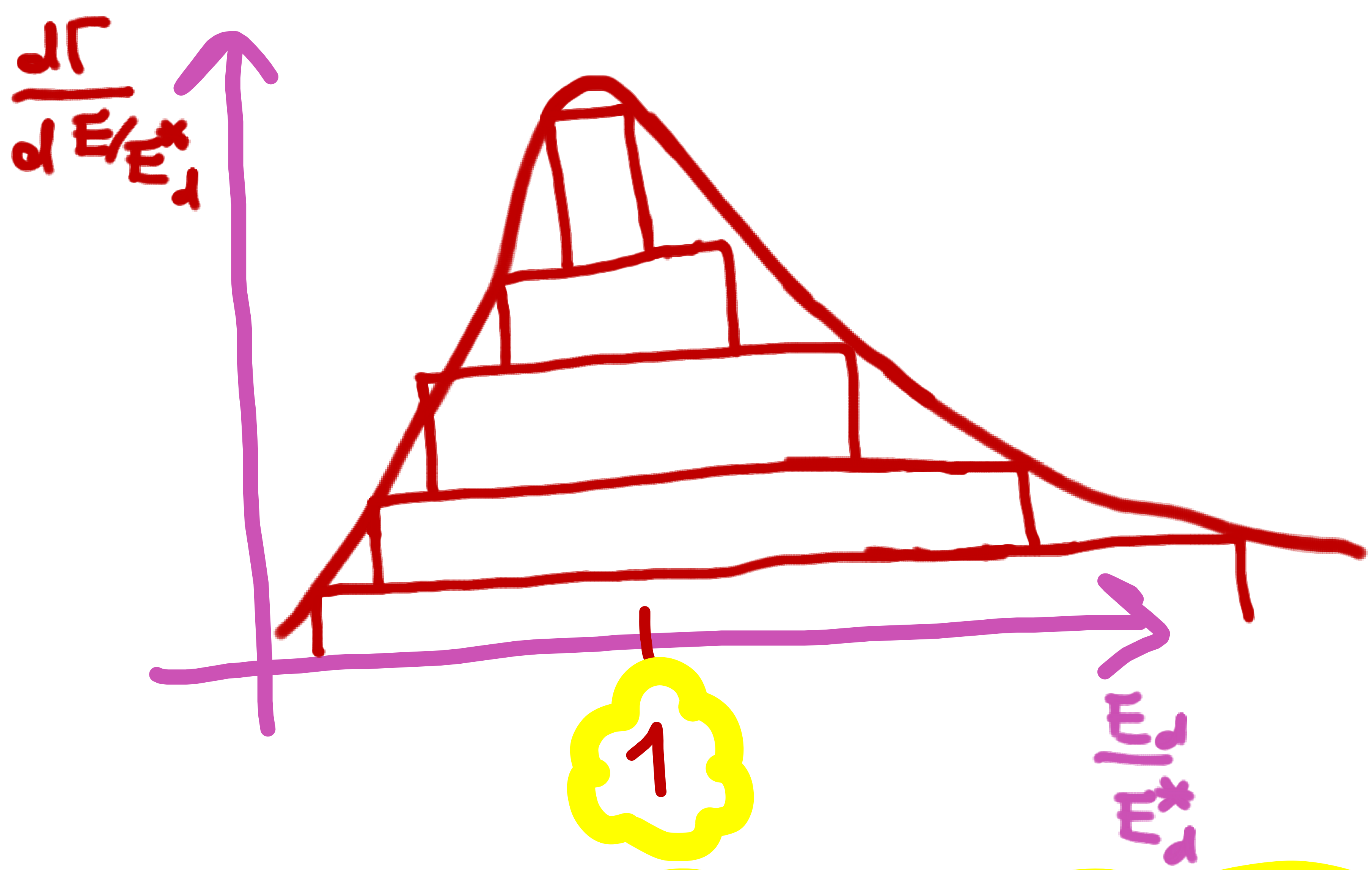
- INSTEAD OF A SCALAR MOTHER ONE CAN TAKE AN UNPOLARIZED ENSEMBLE OF PARTICLES WITH SPIN

- THE DAUGHTER CAN BE MASSIVE

  IF $g(\gamma) = 0$ for $\gamma \geq 2\gamma^* - 1$

  WHERE $\gamma^* = \frac{E_d^*}{m_d}$

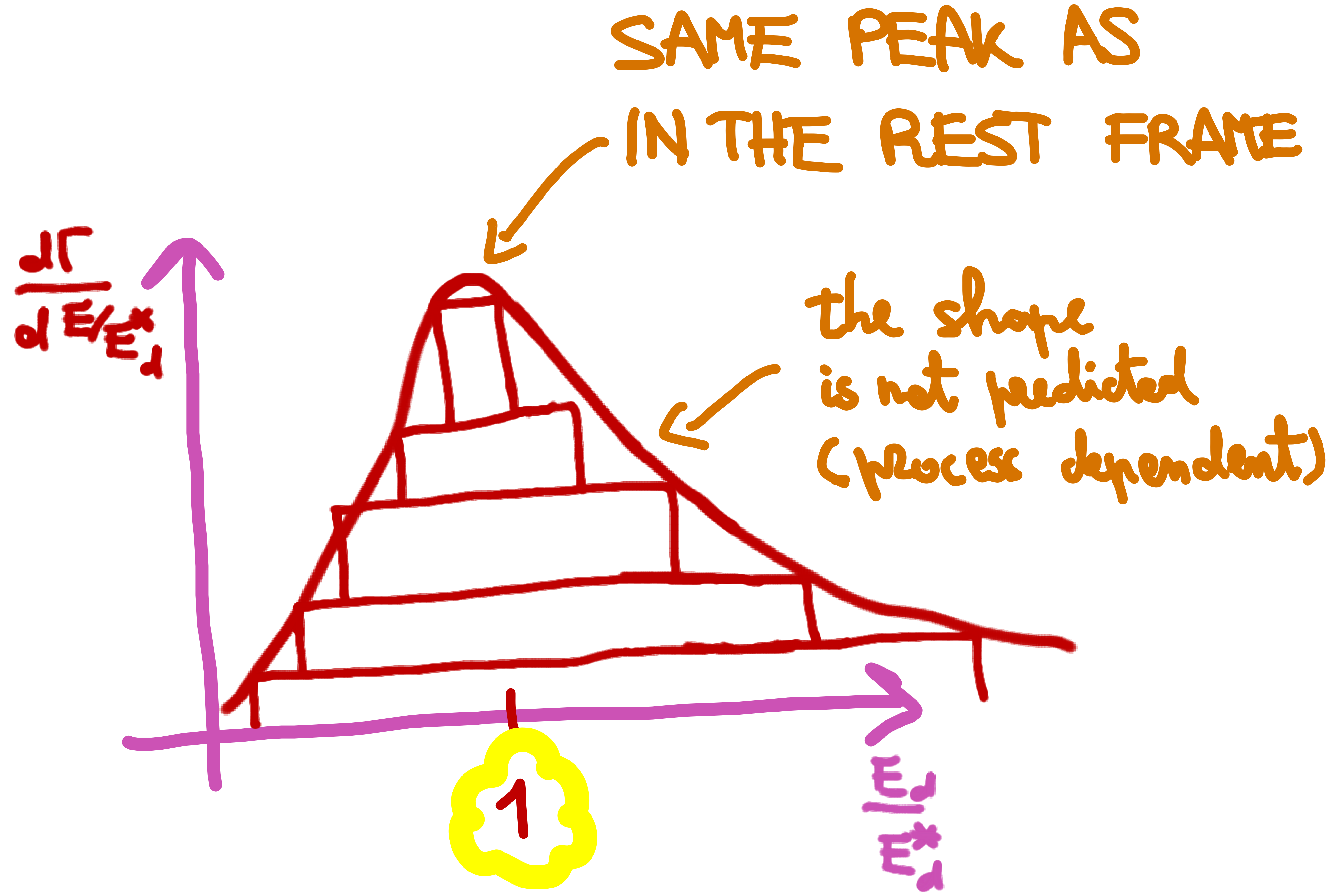
$$E_d^* = \frac{m_M^2 + m_d^2 - m_X^2}{2 m_M}$$



$E = E_d^*$ IS THE PEAK

THE FRAME-DEPENDENT ENERGY DISTRIBUTION ENCODES THE INVARIANT E_d^* IN A VERY SIMPLE WAY

ADVANTAGES (GENERAL: ALMOST ONLY KINEMATICS)

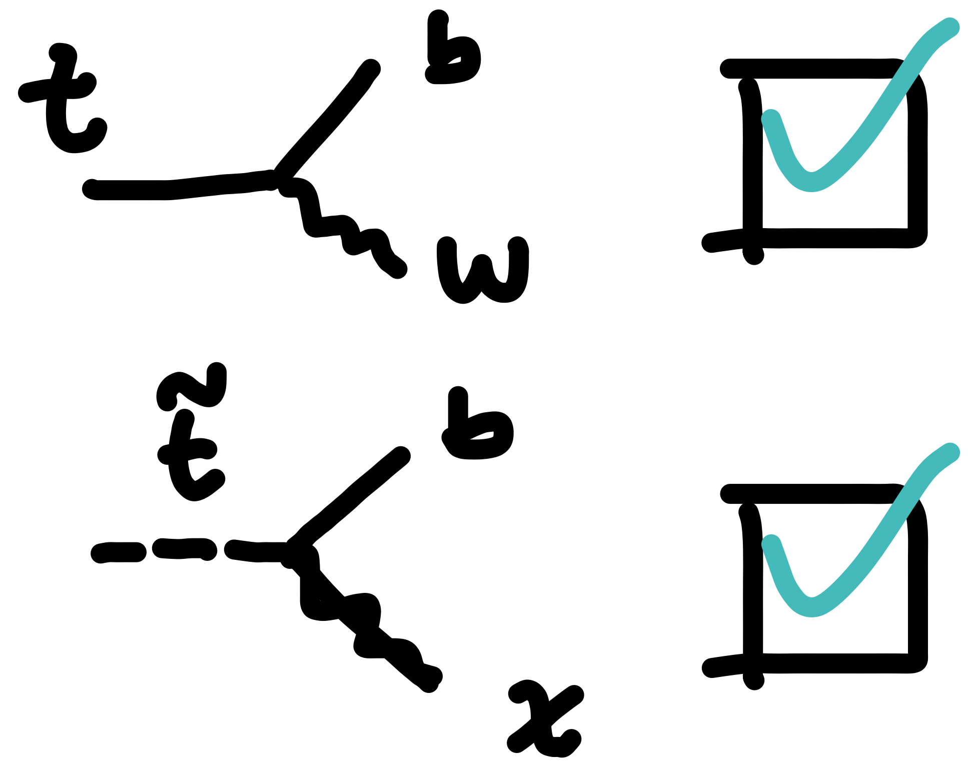


THE ONLY DYNAMICAL ASSUMPTION WAS THE MOTHER TO BE NOT POLARIZED

⇓

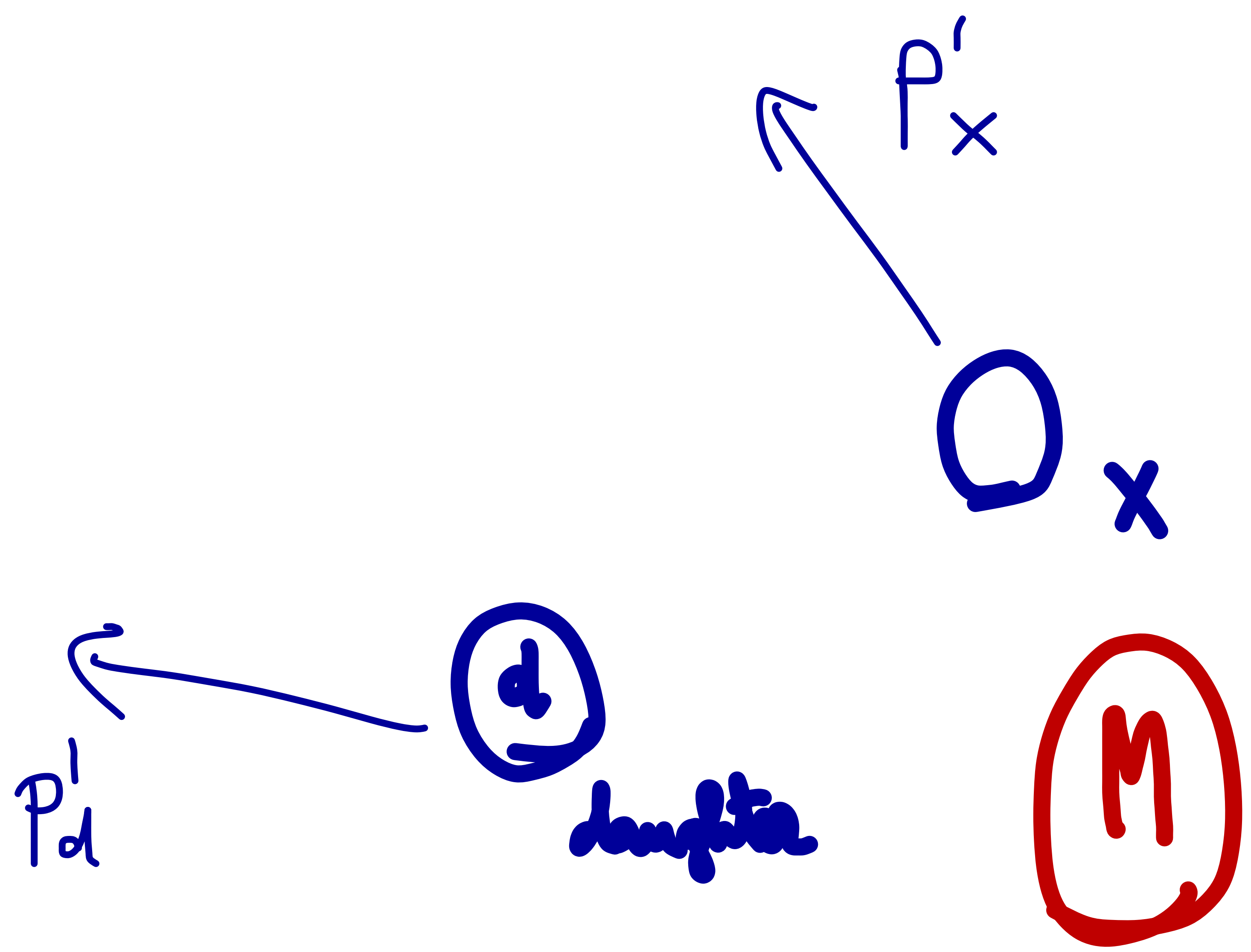
THE RESULT APPLIES FOR BOTH KNOW PARTICLES OF THE SM AND FOR NEW PHYSICS

THE FRAME-DEPENDENT ENERGY DISTRIBUTION ENCODES THE INVARIANT E_d^* IN A VERY SIMPLE WAY

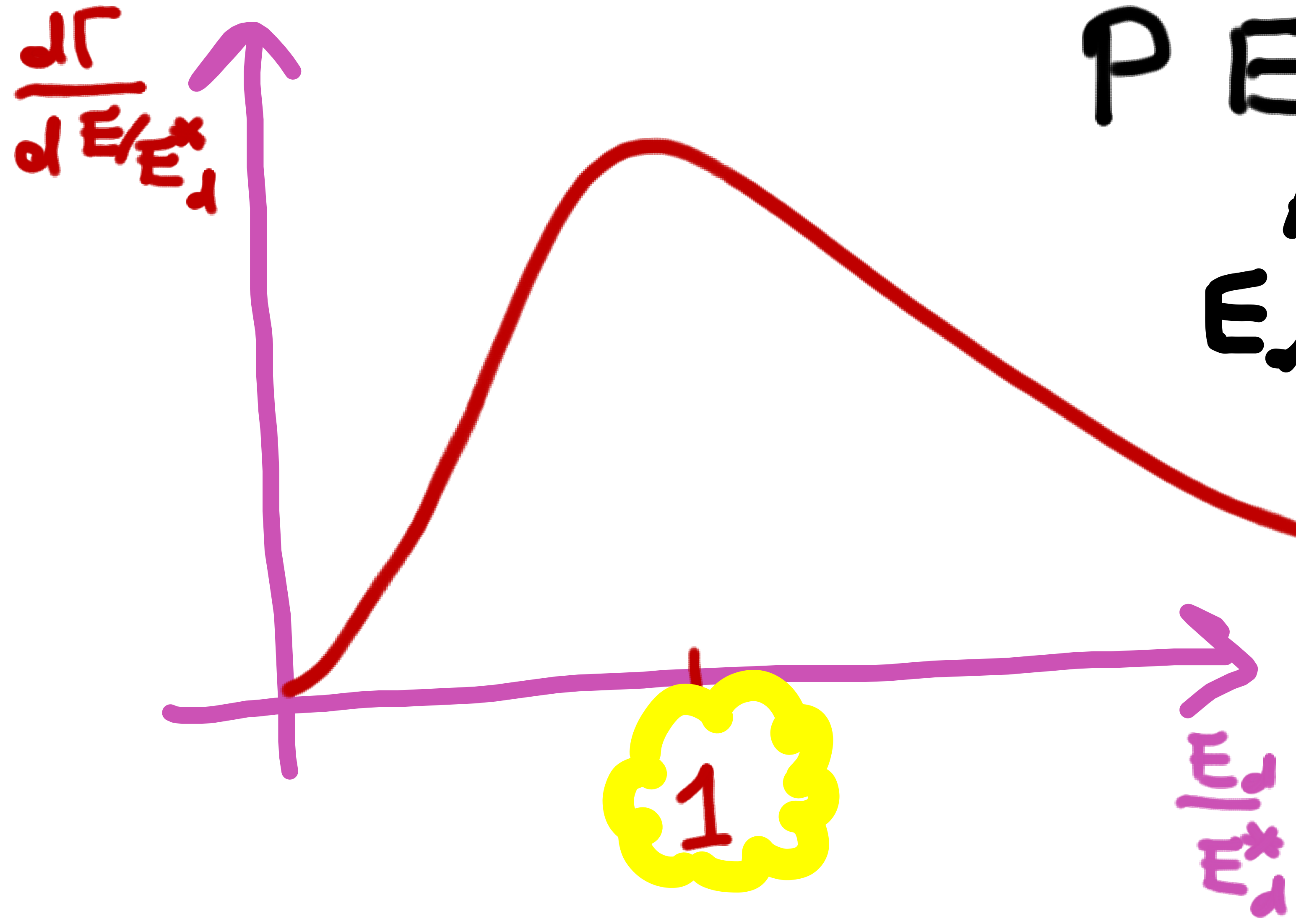


ADVANTAGES

(OVER INVARIANT MASS FOR INSTANCE)



$$E_d^* = \frac{m_M^2 + m_d^2 - m_X^2}{2 m_M}$$



PEAK AT $E_d = E_d^*$

- NO NEED TO MEASURE THE OTHER DECAY PRODUCT

⇓

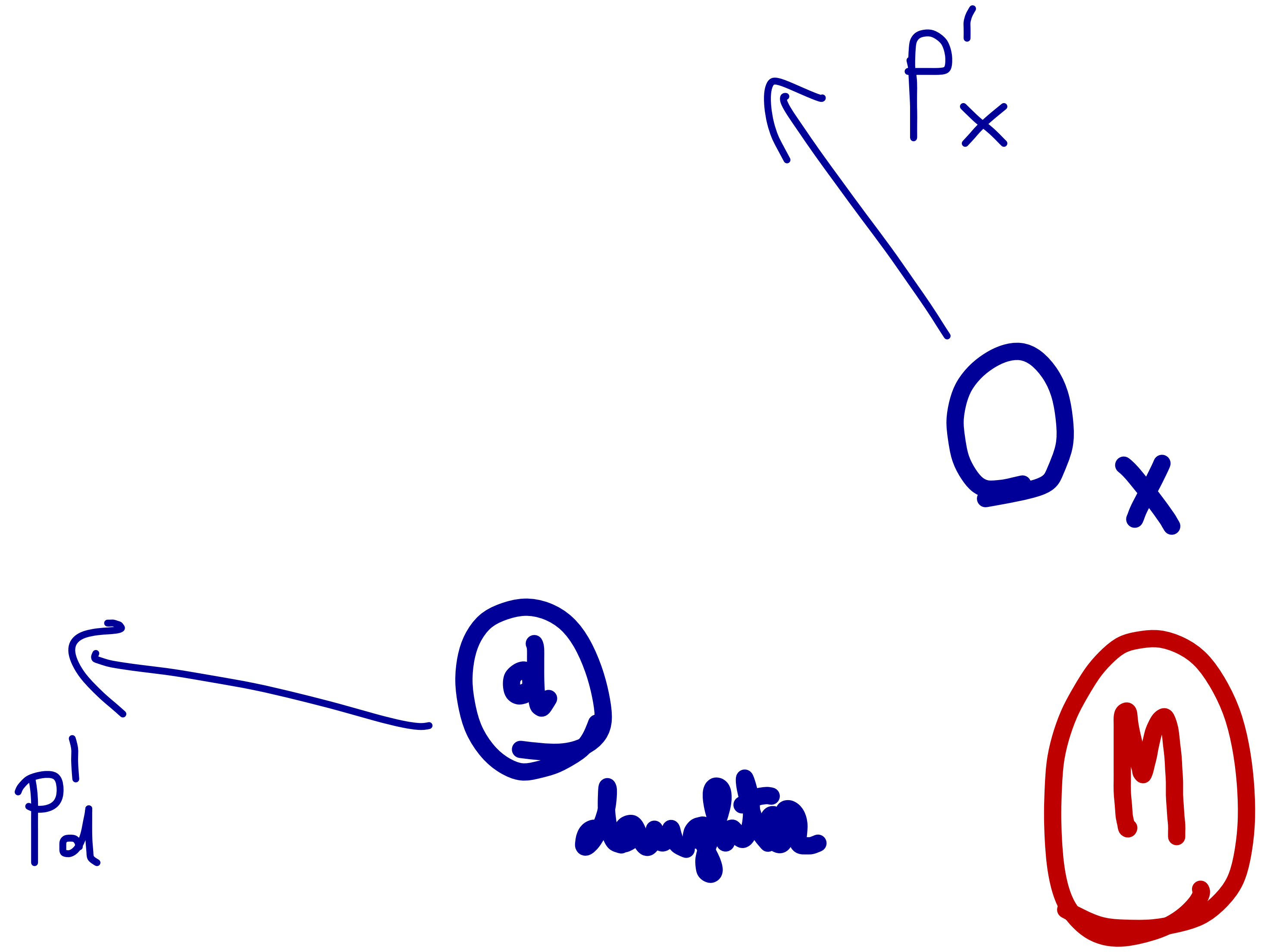
$\tilde{b} \to b \chi^0$

$W \to \ell \nu$

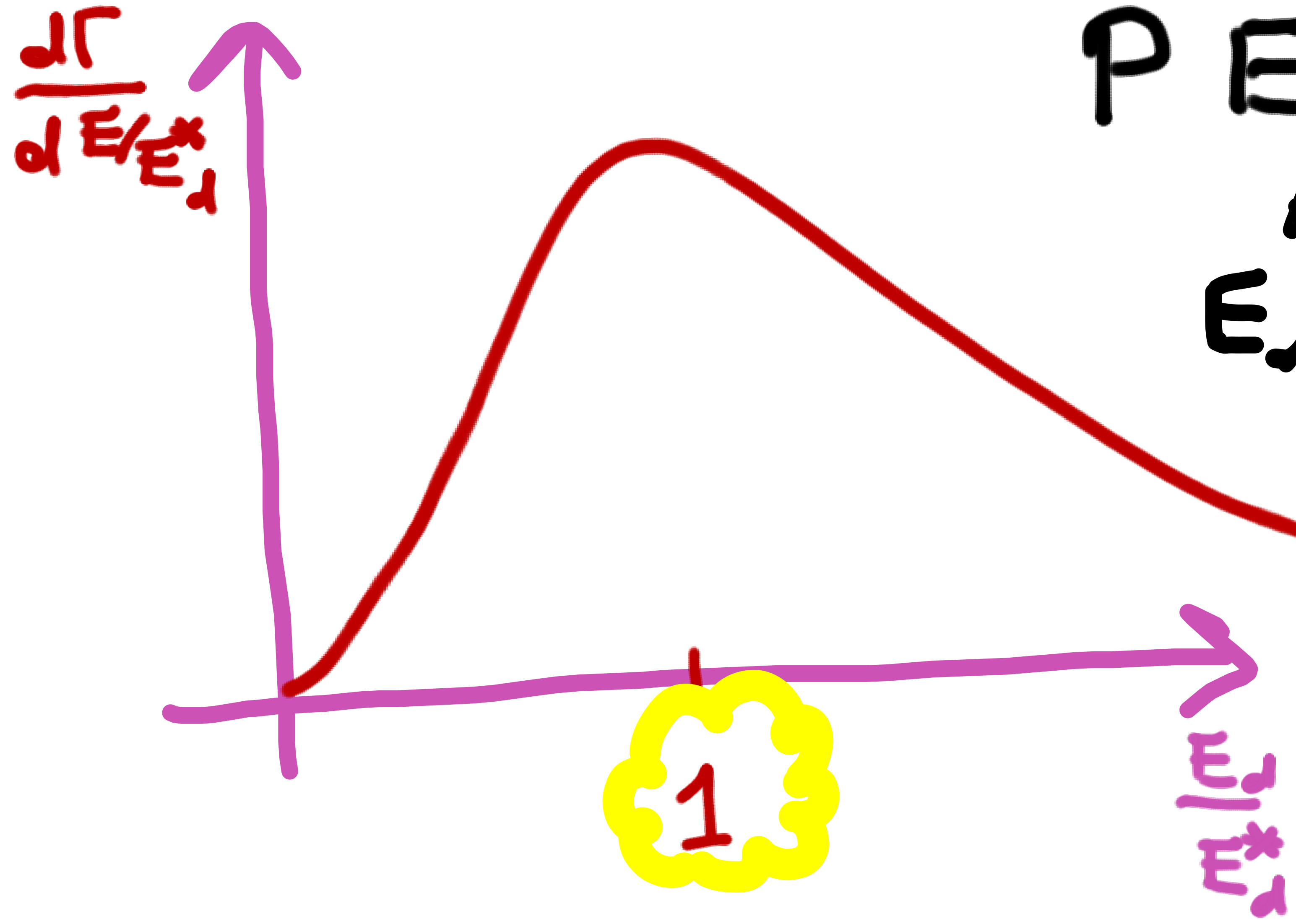
$t \to b W \to b \ell \nu$

ADVANTAGES

OVER MANY TRANSVERSE KINEMATICAL VARIABLES IN USE IN COLLIDER PHYSICS (m_{T2}, ...)

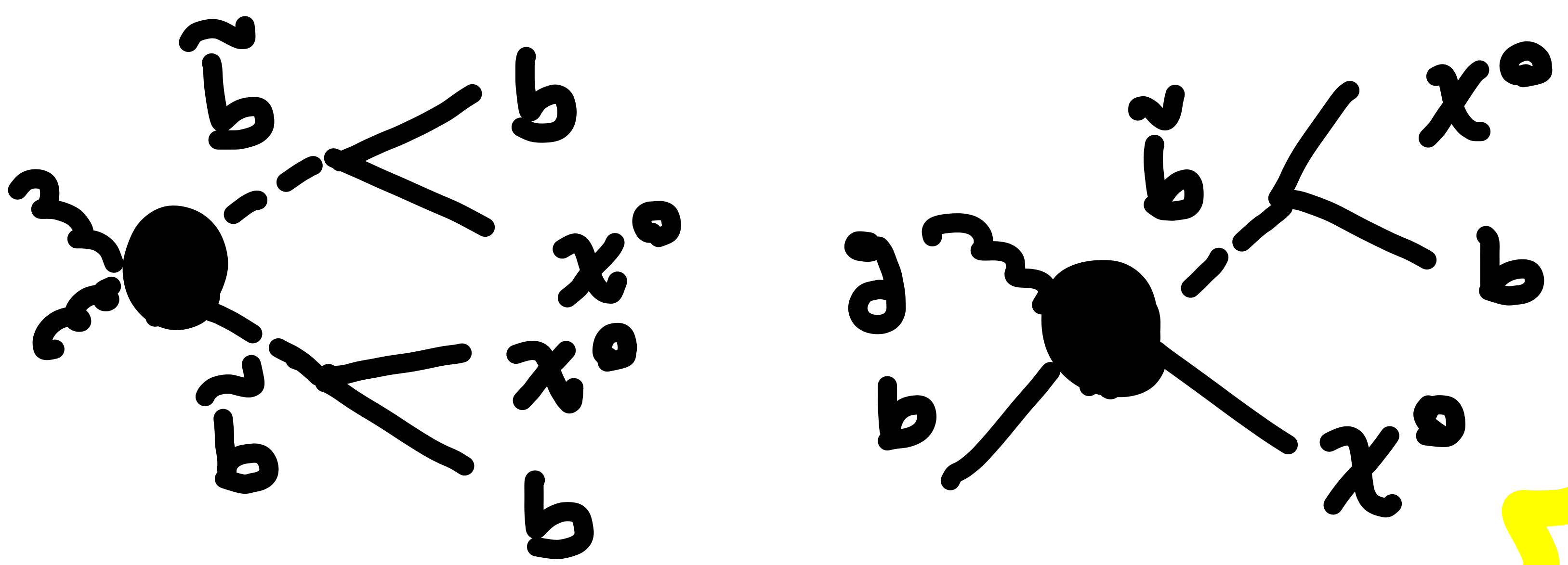


$$E_d^* = \frac{m_M^2 + m_d^2 - m_X^2}{2 m_M}$$



PEAK AT $E_d = E_d^*$

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT



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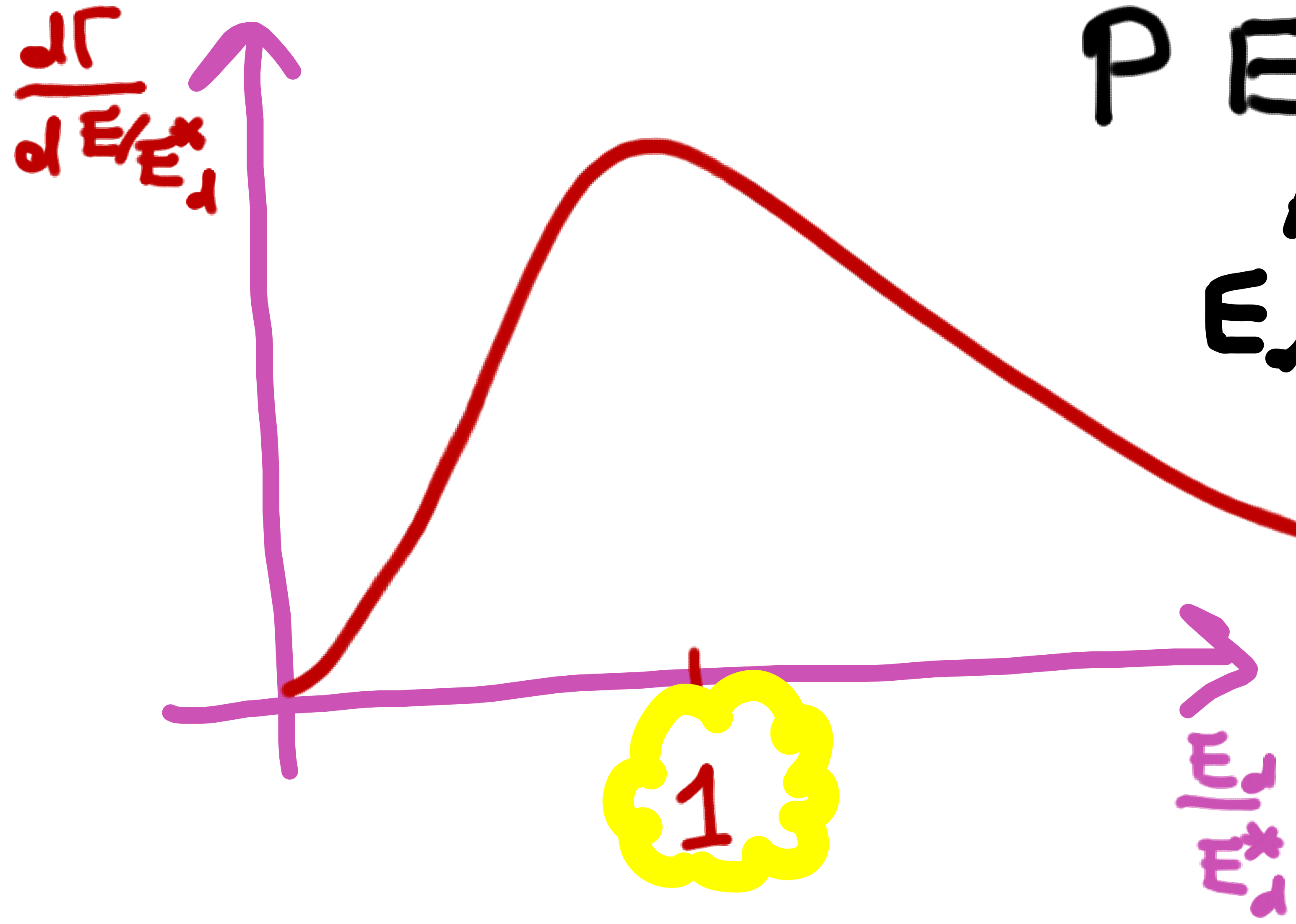

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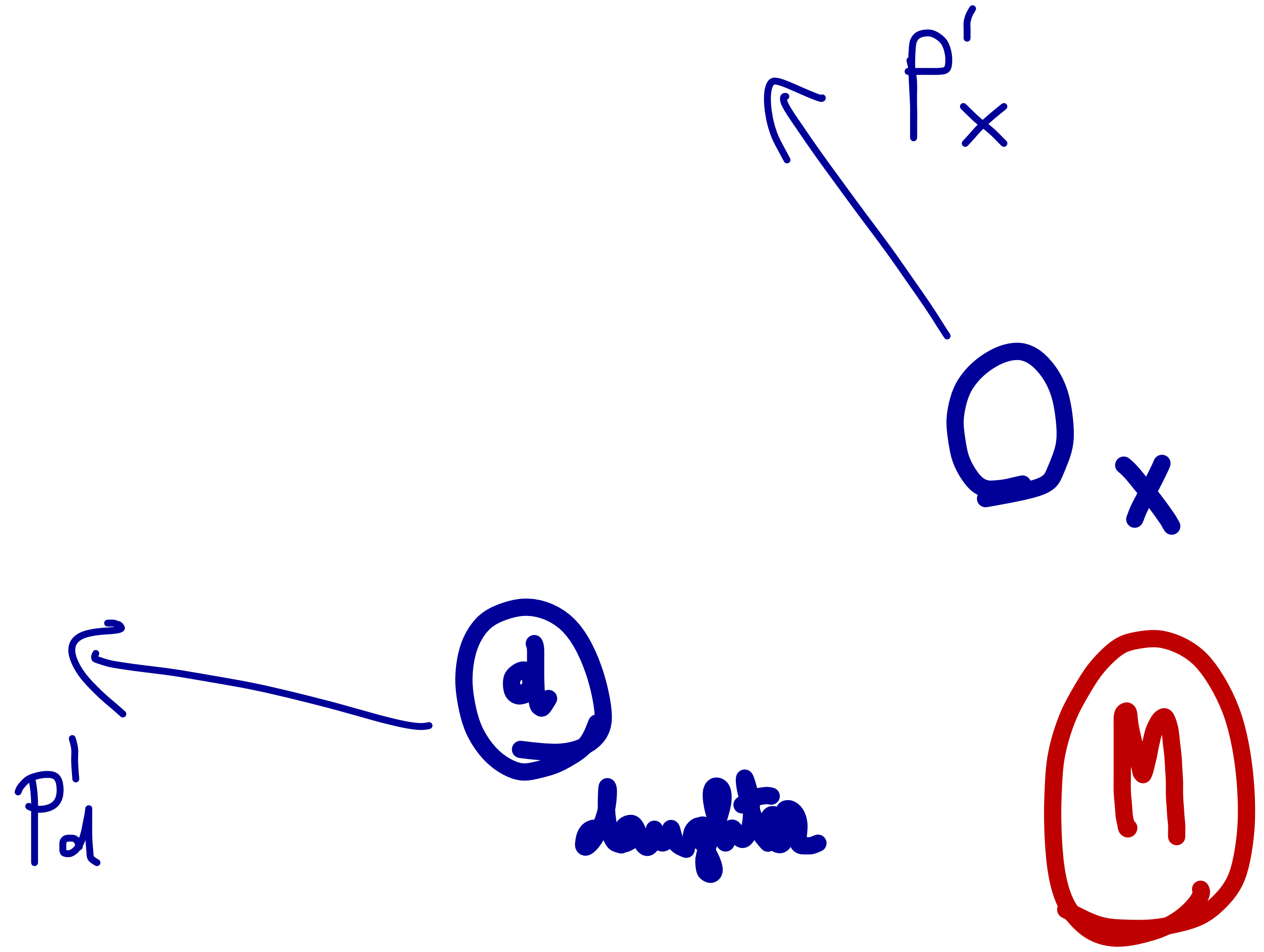

$$E_d^* = \frac{m_M^2 + m_d^2 - m_X^2}{2 m_M}$$

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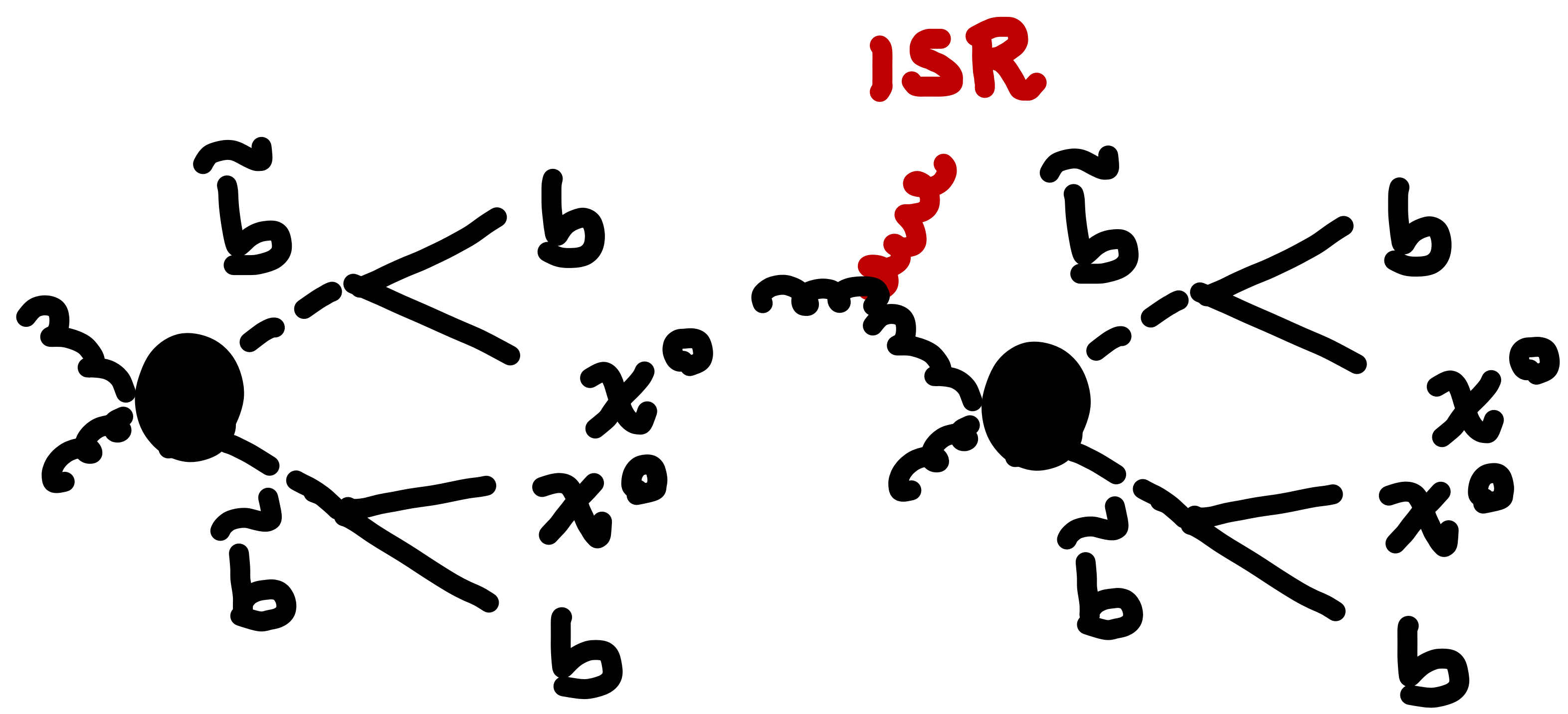
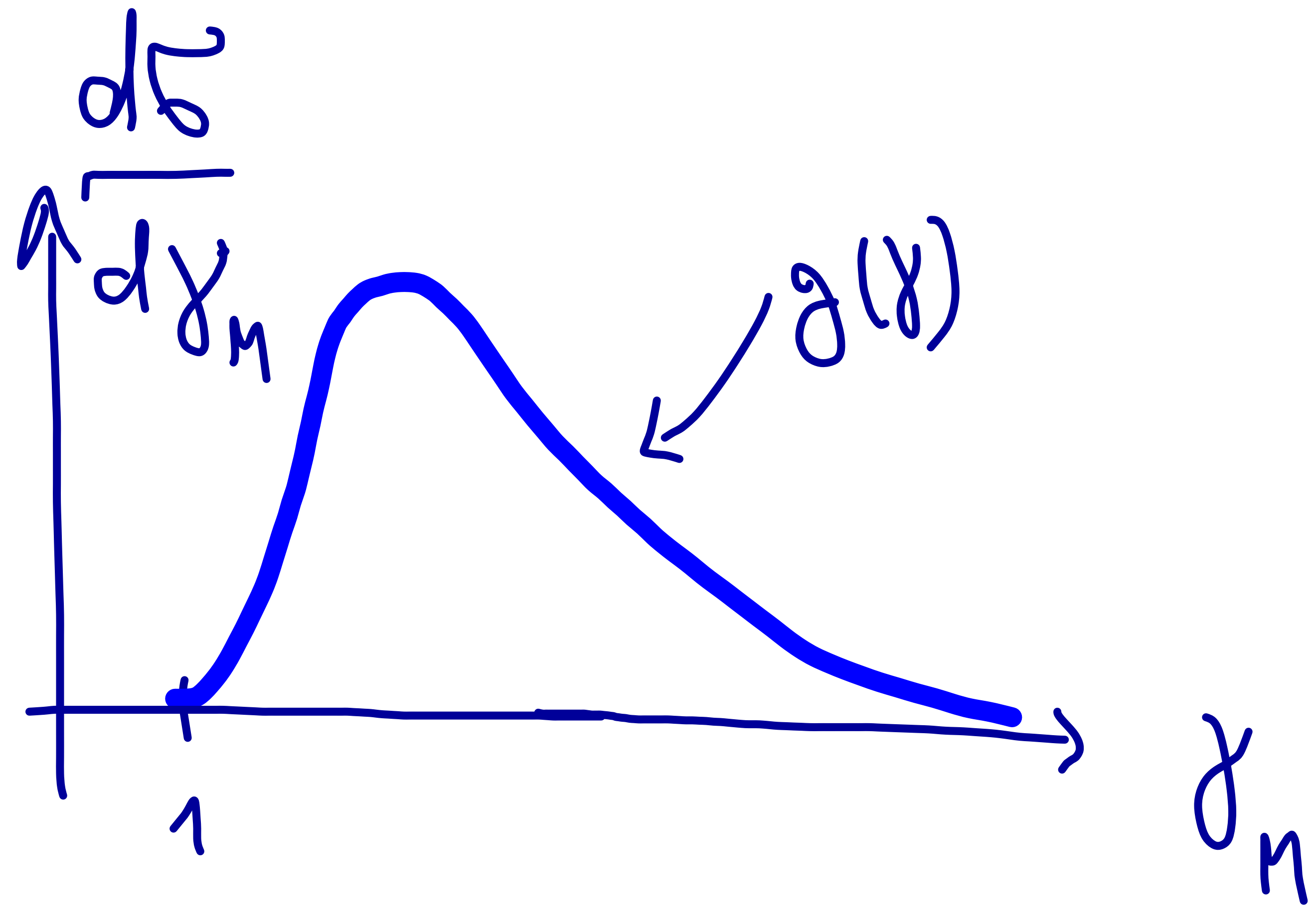
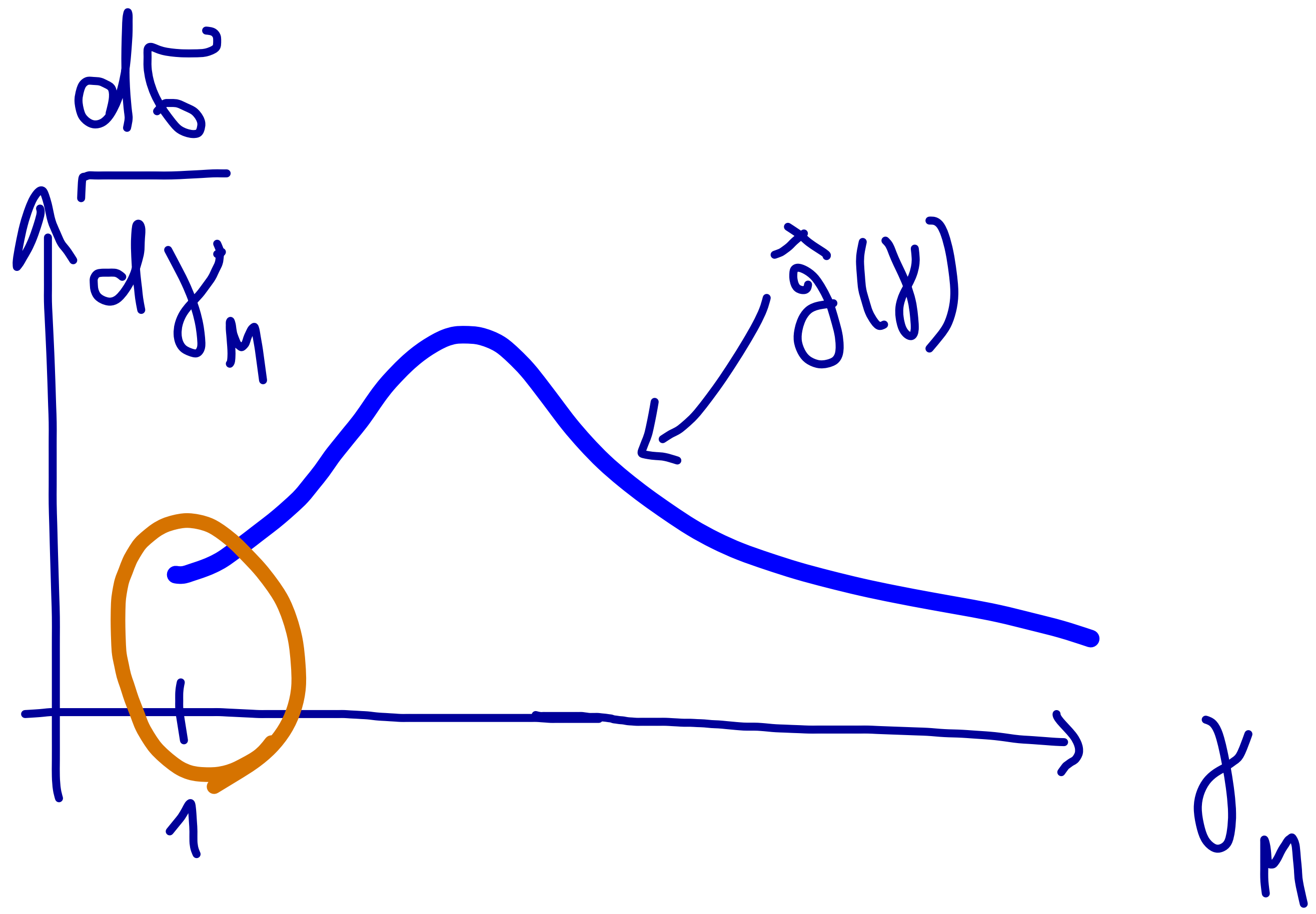


SOME MORE INSIGHTS BY GOING THROUGH AN ANALYTIC PROOF :



$$x := \frac{E_d}{E_d^*}$$

(MASSLESS DAUGHTER)



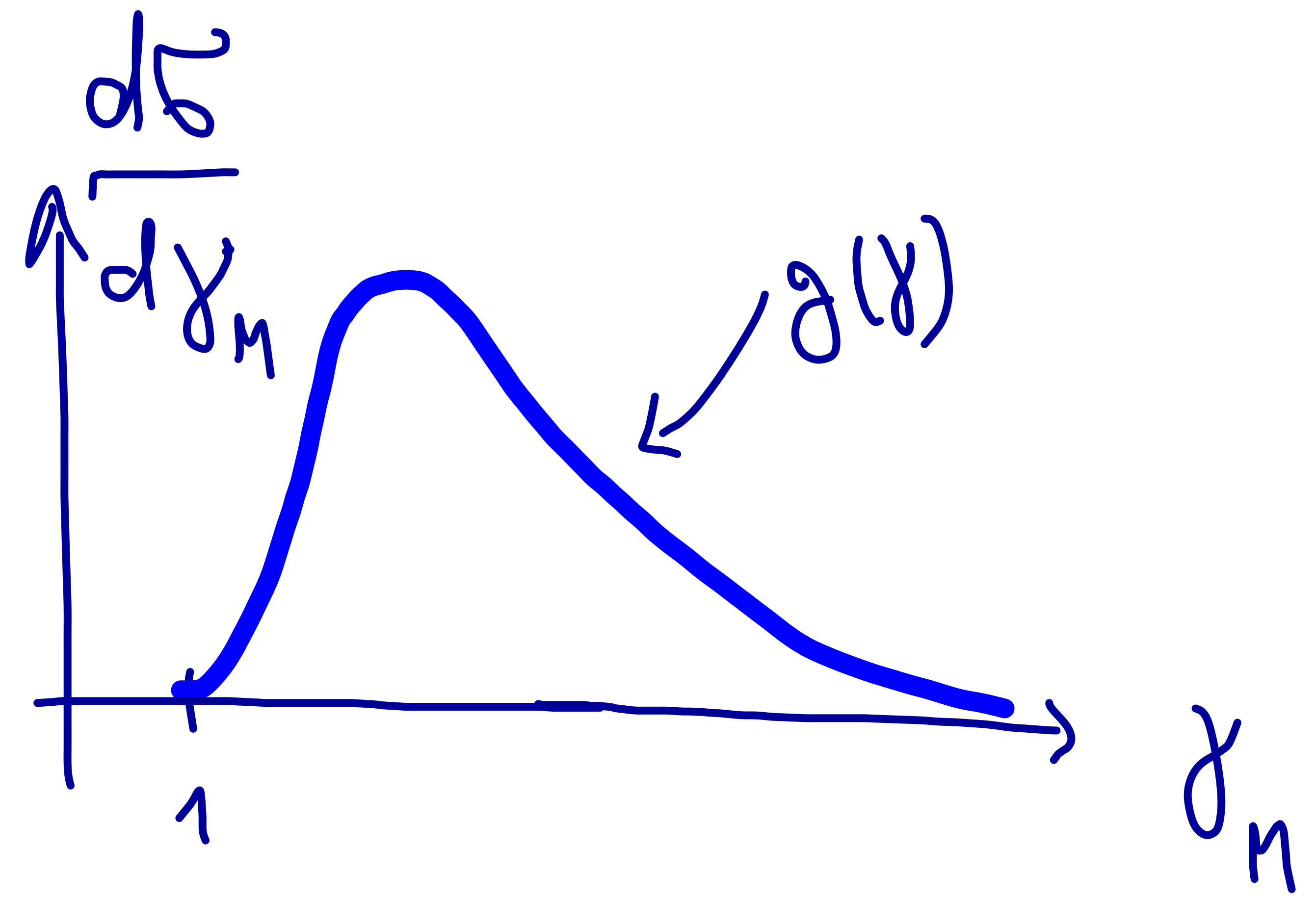
$$f(x) = \frac{1}{\Gamma}\frac{d\Gamma}{dx} = \int_{\frac{1}{2}\left(x+\frac{1}{x}\right)}^{\infty} d\gamma \, \frac{g(\gamma)}{2\sqrt{\gamma^2-1}}$$

$$f'(x) = \frac{\text{sign}(1-x)}{2x}\, g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

$g(1) = 0$

$g(1) \neq 0$ the derivative changes sign $\Rightarrow$ KINK IN THE OBSERVED ENERGY DISTRIBUTION

SOME MORE INSIGHTS BY GOING THROUGH AN ANALYTIC PROOF :



$$x := \frac{E_d}{E_d^*}$$

(MASSLESS DAUGHTER)

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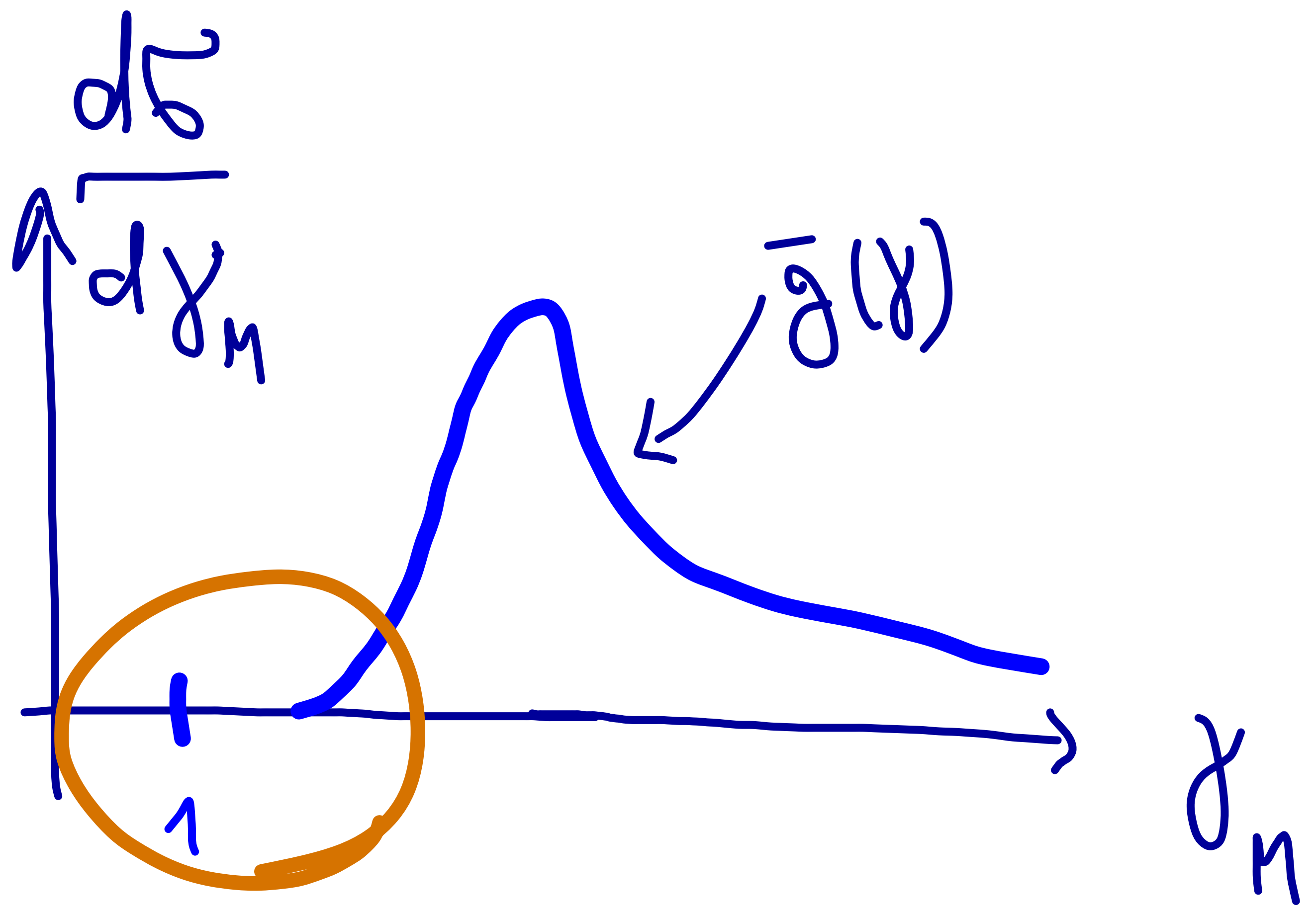
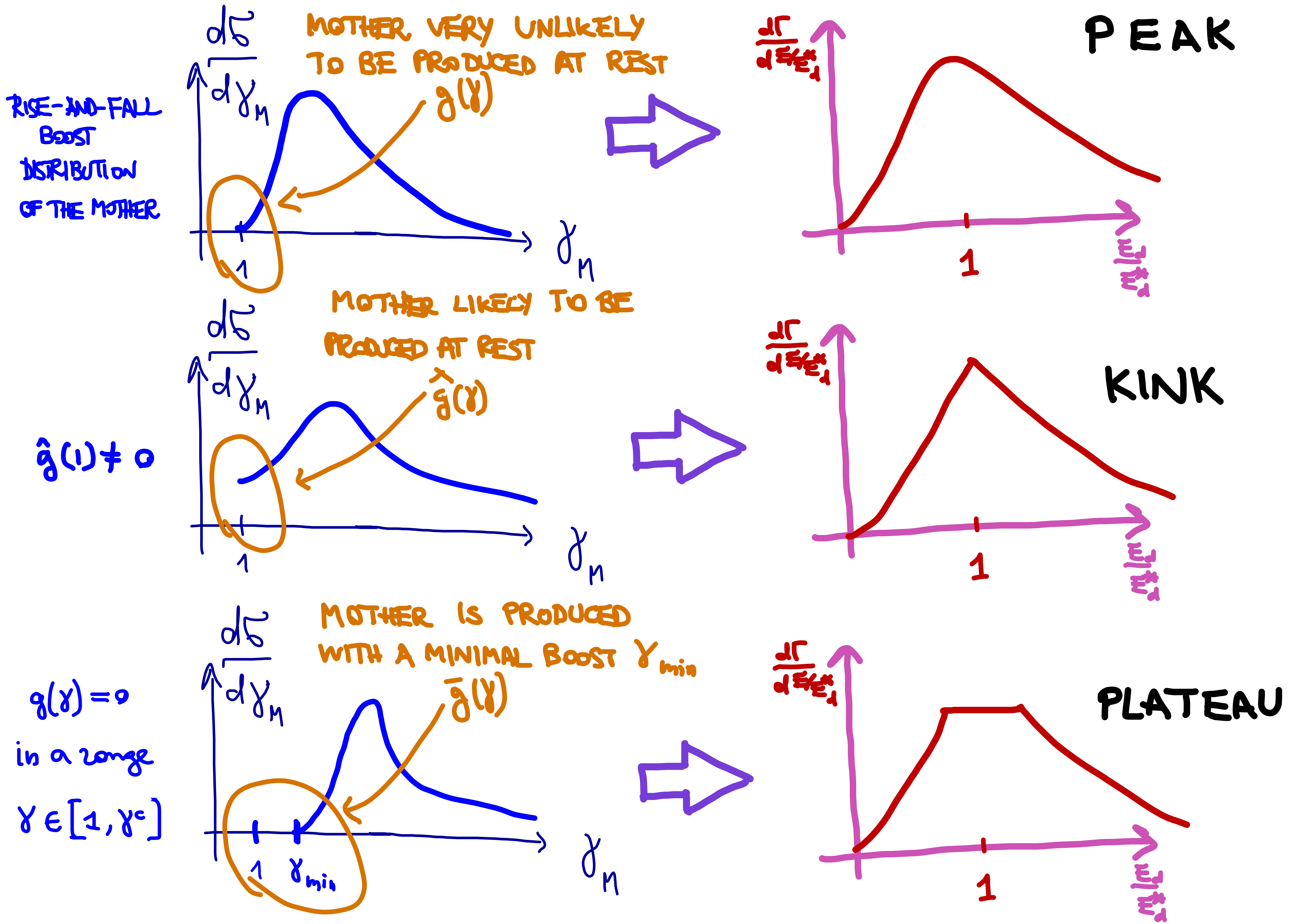


$$f'(x) = \frac{\mathrm{sign}(1-x)}{2x} \, g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

$g(\gamma) = 0$

IN A RANGE $[1, \gamma^c]$ ⇒ PLATEAU IN THE OBSERVED ENERGY DISTRIBUTION

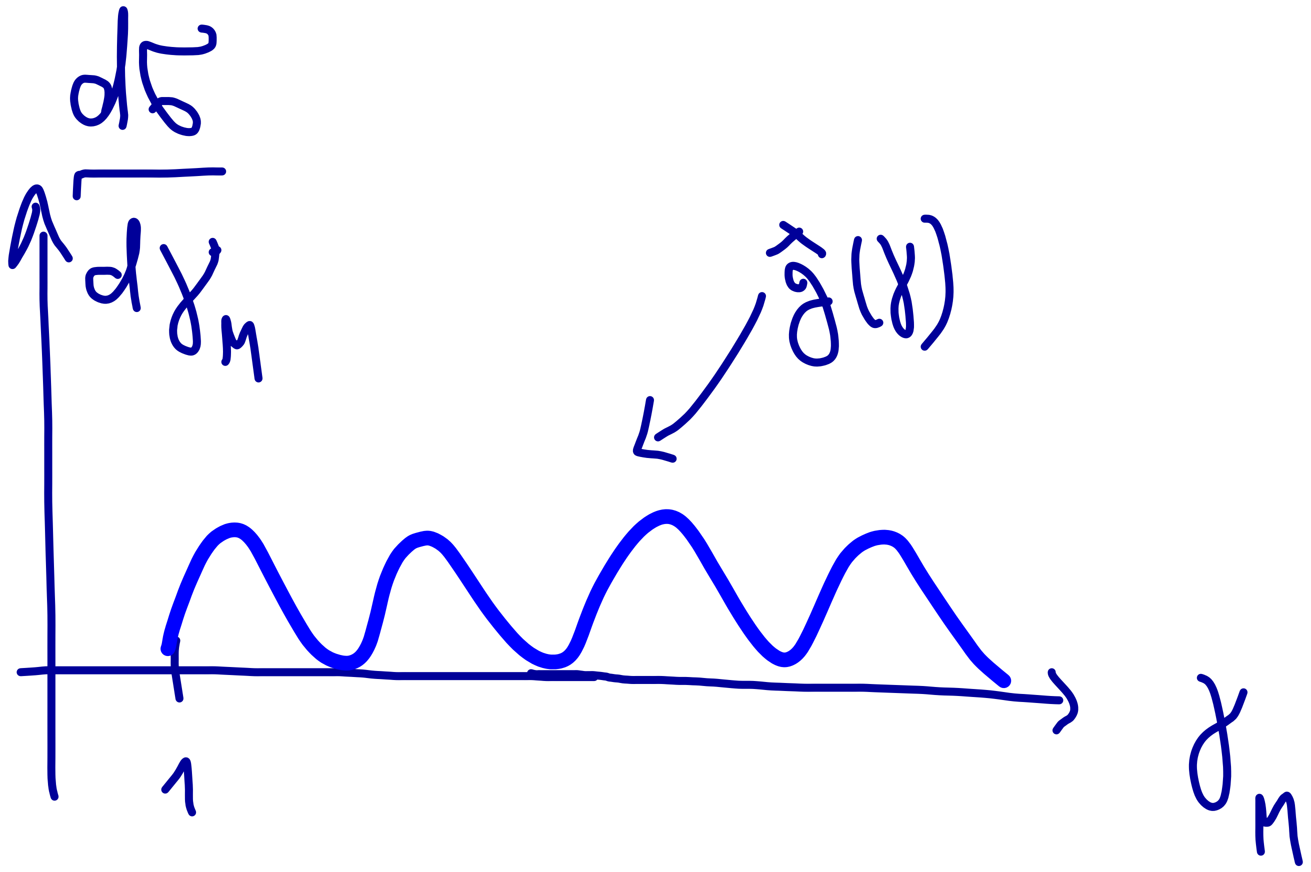


RISE-AND-FALL BOOST DISTRIBUTION OF THE MOTHER
dσ / dγ_M
MOTHER VERY UNLIKELY TO BE PRODUCED AT REST
g(γ)
1
γ_M
PEAK
1
ĝ(1) ≠ 0
MOTHER LIKELY TO BE PRODUCED AT REST
ĝ(γ)
KINK
g(γ) = 0 in a range γ ∈ [1, γ^c]
MOTHER IS PRODUCED WITH A MINIMAL BOOST γ_min
ḡ(γ)
γ_min
PLATEAU

WHEN AND WHY THIS BREAKS DOWN ?

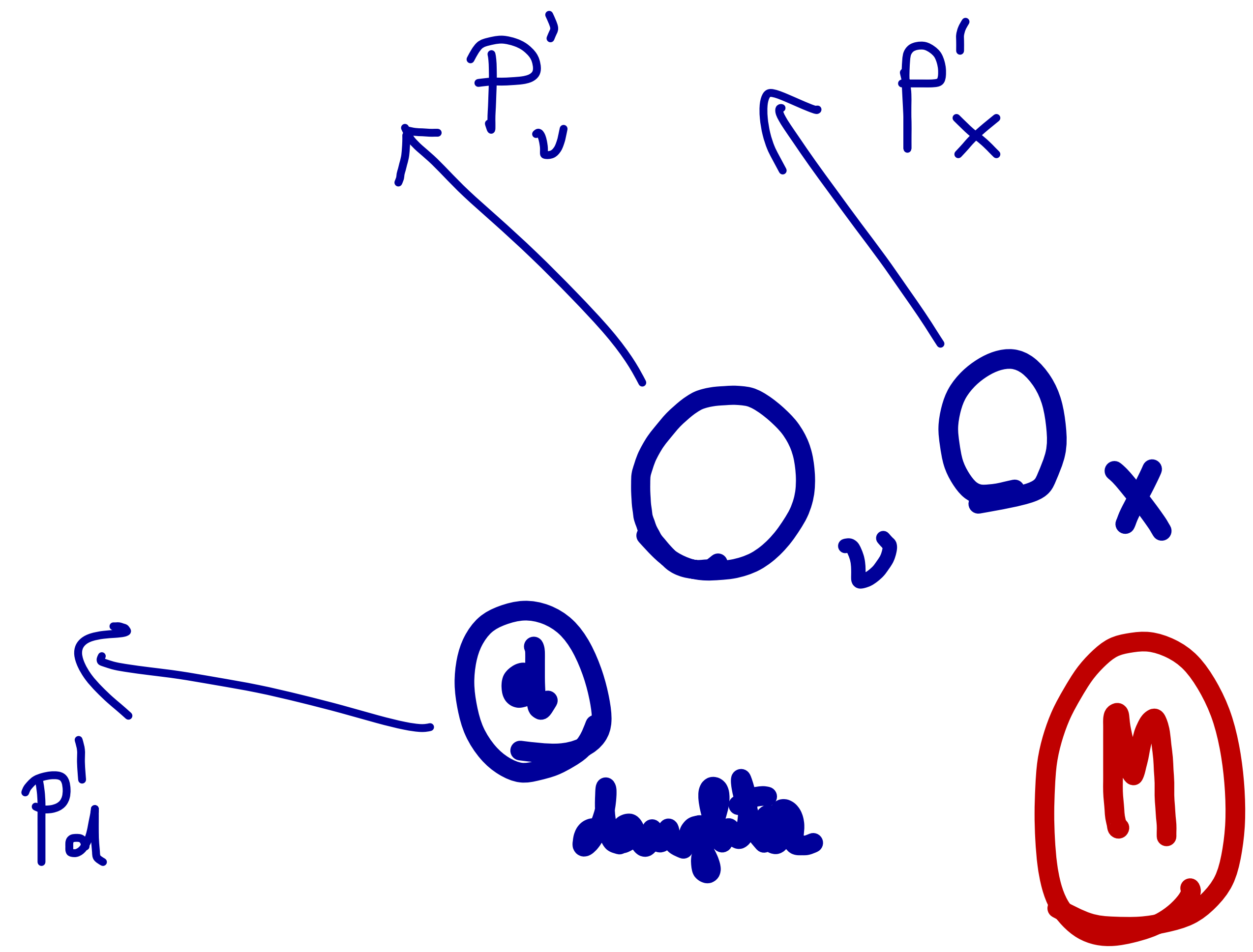
① BOOST DISTRIBUTION OF THE MOTHER WITH SPECIAL FEATURES

(MANY MINIMA, LARGE FLAT PORTIONS, ...)



② THE DECAY WAS NOT TWO-BODY

EXTRA INVISIBLE/UNDETECTED PARTICLES IN THE DECAY



WHEN AND WHY THIS BREAKS DOWN ?

CAVEAT:

$M \to dX$

IS TWO BODY ONLY UP TO <u>EXTRA RADIATION</u>

IF THE FINAL STATES ARE COLORED $M \to dX + \text{gluons}$

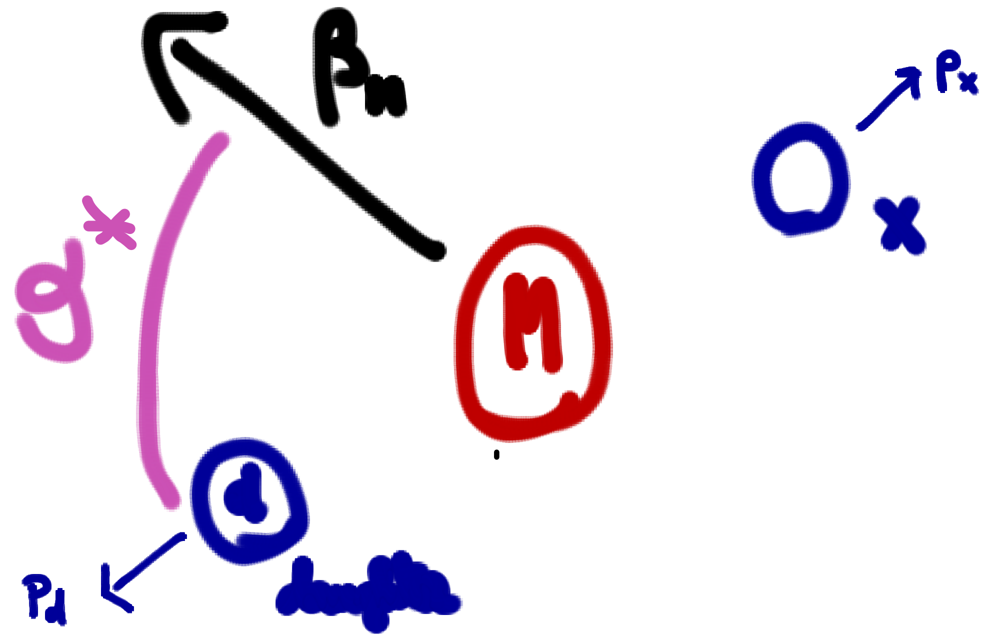
<u>JET CLUSTERING</u> SOLVES THIS ISSUE TO SOME EXTENT

HARD RADIATION MAY BE RESOLVABLE AND <u>EFFECTIVELY</u> GIVE RISE TO A <u>THREE-BODY</u> DECAY

RESOLVABLE RADIATION CAN BE VETOED

WHEN AND WHY THIS BREAKS DOWN ?

③ THE DAUGHTER'S MASS



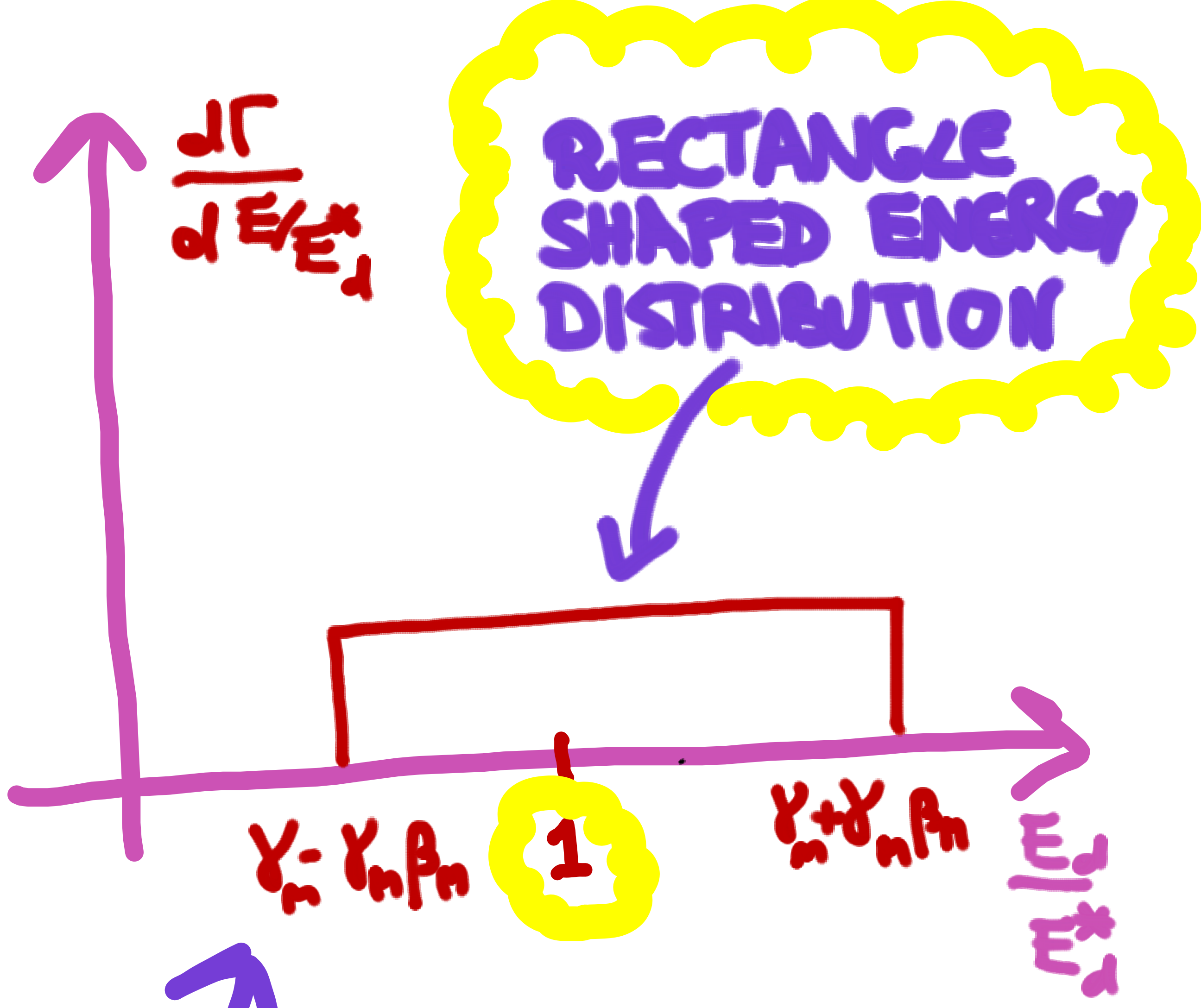
$$E'_d = E^*_d \gamma_M + \cos\theta^* \gamma_M \beta_M p^*_d$$

THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$

(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p^*_d = E^*_d$ (MASSLESS DAUGHTER)

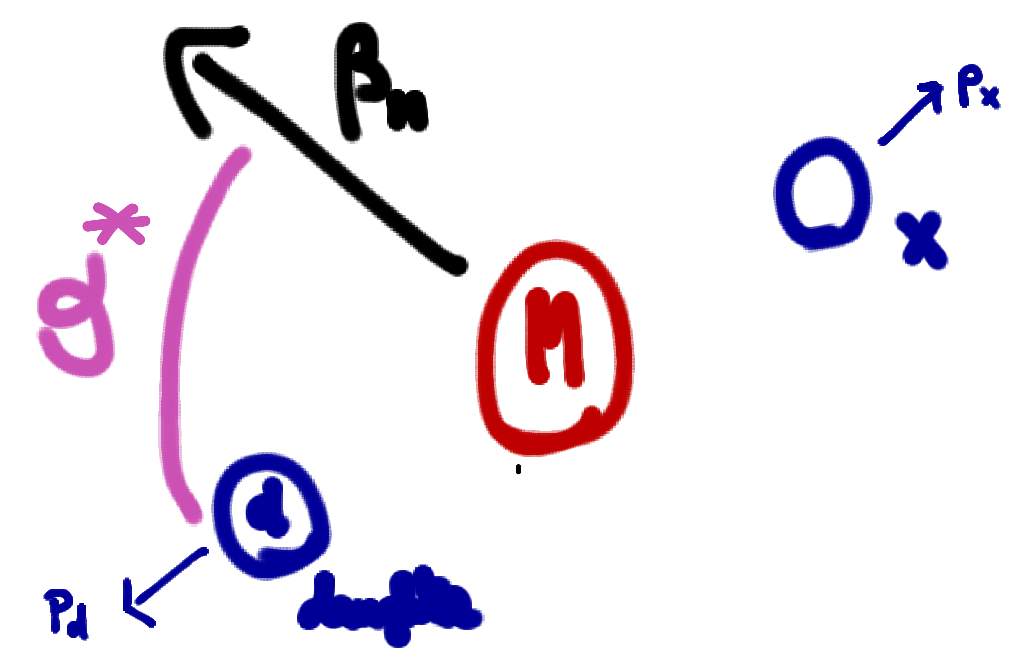
$$E'_{d,min} = E^*_d \left(\gamma_M - \sqrt{\gamma_M^2 - 1}\right) < E^*_d$$



LOWER EDGE OF EACH RECTANGLE **BELOW** E^*

WHEN AND WHY THIS BREAKS DOWN ?

- **THE DAUGHTER'S MASS**



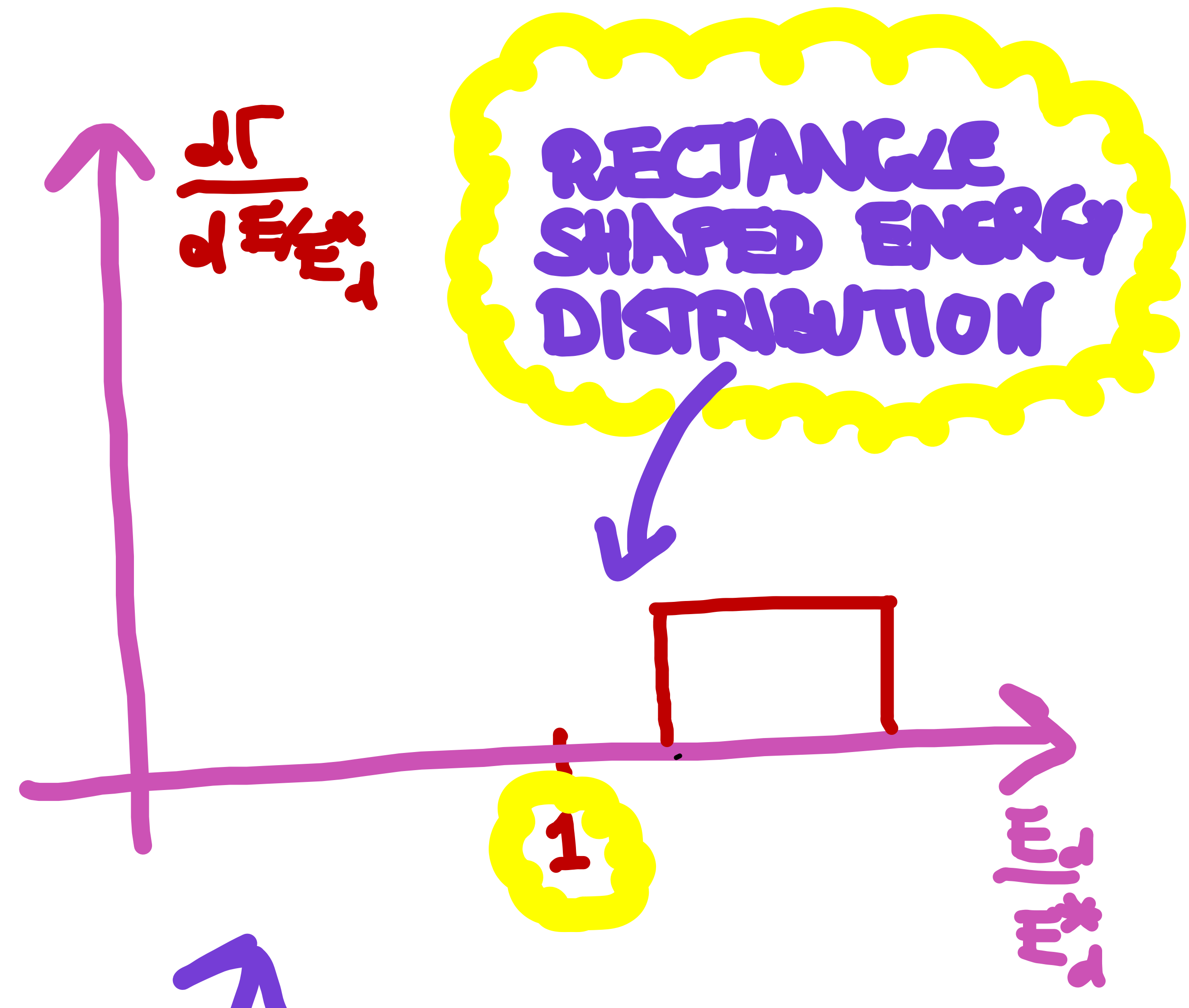
$$E'_d = E^*_d \gamma_M + \cos\vartheta^* \gamma_M \beta_M p^*_d$$

THE MINIMUM OF THIS QUANTITY AT $\vartheta^* = \pi$
(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p^*_d \leq E^*_d$ (MASSIVE DAUGHTER)

$$p^*_d \to 0 \quad E^*_d \to m_d$$

$$E'_d = m_d \gamma_M + \dots$$

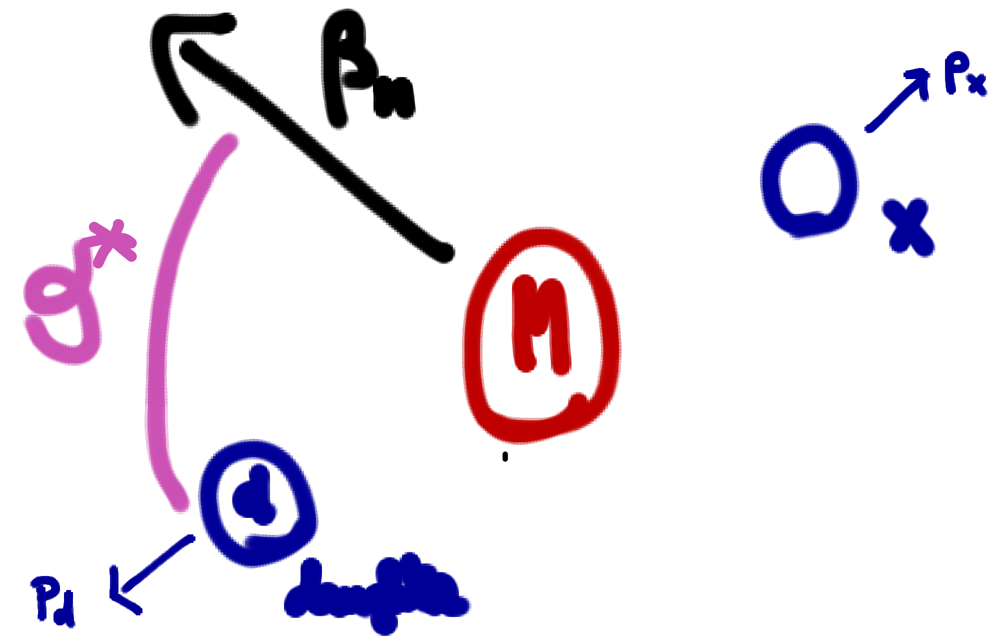


FOR γ_M LARGE GIVES RECTANGLES $E_{d,min} > E^*_d$

LOWER EDGE OF EACH RECTANGLE ABOVE E^*

WHEN AND WHY THIS BREAKS DOWN ?

- THE DAUGHTER'S MASS



$$E'_d = E^*_d \gamma_M + \cos\theta^* \gamma_M \beta_M p^*_d$$

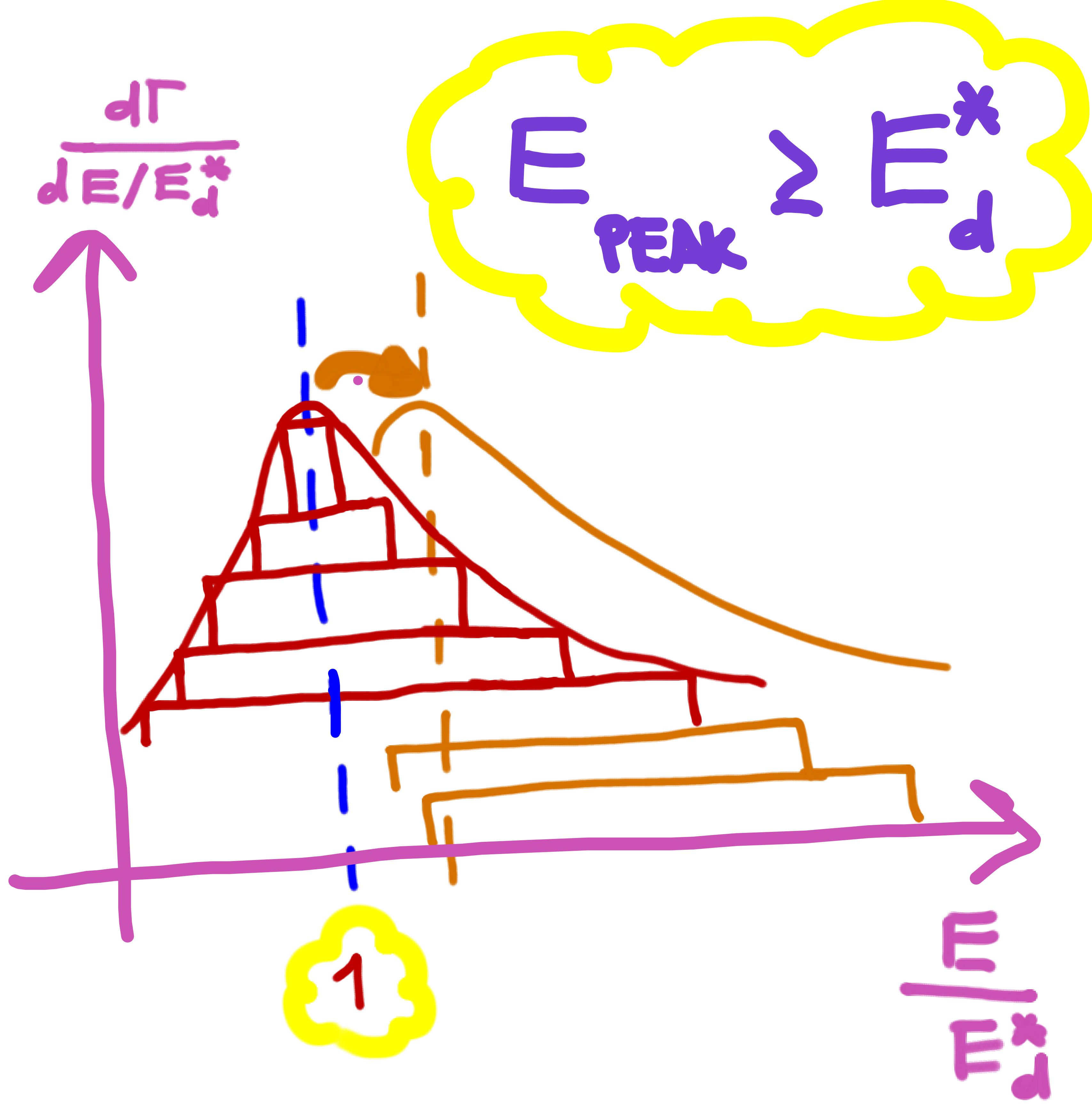
THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$
(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p^*_d \leq E^*_d$ (MASSIVE DAUGHTER)

$$p^*_d \to 0 \quad E^*_d \to m_d$$

$$E'_d = m_d \gamma_M + \dots$$

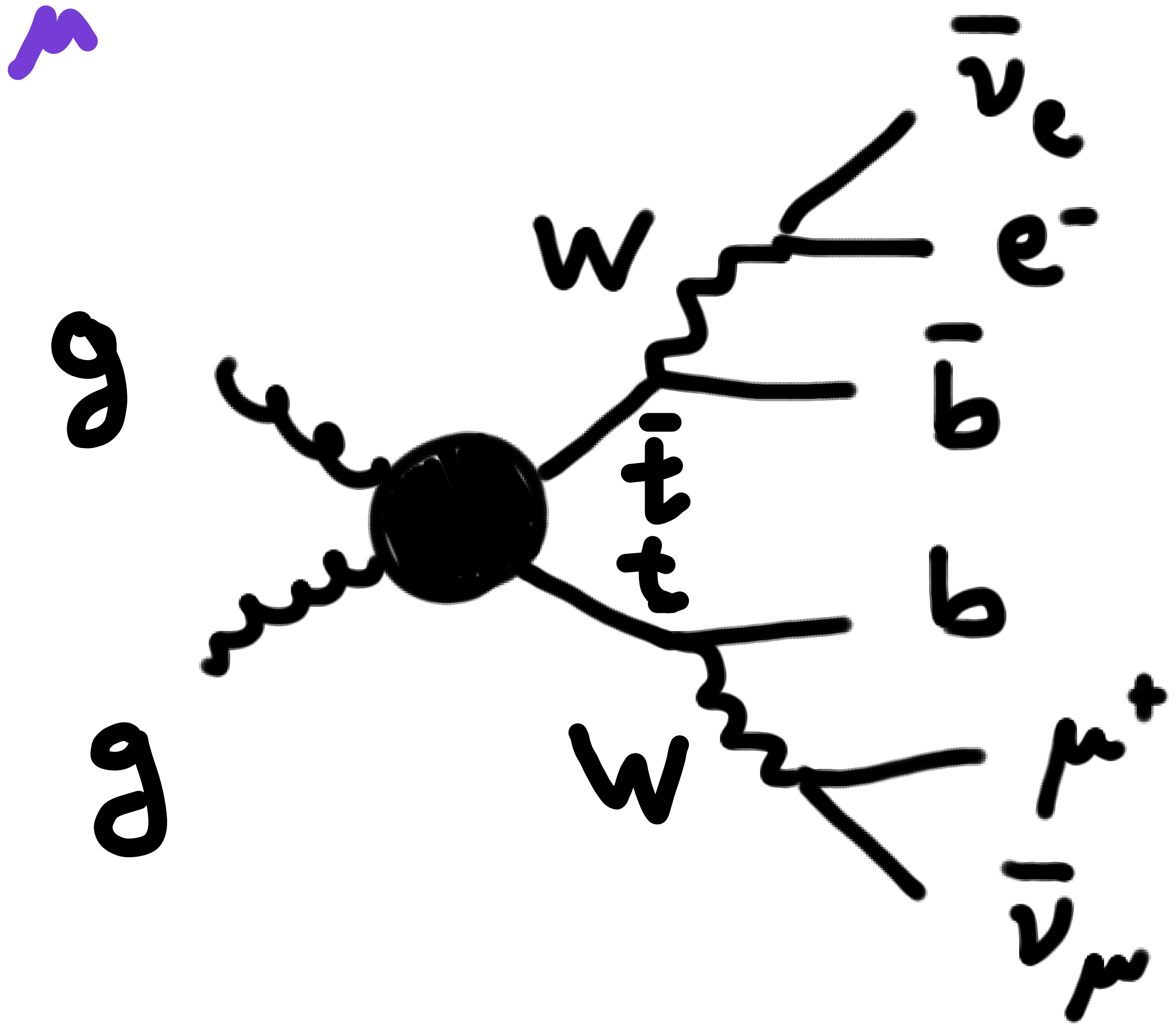
FOR γ_M LARGE GIVES
RECTANGLES $E_{d,min} \geq E^*_d$



APPLICATIONS:

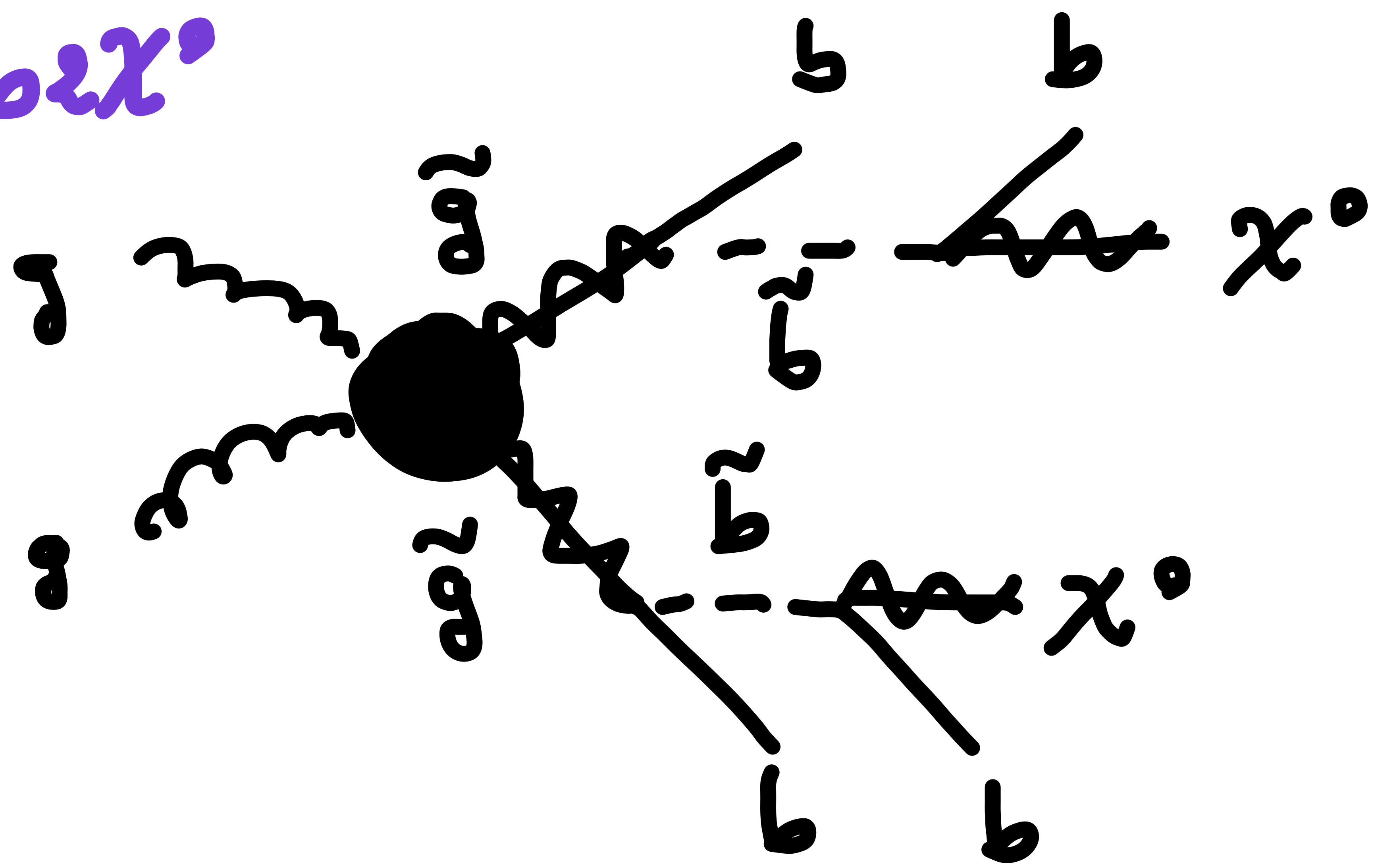
1) $pp \to t\bar{t} \to b\bar{b}\,\mu^+ e^- \bar{\nu}_e \nu_\mu$

m_{top} AS PROOF OF PRINCIPLE



2) $pp \to \tilde{g}\tilde{g} \to b\tilde{b}\,b\tilde{b} \to 4b\,2\chi^0$

$m_{\tilde{g}}$, $m_{\tilde{b}}$, m_χ

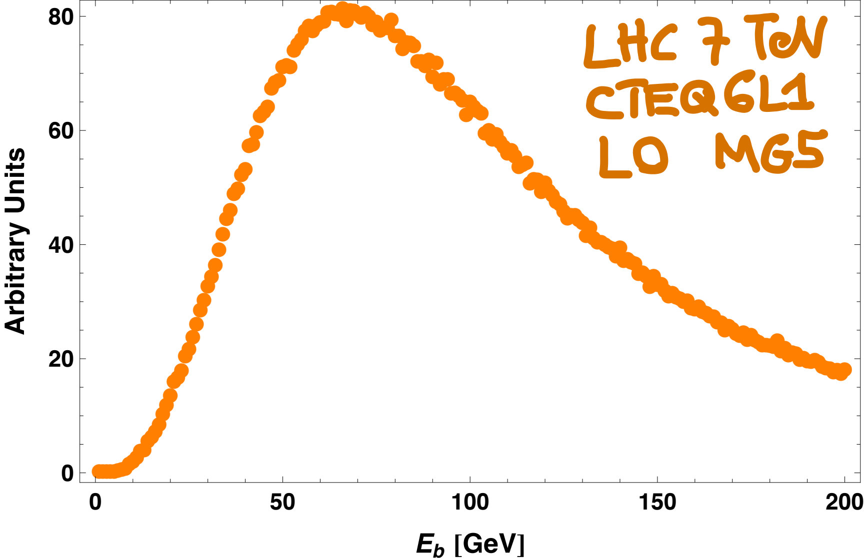


$$pp \to t\bar{t} \to b\bar{b}\, e\bar{\mu}\, \not{E}_T$$

- QCD PAIR PRODUCTION OF $t\bar{t}$ ENSURES THAT THE OVERALL SAMPLE OF TOP DECAYS IS UNPOLARIZED

- $E_b^* = \dfrac{m_t^2 - m_W^2 + m_b^2}{2m_t} \simeq 67\ \text{GeV}$ $\qquad E_b^* \gg m_b$

THE b QUARK CAN BE TAKEN AS MASSLESS


LHC 7 TeV
CTEQ6L1
LO MG5
Arbitrary Units
E_b [GeV]
0
20
40
60
80
0
50
100
150
200

FINDING THE PEAK :

- LOOK BY EYE

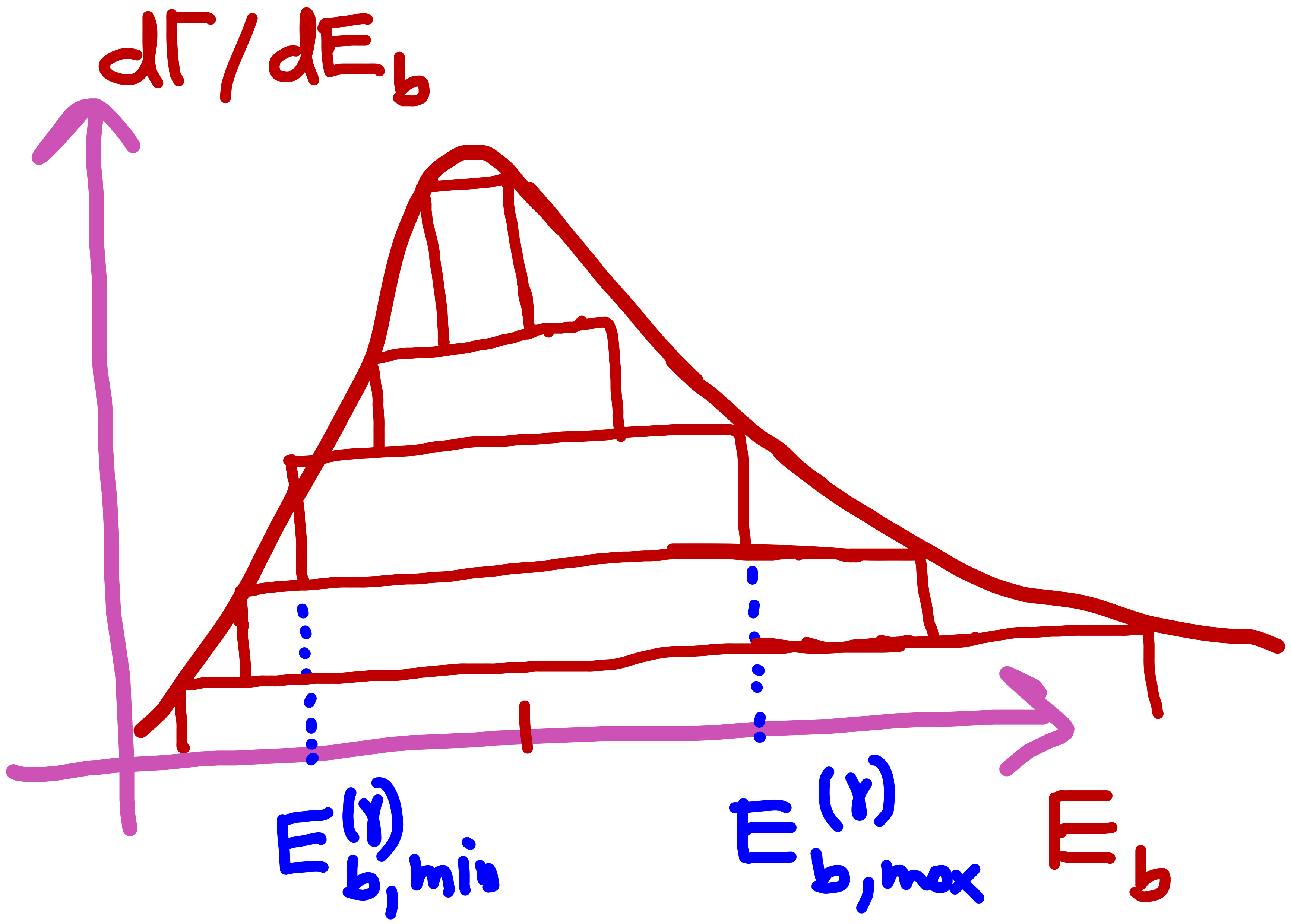
- **FIND A TEMPLATE AND USE IT TO FIT DATA**

  - A TEMPLATE MOTIVATED FROM PRIME PRINCIPLES SEEMS UNATTAINABLE BECAUSE IT DEPENDS ON PARTON DISTRIBUTION FUNCTIONS AND ON THE MATRIX ELEMENT FOR THE PRODUCTION PROCESS

FIND A TEMPLATE AND USE IT TO FIT DATA

$$E_b = E_b^* \left(\gamma_n + \cos\vartheta^* \sqrt{\gamma_M^2 - 1} \right)$$

$$E_{b,\min/\max}^{(\gamma_n)} = E_b^* \left(\gamma_M \mp \sqrt{\gamma^2 - 1} \right)$$



$$\sqrt{E_{b,\min} E_{b,\max}} = E_b^*$$

$d\Gamma/dE_b$

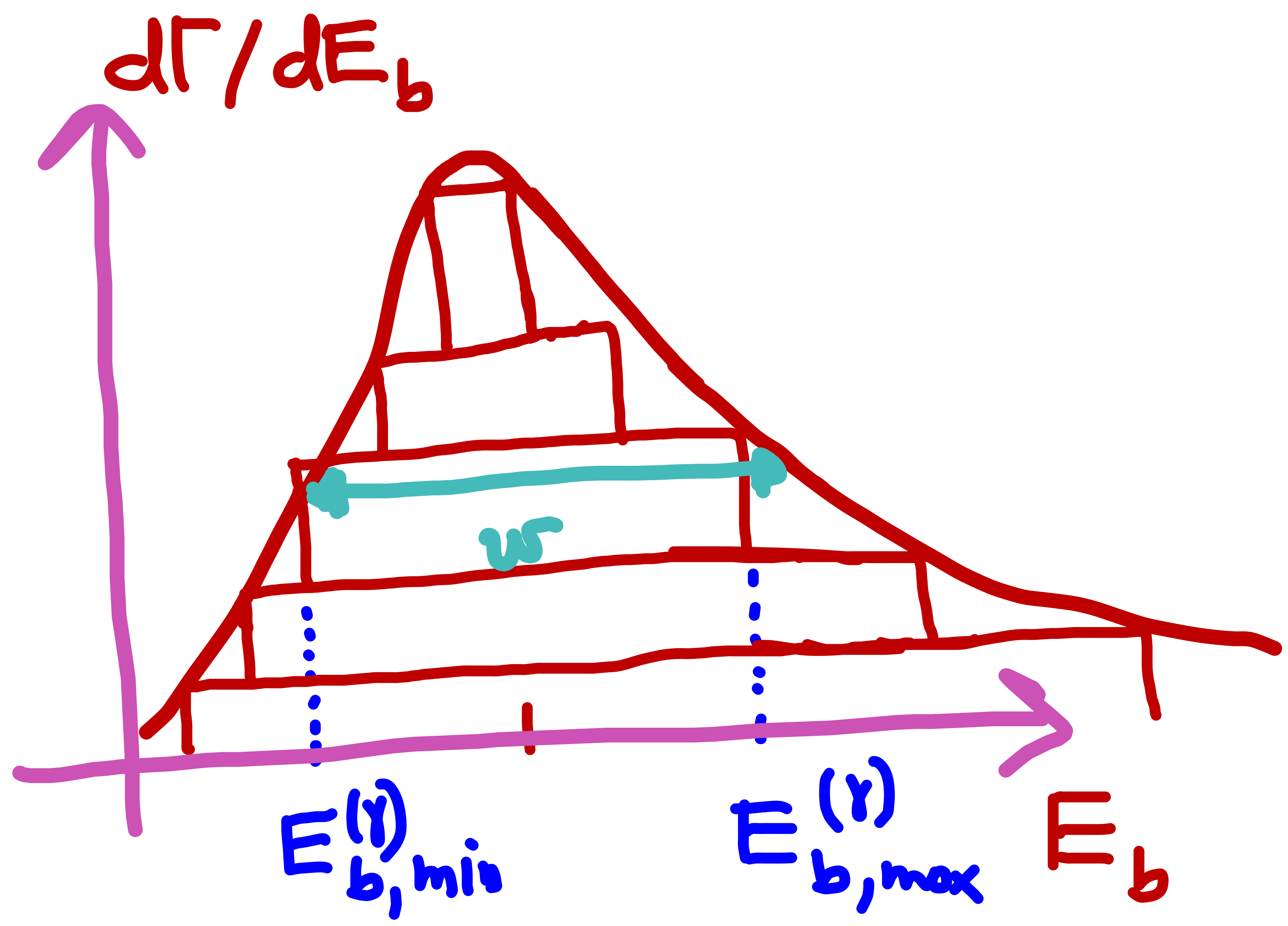
MUST BE A FUNCTION OF

$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$

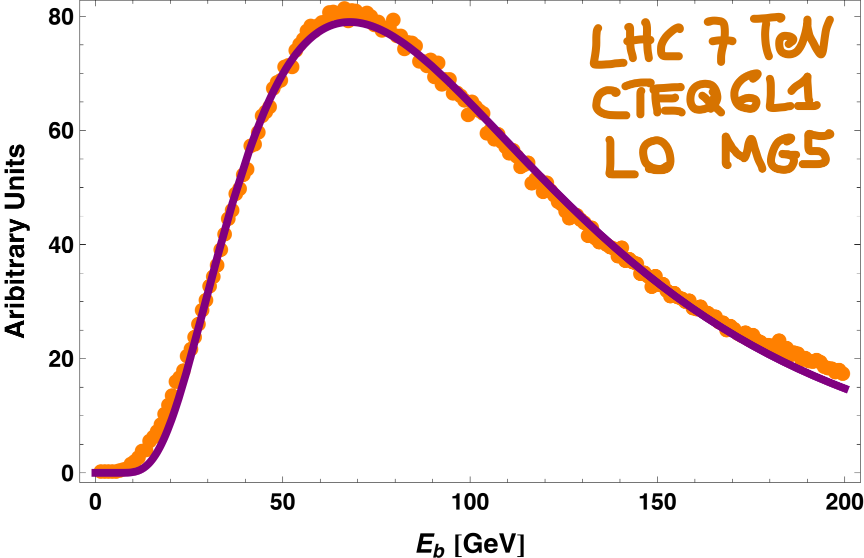
FIND A TEMPLATE AND USE IT TO FIT DATA

- $d\Gamma/dE_b \xrightarrow{E_b \to 0,\infty} 0$ (at least)
- $d\Gamma/dE_b$ max at $E_b = E_b^*$
- IN SOME LIMIT SHOULD BE A δ-FUNCTION (MOTHER AT REST)
- $d\Gamma/dE_b$ MUST BE A FUNCTION OF

$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$



$$d\Gamma/dE \sim \exp\left(-w\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$



LHC 7 TeV
CTEQ6L1
LO MG5
Aribitrary Units
0
20
40
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80
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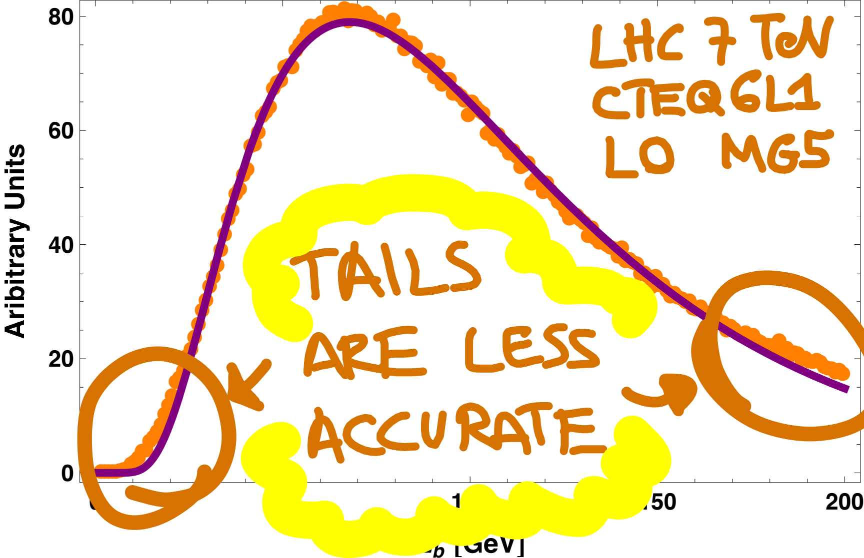
200
E_b [GeV]



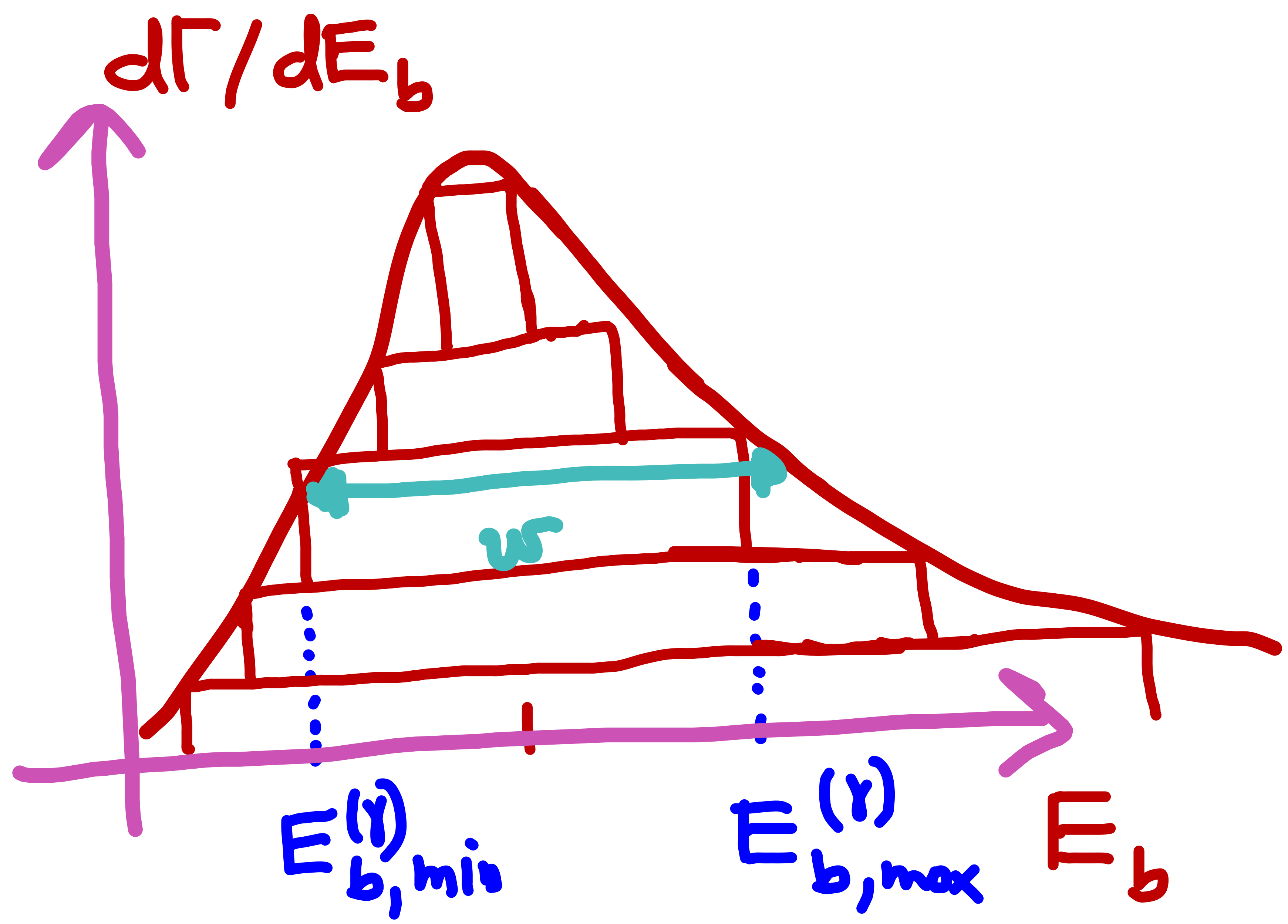
80
60
40
20
0
150
200
Aribitrary Units
[GeV]
LHC 7 TeV
CTEQ6L1
LO MG5
TAILS
ARE LESS
ACCURATE

FIND A TEMPLATE AND USE IT TO FIT DATA

- $d\Gamma/dE_b \xrightarrow{E_b \to 0,\infty} 0$ ☑ ☒
- $d\Gamma/dE_b$ max at $E_b = E_b^*$ ☑
- IN SOME LIMIT SHOULD BE A δ-FUNCTION (MOTHER AT REST) ☑
- $d\Gamma/dE_b$ ☑

  MUST BE A FUNCTION OF

$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$



$$d\Gamma/dE \sim \exp\left(-w\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$

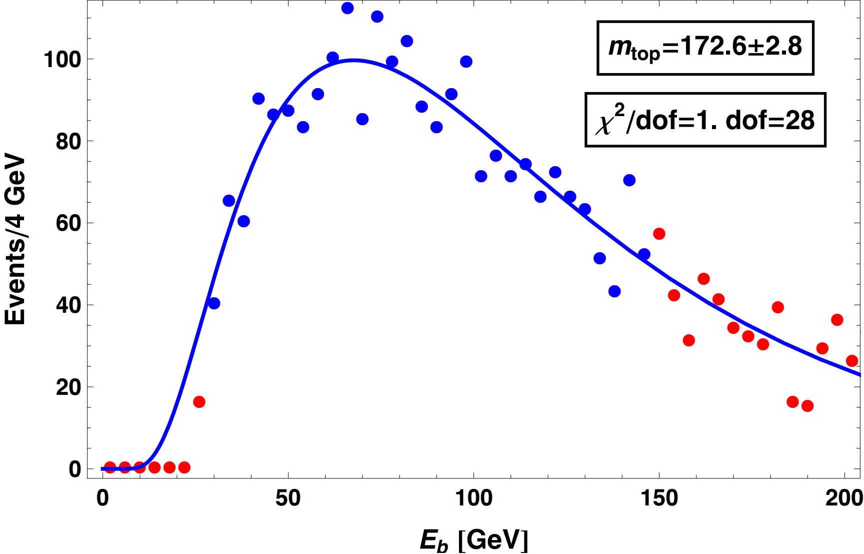
CAN WE MEASURE PARTICLE MASSES ?

- $E_b^* = \dfrac{m_t^2 - m_W^2 + m_b^2}{2m_t} \simeq 67\ \text{GeV}$

FROM THE RESULT OF THE FIT TO THE LEADING ORDER MATRIX ELEMENT WE HAVE AT LEAST A CHANCE

NEED TO EVALUATE :

- DETECTOR EFFECTS ⟶ DELPHES 1.9
- EXTRA QCD RADIATION ⟶ SOFT QCD PYTHIA 6.4
- BIAS FROM EVENT SELECTION ⟶ ATLAS-CONF-2012-097



m_{top}=172.6±2.8
χ^2/dof=1. dof=28
Events/4 GeV
E_b [GeV]
0
20
40
60
80
100
0
50
100
150
200

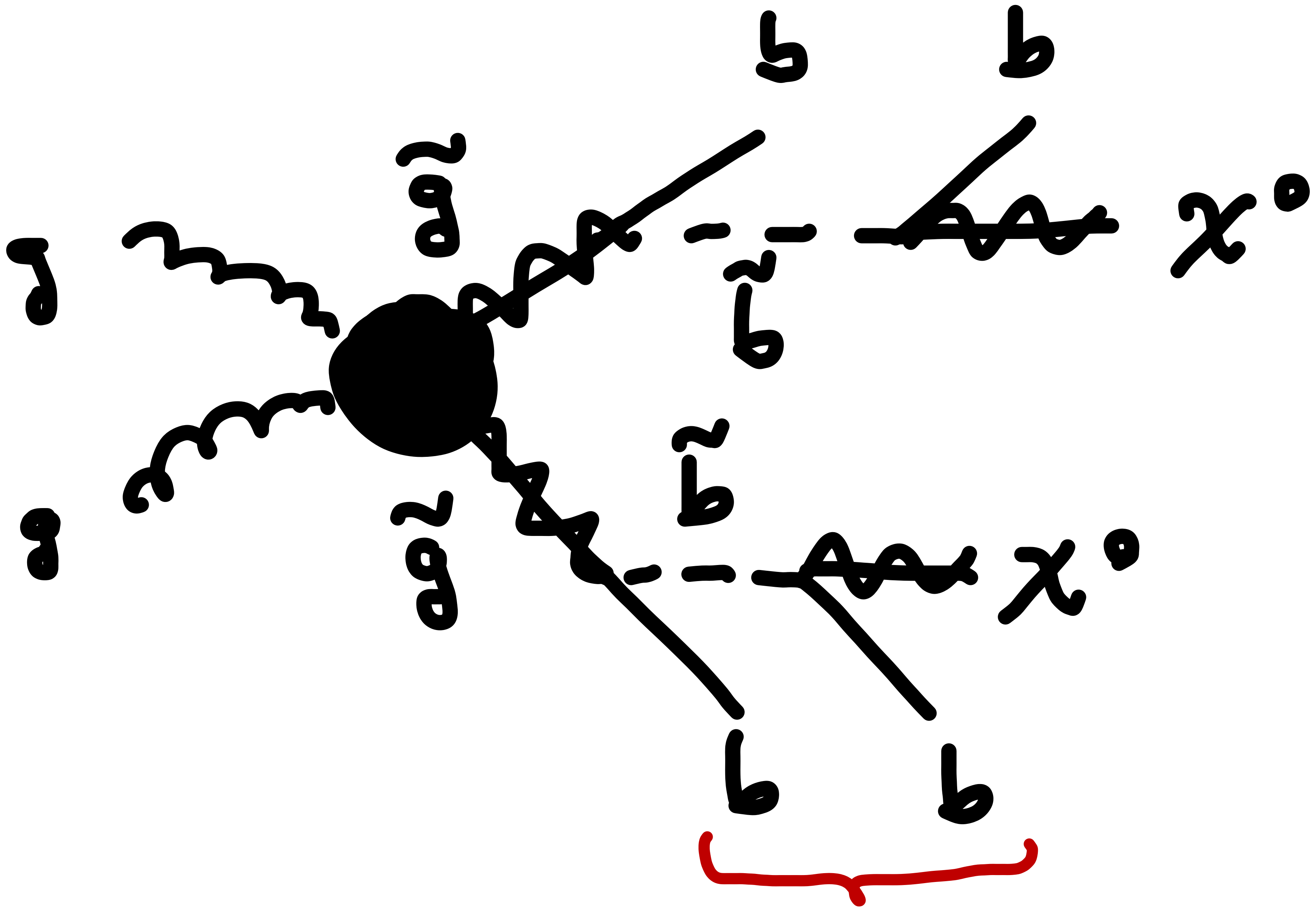
CAN WE MEASURE PARTICLE MASSES ?

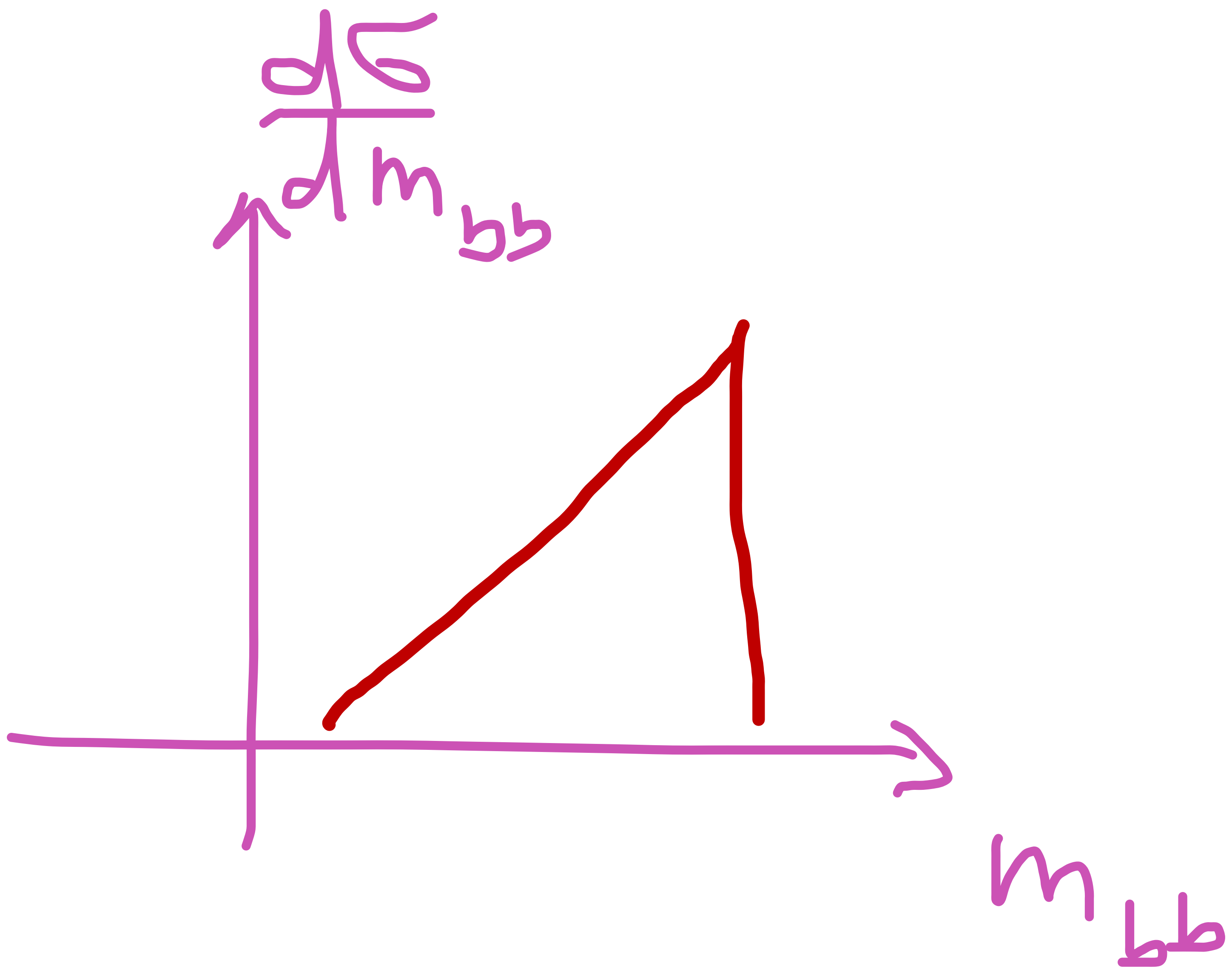
FROM 100 PSEUDO EXPERIMENTS
FOR LHC $\sqrt{s} = 7$ TeV AND $\mathcal{L} = 5/\text{fb}$
WE GET

$$m_{top} = 173.1 \pm 2.5 \text{ GeV}$$

- ALL THE EFFECTS AT LEADING ORDER ARE WELL UNDER CONTROL
- HIGHER ORDER QCD WAS NOT INCLUDED ($\lesssim 10\%$)

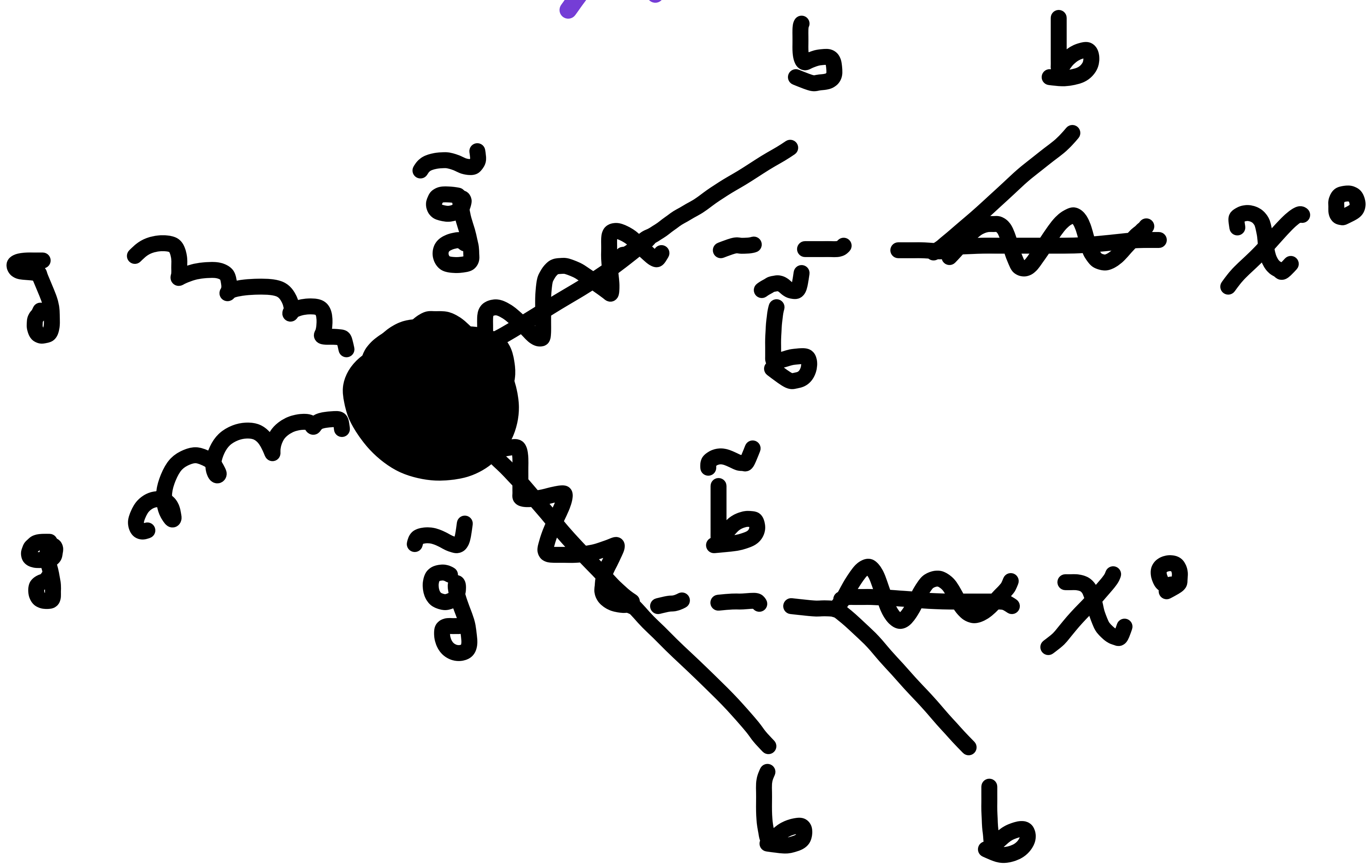
$$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\, b\tilde{b} \to 4b\, \not{E}_T$$





$$m_{bb}^{max} = \sqrt{\frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{m_{\tilde{b}}} \cdot \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{m_{\tilde{b}}}}$$

$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\,b\tilde{b} \to 4b\,\not{E}_T$



TWO-STEPS DECAY $\Rightarrow$

$$E_b^{peak} = \frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{2m_{\tilde{g}}} \qquad E_b^{peak} = \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{2m_{\tilde{b}}}$$

NO COMBINATORICAL ISSUES

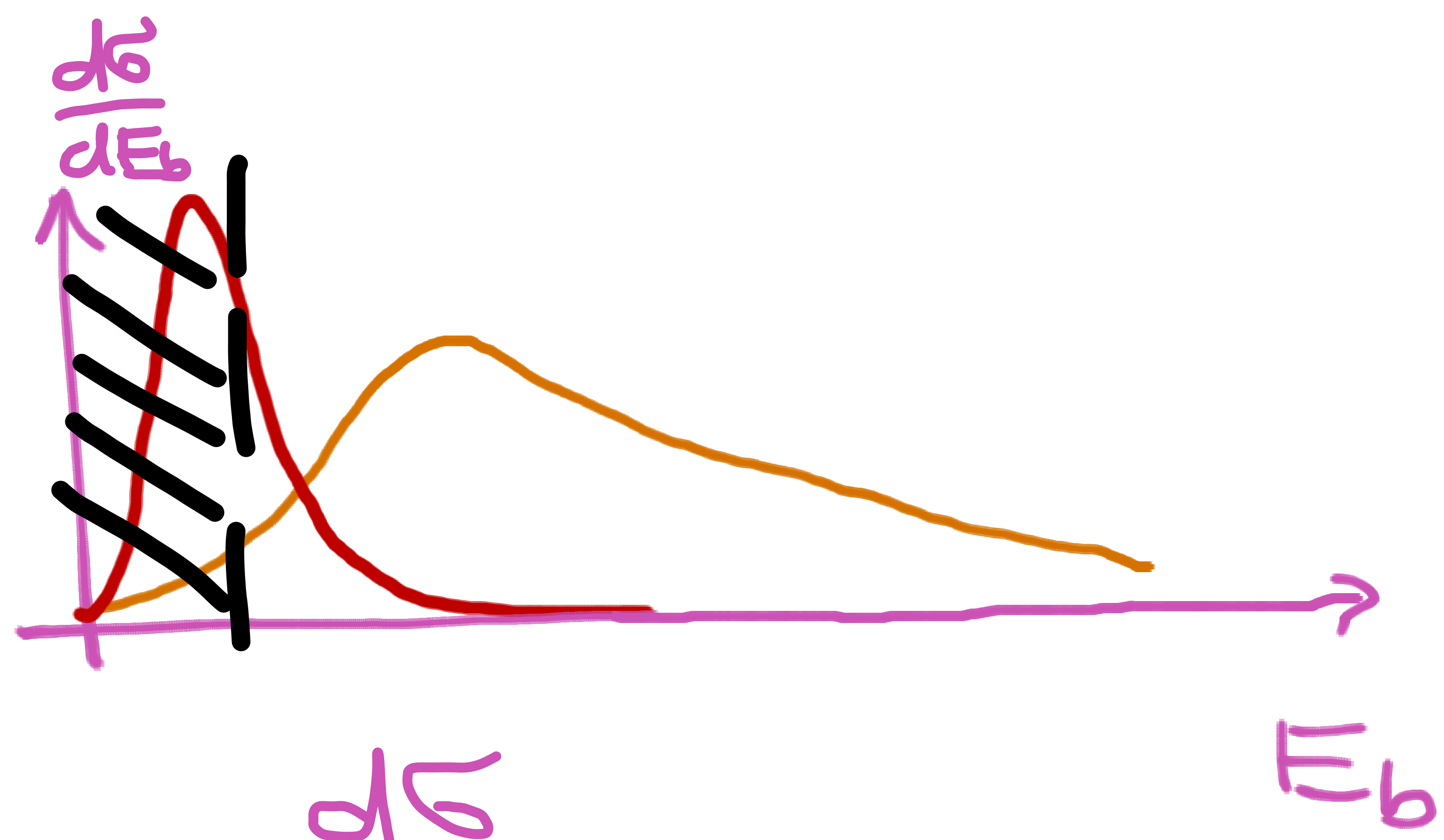
JUST LOOK AT $\frac{d\sigma}{dE_b}$

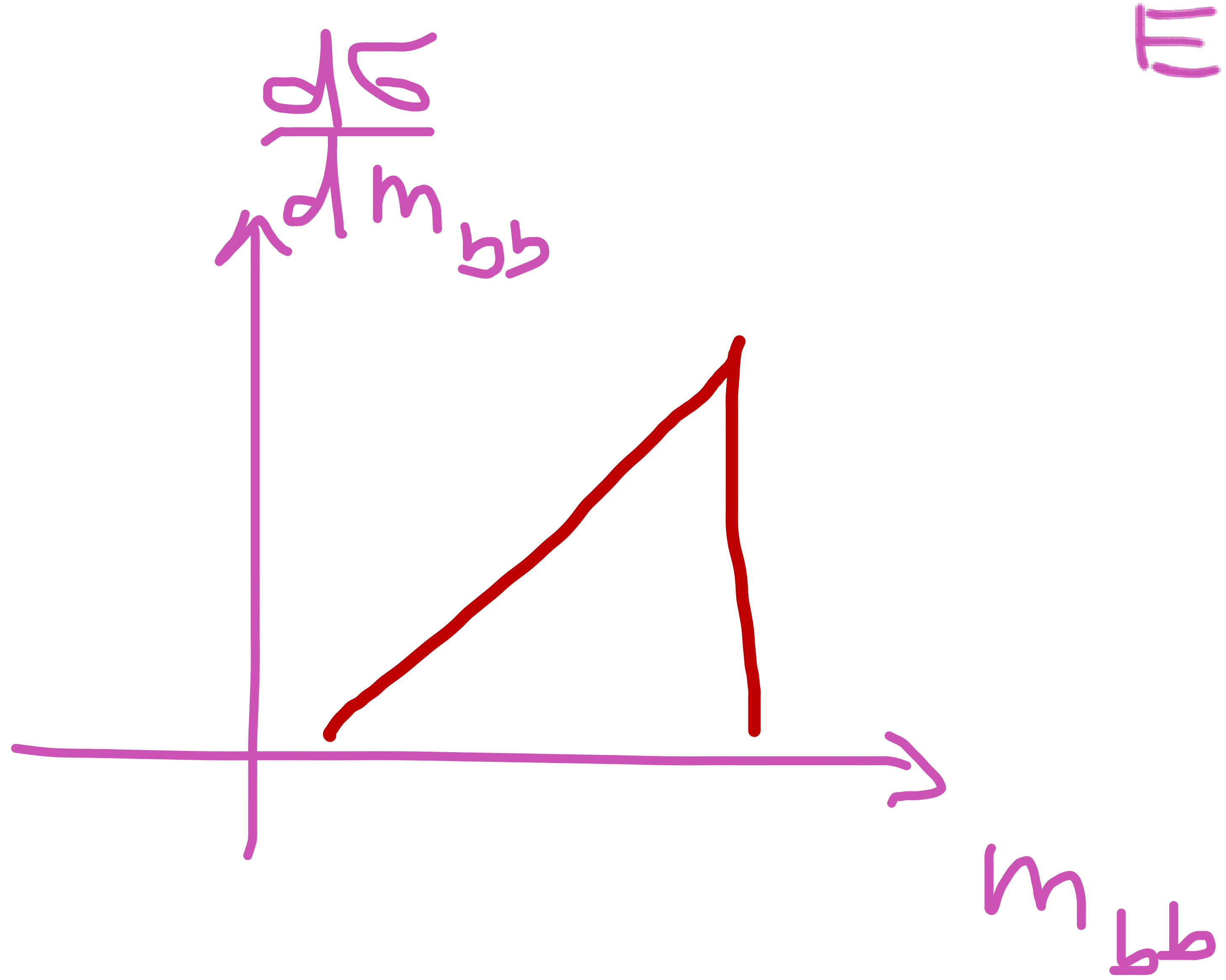
$$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\, b\tilde{b} \to 4b\, \not{E}_T$$

$$E_{b_H} = \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{2 m_{\tilde{b}}}$$

$$E_{b_L} = \frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{2 m_{\tilde{g}}}$$

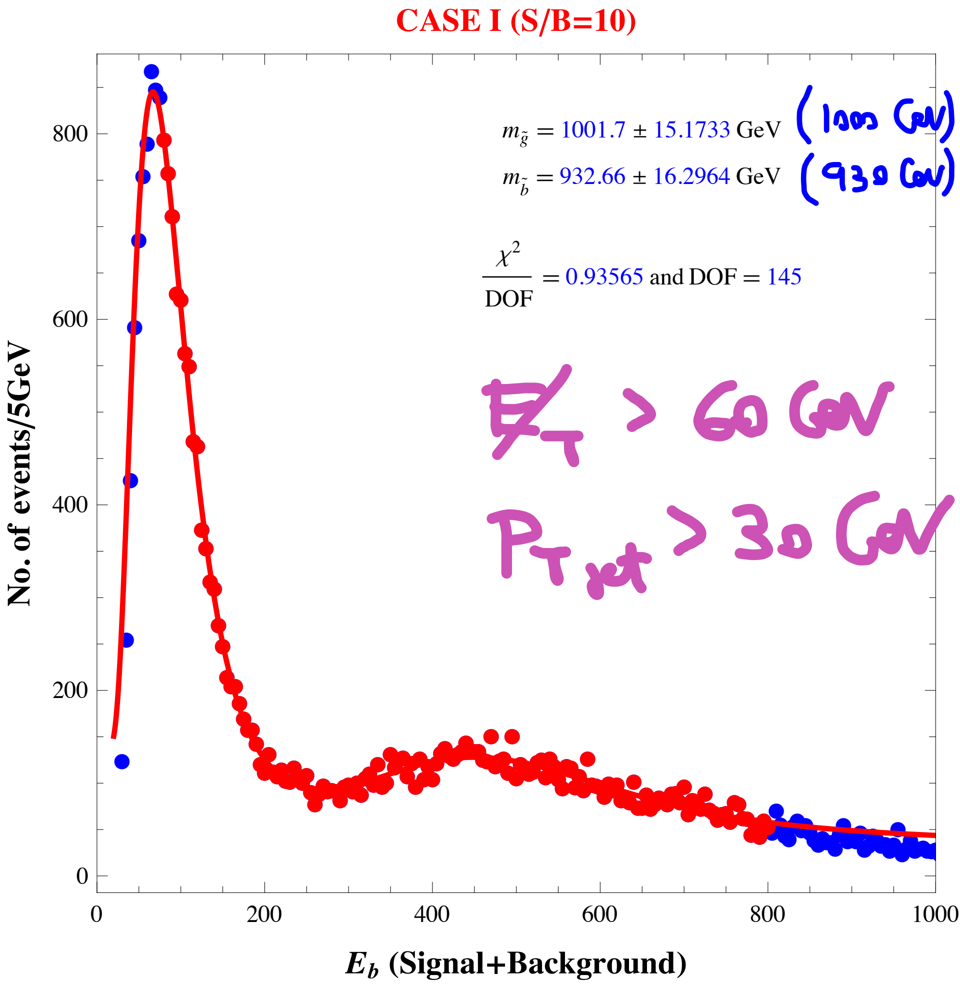
$$m_{bb}^{max} = \sqrt{4 \frac{m_{\chi}}{m_{\tilde{b}}} E_{b_H} E_{b_L}}$$



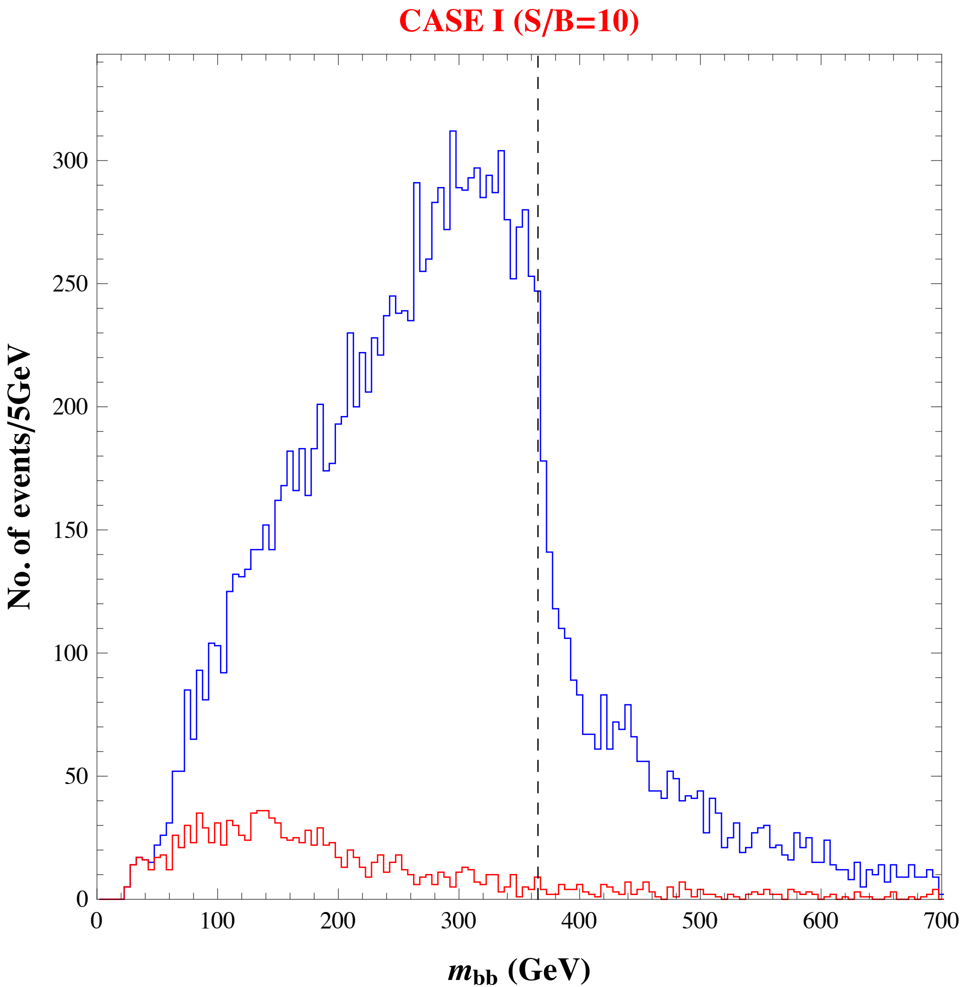


- BACKGROUNDS Z+jets (mostly Z bbbb) & $t\bar{t}b\bar{b}$ (subdom)
- CUTS MAY AFFECT THE ENERGY DISTRIBUTION

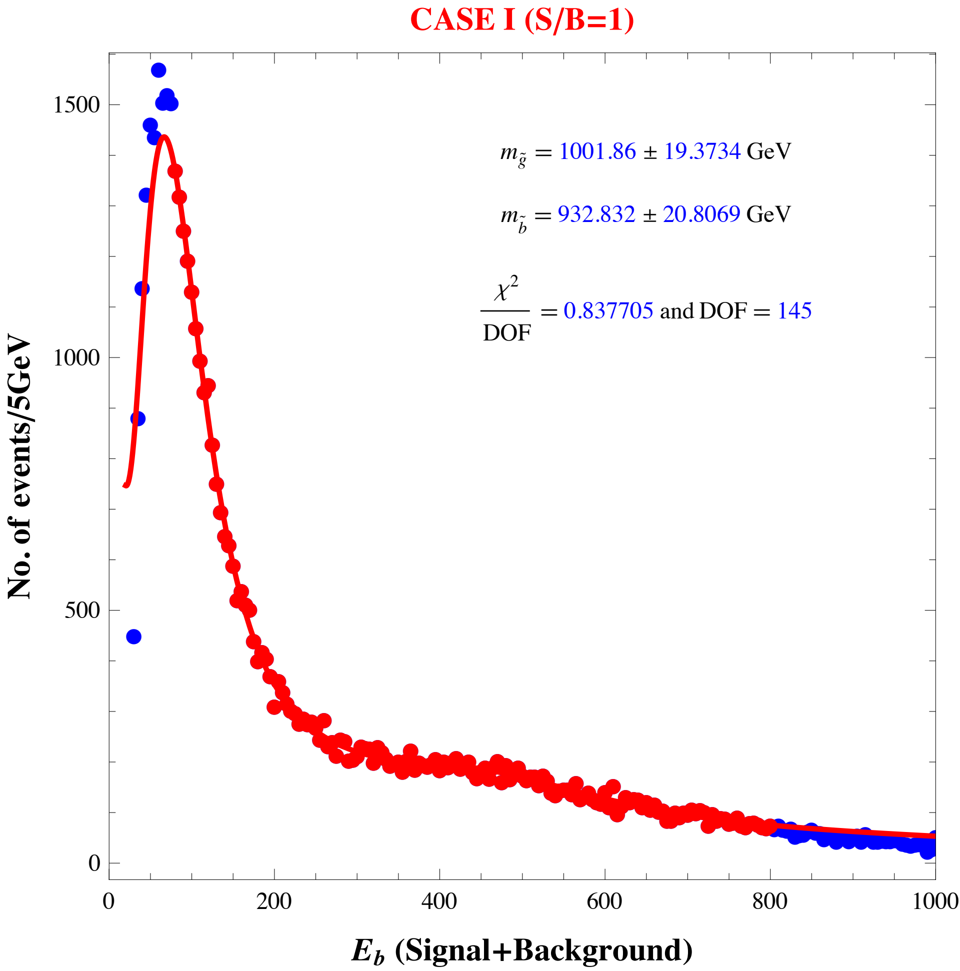
$$P_{T\,jet} > X \Rightarrow E_{jet} > X$$



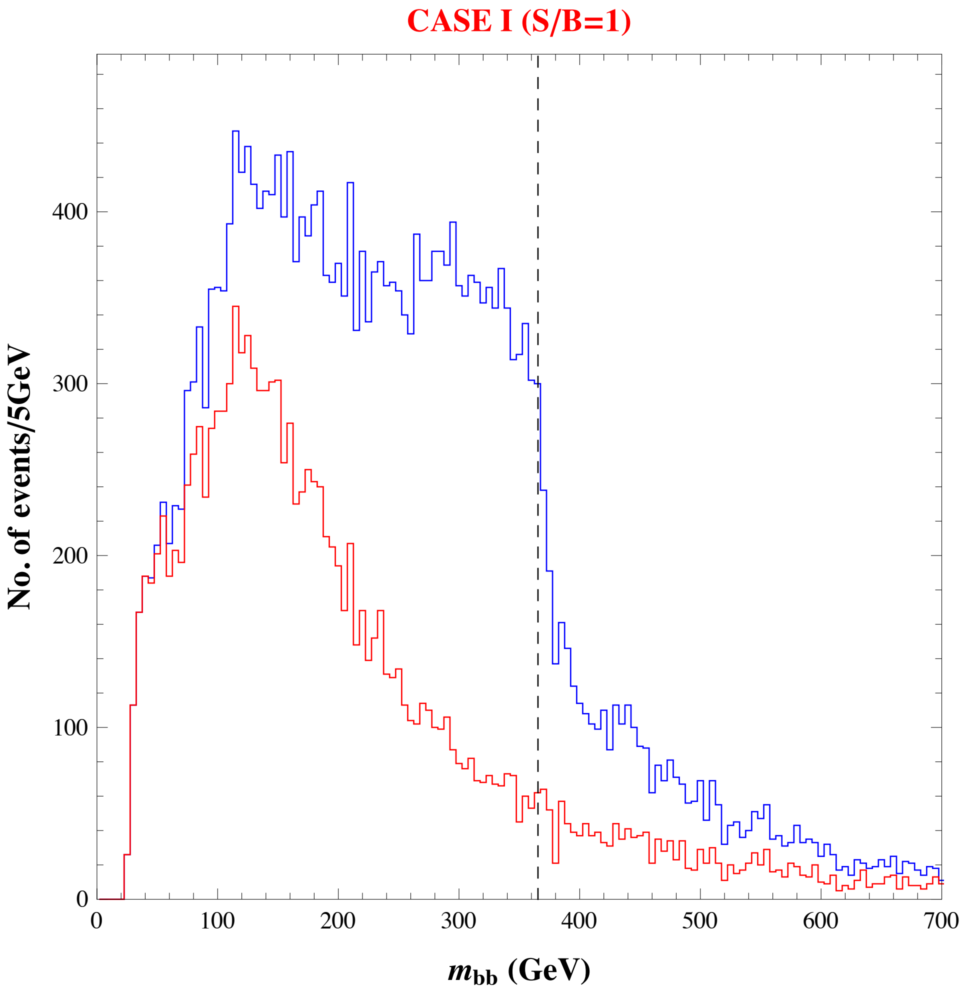
CASE I (S/B=10)
$m_{\tilde{g}} = 1001.7 \pm 15.1733$ GeV
$m_{\tilde{b}} = 932.66 \pm 16.2964$ GeV
$\frac{\chi^2}{\text{DOF}} = 0.93565$ and DOF = 145
No. of events/5GeV
E_b (Signal+Background)
0
200
400
600
800
1000



CASE I (S/B=10)
No. of events/5GeV
0
50
100
150
200
250
300
0
100
200
300
400
500
600
700
m_{bb} (GeV)



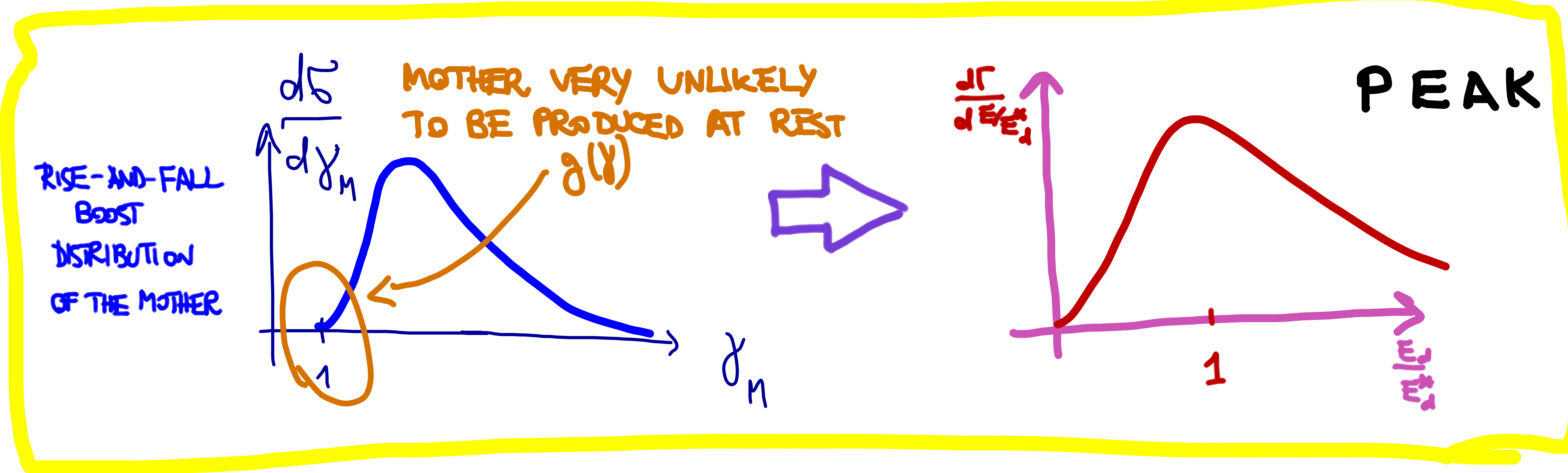
CASE I (S/B=1)
$m_{\tilde{g}} = 1001.86 \pm 19.3734$ GeV
$m_{\tilde{b}} = 932.832 \pm 20.8069$ GeV
$\frac{\chi^2}{DOF} = 0.837705$ and DOF = 145
No. of events/5GeV
1500
1000
500
0
200
400
600
800
1000
E_b (Signal+Background)


CASE I (S/B=1)
No. of events/5GeV
0
100
200
300
400
0
100
200
300
400
500
600
700
m_{bb} (GeV)

Conclusions

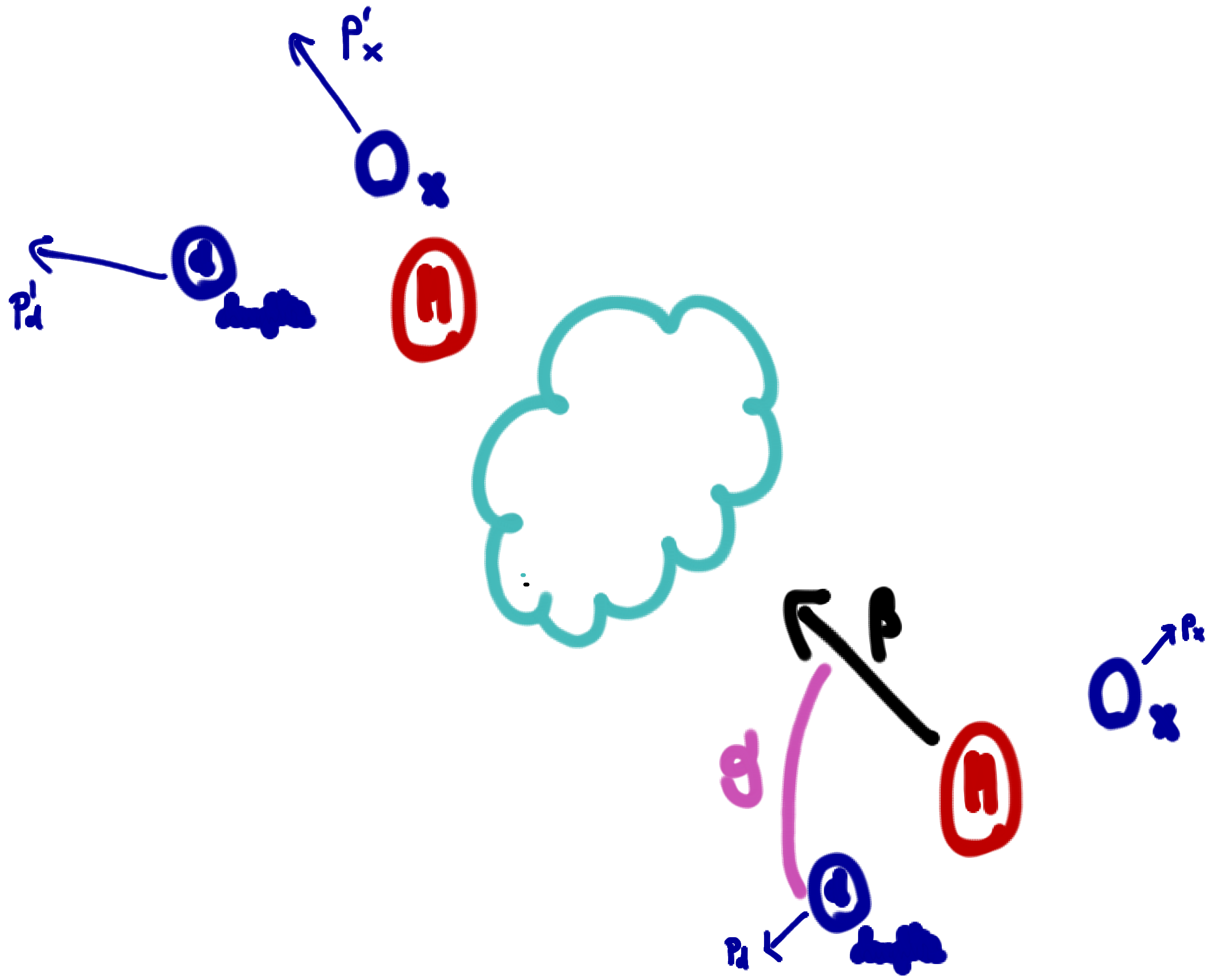
IN PHENOMENOLOGICALLY RELEVANT CASES (HIGH ENERGY COLLIDERS)

THE SPECTRUM OF ENERGY IN TWO BODY DECAYS ENCODES IN A SIMPLE WAY AN INVARIANT OF THE TWO BODY DECAY KINEMATICS



KINKS OR PLATEAUS ARE POSSIBLE AS WELL

Conclusions



THE PEAK OF THE ENERGY DISTRIBUTION IS ROBUST FOR MASSLESS AND MASSIVE DAUGHTERS

$$E_{peak} = \frac{m_M^2 - m_X^2}{2 m_M}$$

$$E_{peak} \geq \frac{m_M^2 - m_X^2 + m_d^2}{2 m_M}$$

LIMITING FACTORS:

- RADIATIVE CORRECTIONS

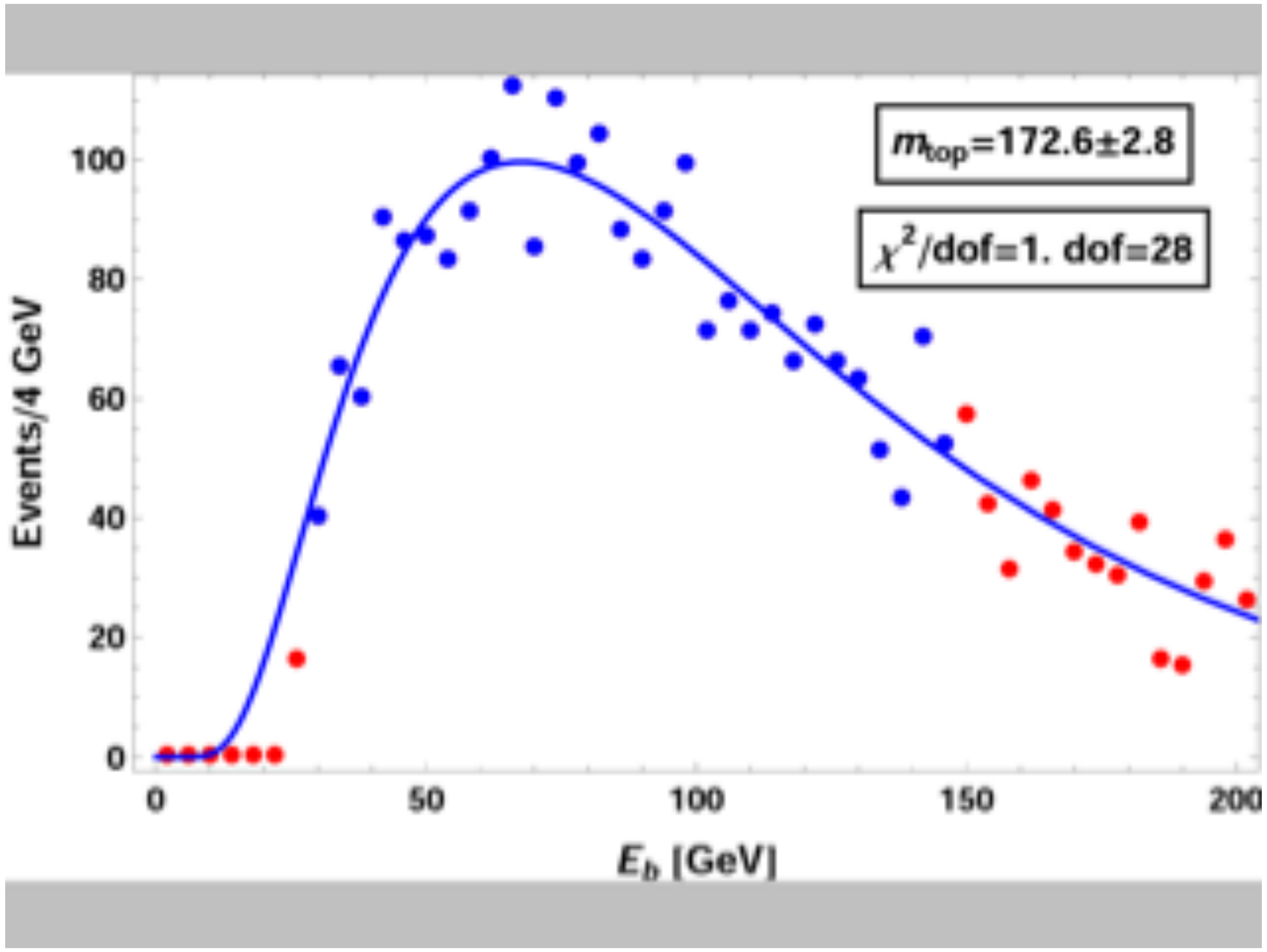
  EXTRA RADIATION MAKES THE DECAY 3-BODY

- TOO LARGE MASS OF THE OBSERVED DAUGHTER

- MAY BE SENSITIVE TO SELECTION CUTS

DESPITE THESE LIMITATIONS THE OBSERVATION CAN BE USED TO MEASURE PARTICLE MASSES WITH 10% ACCURACY OR BETTER

$t \to b \ell \nu$ in $pp \to t\bar{t}$ ⇨ m_{top} FROM $d\sigma/dE_b$

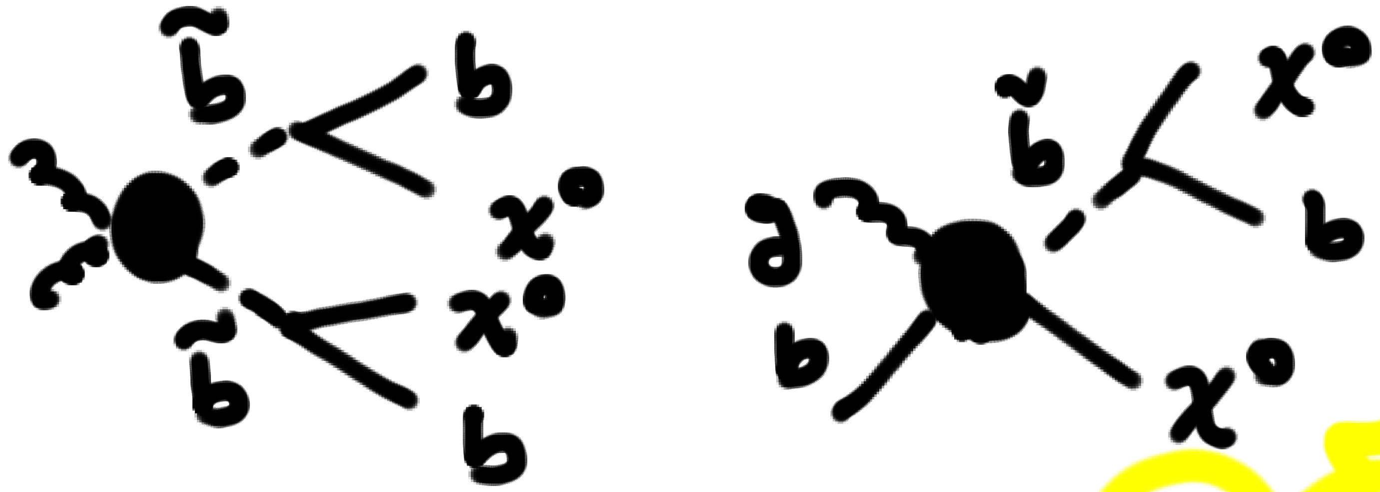


$g \to b\tilde{b} \to bb\chi$
IN PROGRESS

"LOCAL"

SIMPLE

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT



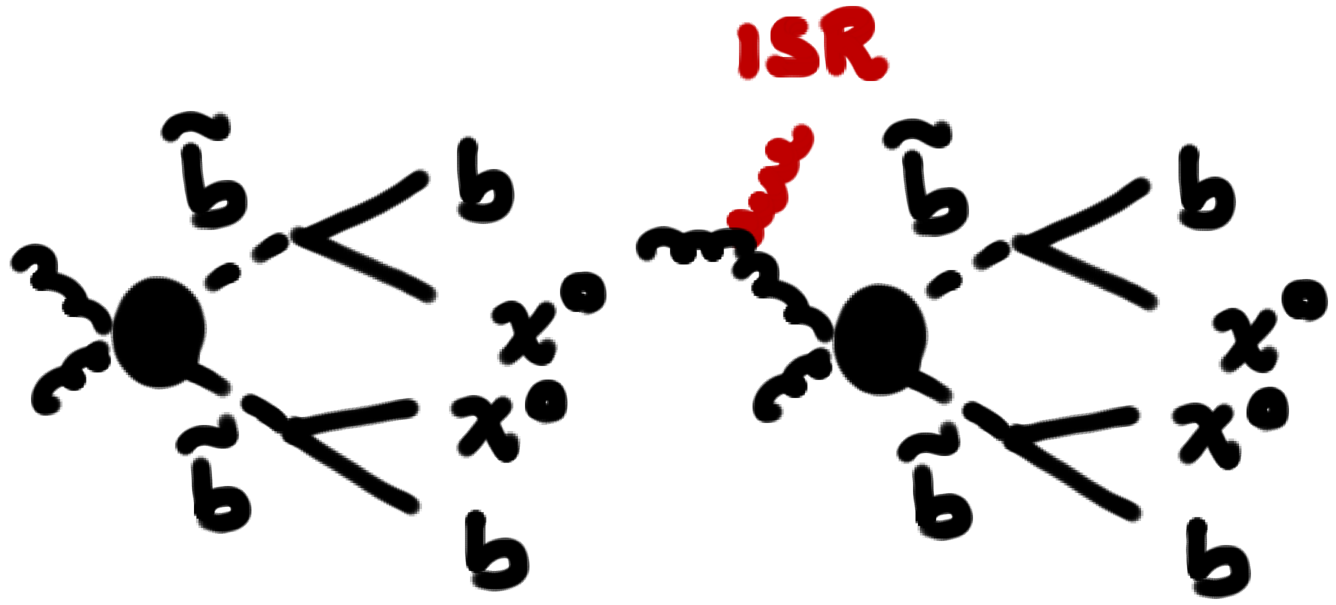
- NO NEED TO MEASURE THE OTHER DECAY PRODUCT

$\tilde{b} \to b \chi^0$

$W \to \ell \nu$

$t \to b W \to b \ell \nu$

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT



ROBUST

Also, since

$$\log E_\gamma = \tfrac{1}{2}(\log E_{\gamma,\min} + \log E_{\gamma,\max}) = \log \mu \qquad (1\text{–}225)$$

it follows that, on logarithmic plots of the energy spectra of these γ-rays, the rest-system energy μ will lie halfway between the extremum energies.

We are particularly concerned with decays that are isotropic in the rest system of the decaying particle, such as the π^0 and Σ^0 decays, which we have previously considered. For these decays, we have already shown that the resultant γ-ray energy distribution function is only a function of the momentum of the primary; indeed this function is a constant which is inversely proportional to this momentum for a given primary, within a range proportional to the momentum of the primary, and vanishes outside this range. Thus, for decays of parent particles with a wide range of primary energies, γ-ray spectra are generated which are made up of a superposition of rectangular spectra, as shown in figure 1–11. Higher energy primaries produce the γ-rays at the extremes of the spectrum. We therefore deduce a second important kinematic property, which holds for two-body decays that produce γ-rays isotropically in the rest system of the decaying primary; viz,

> The energy spectra of γ-rays produced isotropically in the rest system of the decaying primary will be symmetric on a logarithmic plot with respect to $E_\gamma = \mu$ and will peak at $E_\gamma = \mu$.

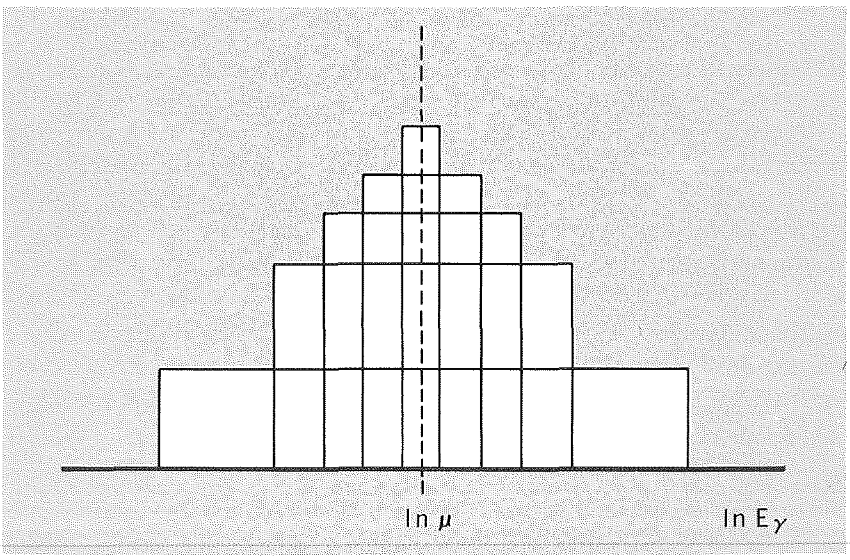


FIGURE 1–11.—Ideal superposition of γ-ray energy spectra from π^0 of Σ^0 particles having discrete values of energy.