Higher-spin algebras and AdS/CFT computations in Vasiliev theory

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(based on the works with V.Didenko, M.Vasiliev, N.Boulanger, D.Ponomarev and Jianwei Mei)

AEI and Lebedev Institute
the work is supported by Alexander von Humboldt Foundation

GGI, 09 May 2013

Outline

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Main features

- "Integrable" Vasiliev theory of infinite-dimensional multiplet of massless/gauge fields of all spins
- Klebanov-Polyakov, Sezgin-Sundell: one and the same Vasiliev theory is dual to the simplest CFT ever and to a phenomenologically interesting interacting model - free/critial vector model
- $\partial^\nu j_{\nu\mu_2,\ldots\mu_s} = 0$ HS 'currents'
- The duality does not rely on supersymmetry
- The spectrum of fields/operators is simpler...

Main features

- HS algebra is infinite-dimensional extension of AdS/conformal algebra $\sim$ Virasoro
- By contrast to 2d-Virasoro HS algebra is rigid
- Rigidity results in CFT being free if the HS symmetry is exact in $d > 2$ (Maldacena-Zhiboedov, 3d)
- If the symmetry is broken in a smart way (deformed) then the CFT is not free — critical vector model (Maldacena-Zhiboedov)

HS symmetry appetizer (Giombi,...; Stanev; Zhiboedov...)

Decoupling of $L_{-2} + \alpha L_{-1}^2$ imposed on $\langle O_\Delta O_{\Delta_1} O_{\Delta_2} \rangle$ relates Δ, Δ_1 and Δ_2

With HS symmetry $L_{-2} + \alpha L_{-1}^2$ gets replaced by ∂^ν

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$\langle j_s j_{s'} O_\Delta \rangle$ $\qquad$ $\Delta = d + 0 - 2 = 2\frac{d-2}{2}$

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$\langle j_s O_{\Delta_1} O_{\Delta_2} \rangle$	$\Delta_1 = \Delta_2$
$\langle j_s j_{s'} O_\Delta \rangle$	$\Delta = d + 0 - 2 = 2\frac{d-2}{2}$
$\langle j_s j_{s'} O_{s'',\Delta} \rangle$	$\Delta = d + s'' - 2?$ conserved current?

More nontrivial info is in Ward identities (Maldacena-Zhiboedov)

One cannot mix massless HS theories with massive.

Either unbroken or completely broken HS symmetry.

Free vector model

$$S = \int d^3x \partial_\mu \phi \partial^\mu \phi$$

$$\phi \otimes \phi = \mathbf{1} + j_0 + j_2 + j_4 + \dots$$

$$j_0 = \phi^2(x)$$

$$j_{\mu_1 \dots \mu_s} = \phi(x)(\overleftarrow{\partial_\mu} - \overrightarrow{\partial_\mu})^s \phi(x) + \dots$$

$$\partial^\nu j_{\nu\mu(s-1)} = 0$$

Free vector model

$$S = \int d^3x \partial_\mu \phi \partial^\mu \phi$$

$$\phi \otimes \phi = \mathbf{1} + \mathbb{J}$$

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Free vector model

$$S = \int d^3x \partial_\mu \phi \partial^\mu \phi$$

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$j_{\mu_1 \dots \mu_s}$ give HS charges

$j^s_\mu = j_\mu{}^{\nu(s-1)} K_{\nu(s-1)}(x)$

K is a Conformal Killing tensor

Free vector model

$$S = \int d^3x \partial_\mu \phi \partial^\mu \phi$$

$\phi \otimes \phi = \mathbf{1} + \mathbb{J}$

$j_{\mu_1 \dots \mu_s}$ give HS charges

$j^s_\mu = j_\mu{}^{\nu(s-1)} K_{\nu(s-1)}(x)^{A(s-1),B(s-1)}$

	$s-1$	
	$s-1$	

(Eastwood)

Free vector model

$$S = \int d^3x \partial_\mu \phi \partial^\mu \phi$$

$$\phi \otimes \phi = \mathbf{1} + \mathbb{J}$$

$j_{\mu_1 \dots \mu_s}$ give HS charges

$$\star j^s_\mu = \Omega^{A(s-1),B(s-1)}$$

	$s-1$	
	$s-1$	

Free vector model

$$S = \int d^3x \partial_\mu \phi \partial^\mu \phi$$

$\phi \otimes \phi = \mathbf{1} + \mathbb{J}$

$j_{\mu_1 \dots \mu_s}$ give HS charges

$\Delta S = \int \phi^{\mu(s)} j_{\mu(s)}$

$\delta \phi^{\mu(s)} = \partial^\mu \xi^{\mu(s-1)}$ − traces

coupling to Fradkin-Tseytlin fields

Free vector model

$$S = \int d^3x \partial_\mu \phi \partial^\mu \phi$$

$\phi \otimes \phi = \mathbf{1} + \mathbb{J}$

$j_{\mu_1 \dots \mu_s}$ give HS charges

$\Delta S = \int \Omega^{A(s-1),B(s-1)} \wedge \bar{W}_{A(s-1),B(s-1)}$

$\delta \bar{W}_{A(s-1),B(s-1)} = D\xi^{A(s-1),B(s-1)}$

conformal HS fields; Vasiliev

We can guess a dual theory to be the theory of $W_{A(s-1),B(s-1)}$

$W^{A,B} \sim \{e^a, \omega^{a,b}\}$

HS algebras

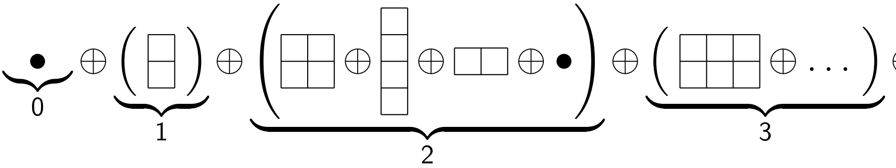
$$[T_{AB}, T_{CD}]_\star = T_{AD}\eta_{BC} + 3\text{more}, \qquad T_{AB} = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$$

- Vasiliev-Eastwood algebra is a quotient of the universal enveloping algebra by a two-sided ideal

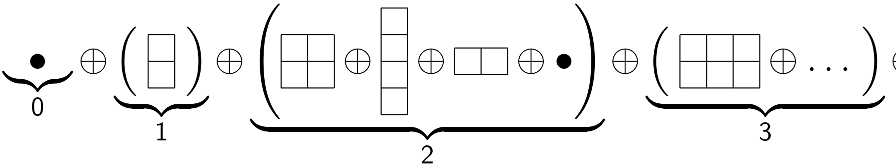
$$W(T) = \sum_s W^{A(k),B(k)} T_{AB(k)} \qquad \begin{array}{|c|c|c|}\hline & s-1 & \\ \hline & & \\ \hline \end{array}$$

- $hs(\nu)$ of $3d$ HS theory too, $U(sl_2)/(C_2-\nu)$
- Originally it came as associative algebras (exact=free=linear equation=associative)
- How rich is the $U(T_{AB})$?

HS algebras: universal enveloping algebra U of T_{AB}



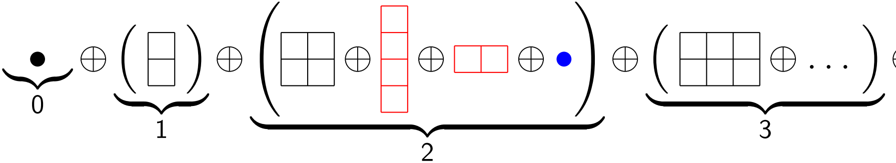
HS algebras: universal enveloping algebra U of T_{AB}



- $\bullet$ is the unit of U
-  represents T^{AB}
- $\bullet\ C_2 = -\frac{1}{2} T_{AB} \star T^{AB}$
-  $= T_{[AB} \star T_{CD]}$
-  $= T_A{}^C \star T_{AC} - \frac{2}{(d+1)} \eta_{AA} C_2$

$W^{A,B} \sim \{e^a, \omega^{a,b}\}$

HS algebras: universal enveloping algebra U of T_{AB}



It is necessary to quotient red guys out, defining a two-sided ideal I, (Eastwood; Vasiliev; Boulanger, E.S.; Iazeolla,Sundell)

HS algebras: universal enveloping algebra U of T_{AB}

$$\underbrace{\bullet}_{0} \oplus \underbrace{\left(\ydiagram{1,1}\right)}_{1} \oplus \underbrace{\left(\ydiagram{2,2} \oplus \quad \oplus \quad \oplus \right)}_{2} \oplus \underbrace{\left(\ydiagram{3,3} \oplus \dots \right)}_{3} \oplus$$

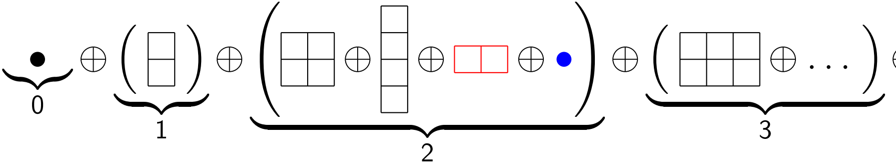
It is necessary to quotient red guys out, defining a two-sided ideal I, (Eastwood; Vasiliev; Boulanger, E.S.; Iazeolla,Sundell)

$$I \cong U \star \left(\ydiagram{1,1,1,1} \oplus \ydiagram{2}\right) \star U \quad .$$

$$hs = U/I$$

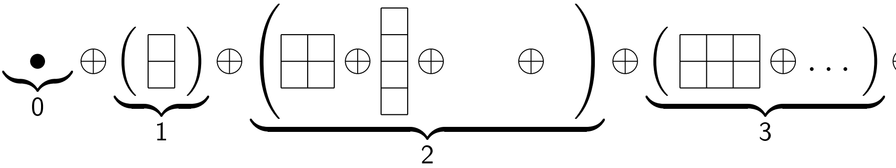
$$C_2 = C_2(\phi)$$

HS algebras: universal enveloping algebra U of T_{AB}



In general one has to get rid of $\square\square$ (Vasiliev,E.S.) as the corresponding gauge field W^{AA} describes partially-massless spin-two field. This fixes all higher Casimirs as certain functions of C_2.

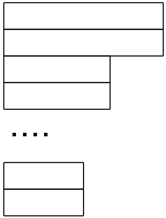
HS algebras: universal enveloping algebra U of T_{AB}



The most general HS algebra can be defined as (Boulanger, E.S.)

$$hs_d(\nu) = U/\left(U \star \{\square\!\square \oplus (C_2 - \nu)\} \star U\right)$$

$$hs(\nu) = \bigoplus$$



List of HS algebras

Large stock of HS algebras $hs_d(\nu)$

$hs_3(\nu)$	exactly $hs(\nu)$ of 3d HS theory
$hs_d(-\frac{(d-1)(d-3)}{4})$	Vasiliev algebra for AdS_{d+1}
$hs_5(\nu)$	discussed by Günaydin
$hs_5(9)$	f.dim (Manvelyan, Mkrtchyan^2, Theisen)
$*$	algebras for p.-m. fields
$*$	algebra of generalized free fields

There are finite-dim. truncations like in $3d$
All algebras are consistent up to the cubic level
Mixed-symmetry fields in general

(Boulanger, E.S., 2011; Boulanger, E.S., Ponomarev, 2012)

Vas~~s~~iliev invariants

Given an adjoint field of HS algebra

$$\delta\Psi_i = [\Psi_i, \xi]$$

we can construct invariants

$$Inv(\Psi_1, ..., \Psi_n) = \sum_{S_n} Tr(\Psi_1 \star ... \star \Psi_n)$$

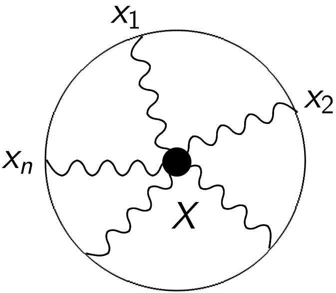
These are invariant under full HS$\supset$conformal symmetry. (Sundell et al)

Invariants can give correlation functions if Ψ is related to boundary-to-bulk propagator B, $\Psi = B \star \delta$ (Sundell, Colombo)

$$\langle j...j \rangle = \sum_{S_n} Tr(\Psi_1 \star ... \star \Psi_n)$$

B is the twisted-adjoint field in the bulk
B generating function of conserved currents on the boundary

Twistorial contact Witten diagram



$$L = \sum_N \int_{AdS} Tr(\Psi^N)$$

X is erased by trace.
Should work in 3d too!!

AdS/CFT dictionary

CFT	AdS
$\langle j \ldots j \rangle$	$Tr(\Psi \star \ldots \star \Psi)$
$[Q, j] = \sum \partial .. \partial j$	$\delta\Psi = [\Psi, \xi]$
$Q\langle j \ldots j \rangle = 0$	$\delta\ Tr(\Psi \star \ldots \star \Psi) \equiv 0$

HS propagators

- $dB + \Omega \star B - B \star \tilde{\Omega} = 0$
- B intertwines HS master-field in the bulk and generating function of conserved currents on the boundary
- B is a projector, $B \star B = B$ (V.Didenko, E.S.; P.Kraus, E.Permutter)
- B is an extremal projector, $e \star B = B \star f = 0$, (R.Gover, A.Waldron; V.Didenko, E.S.)

AdS_4 HS algebra. $sp(4) \sim so(3,2)$

The Vasiliev HS algebra is a Moyal $\star$-product algebra of functions of $sp(4)$ vectors Y_A, $A = 1..4$

$$(f \star g)(Y) = f(Y) \exp\left\{ i \overleftarrow{\partial_A} \epsilon^{AB} \overrightarrow{\partial_B} \right\} g(Y)$$

in particular

$$[Y_A, Y_B]_\star = 2i\epsilon_{AB}$$

and $sp(4)$ generators in HS algebra read

$$T_{AB} = -\frac{i}{4}\{Y_A, Y_B\}_\star$$

Gaussians in $\star$-algebra

$$\Phi(f^{AB}, \xi^A, q) = K \exp i \left(\tfrac{1}{2} Y_A f^{AB} Y_B + \xi^A Y_A + \theta \right)$$

AdS_4 propagators: (Giombi, Yin)

Gaussians in $\star$-algebra

$$\Phi(f^{AB}, \xi^A, q) = K \exp i\left(\tfrac{1}{2} Y_A f^{AB} Y_B + \xi^A Y_A + \theta\right)$$

- $K = \frac{z}{z^2 + (x - \mathrm{x})^2}$
- f^{AB} — wave-vector pointing to the boundary
- ξ^A — parallel-transported polarization
- θ — the phase in Vasiliev equations

AdS_4 propagators: (Giombi, Yin)

Gaussians in $\star$-algebra

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Under the large $so(3,2)$ transformations

$$g^{-1} \star Y_A \star g = \Lambda_A{}^B Y_B$$

$$f = \begin{pmatrix} 0 & F \\ F^T & 0 \end{pmatrix} \qquad \Lambda_M{}^N = \begin{pmatrix} A & -B \\ -C & D \end{pmatrix}$$

we have Möbius-like transformations

$$K' = \frac{K}{\det|A + FC|}\,,$$

$$F' = (A + FC)^{-1}(FD + B)\,,$$

$$\xi' = (A + FC)^{-1}\xi\,.$$

AdS_4 propagators: (Giombi, Yin)

Gaussians in $\star$-algebra

$$\Phi(f^{AB},\xi^A,q) = K \exp i\left(\tfrac{1}{2}Y_A f^{AB} Y_B + \xi^A Y_A + \theta\right)$$

In the string field-theory one is interested in computing $\Phi_1 \star \ldots \star \Phi_n$ (Bars et al)

(f,ξ,θ) is related to $SpH(2M) = Sp(2M) \ltimes H(2M)$ via the generalized Cayley transform

$$(U_1,x_1,c_1)\circ(U_2,x_2,c_2) =$$
$$(U_1U_2,x_1+U_1x_2,c_1+c_2+x_1U_1x_2)$$

(V.Didenko, M.Vasiliev; E.S.,V.Didenko)

Gaussians in $\star$-algebra

$$\Phi(f^{AB}, \xi^A, q) = K \exp i \left(\tfrac{1}{2} Y_A f^{AB} Y_B + \xi^A Y_A + \theta\right)$$

D-brane condition $f^2 = I$ holds for propagators.
Cayley breaks down!!!
Product on square roots of I

$$f_1 \circ f_2 = (f_1 + f_2)^{-1}(2I + f_2 - f_1)$$

Still a $\sqrt{I}$	$(f_1 \circ f_2)^2 = I$
Associativity	$f_1 \circ (f_2 \circ f_3) = (f_1 \circ f_2) \circ f_3$
Forgetful	$f_1 \circ f_2 \circ f_3 = f_1 \circ f_3$

Note on correlators (Rychkov et al, Yin et al)

$$\langle O(x_1,\eta_1)\ldots O(x_n,\eta_n)\rangle = \frac{1}{x_{12}\ldots x_{n1}} f(r_{ab}^{cd}|P,Q,S)$$

n-point correlator of tensor operators depends on conformally invariant ratios r_{ab}^{cd} and conformally invariant tensor structures: P, Q and S

- Q_{bc}^{a} depends on three points
- P_{ab} depends on two points
- S_{bc}^{a} depends on three points and is odd

Main formula: long-trace of projectors

$$Tr(\Phi_1 \star \ldots \star \Phi_n) = \prod_i \frac{1}{|f_i + f_{i+1}|^{1/4}} \exp i \sum_j (Q_j + P_j)$$

$$\begin{aligned}
Q_i &= \frac{1}{8}\xi_i \left(f_{i+1} \circ f_i + f_i \circ f_{i-1} \right) \xi_i \\
P_i &= \frac{1}{4}\xi_i \left(I + f_{i+1} \circ f_i \right) \xi_{i+1} \\
|f_i - f_{i+1}|^{1/2} &= |x_i - x_{i+1}| K_i K_{i+1}
\end{aligned}$$

Already decomposed into Q, P conformal invariants

3-point functions

Yin and Giombi, Sundell and Colombo

$$
\begin{aligned}
\langle j_s j_s j_s \rangle_b &= \frac{4}{\mathrm{x}_{12}\mathrm{x}_{23}\mathrm{x}_{31}} \cos(Q_1 + Q_2 + Q_3)\cos(P_3)\cos(P_1)\cos(P_2) \\
\langle j_s j_s j_s \rangle_f &= \frac{4}{\mathrm{x}_{12}\mathrm{x}_{23}\mathrm{x}_{31}} \sin(Q_1 + Q_2 + Q_3)\sin(P_3)\sin(P_1)\sin(P_2) \\
\langle \tilde{j}_0 j_s j_s \rangle_f &= \frac{\cos(Q^2 + Q^3)}{x_{12}^2 x_{31}^2} S_1 \sin P_1 \\
\langle \tilde{j}_0 \tilde{j}_0 \tilde{j}_0 \rangle_f &= 0
\end{aligned}
$$

n-point functions

$$\langle j_s \ldots j_s \rangle = \cos^n \theta \, \langle j_s \ldots j_s \rangle_b + \sin^n \theta \, \langle j_s \ldots j_s \rangle_f$$

$$\langle j_s \ldots j_s \rangle_b = \sum_{S_n} \frac{4}{\mathrm{x}_{12} \ldots \mathrm{x}_{n1}} \cos \left(\sum_i Q_i \right) \prod_j \cos(P_j)$$

$$\langle j_s \ldots j_s \rangle_f = \sum_{S_n} \frac{4}{\mathrm{x}_{12} \ldots \mathrm{x}_{n1}} (co) \sin_n \left(\sum_i Q_i \right) \prod_j \sin(P_j)$$

V.Didenko, E.S., Jianwei Mei

HS algebra assigns a non-standard normalization to two-point functions

Conclusions

1. There is a plenty of HS algebras
2. Propagators are extremal projectors in HS algebra
3. Propagators form an algebra, infinity of $SpH(2M)$
4. Unbroken Vasiliev theory should reduce to simple contact diagrams
5. The bulk proof of AdS/CFT for HS?
6. Exact HS symmetry works nice
7. Broken HS symmetry?