SMOOTH SOLUTIONS IN "VASILIEV THEORY"

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GRAVITY IN D=2+1

- Einstein-Hilbert action

$$I = \frac{1}{16\pi G} \int \epsilon_{abc} \left(e^a \wedge R^{bc} + \frac{1}{3l^2}\, e^a \wedge e^b \wedge e^c \right)$$

- Field equations

$$\begin{aligned} R_l^{ab} &\equiv d\omega^{ab} + \omega^{ac} \wedge \omega_c{}^b + \frac{1}{l^2}\, e^a \wedge e^b = 0 \qquad \leftarrow \text{constant curvature!} \\ T^a &\equiv de^a + \omega^a{}_b \wedge e^b = 0 \end{aligned}$$

- Rewriting in terms of the metric

$$g_{\mu\nu} = \eta_{ab}\, e^a_\mu e^b_\nu \qquad \Rightarrow \qquad I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(R + \frac{2}{l^2} \right)$$

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  - $\omega_\mu{}^a = \frac{1}{2}\,\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$
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- Chern-Simons formulation on AdS_3

$$e = \frac{l}{2}\left(A - \widetilde{A}\right) \qquad \omega = \frac{1}{2}\left(A + \widetilde{A}\right)$$

HIGHER SPINS IN D=2+1

- Natural generalization of the gravity frame action Blencowe (1989)

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3l^2}\, e\wedge e\wedge e\right) \qquad \text{with} \quad R = d\omega + \omega\wedge\omega$$

- For g = sl(N,R) describes fields of "spin" 2,3,...,N

$$e \;=\; e_\mu{}^{\mathcal{A}}\, J_{\mathcal{A}}\, dx^\mu \;=\; \left(e_\mu{}^a J_a + e_\mu{}^{ab}\, T_{ab} + \cdots\right) dx^\mu$$

$$\omega \;=\; \omega_\mu{}^{\mathcal{A}}\, J_{\mathcal{A}}\, dx^\mu = \left(\omega_\mu{}^a J_a + \omega_\mu{}^{ab}\, T_{ab} + \cdots\right) dx^\mu$$

  - Example: the sl(3,R) algebra

$$[\,J_a\,,\,J_b\,] \;=\; \epsilon_{abc}\, J^c$$

$$[\,J_a\,,\,T_{bc}\,] \;=\; \epsilon^m{}_{a(b}T_{c)m}$$

$$[\,T_{ab}\,,\,T_{cd}\,] \;=\; \sigma\left(\eta_{a(c}\epsilon_{d)bm} + \eta_{b(c}\epsilon_{d)am}\right) J^m$$

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  - More in general: take any Lie algebra **g** with a non-degenerate Killing form and branch it under the adjoint action of sl(2,R)↪**g**

$$\mathfrak{g} \;=\; sl(2,\mathbb{R}) \,\oplus\, \left(\bigoplus_{\ell\,,\,a} \mathfrak{g}^{(\ell,a)}\right)$$

dim = $2\ell + 1$

HIGHER SPINS IN D=2+1

- Simple characterization in terms of Chern-Simons theories (for gauge fields)

$$e = \frac{l}{2}\left(A - \widetilde{A}\right)\,,\ \omega = \frac{1}{2}\left(A + \widetilde{A}\right) \quad \Rightarrow \quad \boxed{S = S_{CS}[A] - S_{CS}[\widetilde{A}]}$$

- Field equations → flatness conditions

$$F = dA + A \wedge A = 0 \qquad\qquad \widetilde{F} = d\widetilde{A} + \widetilde{A} \wedge \widetilde{A}$$

- Simple field equations, but rich space of solutions on AdS
  - Non-trivial topology → black holes — Gutperle, Kraus (2011)
  - Boundary conditions → boundary dynamics, AdS/CFT... — Gaberdiel, Gopakumar (2010)
- How to select "non-singular" solutions? — Gutperle, Kraus (2011); Castro, Gopakumar, Gutperle, Raeymaekers (2011)

OUTLINE

- Coupling ∞ spins: hs[λ] Chern-Simons theories
- Smoothness criteria for asymptotically AdS solutions
- “Analytic continuation” of the sl(N) conical surpluses
- Conclusion

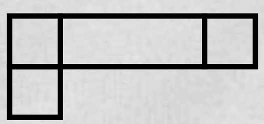
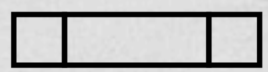
THE GAUGE SECTOR OF THE PROKUSHKIN-VASILIEV MODEL

FRAME-LIKE DESCRIPTION FOR HS

- HS "vielbeins" and "spin connections"

  $e_\mu{}^{a_1 \dots a_{s-1}} \Rightarrow$  $\quad \omega_\mu{}^{b, a_1 \dots a_{s-1}} \Rightarrow$ 

- Everything is traceless, then in D=2+1...

  -  $\approx$  (example: $\omega_\mu{}^a = \frac{1}{2}\, \epsilon^a{}_{bc}\, \omega_\mu{}^{b,c}$)

  - "Vielbeins" and "spin connections" have the same structure

- Structure of the higher-spin generators:

  - $e^{ab\dots}$ traceless $\Rightarrow$ $T_{ab\dots}$ traceless in ab...

  - $e^{ab\dots}$ irreducible $\Rightarrow$ $[\, J_a \,,\, T_{b_1 \dots b_{s-1}} \,] = \epsilon^m{}_{a(b_1} T_{b_2 \dots b_{s-1})m}$

SL(N) HIGHER-SPIN THEORIES

- For sl(3,R) the Jacobi identity fixes the algebra but...
  - $T_{ab} = \sqrt{-\sigma}\left(J_{(a}J_{b)} - \frac{2}{3}\,\eta_{ab}J_cJ^c \right) \;\Rightarrow\; [\,J_a\,,\,T_{bc}\,] = \epsilon^m{}_{a(b}T_{c)m}$
  - 3-dim repr. for J_a $\Rightarrow$ $[\,T_{ab}\,,\,T_{cd}\,] = \sigma\left(\eta_{a(c}\epsilon_{d)bm} + \eta_{b(c}\epsilon_{d)am}\right) J^m$
- Consider traceless and symmetric polynomials in J_a
  - $T_{a_1\dots a_s} \sim J_{(a_1}\,\dots\,J_{a_s)}$ (+ traceless projection in the a_n indices)
  - N-dim repr. for $J_a \Rightarrow$ N^2-1ind. traceless matrices out of T's with s<N

Hoppe (1982)

General lesson to build higher-spin algebras: choose a representation of so(1,2) ≃ sl(2,R) and compute products of the representatives

HIGHEST WEIGHT IRREPS OF SL(2,R)

- sl(2,R) algebra: $[\,J_+\,,\,J_-\,] = 2J_0\,, \qquad [\,J_\pm\,,\,J_0\,] = \pm\,J_\pm$
- Casimir: $C_2 = J_0^2 - \frac{1}{2}(J_+J_- + J_-J_+) = \frac{1}{4}(\lambda^2 - 1)$
- λ≠N ⇒ two pairs of conjugate irreps
  - Realize the generators as e.g.

$$J_+ = y\,\frac{\partial}{\partial x}\,, \qquad J_0 = \frac{1}{2}\left(x\,\frac{\partial}{\partial x} - y\,\frac{\partial}{\partial y}\right), \qquad J_- = -\,x\,\frac{\partial}{\partial y}\,.$$

  - and act on

$$v_i = x^i y^{\lambda-i-1}\,, \qquad \bar{v}_i = x^{\lambda-i-1}y^i\,,$$

$$w_i = x^i y^{-(\lambda+i+1)}\,, \qquad \bar{w}_i = x^{-(\lambda+i+1)}y^i\,,$$

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A FAITHFUL MATRIX REPR. OF HS[λ]

- Irrep of sl(2,R) with highest weight $\frac{1}{2}(\lambda-1)$:

$$(J_+)_{jk} = \delta_{j,\,k+1}\,,$$
$$(J_-)_{jk} = j(j-\lambda)\,\delta_{j+1,\,k}\,,$$
$$(J_0)_{jk} = \frac{1}{2}(\lambda+1-2j)\,\delta_{j,\,k}\,,$$

- Building the hs[λ] generators:

Pope, Romans, Shen (1990)

$$T^\ell_m = (-1)^{\ell-m}\,\frac{(\ell+m)!}{(2\ell)!}\left[\underbrace{J_-,\dots[\,J_-,[\,J_-,}_{\ell-m\ \text{terms}}(J_+)^\ell\,]]\right]$$

- Explicit realization:

A.C., Procházka, Raeymaekers (2013)

$$(T^\ell_m)_{jk} = (-1)^{\ell-m}\sum_{n=0}^{\ell-m}\binom{\ell-m}{n}\frac{[\,\ell\,]_n}{[\,2\ell\,]_n}\,[\,\ell-\lambda\,]_n\,[\,j-m-1\,]_{\ell-m-n}\,\delta_{j,\,k+m}$$

A HS[λ] CHERN-SIMONS THEORY

- The T^ℓ_m satisfy $[\, J_i, T^\ell_m \,] = (i\ell - m)\, T^\ell_{m+i}$
- Trace: $$\mathrm{tr}\, v = \frac{6}{\lambda(\lambda^2-1)} \lim_{N\to\lambda} \sum_{j=1}^{N} v_{jj}$$
- Chern-Simons theory with hs[λ]⊕hs[λ] gauge algebra as a model for the interactions of spins 2,...,∞

Bergshoeff, Blencowe, Stelle (1990); Vasiliev (1991)

- Field equations:

$$F = dA + A \wedge A = 0 \qquad \bar{F} = d\bar{A} + \bar{A} \wedge \bar{A} = 0$$

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- What are the "admissible" connections?

PROPERTIES OF THE HS[λ] MATRICES

- Properties of the hs[λ] matrices:
  - The non-zero elements belong to the diagonal $(T^{\ell}_{m})_{j,\,j-m}$
  - The $(T^{\ell}_{m})_{j,\,j-m}$ are polynomials in j
- Is that enough? What kinds of linear combinations do we have to consider? What are their properties?

PROPERTIES OF THE HS[λ] MATRICES

- Properties of the hs[λ] matrices:

Khesin, Malikov (1996)

  - ∃ N such that $v_{j,\,k} = 0$ if j > k+N
  - The matrix elements along a diagonal, $v_{j,\,j+n}$ for some fixed n, become polynomial in j for sufficiently large j

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- One can perform the substitution $N \to \lambda$ provided that N is large enough

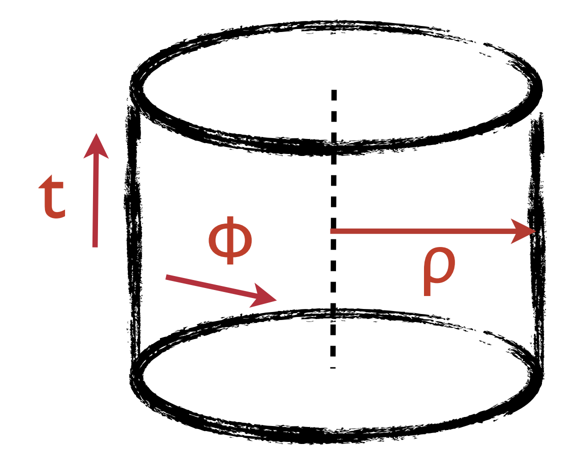
SMOOTH ASYMPTOTICALLY ADS SOLUTIONS

ASYMPTOTICALLY ADS SOLUTIONS

- Focus on a single chiral sector: $F = dA + A \wedge A = 0$
- Radial gauge fixing:

$$A = g^{-1} a(\vec{z}) g \, d\vec{z} + g^{-1} dg$$

$$g = e^{\rho T_0^1}$$



$z \equiv \phi + i t_E$

- Boundary conditions

$$a(z) = \alpha T_1^1 + \frac{12\pi}{c} \sum_{\ell=1}^{\infty} \frac{\alpha^{-\ell}}{N_\ell} W_{\ell+1}(z) \, T_{-\ell}^{\ell} \, , \qquad a(\bar{z}) = 0$$

→ $W_\infty[\lambda]$ algebra on the boundary

Henneaux, Rey (2010)
A.C., Fredenhagen, Pfenninger, Theisen (2010)
Gaberdiel, Hartman (2011)
A.C., Fredenhagen, Pfenninger (2011)

SMOOTH SOLUTIONS?

- How to identify smooth solutions?
  - The metric is no longer gauge invariant: the usual regularity conditions do not apply
  - One can ask that the gauge field A is non singular
- Gauge invariant characterization: holonomies

$$H(A) = \mathcal{P} \exp \oint A = g^{-1} \exp(2\pi a)\, g$$

Gutperle, Kraus (2011)

- Smooth solutions → trivial holonomy around Φ

$$H = e^{2\pi a} = e^{i\varphi_0}\, T^0_0$$

Castro, Gopakumar, Gutperle, Raeymaekers (2011)

A.C., Procházka, Raeymaekers (2013)

HS[λ] PROJECTORS

- How to control exponentials of hs[λ] elements?
- A simple class under control: the gl[λ] projectors

$$P \star P = P \qquad \Rightarrow \qquad e_\star^{cP} = T_0^0 + (e^c - 1)P.$$

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- General hs[λ] element in diagonal gauge

$$a = -\,i\sum_j m_j P_j + \frac{\lambda^2-1}{6}\,i\,\mathrm{tr}\Big(\sum_j m_j P_j\Big)T_0^0$$

$$(P_j)_{kl} = \delta_{jk}\delta_{jl}$$

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THE COUNTERPARTS OF THE SL(N) CONICAL SOLUTIONS

REWRITING OF THE SOLUTION

- General gauge connection that exponentiate to 1

$$a = -\,i\sum_j m_j P_j + \frac{\lambda^2-1}{6}\,i\,\mathrm{tr}\Big(\sum_j m_j P_j\Big)T^0_0$$

- Order the integer m_j and define $s_j = m_j + j$
  - solution specified by an ordered list of integers $s_1 \geq s_2 \geq \dots$
  - the s_j must become polynomials in j for large enough j
- Looking at the matrix realization of $J_0 \sim T^1{}_0$ leads to

$$a = -\,i\sum_j s_j P_j + \frac{\lambda^2-1}{6}\,i\,\mathrm{tr}\Big(\sum_j s_j P_j\Big)T^0_0 - i\,T^1_0$$

CONICAL SOLUTIONS

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- The s_j must become polynomials in j for j large
- Simplest solution: $s_j = S$ for $j > N$
- Smooth connections ↔ Young tableaux
  - length of the rows of Λ → $r_j = s_j - S$

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$\lambda = N$ ⇒ conical solutions of Castro, Gopakumar, Gutperle, Raeymaekers (2011)

CONCLUSIONS & OUTLOOK

- hs[λ] can be conveniently treated as an algebra of infinite matrices
  - Simple discussion of smoothness conditions — Kraus, Perlmutter (2011); A.C., Procházka, Raeymaekers (2013)
- Applications:
  - Drinfeld-Sokolov reduction i.e. asymptotics — Khesin, Malikov (1996); A.C., Fredenhagen, Pfenninger (2011)
  - Conical solutions in Vasiliev theory — A.C., Procházka, Raeymaekers (2013)
- Perspectives:
  - Role on the CFT side of the smooth solution that are not classified by finite Young tableaux? (see talk by J. Raeymaekers)
  - New thermodynamical branches built on these solutions?