Action principles and amplitudes in higher spin gravity

P e r S u n d e l l

in collaborations with

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Outline

- Motivate generalized Hamiltonian action/Poisson sigma model
- Classical geometries, observables and moduli spaces of Vasiliev's 4D HSGRA
- Path integral formulation á la AKSZ
- Quantum amplitudes in AdS4 $\leftrightarrow$ 3D CFTs $\leftrightarrow$ 2D TOSs
- Proposal: Quantum gauge principle

Metric-like formulation

The in some sense least structured approach to higher-spin gravity is to follow metric-like off-shell formulations.

In the case of only symmetric tensors, one has

$$S_\Lambda^{\rm Metric} = \sum_s S_\Lambda^{\rm Fronsdal}[\phi_s] + H\partial/\sqrt{\Lambda}\text{-interactions}$$

- minimal cubic Lorentz couplings require $\Lambda \neq 0$
- cubic vertices classified (but coupling constants free at cubic level)
- quartic vertices understood far less well
- canonical quantization only after redefinitions using $(\partial_t)^2\phi_s \approx (\partial_{\vec{x}})^2\phi_s + \cdots$
- strongly coupled "Born-Infeld tails" require regularizations/re-summations

However, the cubic couplings reveal underlying nonabelian higher spin symmetry algebras

- highly restrictive already at the kinematic level for purely bosonic models
- paves the way to many extended algebras and representations (including fermions and fractional spins)

It seems natural to follow the Noether path into the terra incognita beyond cubics

Frame-like formulation

The Noether approach leads to frame-like off-shell formulations of 4D HSGRA

$$S^{\rm Frame}_{\rm 4D} = S^{\rm Fradkin-Vasiliev}_{\rm 4D,\Lambda}[W] + S^{\rm Extra\ constraints}[V,W] + S^{\rm Matter}_{\rm 4D,\Lambda} + \text{completion}$$

- HSA valued W, $R := dW + W \wedge W$ and Lagrange multiplier V
- Decomposition with "extra" auxiliary fields and curvatures for spin $s \geqslant \frac{5}{2}$

$$W = e + \omega + W^{\rm Extra}\ , \quad R = T + R^{\rm Fronsdal} + R^{\rm Extra} + R^{\rm Weyl}$$

- 2∂ free theory relies on using F-V bilinear form

$$S^{\rm F-V}_{\rm 4D,\Lambda} = \int_{M_4} (R,R)_{\rm F-V} = S^{\rm Lopatin-Vasiliev}_{\Lambda}[e,\omega] + S^{\rm Cubic}[W] + S^{\rm Quartic}[W]$$

- Thus, for $s \geqslant \frac{5}{2}$ one has $W^{\rm Extra} \approx W^{\rm Extra}[e,\omega]$ via

$$S^{\rm Extra\ constraints} = \int_{M_4} V^{\rm Extra} \cdot (R^{\rm Extra})^{\rm lin} + \cdots$$

- Barring Born–Infeld couplings, cubic action completely fixed in gauge sector

Beyond cubics: Vasiliev's on-shell formulation

The inclusion of matter fields and Born–Infeld couplings on-shell is a standard albeit formidable problem; drawing on SUGRA one may expect its solution to involve "hidden" algebraic structures beyond the space-time symmetry/higher spin algebra.

Indeed, Vasiliev's equations of motion for symmetric tensor gauge fields introduce quasi-free associative differential algebras containing Wigner-deformed oscillators.

In 2,3,4D, generalized twistor transforms yield even simpler duality extended systems

$$\widehat{d}\widehat{A}+\widehat{A}\star\widehat{A}+\mathcal{F}(\widehat{B},\widehat{J}^i)\approx 0\ ,\quad \widehat{d}\widehat{B}+\widehat{A}\star\widehat{B}-\widehat{B}\star\widehat{A}\approx 0\ ,\quad \widehat{J}^i\ \text{central}$$

- $(\widehat{A},\widehat{B};\widehat{J}^i)$ horizontal forms of degrees $(1 \bmod 2, 0 \bmod 2; 2)$ on noncommutative total space with base coordinatized by $(X^M, P_M; z^\alpha, \bar{z}^{\dot{\alpha}})$ and fiber by $(y^\alpha, \bar{y}^{\dot{\alpha}}; k, \bar{k})$

$$\widehat{J}^i=\{dX^M dP_M; dz^2, dz^2 k\widehat{\kappa};\ \text{h.c.}\}\ ,\quad k/\widehat{\kappa}\equiv\kappa_y\star\kappa_z\ \text{holom. outer/inner Kleinians}$$

- Bosonic $\frac{1}{2}(1+k\bar{k})$-projection, reality conditions and optional discrete projections
- $\mathcal{F}$ contains free parameters except in the Lorentz-tensorial and parity-invariant Type A and Type B models
- maximally symmetric vacua, unitarizable fluctuation spectra and several branches of exact solutions: instantons, bi-axially symmetric solutions (harmonics, solitons, ...)

For suitable boundary conditions and drawing essentially only on symmetry properties, it has been argued that the quantum effective actions/holographic amplitudes are tree-level exact, corresponding to free conformal theories.

No miracle! ... but a Pandora's box!?

- We shall argue that this somewhat unexpected simplicity is an almost trivial consequence of the fact that Vasiliev's HSGRAs are formulated in terms of differential algebras (rather than generalized metrics).
- Differential algebras can be quantized á la AKSZ as topological field theories within a quite general framework based on generalized Hamiltonian actions *a.k.a.* generalized Poisson sigma models.
- As we shall see, in the context of Vasiliev's 4D HSGRA, the the holographic amplitudes arise within the PSMs from a "ring of topological boundary operators" (rather than from the "bulk Hamiltonian") that are protected from perturbative quantum corrections by a simple mechanism based on conservation of form degree.
- On the other hand, in the context of Prokuskin–Vasiliev's 3D HSGRAs and their CS–like truncations, these operators are not protected, and from what is known in the literature and as far as we have seen, the PSMs reproduce perturbative CS theory (level shift etc).
- But, holographic amplitudes only amount for tiny sub-sectors of PSMs.
- Thus, if one accepts this framework for HSGRAs — and possibly QFTs arising in limits as well as extended "stringy" theories containing HSGRAs — then one stands in front of a vast "PSM landscape".
- We are exploring a "quantum gauge principle" requiring Nature to be described by layers of "n^{th}-quantized" PSMs

Lagrange multipliers and PSM

To create a more direct link between PSMs and HSGRA in the frame-like formulation, let us return to the action principle where we had left it:

- The completion of $S^{\text{Extra constraints}} = \int_{M_4} V^{\text{Extra}} \cdot (R^{\text{Extra}})^{\text{lin}} + \text{h.o.t.}$ poses a peculiar problem as nonabelian HS symmetries transform $(R^{\text{Extra}})^{\text{lin}} \approx 0$ into $\delta S_\Lambda^{\text{L}-\text{V}} \approx 0$, that is, the completion of $S^{\text{Extra constraints}}$ risks putting the theory on-shell on its own (less there exists a second remarkable bilinear form).
- On the other hand, Vasiliev's equations admits perturbative reductions to curvature constraints on commutative base manifolds M of *a priori* unspecified dimensions

  $$(\widehat{A}_{[1]}, \widehat{B}_{[0]}; \widehat{J}^i)|_M = (W+\omega, \Phi \star \tfrac{1}{2}(k+\bar{k}); 0) \ \rightsquigarrow \ dW + Q^W \approx 0 \ , \quad d\Phi + Q^\Phi \approx 0$$

  where Φ is the Weyl zero-form and ω is the canonical Lorentz connection.
- Moreover, in the case of 4D HSGRA the simplest holographic dualities only require a classical "on-shell action" Γ; in fact, we even want all vacuum bubbles to cancel.

Thus, we may lift $S^{\text{Extra constraints}}$ to a "bulk action" on a manifold M_5 with $\partial M_5 = M_4$ and seek a "boundary term" on M_4 producing the amplitudes:

$$S_{\text{tot}} = S_{\text{bulk}} + S_{\text{bndy}} \ , \quad S_{\text{bulk}} = \int_{M_5} \left(V \cdot (dW + Q^W) + U \cdot (d\Phi + Q^\Phi) \right) \ ,$$

$$S_{\text{bndy}} = \int_{M_4} L(W, \Phi, dW, d\Phi) \ , \quad \delta \int_{M_4} L \approx 0 \ , \quad \int_{M_4} L \approx \Gamma \ .$$

Generalized Hamiltonian actions on commutative manifolds

$$S^{\rm PSM}_{\rm tot} = \int_{M_{\hat p+1}} (P_\alpha dX^\alpha + H(X,P)) + \sum_r t_r \oint_{\Sigma_r} L_r(X,dX) \ , \quad \Sigma_r \subseteq \partial M_{\hat p+1}$$

- Variational principle compatible with differentiability of action functional off-shell:

$$P_\alpha|_{\partial M_{\hat p+1}} \equiv 0 \ , \quad H|_{P_\alpha=0} \equiv 0 \quad i.e. \quad H = P_\alpha Q^\alpha(X) + \sum_{n\geqslant 2} P_{\alpha_1}\cdots P_{\alpha_n} \Pi^{\alpha_1\dots\alpha_n}_{(n)}(X)$$

- Topological vertex operators: $\delta L_r \equiv (dX^\alpha + Q^\alpha)(L_r)_{\alpha\beta}\delta X^\beta + d(\cdots)$.
- Gauge symmetries $\Leftrightarrow$ integrable structure in target space:

$$\delta_\epsilon S = \text{tot.der.} \quad \text{under} \quad \delta_\epsilon(X^\alpha, P_\alpha) = d(\epsilon^\alpha, \epsilon_\alpha) + \epsilon\cdot\partial(\partial^\alpha,\partial_\alpha)H$$

$$\Leftrightarrow$$

$$\partial_\alpha H \partial^\alpha H \equiv 0 \quad \Leftrightarrow \quad \mathcal{L}_Q Q \equiv 2Q^2 \equiv 0 \ , \quad \mathcal{L}_Q \Pi_{(n)} + \sum_{n_1+n_2=n-1} \{\Pi_{(n_1)}, \Pi_{(n_2)}\}_{\rm S.B.} \equiv 0$$

- Globally-defined actions from bundle structures based on generalized connections

$$\Gamma^\alpha := \Pi^\alpha_\beta X^\beta \ , \quad \Pi^{\alpha'}_\alpha \partial_{\alpha'}\partial_\beta\partial_\gamma\partial^\delta H \equiv 0 \ , \quad \delta_{\Pi\epsilon}(P_\alpha dX^\alpha + H) \equiv 0 \equiv \delta_{\Pi\epsilon} L_r$$

where Π is nontrivial only in positive form degree and $(1-\Pi)^\alpha_\beta X^\beta$ and P_α form sections.

BV treatment á la AKSZ and PSM landscape

$(X^\alpha, P_\alpha) \to (\mathbb{X}^\alpha, \mathbb{P}_\alpha)$ of fixed total degree = ghost number + form degree

$\rightsquigarrow$ $S^{\rm PSM}_{\rm tot}[\mathbb{X}, \mathbb{P}]$ solves classical and quantum BV equations ($\rightsquigarrow$ existence of BRST current)

Different choices of "trivial pairs"/gauge fixing fermions $\rightsquigarrow$ inequivalent path integrals

- "Topological ground state": $M_{\hat{p}+1} = \mathbb{R}^{\hat{p}+1}$ $\rightsquigarrow$ no radiative corrections
- Single trivial boundary $\rightsquigarrow$ perturbatively defined homotopy $A_\infty(\hat{p})$ algebras
- Multiple trivial boundaries $\rightsquigarrow$ "enveloping/tensoring" of $A_\infty(\hat{p})$ algebras
- Identifications $\rightsquigarrow$ "$A_\infty(\hat{p})$ topologies"
- $\cdots$

The literature contains a number of interesting bulk/boundary examples:

- two-dimensional Type-A, Type-B and Type-C models/QM systems
- three-dimensional $BF + B^3$ models/two-dimensional CFTs
- four-dimensional $BF + B^2$ models/3D CS theories
- higher-dimensional BF models

4D HSGRA amplitudes dual to free CFTs arise within a simple class of TVOs:
L_r depend only on X^αs with degrees $< \frac{1}{2}(\hat{p} - 1)$ inserted on a single boundary $\rightsquigarrow$
perturbatively trivial correlation functions

AKSZ model for duality extended 4D Vasiliev system with zero-form charges

$$S^{\rm AKSZ}_{\rm bulk,\,\hat p=8} = \int_{M_5\times\mathcal{Z}\times\mathcal{Y}} (\mathcal{O}_{\mathcal{Y}})^2 \star \frac{1}{2}(1+k\bar k)\star\left(\widehat{\mathbb{U}}\star\widehat{\mathbb{D}}\widehat{\mathbb{B}} + \widehat{\mathbb{V}}\star(\widehat{\mathbb{F}} + \mathcal{F}(\widehat{\mathbb{B}},\widehat{J}^i) + \widetilde{\mathcal{F}}(\widehat{\mathbb{U}},\widehat{J}^i))\right)\Big|^{gh=0}_{k=\bar k=1}$$

- Integrability/gauge invariance $\Rightarrow$ either $\mathcal{F}$ or $\tilde{\mathcal{F}}$ must be linear
- More general models with $\hat p = 5$ describe 3D HSGRAs with CS sectors.
- Three classes of topological vertex operators:
  - On-shell closed abelian forms on M of degrees $2p$ built from a soldering one-forms $\widehat{E}_{[1]}$ in special geometries obtained by embedding the connection into $\frac{1}{2}(1+ad_k)W$.
  - Decorated Wilson loops in M_4 which reduce for trivial loops to zero-form charges

$$I_n = \int_{\Sigma\times\mathcal{Z}\times\mathcal{Y}} (\mathcal{O}_{\mathcal{Y}})^2 \star (\mathcal{O}_{\mathcal{Z}})^2 \star (\widehat{\Phi}\star\widehat{\kappa})^{\star n}\ , \quad n=1,2,\dots\ ,$$

$$I'_n = \int_{\Sigma\times\mathcal{Z}\times\mathcal{Y}} (\mathcal{O}_{\mathcal{Y}})^2 \star (\mathcal{O}_{\mathcal{Z}})^2 \star (\widehat{\Phi}\star\widehat{\kappa})^{\star n}\star\widehat{\kappa}\star\widehat{\bar\kappa}\ , \quad n=2,4,\dots\ .$$

  - Decorated Wilson loops in M_4 with insertions of straight open Wilson lines in $\mathcal{Z}_4$

$$OWL(\mu,\bar\mu) = exp_\star(i\mu^\alpha(z_\alpha - 2i\widehat{A}_\alpha) + i\bar\mu^{\dot\alpha}(\bar z_{\dot\alpha} - 2i\widehat{\bar A}_{\dot\alpha}))\ ;$$

  in classical perturbation theory
    - ★ $I^{\rm OWL}_n$, n even: both holomorphic and anti-holomorphic OWL-insertions survive
    - ★ $I^{\rm OWL}_n$, n odd: only anti-holomorphic OWL-insertions survive
    - ★ $I'^{\rm OWL}_n$, all n: all OWL-insertions drop out, *i.e.* $I'^{\rm OWL}_n = I'_n$

Perturbative evaluation of zero-form charges

- Formal twistor-space perturbation theory (not keeping track of boundary conditions) indicate that (I_n, I'_n) are not corrected beyond leading order.
- An analysis using twistor-space plane waves and OWL regularization, reveals a more subtle structure:
  - I'_n: no corrections found so far
  - I_n^{OWL}: contain sub-leading corrections
- Proposal: OWL-regularization yields amplitudes that correspond to a deformed CFT whose correlation functions coincide with the free theory for separate points and contain nontrivial contact terms for coinciding points.
- Question: Can the contact terms be removed in a limit?
- Proposal: First formulate the Vasiliev system on a $\mathcal{Z}$-space with finite volume; then send the volume to infinite and the couplings $t_n \to 0$ keeping amplitudes fixed; the resulting limit corresponds to the free theory.
- At the level of the finite-volume analogs of I_n and I'_n, the double-scaling limit is equivalent to deleting the infinite volumes factors in I_n provided that (there exists a gauge in which) the infinite-volume Vasiliev system admits exact solutions for initial data given by boundary-to-bulk propagators in which $\widehat{\Phi}$ is uncorrected.

Particle/soliton scattering and duality

- In fact, there exists a much larger class of such solutions

$$\widehat{\Phi} = \sum_{\sigma=\pm} \tfrac{1}{2}(1 + \sigma\kappa_y \star \bar{\kappa}_{\bar{y}}) \star \Phi_\sigma \ ,$$

$$\widehat{A}_\alpha = \sum_{\sigma=\pm} \sum_{n\geqslant 1} (\Phi_\sigma \star \kappa_y)^n \star \int_{-1}^{1} dt\, f_n(t) \star \left[z_\alpha e^{\frac{t-1}{t+1} z^+ z^-} \right]_{\text{Weyl}} \ ,$$

  where $f_n(t)$ are the same functions as those used in describing exact soliton solutions.
- The resulting regularized expressions for exact zero-form charges read

$$I_n|_{\text{reg}} = \mathrm{Tr}_y \mathrm{Tr}_{\bar{y}} (\Phi \star \kappa_y)^{\star n} = \begin{cases} \mathrm{Tr}_y \mathrm{Tr}_{\bar{y}} ((\Phi \star ad_k(\Phi))^{\star(n/2)}) & n \text{ even} \\ \mathrm{STr}_y \mathrm{Tr}_{\bar{y}} ((\Phi \star ad_k(\Phi))^{\star(n-1)/2} \star \Phi) & n \text{ odd} \end{cases}$$

$$I'_n = \mathrm{Tr}_y \mathrm{Tr}_{\bar{y}} ((\Phi \star \kappa_y)^{\star n} \star \kappa_y \star \bar{\kappa}_{\bar{y}}) = \mathrm{STr}_y \mathrm{STr}_{\bar{y}} (\Phi \star ad_k(\Phi))^{\star(n/2)}$$

- Φ_σ is linear combination of solitons *and* states in LW spaces (in $o(3)$ or $so(2,1)$ covariant bases) $\rightsquigarrow$ “scattering” processes involving *both* particles and solitons.
- In fact, particle and soliton sectors are related by simple duality relation

$$\Phi_{\sigma;\text{particle}} = \Phi_{\sigma;\text{sol}} \star \kappa_y$$

- Also, a direct duality between $\Phi_\pm$ states, *i.e.* squares of scalar/spinor singletons, in the Type A model and $\Phi_\mp$ states in the Type B model.

Conclusions and quantum gauge principle

- There exists an off-shell formulation of 4D HSGRA based on generalized Hamiltonian actions *a.k.a.* Poisson sigma models
- This formulation facilitates the computation of holographic observables using twistor-space methods
- For particular deformations given by topological vertex operators that are functionals of only zero-forms and one-forms, the holographic observables receive no radiative corrections
- Moreover, by taking limits of a finite-volume formulation, or equivalently, by making use of protected exact solutions to the infinite-volume formulation, one finds amplitudes in precise agreement with the free CFT
- The "PSM landscape" is much larger, however, with various types of dualities:
  - particles and solitons in the second-quantized theory
  - some form of underlying first-quantized topological open string
  - some form of overlying third-quantized Poisson sigma model
- To "tame the zoo" we are exploring a "quantum gauge principle" meant to "exclude" any model that does not fall into a larger structure consisting of a sequence of "n^{th}-quantized" PSMs such that

$$\text{Topological summation at level } n \leftrightarrow \text{Radiative corrections at level } n+1$$

$$\text{Cluster decomposition at level } n \leftrightarrow \text{Composite formation at level } n-1$$