Bimetric theory, partial masslessness and conformal gravity

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Based on

- SFH, Angnis Schmidt-May, Mikael von Strauss
  arXiv:1203.5283, 1204.5202,1208:1515, 1208:1797, 1212:4525, 1303.6940
- SFH, Rachel A. Rosen,
  arXiv:1103.6055, 1106.3344, 1109.3515, 1109.3230, 1111.2070

Outline of the talk

Review: Linear and Nonlinear massive spin-2 fields

Ghost-free bimetric theory

Mass spectrum of bimetric theory

Partially Massless bimetric theory

Higher drivative gravity & Conformal gravity

Equivalence between CG and PM bimetric theory

Higher drivative gravity from bimetric theory

Discussion

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Linear massive spin-2 fields

The Fierz-Pauli equation:

Linear massive spin-2 field, $h_{\mu\nu}$, in background $\bar{g}_{\mu\nu}$

$$\bar{\mathcal{E}}^{\rho\sigma}_{\mu\nu}\, h_{\rho\sigma} - \Lambda\Big(h_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu} h^{\rho}_{\rho}\Big) + \frac{m^2_{\rm FP}}{2}\,\big(h_{\mu\nu} - \bar{g}_{\mu\nu} h^{\rho}_{\rho}\big) = 0$$

[Fierz-Pauli, 1939]

- 5 propagating modes (massive spin-2)
- Massive gravity (?)
- What determines $\bar{g}_{\mu\nu}$? (flat, dS, AdS, $\cdots$)
- Nonlinear generalizations?

[The Boulware-Deser ghost (1972)]

Nonlinear massive spin-2 fields

- "Massive gravity" (fixed $f_{\mu\nu}$):

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 \, V(g^{-1} f) \right]$$

- Interacting spin-2 fields (dynamical g and f):

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 \, V(g^{-1} f) \right] + \mathcal{L}(\nabla f)(?)$$

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Bimetric: $\mathcal{L}(\nabla f) = m_f^2 \sqrt{-f} \, R_f(?)$

[Isham-Salam-Strathdee, 1971, 1977]

Generically, both contain a $\mathcal{GHOST}$ at the nonlinear level

[Boulware-Deser, 1972]

Counting modes:

Generic massive gravity:

- Linear modes: 5 (massive spin-2)
- Non-linear modes: 5 + 1 (ghost)

Generic bimetric theory:

- Linear modes: 5 (massive, $\delta g - \delta f$)
  + 2 (massless, $\delta g + \delta f$)
- Non-linear modes: 7 + 1 (ghost)

Complication: Since the ghost shows up nonlinearly, its absence needs to be established nonlinearly

Construction of ghost-free nonlinear theories

Based on "Decoupling limit" analysis:

A specific $V_{dRGT}(\sqrt{g^{-1}\eta})$ was obtained and shown to be ghost-free in a "decoupling limit", also perturbatively in $h = g - \eta$

[de Rham, Gabadadze, 2010; de Rham, Gabadadze, Tolley, 2010]

Non-linear Hamiltonian methods (non-perturbative):

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Non-linear Hamiltonian methods (non-perturbative):

Questions not answerable by "decoupling limit":

- Is massive gravity with $V(\sqrt{g^{-1}\eta})$ ghost-free nonlinearly?

  [SFH, Rosen (1106.3344, 1111.2070)]

- Is it ghost-free for generic fixed $f_{\mu\nu}$?

  [SFH, Rosen, Schmidt-May (1109.3230)]

- Can $f_{\mu\nu}$ be given ghost-free dynamics?

  [SFH, Rosen (1109.3515)]

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Ghost-free bimetric theory

Digression: Elementary symmetric polynomials of $\mathbb{X}$ with eigenvalues $\lambda_1, \cdots, \lambda_4$:

$$
\begin{aligned}
e_0(\mathbb{X}) &= 1, \qquad\qquad e_1(\mathbb{X}) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\,, \\
e_2(\mathbb{X}) &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4, \\
e_3(\mathbb{X}) &= \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\,, \\
e_4(\mathbb{X}) &= \lambda_1\lambda_2\lambda_3\lambda_4 = \det\mathbb{X}\,.
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e_0(\mathbb{X}) &= 1\,, \qquad\qquad e_1(\mathbb{X}) = [\mathbb{X}]\,,\\
e_2(\mathbb{X}) &= \tfrac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]),\\
e_3(\mathbb{X}) &= \tfrac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3])\,,\\
e_4(\mathbb{X}) &= \tfrac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4])\,,\\
e_k(\mathbb{X}) &= 0 \qquad \text{for} \quad k > 4\,,
\end{aligned}
$$

$$[\mathbb{X}] = \mathrm{Tr}(\mathbb{X})\,, \qquad e_n(\mathbb{X}) \sim (\mathbb{X})^n$$

- The $e_n(\mathbb{X})$'s and $\det(\mathbb{1} + \mathbb{X})$:

$$\det(\mathbb{1} + \mathbb{X}) = \sum_{n=0}^{4} e_n(\mathbb{X})$$

- Introduce "deformed determinant" :

$$\widehat{\det}(\mathbb{1} + \mathbb{X}) = \sum_{n=0}^{4} \beta_n \, e_n(\mathbb{X})$$

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- Observation:

$$V(\sqrt{g^{-1}f}) = \sum_{n=0}^{4} \beta_n\, e_n(\sqrt{g^{-1}f}\,)$$

[SFH & R. A. Rosen (1103.6055)]

Ghost-free bi-metric theory

Ghost-free combination of kinetic and potential terms for g & f:

$$\mathcal{L} = m_g^2 \sqrt{-g} R_g - 2m^4 \sqrt{-g} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1} f}) + m_f^2 \sqrt{-f} \, R_f$$

[SFH, Rosen (1109.3515,1111.2070)]

Note,

$$\sqrt{-g} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1} f}) = \sqrt{-f} \sum_{n=0}^{4} \beta_{4-n} \, e_n(\sqrt{f^{-1} g})$$

Hamiltonian analysis: 7 nolinear propagating modes, no ghost!

$$C(\gamma, \pi) = 0 \,, \quad C_2(\gamma, \pi) = \tfrac{d}{dt} C(x) = \{H, C\} = 0$$

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Mass spectrum of bimetric theory

[SFH, A. Schmidt-May, M. von Strauss 1208:1515, 1212:4525]

$$S_{gf} = -\int d^d x \Big[m_g^{d-2}\sqrt{g} R_g - 2m^d \sqrt{g} \sum_{n=0}^{d} \beta_n \, e_n(S) + m_f^{d-2}\sqrt{f} R_f \Big]$$

Three Questions:

- Q1: When are the 7 fluctuations in $\delta g_{\mu\nu}$, $\delta f_{\mu\nu}$ good mass eigenstates? (FP mass)
- Q2: In what sense is this Massive spin-2 field + gravity ?
- Q3: How to characterize deviations from General Relativity?

$$R_{\mu\nu}(g) - \tfrac{1}{2} g_{\mu\nu} R(g) + V^g_{\mu\nu} = T^g_{\mu\nu}$$
$$R_{\mu\nu}(f) - \tfrac{1}{2} f_{\mu\nu} R(f) + V^f_{\mu\nu} = T^f_{\mu\nu}$$

Proportional backgrounds

A1: FP masses exist only around,

$$\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$$

g and f equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu} R(\bar{g}) + \begin{pmatrix} \Lambda_g \\ \Lambda_f \end{pmatrix} \bar{g}_{\mu\nu} = 0 \text{ or } \begin{pmatrix} T^g_{\mu\nu} \\ T^f_{\mu\nu} \end{pmatrix}$$

$$\Lambda_g = \frac{m^d}{m_g^{d-2}} \sum_{k=0}^{d-1} \binom{d-1}{k} c^k \beta_k, \quad \Lambda_f = \frac{m^d}{m_f^{d-2}} \sum_{k=1}^{d} \binom{d-1}{k-1} c^{k+2-d} \beta_k$$

Implication:

$$\Lambda_g = \Lambda_f \quad \Rightarrow \quad c = c(\beta_n, \alpha \equiv m_f/m_g)$$

(Exception: Partially massless (PM) theory)

Mass spectrum around proportional backgrounds

Linear modes:

$$\delta M_{\mu\nu} = \frac{1}{2c}\left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}\right), \quad \delta G_{\mu\nu} = \left(\delta g_{\mu\nu} + \alpha^{d-2} c^{d-4} \delta f_{\mu\nu}\right)$$

$$\bar{\mathcal{E}}^{\rho\sigma}_{\mu\nu}\, \delta G_{\rho\sigma} - \Lambda_g \left(\delta G_{\mu\nu} - \tfrac{1}{2} \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \delta G_{\rho\sigma}\right) = 0\,,$$

$$\bar{\mathcal{E}}^{\rho\sigma}_{\mu\nu}\, \delta M_{\rho\sigma} - \Lambda_g \left(\delta M_{\mu\nu} - \tfrac{1}{2} \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \delta M_{\rho\sigma}\right) + \tfrac{1}{2} m_{\rm FP}^2 \left(\delta M_{\mu\nu} - \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \delta M_{\rho\sigma}\right) = 0$$

The FP mass of δM:

$$m_{\rm FP}^2 = \frac{m^d}{m_g^{d-2}} \left(1 + (\alpha c)^{2-d}\right) \sum_{k=1}^{d-1} \binom{d-2}{k-1} c^k \beta_k$$

Bimetric as massive spin-2 field + gravity

A2: The massless mode is not gravity!

$$G_{\mu\nu} = g_{\mu\nu} + c^{d-4}\alpha^{d-2} f_{\mu\nu}\,, \quad M^G_{\mu\nu} = G_{\mu\rho}\big(\sqrt{g^{-1}f}\big)^{\rho}{}_{\nu} - cG_{\mu\nu}$$

$G_{\mu\nu}$ has no ghost-free matter coupling!

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Hence:

- Gravity: $g_{\mu\nu}$
- Massive spin-2 field: $M_{\mu\nu} = g_{\mu\rho}\big(\sqrt{g^{-1}f}\big)^\rho{}_\nu - cg_{\mu\nu}$
- $m_g >> m_f$: $g_{\mu\nu}$ mostly massless (opposite to massive gr.)

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Hence:

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A3: $M_{\mu\nu} = 0 \Rightarrow$ GR.

$M_{\mu\nu} \neq 0 \Rightarrow$ deviations from GR, driven by matter couplings

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Partial masslessness in FP theory

$$\bar{\mathcal{E}}^{\rho\sigma}_{\mu\nu}\, h_{\rho\sigma} - \Lambda\left(h_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}h^{\rho}_{\rho}\right) + \frac{m^2_{\rm FP}}{2}\left(h_{\mu\nu} - \bar{g}_{\mu\nu}h^{\rho}_{\rho}\right) = 0$$

dS/Einstein backgrounds:

$$\bar{g}_{\mu\nu}: \qquad R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0$$

Higuchi Bound:

$$m^2_{FP} = \tfrac{2}{3}\Lambda$$

New gauge symmetry:

$$\Delta h_{\mu\nu} = (\nabla_\mu\nabla_\nu + \tfrac{\Lambda}{3})\xi(x)$$

Gives 5-1=4 propagating modes

[Deser, Waldron, $\cdots$ (1983-2012)]

Can a nonlinear extension of PM theory exist?

Partial masslessness beyond FP theory

Non-linear PM theory = Nonlinear spin-2 fields with a gauge invariance!

Does it exist? Independent of dS/Einstein backgrounds?

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Known perturbative results around dS:

- Cubic PM vertices ($\sim h^3$) in $d = 4$ *[Zinoviev (2006)]*
- Cubic PM vertices exist only in $d = 3, 4$ with 2 derivatives
  For $d > 4$, higher derivative terms needed.
  [Joung, Lopez, Taronna (2012)]

We will identify a specific bimetric theory as the candidate nonlinear PM theory

Partial masslessness in Bimetric theory

[SFH, Schmidt-May, von Strauss, 1208:1797, 1212:4525]

1) Assume a nonlinear bimetric theory with PM symmetry exists

2) Around $\bar{f} = c^2 \bar{g}$, $\delta M_{\mu\nu}$ satisfies the FP equation. Then the action of PM symmetry must be:

$$\delta M_{\mu\nu} \to \delta M_{\mu\nu} + \left(\nabla_\mu \nabla_\nu + \tfrac{\Lambda}{3}\,\bar{g}_{\mu\nu}\right)\xi(x)\,, \qquad \delta G_{\mu\nu} \to \delta G_{\mu\nu}$$

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- Find the transformation of $\delta g_{\mu\nu}$ & $\delta f_{\mu\nu}$.
- Shift the transf. to dynamical backgrounds $\bar{g}_{\mu\nu}$ & $\bar{f}_{\mu\nu}$
- For the dS-preserving subset $\xi = \xi_0$ (const), this gives,

$$\bar{g}'_{\mu\nu} = (1 + a\xi_0\,)\bar{g}_{\mu\nu}\,, \quad \bar{f}'_{\mu\nu} = (1 + b\xi_0\,)\bar{f}_{\mu\nu}$$

$$\bar{f}' = c'^2(\xi_0)\, \bar{g}' \qquad c' \neq c$$

A symmetry can exist only if $\Lambda_g = \Lambda_f$ does not determine c

Candidate PM bimetric theory in d=4

The necessary condition for the existence of PM symmetry is that c is not determined by $\Lambda_g = \Lambda_f$, or

$$\beta_1 + \left(3\beta_2 - \alpha^2\beta_0\right) c + \left(3\beta_3 - 3\alpha^2\beta_1\right) c^2 + \left(\beta_4 - 3\alpha^2\beta_2\right) c^3 + \alpha^2\beta_3 c^4 = 0$$

This gives the candidate nonlinear PM theory (d=4)

$$\alpha^2\beta_0 = 3\beta_2\,, \qquad 3\alpha^2\beta_2 = \beta_4\,, \qquad \beta_1 = \beta_3 = 0$$

Nonlinear PM bimetric theory

Checks:

- $$m_{\mathrm{FP}}^2 = 2\frac{m^4}{m_g^2}\left(\alpha^{-2} + c^2\right)\beta_2 = \frac{2}{3}\Lambda_g$$
- For $d > 4$, all $\beta_n = 0$. Nonlinear PM bimetric exists only for $d = 3, 4$.
- In $d > 4$ PM is restored by Lanczos-Lovelock terms
- Realization of the ξ_0 gauge transformation in the nonlinear theory on dS.

Full Gauge symmetry of the nonlinear theory not yet known, but expect 6=7-1 propagating modes

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Higher derivative gravity and Conformal gravity

HD gravity:

$$S^{\mathrm{HD}}_{(2)}[g] = m_g^2 \int d^4x \sqrt{g} \left[\Lambda + c_R R(g) - \frac{c_{RR}}{m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right) \right]$$

7 modes: massless spin-2 + massive spin-2 (ghost)

[Stelle (1977)]

d=3: New Massive Gravity (NMG)

[Bergshoeff, Holm, Townsend (2009)]

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Conformal Gravity:

$$S^{\rm CG}[g] = -c \int d^4x \sqrt{g} \left[R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right] ,$$

Weyl Invariance $\Rightarrow$ 6 modes: 2 (massless spin-2) + 4 ghosts

[Riegert (1984), Maldacena (2011)]

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Curvature expansion of bimetric equations

Define

$$S = \sqrt{g^{-1} f}\,, \qquad P_{\mu\nu} = R_{\mu\nu} - \frac{1}{2(d-1)} g_{\mu\nu} R$$

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$$S = \sqrt{g^{-1} f}\,, \qquad P_{\mu\nu} = R_{\mu\nu} - \frac{1}{2(d-1)} g_{\mu\nu} R$$

Solve the bimetric $g_{\mu\nu}$ equation algebraically for $f_{\mu\nu}$, as an expansion in $R_{\mu\nu}(g)/m^2$,

$$S^\mu_{\ \nu} = a\delta^\mu_\nu + \frac{a_1}{m^2} P^\mu_{\ \nu} + \frac{a_2}{m^4}\Big[\Big({P^\mu_{\ \nu}}^2 - PP^\mu_{\ \nu}\Big) + \frac{1}{d-1} e_2(P)\delta^\mu_\nu\Big] + \mathcal{O}(m^{-6})$$

Compute,

$$f_{\mu\nu} = a^2 g_{\mu\nu} + \frac{2aa_1}{m^2} P^\mu_{\ \nu} + \mathcal{O}(m^{-4})$$

Equivalence between CG and PM bimetric theory

[SFH, Schmidt-May, von Strauss, 1303:6940]

- CG equation of motion: The Bach equation (4-derivative),

$$B_{\mu\nu} = 0$$

Propagates 6 modes due to Weyl invariance.

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- In PM bimetric theory, solve the g-equation for $f_{\mu\nu}$. Substitute back in f-equation to get,

  $$B_{\mu\nu} + \mathcal{O}(R^3/m^2) = 0$$

  In the low curvature limit, PM bimetric theory has a gauge symmetry even away from dS and definitely propagates $7 - 1 = 6$ modes! None is a ghost

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- CG eom is the low curvature limit of PM bimetric eom. Conversely, PM bimetric is a ghost-free completion of CG

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Higher derivative gravity from Bimetric theory

- Solve the g-equation for $f = f(g)$. Then,

$$S^{\mathrm{BM}}[g, f(g)] = S^{\mathrm{HD}}[g]$$

- 4-derivative ($\sim R^2$) truncation:

$$S^{\mathrm{BM}}_{(2)}[g, f(g)] = S^{\mathrm{HD}}_{(2)}[g]$$

  The spin-2 ghost in 4-derivative HD gravity is an artifact of this truncation (can be illustrated in a linear theory).

- The correspondence is not an equivalence of the truncated theories (in general). Different truncated EoM's.
- PM bimetric theory again leads to conformal gravity.
- d=3 reproduces NMG.

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- Recurring doubts about 7=5+2 modes in bimetric theory. But in all cases finally confirmed.
- Superluminality (at least two light cones), but no superluminality in the matter sector.
- Accausality ?: Is the Cauchy problem well posed? Yes. No generic shock waves.

  [Izumi, Ong (1304.0211)]
- Stability of classical solutions: Schwarzschild with $f = c^2 g$ has a Gregory-Laflam type instability which goes away in the PM case. Not relevant for astrophysical blackholes

  [Babichev, Fabbri (1304.5992), Brito, Cardoso, Pani (1304.6725)]
- Proof of PM gauge symmetry/6 modes in the candidate PM bimetric theory ?